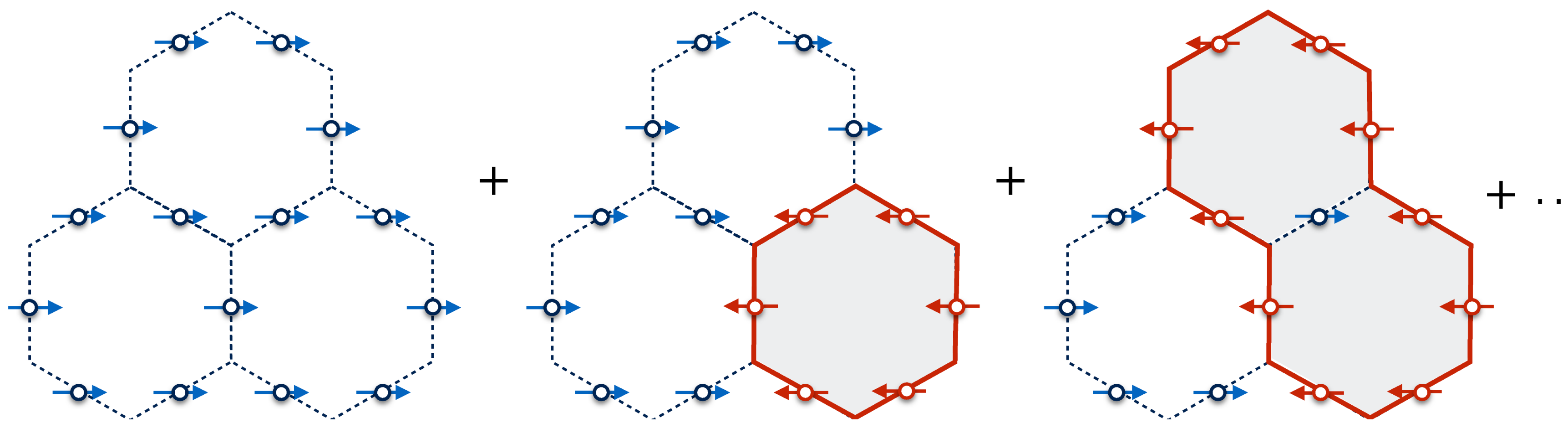
Quantum spin liquid in the semiclassical regime

$\Psi =$ 

Ioannis Rousochatzakis, University of Minnesota
(*now at Loughborough University, UK)

in collaboration with: Y. Sizyuk (ISU) & N. Perkins (UMN)

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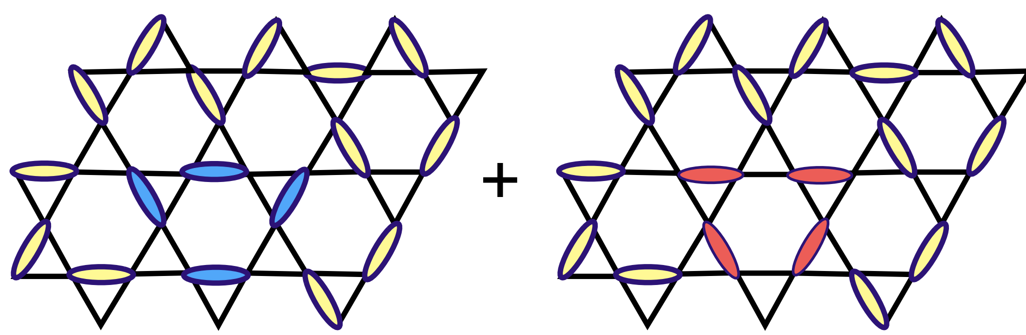
Quantum Spin Liquids (*tutorial by Lucile Savary)

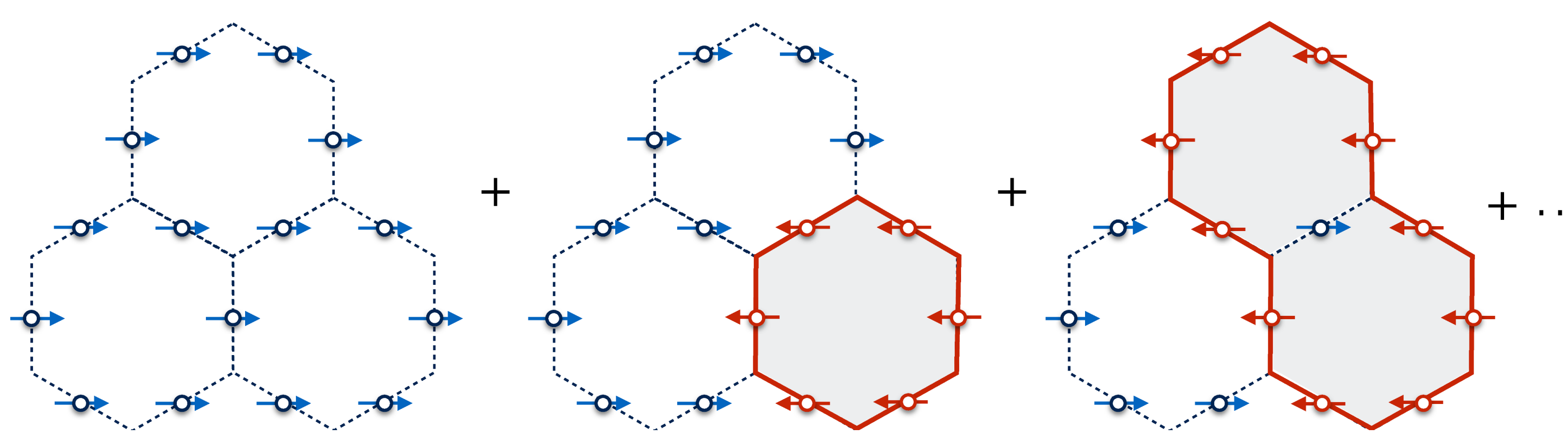
Ψ = massive superposition of product-like states

true for any choice of basis → long-range entanglement

-short-range resonating valence bond (RVB) state P. W. Anderson (1973)

$\Psi =$  $+ \ldots,$ where: $= \frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}$

-Toric code / 'String-net' condensate Kitaev (2003), Levin & Wen (2005), Wegner (1971)

$\Psi =$ 

→ 2 basic ingredients (besides gauge structure):

i) extensive # of states with very low E (in a variational sense)

ii) strong resonance

→ highly frustrated systems with **very low** spins S

Why is it **unusual** to have a QSL at **large** S?

•physics at large S

•classical limit (S=∞)

$$\mathcal{H} = \begin{pmatrix} \square & \cdot & \cdot \\ \cdot & \square & \cdot \\ \cdot & \cdot & \square \end{pmatrix}$$

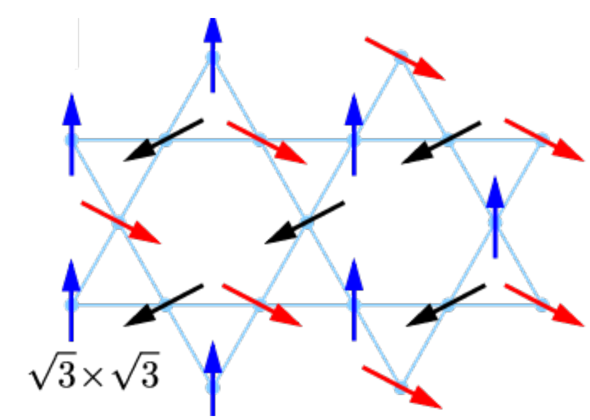
⟵ use as a variational basis

$\mathcal{O}(2^{\alpha N})$

-diagonal elements:
potential energy, in general different for each configuration. Captured by 1/S expansion.

-off-diagonal elements:
tunneling between different classical states. Exponentially small in S.

→for large-S:
degeneracy is lifted by potential energy (order-by-disorder)



Example: large-S kagome

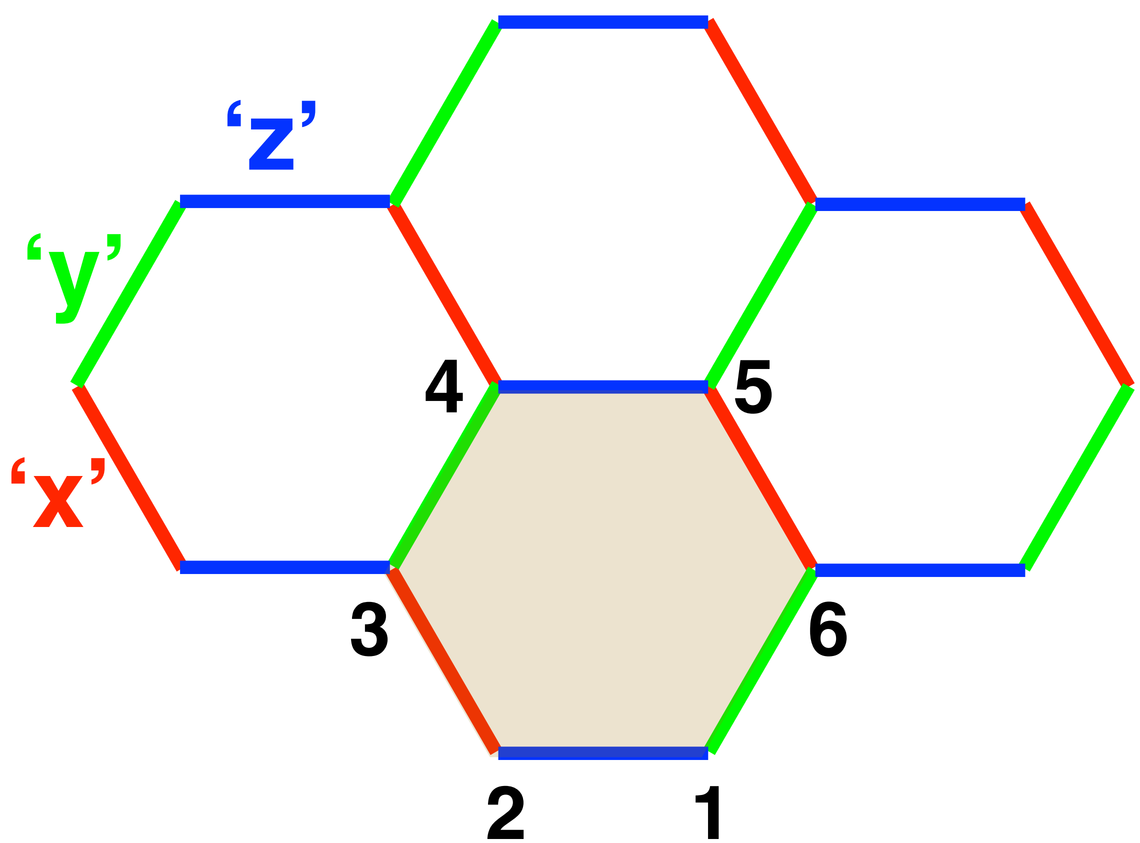
•Alternative scenario (this talk): **local** symmetries & Elitzur's theorem

Elitzur (1975)

potential energy **cannot** lift the infinite degeneracy

Kitaev model on the Honeycomb lattice
with large spin S

$$\mathcal{H} = K\Big(\sum_{\langle ij\rangle \in `x'} S_i^x S_j^x + \sum_{\langle ij\rangle \in `y'} S_i^y S_j^y + \sum_{\langle ij\rangle \in `z'} S_i^z S_j^z \Big)$$



Local symmetries Baskaran, Sen, Shankar (2008)

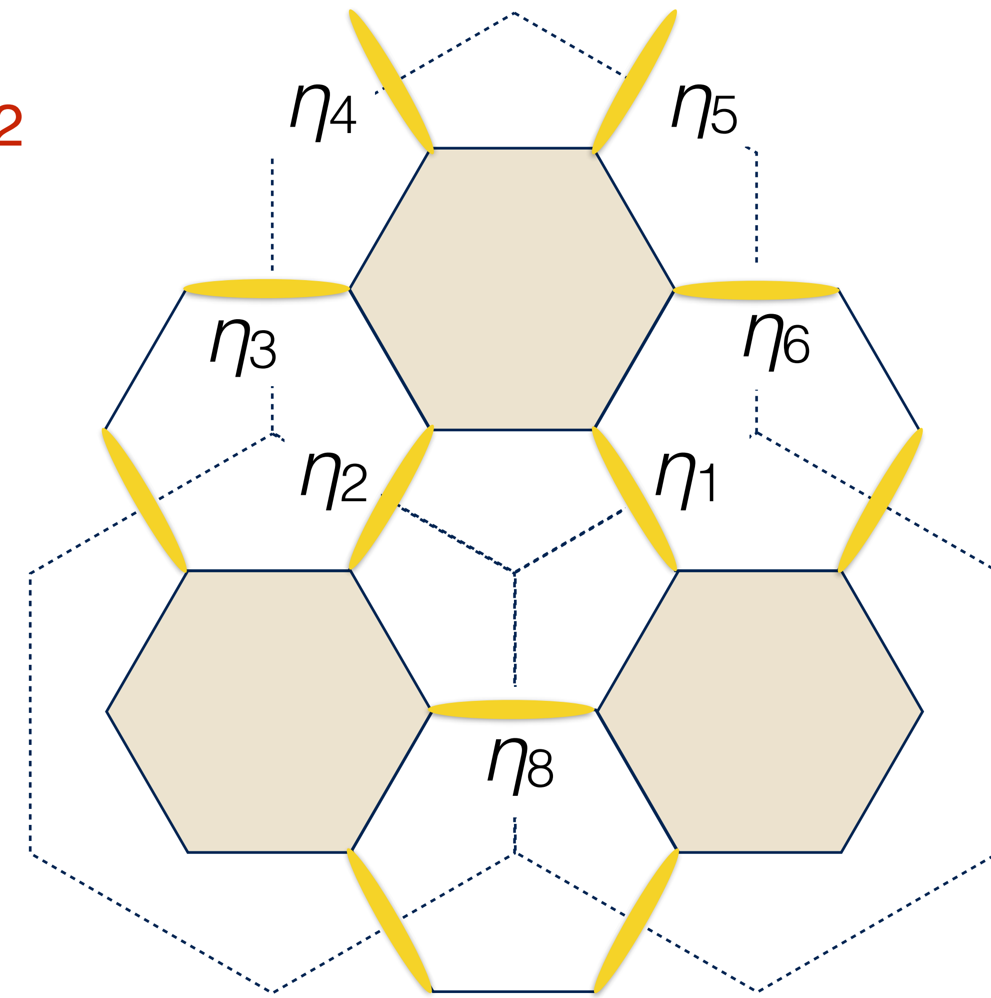
$$W = e^{i\pi\left(S_1^x + S_2^y + S_3^z + S_4^x + S_5^y + S_6^z\right)}$$

→ generalization of Kitaev's plaquette operator

in the following: **K<0**

Overview of main results

•low-E description: **not** in terms of spins-S, but in terms of pseudospins $\eta=1/2$



•η-variables:

sit on the bonds of a honeycomb superlattice, which breaks translational symmetry

•dynamics of η-variables:

Toric code on the honeycomb superlattice

•GS is a Z_2 spin liquid: Topological degeneracy + degeneracy associated with SSB

•Two types of gaps: magnetic fluxes (linear in S) + electric charges (exponentially small in S)

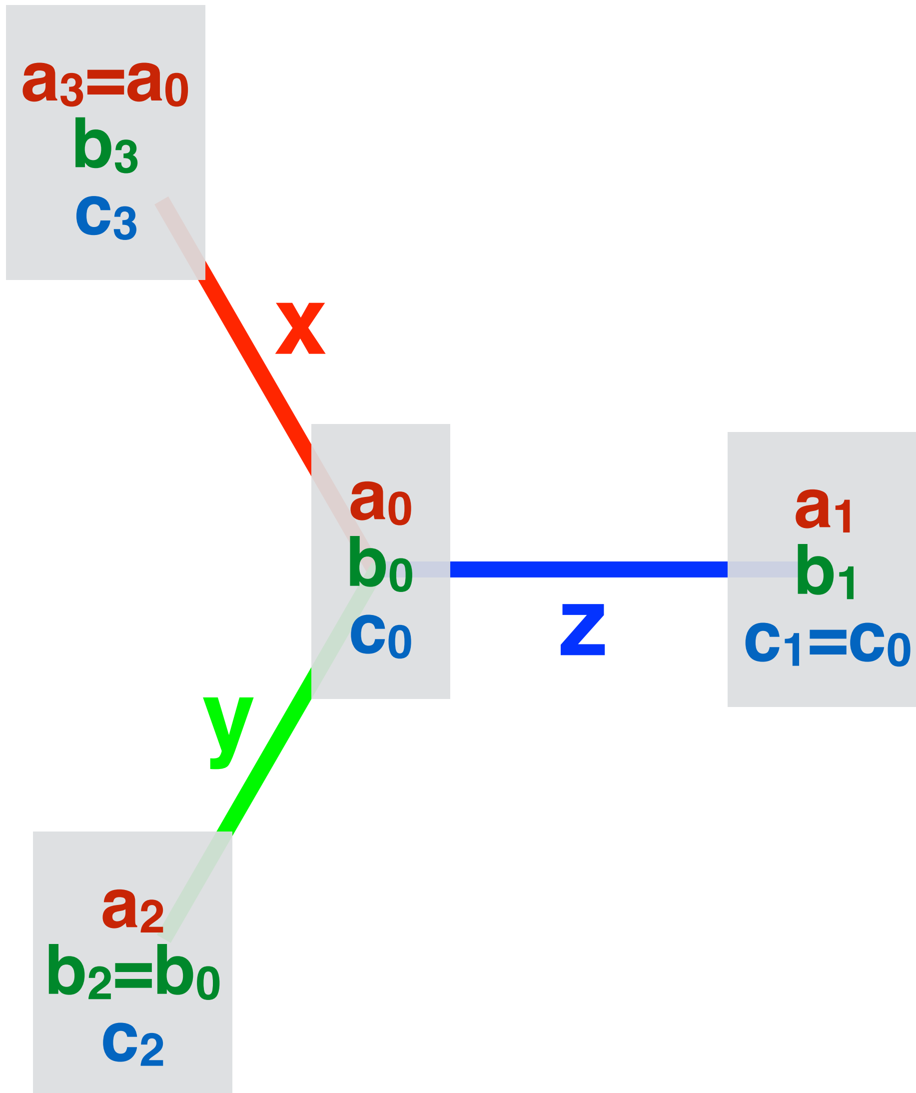
•Description breaks down at S<3/2; **Prediction** for another type of spin liquid at **S=1.**

(all different from the gapless S=1/2 case)

Classical ground state manifold

Chandra, Ramola, Dhar (2010)

IR, Sizyuk, Perkins (2017)



$$E = K(a_0^2 + b_0^2 + c_0^2) = KS^2$$

saturates lower bound

→on the lattice: very rich structure with algebraic correlations

Chandra, Ramola, Dhar (2010)

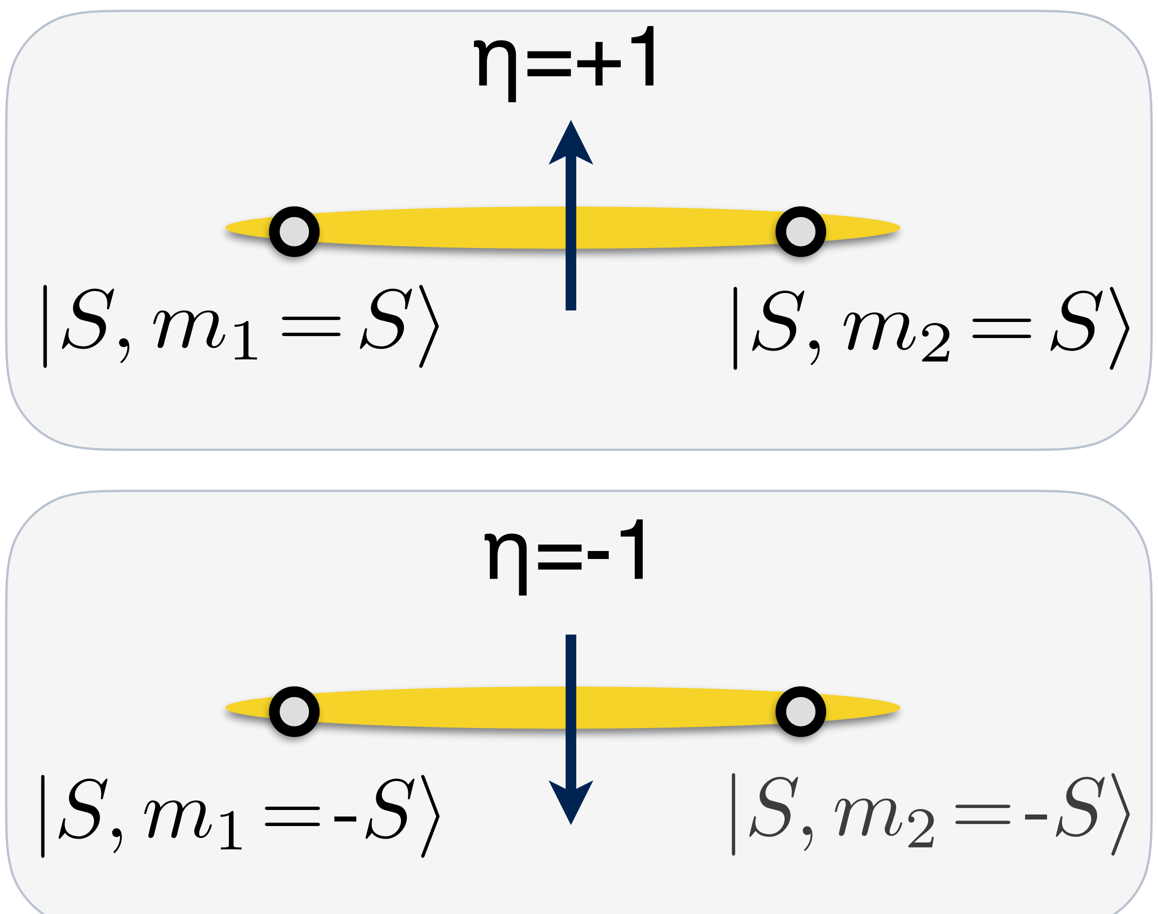
Effect of quantum fluctuations: Potential energy, I

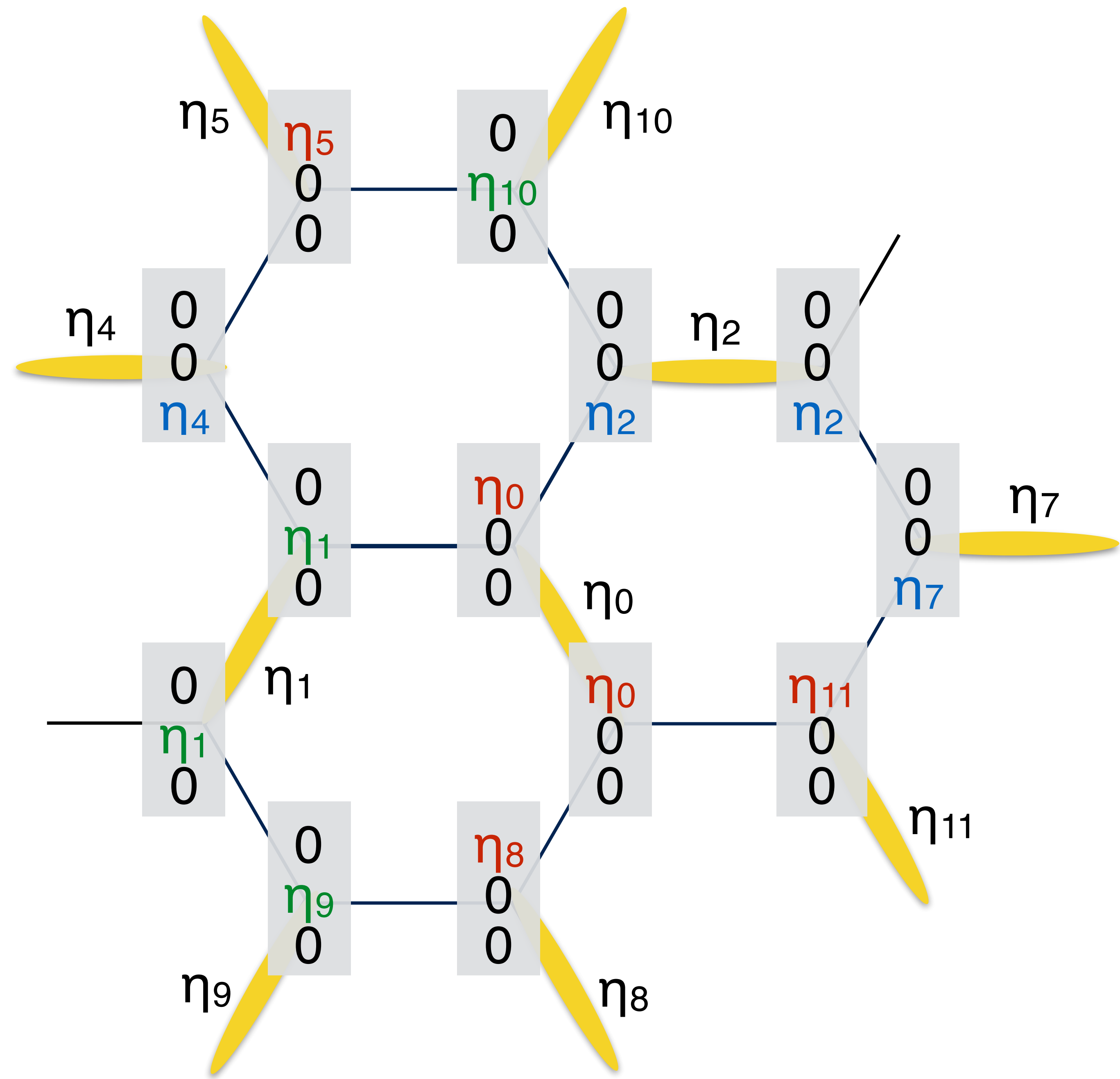
IR, Sizyuk, Perkins (2017)

$$\delta E_{\mathrm{ani}} = -\frac{|K|S}{16}\sum_i (a_i^4 + b_i^4 + c_i^4)$$

→ selects cubic axes

Meaning of η variables

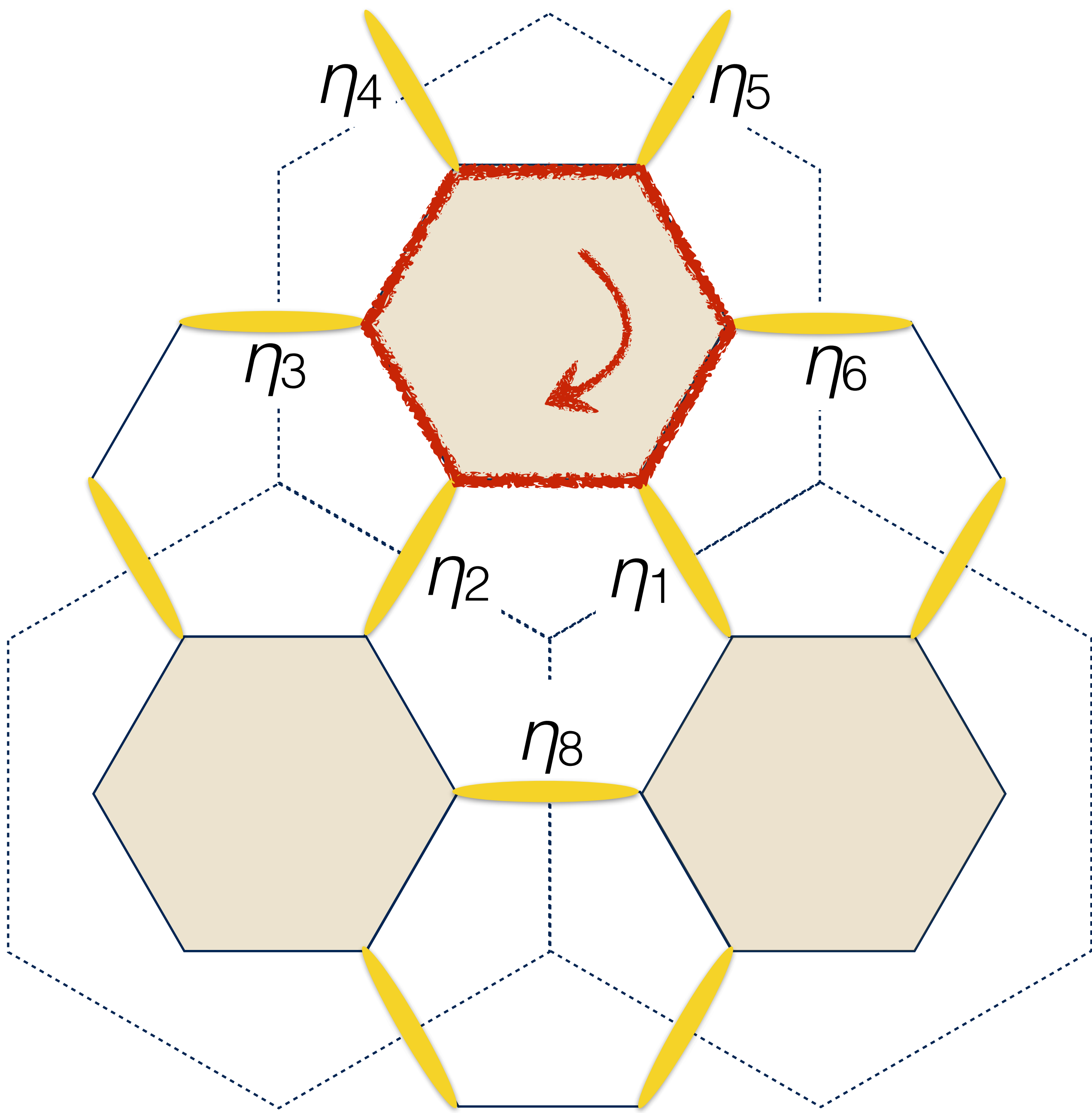




→ 1.662^N ‘Cartesian’ ground states

Baskaran, Sen, Shankar (2008)

Potential energy, II: Selection of dimer pattern (dimer freezing)



•**linear** spin-wave theory:

spin waves propagate along **empty bonds**

Baskaran, Sen, Shankar (2008)

•also true in **interacting** spin wave theory

IR, Sizyuk, Perkins (2018)

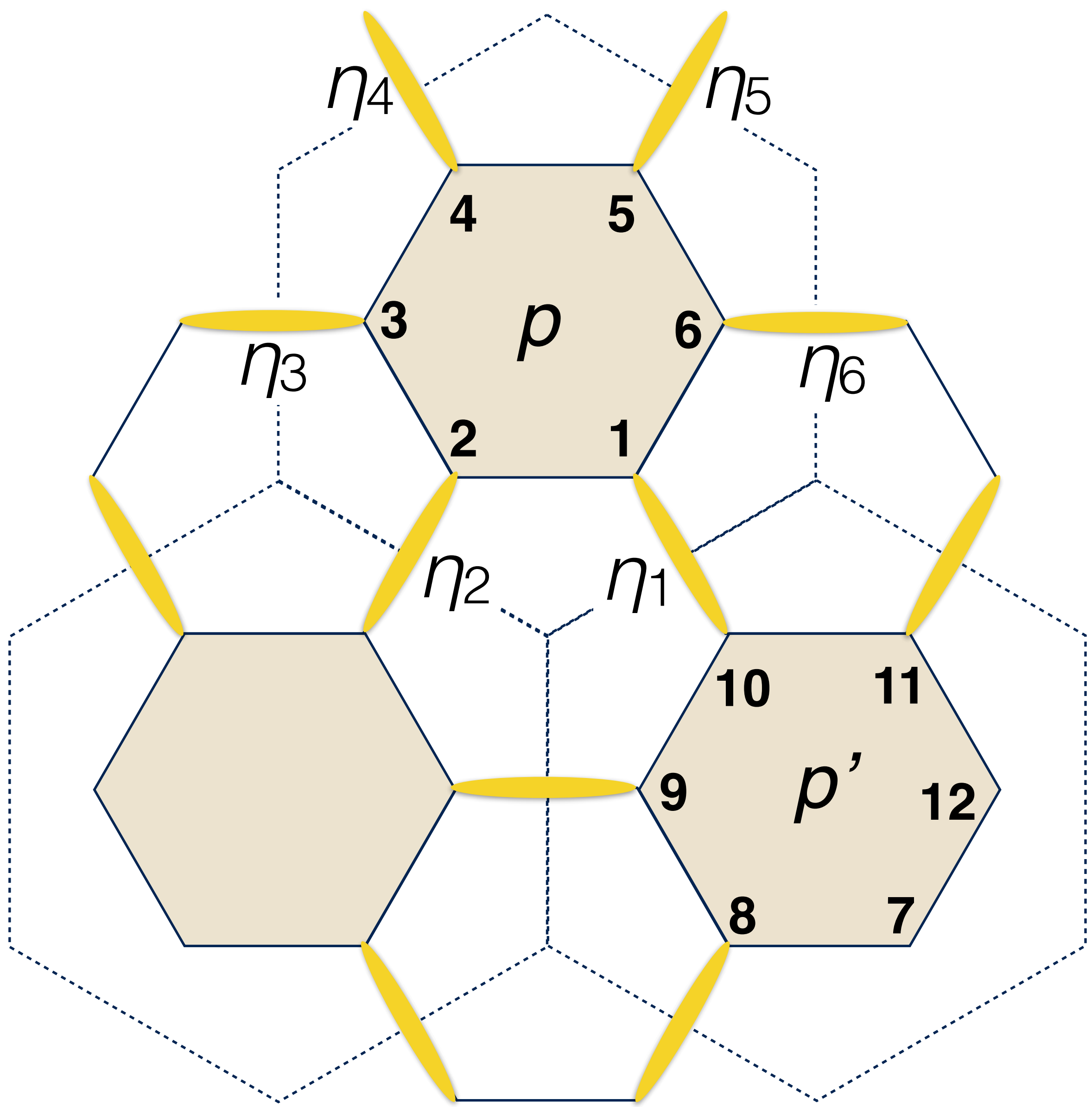
•**shortest** loops have lowest zero-point E

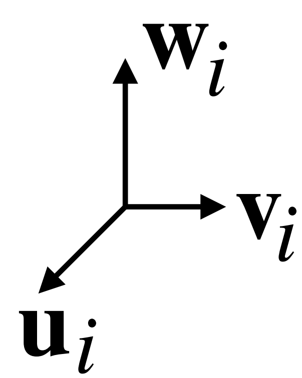
Baskaran, Sen, Shankar (2008)

η-variables live on the bonds of a honeycomb superlattice

Dimer freezing comes with a topological term

IR, Sizyuk, Perkins (2018)





inter-hexagon interactions

$$S_1^w S_{10}^w + \cdots$$

intra-hexagon interactions

$$S_1^u S_2^u + S_2^v S_3^v + S_3^u S_4^u + S_4^v S_5^v + S_5^u S_6^u - B_p S_6^v S_1^v$$

where: $B_p = \eta_1 \eta_2 \cdots \eta_6$

Holstein-Primakoff

$$S_i^\dagger = S_i^u + i S_i^v = (2S - c_i^\dagger c_i)^{1/2}\, c_i$$

$$S_i^w = S - c_i^\dagger c_i$$

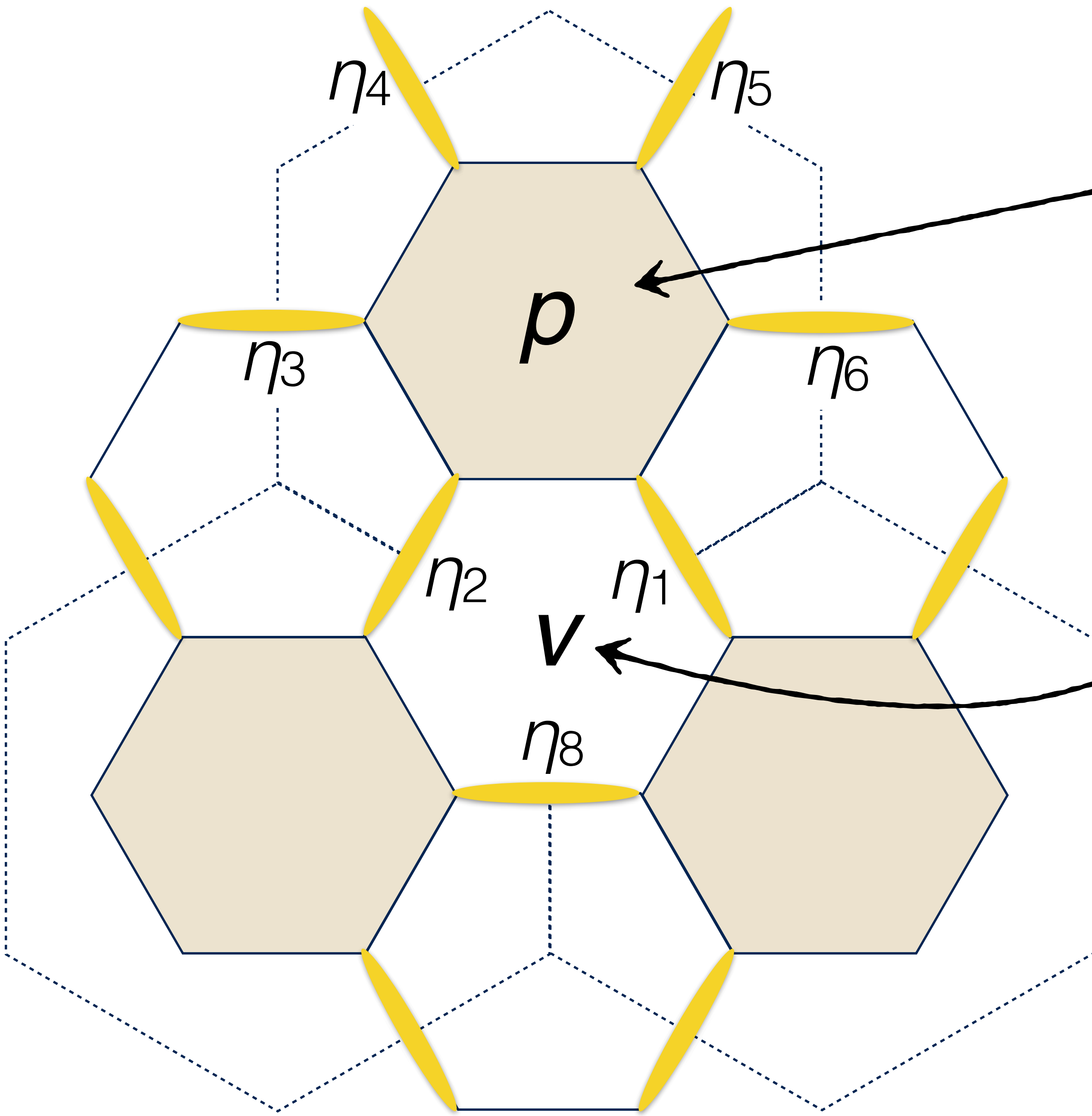
$\rightarrow \mathcal{H}_{\rm pot} = \mathcal{H}_{\rm pot}(\{B_p\})$ same flux pattern $\rightarrow$ same potential energy

$\rightarrow$infinite degeneracy remains (to all orders in 1/S)

$\rightarrow \mathcal{H}_{\rm pot} = J_m \sum_p B_p + \sum_{pp'} J_{pp'} B_p B_{p'} + \cdots$ magnetic term of ‘Toric code’

Kitaev (2003)

Including QM tunneling



$$J_{\mathrm{m}} \eta_1^z \eta_2^z \eta_3^z \eta_4^z \eta_5^z \eta_6^z = J_{\mathrm{m}} \mathbf{B}_p$$

magnetic flux term

$$J_{\mathrm{e}} \; \eta_1^x \eta_2^x \eta_8^x = J_{\mathrm{e}} A_v$$

electric charge term

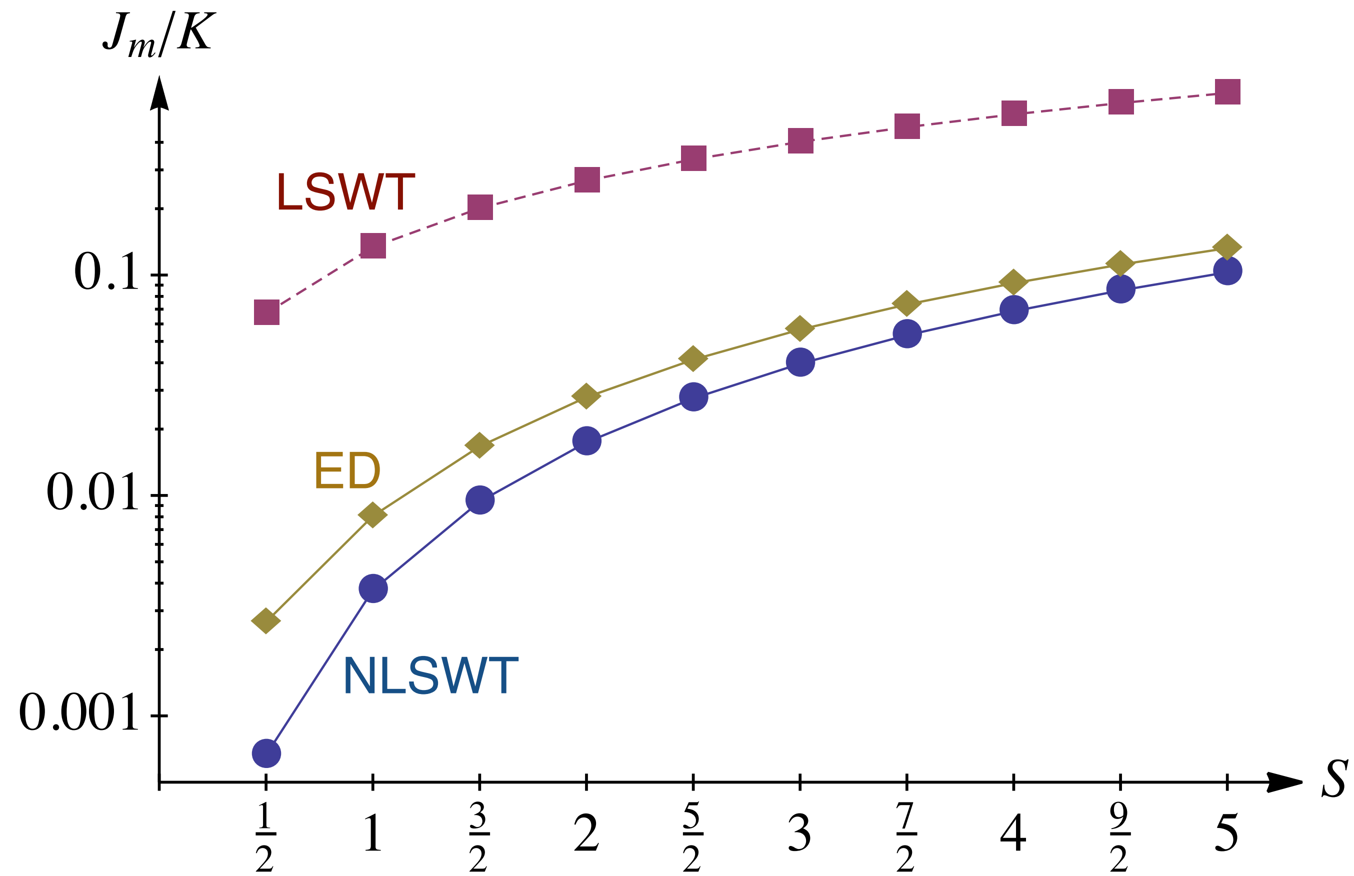
→Toric code Hamiltonian

Kitaev (2003), Levin & Wen (2005), Wegner (1971)

*tunneling restores local symmetries **W** in non-empty hexagons!

J_m and J_e versus S

J_m scales roughly linearly with S
(for large S)



J_e scales exponentially with S:

$$J_e = 3 \times 2^{5-18S} S^{5-6S} [(2S-1)!]^3 K$$

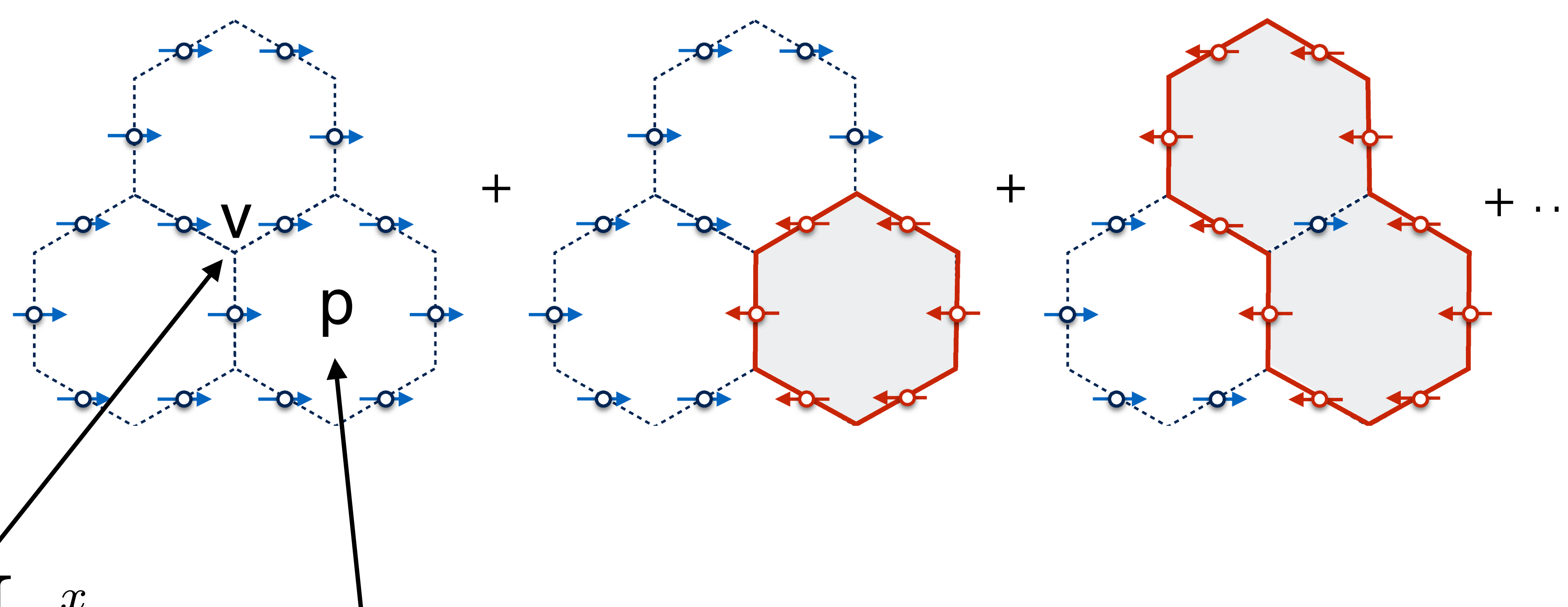
Ground states

Kitaev (2003), Levin & Wen (2005)

$$\mathcal{H}_{\rm eff} = J_m \sum_p \mathbf{B}_p + J_e \sum_v \mathbf{A}_v + \cdots$$

for K<0: $J_m<0$, $J_e<0$ → ground states: $B_p=1$, $A_v=1$ for all p & v

$\Psi =$

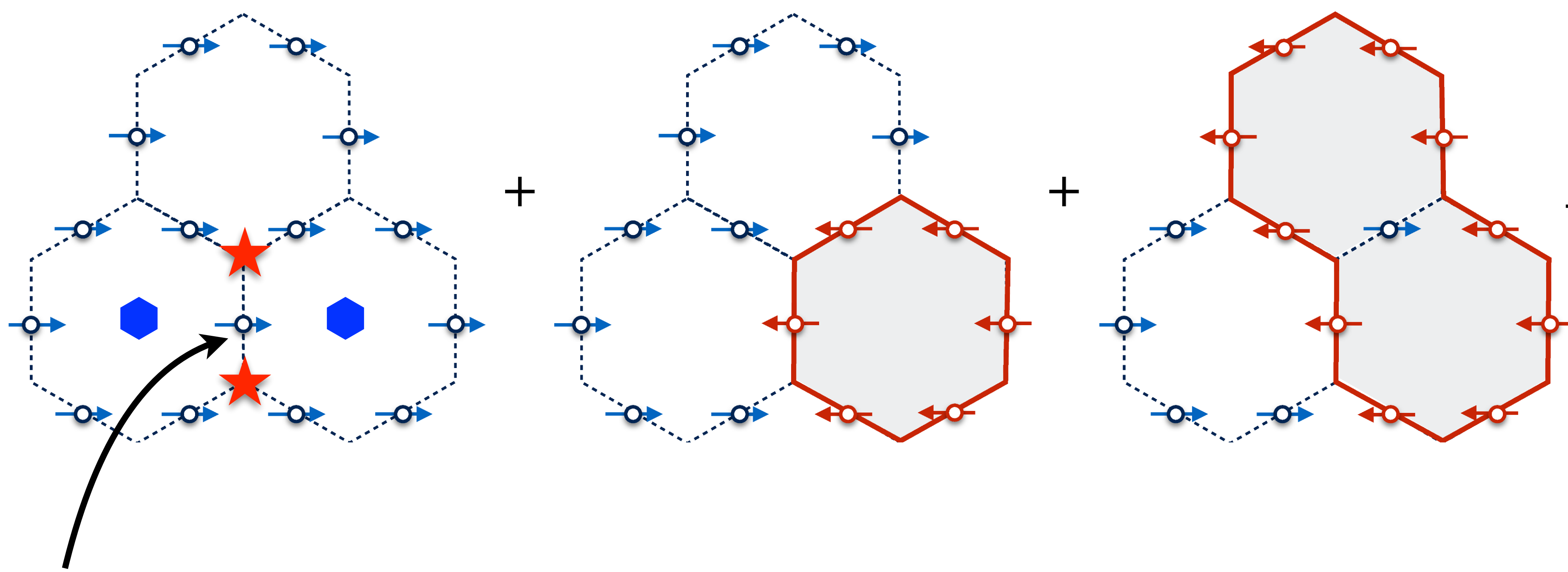


$$\mathbf{A}_v = \prod \eta^x \qquad \mathbf{B}_p = \prod \eta^z$$

-GS degeneracy depends on the topology of the system (4 GS's on a torus)

Elementary excitations are fractionalized

Kitaev (2003)

$\Psi =$  $+ \cdots$

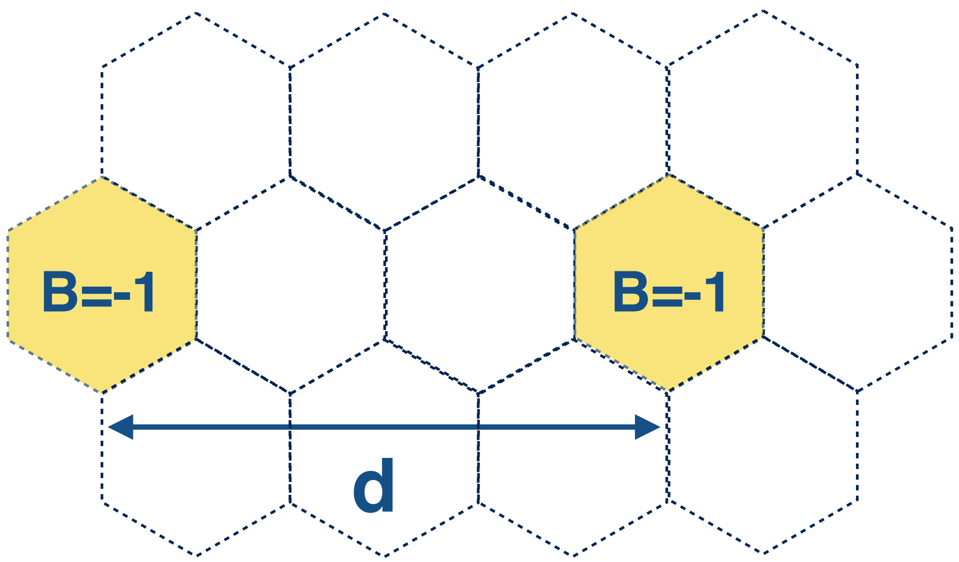
-apply η^z → **two** A=-1 charges ($E=2J_e$)

-apply η^x → **two** B=-1 fluxes ($E=2J_m$)

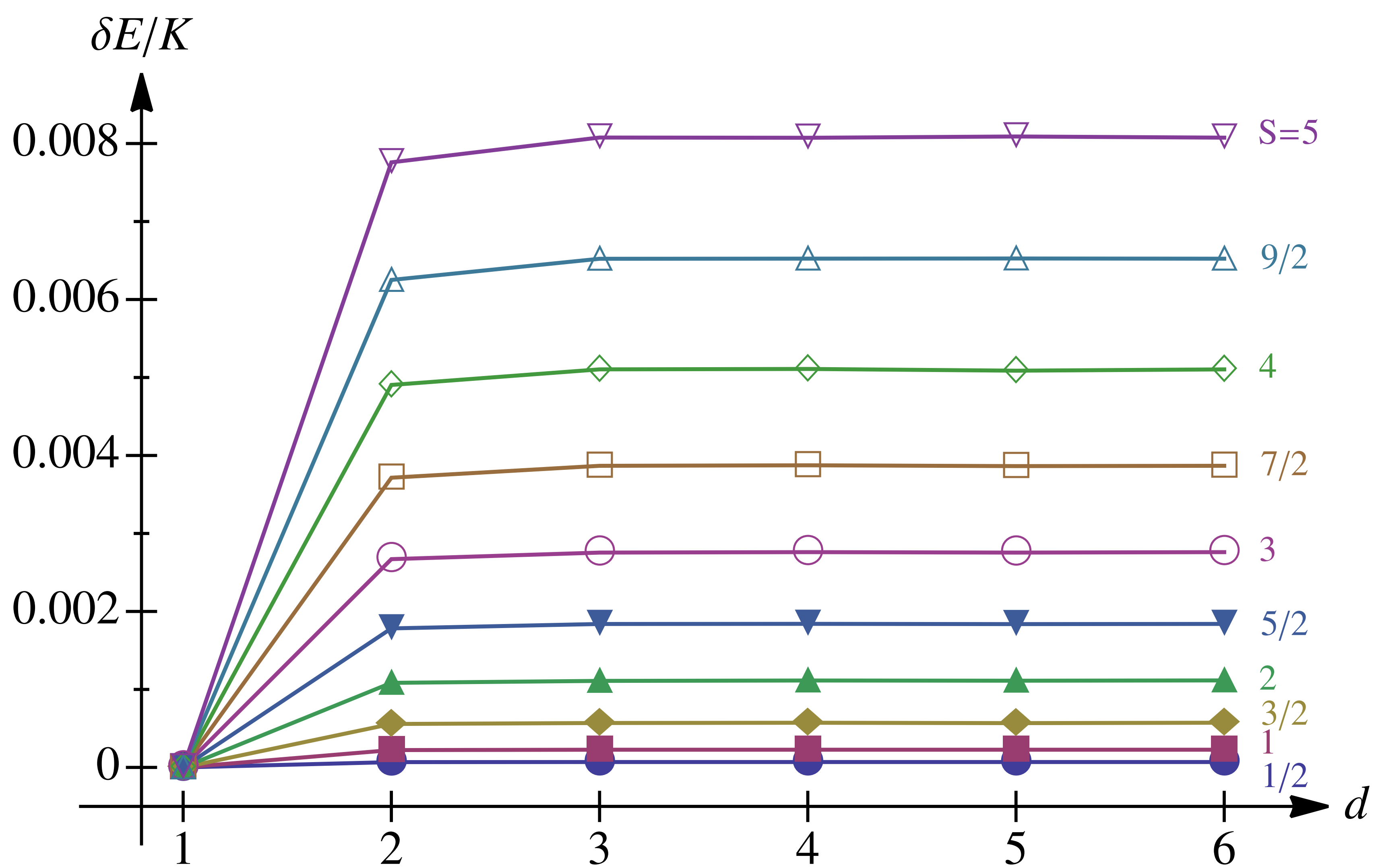
→ can separate the two particles with a finite energy cost

Deconfinement

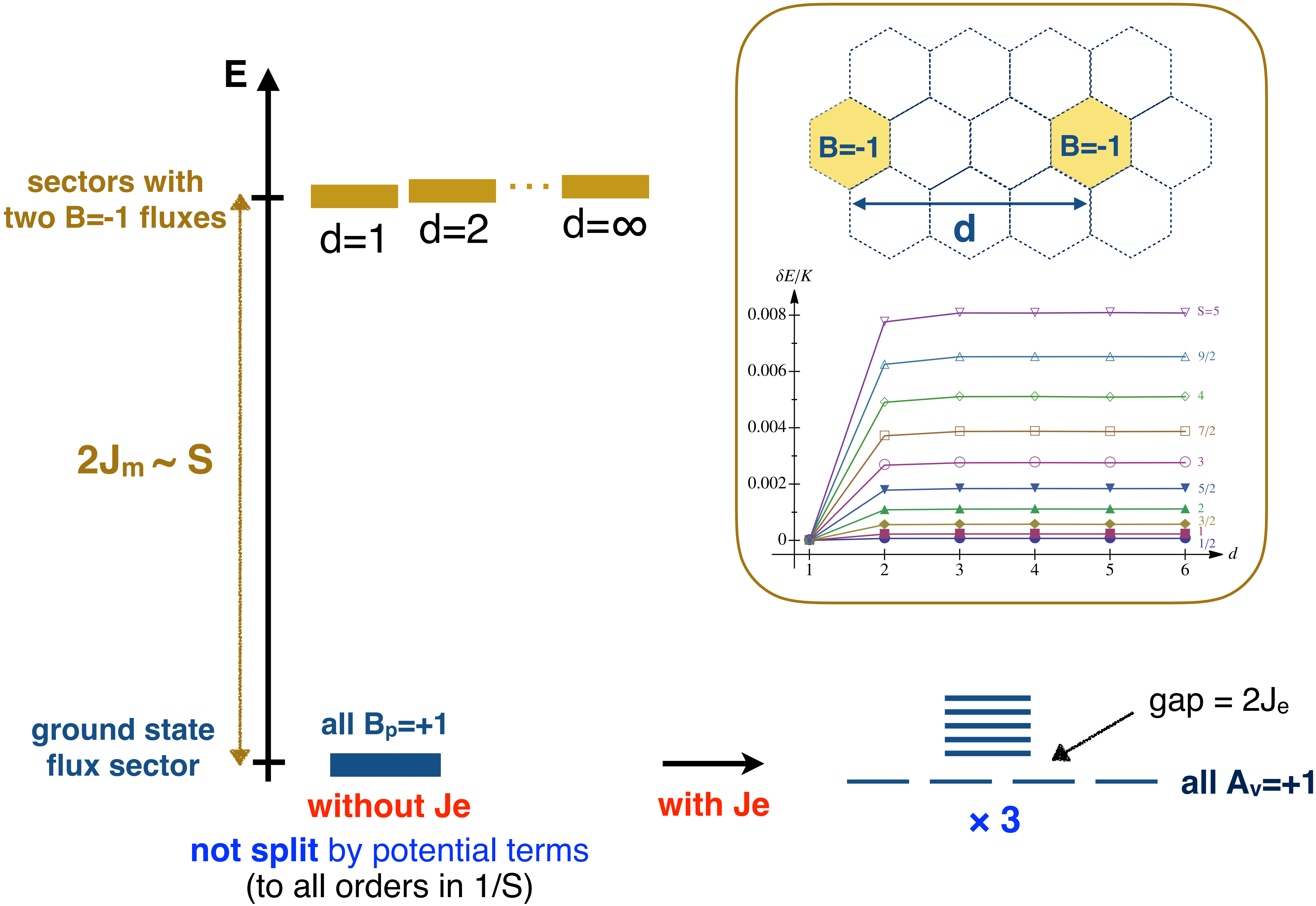
IR, Sizyuk, Perkins (2018)



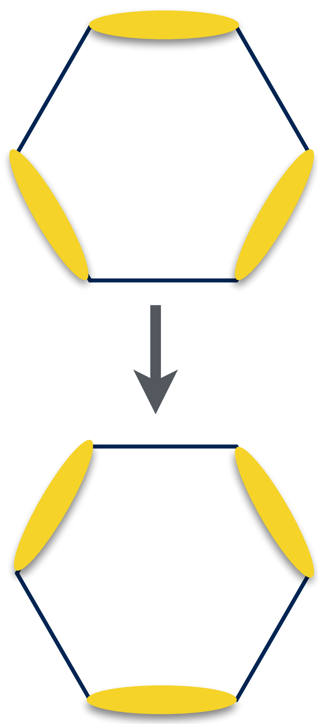
*calculated with an
iterative ED method

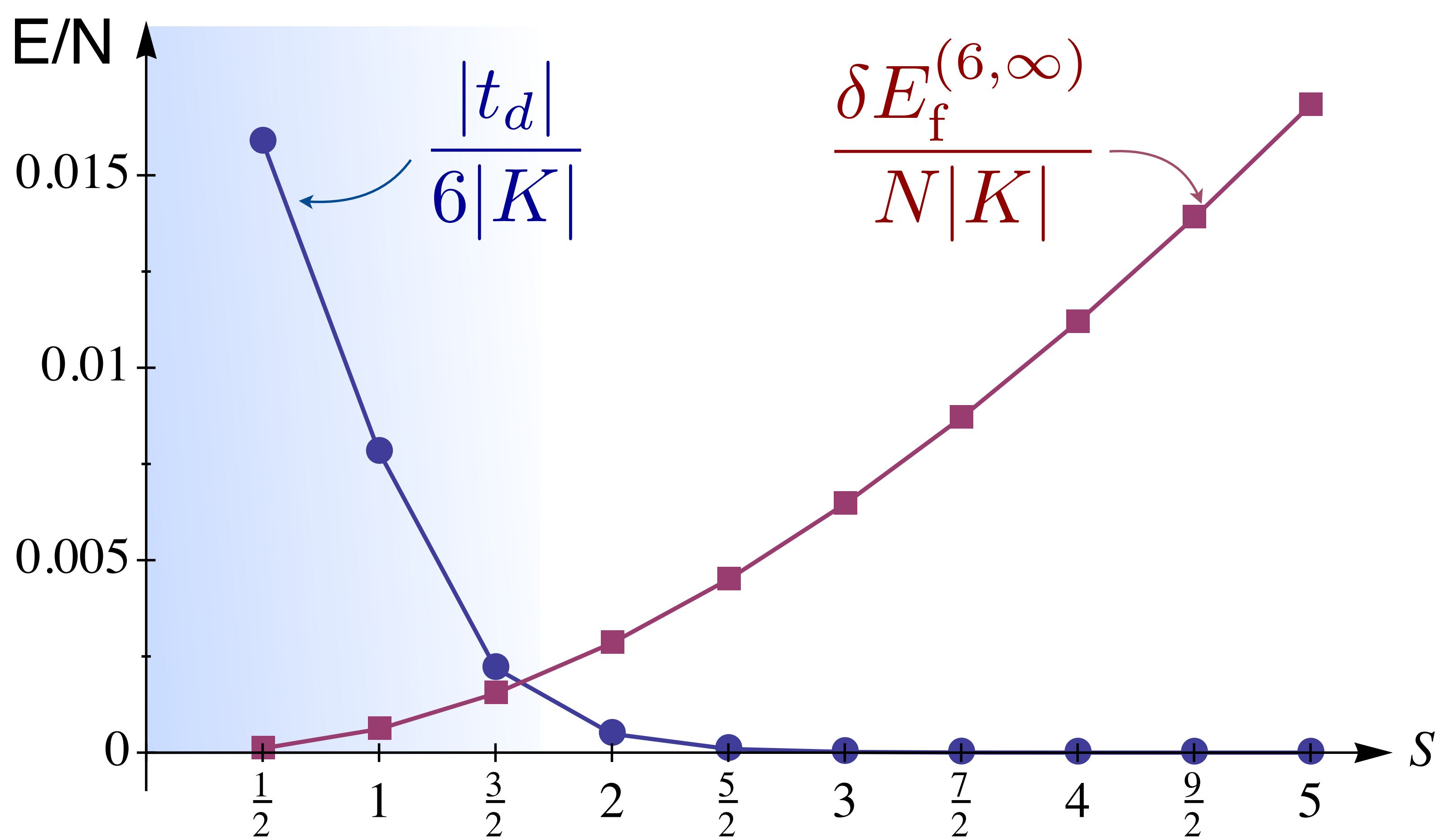


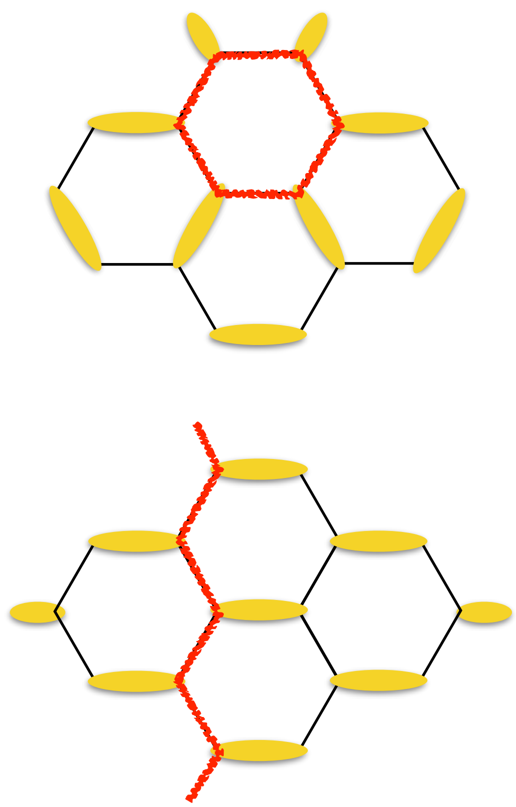
Overall spectrum (skipping spin wave modes)



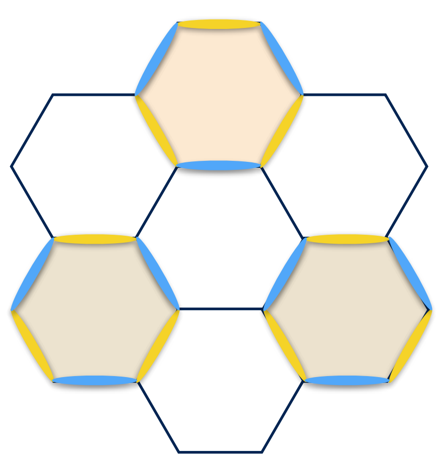
Breakdown of Toric description at S < 3/2







S≲2: dimer positions + **η**-variables on top



prediction for S=1: another type of gapped spin liquid

dimers form resonating plaquette VBC, **η**-variables fluctuate on top.

U.S. DEPARTMENT OF ENERGY

Thank you very much for your attention!