COMP2121A: Discrete Mathematics
Problem Solving Session 3 9 Oct, 2018

1. Suppose A and B are finite sets. Prove that if there is an injection from A to B and an injection from B to A, then there is a bijection from A to B.

   Solution: Since there is an injection from A to B, we have that $|A| \leq |B|$. For the same reason, $|B| \leq |A|$. Therefore, $|A| = |B|$ and a bijection from A to B exists.

2. How many ways are there to distribute five balls into three different boxes, labeled by A, B and C, if

   (a) the balls are different?

   (b) the balls are identical?

   (c) the balls are different and the following is satisfied: either box A is empty or box B is empty.

   Solution:

   (a) 3^5

   (b) $\binom{5+3-1}{5}$

   (c) $2^5 + 2^5 - 1^5$

3. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Show that if six integers are chosen from S, then there must exist two chosen integers whose sum is 11.

   Solution: Divide the set S into five subsets such that the integers of each subset add up to 11: $\{1, 10\}$, $\{2, 9\}$, $\{3, 8\}$, $\{4, 7\}$, $\{5, 6\}$. By the pigeon hole principle, among the six integers chosen from S, at least two of them come from the same subset.

4. A is a set of n elements. How many different equivalence relations on A are there with exactly r equivalent classes?

Solution: This is equivalent to count the number of different partitions of A into r subsets. By inclusion-exclusion we have the following desired value.

$$\frac{1}{r!}\sum_{i=1}^{r}(-1)^{r-i}\binom{r}{i}\cdot i^n.$$