A governance valuation framework

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1 Voting distribution

Suppose you are in a governance system where voting power is denoted by a token across n token holders. Let the token holder stake be denoted by a vector

$$\mathbf{s} = (s_1, s_2, \ldots, s_n)$$

where $\sum s_j = 1$ and $s_j > 0$ for all $j = 1, \ldots, n$. Now, define a *vote* as an n-dimensional vector of binary digits

$$\mathbf{v} = (v_1, v_2, \ldots, v_n)$$

where $v_j \in \{0, 1\}$. Given a real positive number $M \in (0, 1]$, and some distribution $\mathbf{s}$, and some vote $\mathbf{v}$ such that

$$\mathbf{s} \cdot \mathbf{v} > M$$

we will say that "the vote passes". Otherwise, "the vote is rejected." We say that M is the "majority threshold" or "majority required for the vote to pass".

2 Decisive votes

For some token holders, in some votes, their stake and vote is the difference between a proposal passing or not passing. Let us formalize this as follows.

Suppose $\mathbf{v}$ is some vote, and define $\mathbf{v_0}(j) := (v_1, v_2, \ldots, v_j, \ldots, v_n), v_j = 0$, the vote where the j-th digit is set to 0. Similarly, define $\mathbf{v_1}(j) := (v_1, v_2, \ldots, v_j, \ldots, v_n), v_j = 1$. Given a stakeholder distribution $\mathbf{s}$ and a vote $\mathbf{v}$, we say that token holder j's vote was *decisive* if and only if the following condition holds.

$$\mathbf{s} \cdot \mathbf{v_0}(j) \leq M \textit{ but } \mathbf{s} \cdot \mathbf{v_1}(j) > M \tag{1}$$

Intuitively, a token holder's stake s_j in a particular vote $\mathbf{v}$ is decisive if and only if it has the power to overturn the outcome of the vote. Note that we can shorten the characterization of a decisive vote as follows. A token holder j is decisive to a vote with respect to token distribution $\mathbf{s}$ if and only if

$$M - s_j < \mathbf{s} \cdot \mathbf{v_0}(j) \leq M \tag{2}$$

The last inequality follows by definition from (1). The strict inequality follows by subtracting s_j from both sides of the latter inequality in (1), noting that $\mathbf{s} \cdot \mathbf{v_1}(j) - s_j = \mathbf{s} \cdot \mathbf{v_0}(j)$.

3 Calculating decisiveness

Let $\mathcal{V}_n$ be set of all possible votes on n token holders, that is, the set of all n-dimensional vectors of binary digits. Given some stake distribution $\mathbf{s}$, and given some token holder j, let $\mathcal{D}_n(j)$ be the set of decisive votes with respect to j. Then define the decisiveness of j with respect to $\mathbf{s}$ as

$$D(\mathbf{s}, j) := |\mathcal{D}_n(j)|/|\mathcal{V}_n|.$$

Note also that $|D(\mathbf{s}, j)| \leq 1$ and $|\mathcal{V}_n| = 2^n$.