COMP 3711 – Spring 2019
Tutorial 7

1. Let $G = (V, E)$ be an undirected graph where V is the set of vertices and E is the set of edges.
   Assume that there are no self-loops or duplicated edges.
   Answer all questions below as a function of $|V|$, the number of vertices.

   a) What is the maximum number of edges in G?
   b) What is the maximum number of edges in G if two vertices have degree 0.
   c) What is the maximum number of edges that an acyclic graph G can have?
   d) What is the minimum number of edges in G if G is a connected graph and contains at least one cycle?
   e) What is the minimum possible degree a vertex in a connected graph G can have?
   f) What is the maximum length of any simple path in G?

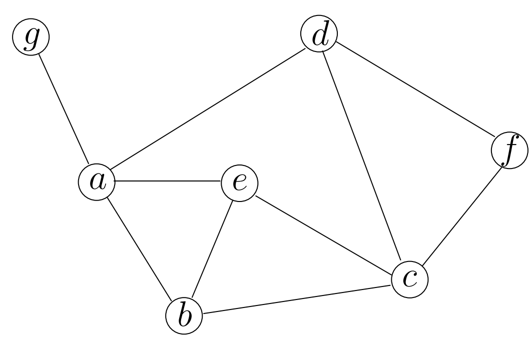
2. Let $G = (V, E)$ be a connected undirected graph. Prove that

$$\log(E) = \Theta(\log V).$$

   Note: we implictly use this fact in many of our analyses in class.

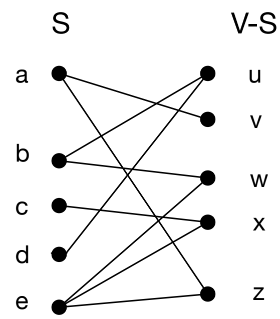
3. The adjacency list representation of a graph G, which has 7 vertices and 10 edges, is:

$$\begin{array}{ll} a :\to d, e, b, g & b :\to e, c, a \\ c :\to f, e, b, d & d :\to c, a, f \\ e :\to a, c, b & f :\to d, c \\ g :\to a & \end{array}$$



   (a) Show the breadth-first search tree that is built by running BFS on graph G with the given adjacency list, using vertex a as the source.
   (b) Indicate the edges in G that are NOT in the BFS tree in part (a) by dashed lines.
   (c) Show the depth-first search tree that is built by running DFS on graph G with the given adjacency list, using vertex a as the source.
   (d) Indicate the edges in G which are NOT in the DFS (c) by dashed lines.

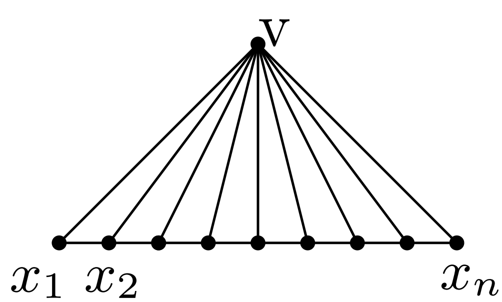
4. An (undirected) graph $G = (V, E)$ is *bipartite* if there exists some $S \subset V$ such that, for every edge $\{u, v\} \in E$, either
(i) $u \in S$, $v \in V - S$ or
(ii) $v \in S$, $u \in V - S$.



Let $G = (V, E)$ be a connected graph. Design an $O(|V| + |E|)$ algorithm that checks whether G is bipartite. *Hint: Run BFS.*

5. In the Fan Graph F_n, node v is connected to all the nodes and the other connections are given by the adjacency lists below.

$$v : x_1, x_2, \ldots, x_n, \qquad x_1 : v, x_2$$
$$x_n : v, x_{n-1} \qquad \forall i \neq 1, n, \quad x_i : v, x_{i-1}, x_{i+1}$$



(a) : Describe the tree that is output when BFS is run on F_n starting from initial vertex v; (ii) initial vertex x_1;
(iii) x_n; (iv) Other x_i.

(b) : Describe the tree that is output when DFS is run on F_n starting from initial vertex v; (ii) initial vertex x_1;
(iii) x_n; (iv) Other x_i.