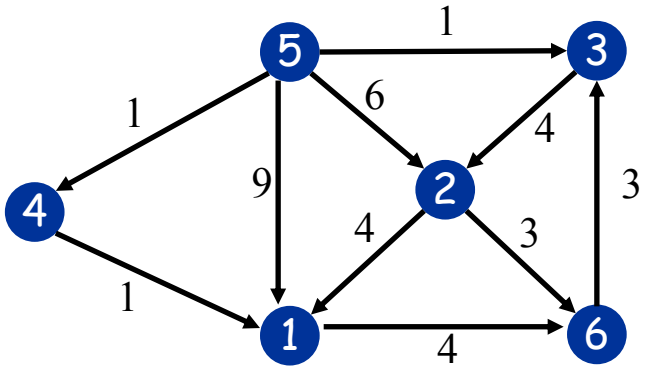
# Floyd-Warshall Example

Version of April 11, 2019

# Dynamic Programming: Solution 3 (Floyd-Warshall)

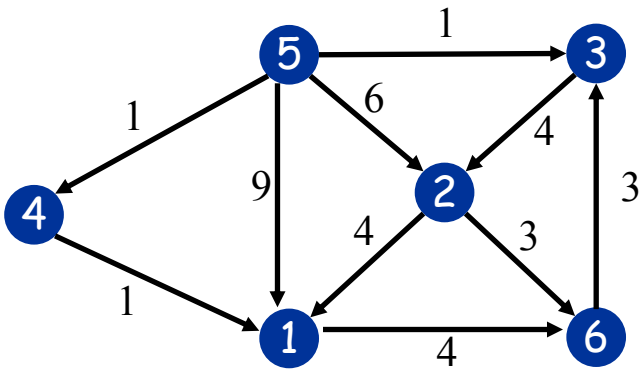
Def: $d_{ij}^{(k)}$ = length of the shortest path from $i$ to $j$ that such that all intermediate vertices on the path (if any) are in the set $\{1,2,\ldots,k\}$.



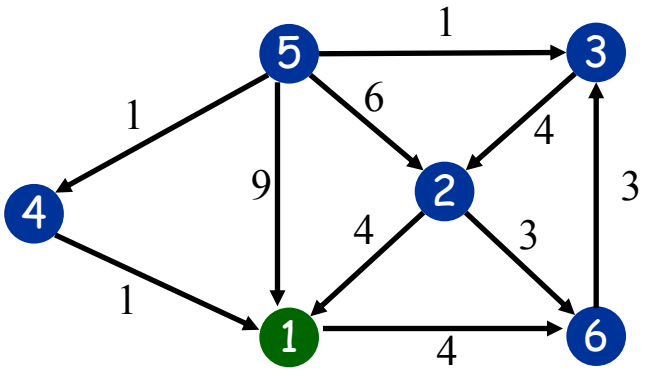

Initially: $d_{ij}^{(0)} = w(i,j)$

Goal: $D^{(n)}$

$$d_{ij}^{(k)} = min\left\{d_{ij}^{(k-1)},\quad d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right\}$$




$$L^{(0)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ \infty & 4 & 0 & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 & \infty & \infty \\ 9 & 6 & 1 & 1 & 0 & \infty \\ \infty & \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

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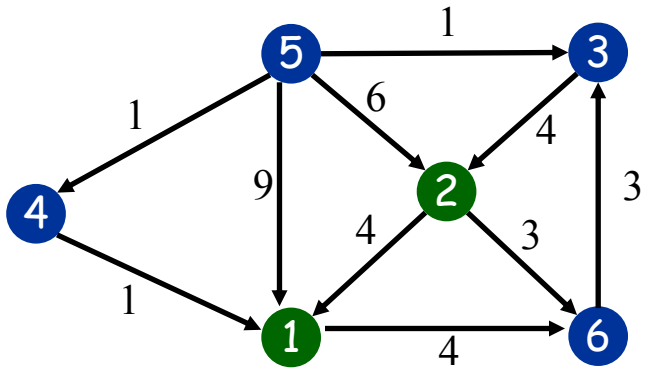
$$L^{(1)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ \infty & 4 & 0 & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 & \infty & 5 \\ 9 & 6 & 1 & 1 & 0 & 13 \\ \infty & \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$d_{ij}^{(1)} = min\left\{d_{ij}^{(0)}, \quad d_{i1}^{(0)} + d_{1j}^{(0)}\right\}$$

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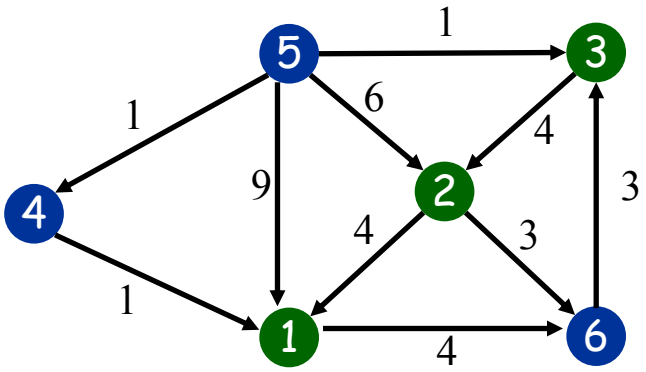
$$d_{ij}^{(1)} = min\left\{d_{ij}^{(0)},\quad d_{i1}^{(0)} + d_{1j}^{(0)}\right\}$$

$$L^{(2)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ 8 & 4 & 0 & \infty & \infty & 7 \\ 1 & \infty & \infty & 0 & \infty & 5 \\ 9 & 6 & 1 & 1 & 0 & 9 \\ \infty & \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$d_{ij}^{(2)} = min\left\{d_{ij}^{(1)},\quad d_{i2}^{(1)} + d_{2j}^{(1)}\right\}$$



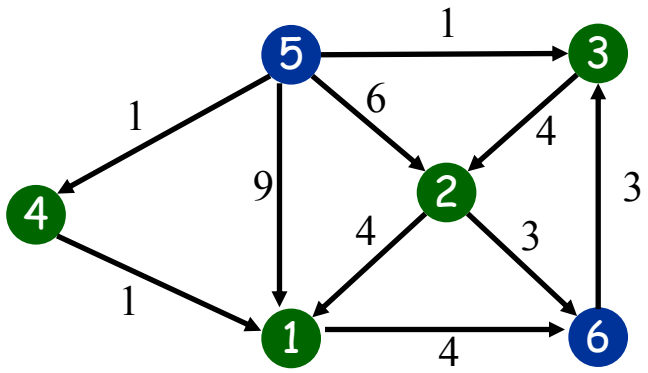

$$L^{(0)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ \infty & 4 & 0 & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 & \infty & \infty \\ 9 & 6 & 1 & 1 & 0 & \infty \\ \infty & \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$L^{(2)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ 8 & 4 & 0 & \infty & \infty & 7 \\ 1 & \infty & \infty & 0 & \infty & 5 \\ 9 & 6 & 1 & 1 & 0 & 9 \\ \infty & \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$d_{ij}^{(2)} = min\left\{d_{ij}^{(1)},\quad d_{i2}^{(1)} + d_{2j}^{(1)}\right\}$$

$$L^{(3)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ 8 & 4 & 0 & \infty & \infty & 7 \\ 1 & \infty & \infty & 0 & \infty & 5 \\ 9 & 5 & 1 & 1 & 0 & 8 \\ 11 & 7 & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$d_{ij}^{(3)} = min\left\{d_{ij}^{(2)},\quad d_{i3}^{(2)} + d_{3j}^{(2)}\right\}$$

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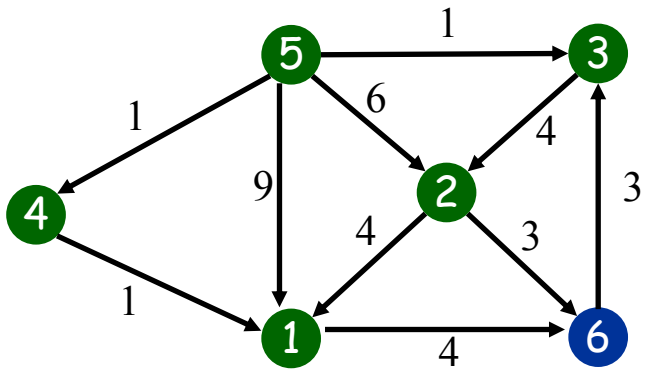
$$d_{ij}^{(3)} = min\left\{d_{ij}^{(2)},\quad d_{i3}^{(2)} + d_{3j}^{(2)}\right\}$$

$$L^{(4)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ 8 & 4 & 0 & \infty & \infty & 7 \\ 1 & \infty & \infty & 0 & \infty & 5 \\ 2 & 5 & 1 & 1 & 0 & 6 \\ 11 & 7 & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$d_{ij}^{(4)} = min\left\{d_{ij}^{(3)},\quad d_{i4}^{(3)} + d_{4j}^{(3)}\right\}$$




$$L^{(0)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ \infty & 4 & 0 & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 & \infty & \infty \\ 9 & 6 & 1 & 1 & 0 & \infty \\ \infty & \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

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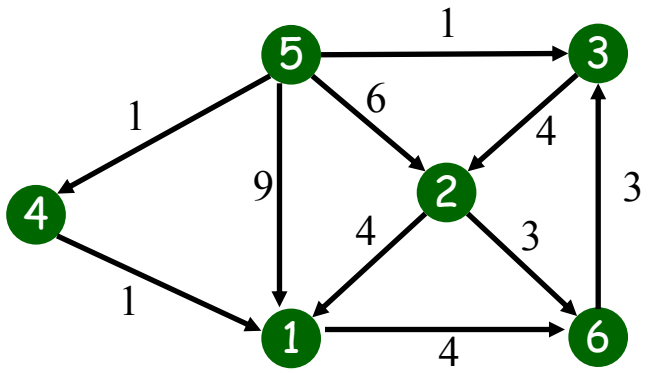
$$d_{ij}^{(4)} = min\left\{d_{ij}^{(3)},\quad d_{i4}^{(3)} + d_{4j}^{(3)}\right\}$$

$$L^{(5)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ 8 & 4 & 0 & \infty & \infty & 7 \\ 1 & \infty & \infty & 0 & \infty & 5 \\ 2 & 5 & 1 & 1 & 0 & 6 \\ 11 & 7 & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$d_{ij}^{(5)} = min\left\{d_{ij}^{(4)},\quad d_{i5}^{(4)} + d_{5j}^{(4)}\right\}$$



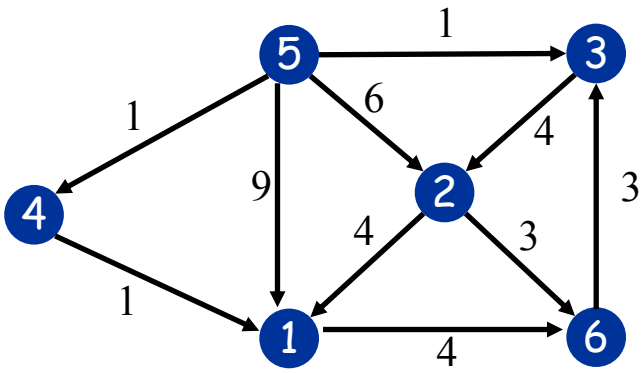

$$L^{(0)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ \infty & 4 & 0 & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 & \infty & \infty \\ 9 & 6 & 1 & 1 & 0 & \infty \\ \infty & \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$L^{(5)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ 8 & 4 & 0 & \infty & \infty & 7 \\ 1 & \infty & \infty & 0 & \infty & 5 \\ 2 & 5 & 1 & 1 & 0 & 6 \\ 11 & 7 & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$d_{ij}^{(5)} = min\left\{d_{ij}^{(4)},\quad d_{i5}^{(4)} + d_{5j}^{(4)}\right\}$$

$$L^{(6)} = \begin{pmatrix} 0 & 11 & 7 & \infty & \infty & 4 \\ 4 & 0 & 6 & \infty & \infty & 3 \\ 8 & 4 & 0 & \infty & \infty & 7 \\ 1 & 12 & 8 & 0 & \infty & 5 \\ 2 & 5 & 1 & 1 & 0 & 6 \\ 11 & 7 & 3 & \infty & \infty & 0 \end{pmatrix}$$

$$d_{ij}^{(6)} = min\left\{d_{ij}^{(5)},\quad d_{i6}^{(5)} + d_{6j}^{(5)}\right\}$$




$$L^{(0)} = \begin{pmatrix} 0 & \infty & \infty & \infty & \infty & 4 \\ 4 & 0 & \infty & \infty & \infty & 3 \\ \infty & 4 & 0 & \infty & \infty & \infty \\ 1 & \infty & \infty & 0 & \infty & \infty \\ 9 & 6 & 1 & 1 & 0 & \infty \\ \infty & \infty & 3 & \infty & \infty & 0 \end{pmatrix}$$

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