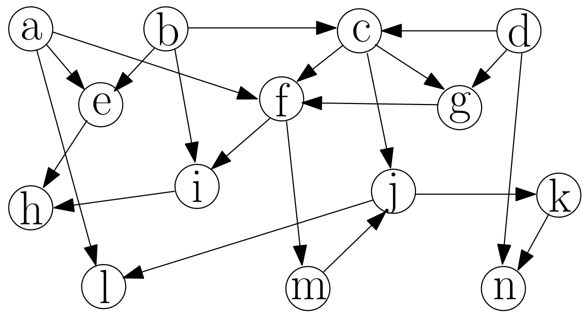
COMP 3711 – Spring 2019
Tutorial 8

1. Give a topological ordering of the following graph.



2. Let $T = (V, E)$ be a tree and $e = (u, v) \in E$. Show that removing e from T leaves a graph with exactly two connected components with one component containing u and the other containing v.

3. Let $G = (V, E)$ be a weighted graph with non-negative distinct edge weights. In class we showed that T is the *unique* MST of G.

   Now replace every weight $w(u, v)$ with its square $(w(u, v))^2$.

   (a) Is T still a MST of G with the new weights? Either prove that it is or give a counterexample

   (b) Next consider a shortest path $u \to v$ in the original graph. Is this path still a shortest path with the new weights? Either prove that it is or give a counterexample

4. Let G be a connected undirected graph with distinct weights on the edges, and let e be an edge of G.

   Suppose e is the largest-weight edge in some cycle of G.

   Show that e cannot be in the MST of G.

5. It is not difficult to see that if e in a minimum weight edge in G then e is always an edge in *some* Minimum Spanning Tree for G. Prove that if e is a maximum weight edge, the corresponding statement is not correct. That is, it is possible that e *does* belong to a MST of G. It is also possible that e does not belong to any MST for G.

6. Let $G = (V, E)$ be a connected undirected graph in which all edges have weight either 1 or 2. Give an $O(|V| + |E|)$ algorithm to compute a minimum spanning tree of G. Justify the running time of your algorithm. (*Note:* You may either present a new algorithm or just show how to modify an algorithm taught in class.)