07.11.2007

$$\int_{\partial V} \vec{n} \cdot \left[\vec{F} \times (\vec{\nabla} \times \vec{G}) - \vec{G} \times (\vec{\nabla} \times \vec{F}) \right] dA$$

in ein Volumenintegral umformen.
2'te Green Identität

$$= \int_V \vec{\nabla} \cdot \left[\vec{F} \times (\vec{\nabla} \times \vec{G}) - \vec{G} \times (\vec{\nabla} \times \vec{F}) \right] dA$$

$\vec{\nabla} \times \vec{G} = \vec{G_1}$ $\quad$ $\vec{\nabla} \times \vec{F} = \vec{F_1}$

$$= \int_V \vec{\nabla} \cdot \left[\vec{F} \times \vec{G_1} \right] - \vec{\nabla} \cdot \left[\vec{G} \times \vec{F_1} \right] dA = \int_V -\vec{F} \cdot (\vec{\nabla} \times \vec{G_1}) + \underbrace{\vec{G_1} \cdot (\vec{\nabla} \times \vec{F})} + \vec{G} \cdot (\vec{\nabla} \times \vec{F_1}) - \underbrace{\vec{F_1} \cdot (\vec{\nabla} \times \vec{G})} dA$$

$$= \int_V -\vec{F} \cdot (\vec{\nabla} \times \vec{G_1}) + \vec{G} \cdot (\vec{\nabla} \times \vec{F_1}) \, dA = \int_V -\vec{F} \cdot (\vec{\nabla} \times (\vec{\nabla} \times \vec{G})) + \vec{G} \cdot (\vec{\nabla} \times (\vec{\nabla} \times \vec{F})) \, dA$$

$$= \int_V -\vec{F} \cdot \left(\vec{\nabla}(\vec{\nabla} \cdot \vec{G}) - \nabla^2 \vec{G} \right) + \vec{G} \cdot \left(\vec{\nabla}(\vec{\nabla} \cdot \vec{F}) - \nabla^2 \vec{F} \right) dA$$

$\vec{\nabla} \cdot \vec{F} = h$ $\quad$ $\vec{\nabla} \cdot \vec{G} = k$

$$-\vec{F} \cdot \vec{\nabla} k + \vec{G} \cdot \vec{\nabla} h$$

$$\vec{\nabla} \cdot (f \vec{g}) = f \, \vec{\nabla} \cdot \vec{g} + \vec{g} \cdot \vec{\nabla} f$$

$$\vec{\nabla} \cdot (g \vec{f}) = g \, \vec{\nabla} \cdot \vec{f} + \vec{f} \cdot \vec{\nabla} g$$

$$\vec{\nabla} \cdot (f \vec{g}) - \vec{\nabla} \cdot (g \vec{f}) = f \, \vec{\nabla} \cdot \vec{g} + \vec{g} \cdot \vec{\nabla} f - g \, \vec{\nabla} \cdot \vec{f} - \vec{f} \cdot \vec{\nabla} g$$

$$\vec{\nabla} \cdot (h \vec{G} - k \vec{F}) = h \, \vec{\nabla} \cdot \vec{G} + \vec{G} \cdot \vec{\nabla} h - k \, \vec{\nabla} \cdot \vec{F} - \vec{F} \cdot \vec{\nabla} k$$

$$= (\vec{\nabla} \cdot \vec{F})(\vec{\nabla} \cdot \vec{G}) + \vec{G} \cdot \vec{\nabla} (\vec{\nabla} \cdot \vec{F}) - (\vec{\nabla} \cdot \vec{G})(\vec{\nabla} \cdot \vec{F}) - \vec{F} \cdot \vec{\nabla} (\vec{\nabla} \cdot \vec{G})$$

$$= \vec{G} \cdot \vec{\nabla} (\vec{\nabla} \cdot \vec{F}) - \vec{F} \cdot \vec{\nabla} (\vec{\nabla} \cdot \vec{G})$$

$$= \int_V \vec{\nabla} \cdot \left[(\vec{\nabla} \cdot \vec{F}) \vec{G} - (\vec{\nabla} \cdot \vec{G}) \vec{F} \right] + \vec{F} \nabla^2 \vec{G} - \vec{G} \nabla^2 \vec{F} \, dA$$

26.09.2007

$$\vec{f} = f(\varrho)\vec{e_\varrho}$$

$\nabla^2 \vec{f} = ?$ Hinweis $\vec{\nabla} \times (\vec{\nabla} \times \vec{f}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{f}) - \nabla^2 \vec{f}$

$$\nabla^2 \vec{f} = \vec{\nabla}(\vec{\nabla} \cdot \vec{f}) - \vec{\nabla} \times (\vec{\nabla} \times \vec{f}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{f})$$

$\vec{\nabla} \times \vec{f} = \vec{0}$ $\quad \vec{\nabla} \cdot \vec{f} = \frac{1}{\varrho} \partial_\varrho (\varrho f_\varrho) = \frac{1}{\varrho}\left[f(\varrho) + f'(\varrho)\varrho \right] = \frac{f(\varrho)}{\varrho} + f'(\varrho)$

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{f}) = \vec{e_\varrho}\, \partial_\varrho \left(\frac{f(\varrho)}{\varrho} + f'(\varrho) \right) = \vec{e_\varrho} \left(\frac{f'(\varrho)\varrho - f(\varrho)}{\varrho^2} + f''(\varrho) \right)$$

$$\nabla^2 \vec{f} = \vec{e_\varrho} \left(\partial_\varrho^2 f(\varrho) + \frac{\partial_\varrho f(\varrho)}{\varrho} - \frac{f(\varrho)}{\varrho^2} \right)$$

Linear - homogen - anisotropes Medium (12.1.12 Aufgabensammlung Skriptum)

$$\underset{\sim}{\varepsilon} = \left(2.6\, \vec{e_x} \otimes \vec{e_x} + 1.2\, \vec{e_y} \otimes \vec{e_y} + 1.7\, \vec{e_z} \otimes \vec{e_z} \right) \varepsilon_0$$

$\measuredangle$ zwischen $\vec{D}$ und $\vec{E}$

$$\vec{E} = E_0 \, \frac{\vec{e_x} - 4\vec{e_y} + 2\vec{e_z}}{\sqrt{21}}$$

$$\vec{D} = \underset{\sim}{\varepsilon}\, \vec{E} = \frac{\varepsilon_0 E_0}{\sqrt{21}} \left(2.6\, \vec{e_x} - 4.8\, \vec{e_y} + 3.4\, \vec{e_z} \right)$$

$$\vec{D} \cdot \vec{E} = |\vec{D}| \cdot |\vec{E}| \cdot \cos(\alpha)$$

$$\cos(\alpha) = \frac{\vec{D} \cdot \vec{E}}{|\vec{D}| \cdot |\vec{E}|} = \qquad \alpha = 13.961^\circ$$

Ein stationäres Magnetfeld im leeren Raum 9.11.2005

ist in Kreiszylinderkoordinaten durch das Vektorpotential

$\vec{A} = K \cdot \ln(\varrho / a)\, \vec{e_z}$, K und a const. gegeben.

Bestimmen Sie für dieses Feld ein Magnetisches Skalarpotential

$$\vec{B} = \vec{\nabla} \times \vec{A} = -\vec{e_\alpha}\, \partial_\varrho A_z = -\vec{e_\alpha} \cdot \left(K \frac{\frac{1}{a}}{\frac{\varrho}{a}} \right) = \frac{-K}{\varrho} \vec{e_\alpha}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{-K}{\varrho \mu_0} \vec{e_\alpha}$$

$$\vec{H} = -\vec{\nabla} \varphi_m \qquad \vec{\nabla} \varphi_m = \frac{K}{\varrho \mu_0} \vec{e_\alpha}$$

$$= \frac{1}{\varrho} \partial_\alpha \varphi_m \vec{e_\alpha}$$

$$\frac{1}{\cancel{\varrho}} \partial_\alpha \varphi_m = \frac{K}{\cancel{\varrho} \mu_0}$$

$$\partial_\alpha \varphi_m = \frac{K}{\mu_0}$$

$$\varphi_m = \frac{K}{\mu_0} \alpha + const$$

7.11.2007

(i) $\vec{F} = \vec{p} \cdot \vec{\nabla} \vec{E}$ und $\vec{T} = \vec{p} \times \vec{E}$

$$\vec{p} = p\,\vec{e_r} \qquad \vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \vec{e_r}$$

$$\vec{\nabla} \otimes \vec{E} = \left(\vec{e_r} \partial_r + \vec{e_\theta} \frac{\partial_\theta}{r} + \vec{e_\alpha} \frac{\partial_\alpha}{r \sin\theta} \right) \otimes \left(\frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2} \vec{e_r} \right)$$

$$\vec{\nabla} \otimes \vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{(-2)}{r^3} \vec{e_r} \otimes \vec{e_r}$$

$$\vec{p} \cdot (\vec{\nabla} \otimes \vec{E}) = \frac{-pQ}{2\pi\varepsilon_0 r^3} \vec{e_r} \qquad \vec{T} = \vec{p} \times \vec{E} = \vec{0} \quad (\vec{e_r} \times \vec{e_r} = 0)$$

(ii)

$$\vec{F} = \frac{\cancel{\varepsilon_0} \alpha Q}{4\pi\varepsilon_0 r^2} \vec{e_r} \cdot (\vec{e_r} \otimes \vec{e_r}) \frac{-2Q}{4\pi\varepsilon_0 r^3} = \frac{-2Q^2\alpha}{16\pi^2\varepsilon_0 r^5} \vec{e_r}$$

$$= \frac{-Q^2\alpha}{8\pi^2\varepsilon_0 r^5} \vec{e_r}$$

$$T = \vec{p} \times \vec{E} = \vec{0} \qquad \{\vec{E} \times \vec{E}\} = 0$$

$$\vec{\nabla}\cdot\vec{D} = \rho = \frac{1}{r^2}\partial_r(r^2 D_r)$$

$$\vec{D} = D_r\vec{e_r}$$

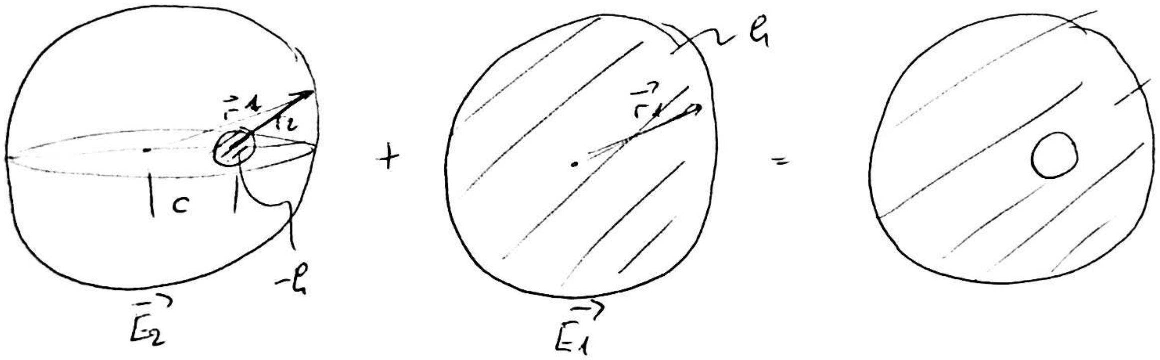
$$\rho r^2 = \partial_r(r^2 D_r)$$

$$r^2 D_r = \rho\frac{r^3}{3}$$

$$D_r = \frac{\rho r}{3}$$

$$\vec{D} = \frac{\rho r}{3}\vec{e_r}$$

$$\vec{D} = \varepsilon_0\vec{E} \qquad \underline{\vec{E_1} = \frac{\vec{D}}{\varepsilon_0} = \frac{\rho r}{3\varepsilon_0}\vec{e_r}} = \frac{\rho}{3\varepsilon_0}\vec{r_1}$$



$$\vec{r_1} = x\vec{e_x} + y\vec{e_y} + z\vec{e_z} \qquad \vec{E_2} = \frac{-\rho}{3\varepsilon_0}\vec{r_2}$$

$$\vec{r_2} = (x-c)\vec{e_x} + y\vec{e_y} + z\vec{e_z}$$

$$\vec{E} = \vec{E_1} + \vec{E_2} = \frac{\rho}{3\varepsilon_0}\left(\vec{r_1} - \vec{r_2}\right) = \frac{\rho}{3\varepsilon_0}\Big(\underbrace{x\vec{e_x} + y\vec{e_y} + z\vec{e_z} - (x-c)\vec{e_x} - y\vec{e_y} - z\vec{e_z}}_{c\vec{e_x}}\Big)$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}c\vec{e_x}$$

9.11.2005

$$B(x,t) = \sum_{n=0}^{\infty} f_n(x)\, g_n(t) = \underbrace{f_0(x)\, g_0(t)}_{0} + \sum_{n=1}^{\infty} f_n(x)\, g_n(t) =$$

$g_n(0) = 1$

$$B(0,t) = \sum_{n=1}^{\infty} f_n(x)\, g_n(0) = \sum_{n=1}^{\infty} \hat{B}_n \sin\left(\frac{n\pi x}{a}\right)$$

Ansatz für $g_n(t)$

$$\boxed{f_n(x) = \hat{B}_n \sin\left(\frac{n\pi x}{a}\right)}$$

$$g_n(t) = A e^{\lambda t} \quad A=1$$

$$= e^{\lambda t}$$

$$\partial_x^2 B = \sum - \hat{B}_n \sin\left(\frac{n\pi x}{a}\right)\left(\frac{n\pi}{a}\right)^2 g_n(t)$$

$$\partial_t B = \cancel{\ldots}\, \partial_t \sum f_n(x)\, g_n(t) = \sum f_n(x)\, \lambda e^{\lambda t}$$

$$\sum_{n=1}^{\infty} - \hat{B}_n \sin\left(\frac{n\pi x}{a}\right)\left(\frac{n\pi}{a}\right)^2 e^{\lambda t} = \mu\gamma \sum_{n=1}^{\infty} \hat{B}_n \sin\left(\frac{n\pi x}{a}\right) \lambda e^{\lambda t}$$

$$-\underbrace{\hat{B}_n}\, \underbrace{\sin\left(\frac{n\pi x}{a}\right)}\left(\frac{n\pi}{a}\right)^2 \underbrace{e^{\lambda t}} = \mu\gamma\, \underbrace{\hat{B}_n}\, \underbrace{\sin\left(\frac{n\pi x}{a}\right)} \lambda \underbrace{e^{\lambda t}}$$

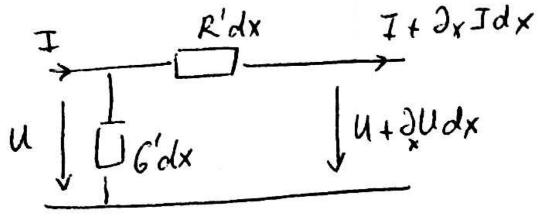
$$-\left(\frac{n\pi}{a}\right)^2 = \mu\gamma\lambda \qquad \lambda = \frac{-n^2\pi^2}{a^2\mu\gamma}$$

$$\boxed{g_n(t) = e^{\frac{-n^2\pi^2 t}{a^2\mu\gamma}}}$$

Gleichstromkabel

A 5.3.11

Ersatzschaltung



$$\cancel{I} = \cancel{I} + \partial_x I \, \cancel{dx} + G' \cancel{dx}\, U$$

$$\partial_x I + G' U = 0$$

$$\cancel{U} = \cancel{U} + \partial_x U \, \cancel{dx} + I R' \cancel{dx}$$

$$\partial_x U + R' I = 0$$

$$\partial_x^2 U + R' \partial_x I = 0$$

$$\partial_x I = \frac{-\partial_x^2 U}{R'}$$

$$\frac{-\partial_x^2 U}{R'} + G' U = 0$$

$$-\partial_x^2 U + G' R' U = 0$$

$$\partial_x^2 U - R' G' U = 0$$

$$\partial_x^2 U = R' G' U = \alpha^2 U \quad \text{mit} \quad \alpha = \sqrt{R' G'}$$

R.B

$x = 0 \qquad U = U_0$

$x = l \qquad U(l) = R_A I(l)$

$$U = \frac{-R_A}{R'} \frac{dU}{dx}$$

Lösungs Ansatz $\quad U(x) = C_1 e^{\alpha x} + C_2 e^{-\alpha x}$

$$U(0) = C_1 + C_2 = U_0 \qquad U(l) = C_1 e^{\alpha l} + C_2 e^{-\alpha l} =$$

$$\frac{dU}{dx} = \alpha C_1 e^{\alpha x} - \alpha C_2 e^{-\alpha x}$$

$$\left(C_1 e^{\alpha l} + C_2 e^{-\alpha l}\right) = \frac{-R_A}{R'} \left(C_1 e^{\alpha l} - C_2 e^{-\alpha l} \right) \alpha$$

$$= -\beta \left(C_1 e^{\alpha l} - C_2 e^{-\alpha l} \right)$$

$$C_1 e^{\alpha l} + C_2 e^{-\alpha l} = -\beta C_1 e^{\alpha l} + \beta C_2 e^{-\alpha l}$$

$$C_1 e^{\alpha l} (1 + \beta) + C_2 e^{-\alpha l} (1 - \beta) = 0$$

$$C_1 = U_0 - C_2 \qquad (U_0 - C_2) e^{\alpha l} (1 + \beta) + C_2 e^{-\alpha l} (1 - \beta) = 0$$

$$U_0 e^{\alpha l}(1+\beta) - C_2 e^{\alpha l}(1+\beta) + C_2 e^{-\alpha l}(1-\beta) = 0$$

$$U_0 e^{\alpha l}(1+\beta) = C_2 \left(e^{\alpha l}(1+\beta) - e^{-\alpha l}(1-\beta) \right)$$

$$C_2 = \frac{U_0 e^{\alpha l}(1+\beta)}{e^{\alpha l}(1+\beta) - e^{-\alpha l}(1-\beta)}$$

$$C_1 = U_0 - C_2 = U_0 - \frac{U_0 e^{\alpha l}(1+\beta)}{e^{\alpha l}(1+\beta) - e^{-\alpha l}(1-\beta)}$$

$$= U_0 \left[1 - \frac{e^{\alpha l}(1+\beta)}{e^{\alpha l}(1+\beta) - e^{-\alpha l}(1-\beta)} \right]$$

$$C_1 = U_0 \left[\frac{\cancel{e^{\alpha l}(1+\beta)} - e^{-\alpha l}(1-\beta) \cancel{- e^{\alpha l}(1+\beta)}}{e^{\alpha l}(1+\beta) - e^{-\alpha l}(1-\beta)} \right]$$

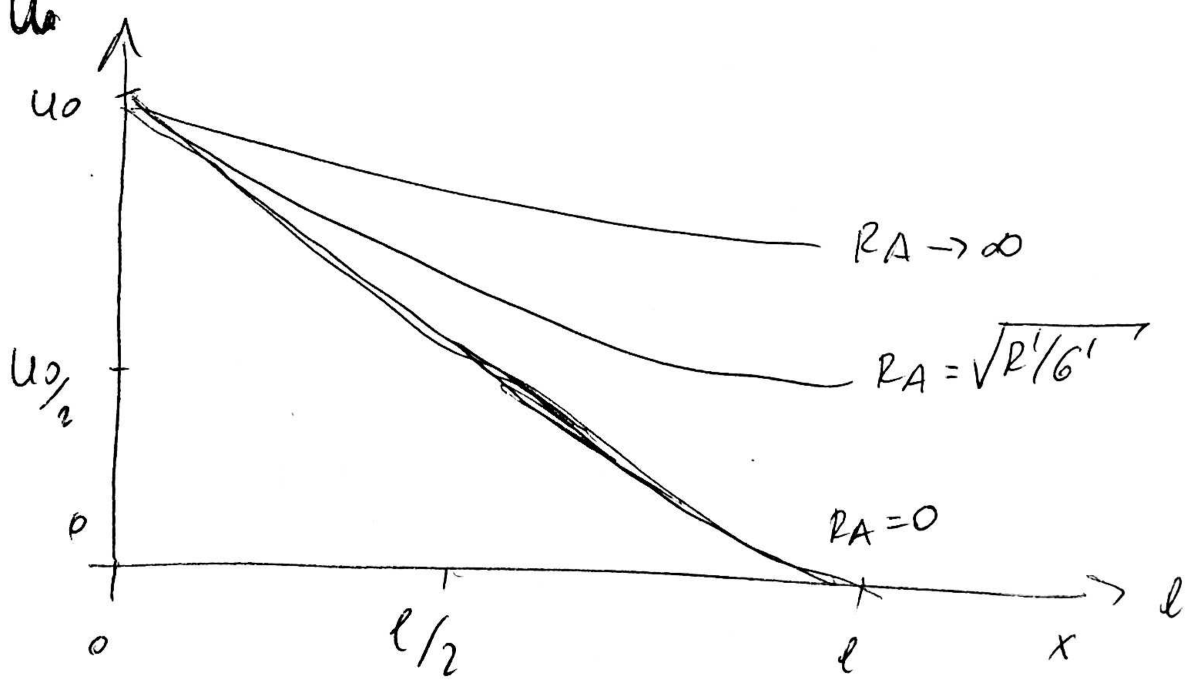
$$U(x) = C_1 e^{\alpha x} + C_2 e^{-\alpha x}$$

$$= U_0 \left[\frac{-e^{-\alpha l}(1-\beta) e^{\alpha x} + e^{\alpha l}(1+\beta) e^{-\alpha x}}{e^{\alpha l}(1+\beta) - e^{-\alpha l}(1-\beta)} \right]$$

$$= U_0 \left[\frac{(1+\beta) e^{\alpha(l-x)} + (1-\beta) e^{-\alpha(l-x)}}{e^{\alpha l}(1+\beta) - e^{-\alpha l}(1-\beta)} \right]$$

$$e^{x} = \cosh(x) + \sinh(x) \qquad e^{-x} = \cosh(x) - \sinh(x)$$

$$= U_0 \frac{\sinh[\alpha(l-x)] + \beta \cosh[\alpha(l-x)]}{\sinh(\alpha l) + \beta \cosh(\alpha l)}$$



30.10.2007

TEM Wellenimpedanz

$U = 5\,\Delta\varphi$ Potentialflächen

$\Psi' = 40\,\varepsilon_0\,\Delta\varphi$ Flussröhren

$Q = CU$ $$C' = \frac{Q'}{U} = \frac{\Psi'}{U} = \frac{40\,\varepsilon_0\,\Delta\varphi}{5\,\Delta\varphi} = 8\,\varepsilon_0$$

$$L'C' = \mu\varepsilon = \mu_0\varepsilon_0 \qquad L' = \frac{\mu_0\varepsilon_0}{C'} = \frac{\mu_0}{8}$$

$$Z_W = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{\mu_0}{64\varepsilon_0}} = 47.091\,\Omega$$

Ein Kugelschale mit radius R;

$$\sigma = \sigma_0 \cos(\theta)$$

geladen.

Ges: Elektrische Dipolmoment?