



COMP 2012H Honors Object-Oriented Programming and Data Structures

Topic 6: Recursion

Dr. Desmond Tsoi

Department of Computer Science & Engineering
The Hong Kong University of Science and Technology
Hong Kong SAR, China



Recursion

- In programming, **recursion** means that a function calls **itself**!
- Although it looks strange in the beginning, solving a programming task by **recursion** renders the program
 - ▶ easier to **write**
 - ▶ easier to **read** (understand)
 - ▶ **shorter** (in codes).

Implement a Recursive Solution

- 1 **Decompose** the **problem** into **sub-problems** — which are smaller examples of the same problem — plus some **additional work** that “**glues**” the solutions of the sub-problems together.
- 2 The **smallest sub-problem** has a **non-recursive** solution.

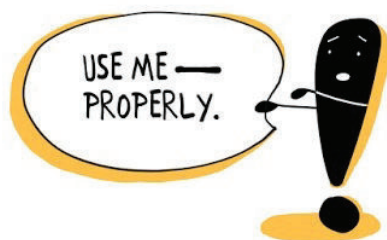
Example: Factorial Function

Definition of the **factorial function** you learn in high school:

$$n! = n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$$

Recursive Definition of Factorial Function

- $0! = 1$
- $n! = n \times (n-1)!$ if $n > 0$
- To find the value of $n!$, first find the value of $(n-1)!$ and then multiply the result with n .



Example: Factorial Recursive Function

```
int factorial(int n)    /* factorial.cpp */
{
    if (n < 0)           // Error checking
        return -1;
    else if (n == 0)     // Base case; ending case too!
        return 1;
    else                 // Recursive case
        return n * factorial(n-1);
}
```

Or, equivalently,

```
int factorial(int n)    /* factorial2.cpp */
{
    if (n < 0)           // Error checking
        return -1;
    if (n == 0)          // Base case; ending case too!
        return 1;
    return n * factorial(n-1); // Recursive case
}
```

How the Recursive Factorial Function Works?

```
factorial(3) :  
  3 < 0    false  
  3 == 0   false  
  3 * factorial(2)  
    factorial(2) :  
      2 < 0    false  
      2 == 0   false  
      2 * factorial(1)  
        factorial(1) :  
          1 < 0    false  
          1 == 0   false  
          1 * factorial(0)  
            factorial(0) :  
              0 < 0    false  
              0 == 0   true  
              return 1  
            return 1*1 = 1  
          return 2*1 = 2  
        return 3*2 = 6
```

Factorial Function: Recursive vs. Non-Recursive

```
int factorial(int n)    /* factorial.cpp */  
{  
    if (n < 0)          // Error checking  
        return -1;  
    else if (n == 0)    // Base case; ending case too!  
        return 1;  
    else                // Recursive case  
        return n * factorial(n-1);  
}
```

```
int factorial(int n)    /* non-recursive-factorial.cpp */  
{  
    int result = 1;     // Default value for n = 0 or 1  
    if (n < 0)          // Error checking  
        return -1;  
  
    for (int j = 2; j <= n; j++) // When n >= 2  
        result *= j;  
    return result;  
}
```

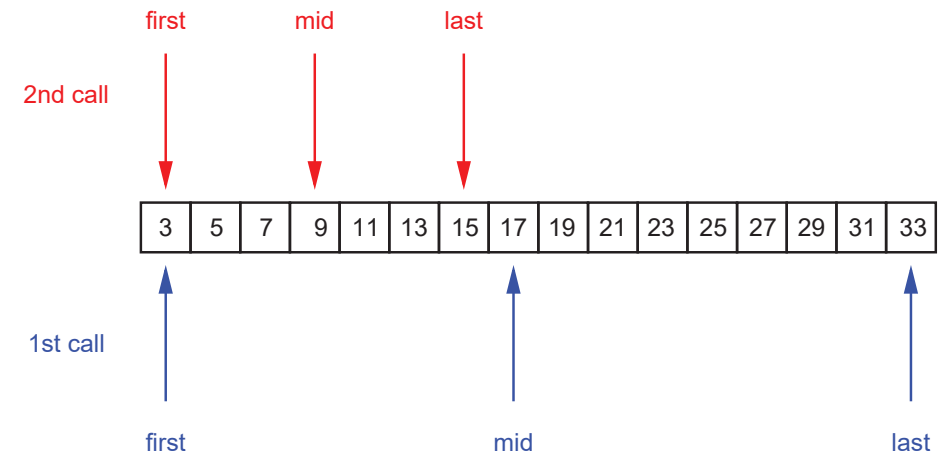
Infinite Recursion!

- We have to be careful that a recursion will eventually end up with a non-recursive base case.
- Otherwise, we will get infinite recursion!

```
int factorial(int n)  
{  
    // Forget the base case, which is the ending case too!  
    return n * factorial(n-1);  
}
```

```
int factorial(int n)  
{  
    // Forget checking if n < 0  
    if (n == 0)  
        return 1;  
  
    // Infinite recursion for negative n  
    return n * factorial(n-1);  
}
```

Binary Search



binary search for the value 9

Example: Recursive Solution of Binary Search

```
const int NOT_FOUND = -1;    /* File: bsearch.cpp */
int bsearch(const int data[ ], // sorted in ascending order
            int first,        // lower bound index
            int last,         // upper bound index
            int value)        // value to search
{
    if (last < first)         // Base case #1
        return NOT_FOUND;

    int mid = (first + last)/2;

    if (data[mid] == value)
        return mid;          // Base case #2

    else if (data[mid] > value) // Search the lower half
        return bsearch(data, first, mid-1, value);

    else
        // Search the upper half
        return bsearch(data, mid+1, last, value);
}
```

Example: Non-Recursive Solution of Binary Search

```
const int NOT_FOUND = -1;    /* File: non-recursive-bsearch.cpp */
int bsearch(const int data[ ], // sorted in ascending order
            int size,          // number of data in the array
            int value)         // value to search
{
    int first = 0;
    int last = size - 1;

    while (first <= last)
    {
        int mid = (first + last)/2;
        if (data[mid] == value)
            return mid;        // Value found!
        else if (data[mid] > value)
            last = mid - 1;     // Set up for searching the lower half
        else
            first = mid + 1;    // Set up for searching the upper half
    }

    return NOT_FOUND;
}
```

Disadvantages of Recursion

- The greater programming productivity is achieved at the expenses of the more computing resources. To run recursion, it usually requires
 - ▶ more memory
 - ▶ more computational time
- The reason is that whenever a function is called, the computer
 - ▶ has to memorize its **current state**, and **passes control** from the **caller** to the **callee**.
 - ▶ sets up a new data structure (you may think of it as a scratch paper for rough work) called **activation record** which contains information such as
 - ★ **where** the **caller** stops
 - ★ what **actual parameters** are passed to the **callee**
 - ★ create new **local variables** required by the **callee** function
 - ★ the **return value** of the function at the end
 - ▶ removes the **activation record** of the **callee** when it finishes.
 - ▶ passes control back to the **caller**.

That's all!

Any questions?



Further Reading



Example: Fibonacci Numbers

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, ...

Fibonacci (1202) investigated how fast rabbits could breed:

- A **newly-born** pair of rabbits, one male, one female, are put in a field.
- Rabbits **mate at the age of one month** so that at the **end of its 2nd month**, a female can **produce another pair** of rabbits.
- Suppose that our rabbits **never die**.
- Suppose the female always produces **one new pair** (one male, one female) **every month** from the 2nd month on.
- **How many pairs will there be in one year?**

Question : What is special with the above numbers?

Answer : Except for the first 2 numbers, each number is the **sum of the last 2 numbers** in the sequence.

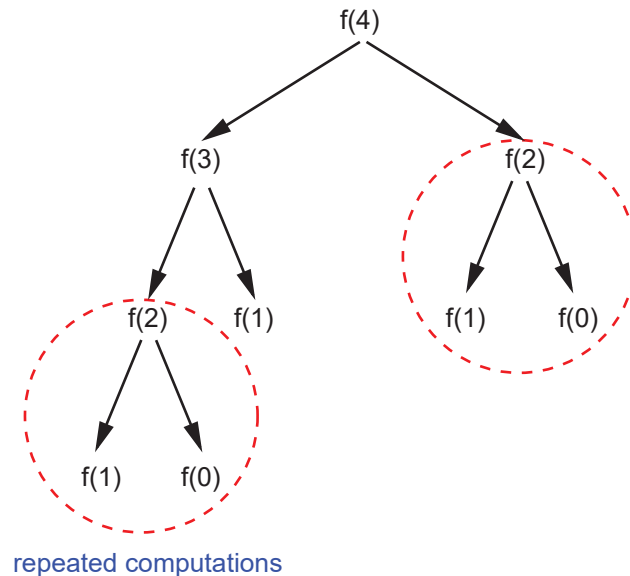
Example: Fibonacci Function as a Recursion

```
int fibonacci(int n)    /* File: fibonacci.cpp */
{
    if (n == 0)          // Base case #1
        return 0;

    if (n == 1)          // Base case #2
        return 1;

    return fibonacci(n-1) + fibonacci(n-2);
}
```

Inefficiency of Recursive Fibonacci Function



Example: Non-Recursive Fibonacci Function

```
int fibonacci(int n)      /* non-recursive-fibonacci.cpp */
{
    int fn;                // keep track of f(n)
    int fn_1 = 1;          // keep track of f(n-1)
    int fn_2 = 0;          // keep track of f(n-2)

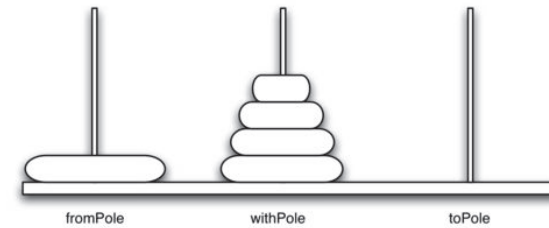
    if (n == 0) return 0; // Base case #1
    if (n == 1) return 1; // Base case #2

    for (int j = 2; j <= n; j++)
    {
        fn = fn_1 + fn_2; // f(n) = f(n-1) + f(n-2)

        // Prepare for the calculation of the next fibonacci number
        fn_2 = fn_1;      // f(n-2) = f(n-1)
        fn_1 = fn;        // f(n-1) = f(n)
    }

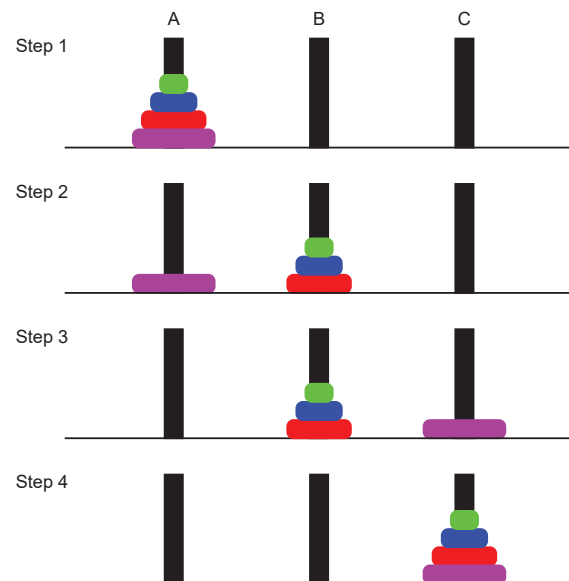
    return fn;
}
```

Example: Tower of Hanoi Game



- It consists of 3 **pegs**, and a stack of **discs of different sizes**.
- It **starts** with all discs stacked up on one peg with smaller discs sitting on top of bigger discs.
- The **goal** is to **move** the entire stack of discs to another peg, making use of the remaining peg.
- **Rules:**
 - ▶ only one disc may be moved at a time
 - ▶ no disc may be placed on top of a smaller disc

Recursive Solution of Tower of Hanoi



Example: Recursive Solution of Tower of Hanoi

```
#include <iostream>      /* File: toh.cpp */
using namespace std;

void tower_of_hanoi(int num_discs, char pegA, char pegB, char pegC)
{
    if (num_discs == 0) // Base case
        return;

    tower_of_hanoi(num_discs-1, pegA, pegC, pegB);

    cout << "move disc " << num_discs
          << " from peg " << pegA << " to peg " << pegC << endl;

    tower_of_hanoi(num_discs-1, pegB, pegA, pegC);
}
```

Example: Counting Zeros in an Integer

- **Example:** for the integer 120809, there are 2 zeros.
- **Basic idea:**
 - ▶ Break down the number into **quotient** and **remainder**.
 - ▶ Count the number of zeros in **quotient** and **remainder**.

```
int num_zeros(int n)    /* File: num-zeros.cpp */
{
    if (n == 0)          // Base case #1
        return 1;

    else if (n < 10 && n > 0) // Base case #2
        return 0;

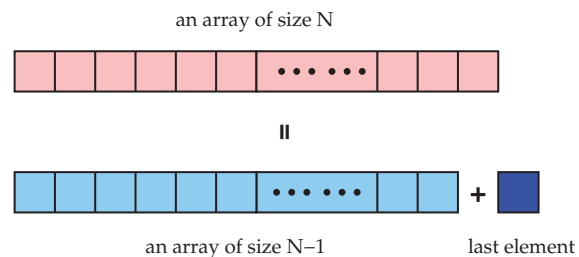
    else
        return num_zeros(n/10) + num_zeros(n%10);
}
```

Example: Factoring

- **Goal:** find how many times **factor** m appears in the **integer** n .
- **Example:** if $n = 48$ and $m = 4$, since $48 = 4 \times 4 \times 3$, the answer is 2.
- **Basic idea:**
 - ▶ Divide n by m **until** the remainder is non-zero.
 - ▶ **Increment** the count by 1 for every successful division.

```
int num_factors(int n, int m) /* File: factor.cpp */
{
    if (n % m != 0)          // Base case
        return 0;
    else
        return 1 + num_factors(n/m, m);
}
```

Array and Recursion



- **Array** is a **recursive data structure** in nature.
- For many problems, one may define a recursion on an array of size N which
 - ▶ will call **itself** with only $N - 1$ elements (either the top $N - 1$ or the last $N - 1$ elements),
 - ▶ with some **extra codes** to deal with the **remaining** element (last or first element).

Example: Sum Up Array Elements

```
#include <iostream> /* File: array-sum.cpp */
using namespace std;

// Summing up x[0] + x[1] + ... + x[num_elements-1]
int array_sum(const int x[], int num_elements)
{
    if (num_elements <= 0) return 0; // Base case
    return array_sum(x, num_elements-1) + x[num_elements-1];
}

int main()
{
    int a[] = { 1, 2, 3, 4, 5, 6 };
    int n;          // #elements in an array to sum
    while (cin >> n)
        cout << array_sum(a, n) << endl;
    return 0;
}
```

Question: What happens if you pass a value bigger than the size of the array size to n ?