Unobserved Networks and Common Factors in Macroeconomic Fluctuations

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Motivation

- Output and inflation of different countries/sectors show strong comovement
  - Are these common shocks or shocks originated in one sector/country that spillover?
- Two approaches to address these questions:
  1. **Empirically**: Factor models/VAR to study the importance of agg. shocks vs spillovers
  2. **Structurally**: Constructing and estimating detailed macro/trade models with linkages
- Getting this answer right matters for policy.
  - Compare a downturn due to an aggregate credit crunch vs due to a few large firms collapsing
  - e.g., still debate about post-84 business cycles' properties: shocks' correlation vs network structure (Garin, Pries, Sims, 2018 vs vom Lehn and Wimberry, 2023)

This paper

We propose a semi-structural approach to jointly estimate general unobserved networks and unobserved common factor(s).

What We Do

- Propose the factor-augmented-sparse-vector-autoregression (FASVAR) to estimate the underlying network structure and unobserved common factor(s)
  - imposing sparsity conditions on the network and a sparsity pattern (‘group sparsity’)
  - and a rank restriction on the vector of common factors
- Our simulation exercises show that
  - FASVAR is superior (in and out of sample) to factor models and VAR-Lasso, especially when networks are asymmetric and when aggregate factors exist
  - Our reduced-form approach is able to recover structural networks from seminal models
- Applications
  - US sectoral output: can recover networks with a similar structure to IO matrices (star suppliers) using only sectoral output growth.
  - Commodity price comovement puzzle: different from previous findings, comovement is driven by spillovers, not global factors

Literature

- **Methodological**: Factor models (Geweke, 1977; and Sargent and Sims, 1977), Vector Autoregression (VAR) models (Sims, 1980), Factor-VAR (Bernanke, Boivin, and Eliasz, 2005), Panel Models with Interactive FE (Moon and Weidner, 2023)
  - Our FASVAR approach embeds these approaches (factors & networks). Can better decompose sources of fluctuations and estimate network
- **Semi-structural applications**: Causes of the Great Recession (Altinoglu, 2019; Li and Martin, 2019); Reduction in comovement post-1984 (Foerster, Sarte, and Watson, 2011, Garin et al., 2019, vom Lehm and Wimberry, 2022)
  - Our approach is agnostic, does not need to assume a specific network exists, and can better guide theory

A motivating example

A Well-Known Structural Model: Long and Plosser (1983)

- There is a representative firm in each of the N competitive sectors of the economy
- Firms hire labor (L_{it}) and purchase intermediates (M_{ijt-1}) to produce output $Y_{it} = F(L_{it}, M_{ijt-1}, \epsilon_t)$, where ϵ_t is a shock to sectoral productivity.
- The representative household supplies labor and consumes all the goods in the economy
- The competitive equilibrium of this economy (Cobb-Douglas and log utility) yields

$$d \ln \mathbf{Y}_t = \mathbf{A}^T d \ln \mathbf{Y}_{t-1} + \epsilon_t,$$

in which $d \ln \mathbf{Y}_t$ is the vector of sectoral sales growth, $\mathbf{A}$ is the input-output matrix, and ϵ_t is the vector of serially uncorrelated productivity shocks. aggregate shocks manifest in non-zero off-diagonal elements in the covariance $\Sigma_{\epsilon\epsilon}$

A Well-Known Structural Model: Long and Plosser (1983)

Let

$$d \ln \mathbf{Y}_t = \mathbf{A}^T d \ln \mathbf{Y}_{t-1} + \epsilon_t,$$

the underlying DGP for sectoral output growth. For **A**, we use the US input-output structure (BEA 71 sectors in 2014). We assume that

$$\epsilon_{it} = \underbrace{\Pi_t}_{\text{aggregate shock}} + \underbrace{u_{it}}_{\text{idiosyncratic shock}}$$

.

Then, simulate long series of $d \ln Y_t$.

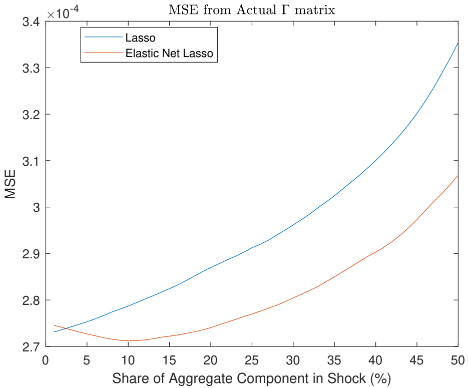
1. How well do VAR-Lasso methods estimate the underlying network **A**?
2. How well FM estimate the structure of ϵ_t: aggregate vs. idiosyncratic shocks?

Carvalho (2007)

VAR-Lasso bias

How well do VAR-Lasso methods estimate the underlying network USA IO network **A**?

Figure: MSE estimated network for different importance of the 'common factor'

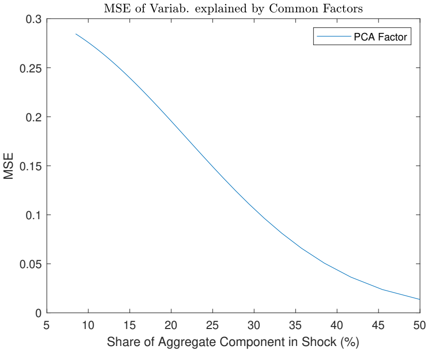


VAR-Lasso overstates linkages when common shocks are more relevant. Also, VAR-Lasso does a poor job of recovering the asymmetry in the US IO network (few star-supplier sectors).

Factor model bias

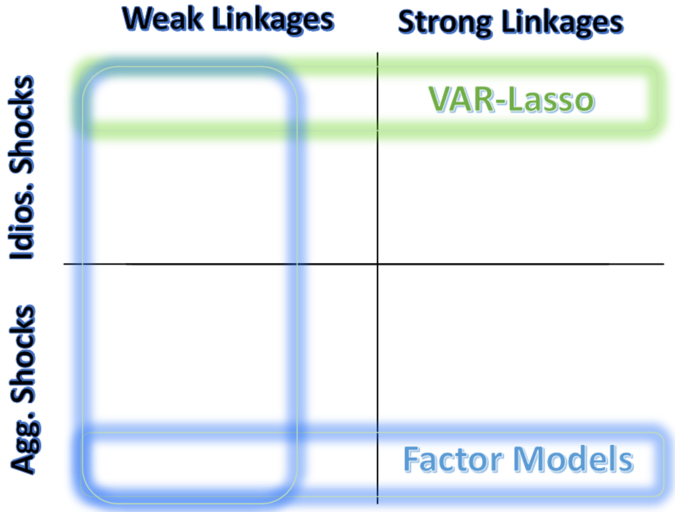
How well FM estimate the structure of ϵ_t: aggregate vs. idiosyncratic shocks?

Figure: MSE in implied common factor importance

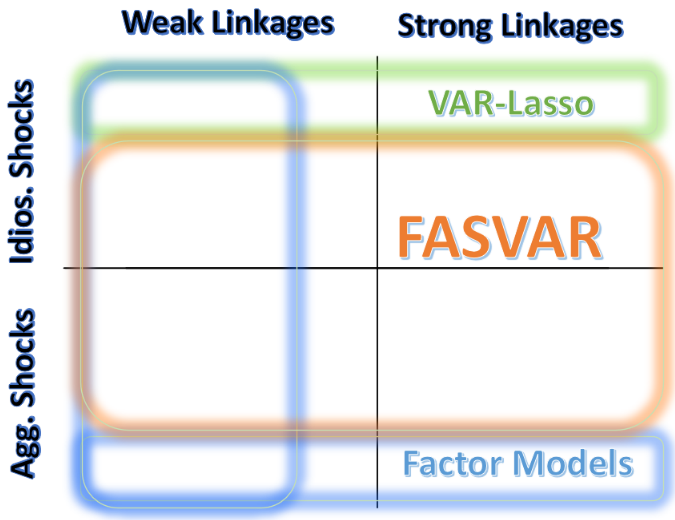


Given the US network, factor models do a poor job of decomposing the role of aggregate vs. idiosyncratic shocks, especially when the underlying shocks are mainly idiosyncratic. As we show later, the bias increases with the degree of network centrality

Summary


Weak Linkages
Strong Linkages
Idios. Shocks
Agg. Shocks
VAR-Lasso
Factor Models

Summary



Weak Linkages
Strong Linkages
Idios. Shocks
Agg. Shocks
VAR-Lasso
FASVAR
Factor Models

FASVAR

Model

Let the dynamics of unit i at time t (e.g., firm i's output growth or country i's exchange rate) be expressed as

$$y_{i,t} = \sum_s \sum_j A_{s,ij} y_{j,t-s} + \Pi_t + u_{i,t},$$

$$\Pi_{it} = \Lambda_i \mathbf{F}_t$$

- $\sum_s \sum_j A_{s,ij} y_{j,t-s}$ accounts for spillovers between *units* (unobserved network)
- $\Pi_{it} = \Lambda_i \mathbf{F}_t$ is the unobserved common factor structure
- u_{it} are unit-specific idiosyncratic *errors*, which are orthogonal to $y_{j,t-s}$ and Π_t

Estimating Unobserved Networks and Common Factors - Low Dimensional

Moon and Weidner (2023) estimator applies when $\mathbf{A}$ is known up to a fixed number of unknown constants (e.g., $\mathbf{A} = \beta\mathbf{B}$) where β captures the intensity of spillovers.

Minimize the convex objective function with respect to β and Π,

$$argmin_{\beta,\Pi}\left(\underbrace{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{i,t}-\sum_{s=1}^{P}\sum_{j=1}^{N}\beta B_{s,ij}y_{j,t-s}-\Pi_{it}\right)^2}_{OLS}+\underbrace{\lambda p(\Pi)}_{Rank}\right),$$

where $\lambda \geq 0$ is a tuning parameter selected in a data driven manner and $p(\Pi)$ is the sum of the singular values of Π. Moon and Weidner (2023) establish $\sqrt{NT}$ consistency of β.

Estimating Unobserved Networks and Common Factors - High Dimensional

If **A** is not known up to a fixed number of unknown constants we have a high dimensional problem.

Propose to minimize a convex objective function such as the following with respect to $(A_s)_{s=1,\ldots,P}$ and Π,

$$argmin_{A,\Pi}\left(\underbrace{\sum_{i=1}^{N}\sum_{t=1}^{T}\left(y_{i,t}-\sum_{s=1}^{P}\sum_{j=1}^{N}A_{s,ij}y_{j,t-s}-\Pi_{it}\right)^2}_{OLS}+\underbrace{\lambda_1\sum_{i=1}^{N}\sum_{j=1}^{N}p_1(i,j)}_{Sparsity}+\underbrace{\lambda_2 p_2(\Pi)}_{Rank}\right),$$

where $\lambda_1 \geq 0, \lambda_2 \geq 0$ are tuning parameters.

The sparsity function is $p_1(i,j)=\left(\sum_{s=1}^{P}\left(A_{s,ij}-A_{0,ij}\right)^2\right)^{1/2}$. The matrix $\mathbf{A}_0$ could be zeros or a prior network up to a fixed number of constants. Additional convex restrictions can be added to **A**.

Simulation Results

Varying the network structure

Framework

- Simulate data in order to check the in-sample and out-of-sample fit of our proposed model compared to VAR-Lasso's and factor models
- Simulated data allows us to know the "actual" parameters we are estimating
- Two strands of results:
  1. Model fit: Compare in and out of sample forecast and the accuracy of network estimation.
     a) FASVAR outperforms in and out of sample Lasso estimators. It also estimates with less error the actual network.
  2. Common factor estimation: Compare the variability explained by common factors in the simulated data with the one estimated by our method.
     b) Factor Models show an important bias under the existence of concentrated networks.

Data Generating Process (DGP)

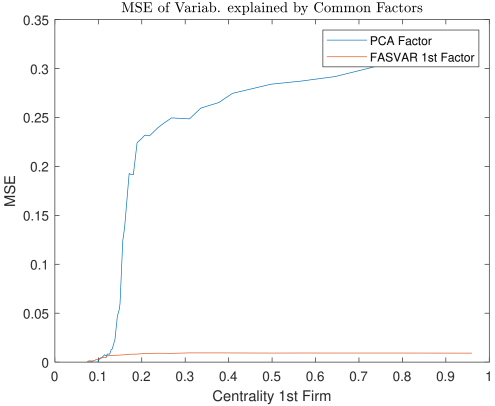
Assume the following DGP for $\mathbf{Y_t}$

$$\mathbf{Y_t} = \mathbf{Y_{t-1}} \mathbf{A} + \boldsymbol{\Pi}_t + \mathbf{U_t}.$$

- $\mathbf{A}$ is a row-normalized matrix of connections with varying degrees of sparseness
- Π, is a $T \times N$ matrix of with one common factor following a standard normal i.i.d and factor loadings equal to 1 ($rank(\Pi_0) = 1$)
- $\mathbf{U}$ follows a normal i.i.d error structure

Different degree of star supplier networks - variability explained by common factors

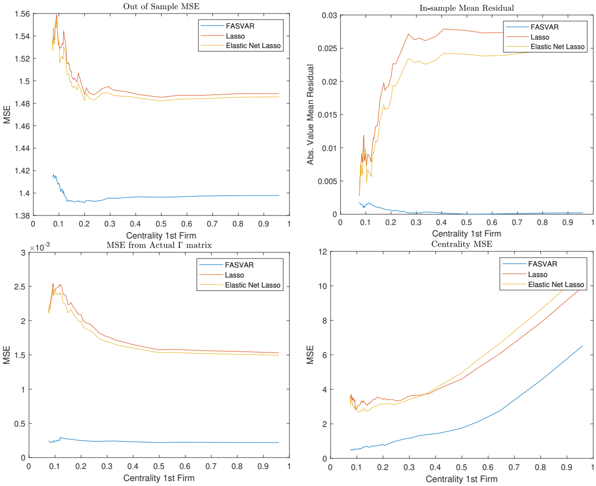
Figure: Bias in Common Factor Estimation



Theoretical example Metrics to compare

Different degree of star supplier networks - FASVAR vs VAR-Lasso Fit

Figure: Bias in VAR-Lasso



Metrics to compare

FASVAR - Long and Plosser (1983)

Given network, vary shocks' structure

Long and Plosser (1983)

Let

$$d \ln Y_t = \mathbf{A}^T d \ln Y_{t-1} + \epsilon_t,$$

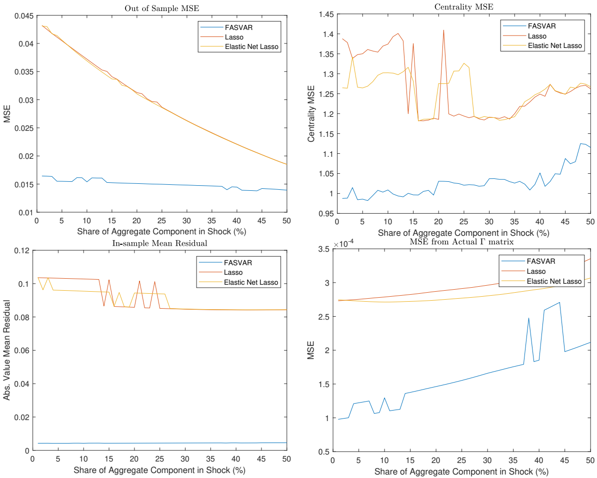
the underlying DGP for sectoral output growth. We use the US input-output structure (BEA 71 sectors in 2014) and assume a stochastic process for sectoral productivity $\epsilon_t = \Pi_t + \mathbf{U}_t$. We vary the degree of shocks' correlation (aggregate factor). Then, simulate long series of

$d \ln Y_t$.

1. How much better does FASVAR do compared to VAR-Lasso (network estimation $\mathbf{A}$)?
2. How much better does FASVAR do compared to FM (ϵ_t: aggregate vs. idiosyncratic shocks)?

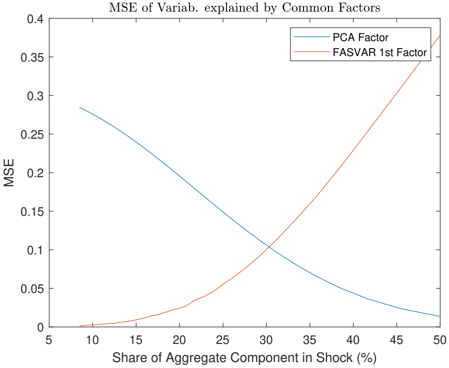
Carvalho (2007)

Different magnitude of Agg. Shocks - FASVAR vs VAR-Lasso



Better in and out of sample fit. Can recover the network entries and their asymmetric structure better than alternatives

Different magnitude of Agg. Shocks - Variability Explained by Common Factors



Given the US IO tables **A** (70 sectors), and the LP83 economy, FASVAR works better than Factor Models, only when the aggregate shock is not super large

Application to actual data

US sectoral output growth

Data

- $\mathbf{Y}_t$: Monthly industrial production (IP) indices for $N = 138$ US sectors for $T = 421$ months from January 1973 to April 2008 (from Foerster, Sarte, and Watson, 2011)

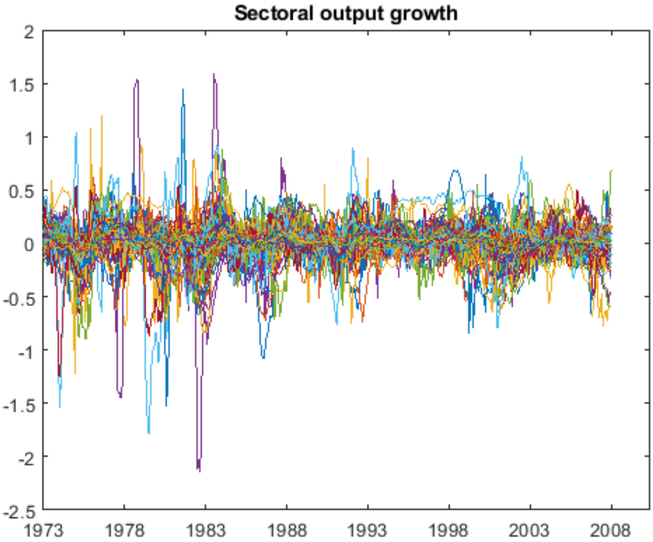
- $N \times N$ IO matrix

- Assume a Long and Plosser (1983) structure

$$d \ln Y_t = \mathbf{A}^T d \ln Y_{t-1} + \Pi_t + \mathbf{U}_t,$$

Do the data support the model? Use Moon and Weidner (2023) and our FASVAR

US sectoral output growth over time



Moon and Weidner (2023): Estimated β

- Assume $\mathbf{A} = \beta \mathbf{M}^\top$ where $\mathbf{M}$ is the observed IO matrix.

$$d \ln Y_t = \beta \mathbf{M}^\top d \ln Y_{t-1} + \Pi_t + \mathbf{U}_t,$$

- Estimates of β

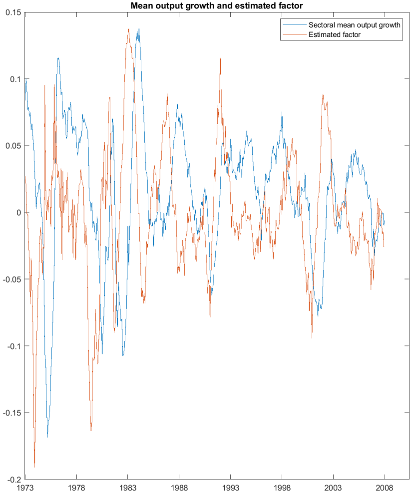
	OLS	M+W
β	1.9763	1.7304
	(0.0321)	(0.0566)
Obs	57,960	57,960

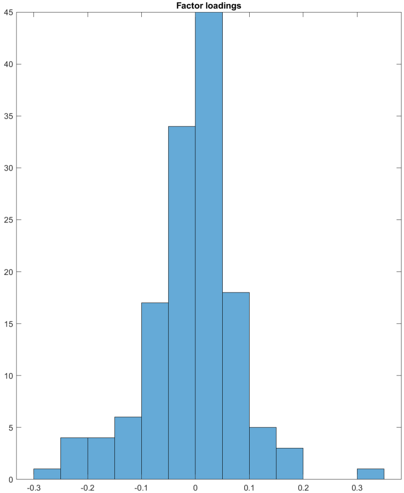
- Mean column sum of $\mathbf{M}$ is 0.3987.

- So for the mean sector, a 1pp increase in the growth of each supplying sector leads to a $0.3987 \times 1.7792 = 0.7094$pp increase in growth.

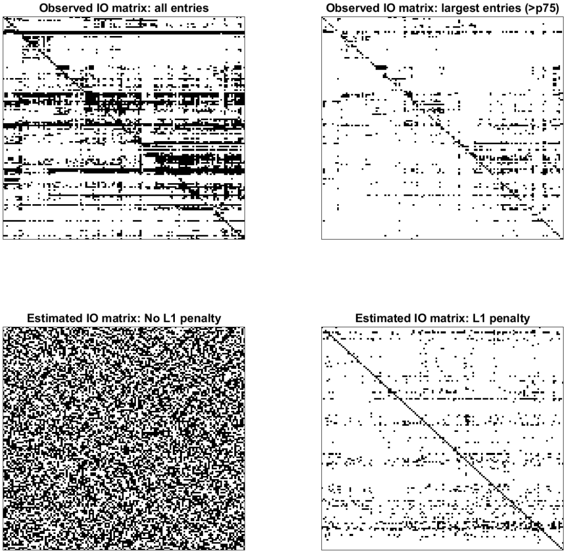
- Obtain three factors.

Moon and Weidner (2023): First estimated factor and loadings





FASVAR: Estimated **A** matrix



Work in progress

- Establish asymptotic properties of FASVAR. Expect to obtain oracle like rate of convergence similar to those established for LASSO etc.

- Consider alternative models, e.g., conditional quantiles.

- Take empirical application more seriously, including decomposition of the relative importance of IO and investment networks in determining sectoral growth.

Conclusion

- We propose an econometric approach that separately identifies general unobserved networks from unobserved common factor(s)
- Simulation exercises show that our methodology outperforms VAR-Lasso estimators and factor models: when networks are asymmetric and under the existence of aggregate factors.
- Applied to US sectoral output, our approach highlights the relevance of linkages and spillovers

Appendix

A Well-Known Structural Model: Carvalho (2007)

There is labor, capital, and intermediate inputs. Capital depreciates in one period and is produced using same sector's output. We have

$$d \ln Y_t = (I - \mathbf{A})^{-1} \alpha_d \ln Y_{t-1} + (I - \mathbf{A})^{-1} \epsilon_t,$$

where α_d is a matrix with sectoral capital shares in its diagonal and zero otherwise.

Back

Metrics to Compare Models - Fit

To check out how each model fits the data, we use these metrics to compare FASVAR with VAR-Lasso estimators.

1. MSE Out of sample: Forecast out of sample and estimate the MSE[1].
2. In sample fit: Mean in sample residuals.
3. MSE from actual $\mathbf{A}$ matrix: Error measure over the $\mathbf{A}$ matrix generated in the DGP. Specifically, the MSE between the estimated $\mathbf{A}$ and $\mathbf{A}_0$ from the DGP.
4. Centrality MSE: MSE between the centrality of the network from DGP and the estimated network by FASVAR and VAR-Lasso.

back

[1] FASVAR doesn't have out of sample data on the common factor, it has a disadvantage in this strand in comparison to VAR-Lasso's.

Metrics to Compare Models - Common Factor Variability

Goal: compare FASVAR with a PCA/Factor Model in terms of how well they approximate the actual importance of the aggregate factor in driving Y_t

1. We follow Foerster, Sarte and Waton (2011) to get the variance explained by the aggregate factor component.
   1. $R^2(F) = \bar{w}' \Lambda \Sigma_{FF} \Lambda' \bar{w} / \sigma_g^2$.
   2. $\bar{w}$ are the weights of each sector in the economy. Λ are FM coefficients. Σ_{FF} is the variance-covariance matrix of a FM. σ_g^2 is the variance of aggregate output in the economy.
2. MSE of individual R^2s
   1. Estimate a FM for each individual sector with (i) FASVAR's estimated factor; (ii) PCA's estimated factor; (iii) Π_0 (from the DGP) estimated factor.
   2. Calculate the MSE between the R^2 of (i) and (iii); and (ii) and (iii). Not an aggregate measure.

back

Bias in factor models

Suppose the data generation process for X is given by

$$X = AX + v, \tag{1}$$

where A is a N by N matrix that maps into the economy's network structure (adjacency matrix). The vector v represents iid shocks with zero mean and variance covariance matrix $\Sigma_{vv} = \sigma_v^2 I_n$, where I_n is the n dimensional identity matrix. Now assume that X can be represented as a factor model

$$X = \Lambda F + u, \tag{2}$$

where F is a vector of r dimensional common factors, Λ is a N by r matrix of coefficients, and u is the vector of mutually independent and identical distributed disturbances, where $\Sigma_{uu} = \sigma_{uu}^2 I_n$. The common factors and the disturbance are mutually uncorrelated. The factor model implies that $\Sigma_{YY} = \Lambda \Sigma_{FF} \Lambda + \Sigma_{uu}$. As Σ_{uu} is a diagonal matrix, the only source of comovement between sectors is due to common shocks in F.

Bias in factor models

Our approach consists in studying under what condition equation the DGP can be represented by a factor model. By well-represented we mean that the factor model is able to estimate the importance of idiosyncratic or common factors in an accurate way. Impose that they are equivalent in terms of the implied X

$$\begin{aligned} \Lambda F + u = & \quad (I - A)^{-1} v \\ (I - A)(\Lambda F + u) = & \quad v. \end{aligned}$$

Let's make use of $\mathbb{E}(vv') = \Sigma_{vv}$, $\mathbb{E}(uu') = \Sigma_{uu}$ and the fact that F and u are mutually independent to obtain

$$(I - A)\left(\Lambda \mathbb{E}(FF')\Lambda' + \Sigma_{uu})'\right)(I - A') = \quad \Sigma_{vv}. \tag{3}$$

We motivate our results with two examples. First, we consider a complete network in which all sectors are equally connected to everyone else. Second, we consider a star network, where only one industry supplies all the intermediates to the economy.

Bias in factor models: the complete network

We now ask, how does the underlying structure of A affect the equivalence? For the complete network (every element in A is the same) we have

$$\sigma_f^2 \lambda \lambda' + \sigma_u^2 I_n = \sigma_v^2 (I - A)^{-2}$$

which becomes

$$\lambda_i \lambda_j = \frac{1}{n\sigma_f^2} \mathbf{A} \sigma_v^2 \qquad \forall i \neq j$$

$$\lambda_i^2 = \frac{\sigma_v^2 (1 + \frac{1}{n}\mathbf{A}) - \sigma_u^2}{\sigma_f^2} \qquad \forall i.$$

It then is straightforward to see that $\sigma_u^2 = \sigma_v^2$ is a solution to the system

$$\frac{1}{n\sigma_f^2} \mathbf{A} \sigma_v^2 = \frac{\sigma_v^2 (1 + \frac{1}{n}\mathbf{A}) - \sigma_u^2}{\sigma_f^2}$$
$$\sigma_u^2 = \sigma_v^2.$$

This result indicates that in a completely symmetric network model, factor models can precisely decompose the importance of aggregate and idiosyncratic disturbances.

Bias in factor models: the star-supplier network

For the star-supplier network (one sector is connected to all others) we have

$$\sigma_f^2 \lambda\lambda' + \sigma_u^2 I_n = \sigma_v^2 (I - A)^{-2}$$

which becomes

$$\begin{aligned}
\sigma_f^2 \lambda_1 \lambda_1 + \sigma_u^2 &= \frac{\sigma_v^2}{(1-\beta)^2} \\
\sigma_f^2 \lambda_1 \lambda_2 &= \sigma_v^2 \frac{\beta}{(1-\beta)^2} \\
\sigma_f^2 \lambda_2 \lambda_2 + \sigma_u^2 &= \sigma_v^2 \left(\frac{\beta^2}{(1-\beta)^2} + 1\right) \\
\sigma_f^2 \lambda_2 \lambda_3 &= \sigma_v^2 \frac{\beta^2}{(1-\beta)^2}.
\end{aligned}$$

It then is straightforward to see that–when σ_{uj} varies across sectors—is a solution to the system implies $\sigma_{u1} = 0$ and $\sigma_{uj} = \sigma_v \; \forall j \neq 1$. However, an important bias is present. Factor models heavily underestimate the actual importance of idiosyncratic shocks coming from the star supplier sector ($\sigma_{u1} = 0$).

Back

Estimating Unobserved Networks and Common Factors

- Objective is to propose a reduced form approach of estimating unobserved networks and common factors jointly.
- Basics: Reduced form estimation of unobserved networks has been studied (cite). Variations of Lasso estimators are typically used (VAR-Lasso):

$$argmin_A \left(\underbrace{\sum_{i=1}^{N} (\mathbf{y_i} - \mathbf{x_i}\mathbf{A})^2}_{OLS} + \underbrace{\lambda_1 \sum_{i=1}^{N} |A_i|}_{Penalization\ term\ -\ Sparsity} \right) \tag{4}$$

- $\mathbf{y_i}$ is a $T \times N$ matrix with the Commodity Price Index (cyclical component) for each firm/sector/country $i \in \{1, \ldots N\}$ at each period $t \in \{1, \ldots, T\}$. $\mathbf{x_i}$ is the lag of $\mathbf{y_i}$. $\mathbf{A}$ is an $N \times N$ estimated matrix of connections.

Estimating Unobserved Networks and Common Factors

- Big amount of parameters to be estimated requires to impose some structure over the optimization problem to improve accuracy.

$$argmin_A \left(\underbrace{\sum_{i=1}^{N} (\mathbf{y_i} - \mathbf{x_i}\mathbf{A})^2}_{OLS} + \underbrace{\lambda_1 \sum_{i=1}^{N} |A_i|}_{Penalization - Sparsity} \right) \tag{5}$$

- λ_1 is a penalization term that imposes sparsity over **A** matrix.
- If we estimated the network with a *Lasso* type estimator, it might *include partially a common factor structure*

Estimating Unobserved Networks and Common Factors

- Basics: Unobserved common factors are typically estimated with PCA and Factor Models:

$$\mathbf{X_i} = \Lambda_i \mathbf{F_i} + u_t \tag{6}$$

- Where $\mathbf{F_i}$ describes common factors found by a PCA and the equation describes a factor model.
- Problem: The estimation of unobserved common factors is biased under the existence of networks, specially concentrated networks (link to appendix)
- Proposed solution: A reduced form approach to jointly estimate unobserved networks and common factors.

Application: Commodity Prices

Commodity price comovement

Application to commodity prices

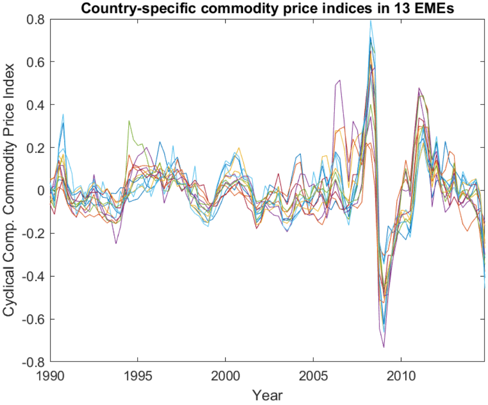


Figure: Commodity Price Index for 13 EMEs, Fernandez et al. (2018)

- Visually, there is a relevant common factor structure, but is it a common factor or

Real Data Application

- We test our reduced form methodology using cross-country data on commodity price indexes, based on Fernandez et al. (2018).
- **Data:** Commodity Price Indexes for 60 countries in 140 periods (quarterly from 1980 until 2014).
- **Their results:** Relying on a factor model, they start by documenting that one common factor accounts $\approx 77\%$ of the variance in the Commodity Price Indexes across a subset of 13 EMEs.
- **What we do:** Using the same data, apply FASVAR and compare the relevance of a FASVAR's common factor with their result.

Results: Common factors while considering spillovers

Table: Variability (R^2) explained by common factor (in %)

	PCA	FASVAR	FASVAR-Prior
Mean R^2 (unweighted)	77.05	30.35	31.95

- A factor model estimates $\approx 77\%$ of variability explained by a common factor. FASVAR estimates $\approx 30\%$.
- Our proposed methodology implies that cross-country spillover effects could be important

Counterfactuals with FASVAR

For each period t we have

$$\widehat{\mathbf{y}}_{\mathbf{t}} = \mathbf{x}_{\mathbf{t}}\widehat{\mathbf{A}} + \hat{\Pi}_t,$$

where $\widehat{\mathbf{y}}_{\mathbf{t}}$ is a vector of dimension N (countries) of predicted prices for commodities. In our counterfactual we define

$$\widetilde{\mathbf{y}}_t = \mathbf{x}_{\mathbf{t}}\widetilde{\mathbf{A}} + \hat{\Pi}_t,$$

We then compare the counterfactual distribution of commodity price volatility, $mean(SD(\widetilde{\mathbf{y}}_i))$, and comovement among $\widetilde{\mathbf{y}}_i$ for all i, with the implied by the benchmark estimation

Counterfactuals

Table: Volatility and comovement: counterfactual networks

	FASVAR $\widehat{\mathbf{A}}$	$\widetilde{A}_{ij} = \frac{1}{N}\ (\forall i, j)$	$Diag(sum(\widehat{\mathbf{A}}))$	$\widehat{\mathbf{A}}_{ij} = 0$ (all $i \neq j$)
Mean Volatility	0.1840	0.1552	0.7825	0.1322
Mean Correlation	0.7772	0.8948	0.5728	0.6483

- Diversification in linkages (complete network) would reduce volatility (by 3 percentage points) and increase comovement (by 15%)
- An island network would increase volatility (by 70 percentage points) and decrease comovement (by 26%)
- Severing off-diagonal elements would reduce volatility (by 5 percentage points) and decrease comovement (by 17%)

Results: Analyzing the EMEs network

Table: Countries with Higher Pagerank Centrality - 13 EMEs[2] subset

FASVAR	Chile	Mexico	Brazil	Ukraine	Ecuador	Venezuela	Peru	Bulgaria	Malaysia	Argentina
Pagerank central.	29.9585	22.4069	11.3563	9.7680	6.1406	5.4481	3.7559	3.3702	1.9398	1.6100
Trade Network	Chile	Brazil	Peru	Argentina	Mexico	Ecuador	Colombia	Russia	Ukraine	Malaysia
Pagerank central.	7.8927	7.8815	7.8673	7.8661	7.8519	7.8456	7.8445	7.8164	7.7931	7.7679

[2] These countries are: Argentina, Brazil, Bulgaria, Chile, Colombia, Ecuador, Malaysia, Mexico, Peru, Russia, South Africa, Ukraine and Venezuela.