

# topography and mimicry of a spin liquid on a triangular lattice

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# topographic mimicry team

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Steven White

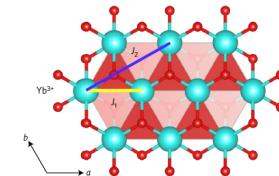
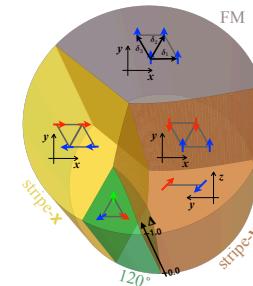


Pavel Maksimov



# plan

## I. anisotropic $\Delta$ -antiferromagnets, intro



YbMgGaO<sub>4</sub>

## II. disorder-induced mimicry in YMGO

PRL 119, 157201 (2017)

PHYSICAL REVIEW LETTERS

week ending  
13 OCTOBER 2017

### Disorder-Induced **Mimicry** of a Spin Liquid in YbMgGaO<sub>4</sub>

Zhenyue Zhu, P. A. Maksimov, Steven R. White, and A. L. Chernyshev

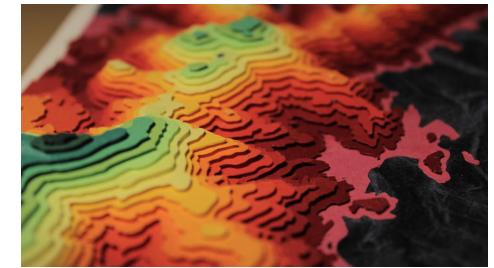


## III. phase diagram and **topography** of spin-liquid phase

PHYSICAL REVIEW LETTERS 120, 207203 (2018)

### **Topography** of Spin Liquids on a Triangular Lattice

Zhenyue Zhu, P. A. Maksimov, Steven R. White, and A. L. Chernyshev



## IV. duality, **truality** (= true reality), etc. [to be unpublished]



# credits

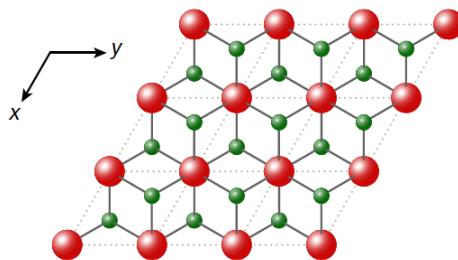
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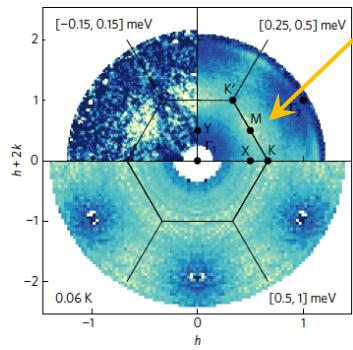
# story

$$\mathcal{H} = \sum_{\langle ij \rangle} [J_{zz} S_i^z S_j^z + J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) + J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) - \frac{iJ_{z\pm}}{2} (\gamma_{ij}^* S_i^+ S_j^z - \gamma_{ij} S_i^- S_j^z + \langle i \leftrightarrow j \rangle)]$$

YbMgGaO<sub>4</sub>



broad intensity at **M**-points



From: Martin Mourigal

Date: June 24, 2016 at 5:37:52 AM GMT+2

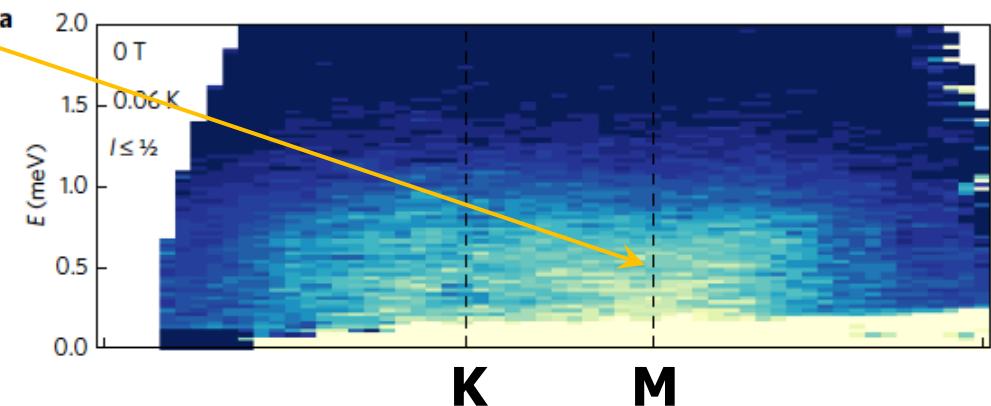
To: Sasha Chernyshev

Dear Sasha,

...

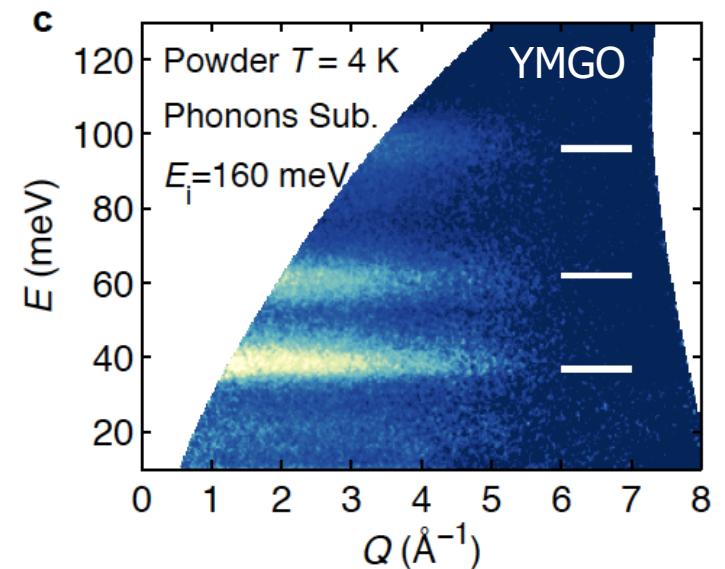
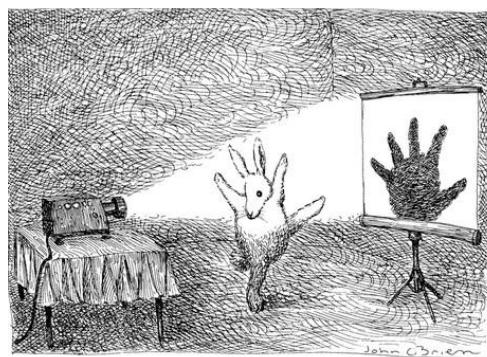
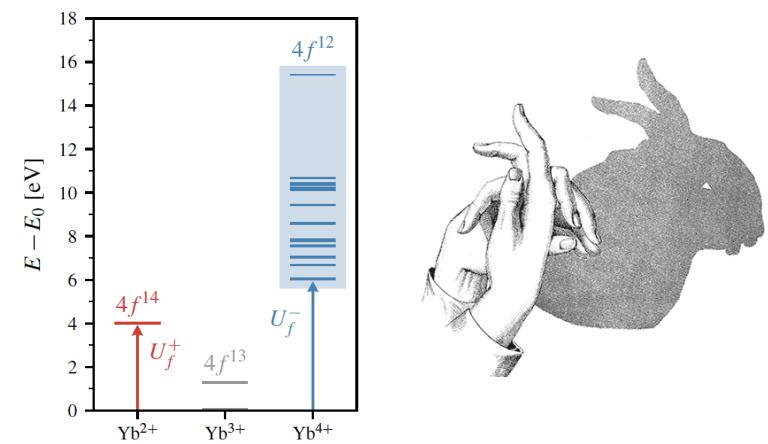
Also, can I confidentially send you a manuscript on a triangular lattice compound that we are about to submit?

Best,  
Martin



# is it crazy?

- rare-earth,  $\text{Yb}^{3+}$ ,  $J=7/2$ , + crystal-field splitting
- lowest doublet, effective  $S=1/2$

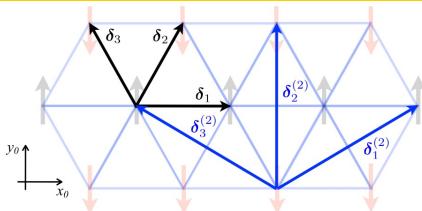


- **projection** of the large moment degrees of freedom onto the manifold of effective  $S=1/2$  states
- **agnostic approach:** spin-spin interactions = **anything** allowed by lattice symmetry & (mostly) nearest-neighbor, because  $f$ -electrons



# humanizing the hamiltonian, I

$$J=2J_{\pm}, \Delta=J_{zz}/2J_{\pm}$$



- triangular lattice symmetries  $\Rightarrow$  **four** terms
- XXZ part  $\Rightarrow$  friendlier form
- "extended" form  $\Rightarrow$  more intuition

[42] For a less humane form of (1), see

$$\mathcal{H} = \sum_{\langle ij \rangle} [J_{zz} S_i^z S_j^z + J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+)] \text{ xxz} \quad \gamma_{ij} = 1, e^{2\pi i/3}, e^{-2\pi i/3}$$

$$+ J_{\pm\pm} [\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-] - \frac{i J_{z\pm}}{2} [\gamma_{ij}^* S_i^+ S_j^z - \gamma_{ij} S_i^- S_j^z + (i \leftrightarrow j)]$$

anisotropic, bond-dependent

[56] For a more humane form of (2), see

$$\mathcal{H} = \sum_{\langle ij \rangle} [J [S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z]] \text{ xxz} \quad \tilde{\varphi}_{\alpha} = \{0, -2\pi/3, 2\pi/3\}$$

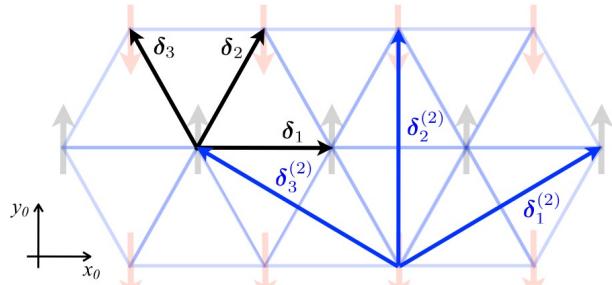
$$+ 2J_{\pm\pm} [\cos \tilde{\varphi}_{\alpha} (S_i^x S_j^x - S_i^y S_j^y) - \sin \tilde{\varphi}_{\alpha} (S_i^x S_j^y + S_i^y S_j^x)]$$

$$+ J_{z\pm} [\cos \tilde{\varphi}_{\alpha} (S_i^y S_j^z + S_i^z S_j^y) - \sin \tilde{\varphi}_{\alpha} (S_i^x S_j^z + S_i^z S_j^x)]$$

anisotropic, bond-dependent



# humanizing the hamiltonian, II



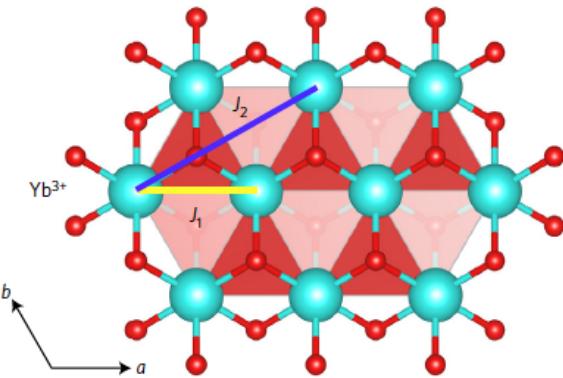
[x-bond]

- triangular lattice symmetries  $\Rightarrow$  **four** terms
- parametrization of the exchange matrix

$$\hat{\mathcal{H}}_{12} = \mathbf{S}_1^0 \begin{pmatrix} J_{xx} & J_{xy} & J_{xz} \\ J_{yx} & J_{yy} & J_{yz} \\ J_{zx} & J_{zy} & J_{zz} \end{pmatrix} \mathbf{S}_2^0 \quad \Rightarrow \quad \hat{\mathcal{H}}_{12} = \mathbf{S}_1^0 \begin{pmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & J_{yz} \\ 0 & J_{zy} & J_{zz} \end{pmatrix} \mathbf{S}_2^0$$

- parametrization:  $J_{zz} = \Delta \cdot J_1$ ,  $J_1 = (J_{xx} + J_{yy})/2$ ,  $J_{\pm\pm} = (J_{xx} - J_{yy})/4$ ,  $J_{zy} = J_{yz} = J_{z\pm}$

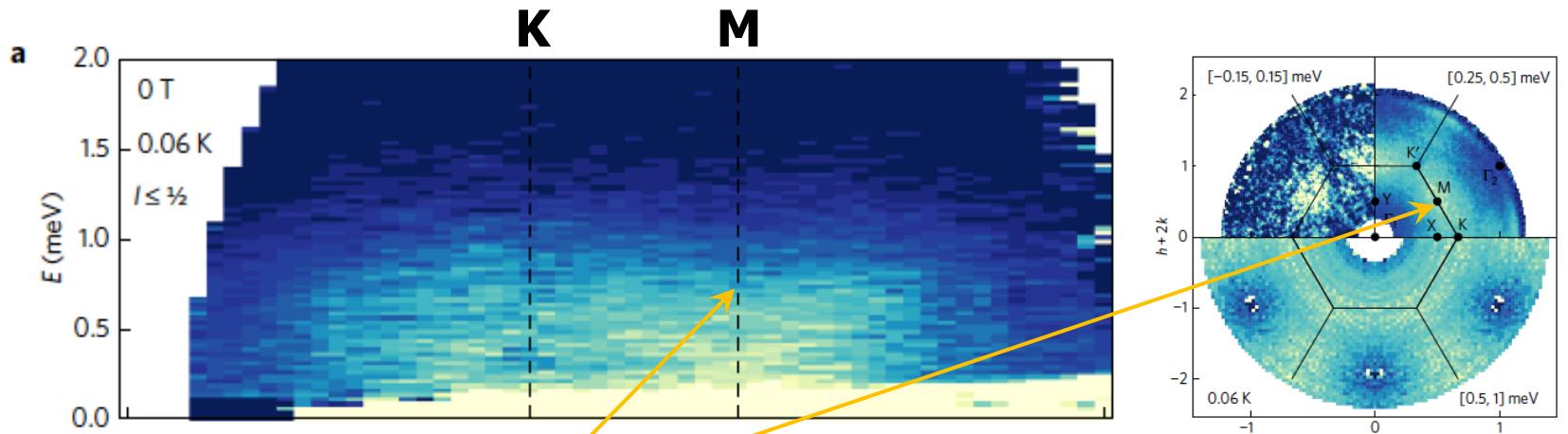
$$\begin{aligned} \mathcal{H} = \sum_{\langle ij \rangle} J [S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z] &\quad \tilde{\varphi}_\alpha = \{0, -2\pi/3, 2\pi/3\} \\ &+ 2J_{\pm\pm} [\cos \tilde{\varphi}_\alpha (S_i^x S_j^x - S_i^y S_j^y) - \sin \tilde{\varphi}_\alpha (S_i^x S_j^y + S_i^y S_j^x)] \\ &+ J_{z\pm} [\cos \tilde{\varphi}_\alpha (S_i^y S_j^z + S_i^z S_j^y) - \sin \tilde{\varphi}_\alpha (S_i^x S_j^z + S_i^z S_j^x)] \end{aligned}$$



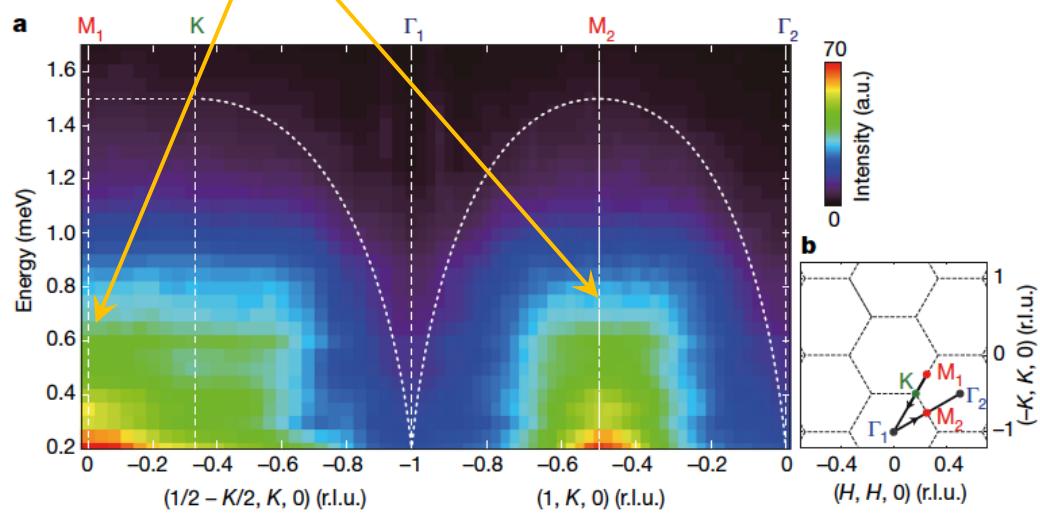
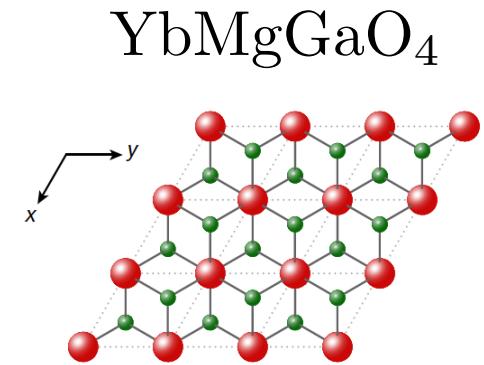
mimicry



# experiments, I



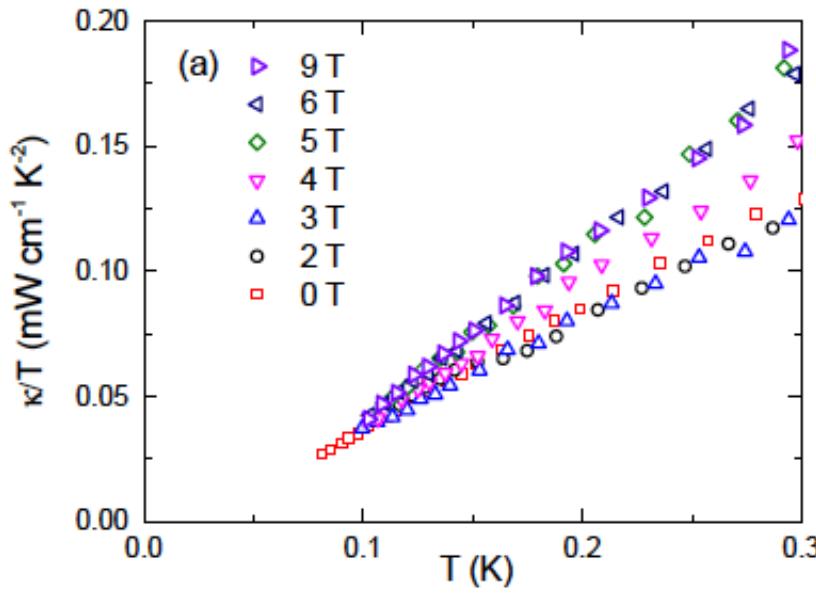
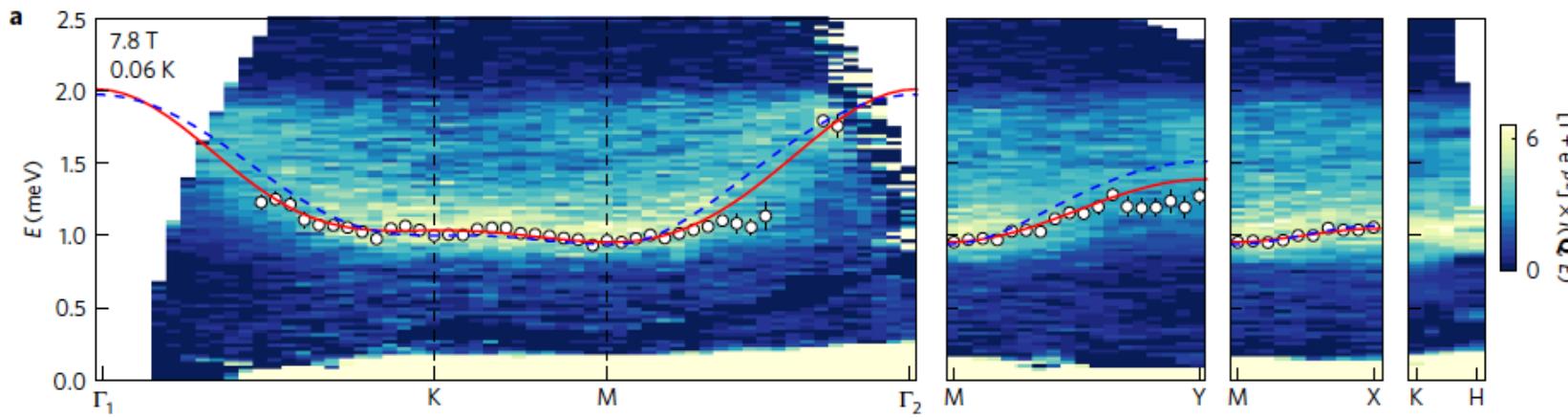
broad intensity at **M**-points



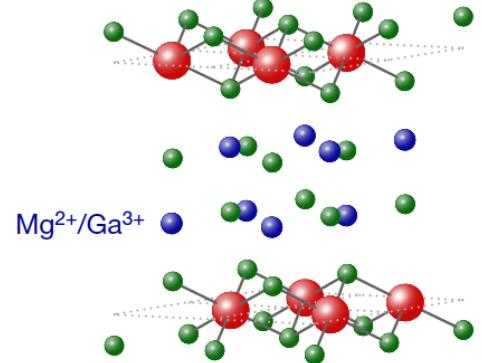
- no order
- broad features in  $S(\mathbf{q}, \omega)$
- spin-liquid?



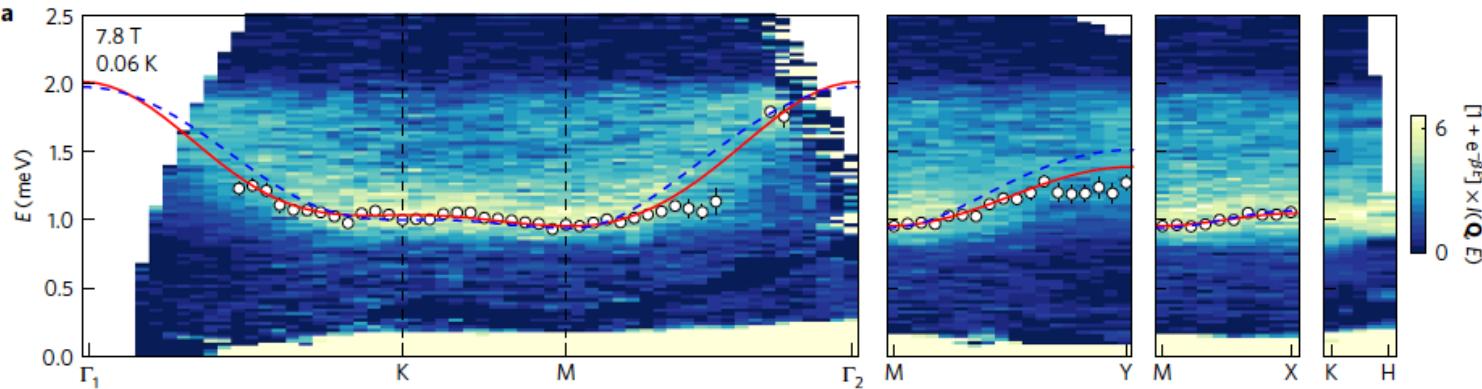
# experiments, II



- features remain broad above saturation
- no thermal conductivity by spin excitations
- low- $T$  freezing
- disorder? [mixing of interplane  $\text{Ga}^{3+}$  and  $\text{Mg}^{2+}$ ]



# (early) parameters: simpler model!



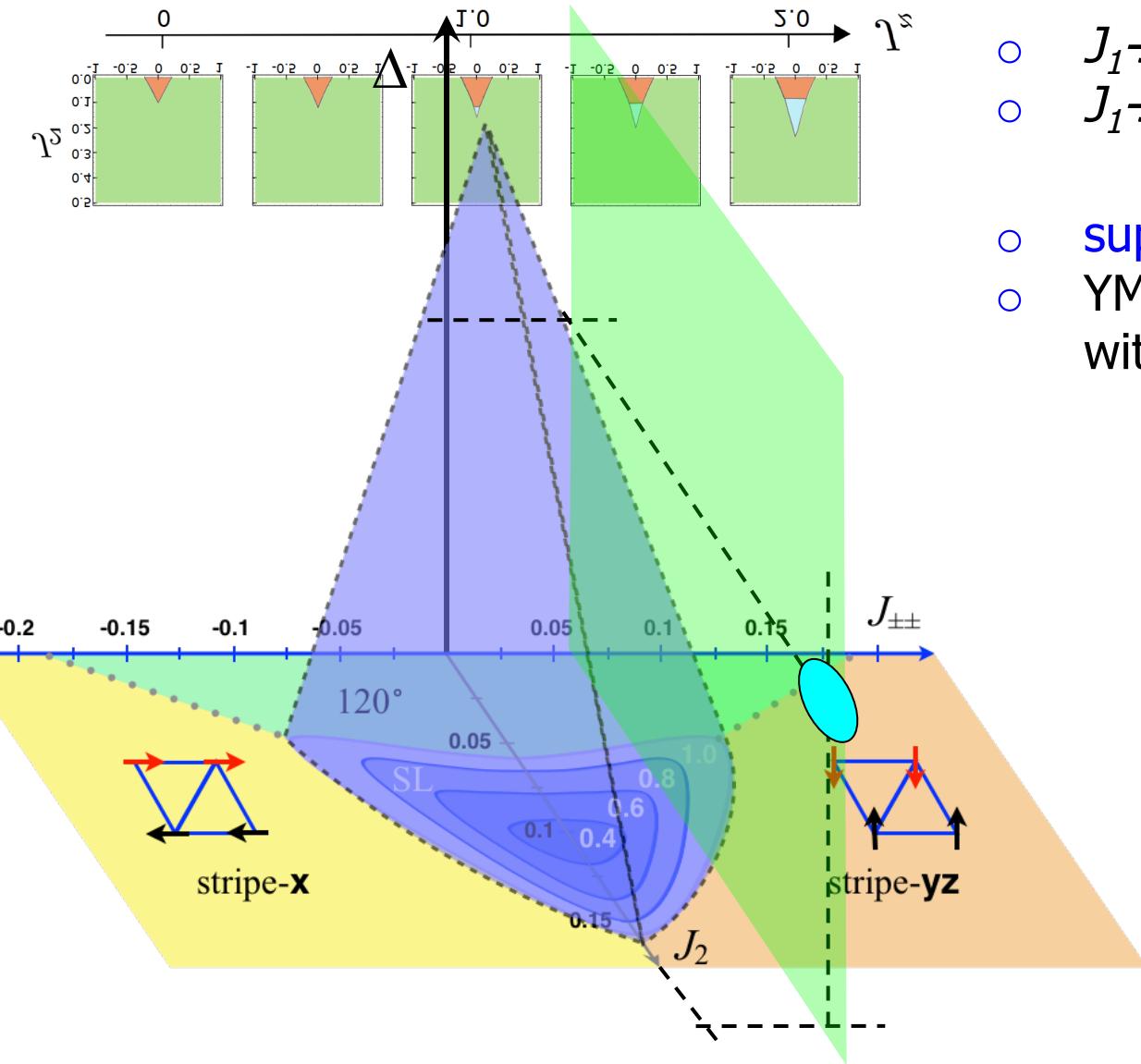
- **neutrons, ESR:**  $J_1 = 0.218(8)\text{meV}$ ,  
 $\Delta=0.58(2)$ ,  $\mathbf{J}_2 = \mathbf{0.22(2)J}_1$ ,  
 $J_{\pm\pm} = 0.06J_1$ ,  $J_{z\pm} \approx 0.0$
- **mostly  $J_1$ - $J_2$  XXZ**  
+ **subleading**  $J_{\pm\pm}$   
+ **negligible**  $J_{z\pm}$

$$\mathcal{H} = \mathcal{H}_{\text{XXZ}}^{J_1-J_2} + \mathcal{H}_{\text{pd}}$$

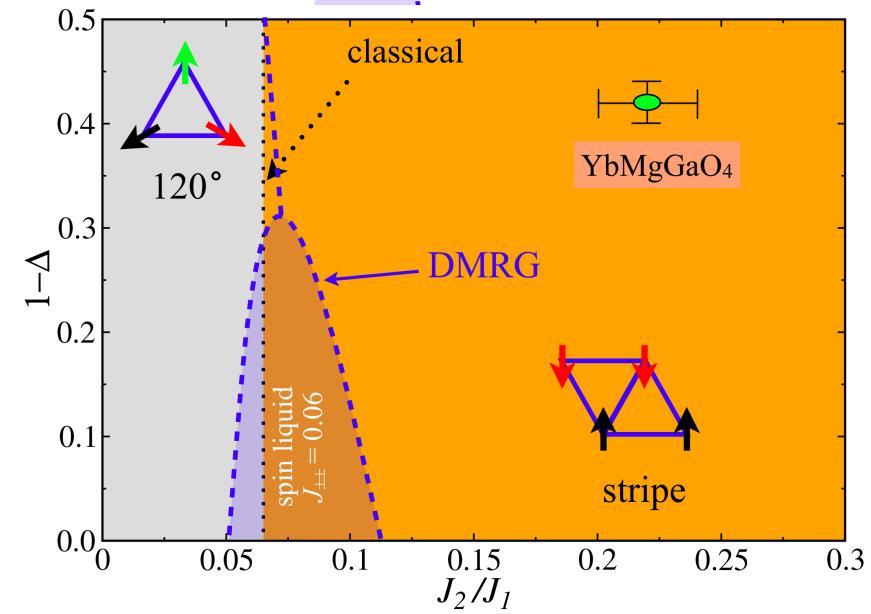
$$\mathcal{H}_{\text{XXZ}}^{J_1-J_2} = \sum_{\langle ij \rangle_n} J_n (S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z)$$

$$\mathcal{H}_{\text{pd}} = J_{\pm\pm} \sum_{\langle ij \rangle} (e^{i\tilde{\varphi}_\alpha} S_i^+ S_j^+ + e^{-i\tilde{\varphi}_\alpha} S_i^- S_j^-)$$

# $J_1$ - $J_2$ -XXZ- $J_{\pm\pm}$ , DMRG?

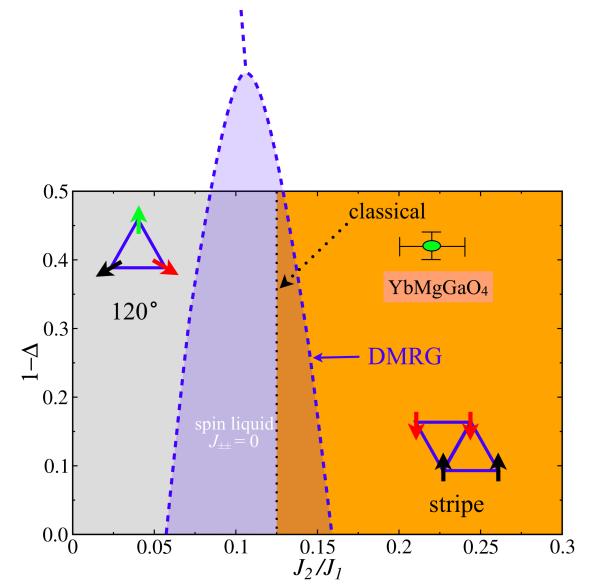
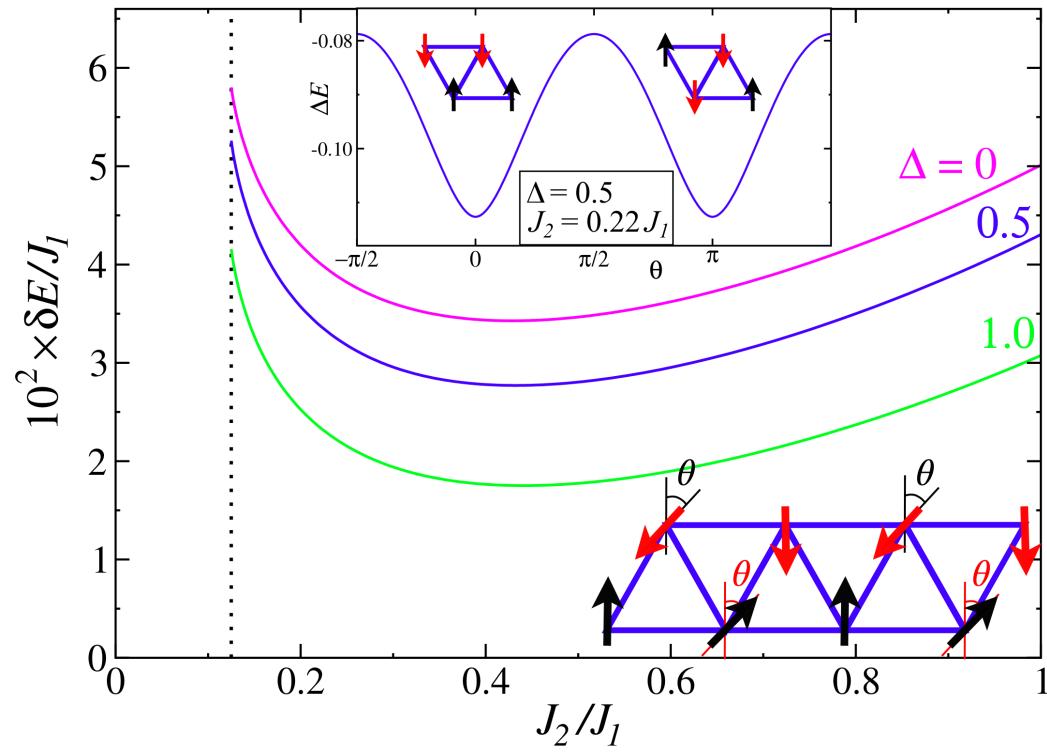


- $J_1$ - $J_2$  model has a spin liquid phase [ $J_2 = 0.06 \dots 0.15$ ]
- $J_1$ - $J_2$ -XXZ- $J_{\pm\pm} \Rightarrow$  **extended spin liquid phase**  
(extent in  $\Delta=[1.0 \dots 0.3]$ ,  $J_{\pm\pm}=[-0.1 \dots 0.1]$  )
- supported by variational MC (Iaconis *etal.*, '18)
- YMGO?: **well outside an SL, in a stripe phase**  
with a large ordered moment  $\langle S \rangle \approx 0.42$  [DMRG]



# how can stripe look like an “SL”? a scenario:

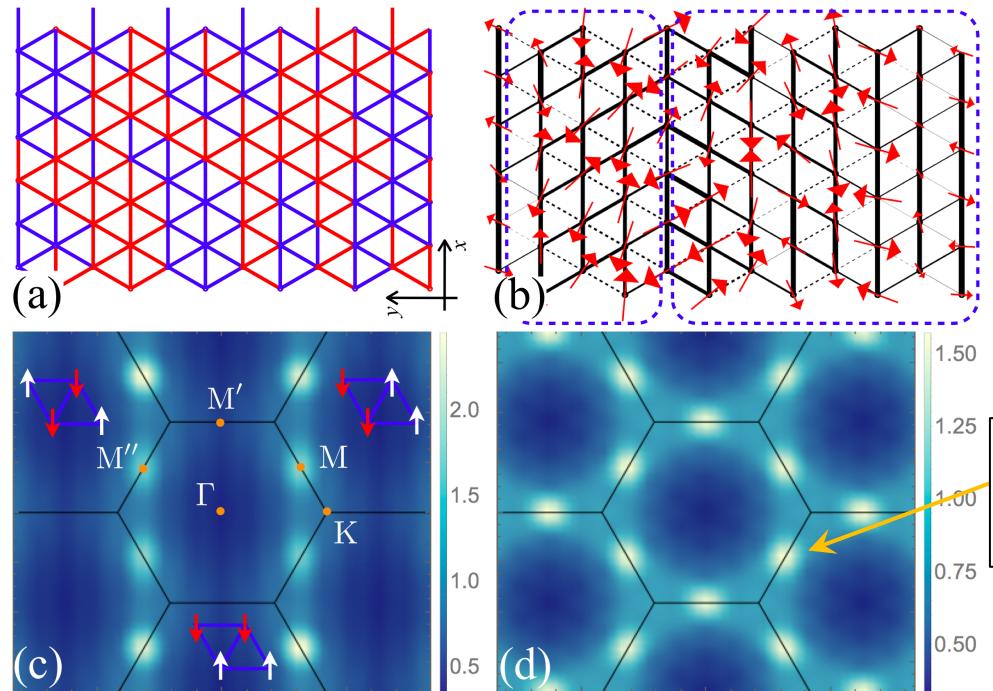
- pure  **$J_1$ - $J_2$ -XXZ** limit: stripes are selected by an order-by-disorder effect [from a classically degenerate manifold of (spiral) states]
- $\Rightarrow$  tunneling barrier between stripe different phases: about **0.03 $J_1$**  per spin



# + disorder ... = SL mimicry

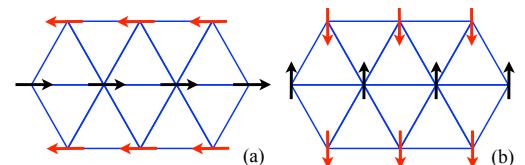
$$\hat{\mathcal{H}}_{12} = \mathbf{S}_1^0 \begin{pmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & J_{yz} \\ 0 & J_{zy} & J_{zz} \end{pmatrix} \mathbf{S}_2^0$$

$$J_{\pm\pm} = (J_{xx} - J_{yy})/4$$



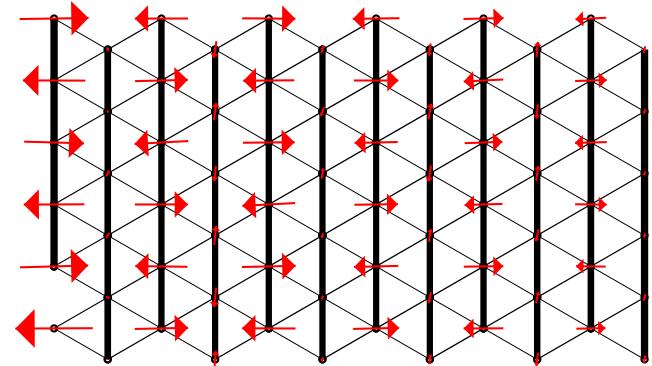
- subleading  $\mathbf{J}_{\pm\pm}$  terms are a (small) difference of the diagonal terms in the exchange matrix
- disorder in the latter (20%, consistent with experiments)  
⇒ **random pinning fields for stripes**

Y. Li *et al*, PRL 118, 107202 (2017).



- DMRG ⇒ stripe clusters, or **stripe-superposed states**, yielding  $S(\mathbf{q})$  similar to experiments

broad intensity at **M**-points;  
short-range stripe order



see also E. Parker and L. Balents, PRB 97, 184413 (2018);  
Z. Ma *et al*, PRL 120, 087201 (2018).

**YMGO** is in a stripe phase with large ordered moment;  
disorder makes it look like a spin-liquid

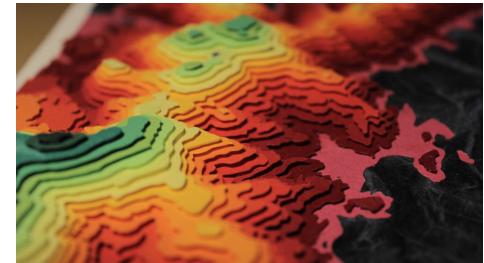
isomorphism

holy grail

# topography

reincarnation

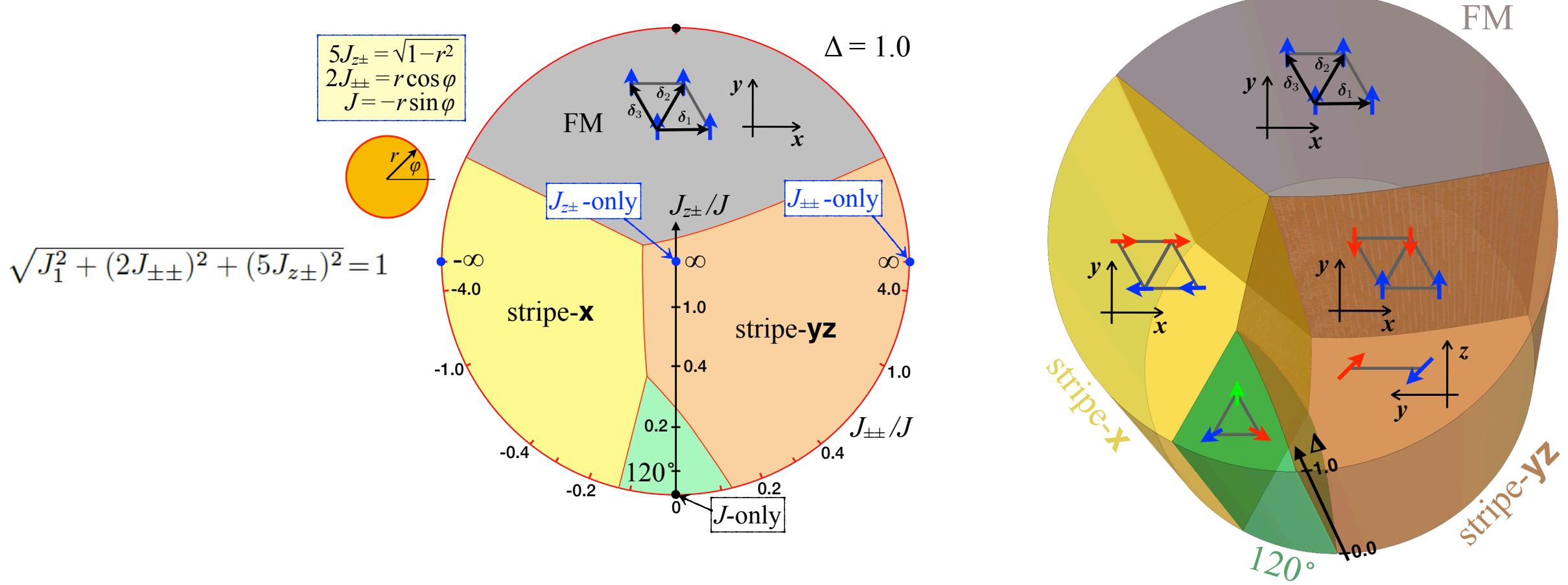
redemption



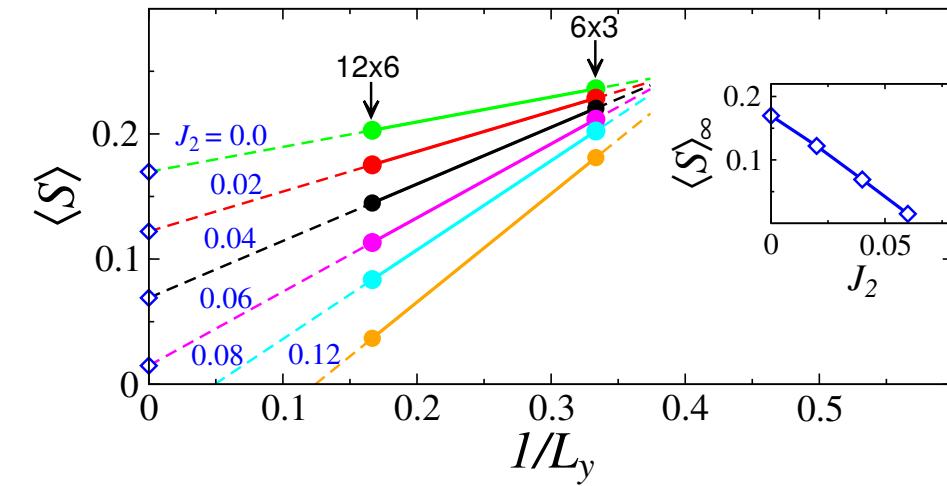
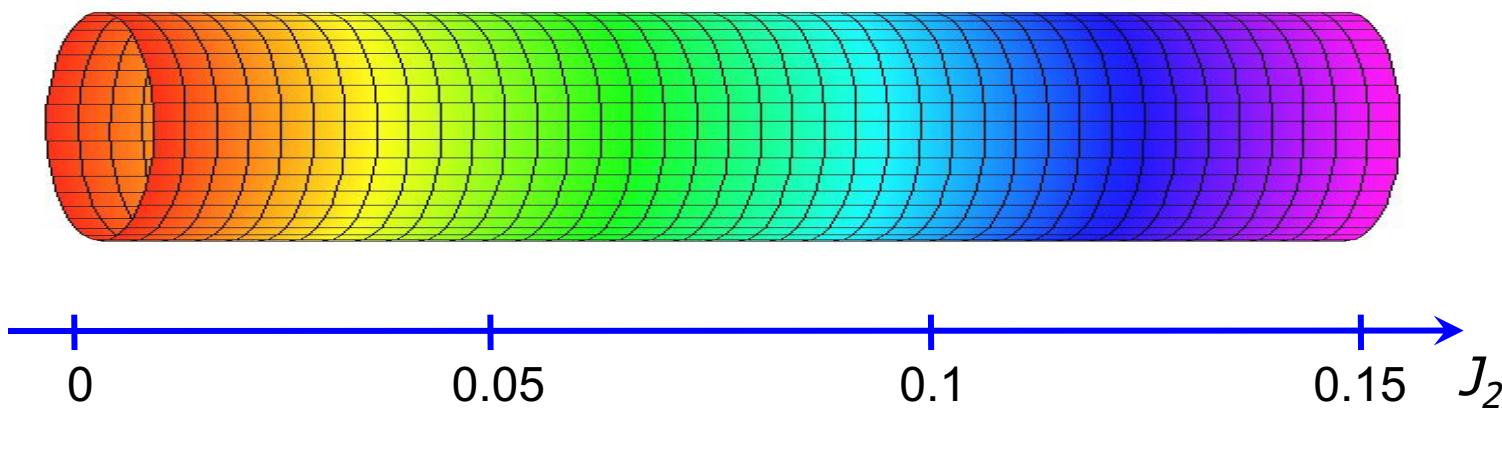
# classical phase diagram

- **3D parameter space:**  $XXZ$ - $\Delta$ ,  $J_{\pm\pm}$ , and  $J_{z\pm}$
- ⇒ **mapping on a cylinder**, [“2” and “5” to exaggerate  $J_{\pm\pm} \sim J_{z\pm} \sim J$  region]
- **FM,  $120^\circ$ , and stripe states\***, weak  $\Delta$ -dependence (+ few subtleties)

**top** = Heisenberg  
**bottom** = XY



# $S=1/2$ , DMRG: 3D phase space, how?

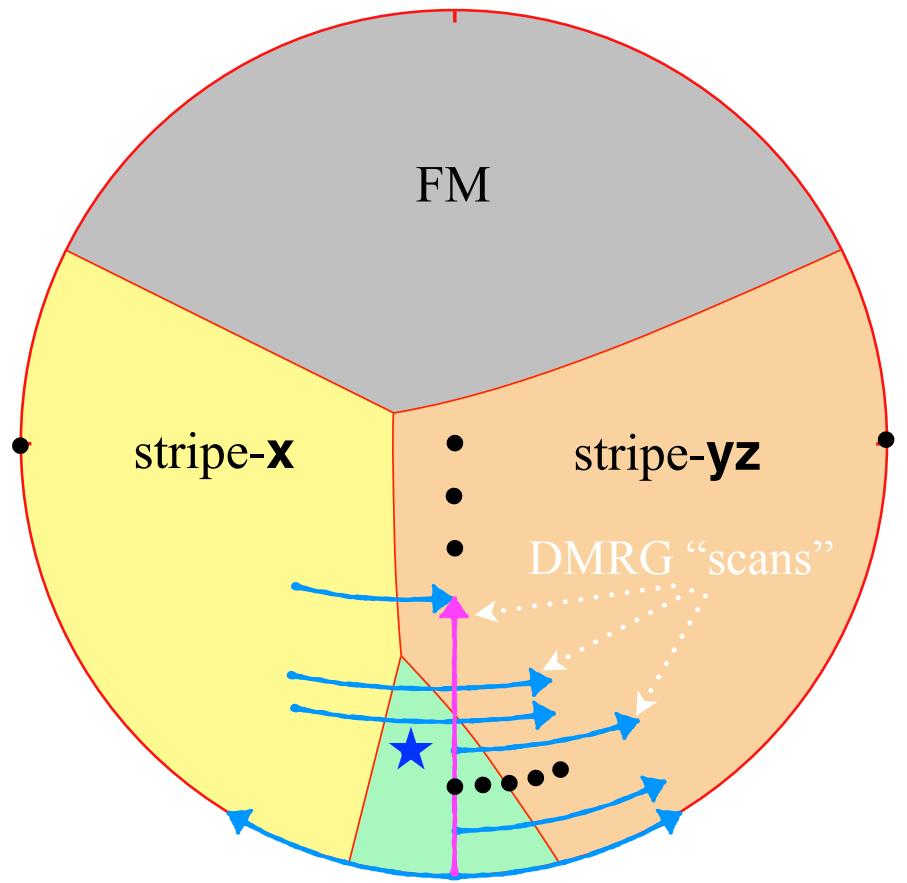


- **1D long-cylinder scans ["scans"]:** vary one parameter, play with BCs/range, read orders
- "all-fixed" parameters ["**non-scans**"], 12x6, 20x6 clusters
- $1/L_y$ -scaling, fixed aspect-ratio clusters
- decay of correlations off boundary, exp vs power-law
- $S(\mathbf{q})$  structure factor

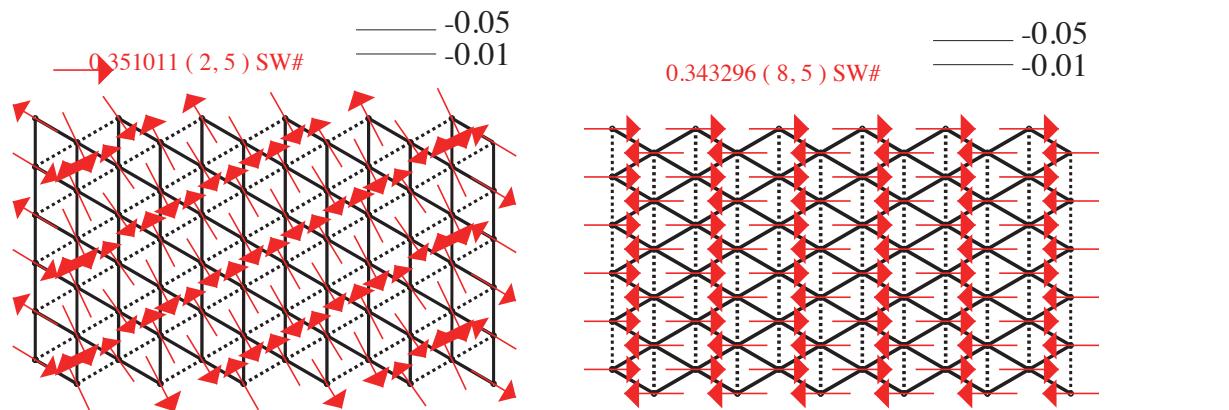
[S. R. White and SC, PRL 2007]



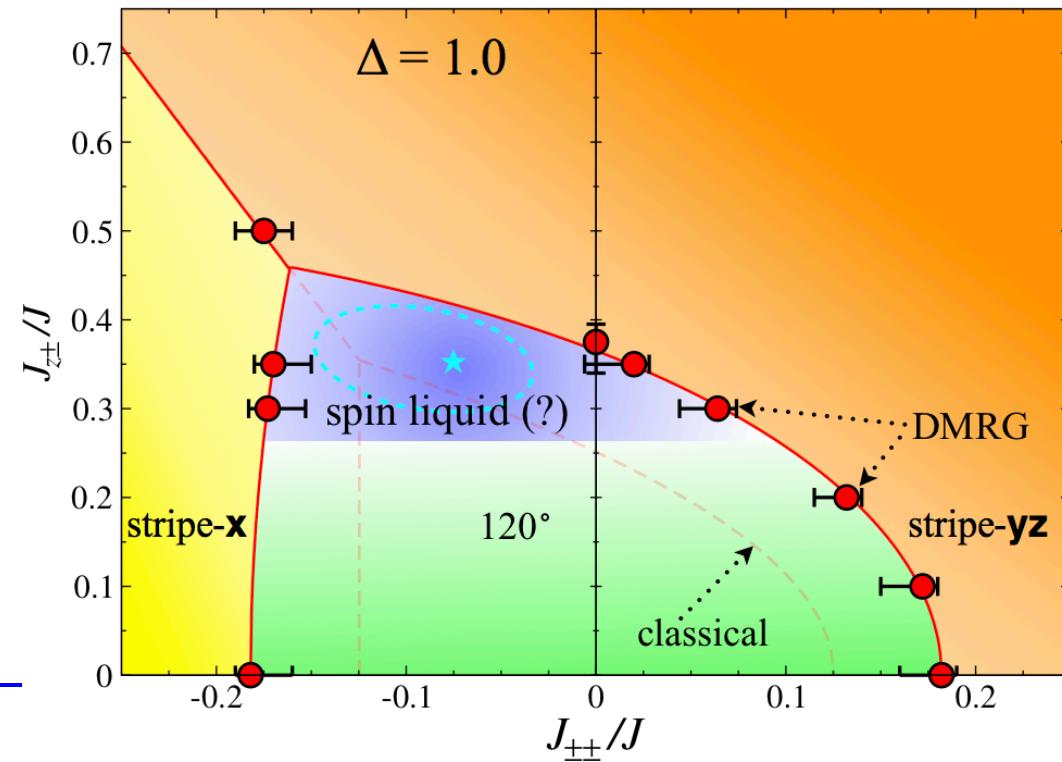
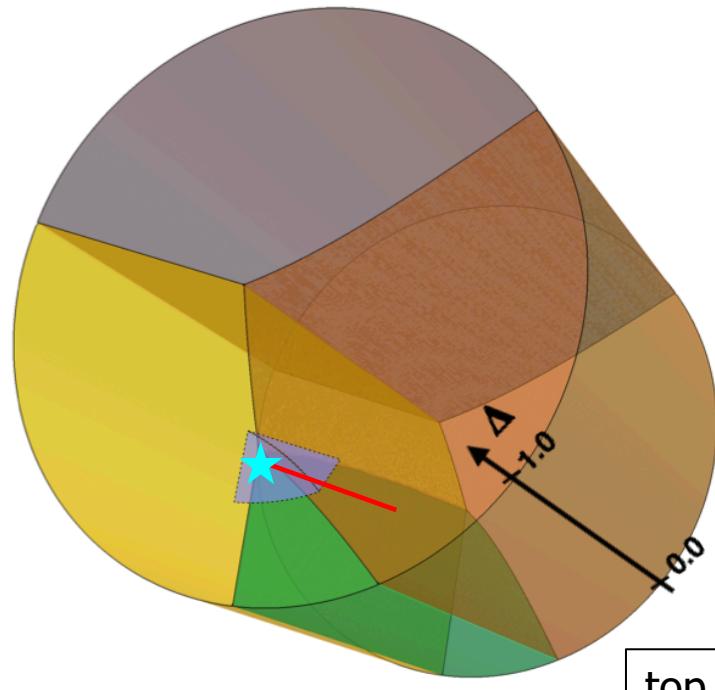
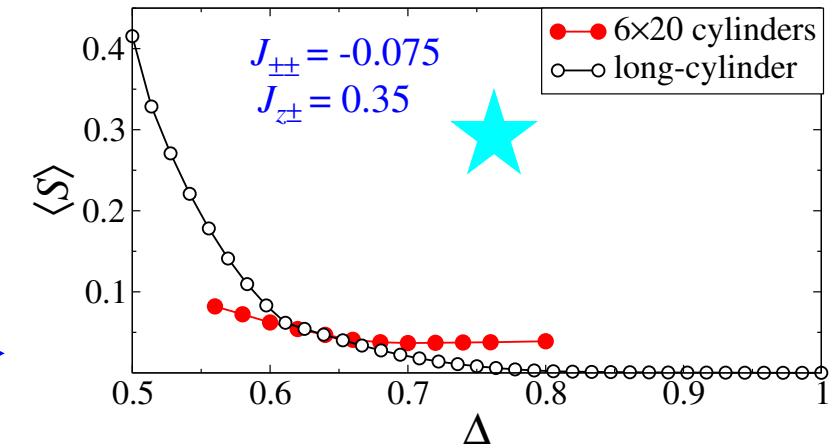
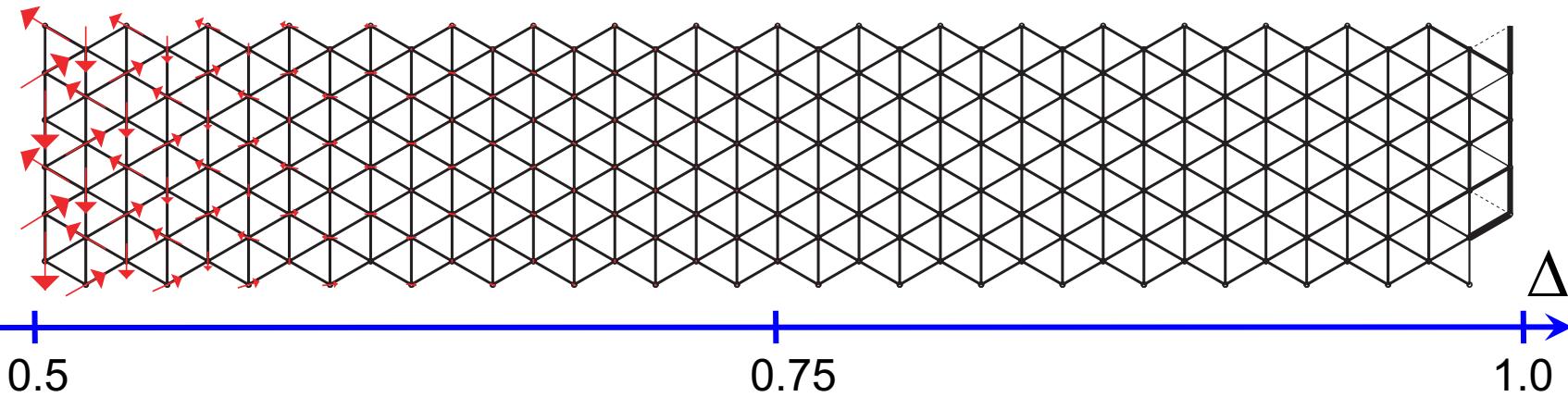
# DMRG: scans, non-scans, large $J_{\pm\pm}$ , $J_{z\pm}$



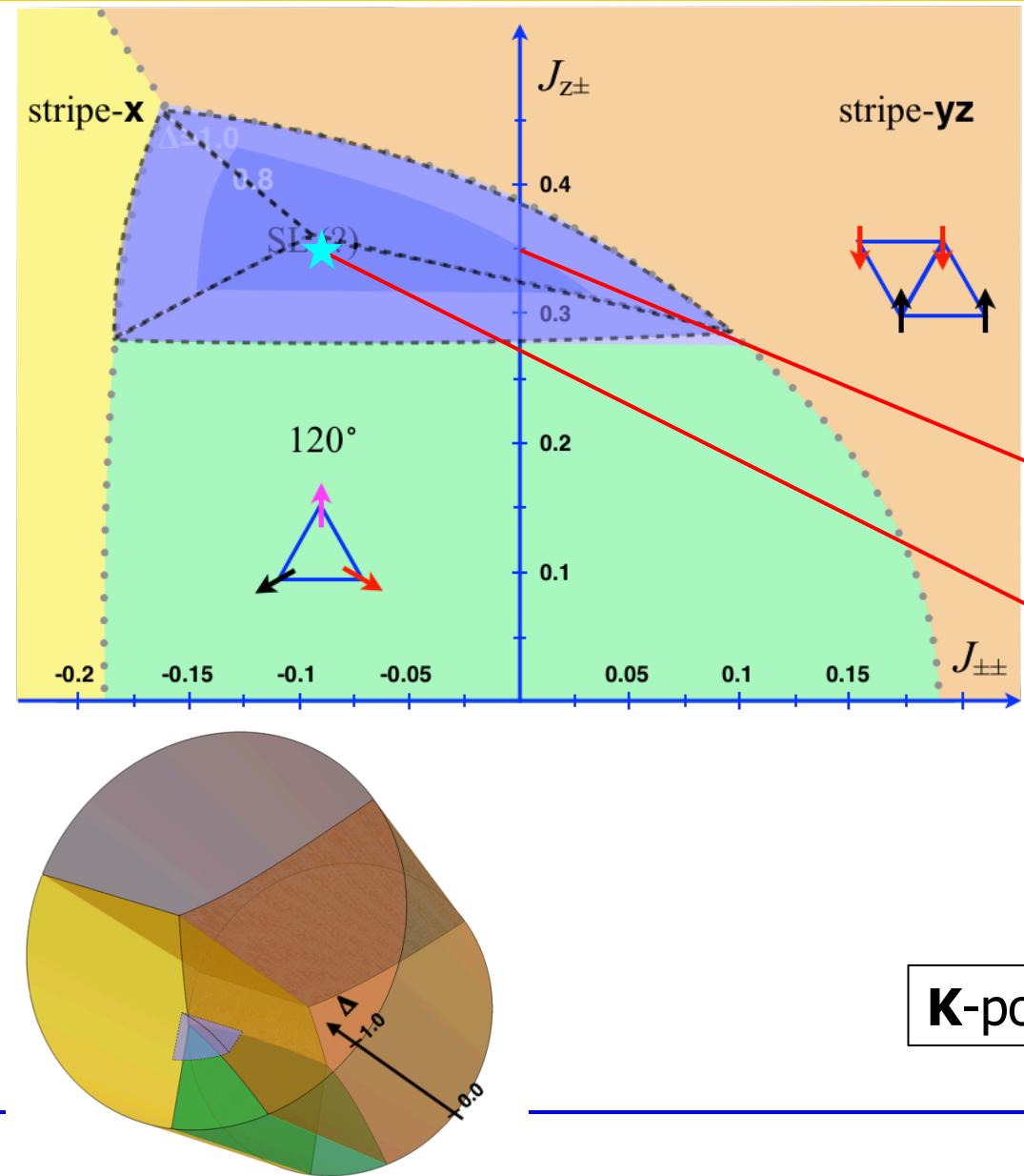
- some regions: obvious continuity
- strong stripe orders; little quantum fluctuations
- ⇒ potential spin liquid regions are near the 120° state
- ⇒ **SKIP MOST DMRG results ...**



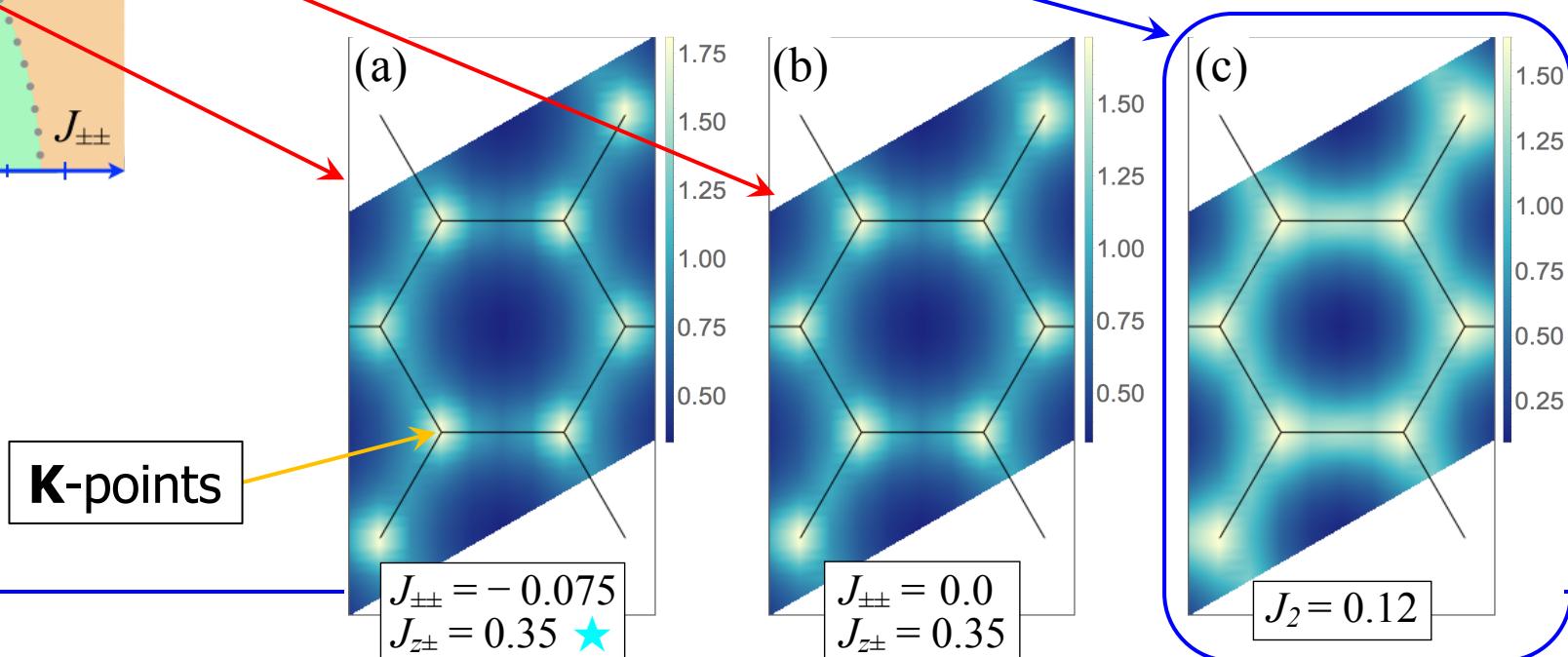
# DMRG, spin liquid??



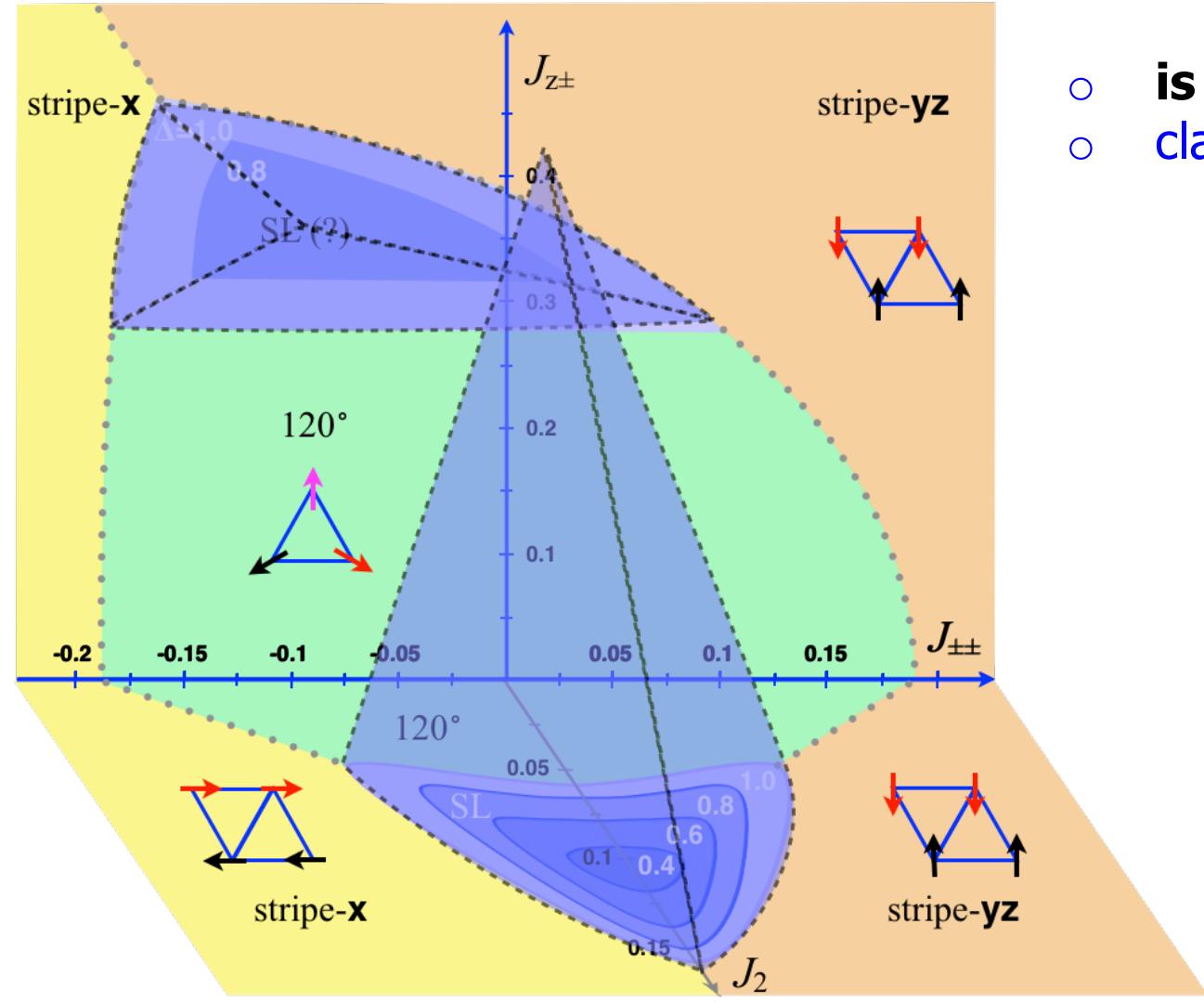
# spin-liquid topographic map



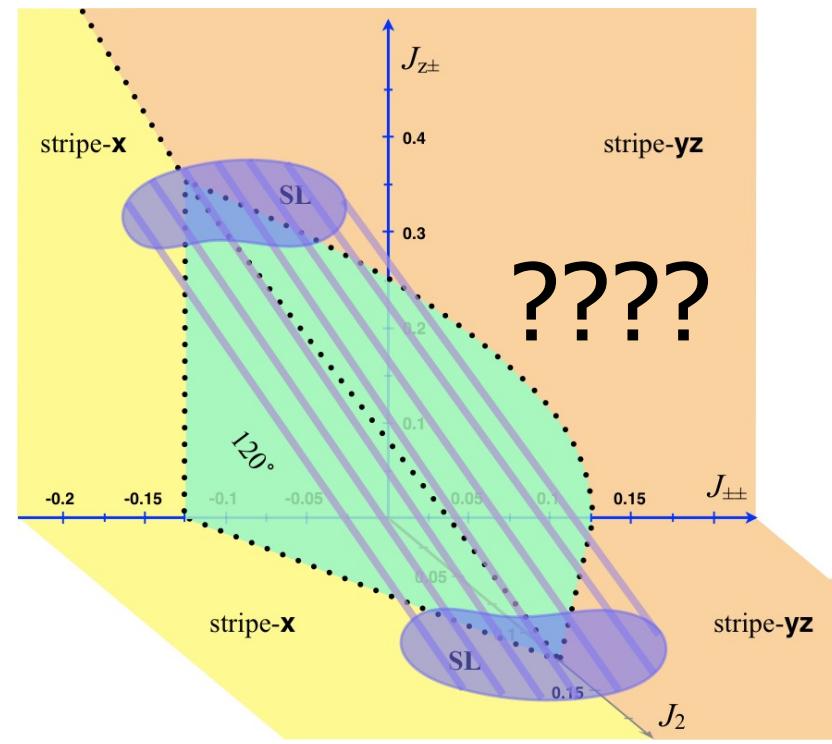
- strong evidence: region ( $J_{z\pm} \approx [0.3 \dots 0.5]$ ,  $J_{\pm\pm} \approx [-0.2 \dots +0.1]$ ,  $\Delta = [1.0 \dots 0.7]$ )  
**is a spin liquid**
- what kind of SL?
- maximal at  $\Delta = 1.0 \Rightarrow$  **not** driven by anisotropic terms
- $S(\mathbf{q})$ : isotropic, no off-diagonal  $S^{\alpha\beta}(\mathbf{q})$
- **$S(\mathbf{q})$  is similar to  $J_1$ - $J_2$  SL region**



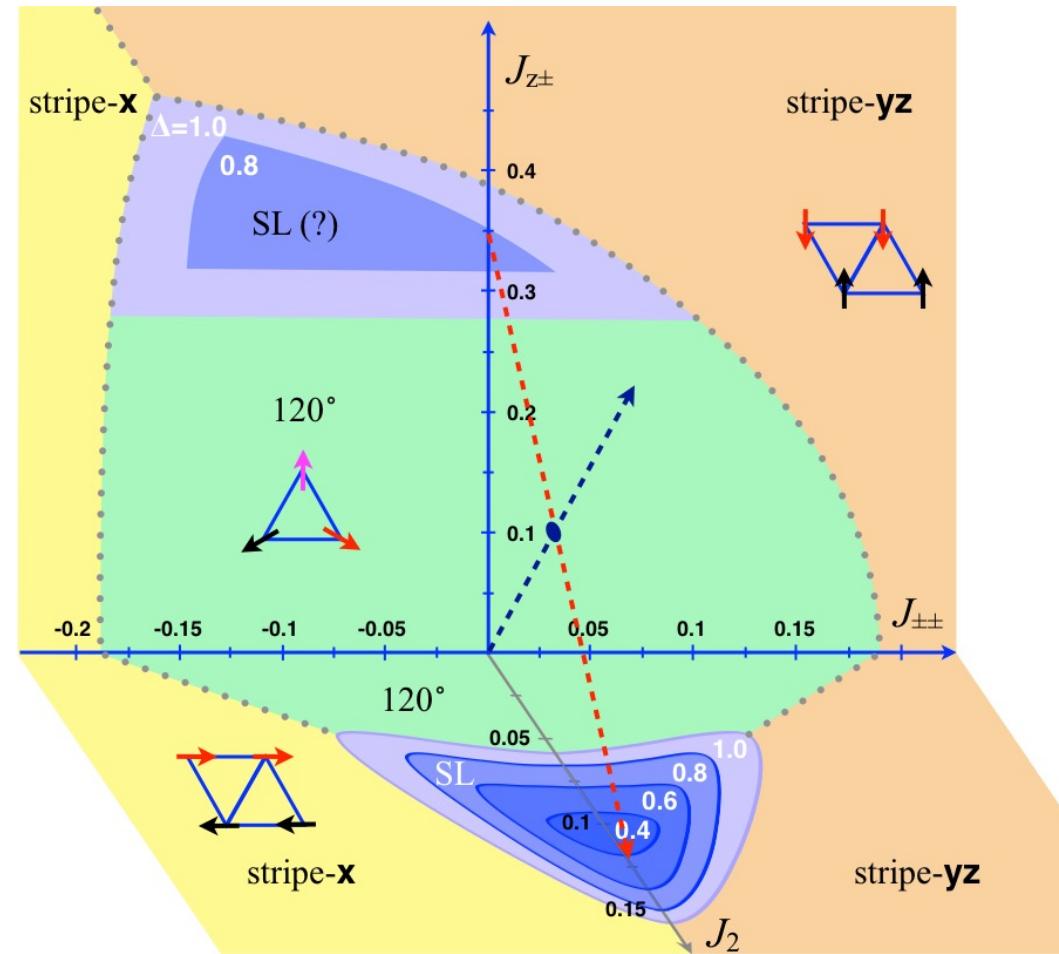
# connection with the SL in $J_1$ - $J_2$ -XXZ- $J_{\pm\pm}$ ?



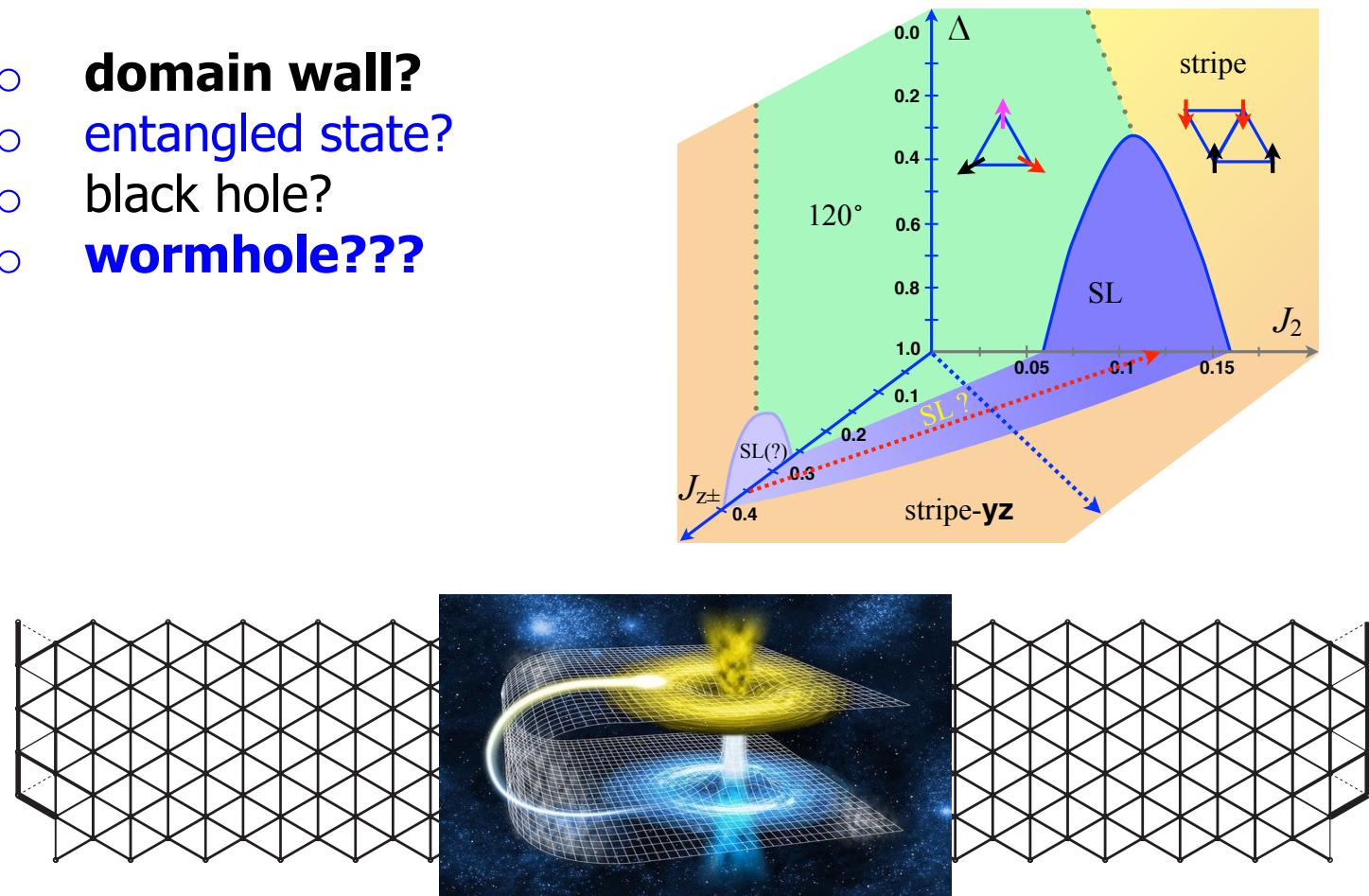
- **is there a connection between SL phases?**
- classically, connection along the rim of the 120° phase



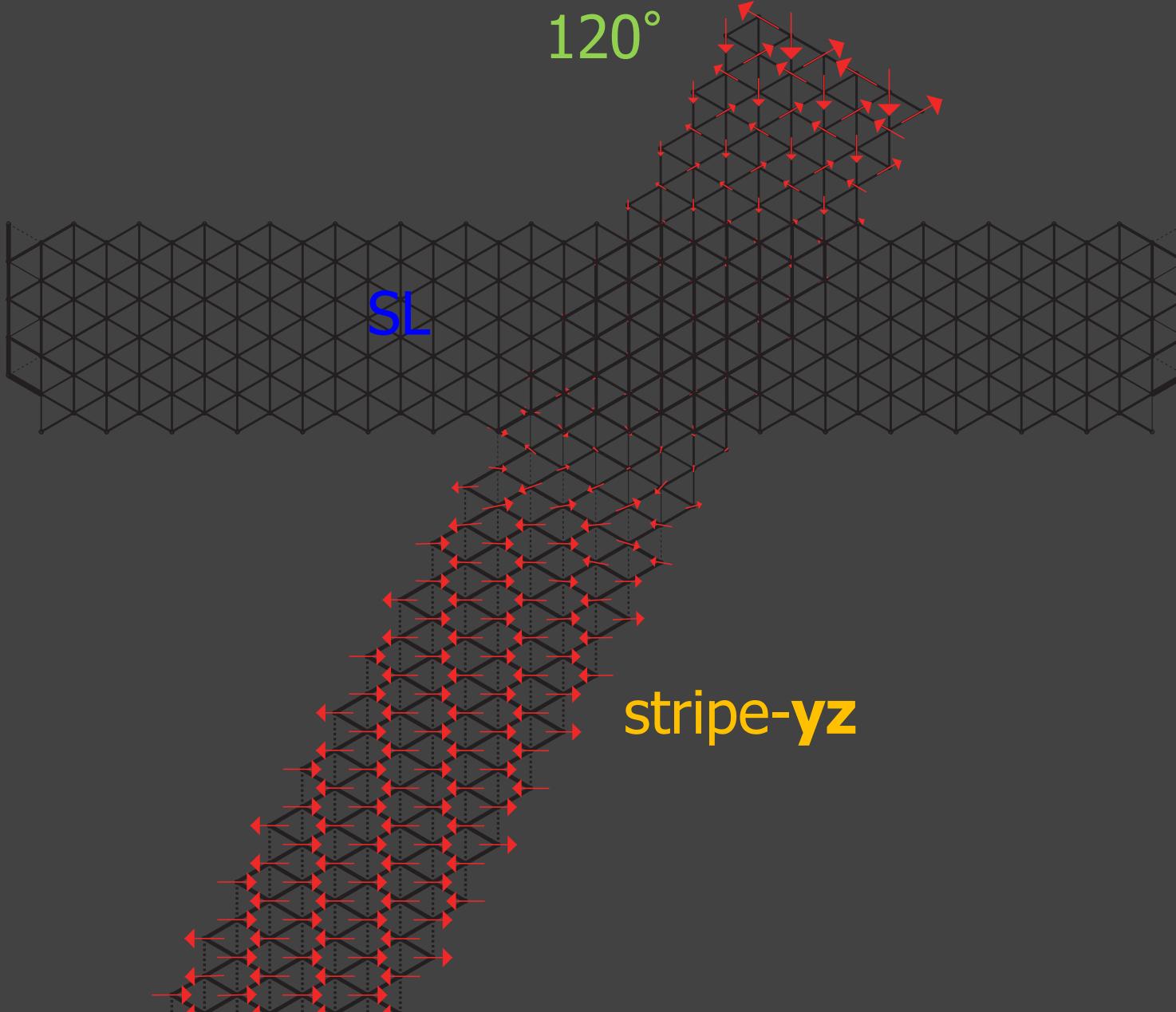
# 4D-extension, from SL to SL



- a DMRG scan from “anisotropic” SL to  $J_1$ - $J_2$  SL
- what happens when two spin liquids meet?
- domain wall?
- entangled state?
- black hole?
- wormhole???

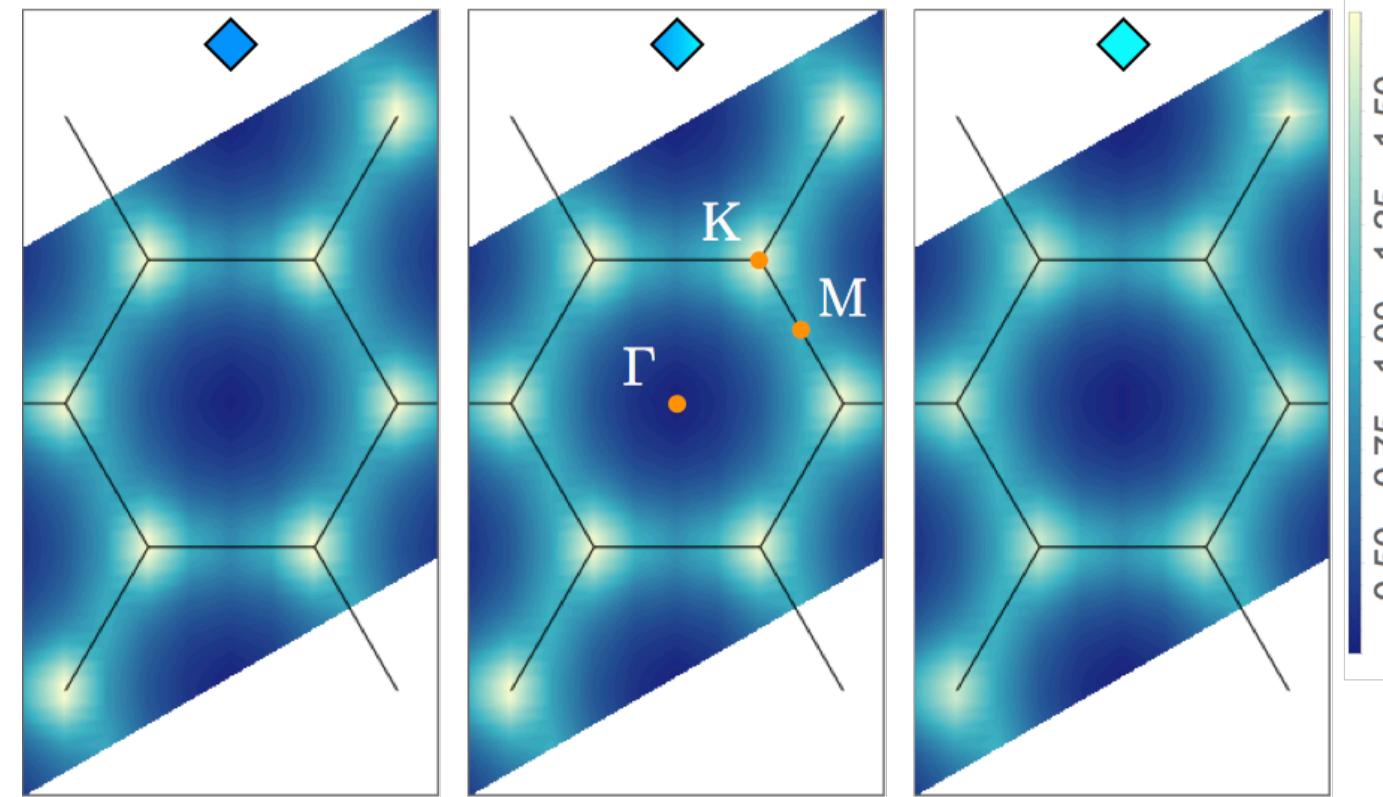
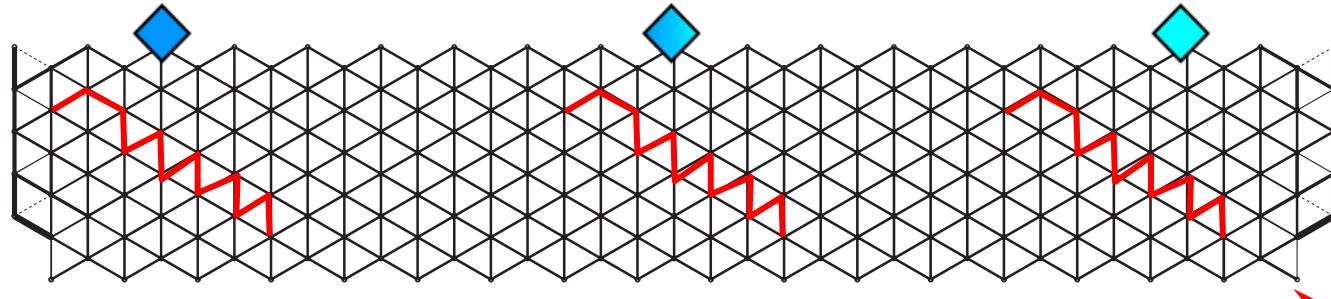


$J$ - $J_{z\pm}$  model

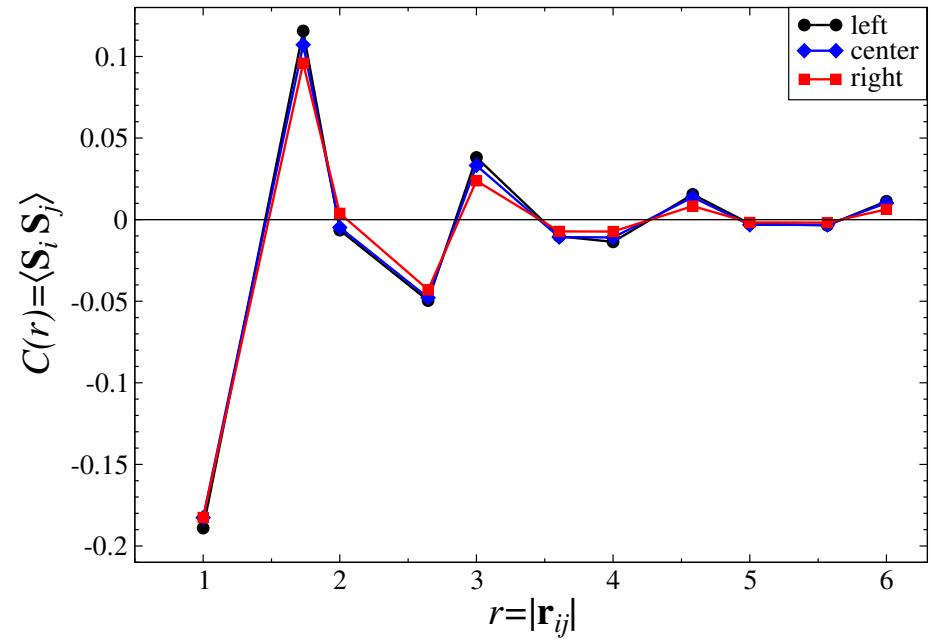


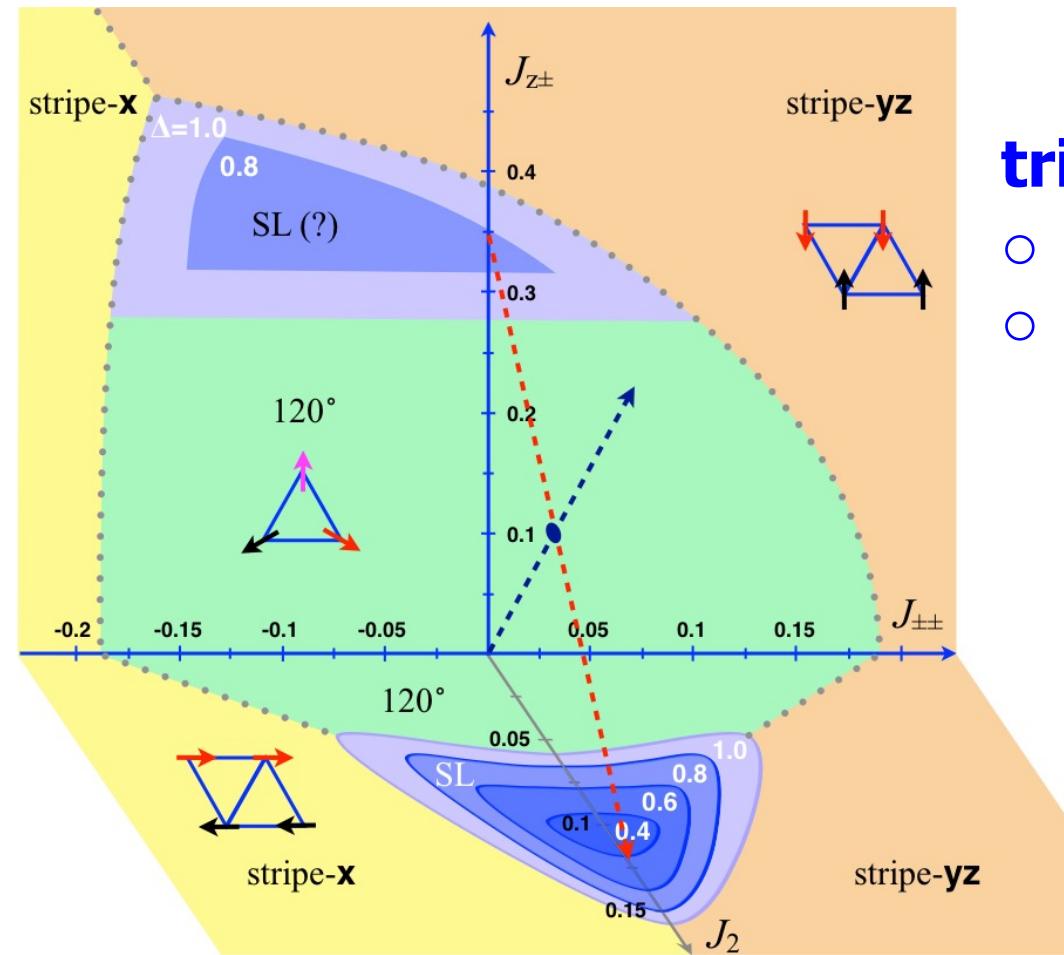
$J_1$ - $J_2$  model

# spin-liquid isomorphism



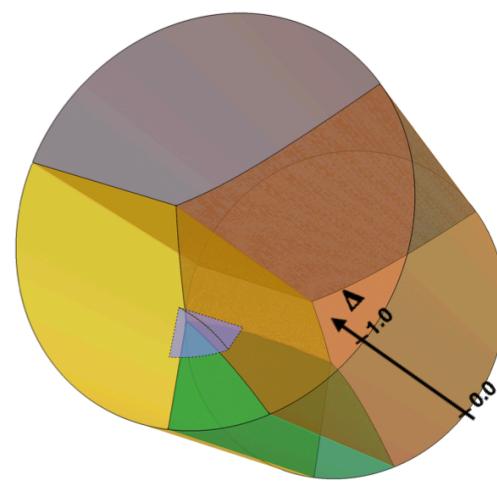
- no order all the way through the “SL scan”  
[no VBS or chiral order too]
  - bonds = n.-n.  $\langle \mathbf{S}_i \mathbf{S}_j \rangle$ , no transition
  - $S(\mathbf{q})$  is nearly identical [ $<5\%$  variation]
  - real-space correlations are the same
- ⇒ **spin liquids are isomorphic**  
(= same phase)





## triangular lattice:

- new SL in anisotropic-exchange  $n$ - $n$  model
- it is isomorphic to the one in the  $J_1$ - $J_2$  model

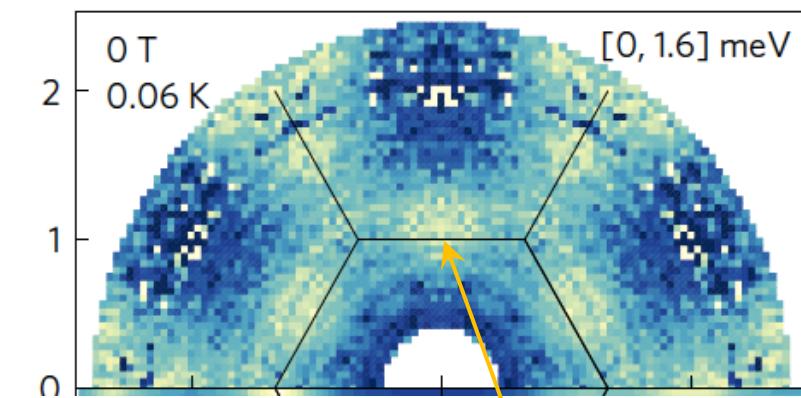
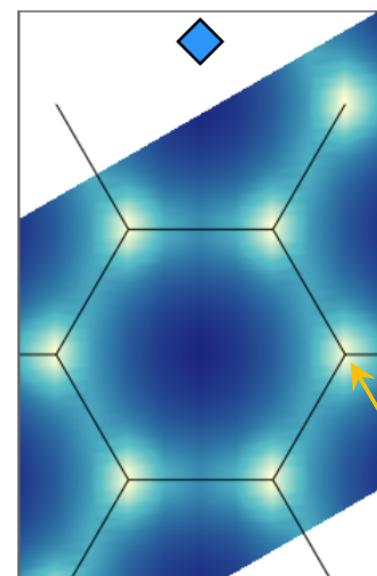
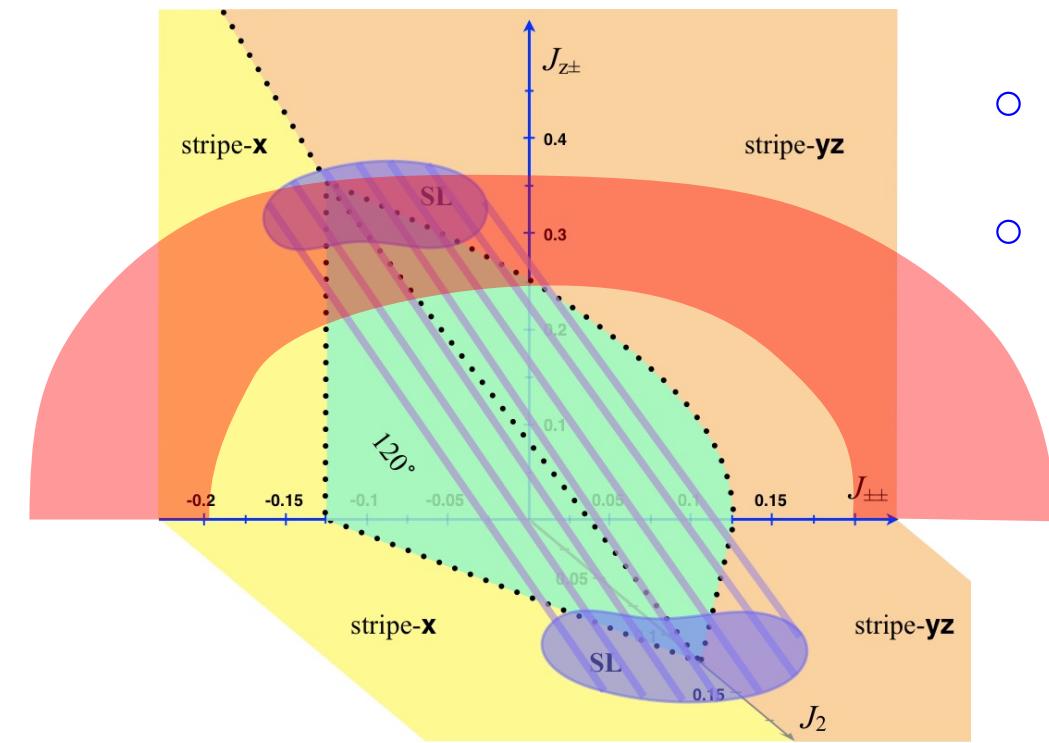
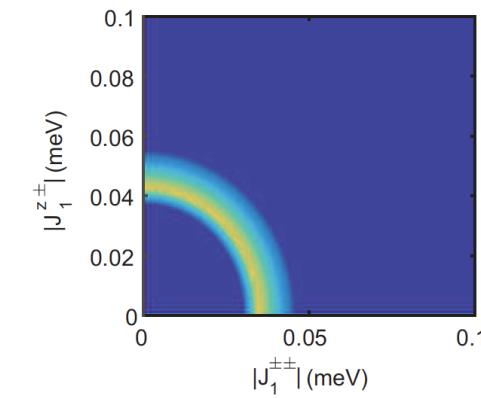


# “where is disorder-free YMGO?”, II

$$J_1^{zz} = 0.13(1) \text{ meV}, \quad J_1^{\pm} = 0.08(1) \text{ meV}, \\ J_2/J_1 = 0.18(7), \quad \sqrt{|J_1^{\pm\pm}|^2 + |J_1^{z\pm}|^2} \lesssim 0.05 \text{ meV}$$

$\Rightarrow \Delta=0.8(1), J_2 = 0.18(7)J_1, J_{\pm\pm} \approx 0.2-0.3, J_{z\pm} \approx 0.25-0.35$

- closer to isotropic  $XXZ$ , larger bond-dependent terms, tied in a “ring of fire”
- **outside SL, deep in the stripe phase**
- **even if: wrong SL! [ $S(\mathbf{q})$  different]**



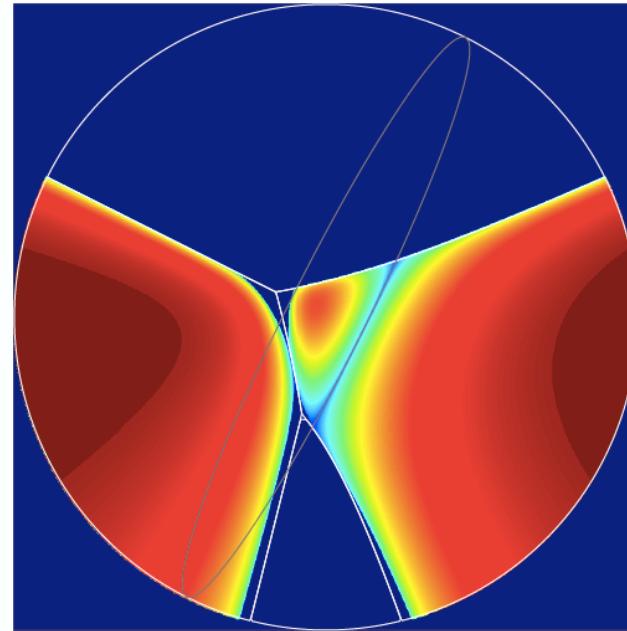
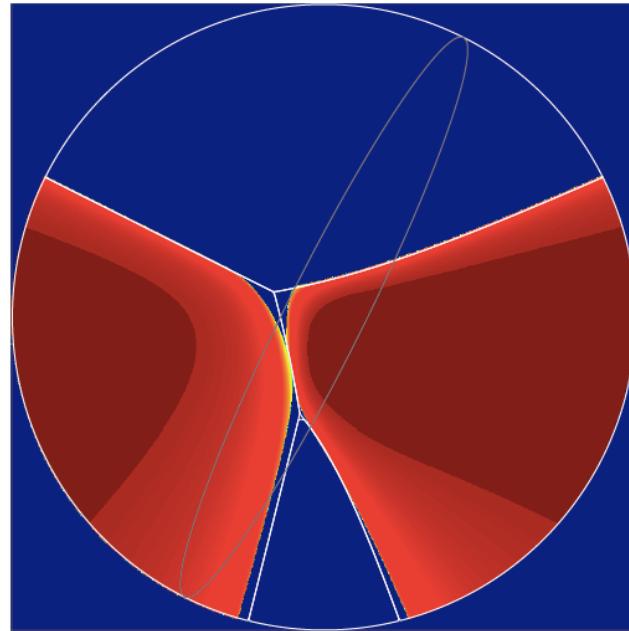
K-points

M-points

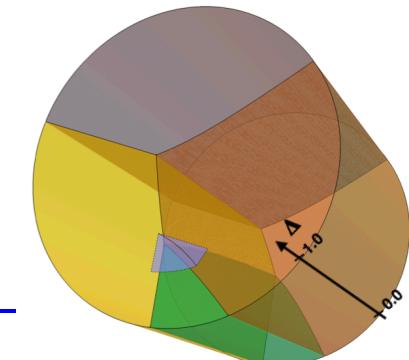
truality

# stripe phases, ordered moment vs $T_N$

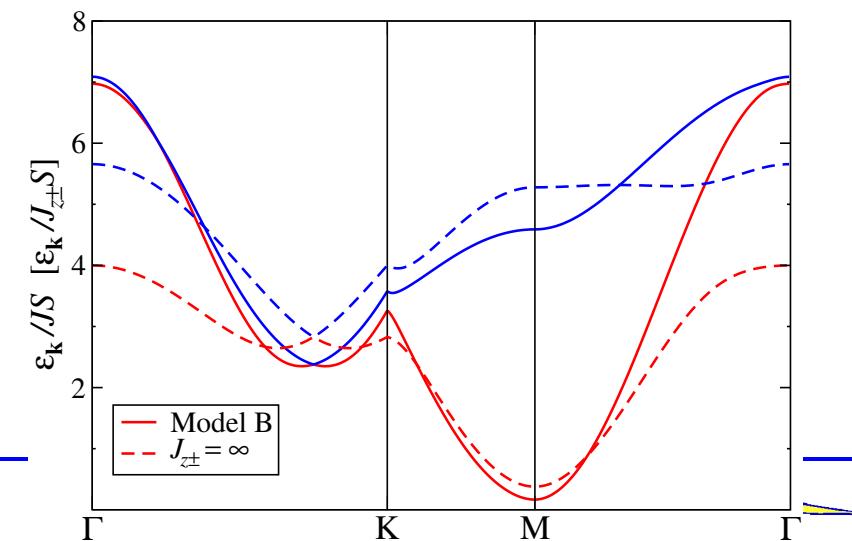
$\langle S \rangle$



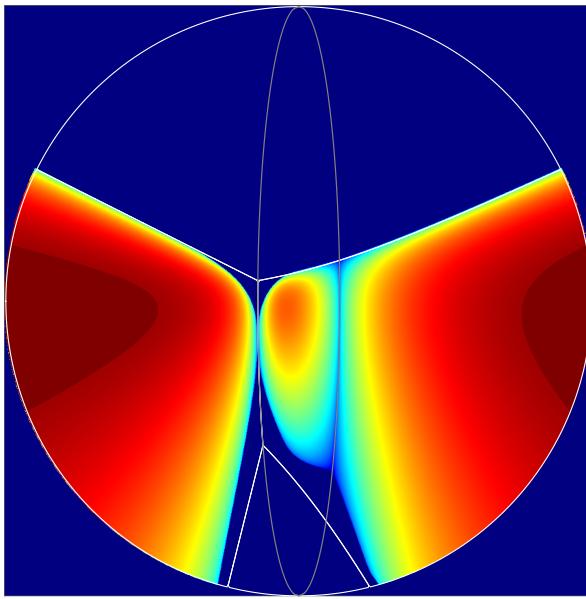
$\Delta = 0.0$



- **stripe phases:** ordered moment is nearly classical
- **Neel temperature is strongly suppressed around a weird line**
- **spectrum has a pseudo-Goldstone mode along it**
- **similar for other  $\Delta$ 's**



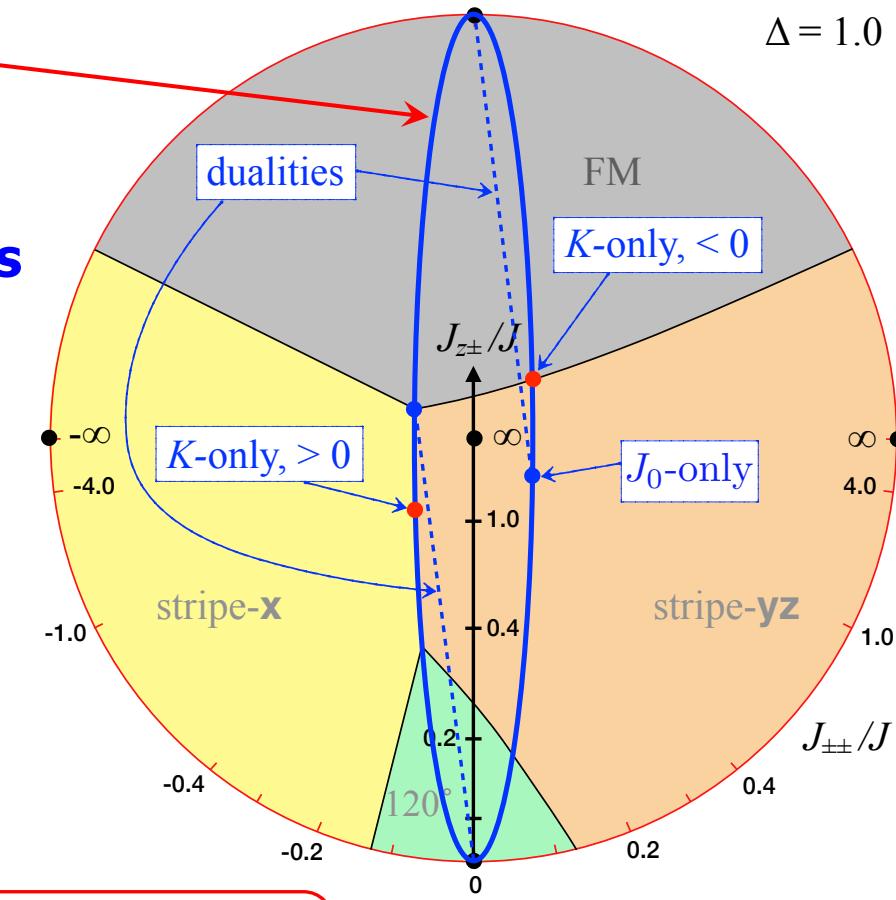
# $J$ - $K$ - $\Gamma$ - $\Gamma'$ model correspondence



- for  $\Delta=1.0$  and  $J_{z\pm}=2\sqrt{2}J_{\pm\pm}$  line  
the model becomes  $J$ - $K$  model
- hence, pseudo-goldstone modes
- yz-stripe sector  $\Rightarrow$  duality to FM  
 $\Rightarrow$  nearly classical state
- elsewhere (not on that line):  
 $\Rightarrow$  equivalent to  $J$ - $K$ - $\Gamma$ - $\Gamma'$  model

$$\begin{aligned}\mathcal{H} = \sum_{\langle ij \rangle} & [J_{zz} S_i^z S_j^z + J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) \\ & + J_{\pm\pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) \\ & - \frac{iJ_{z\pm}}{2} (\gamma_{ij}^* S_i^+ S_j^z - \gamma_{ij} S_i^- S_j^z + \langle i \leftrightarrow j \rangle)]\end{aligned}$$

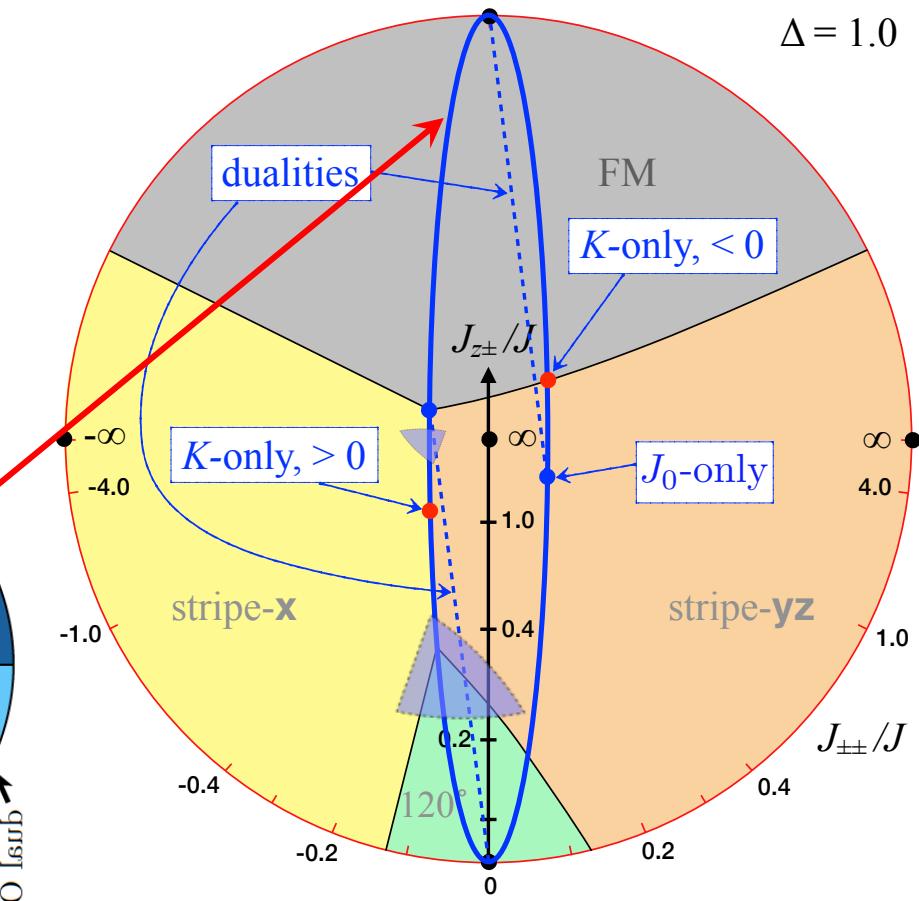
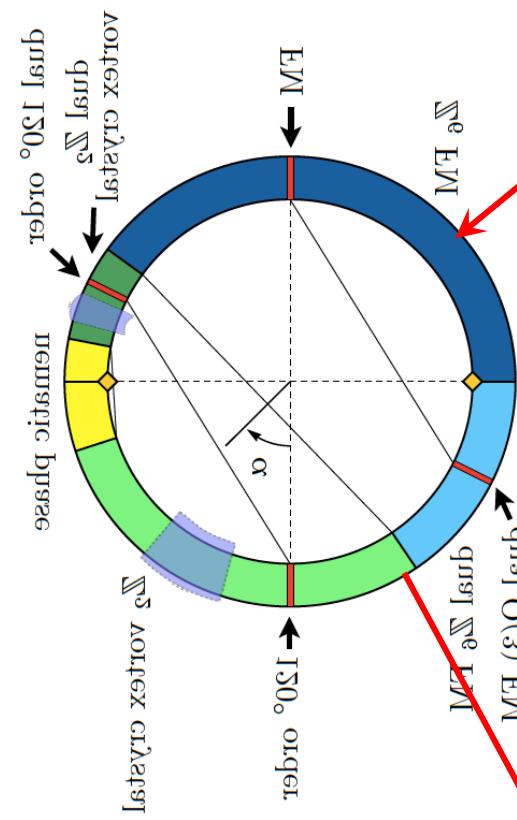
$$\Rightarrow \mathcal{H} = -\tilde{J} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$



# dualities

- triangular-lattice  **$J$ - $K$**  model:  
**no Kitaev-like solution, still an SL phase**
- $J$ - $K$**  model on triangular lattice has been studied before  
**[missed SL]**
- there should exist a **dual SL phase**

$$\mathcal{H} = -\tilde{J} \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - K \sum_{\gamma \parallel \langle ij \rangle} S_i^\gamma S_j^\gamma$$



# conclusions

- ✓ **emergent consensus:** YMGO spin-liquid-like response is disorder-induced, not intrinsic  $\Rightarrow$  [generic for ObD, + plain=strong disorder]
- ✓ **3D/4D quantum phase diagram:** SL phase,  $\Rightarrow$  isomorphism to  $J_1$ - $J_2$  SL state; framework for the triangular-lattice based systems
- ✓  **$J$ - $K$  correspondence:** “dual SL”; suppressed  $T_N$
- ✓ life does not end where spin-liquid does