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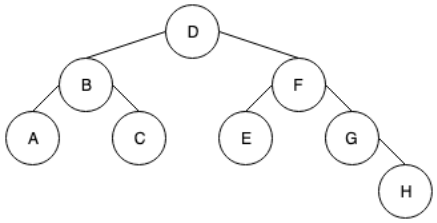
Problem 1:

a)

i/j	1	2	3	4	5	6	7	8
1	5	17	19	36	48	64	70	81
2		7	9	25	33	49	55	66
3			1	10	18	34	40	49
4				8	16	32	38	46
5					4	14	18	26
6						6	10	18
7							2	7
8								3

i/j	1	2	3	4	5	6	7	8
1	1	2	2	2	2	4	4	4
2		2	2	4	4	4	4	4
3			3	4	4	4	4	6
4				4	4	4	4	6
5					5	6	6	6
6						6	6	6
7							7	8
8								8

b)



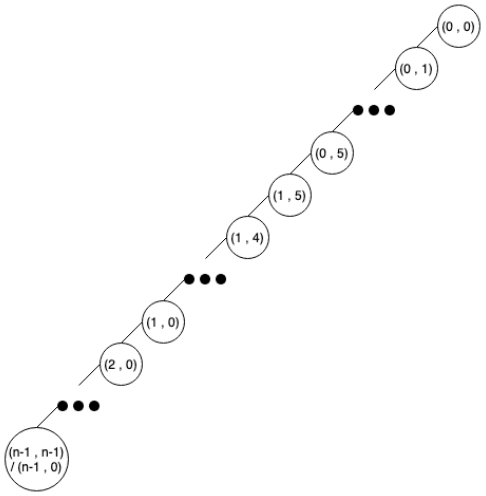
Problem 2

Problem 3

a)

i. $e = (u_{i,j}, v_{i,j+1}) \Rightarrow j \leq n - 2$,
$e = (u_{i,j}, v_{i+1,j}) \Rightarrow (i = 2k , j = n-1) \text{ OR } (i = 2k+1, j = 0)$ $k = 0 \ldots n$.

ii.



b)

i. $e = (u_{i,j}, v_{i+1,j}) \Rightarrow i \leq n - 2$
$e = (u_{i,j}, v_{i,j+1}) \Rightarrow i = 0, j < n - 2$

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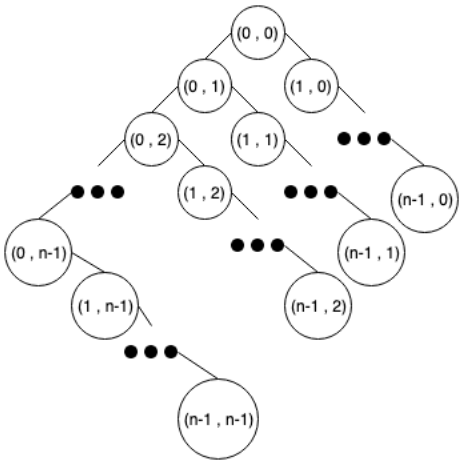
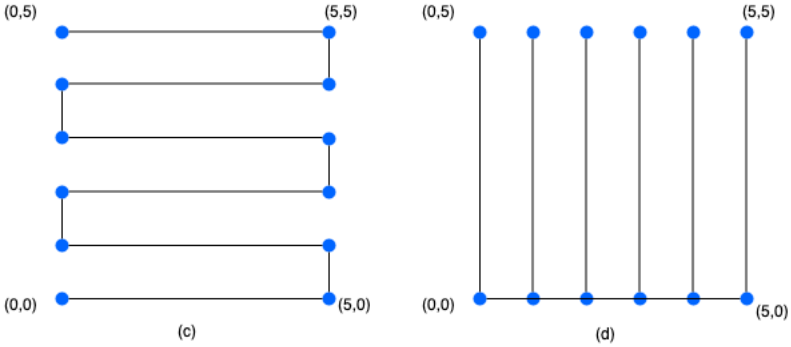
ii.



c)

d)



e)

i. DFS:

$e = (u_{i,j}, v_{i+1,j}) \Rightarrow i < n - 2$

$e = (u_{i,j}, v_{i,j+1}) \Rightarrow (i = 0, j = 2k+1) \text{ OR } (i = n-1, j = 2k) \; k = 0 \ldots n$.

BFS:

$e = (u_{i,j}, v_{i,j+1}) \Rightarrow i \leq n - 2, j = 0$

$e = (u_{i,j}, v_{i+1,j}) \Rightarrow j < n - 2$

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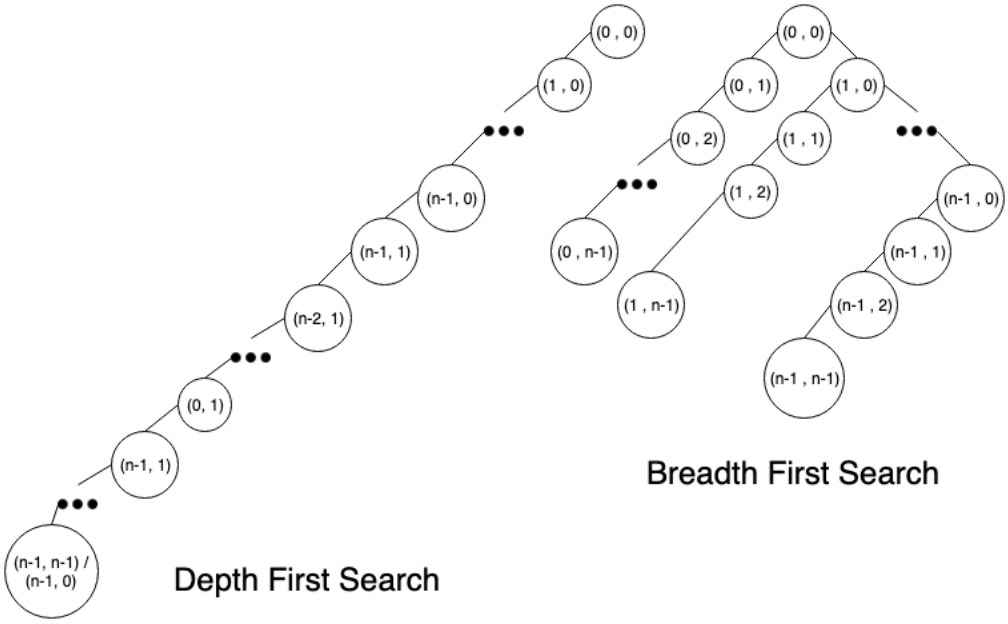
ii.



Problem 4

CycleDetection(G, v):

```
for each vertex u ∈ V do
     u.color←white
     u.p←nil
for each vertex u ∈ V do
     if u.color = white then DFS-Visit(u, v)
return "No Cycle"
```

DFS-Visit(u, vertex):

```
u.color←gray
for each v ∈ Adj[u] do
     if v.color = white then
          v.p←u
          DFS-Visit(v)
     else if v ≠ u.p then
          while u ≠ v do
               Cycle[]←u
               u←u.p
          Cycle[]←v
u.color←black
if Cycle[] contain vertex
     output "Cycle found:"
     for each u ∈ Cycle[]
          output u
     return
```

The algorithm mentioned in class is O(|V|) since there is early termination, the algorithm will end when the first cycle has been found. If we are going to found all the cycle until the target vertex inside the cycle, the run time will just like DFS algorithm: O(|E| + |V|).

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Problem 5

1. If Edge e = (u, v) does not belong to the MST, then there must be an Edge e' = (u', v) weight less than e = (u, v). In term of using Kruskal's Algorithm, all the edges have been gone through according to their weight in ascending order. A cheaper edge to node v (i.e. Edge e') will always been chosen earlier. And the more expense edge will never be choosing afterward (i.e. Edge e).
   Or in other way, if Edge e = (u, v) is not in the MST, adding it to the MST must form a cycle. Since there must be another path travel from u to v. By removing the heaviest edge in the cycle can produce an MST. Let's assume that the original MST is correct, if we remove any edge other than Edge e, the cost of the MST will increase. This mean all other edges in the cycle are cheaper than Edge e.
2. ```
   For each edge in E do
        if weight(edge) > weight(e) then
             remove edge from E
   For each edge in E do
        connect edge to Tree
   if Tree is connected then
        e is not in the MST
   else
        e is in the MST
   ```
   ```