Search for SU(2) non-Abelian chiral spin liquids Using iPEPS

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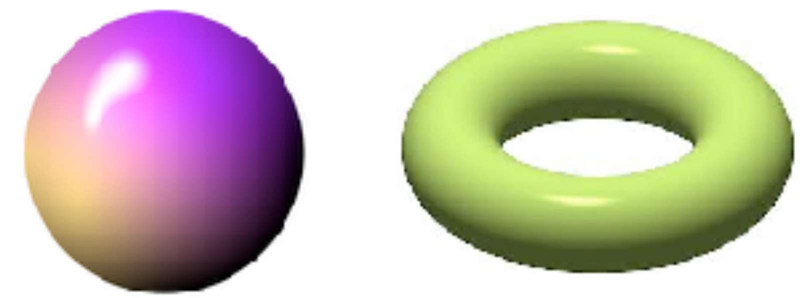
- Motivations from the physics of the Fractional Quantum Hall effect
- «Projected Entangled Pair States» (PEPS) Ansätze and the iPEPS method
- Non-Abelian chiral spin liquid in a frustrated spin-1 Heisenberg model

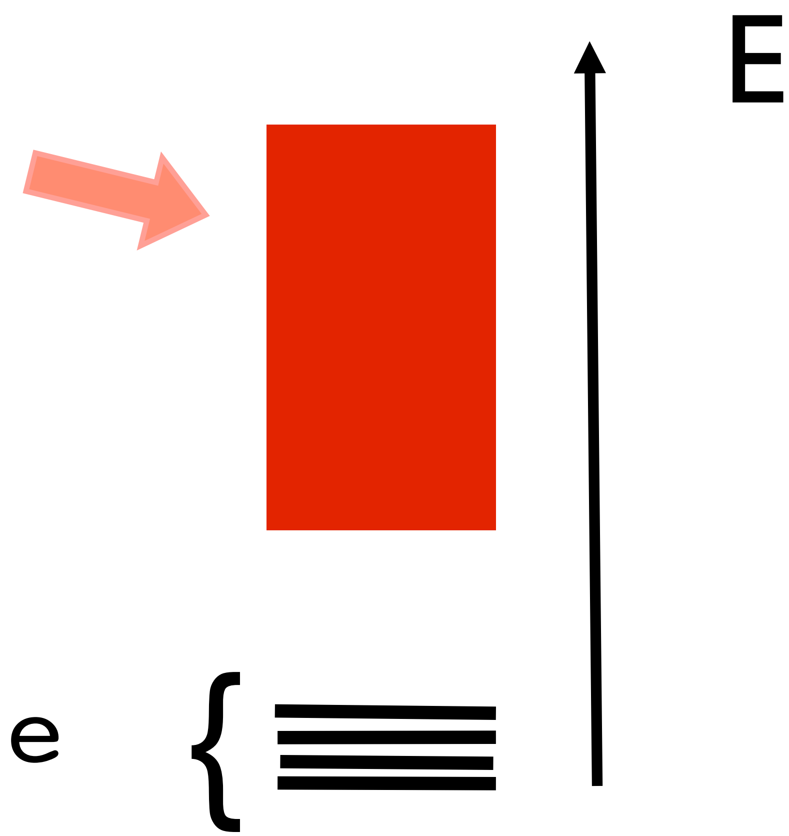
«Topological liquids»
beyond the
«order parameter» paradigm

X. G. Wen

* no spontaneous broken symmetry
* no local order but...
* Topological order



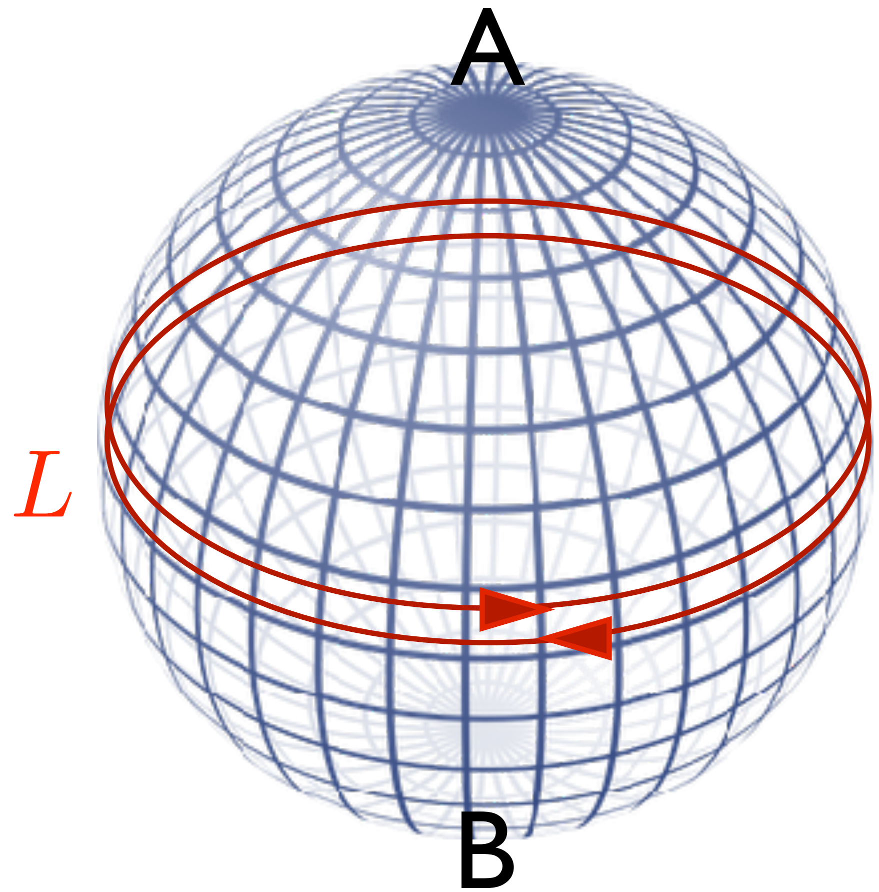
Excitations are fractional
Abelian or non-Abelian «anyons»



GS degeneracy
depends on **topology** of space

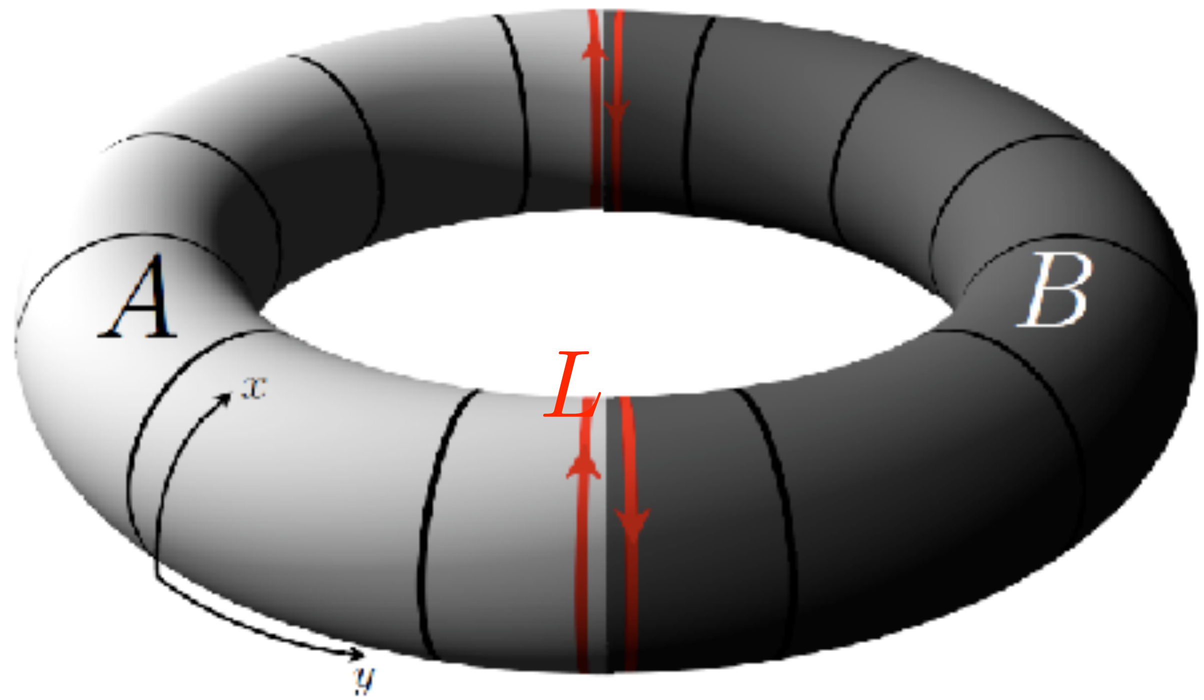
non-Abelian «anyons» needed for quantum computing !

Chiral topological liquids : edge states



Li & Haldane PRL 2008

Regnault, Bernevig & Haldane, 2009



Lauchli et al., 2009

Long Range Entanglement ➡ edge modes are protected !

Described by $SU(2)_k$ Conformal Field Theories (CFT)

Chiral topological spin liquids

- Topological chiral SL are genuine in the field of the Fractional Quantum Hall effect
- Spin analogs on the lattice ?
- Non-Abelian SL analog of non-Abelian FQH states ?

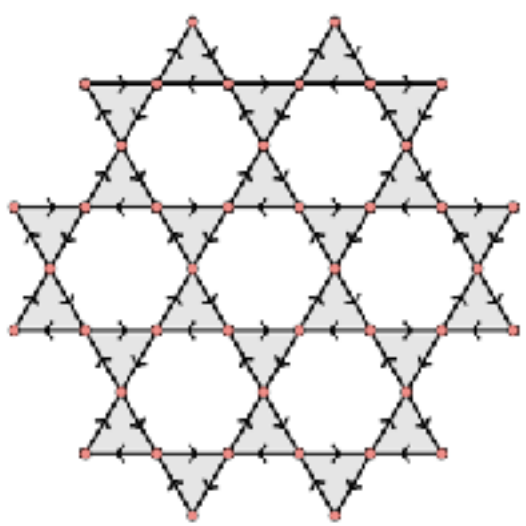
$\nu = \frac{1}{2}$ FQHS on a lattice (Kalmeyer-Laughlin, 1987):

(N=Nsites/2)

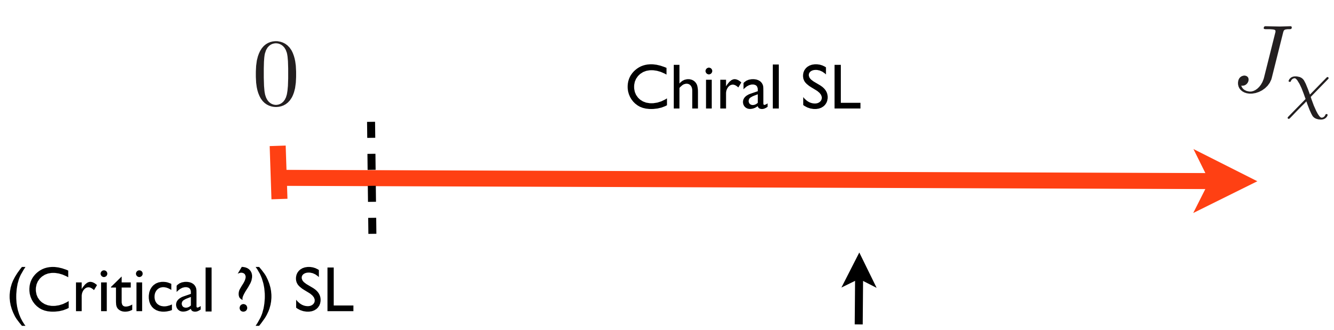
→ Paradigmatic "*Abelian* chiral spin liquid" (CSL)

Chiral SL in microscopic spin-1/2 models

B. Bauer, L. Cincio, B. P. Keller, M. Dolfi, G. Vidal, S. Trebst, A. W. W. Ludwig
Nature Communications 5, 5137 (2014)



$$H = J_{\mathrm{HB}} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + J_\chi \sum_{i,j,k \in \triangle} \vec{S}_i \cdot (\vec{S}_j \times \vec{S}_k)$$



Edge modes : chiral c=1 CFT (SU(2) $_1$ WZW)

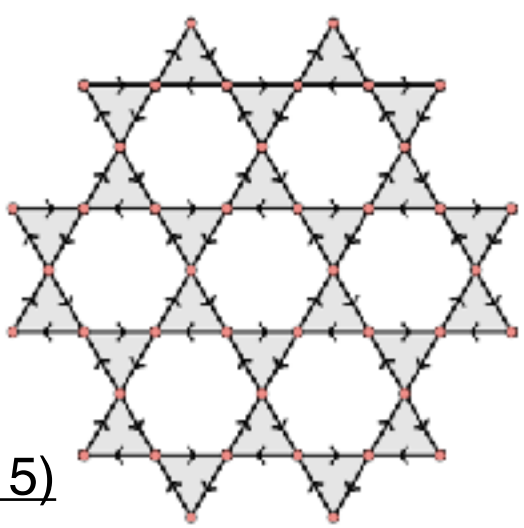
Chiral SL in microscopic spin-1/2 models (II)

- Triangular lattice

A. Wietek, A.M. Lauchli, Phys. Rev. B 95, 035141 (2017)

Shou-shu Gong et al., , Phys. Rev. B 96, xx (2017)

- Spontaneous symmetry breaking in J_1-J_2-J_3 model



S. Gong, W. Zhu, D.N. Sheng, Nat. Sci. Rep. 4, 6317 (2014)

Yin-Chen He et al., PRL 112 (2014)

A. Wietek, A. Sterdyniak, A.M. Lauchli, Phys. Rev. B 92, 125122 (2015)

Goal : search for non-Abelian CSL !

Non-Abelian CSL based on bosonic Moore-Read FQHS

New Journal of Physics
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Deutsche Physikalische Gesellschaft

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Exact parent Hamiltonians of bosonic and fermionic Moore–Read states on lattices and local models

I. Glasser, I. Cirac, G. Sierra & A. Nielsen (2015)

Bosonic Moore-Read Pfaffian state at $\nu = 1\,(q = 1)$

$$\psi(w_1, \ldots, w_M) \propto \prod_{i<j} (w_i - w_j)^q \mathrm{Pf}\left[\frac{1}{w_i - w_j}\right] e^{-\frac{1}{4}\sum_i |w_i|^2}$$

non-Abelian anyons : $\sigma \times \sigma = 1 + \Psi$

like Kitaev's honeycomb non-Abelian phase

Can be written as CFT correlator of $SU(2)_2$ CFT

→ Spin-1 singlet wave function

Chiral spin-1 frustrated Heisenberg model on the square lattice

Parent Hamiltonian is long range : truncation needed !

$$H = J_1 \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle\langle k,l \rangle\rangle} \mathbf{S}_k \cdot \mathbf{S}_l$$
$$+K_1 \sum_{\langle i,j \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 + K_2 \sum_{\langle\langle k,l \rangle\rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2$$
$$+K_c \sum_{\square} [\mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k) + \mathbf{S}_j \cdot (\mathbf{S}_k \times \mathbf{S}_m)$$
$$+\mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_m) + \mathbf{S}_i \cdot (\mathbf{S}_k \times \mathbf{S}_m)]$$

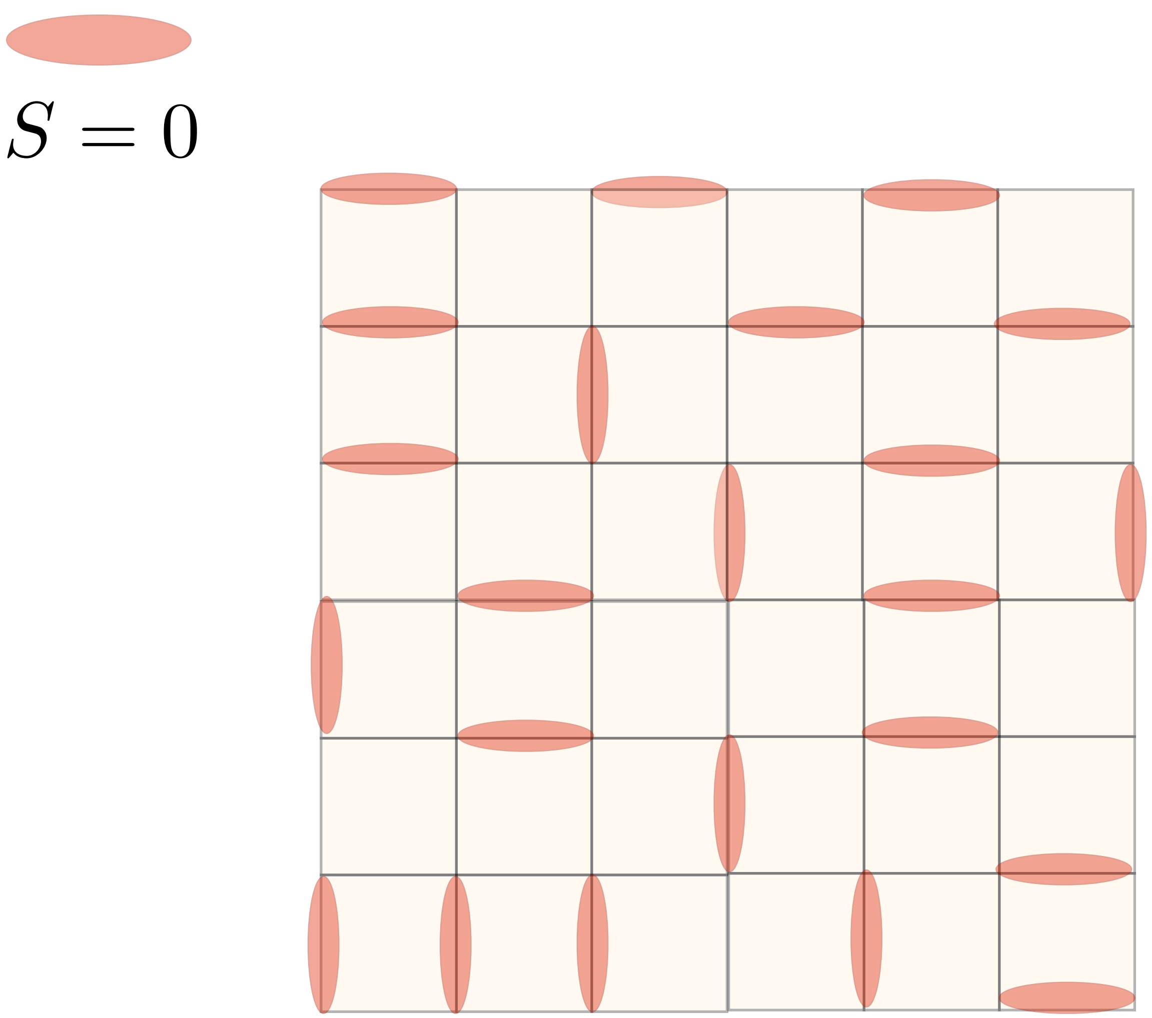
Spin-1 chiral HAFM

with fine tuned amplitudes !

$J_1 = 1$ $\quad J_2 = 0.623$ $\quad K_1 = -0.176$ $\quad K_2 = 0.323$ $\quad K_c = 0.464$

optimize overlap $\langle \Psi_{\mathrm{GS}} | \Psi_{\mathrm{MR}} \rangle$

A «toy» spin liquid: the RVB spin liquid



Equal-weight superposition of NN singlet coverings

spin-1/2 RVB

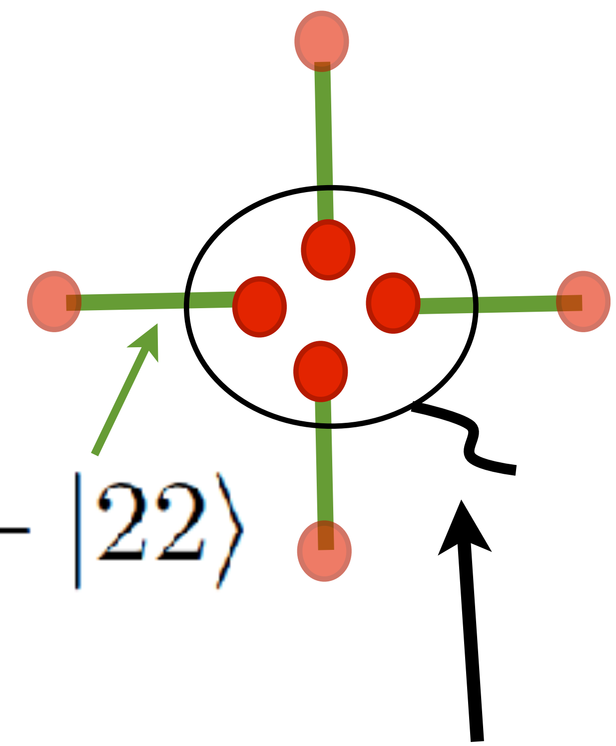
P. Fazekas and P.W. Anderson
Philosophical Magazine **30**, 423-440 (1974)

The spin-1/2 RVB can be written as a PEPS !

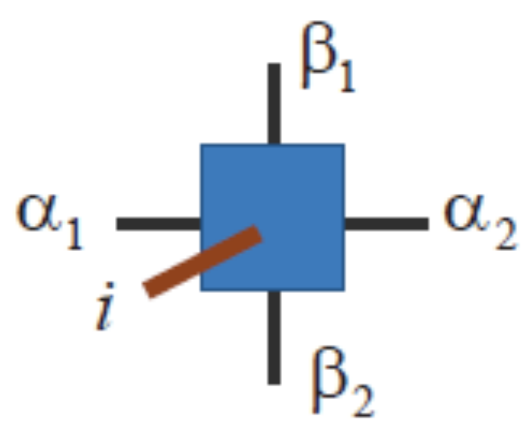
virtual states: $1/2 \oplus 0$

(D=3)

$$|\mathcal{S}\rangle = |01\rangle - |10\rangle + |22\rangle$$



Project onto physical subspace S=1/2 (d=2)



PEPS tensor

Gauge symmetry: topological order is encoded at the tensor level

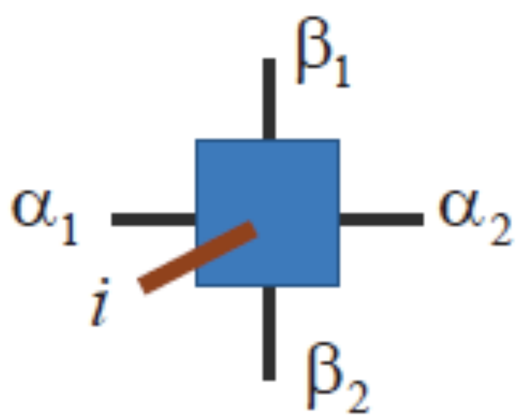
The PEPS as a variational ansatz

$$|\Psi\rangle = \sum C_{i_1, i_2, \ldots, i_N} |i_1, i_2, \ldots, i_N\rangle$$

$$A^{i}_{\alpha_1,\alpha_2;\beta_1,\beta_2}$$

$$i = \{1, \cdots, d_{\text{phys}}\}$$

$$\alpha, \beta = \{1, \ldots, D\}$$



dimension of auxilary (or virtual) space

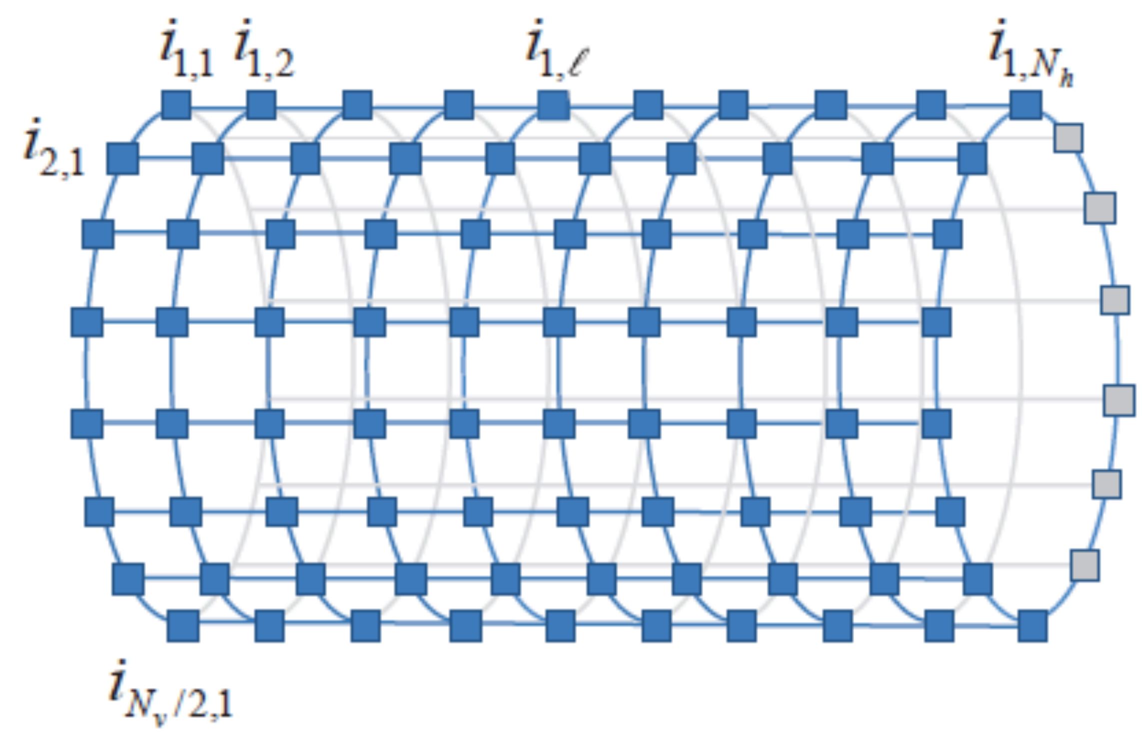
I. Cirac
F. Verstraete
G. Vidal

$N = N_v N_h$

Coefficients $C_{\{i_{1,1},\ldots,i_{N_v,N_h}\}}$ of the wavefunction

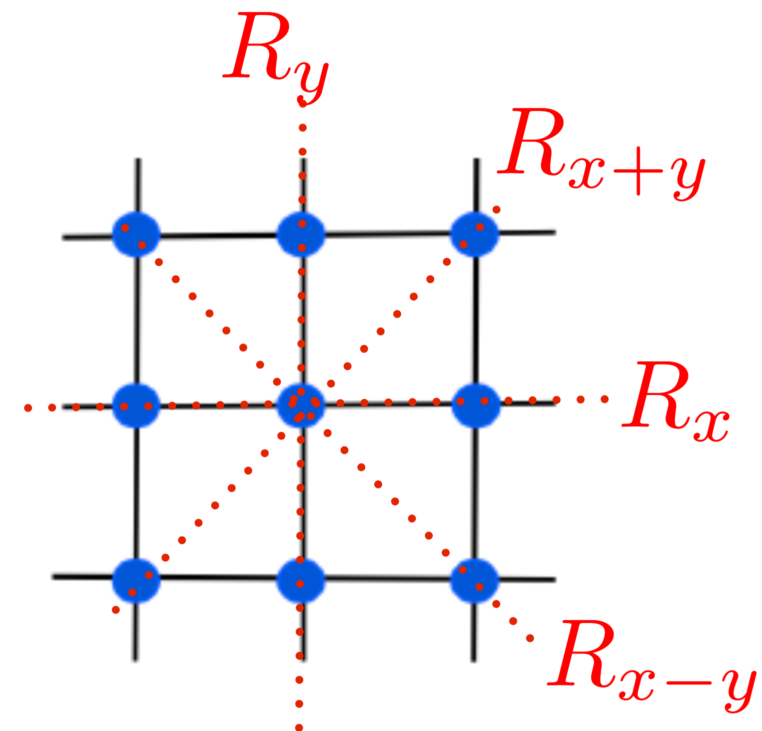
↓

"contract" product of tensors



Simple requirement for a chiral PEPS

Necessary conditions:



$\forall \; G_i \in C_{4v}$ reflexion symmetries

$G_i|\Psi\rangle = |\Psi^*\rangle$ *Time-reversed partner*

Realized for a PEPS ansatz with the form:

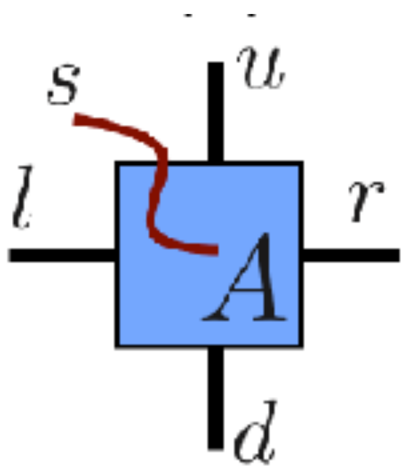
$$\Psi = \Psi_s + i\,\Psi_g$$

$s + ig$ symmetry $\qquad d_{x^2-y^2} + id_{xy}$ symmetry

$A_1 + iA_2 \qquad B_1 + iB_2$

General construction using a classification of SU(2)-invariant PEPS

M. Mambrini, R. Orus & DP, Phys. Rev. B 94, 205124 (2016)



* virtual space

* Irreps of point group
(C4v for square lattice)

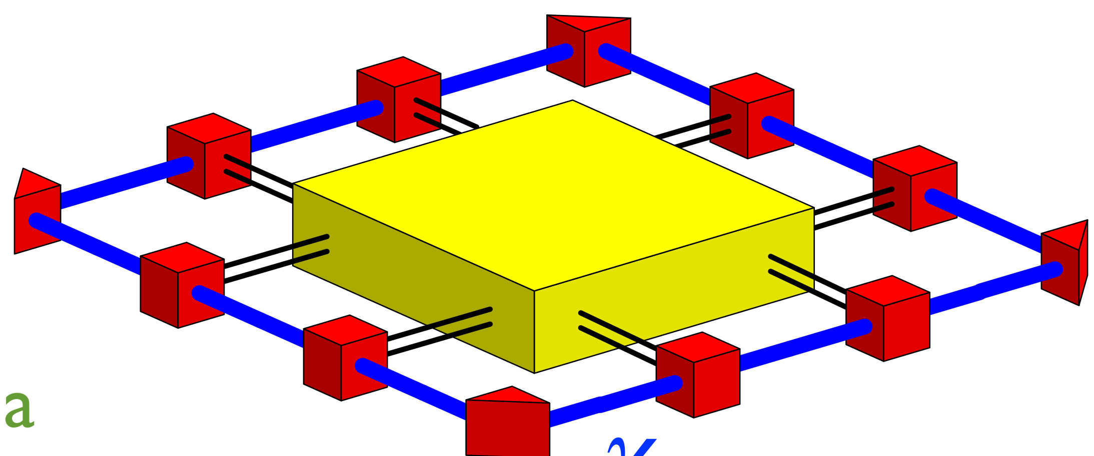
Chiral PEPS ansatz: $$A = A_R + iA_I$$

$$A_R = \sum_\alpha \lambda_\alpha A_\alpha^{(A_1)} \quad A_I = \sum_\beta \gamma_\beta A_\beta^{(A_2)}$$

iPEPS method

$$\langle \Psi_{\rm PEPS} | H_{\Box} | \Psi_{\rm PEPS} \rangle$$

$$||$$



- Environment constructed by renormalization of the corner transfer matrix (CTM)

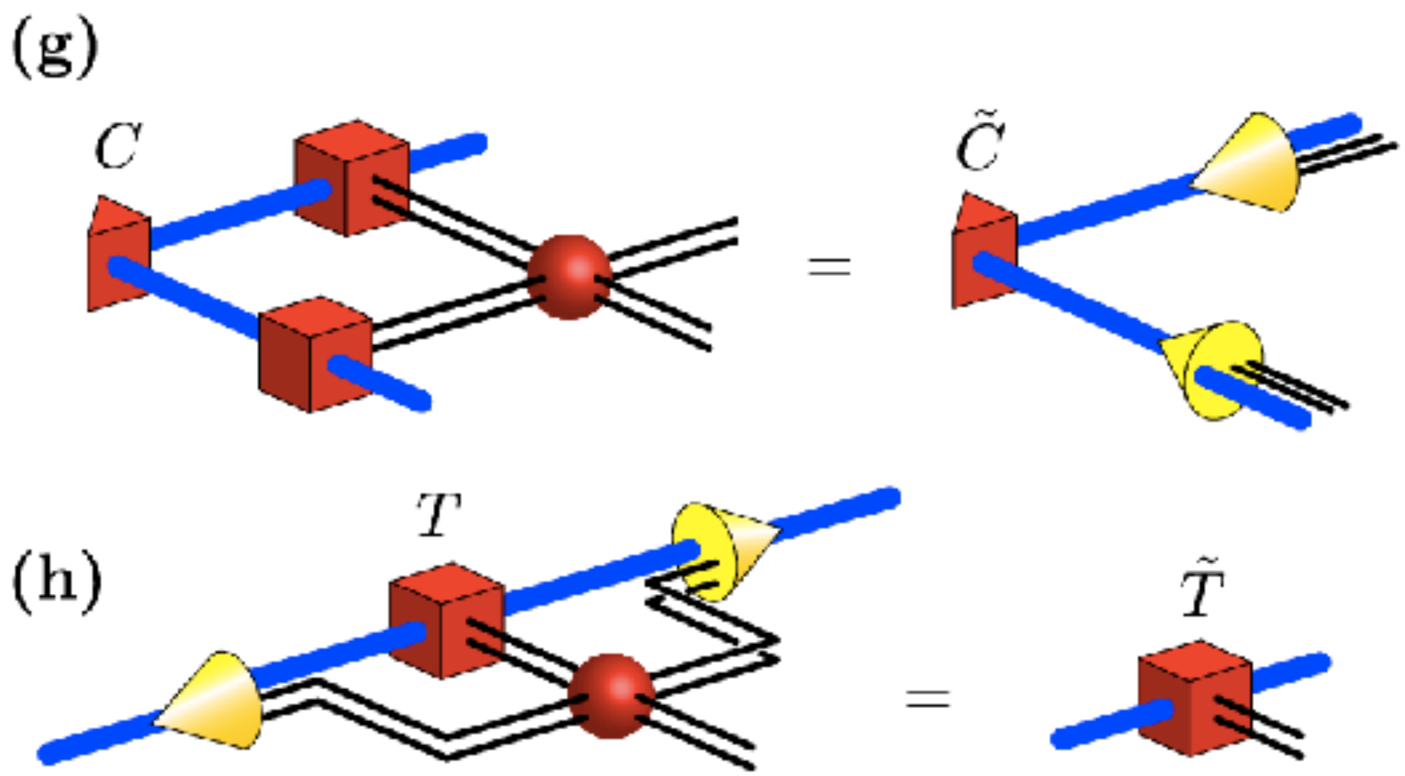
T. Nishino & K. Okunichi, J. Phys. Soc. J. **65**, 891 (1996)
R. Orus & G. Vidal, Phys. Rev. B **80**, 094403 (2009)

- Variational optimisation scheme based on a conjugate gradient method

used for non-chiral AFM: L. Vanderstraeten, J. Haegeman, P. Corboz, F. Verstraete, Phys. Rev. B **94**, 155123 (2016)
DP & M. Mambrini, Phys. Rev. B **96**, 014414 (2017)

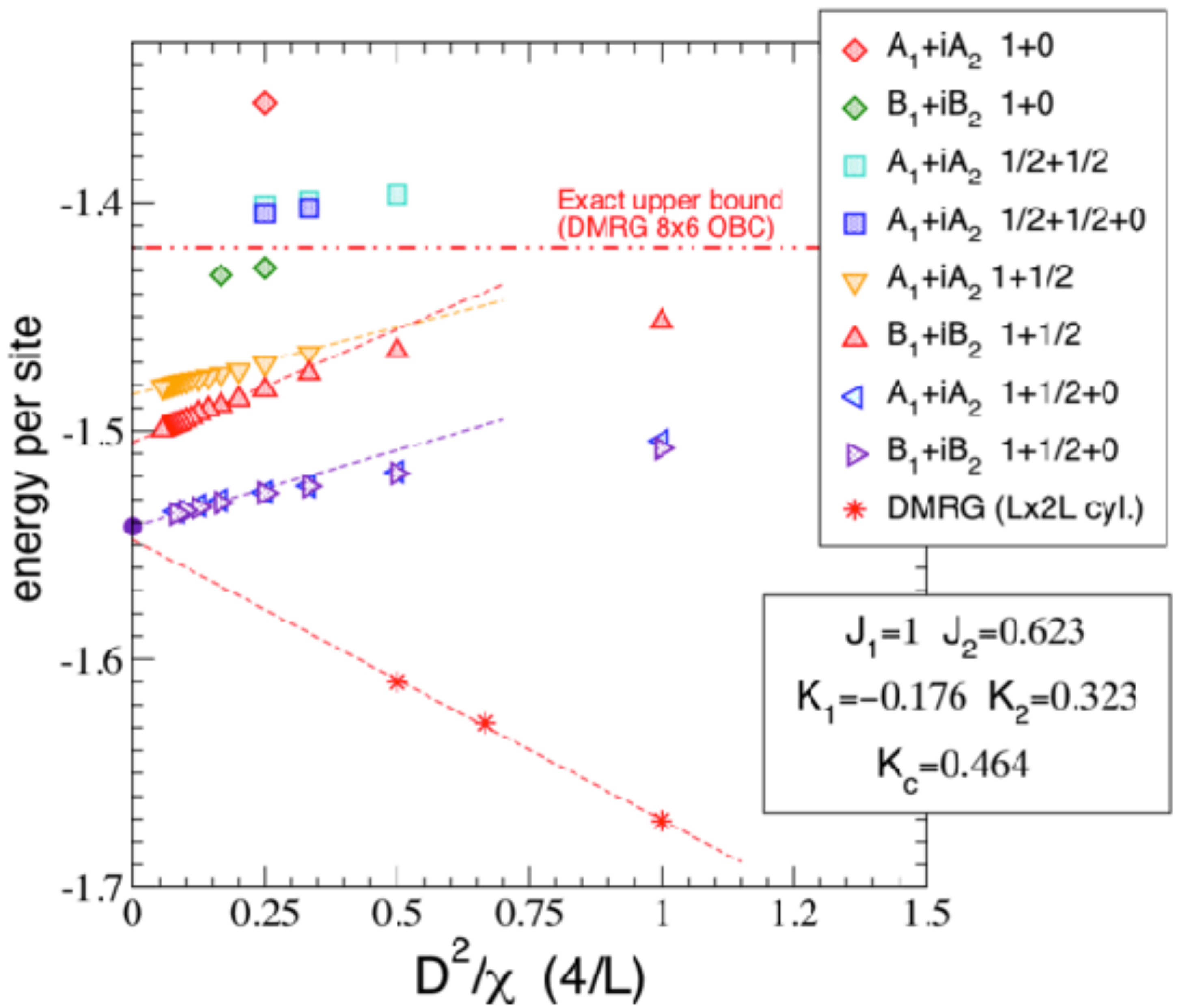
used for spin-1/2 chiral AFM: DP, Phys. Rev. B **96**, 121118 (2017)

CTM Renormalization Group algorithm



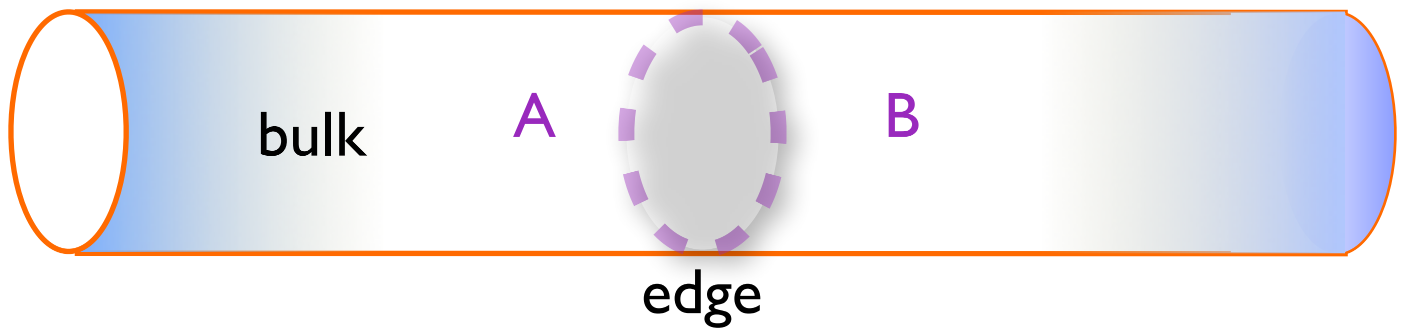
- Specific features for symmetric PEPS providing better stability :
    - The CTM is Hermitian -> SVD replaced by ED (better stability)
    - All C (corner) and T (edges) matrices are the same
    - SU(2) symmetry preserved in the CTM fixed point.

Variational energy



χ = environment dimension

Entanglement spectrum



$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

Li & Haldane conjecture : $\rho_A = \exp(-H_b)$

One-to-one correspondence between ES and edge mode spectrum

Use PEPS bulk-edge correspondence

$$\rho_A = U\sigma_b^2 U^\dagger$$

$$\sigma_b^2 = \exp(-H_b^{\mathrm{edge}})$$

Compare spectrum of H_b^{edge}

to predictions of chiral CFT's

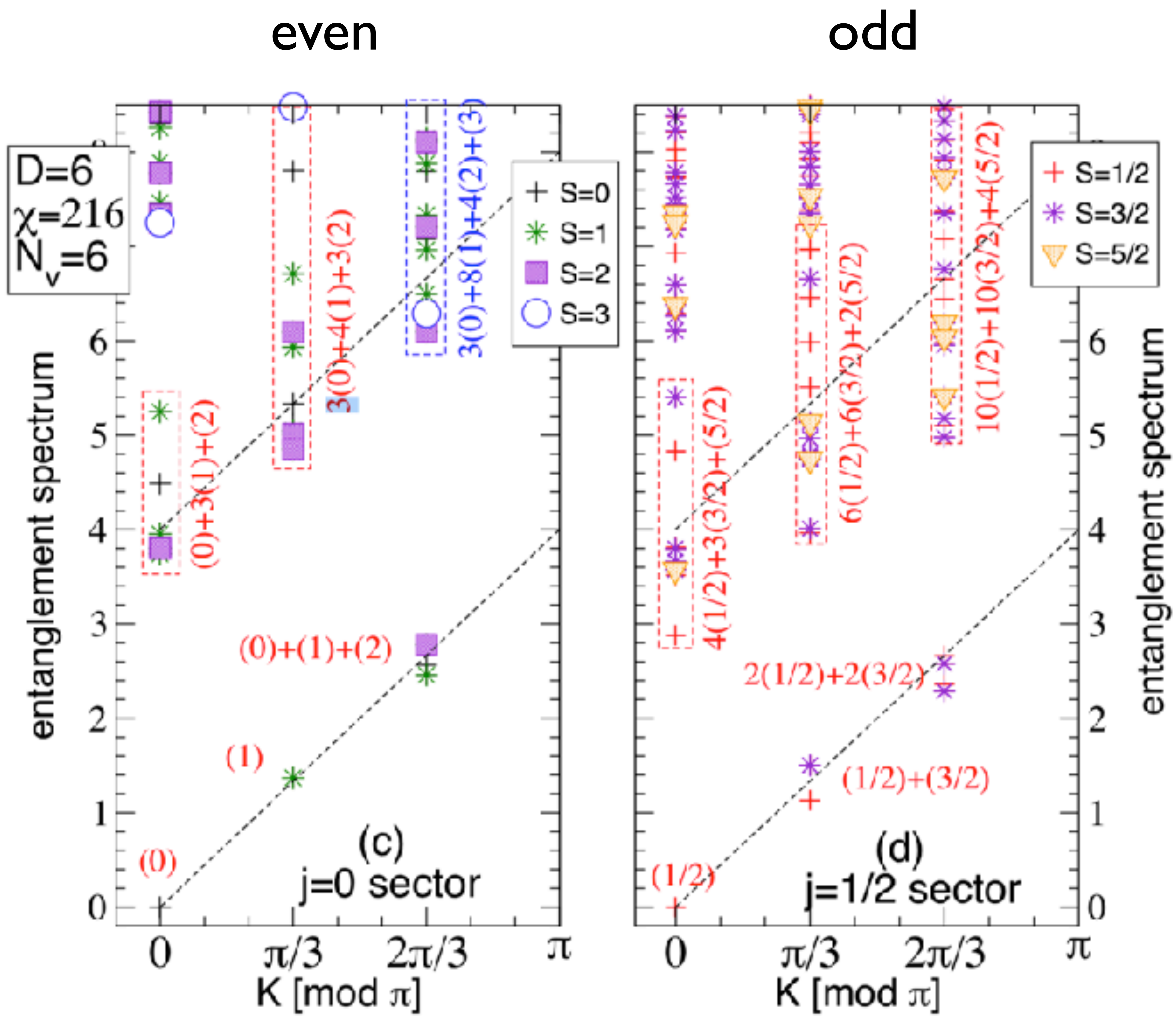
Conformal tower of $SU(2)_2$ CFT

Central charge c=3/2
Majorana (Ising) + boson

Conformal tower:

$n\backslash j$	0	$\frac{1}{2}$	1
0	(0)	$(\frac{1}{2})$	(1)
1	(1)	$(\frac{1}{2})+(\frac{3}{2})$	$(0)+(1)$
2	$(0)+(1)+(2)$	$2(\frac{1}{2})+2(\frac{3}{2})$	$(0)+2(1)+(2)$
3	$(0)+3(1)+(2)$	$4(\frac{1}{2})+3(\frac{3}{2})+(\frac{5}{2})$	$2(0)+3(1)+2(2)$
4	$3(0)+4(1)+3(2)$	$6(\frac{1}{2})+6(\frac{3}{2})+2(\frac{5}{2})$	-
5	$3(0)+8(1)+4(2)+(3)$	$10(\frac{1}{2})+10(\frac{3}{2})+4(\frac{5}{2})$	-

Entanglement spectrum



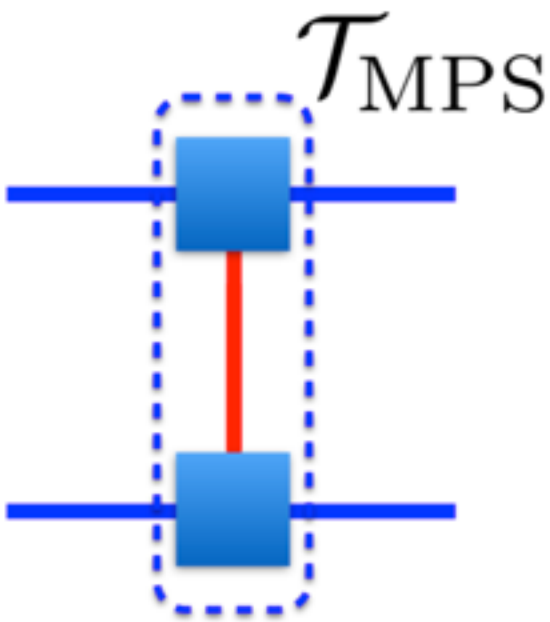
"tower of states" of chiral $SU(2)_2$ CFT

non-Abelian Moore-Read Chiral SL

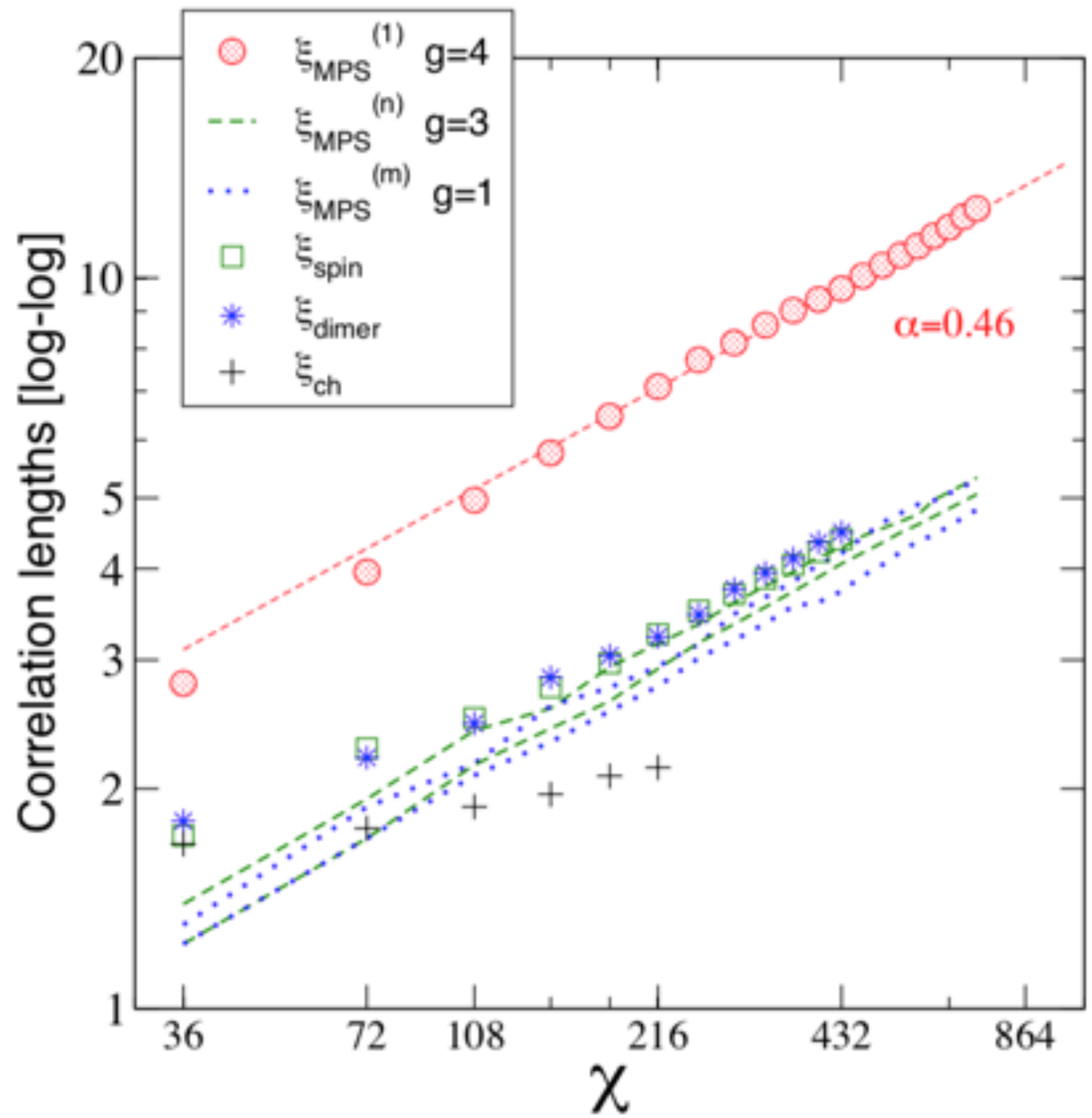
Correlation lengths

(S=1)

Transfer matrix

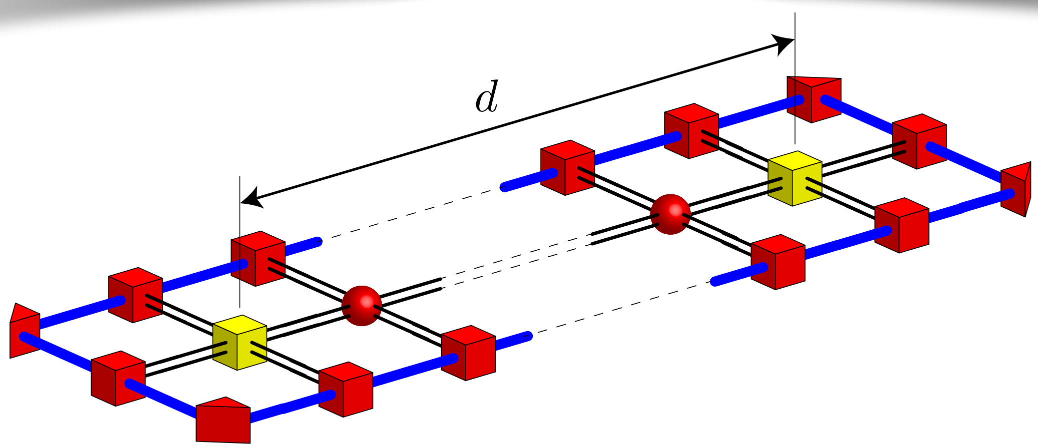


$$\xi_n = -1/\ln(\lambda_n/\lambda_{\max})$$

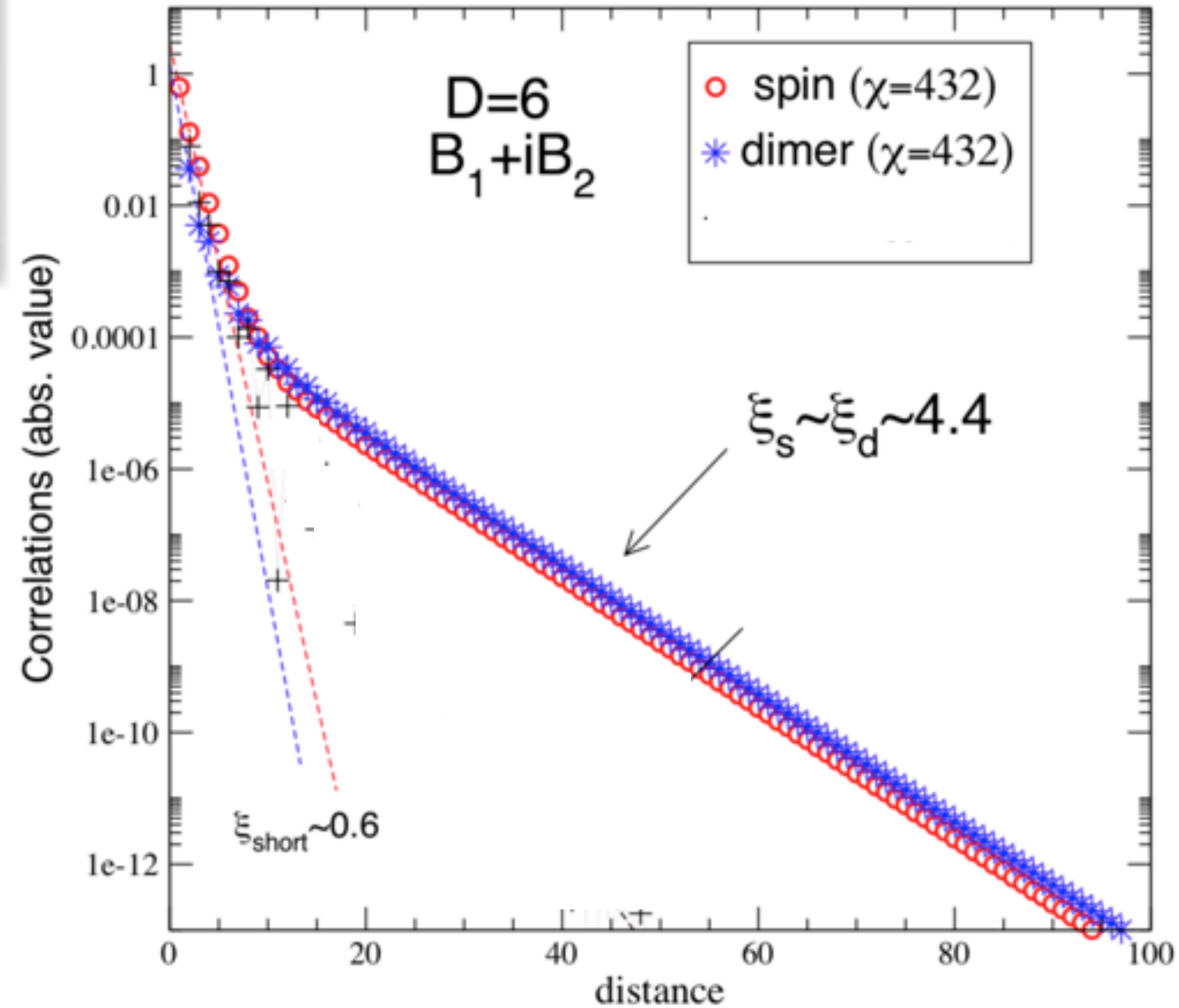


Diverging correlation length !

Spin-spin and dimer-dimer correlations



$$C_S(d) = \sum_{\xi_{\min} \leq \xi \leq \xi_{\max}} w(\xi) \exp\left(-d/\xi\right)$$



short-range

"gossamer" tail

- For $\chi \rightarrow \infty$ the distribution of ξ becomes continuous
- $w(\xi)$ decays rapidly with ξ
- The weight of $w(\xi)$ decreases rapidly with D :"gossamer tail"

Summary

- Chiral PEPS offer *conceptual* / quantitative understanding of quantum antiferromagnets with gapped topological chiral GS, spin analogs of Abelian and non-Abelian FQHS

- "Gossamer LR tail" in correlations expected from no-go theorem but not an obstruction to describe gapped CSL with PEPS

- Virtual degrees of freedom play «physical role» at boundary : chiral edge states

Outlook

- Work-out full phase diagram / search for spontaneous TR symmetry breaking in TR-invariant models

- Larger spin-S : Fibonacci non-Abelian CSL ?

- Other symmetries like SU(N)

PEPS spin-1/2 chiral spin liquids

DP, J. Ignacio Cirac and Norbert Schuch,
Phys. Rev. B 91, 224431 (2015)

DP, Norbert Schuch and Ian Affleck,
Phys. Rev. B 93, 174414 (2016)
(Editor's suggestion)

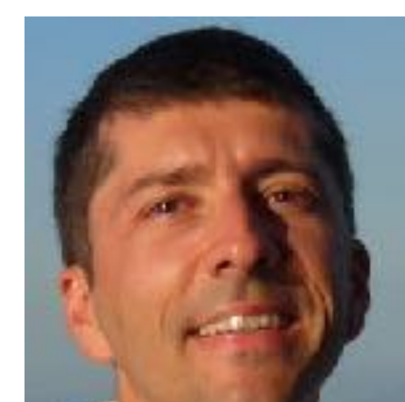
Classification of SU(2) spin liquids

Matthieu Mambrini, Roman Orus & DP,
Phys. Rev. B **94**, 205124 (2016)

Chiral SL in frustrated models with iPEPS

DP, Phys. Rev. B 96, 121118 (2017)

Ji-Yao Chen, L. Vanderstraeten, S. Capponi & DP, arXiv soon !



Topological Phases in Condensed Matter and Ultracold Atoms Systems

October 02 - 12, 2018

Web site

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The 2016 Nobel Prize in Physics was awarded to pioneering work opening the field of topological phases of matter. This field has matured later on in the study of the fractional quantum Hall effect, which continues to deliver exciting physics, in the form of non-abelian excitations and the observation of neutral edge modes. Inspired by the quantum Hall effect, the study of non-abelian particles has branched into different topics, such as the study of topological phases emerging in (spin) lattice models and recently topological insulators and superconductors. During recent years, the field of topological phases has been boosted by the possible application to quantum computing. Implementing topological quantum computation in realistic experimental systems is one of the holy grails of the community.

Main topics will include

- Topological phases & Quantum Simulations
- Strongly Correlated Electrons & Cold Atoms
- Machine Learning & Quantum Computation
- Non-equilibrium Physics

Eminent scientists will animate the workshop. These include (tentative):

Maissam Barkeshli (Maryland Univ.), Federico Becca (SISSA), Emil Bergholtz (Univ. Stockholm), Philippe Corboz (Amsterdam Univ.), Benoit Doucot (LPTHE Paris), Matthew Fisher (UCSB), Tarun Grover (KITP UCSB), Markus Heyl (MPI-PKS Dresden), Johannes Knolle (Cambridge), Dima Kovrizhin (Oxford), Andreas Laeuchli (Univ. Innsbruck), David Luitz (MPI-PKS Dresden), Roger Melko (Univ. Waterloo), Frédéric Mila (EPFL, Lausanne), Adam Nahum (Oxford), Titus Neupert (Univ. of Zurich), Masaki Oshikawa (ISSP, Tokyo), Zlatko Papic (Dresden), Sid Parameswaran (Oxford), K. Penc (Univ. of Budapest), Norbert Schuch (MPI-Garching), Ali Yazdani (Princeton).

Organization Committee and Scientific Committee

Roderich Moessner (Max Plank Inst. Dresden DE), Didier Poilblanc (IRSAMC Toulouse, FR), Nicolas Regnault (LPA ENS Paris, FR)

Application and registration

https://www.azur-colloque.fr/DR14/inscription/preinscription/108
Contact and organization : Malika Bentour, topo2018@irsamc.ups-tlse.fr
Deadline Application : 31/08/2018
Registration Fees (includes lodging, lunches, breakfasts and coffee breaks) : 750 euros
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