HKUST – Department of Computer Science and Engineering

COMP 2711: Discrete Mathematical Tools for Computer Science

T3B Quiz 1 (40 minutes)

Problem 1: For each of the following pairs of logic statements, either prove that the two statements are logically equivalent, or give a counterexample. In your proof, you may use either a truth table or logic laws. A counterexample should consist of a truth setting of the variables and the truth values of the statements under the setting.

(a) $(q \wedge r) \to p$ and $\neg q \vee \neg r \vee p$

(b) $(q \wedge r) \to p$ and $\neg p \to (q \to \neg r)$

(c) $(q \to p) \wedge (r \to p)$ and $(q \wedge r) \to p$

(d) $(q \wedge \neg r) \to (p \wedge \neg p)$ and $q \to r$

Solution : (a) Equivalent.

$$\begin{aligned} (q \wedge r) \to p &\equiv \neg(q \wedge r) \vee p \quad && (s \to t \equiv \neg s \vee t) \\ &\equiv \neg q \vee \neg r \vee p && \text{(by DeMorgan's law)} \end{aligned}$$

(b) Equivalent.

$$\begin{aligned} \neg p \to (q \to \neg r) &\equiv p \vee (\neg q \vee \neg r) \quad && (s \to t \equiv \neg s \vee t) \\ &\equiv (q \wedge r) \to p && \text{(by part (a))} \end{aligned}$$

(c) Not equivalent. Counter example: $q = T, r = F, p = F$. The first statement is false, while the second statement is true.

(d) Equivalent.

$$\begin{aligned} (q \wedge \neg r) \to (p \wedge \neg p) &\equiv \neg(q \wedge \neg r) \vee (p \wedge \neg p) \\ &\equiv \neg q \vee r \vee F \\ &\equiv \neg q \vee r \\ &\equiv q \to r \end{aligned}$$

Problem 2: Express the negations of each of these statements so that all negation symbols immediately precede predicates.

(a) $\forall x \exists y \forall z\ T(x, y, z)$

(b) $\forall x \exists y\ P(x, y) \vee \forall x \exists y\ Q(x, y)$

(c) $\forall x \exists y\ (P(x, y) \wedge \exists z\ R(x, y, z))$

(d) $\forall x \exists y\ (P(x, y) \to Q(x, y))$

Solution : As we push the negation symbol toward the inside, each quantifier it passes must change its type. For logical connectives we either use De Morgan's laws or recall that $\neg(p \to q) \equiv p \wedge \neg q$.

(a)

$$\begin{aligned} \neg\forall x \exists y \forall z\ T(x, y, z) &\equiv \exists x \neg\exists y \forall z\ T(x, y, z) \\ &\equiv \exists x \forall y \neg\forall z\ T(x, y, z) \\ &\equiv \exists x \forall y \exists z\ \neg T(x, y, z) \end{aligned}$$

(b)

$$\begin{aligned} \neg(\forall x \exists y\ P(x, y) \vee \forall x \exists y\ Q(x, y)) &\equiv \neg\forall x \exists y\ P(x, y) \wedge \neg\forall x \exists y\ Q(x, y) \\ &\equiv \exists x \forall y\ \neg P(x, y) \wedge \exists x \forall y\ \neg Q(x, y) \end{aligned}$$

(c)

$$\begin{aligned} \neg\forall x \exists y\ (P(x, y) \wedge \exists z\ R(x, y, z)) &\equiv \exists x \forall y\ \neg(P(x, y) \wedge \exists z\ R(x, y, z)) \\ &\equiv \exists x \forall y\ (\neg P(x, y) \vee \neg\exists z\ R(x, y, z)) \\ &\equiv \exists x \forall y\ (\neg P(x, y) \vee \forall z\ \neg R(x, y, z)) \end{aligned}$$

(d)

$$\begin{aligned} \neg\forall x \exists y\ (P(x, y) \to Q(x, y)) &\equiv \exists x \forall y\ \neg(P(x, y) \to Q(x, y)) \\ &\equiv \exists x \forall y\ (P(x, y) \wedge \neg Q(x, y)) \end{aligned}$$

Problem 3: Let p, q, and r be the propositions

p: Grizzly bears have been seen in the area.

q: Hiking is safe on the trail.

r: Berries are ripe along the trail.

Write these propositions using p, q, and r and logical connectives (including negations).

(a) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

(b) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail.

(c) Grizzly bears have not been seen in the area is necessary for safe hiking on the trail.

Solution : (a) $r \to (q \leftrightarrow \neg p)$.

(b) $(p \wedge r) \to \neg q$.

(c) $q \to p$.

Problem 4: Suppose that the domain of the propositional function $P(x)$ consists of -5, -3, -1, 1, 3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

(a) $\neg \forall x\ P(x)$.

(b) $\exists x\ ((x \geq 0) \wedge P(x))$.

(c) $\forall x\ ((x < 0) \to P(x))$.

Solution : (a) $\neg(P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5))$.

(b) $P(1) \vee P(3) \vee P(5)$.

(c) $P(-5) \wedge P(-3) \wedge P(-1)$.