

COMP 3711 – Spring 2019
Tutorial 1
For Personal Revision

1. For each of the following statements, answer whether the statement is true or false.
 - (a) $1000n + n \log n = O(n \log n)$.
 - (b) $n^2 + n \log(n^3) = O(n \log(n^3))$.
 - (c) $n^3 = \Omega(n)$.
 - (d) $n^2 + n = \Omega(n^3)$.
 - (e) $n^3 = O(n^{10})$.
 - (f) $n^3 + 1000n^{2.9} = \Theta(n^3)$
 - (g) $n^3 - n^2 = \Theta(n)$
2. For each pair of expressions (A, B) below, indicate whether A is O , Ω , or Θ of B . Note that zero, one, or more of these relations may hold for a given pair; list all correct ones. Justify your answers.
 - (a) $A = n^3 + n \log n$; $B = n^3 + n^2 \log n$.
 - (b) $A = \log \sqrt{n}$; $B = \sqrt{\log n}$.
 - (c) $A = n \log_3 n$; $B = n \log_4 n$.
 - (d) $A = 2^n$; $B = 2^{n/2}$.
 - (e) $A = \log(2^n)$; $B = \log(3^n)$.
3. Suppose $T_1(n) = O(f(n))$ and $T_2(n) = O(f(n))$. Which of the following are true? Justify your answers.
 - (a) $T_1(n) + T_2(n) = O(f(n))$
 - (b) $\frac{T_1(n)}{T_2(n)} = O(1)$
 - (c) $T_1(n) = O(T_2(n))$
4. Let $f(n)$ and $g(n)$ be non-negative functions. Using the basic definition of Θ -notation, prove that
$$\max(f(n), g(n)) = \Theta(f(n) + g(n)).$$
5. In the analysis of the max-subarray algorithms we (will) see nested loops of the form

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for i = 1 to n
  for j = i to n
    for k = i to j
      do one unit of work
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In class we give an intuitive explanation as to why this code performs $\Theta(n^3)$ units of work.

Prove this fact rigorously.