

# Divide and Conquer for Integer Multiplication

## A Review

# Outline

- Quick D&C Integer Multiplication Review
- High level Example for 4X4 D&C simple integer multiplication
- High level Example for 4X4 D&C Karatsuba integer multiplication
- Full worked example of 4X4 simple integer multiplication
- Full worked example of 4X4 Karatsuba integer multiplication

# Background

- Straightforward “long multiplication” of 2  $n$ -bit integer words requires  $O(n^2)$  time (scalar multiplications and additions).
- In class, we saw how to use divide and conquer ideas to develop two different multiplication algorithms.
- These slides assume you know the algorithms

- The first algorithm satisfied the recurrence

$$\text{For } n > 1, \quad T(n) = 4T(n/2) + n. \quad T(1) = 1$$

$$\Rightarrow T(n) = O(n^2)$$

- The 2<sup>nd</sup>, Karatsuba Multiplication, satisfied the recurrence

$$\text{For } n > 1, \quad T(n) = 3T(n/2) + n. \quad T(1) = 1$$

$$\Rightarrow T(n) = O(n^{\log_2 3}) = O(n^{1.585\dots})$$

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# The first divide-and-conquer algorithm for integer multiplication

4 bit by 4 bit example

$$7 \quad 13 \quad = \quad 91$$

$$0100 \quad 1101 \quad = \quad 01011011$$

$$\mathbf{1} \cdot 2^2 + \mathbf{3} \quad \mathbf{3} \cdot 2^2 + \mathbf{1} \quad = \quad \mathbf{1} \cdot \mathbf{3} \times 2^4 + [\mathbf{1} \cdot \mathbf{1} + \mathbf{3} \cdot \mathbf{3}] \times 2^2 + \mathbf{3} \cdot \mathbf{1}$$

$$\boxed{a2^2 + b} \times \boxed{c2^2 + d} = \boxed{ac \times 2^4 + [ad + bc] \times 2^2 + bd}$$

# The first divide-and-conquer algorithm for integer multiplication

4 bit by 4 bit example

$$(a2^2 + b) \times (c2^2 + d) = ac \times 2^4 + [ad + bc] \times 2^2 + bd$$

The diagram illustrates the decomposition of the multiplication of two 4-bit integers into four 2-bit multiplications. The equation  $(a2^2 + b) \times (c2^2 + d) = ac \times 2^4 + [ad + bc] \times 2^2 + bd$  is shown. Below the equation, four lines radiate from the right-hand side expression to the subproblems:  $ac$ ,  $ad$ ,  $bc$ , and  $bd$ . The subproblems  $ad$  and  $bc$  are combined to form the middle term  $[ad + bc] \times 2^2$  in the final result.

# The first divide-and-conquer algorithm for integer multiplication

4 bit by 4 bit example

$$(a2^2 + b) \times (c2^2 + d) = ac \times 2^4 + [ad + bc] \times 2^2 + bd$$

$ac$

$ad$

$bc$

$bd$

$$a = a_12^1 + a_0$$

$\times$

$$c = c_12^1 + c_0$$

$$a_12^1 + a_0$$

$\times$

$$d_12^1 + d_0$$

$$b_12^1 + b_0$$

$\times$

$$c_12^1 + c_0$$

$$b_12^1 + b_0$$

$\times$

$$d_12^1 + d_0$$

# The first divide-and-conquer algorithm for integer multiplication

4 bit by 4 bit example

$$(a2^2 + b) \times (c2^2 + d) = ac \times 2^4 + [ad + bc] \times 2^2 + bd$$

$ac$

$$a = a_12^1 + a_0 \times c = c_12^1 + c_0$$

$$a_1c_12^2 + [a_1c_0 + a_0c_1]2^1 + a_0c_0$$

$ad$

$$a_12^1 + a_0 \times d_12^1 + d_0$$

$$a_1d_12^2 + [a_1d_0 + a_0d_1]2^1 + a_0d_0$$

$bc$

$$b_12^1 + b_0 \times c_12^1 + c_0$$

$$b_1c_12^2 + [b_1c_0 + b_0c_1]2^1 + b_0c_0$$

$bd$

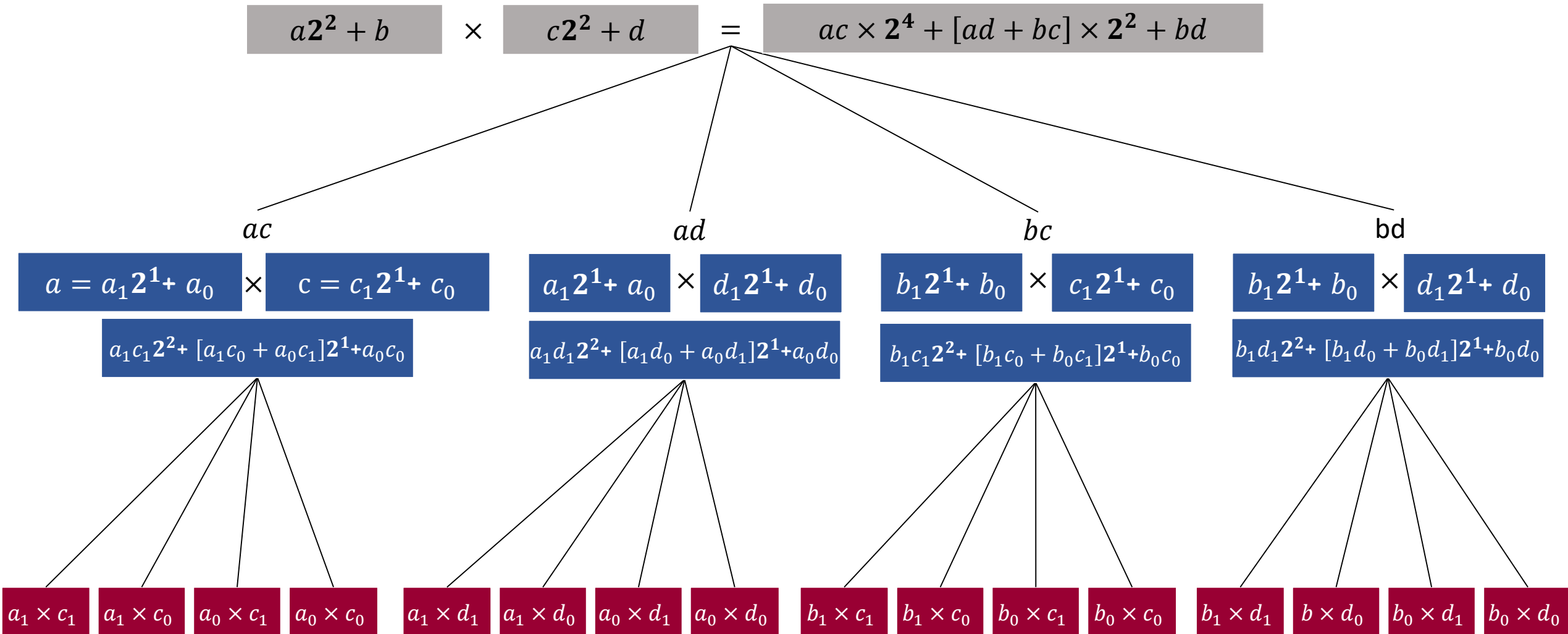
$$b_12^1 + b_0 \times d_12^1 + d_0$$

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# The first divide-and-conquer algorithm for integer multiplication

4 bit by 4 bit example



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# Karatsuba Multiplication

4 bit by 4 bit example

7

13

=

91

0100

1101

=

01011011

$$1 \cdot 2^2 + 3$$

$$3 \cdot 2^2 + 1$$

=

$$1 \cdot 3 \times 2^4 + [1 \cdot 1 + 3 \cdot 3] \times 2^2 + 3 \cdot 1$$

Old  
Method

$$a2^2 + b$$

×

$$c2^2 + d$$

=

$$ac \times 2^4 + [ad + bc] \times 2^2 + bd$$

$$1 \cdot 2^2 + 3$$

$$3 \cdot 2^2 + 1$$

=

$$1 \cdot 3 \times 2^4 + [(1 + 3)(3 + 1) - 1 \cdot 3 - 3 \cdot 1] \times 2^2 + 3 \cdot 1$$

Rewrite  
of same  
expression

$$a2^2 + b$$

×

$$c2^2 + d$$

=

$$ac \times 2^4 + [(a + b) \times (c + d) - ac - bd] \times 2^2 + bd$$

# Karatsuba Multiplication

4 bit by 4 bit example

$$\boxed{a2^2 + b} \times \boxed{c2^2 + d} = \boxed{(i) \times 2^4 + [(ii) - (i) - (iii)] \times 2^2 + (iii)}$$

The diagram illustrates the Karatsuba multiplication algorithm for a 4-bit by 4-bit example. The main equation is shown at the top, with three sub-problems branching out from the equals sign. The sub-problems are labeled (i), (ii), and (iii). The sub-problem (ii) is further expanded to show its calculation.

$(i) = ab$

$(ii) = (a + b) \times (c + d)$

$(iii) = bd$

# Karatsuba Multiplication

4 bit by 4 bit example

$$a2^2 + b \times c2^2 + d = (i) \times 2^4 + [(ii) - (i) - (iii)] \times 2^2 + (iii)$$

$$(i) = ab$$

$$a = a_1 2^1 + a_0 \times c = c_1 2^1 + c_0$$

$$(ii) = (a + b) \times (c + d)$$

$$a + b = (a_1 + b_1) 2^1 + (a_0 + b_0) \times c + d = (c_1 + d_1) 2^1 + (c_0 + d_0)$$

$$(iii) = bd$$

$$b = b_1 2^1 + b_0 \times d = d_1 2^1 + d_0$$

# Karatsuba Multiplication

4 bit by 4 bit example

$$a2^2 + b \times c2^2 + d = (i) \times 2^4 + [(ii) - (i) - (iii)] \times 2^2 + (iii)$$

$$(i) = ab$$

$$a = a_1 2^1 + a_0 \times c = c_1 2^1 + c_0$$

$$(ii) = (a + b) \times (c + d)$$

$$a + b = (a_1 + b_1) 2^1 + (a_0 + b_0) \times c + d = (c_1 + d_1) 2^1 + (c_0 + d_0)$$

$$(iii) = bd$$

$$b = b_1 2^1 + b_0 \times d = d_1 2^1 + d_0$$

$$a_1 \times c_1 \quad (a_1 + a_0)(c_1 + c_0) \quad a_0 \times c_0$$

$$(a_1 + b_1)(c_1 + d_1) \quad [(a_1 + b_1) + (a_0 + b_0)][(c_1 + d_1) + (c_0 + d_0)] \quad (a_0 + b_0)(c_0 + d_0)$$

$$b_1 \times d_1 \quad (b_1 + b_0)(d_1 + d_0) \quad b_0 \times d_0$$

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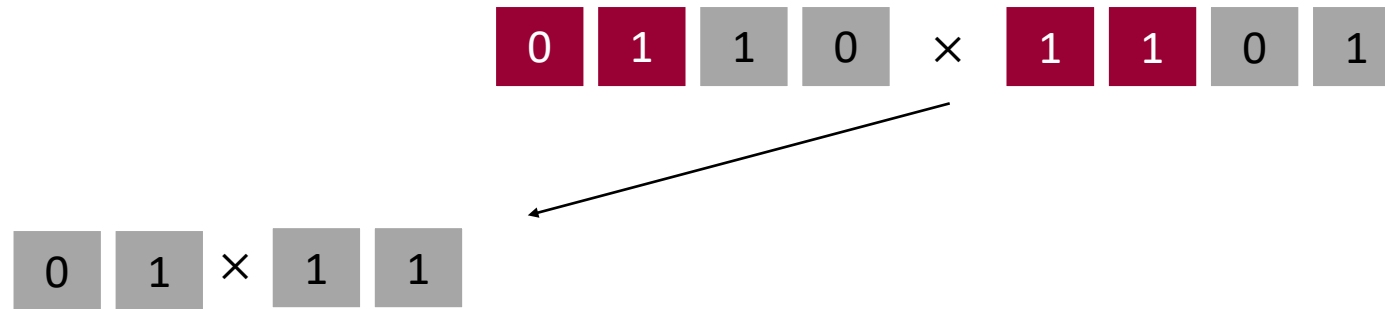
First divide and conquer algorithm for multiplication : Example

$$\begin{array}{|c|c|c|c|} \hline 0 & 1 & 1 & 0 \\ \hline \end{array} \times \begin{array}{|c|c|c|c|} \hline 1 & 1 & 0 & 1 \\ \hline \end{array}$$

$$(a2^2 + b)(c2^2 + d) = ac \times 2^4 + [ad + bc] \times 2^2 + bd$$

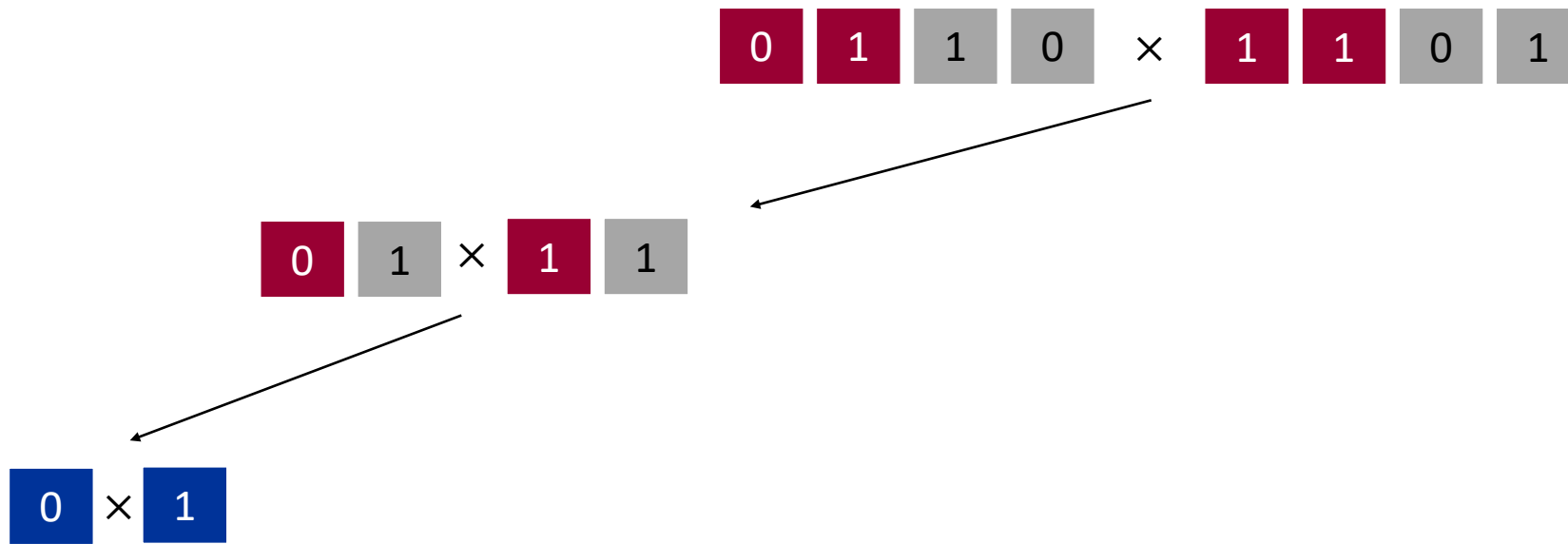


## First divide and conquer algorithm for multiplication : Example



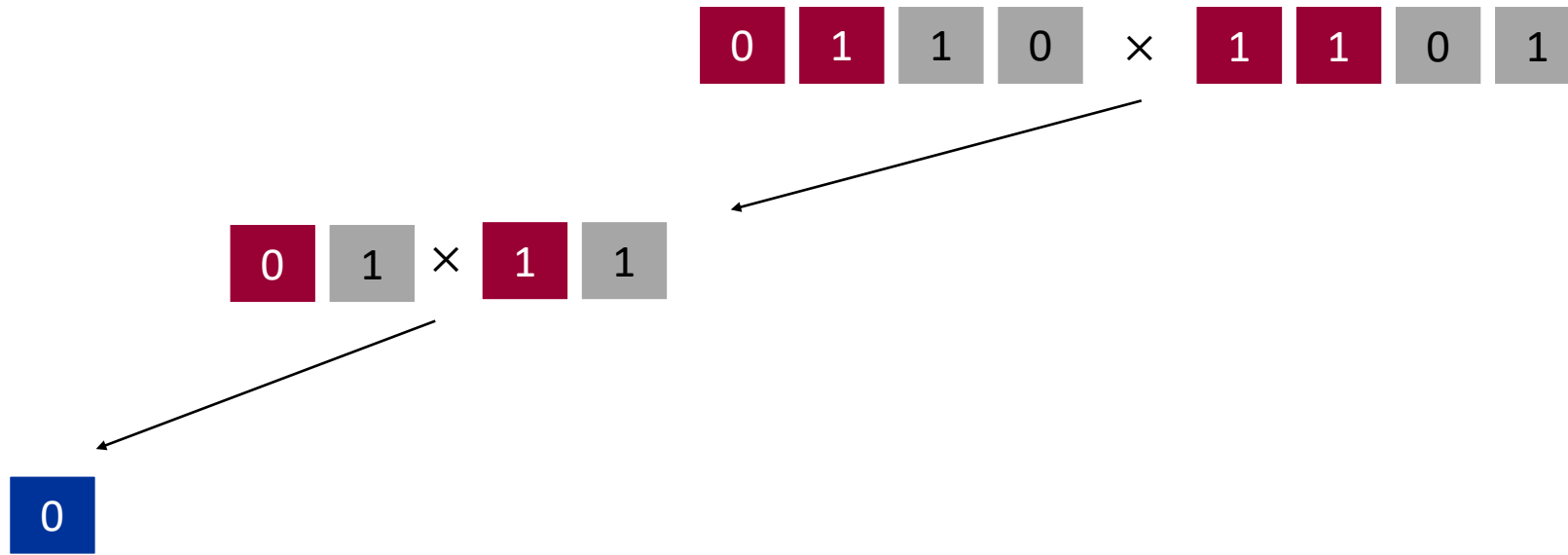
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## First divide and conquer algorithm for multiplication : Example



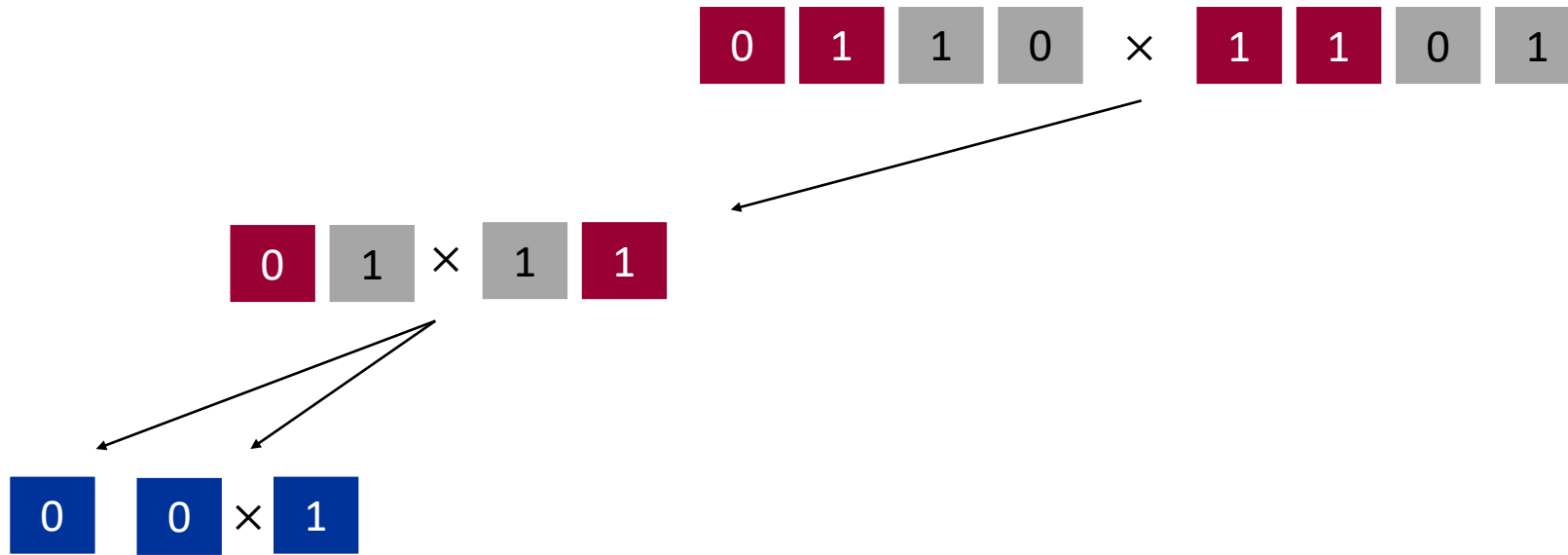
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## First divide and conquer algorithm for multiplication : Example



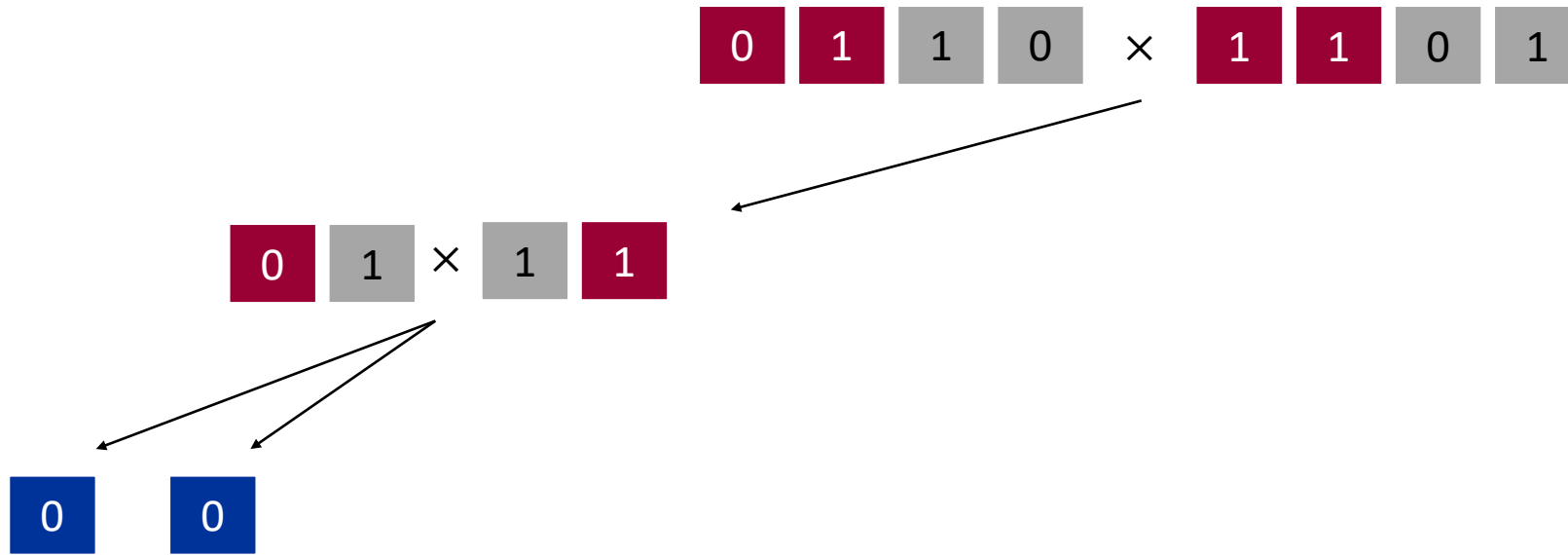
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## First divide and conquer algorithm for multiplication : Example



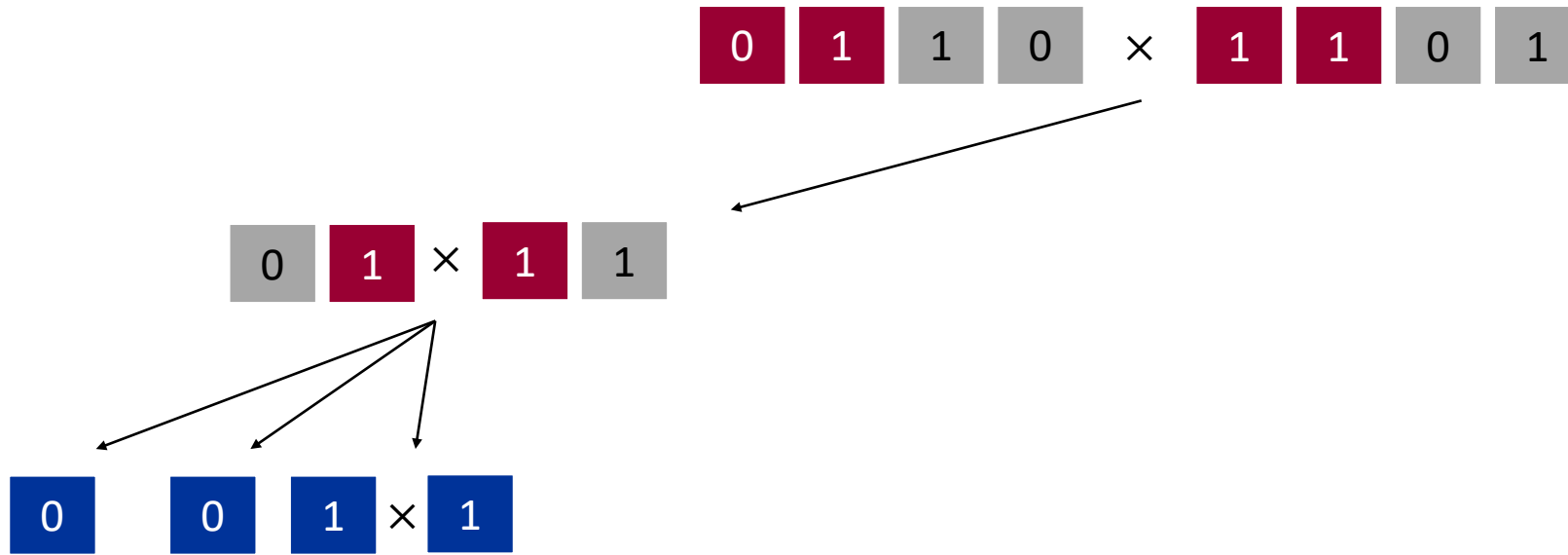
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## First divide and conquer algorithm for multiplication : Example



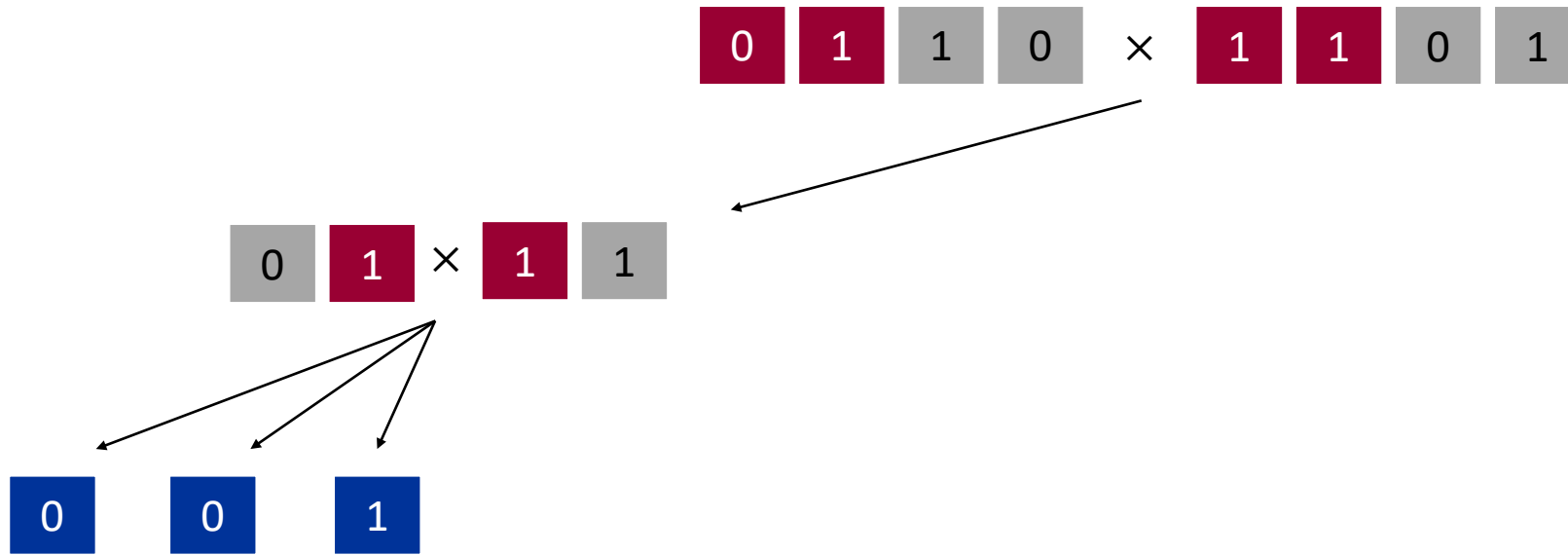
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## First divide and conquer algorithm for multiplication : Example



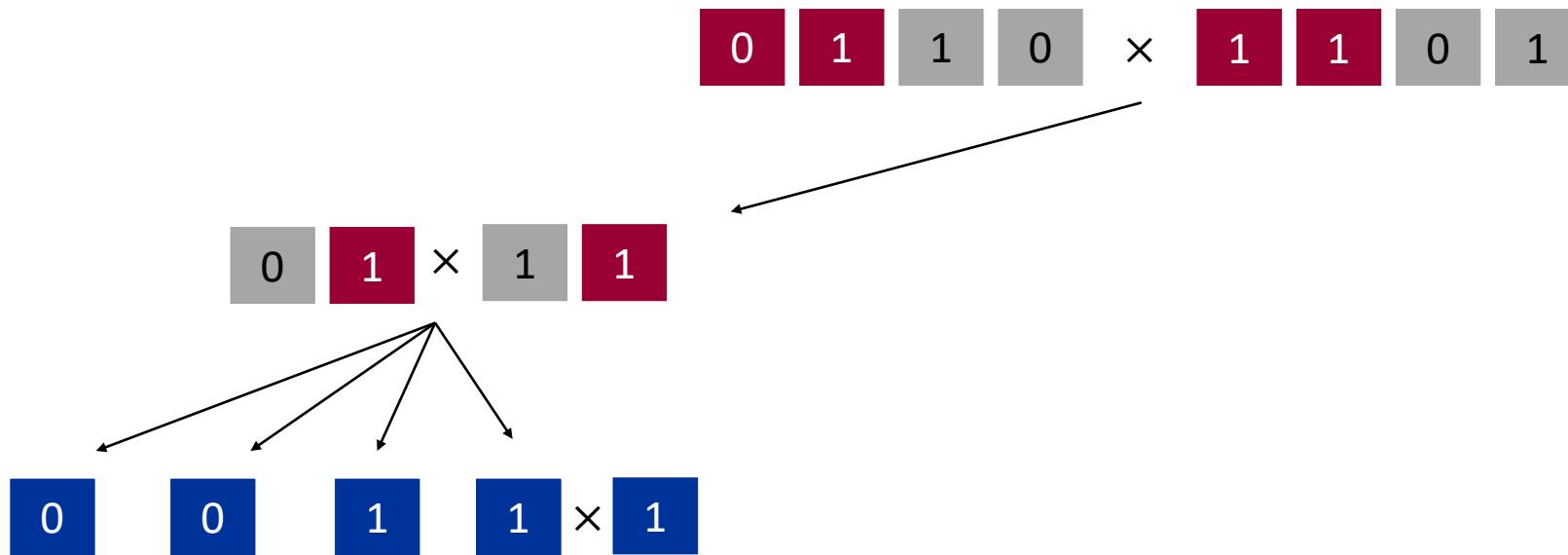
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## First divide and conquer algorithm for multiplication : Example



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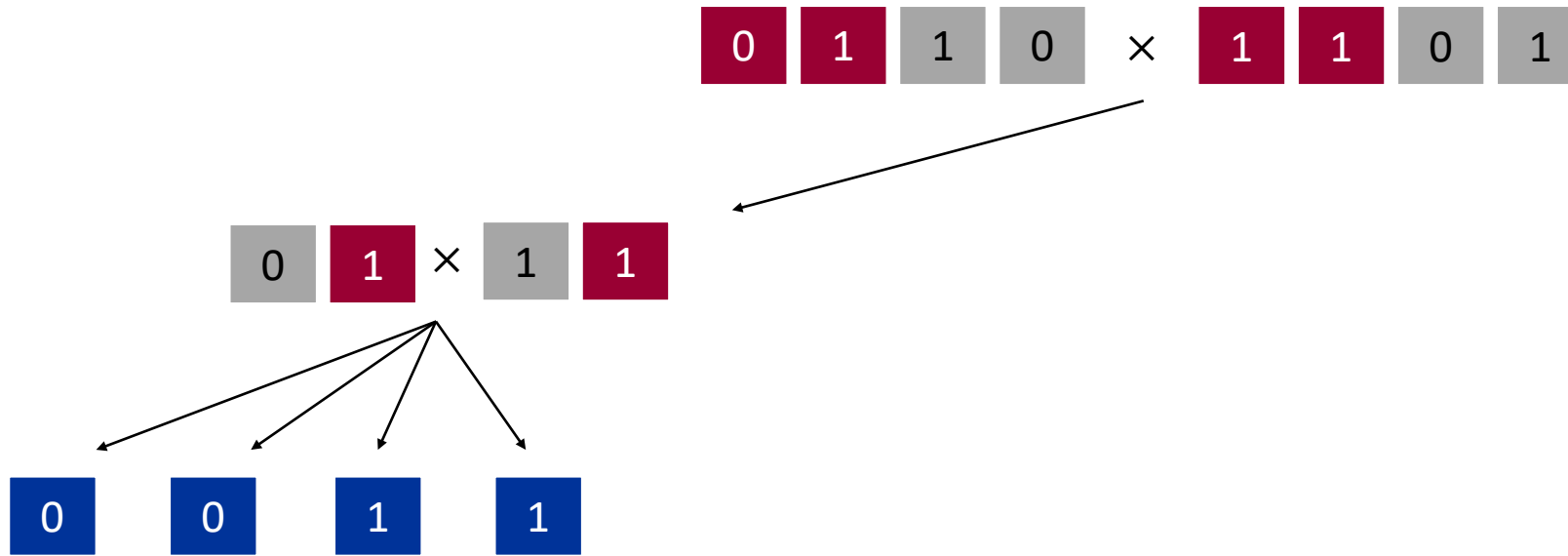
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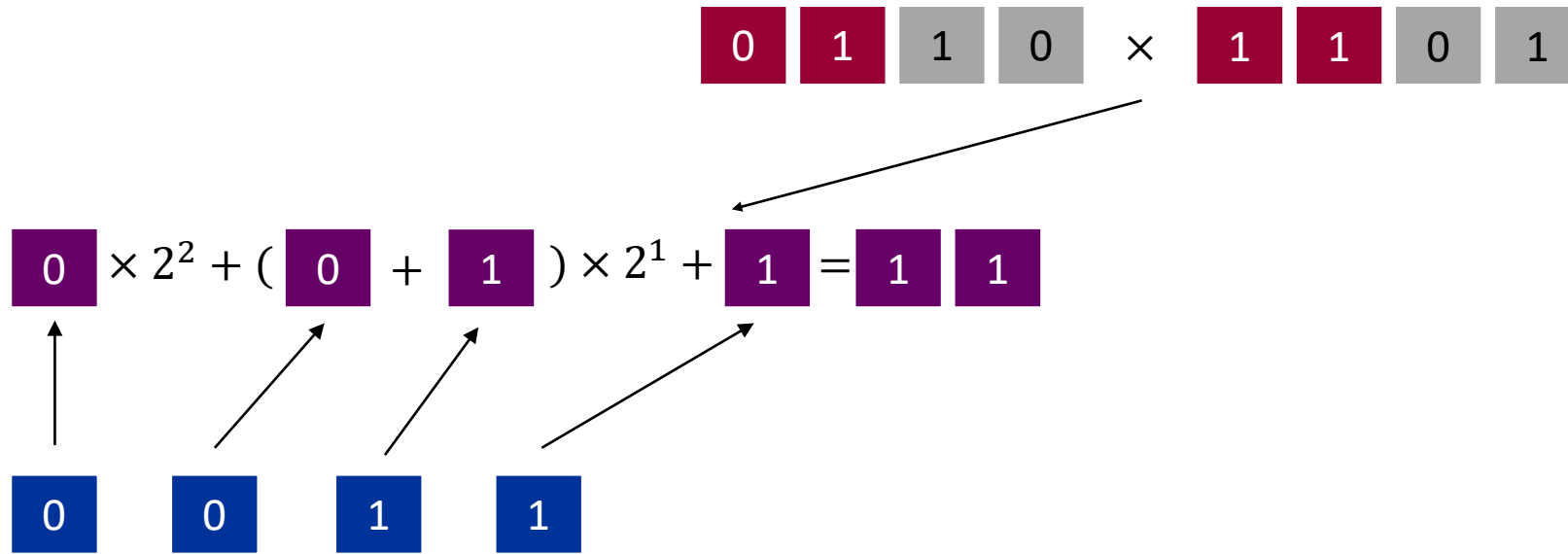


## First divide and conquer algorithm for multiplication : Example



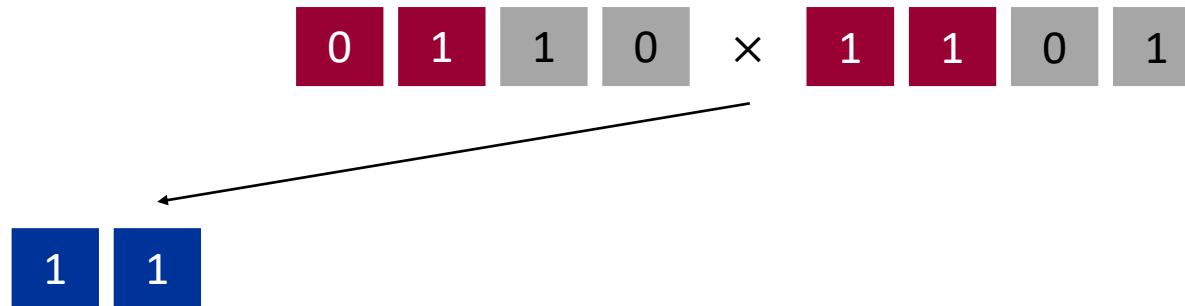
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# First divide and conquer algorithm for multiplication : Example



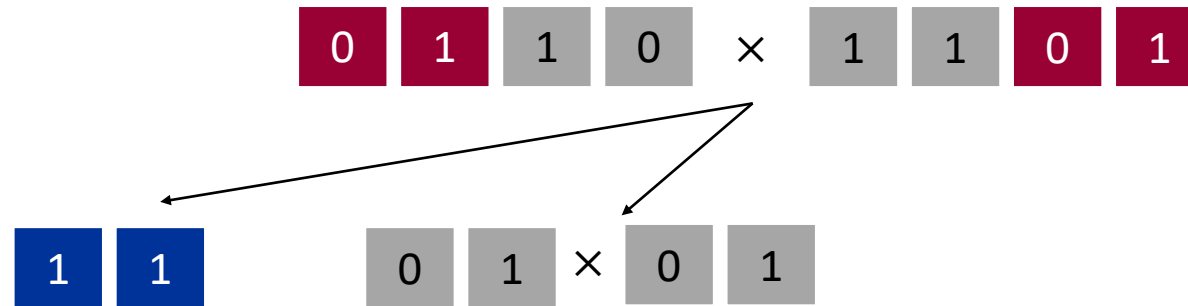
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First divide and conquer algorithm for multiplication : Example



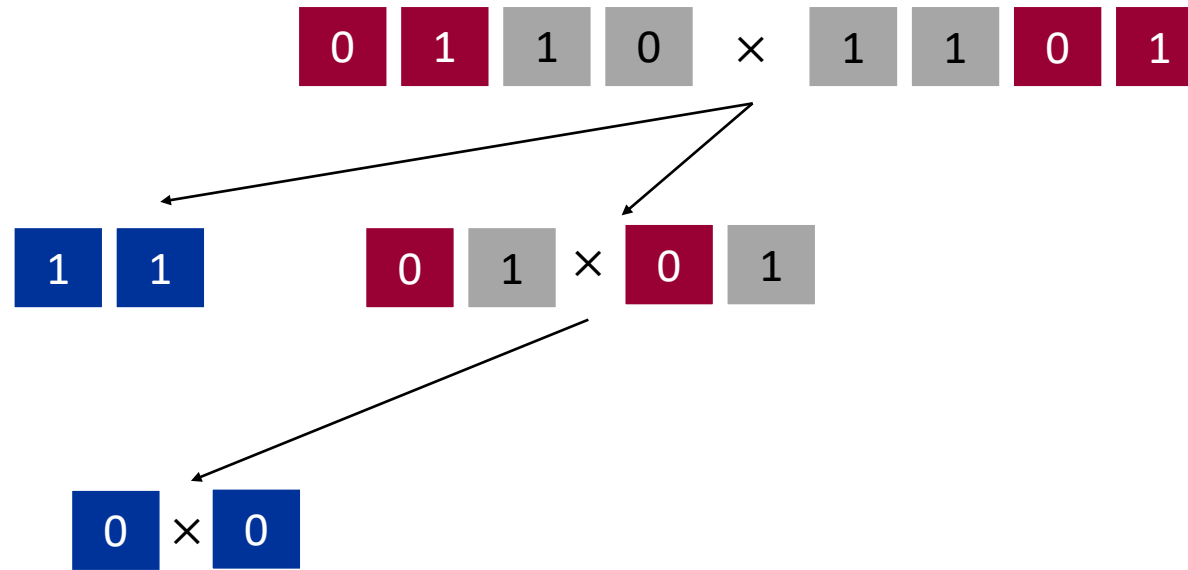
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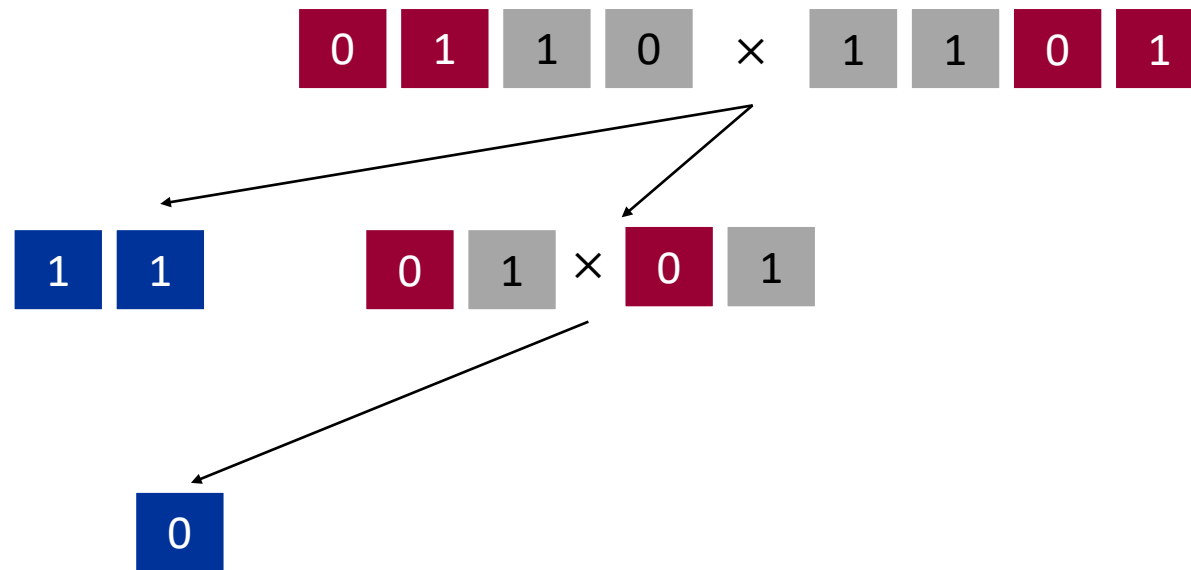
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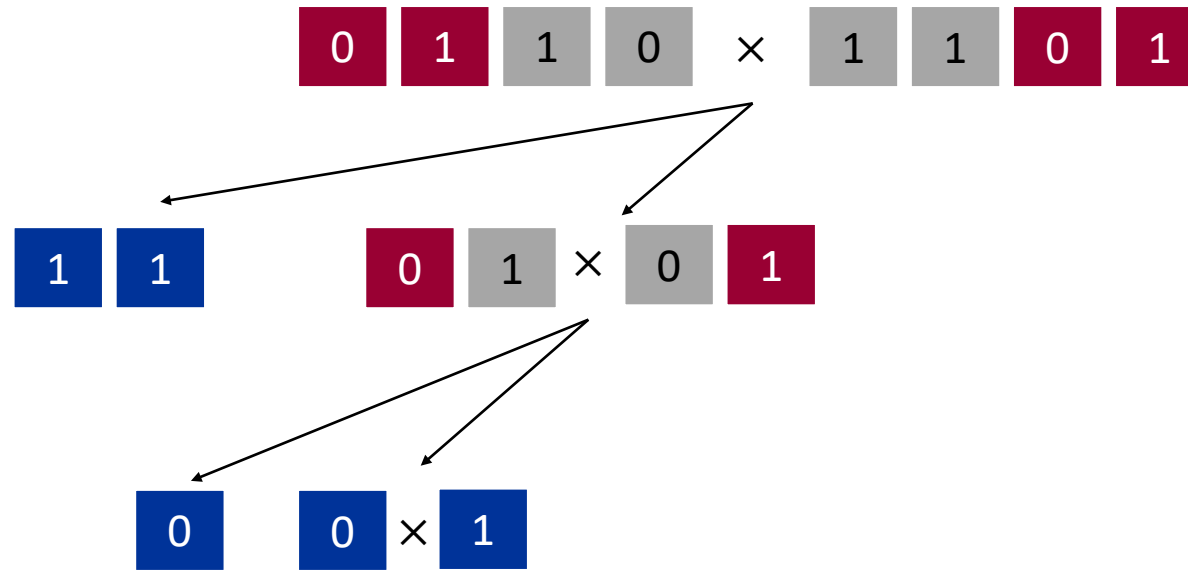
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First divide and conquer algorithm for multiplication : Example



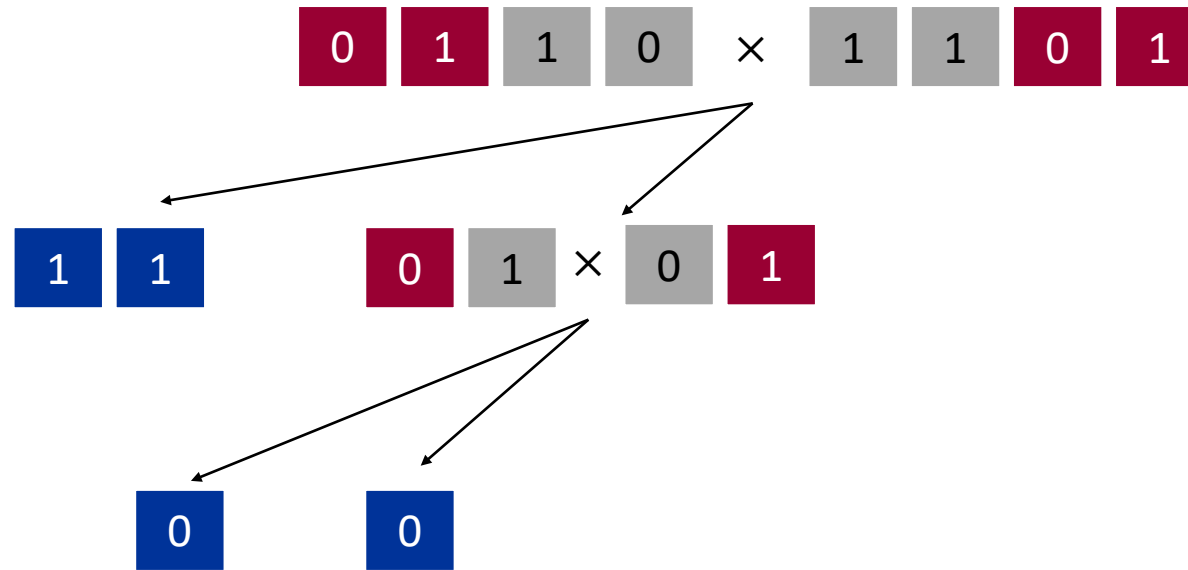
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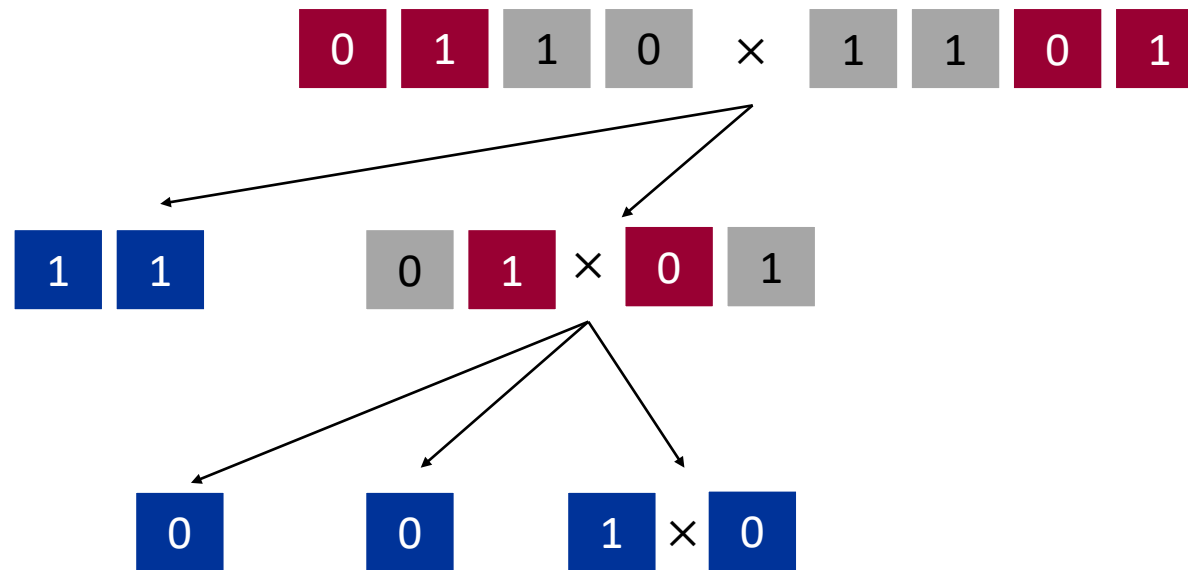
First divide and conquer algorithm for multiplication : Example



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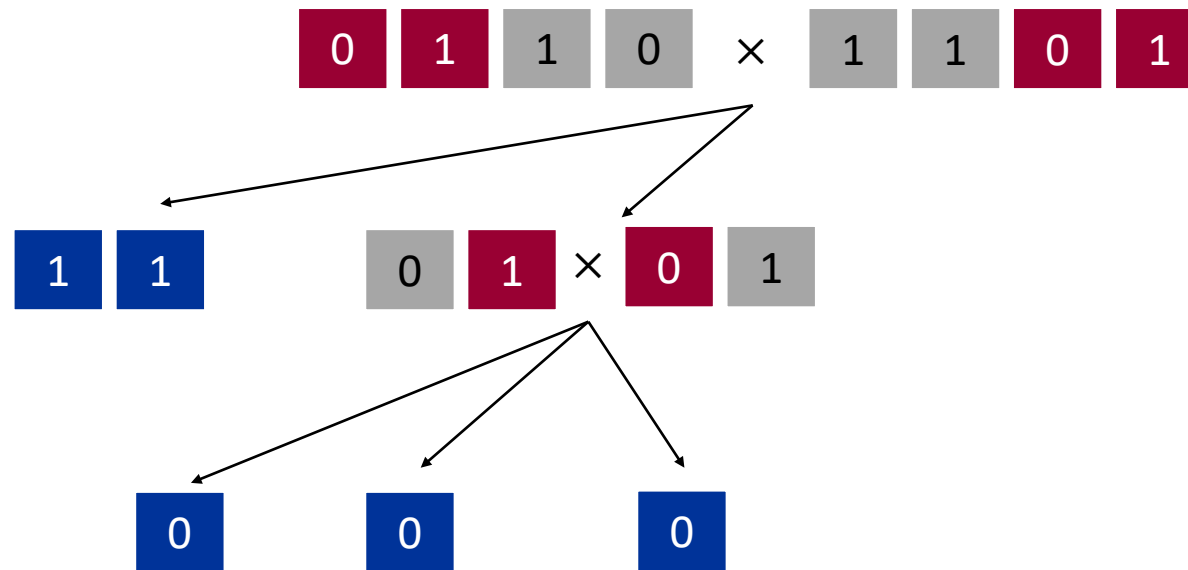


First divide and conquer algorithm for multiplication : Example



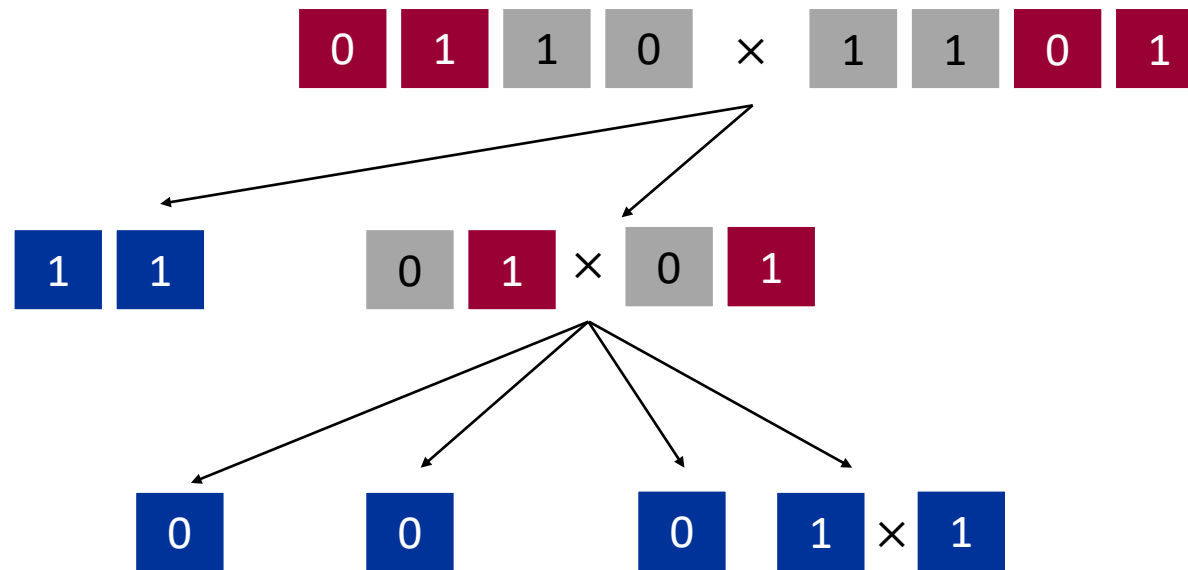
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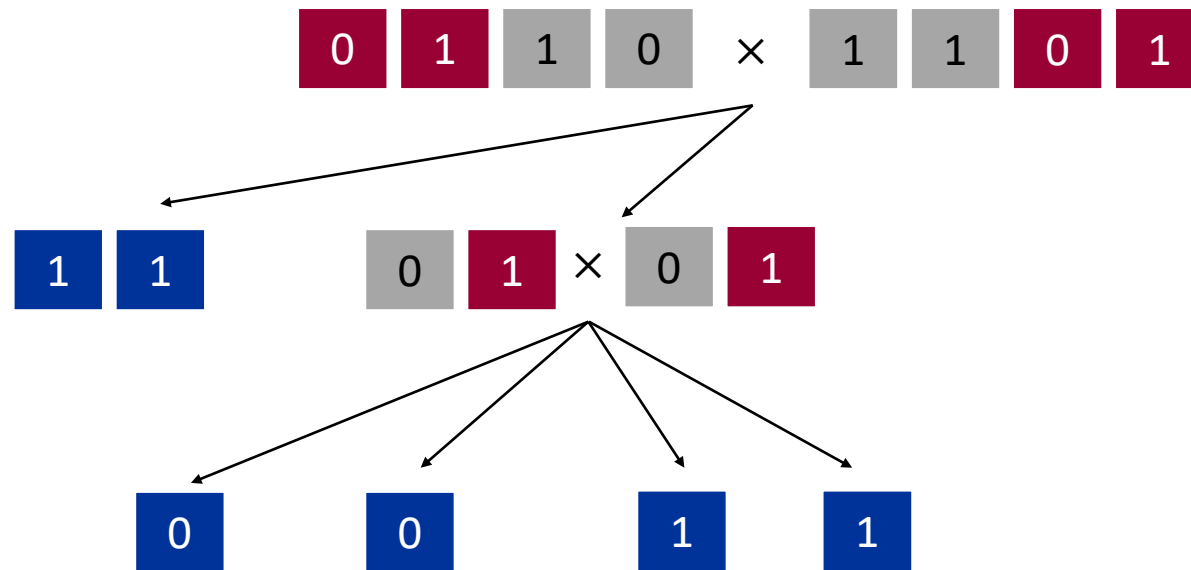
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First divide and conquer algorithm for multiplication : Example



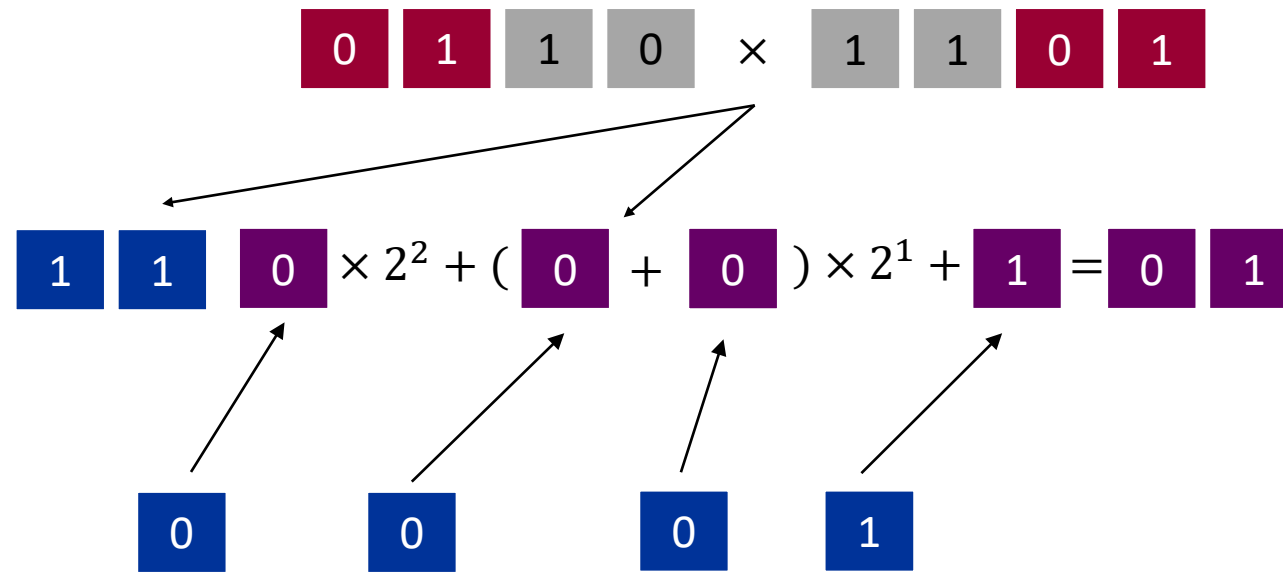
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First divide and conquer algorithm for multiplication : Example

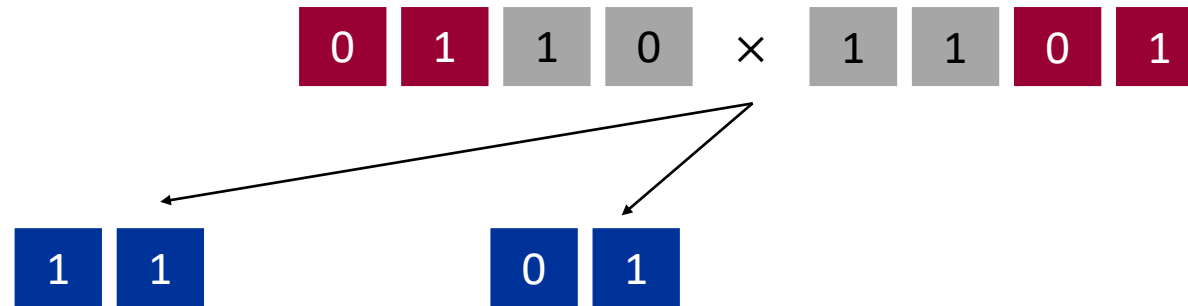


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## First divide and conquer algorithm for multiplication : Example

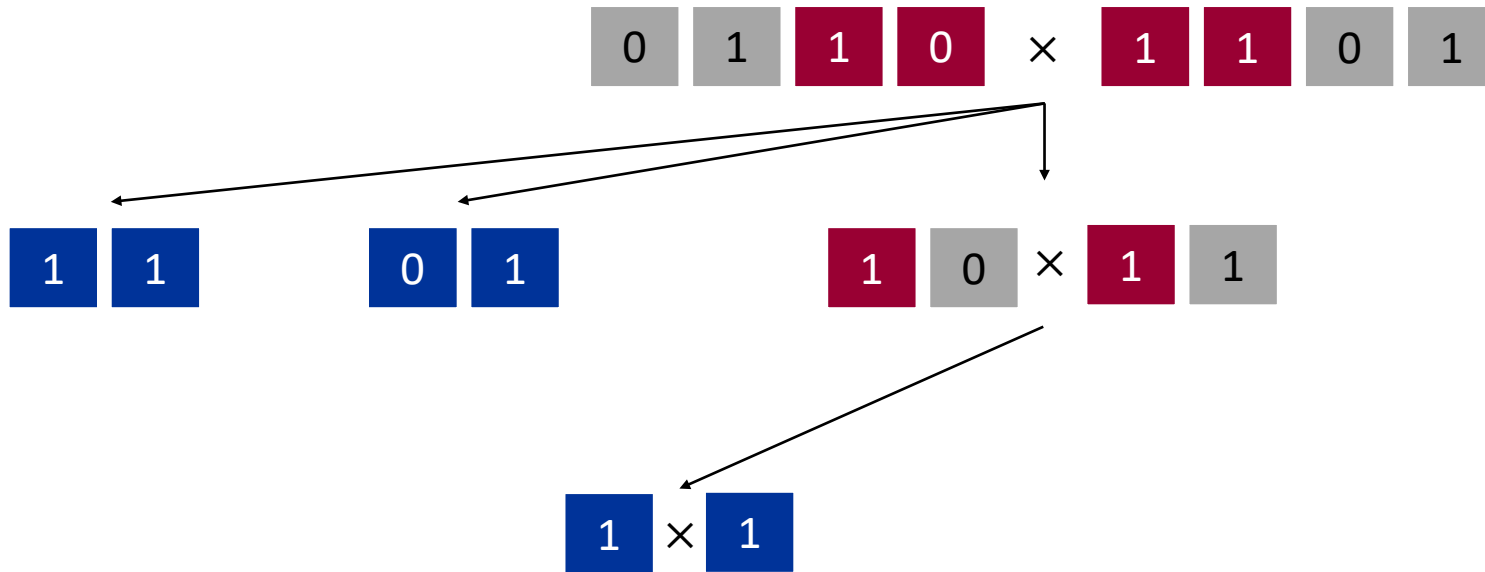


First divide and conquer algorithm for multiplication : Example



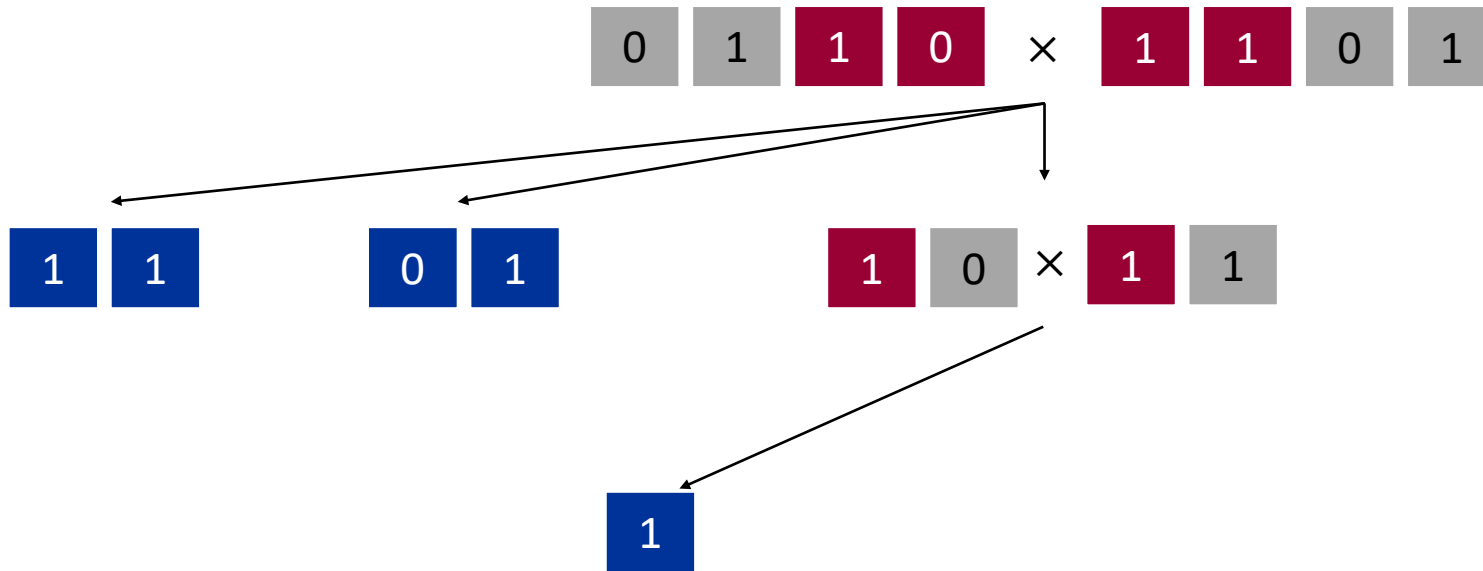
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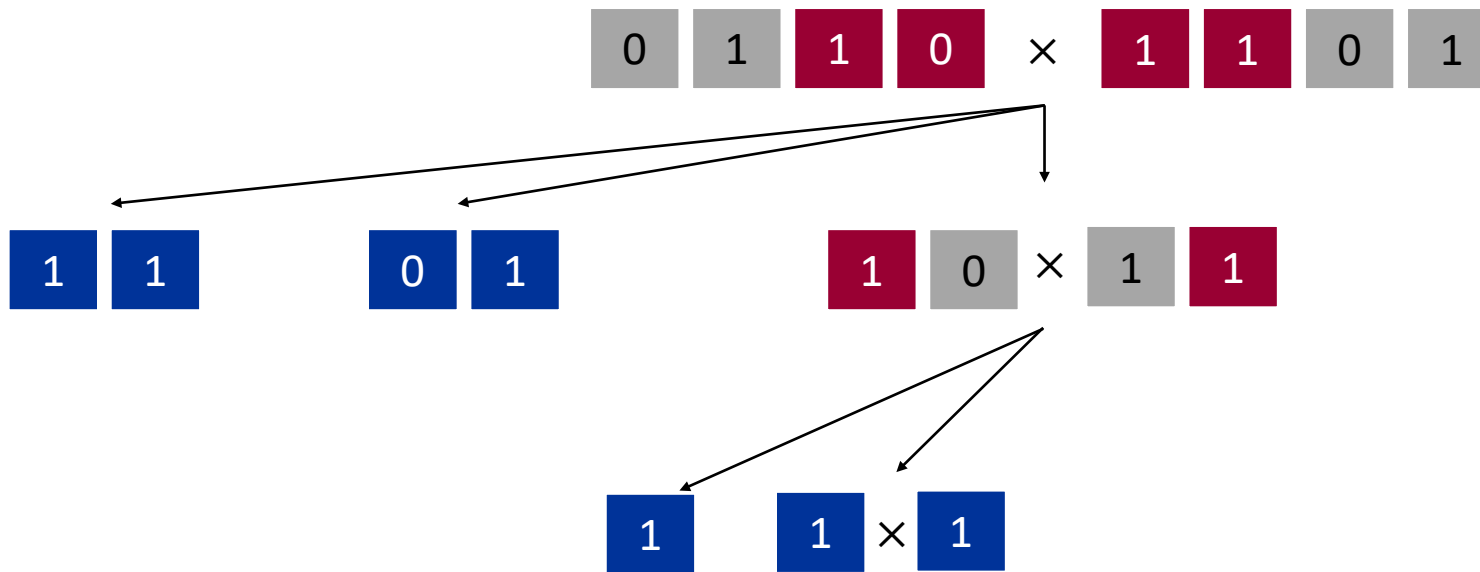
# First divide and conquer algorithm for multiplication : Example



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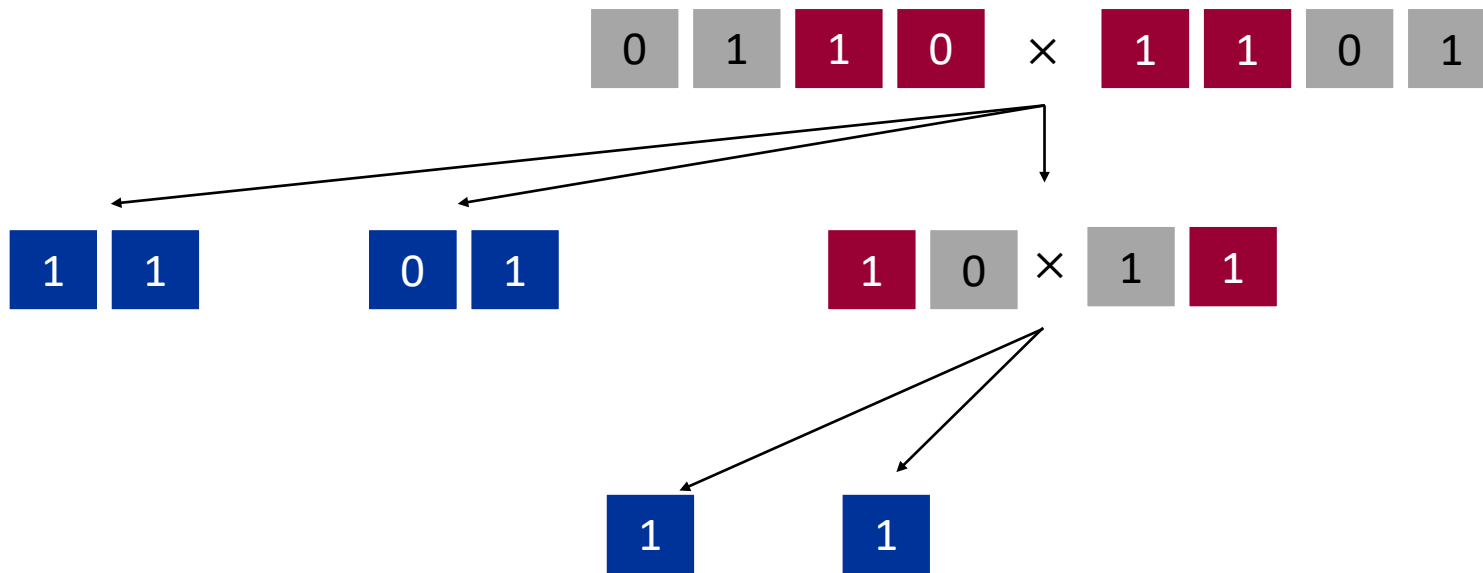


# First divide and conquer algorithm for multiplication : Example



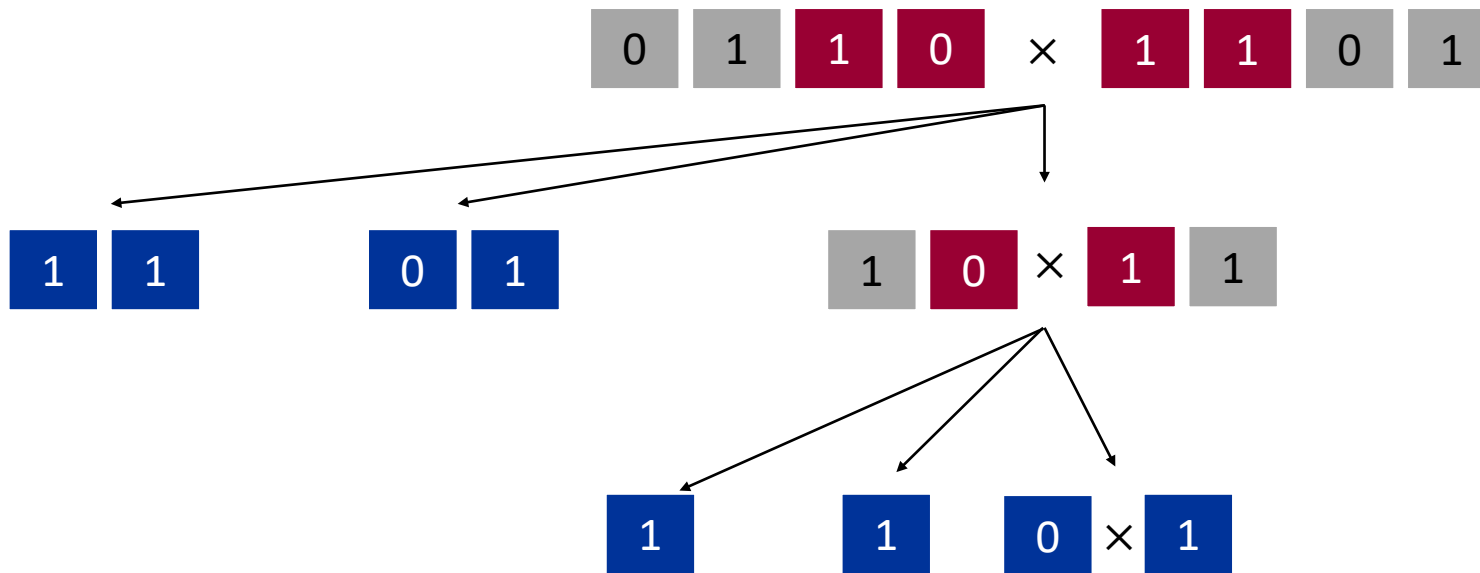
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# First divide and conquer algorithm for multiplication : Example



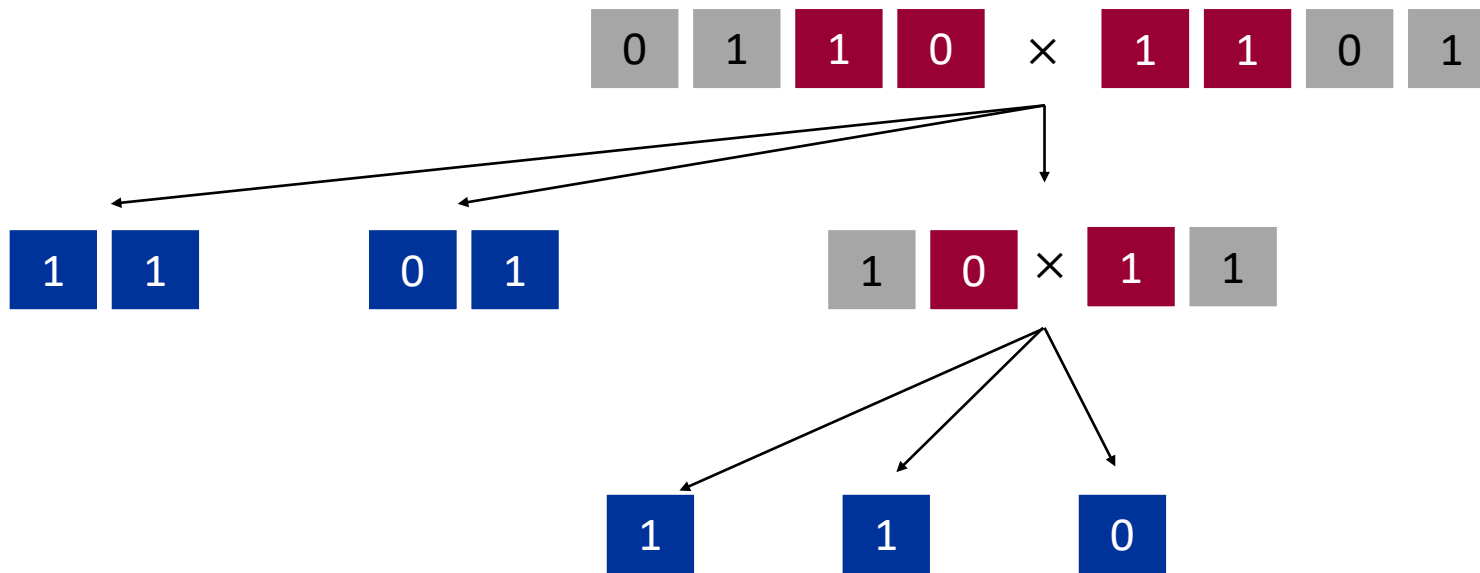
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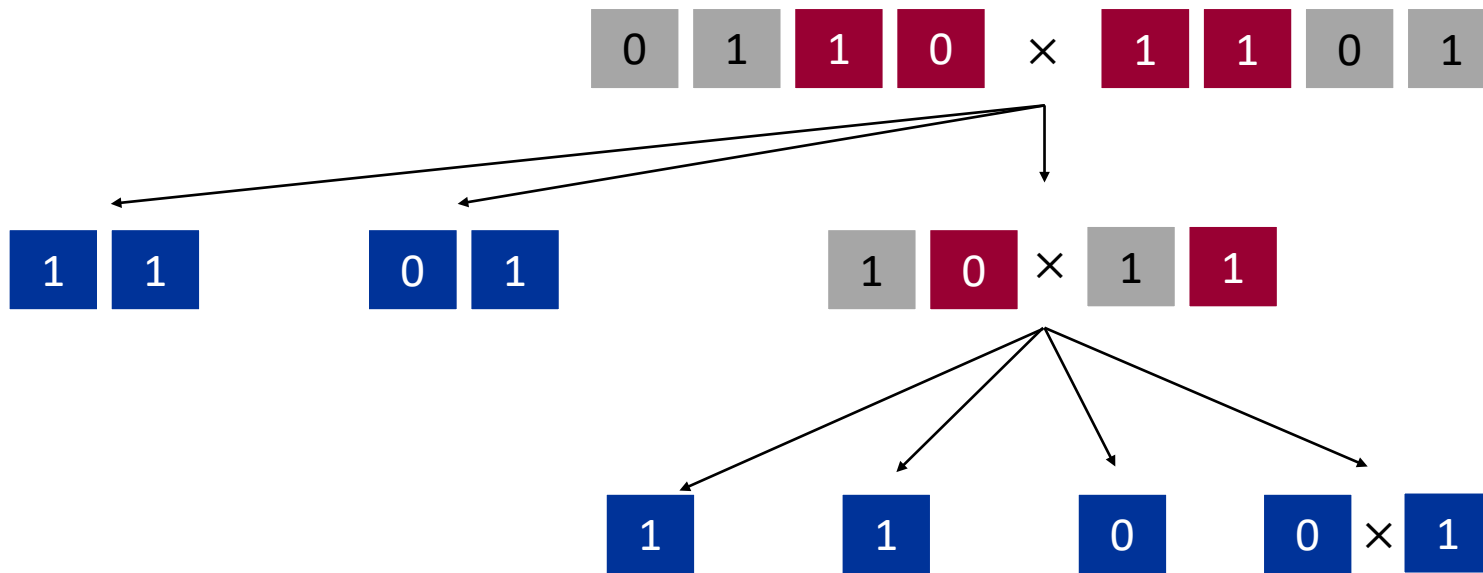
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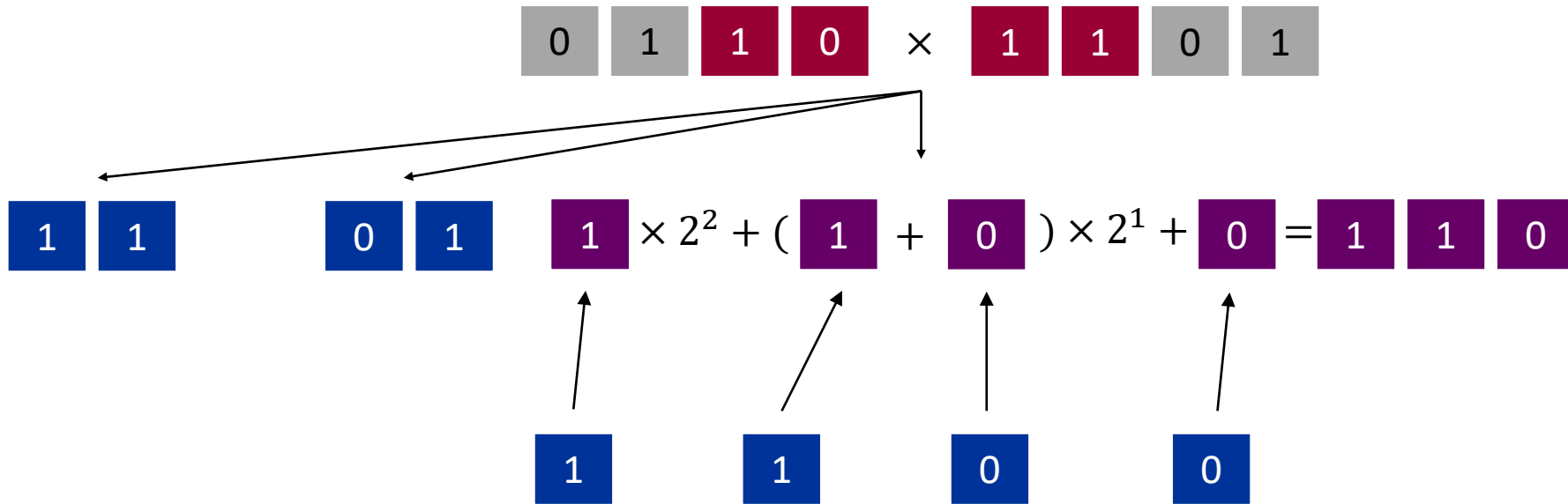
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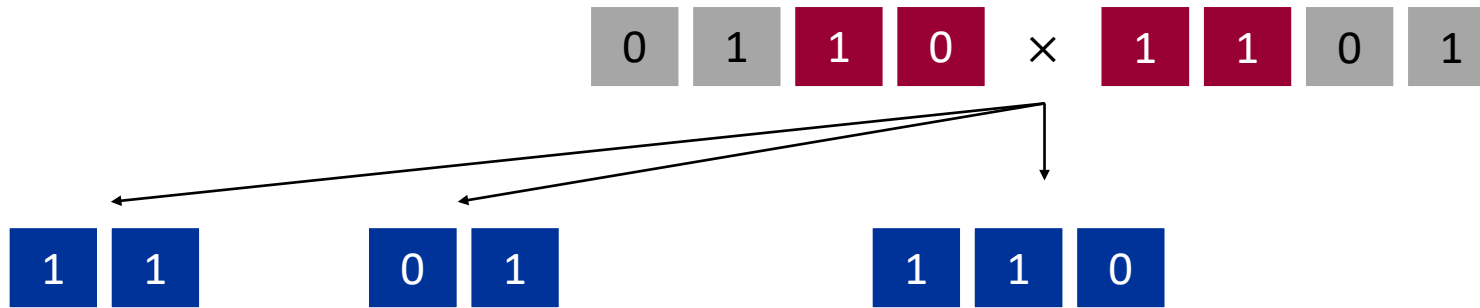
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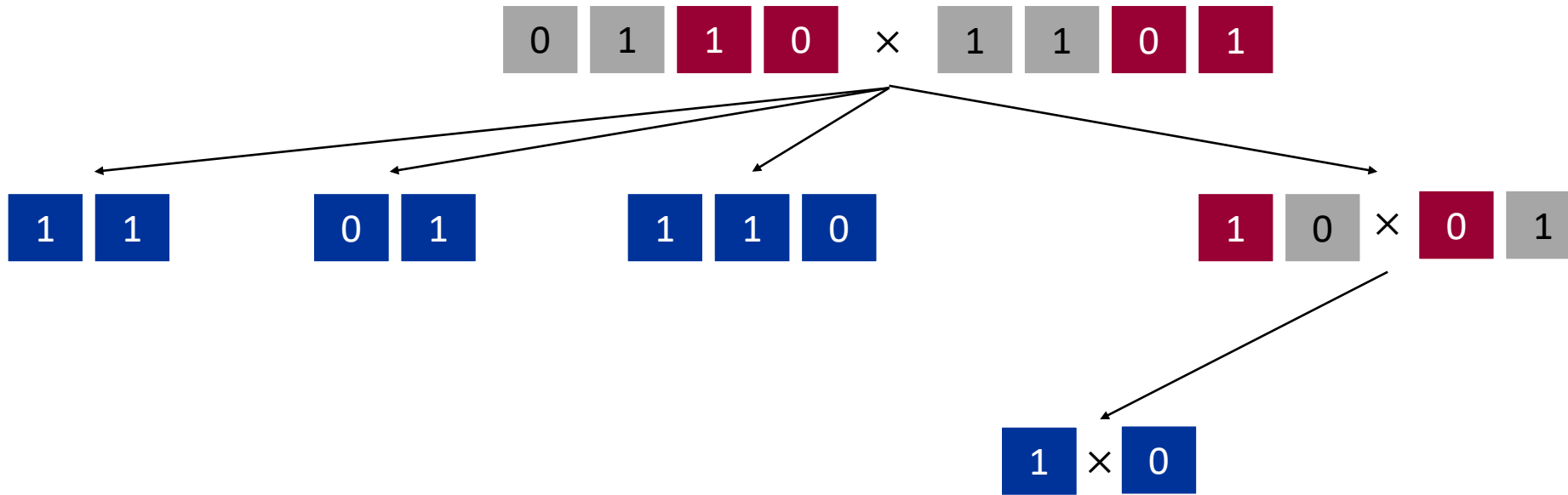
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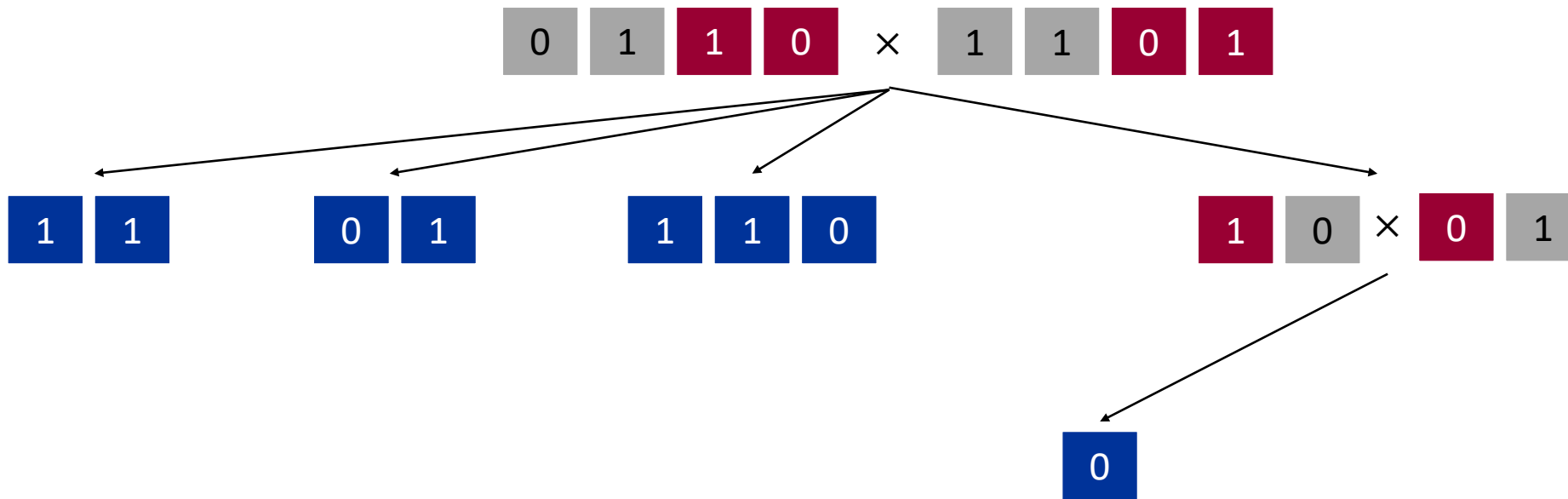
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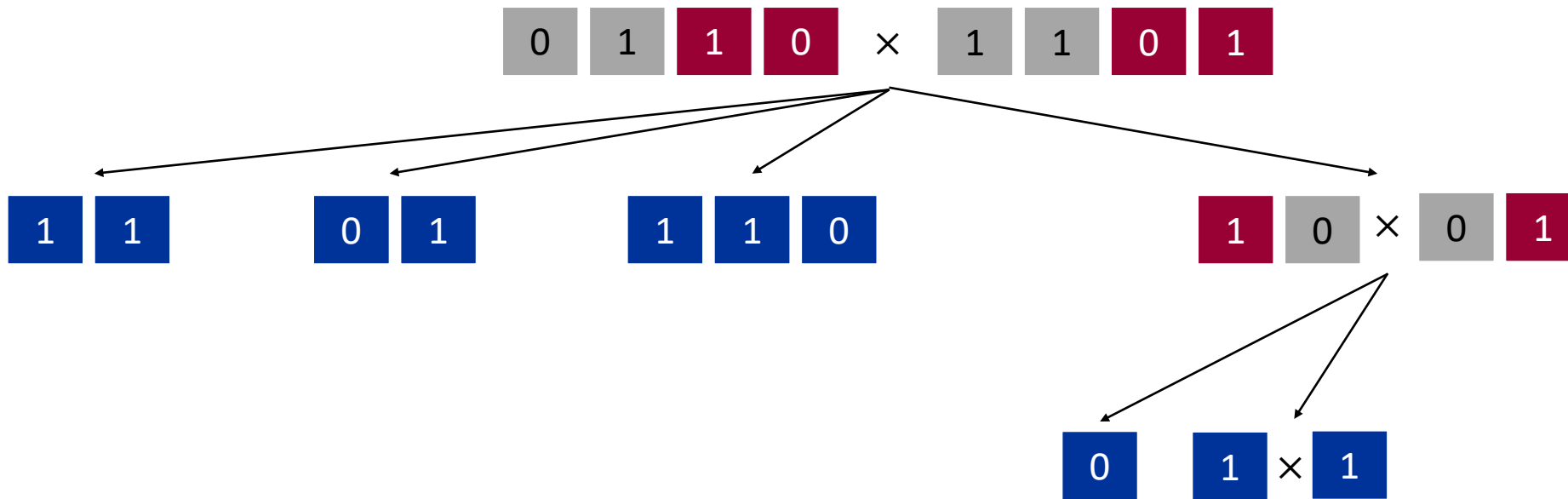


# First divide and conquer algorithm for multiplication : Example



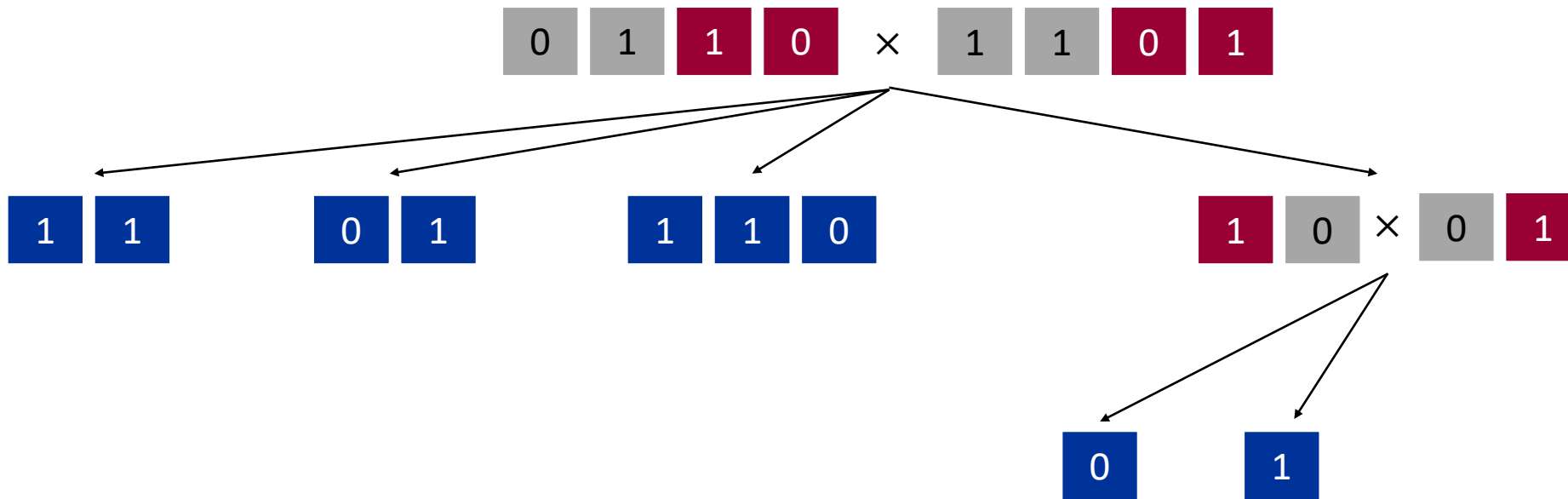
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# First divide and conquer algorithm for multiplication : Example



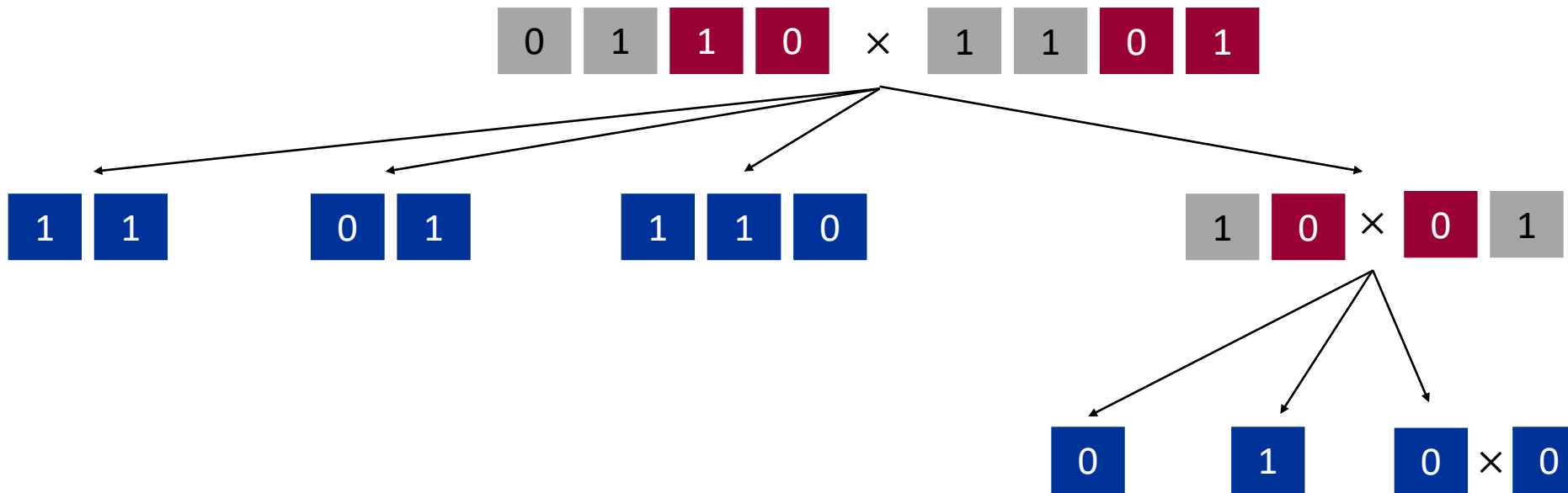
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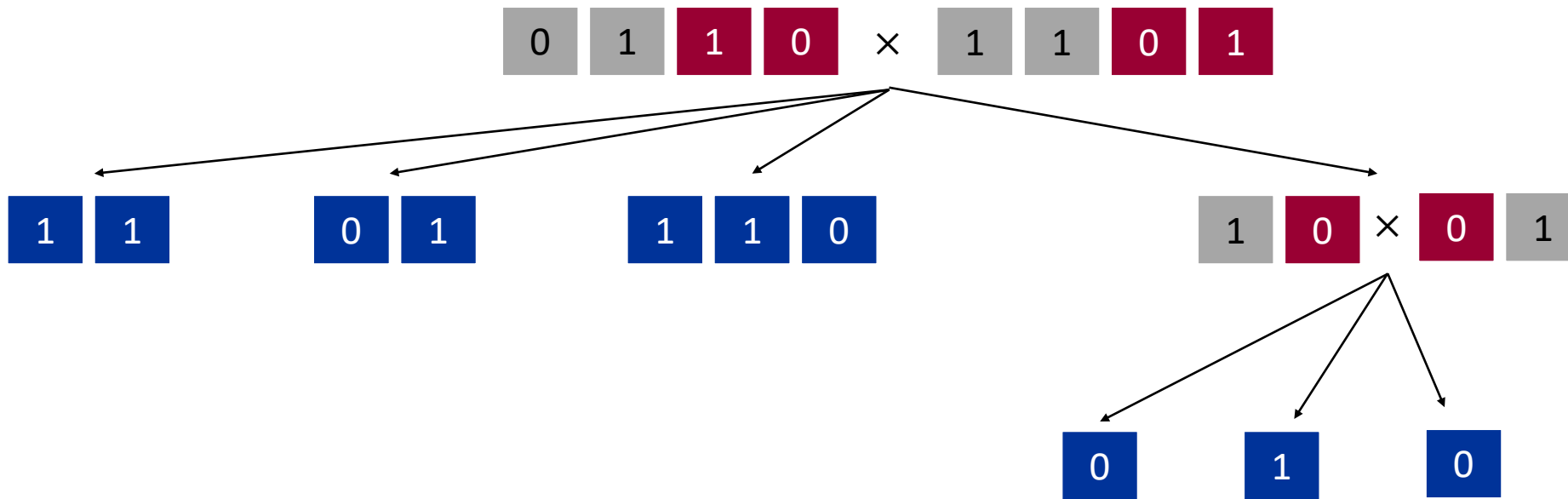
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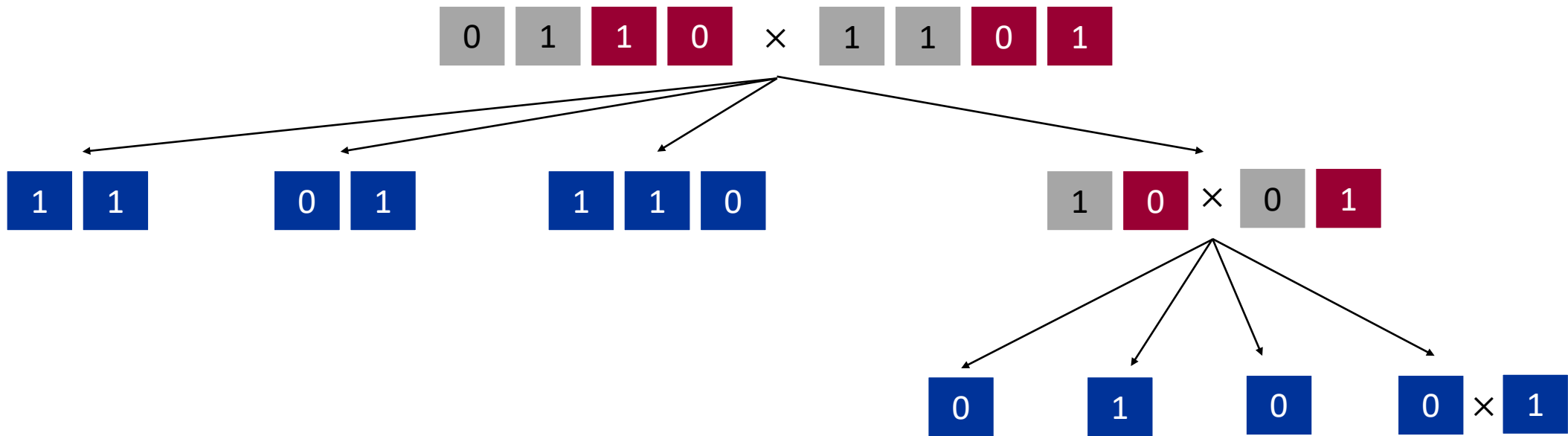
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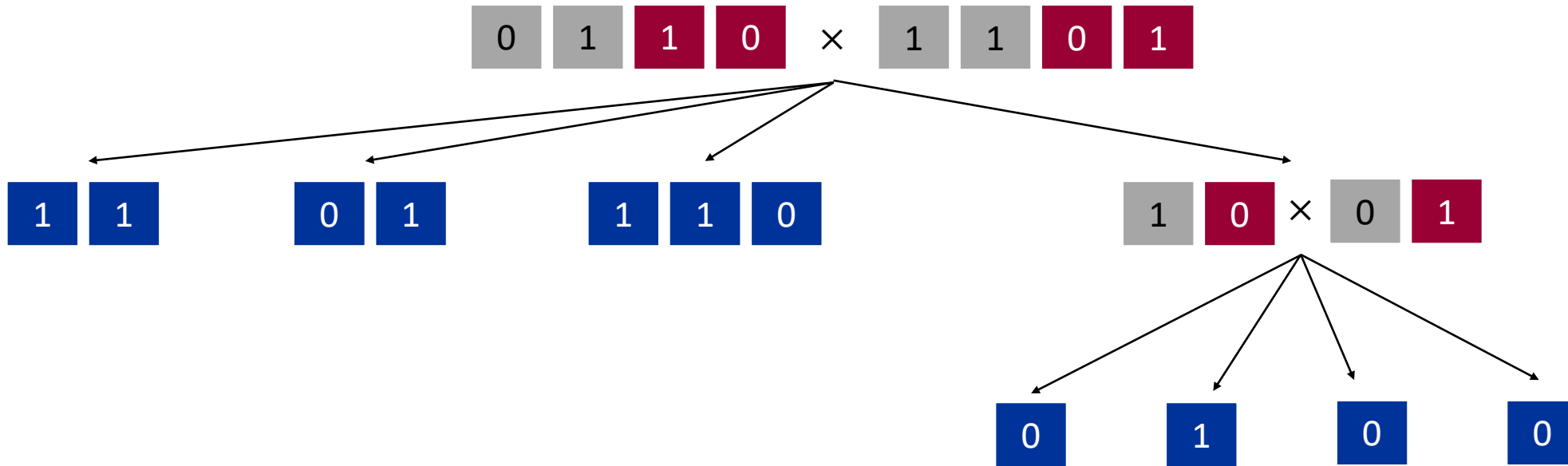
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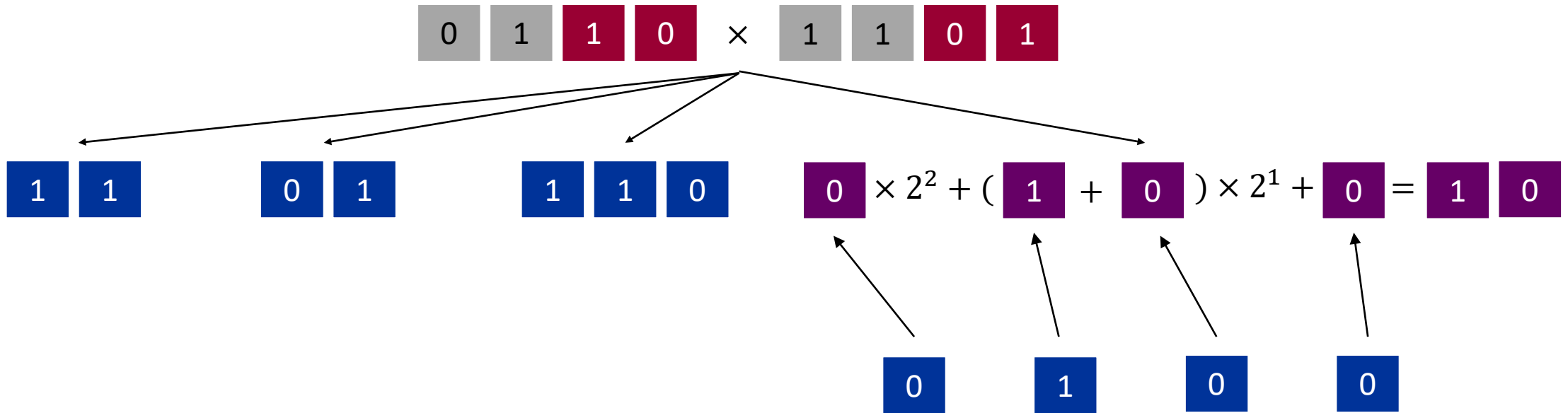
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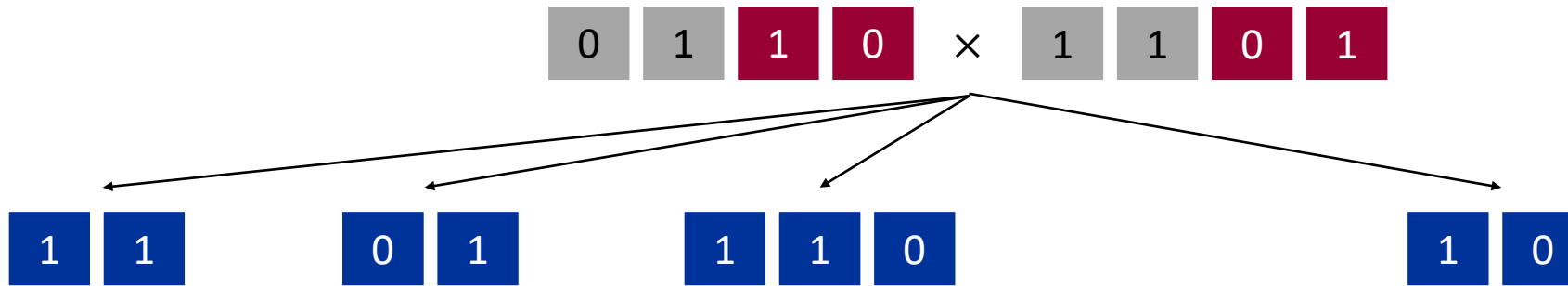
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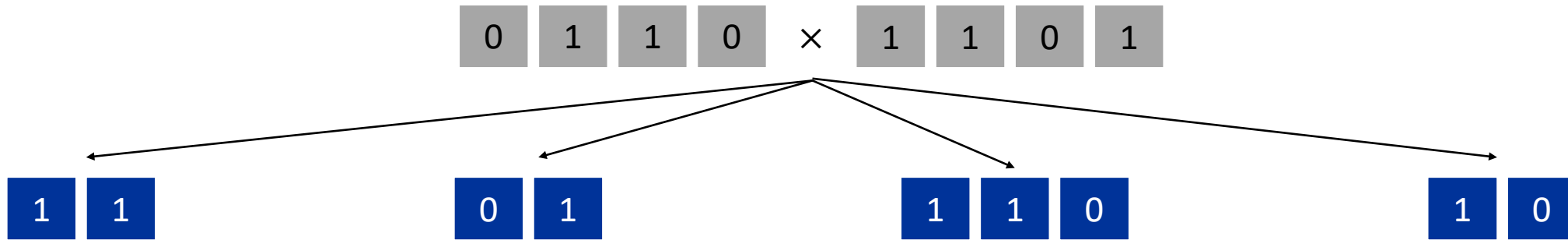


## First divide and conquer algorithm for multiplication : Example



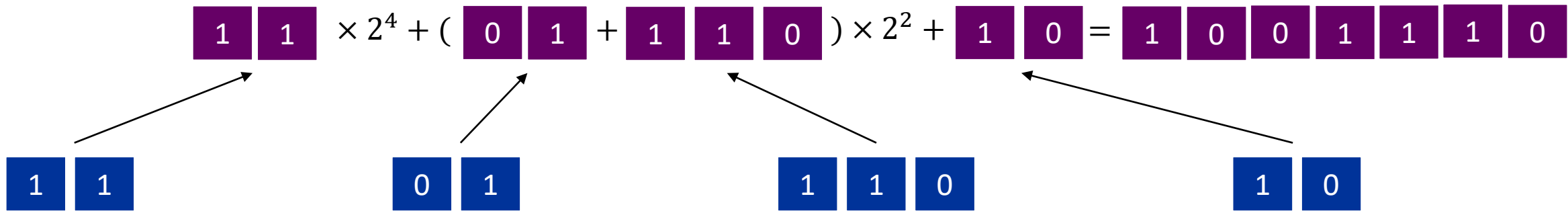
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First divide and conquer algorithm for multiplication : Example

$$\begin{array}{|c|c|c|c|} \hline 0 & 1 & 1 & 0 \\ \hline \end{array} \times \begin{array}{|c|c|c|c|} \hline 1 & 1 & 0 & 1 \\ \hline \end{array} = \begin{array}{|c|c|c|c|c|c|c|} \hline 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ \hline \end{array}$$

$$(a2^2 + b)(c2^2 + d) = ac \times 2^4 + [ad + bc] \times 2^2 + bd$$

# Outline

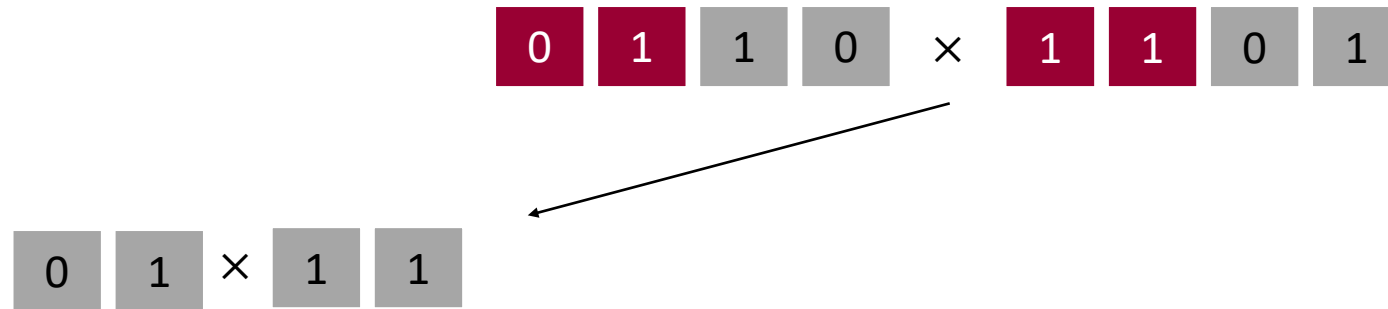
- Quick D&C Integer Multiplication Review
- High level Example for 4X4 D&C simple integer multiplication
- High level Example for 4X4 D&C Karatsuba integer multiplication
- Full worked example of 4X4 simple integer multiplication
- Full worked example of 4X4 Karatsuba integer multiplication

## Karatsuba Multiplication: Example

$$\begin{array}{|c|c|c|c|} \hline 0 & 1 & 1 & 0 \\ \hline \end{array} \times \begin{array}{|c|c|c|c|} \hline 1 & 1 & 0 & 1 \\ \hline \end{array}$$

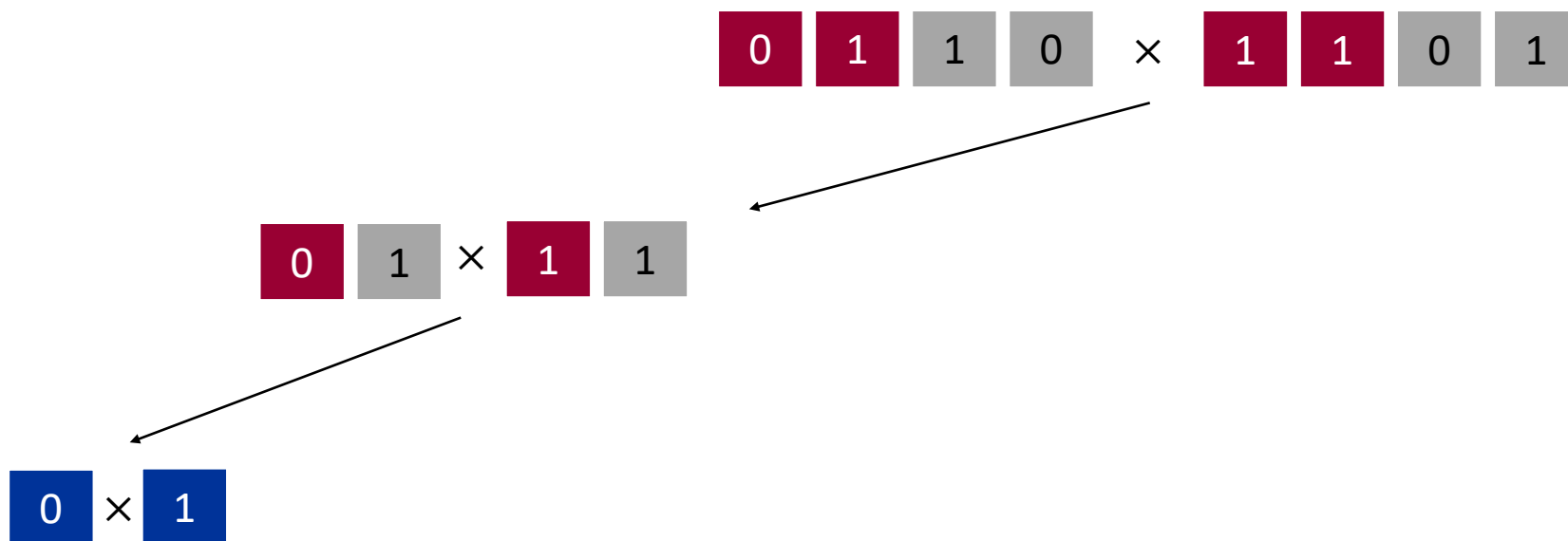
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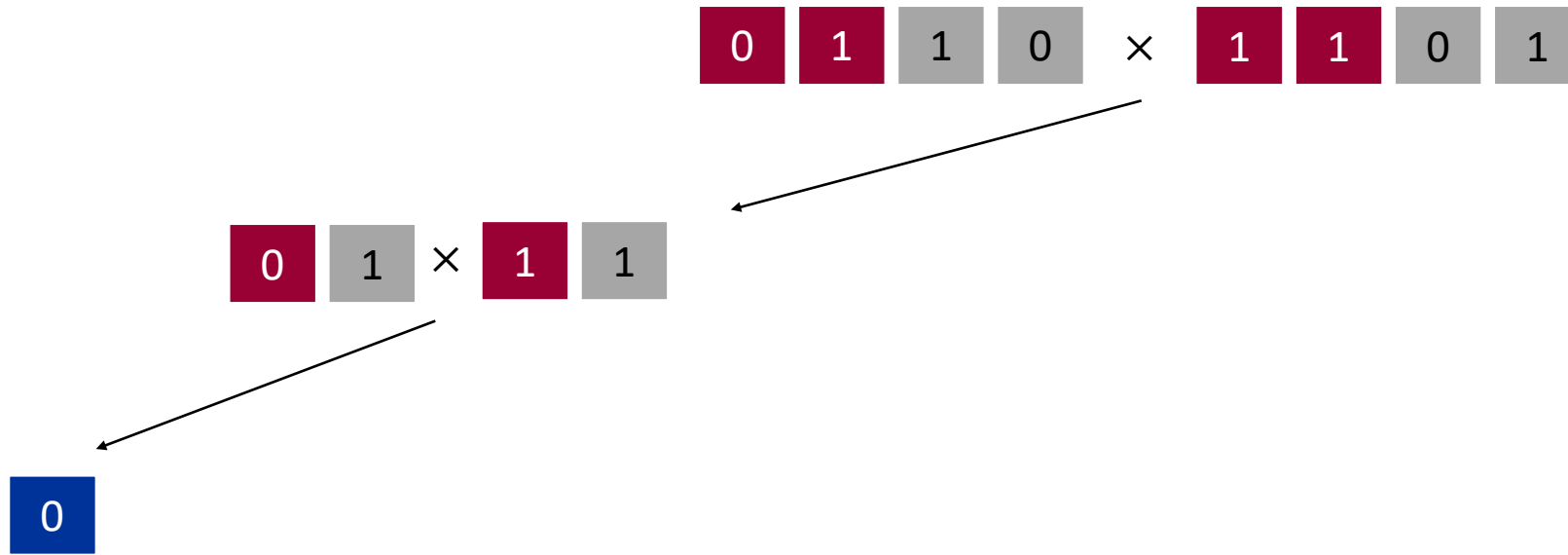
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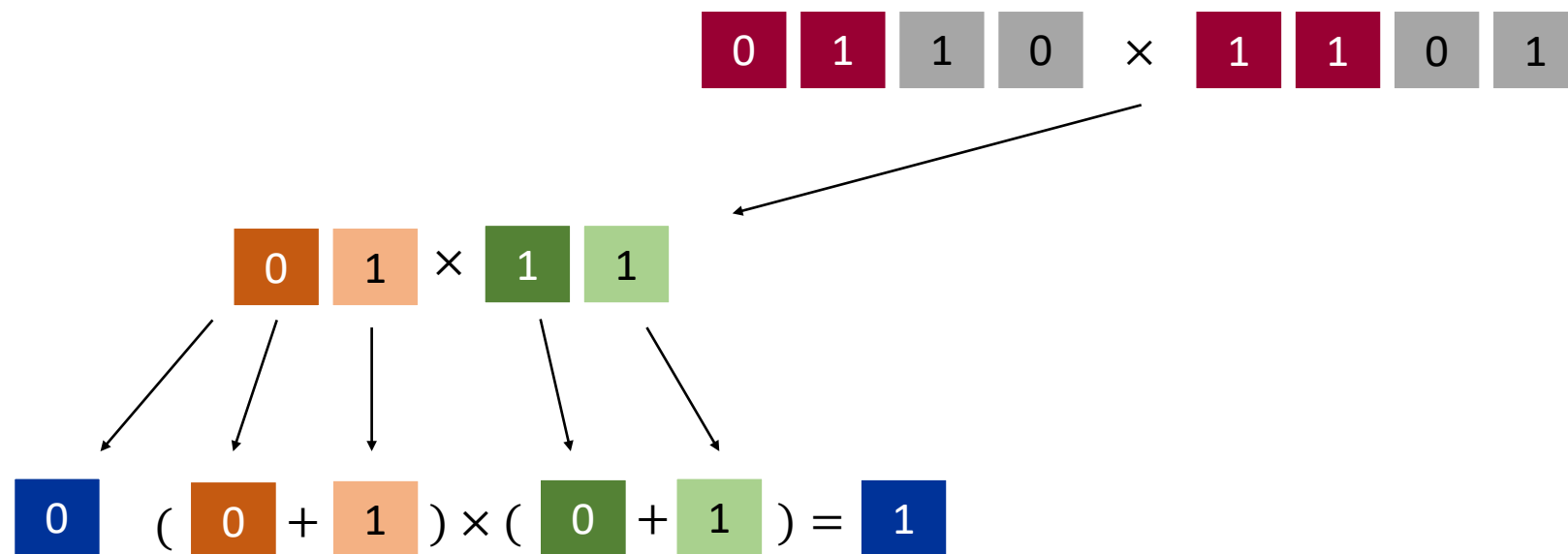


# Karatsuba Multiplication: Example



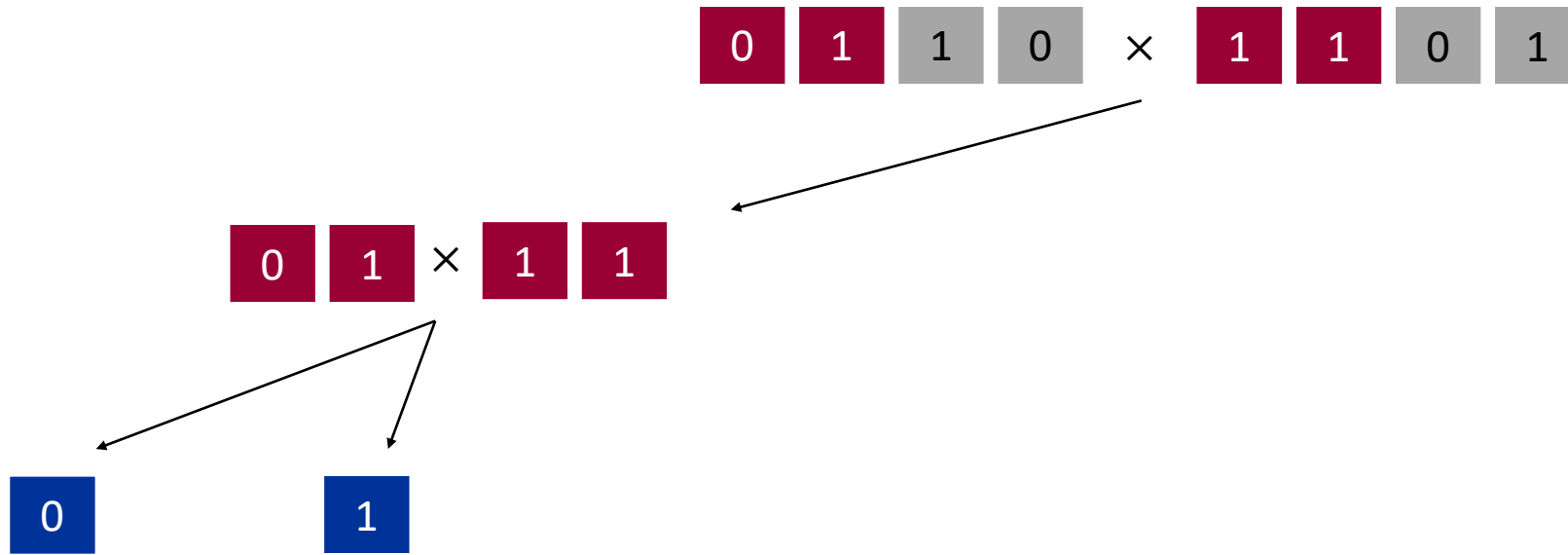
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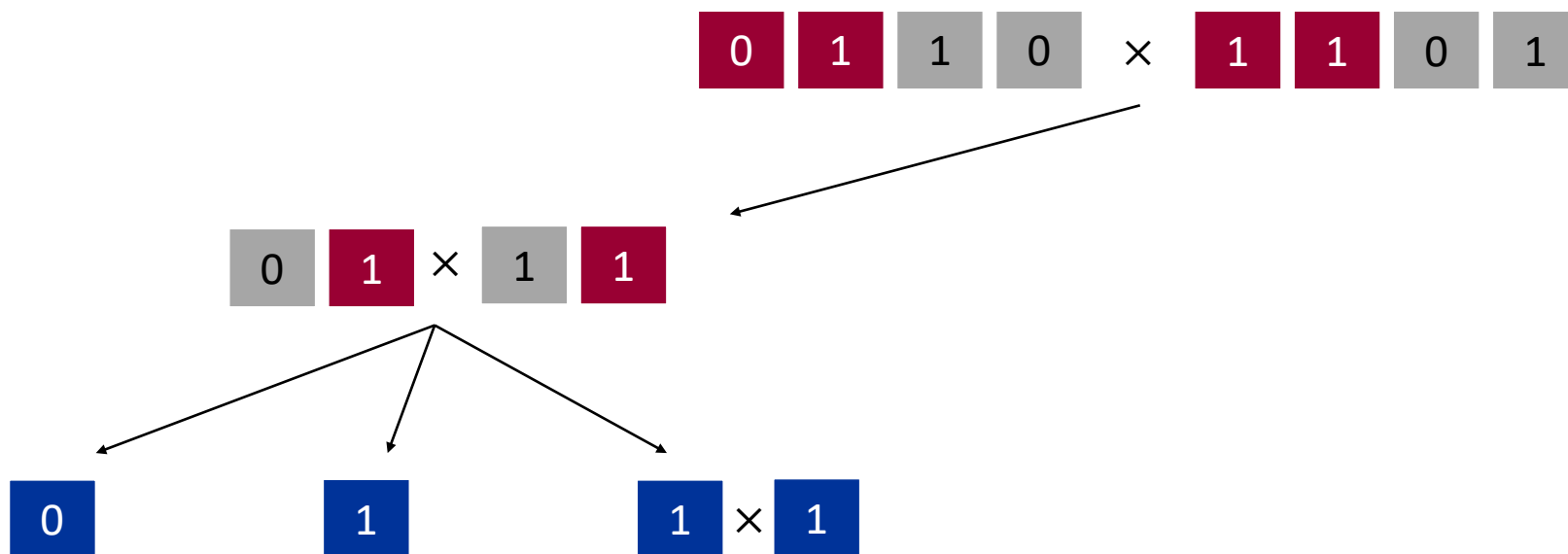
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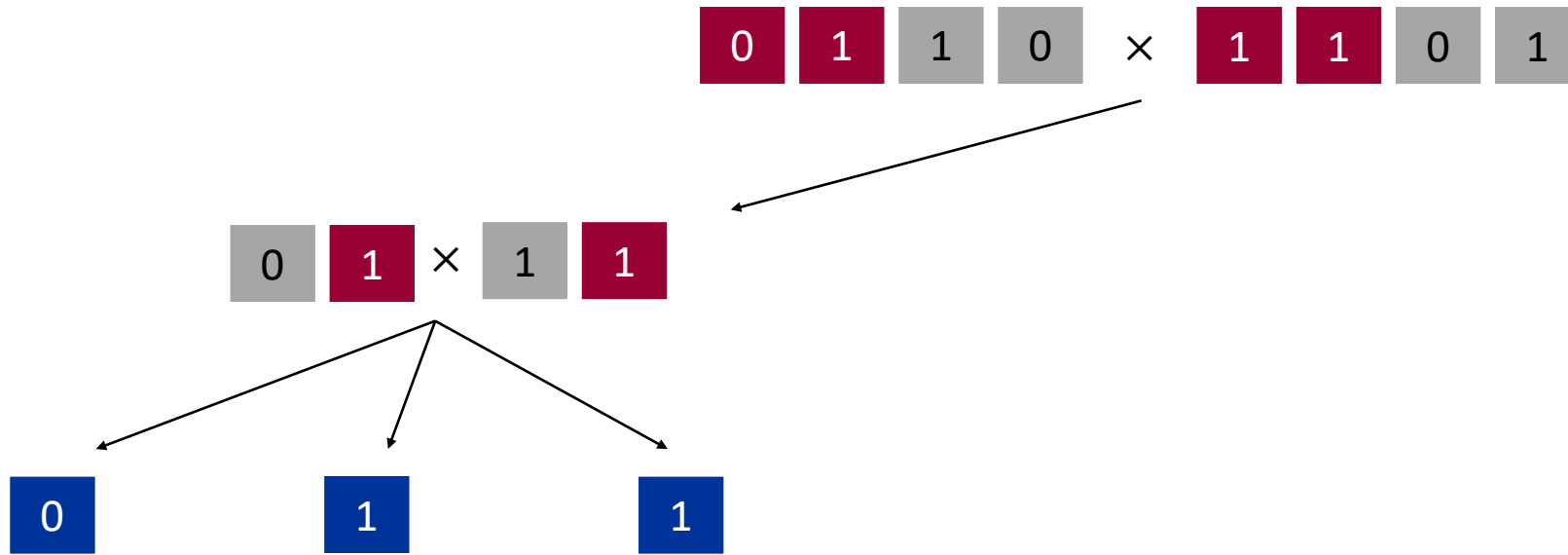


$$(a2^2 + b)(c2^2 + d) = ac \times 2^4 + [(a + b) \times (c + d) - ac - bd] \times 2^2 + bd$$

# Karatsuba Multiplication: Example

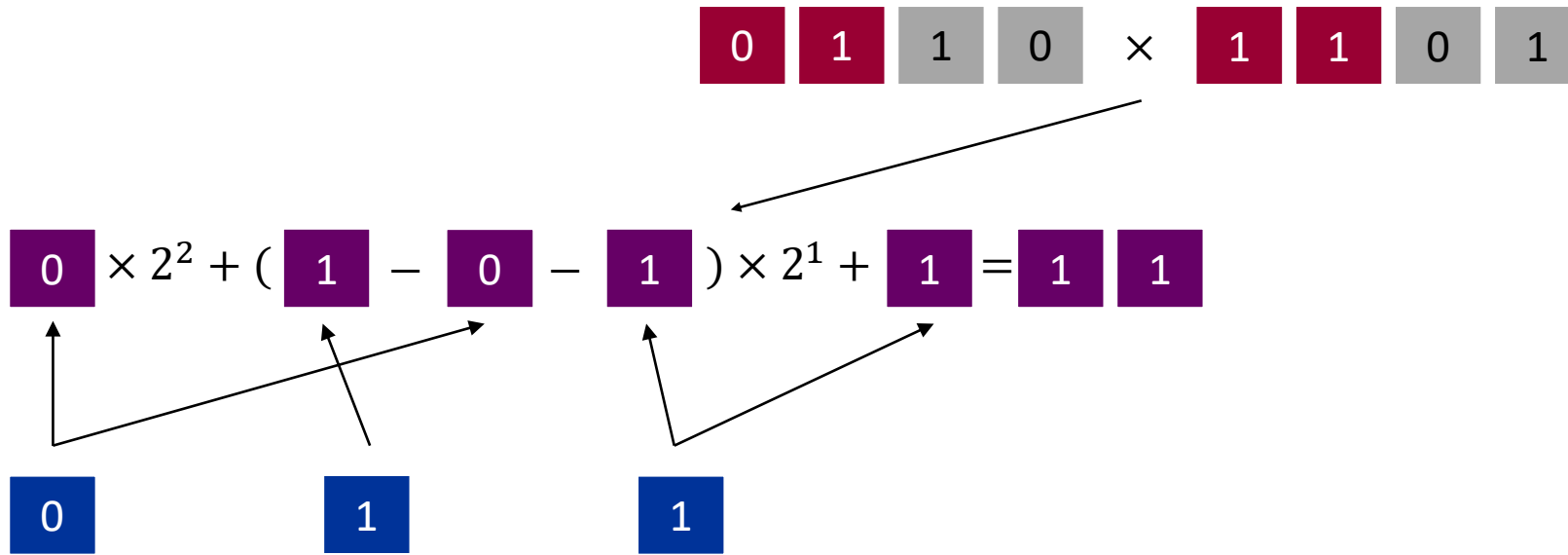


# Karatsuba Multiplication: Example



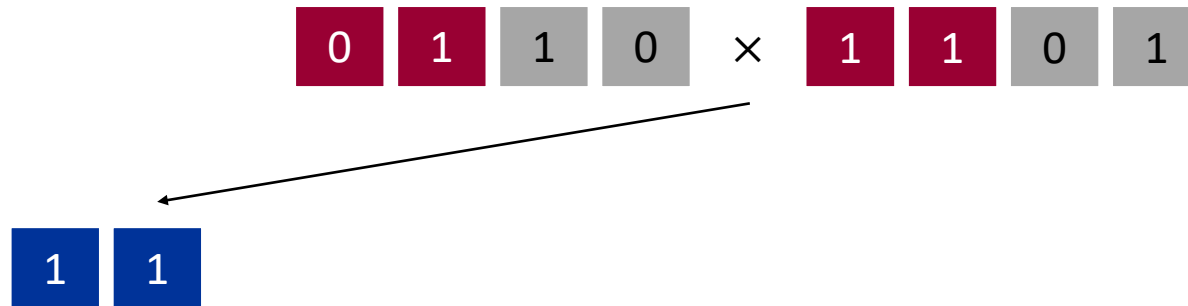
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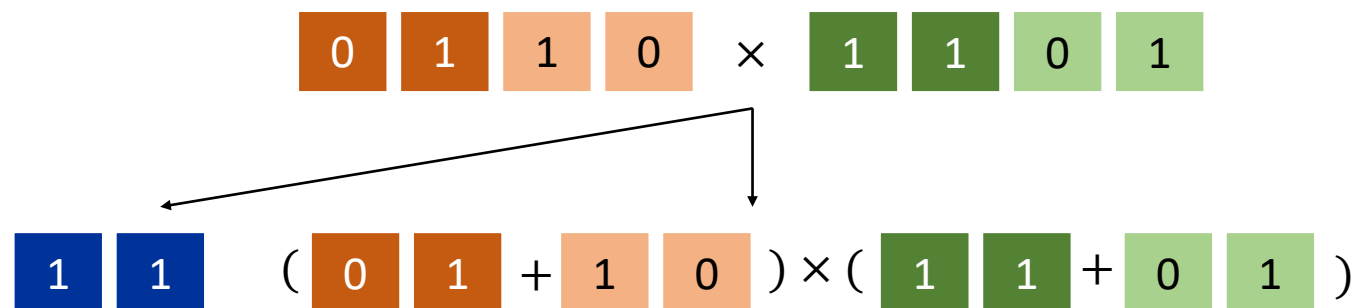
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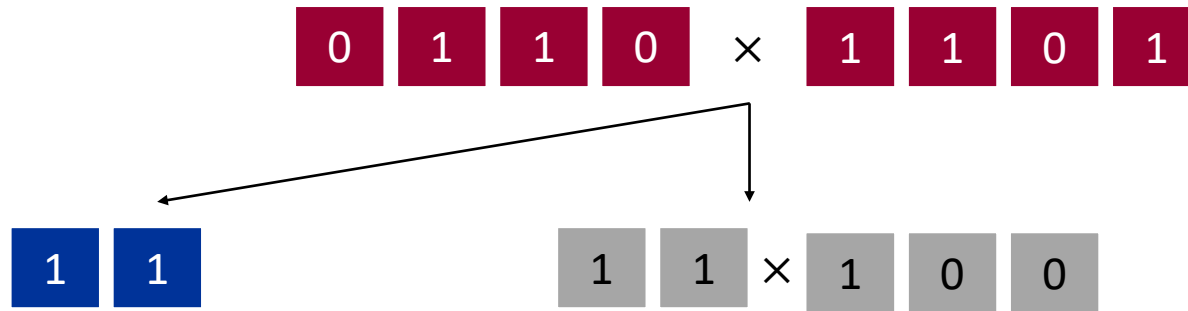
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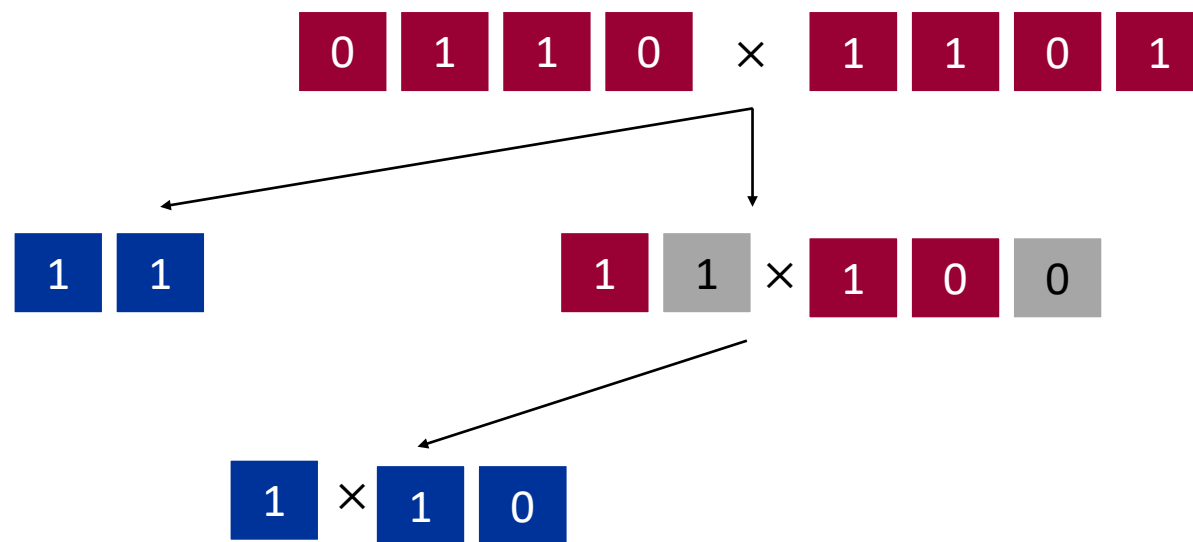


# Karatsuba Multiplication: Example



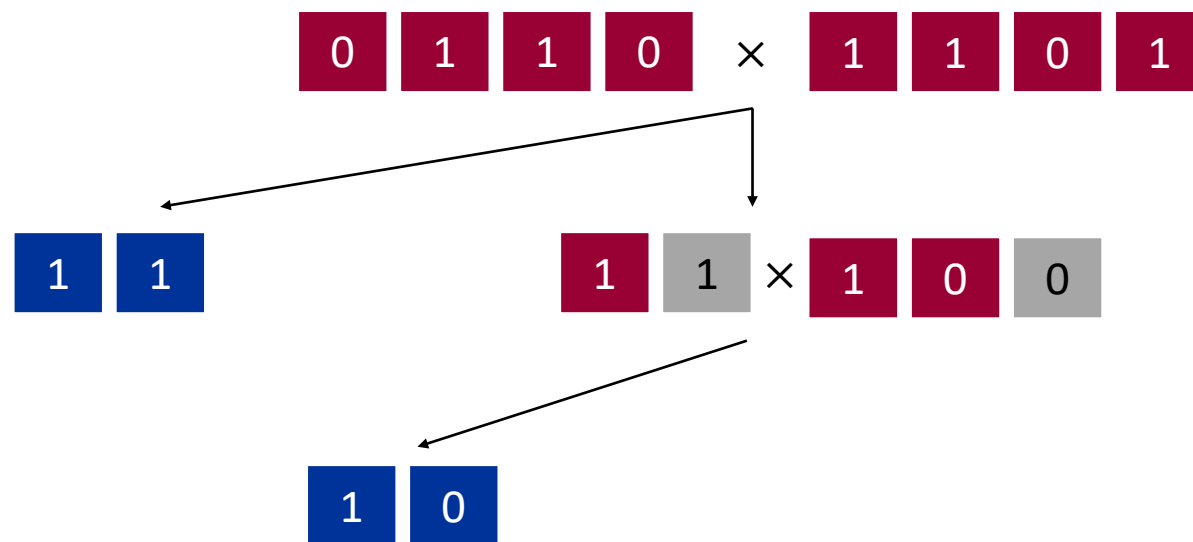
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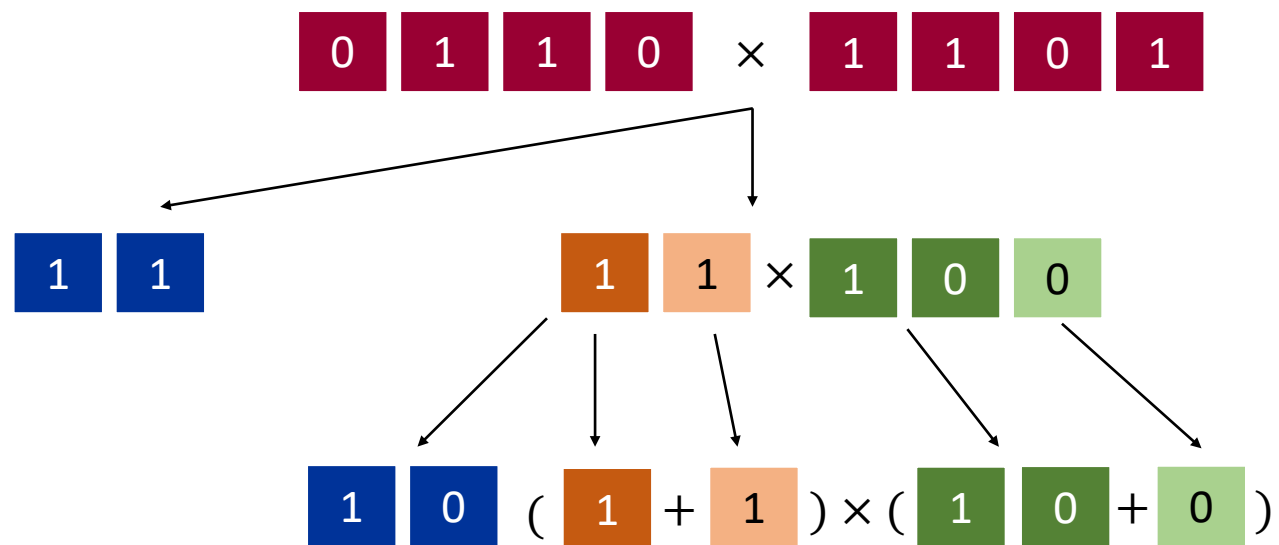
$$(a2^2 + b)(c2^2 + d) = ac \times 2^4 + [(a + b) \times (c + d) - ac - bd] \times 2^2 + bd$$

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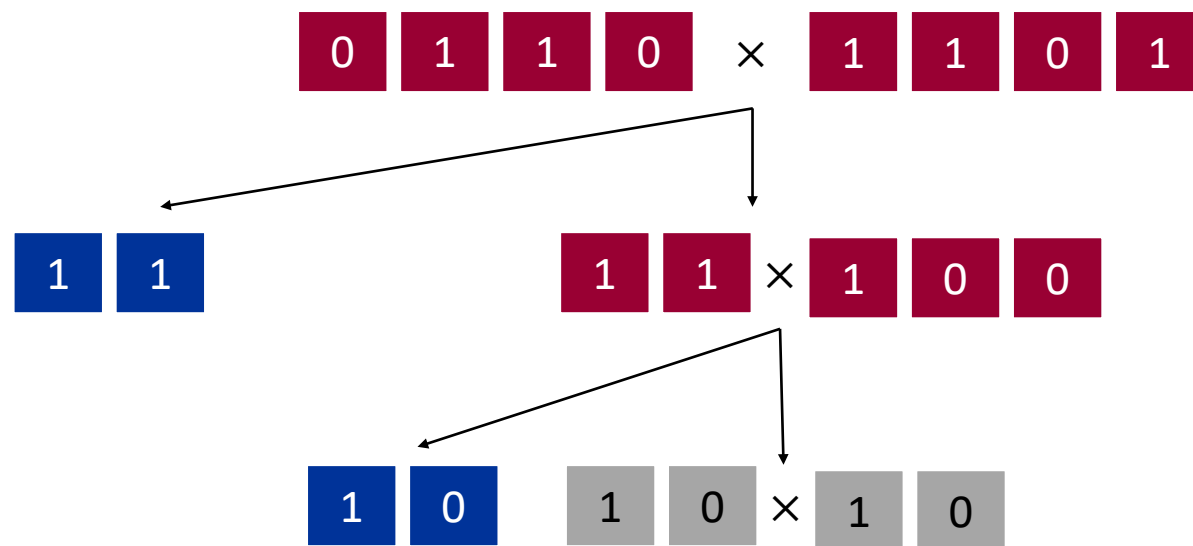
$$(a2^2 + b)(c2^2 + d) = ac \times 2^4 + [(a + b) \times (c + d) - ac - bd] \times 2^2 + bd$$

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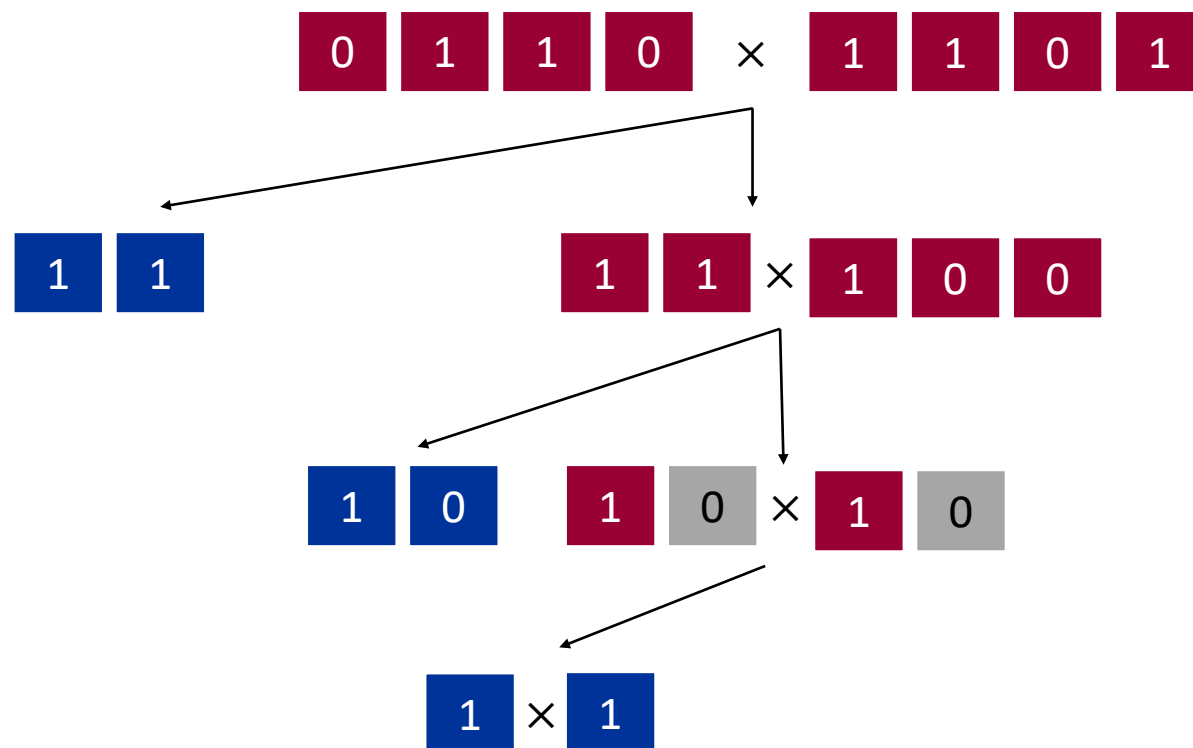
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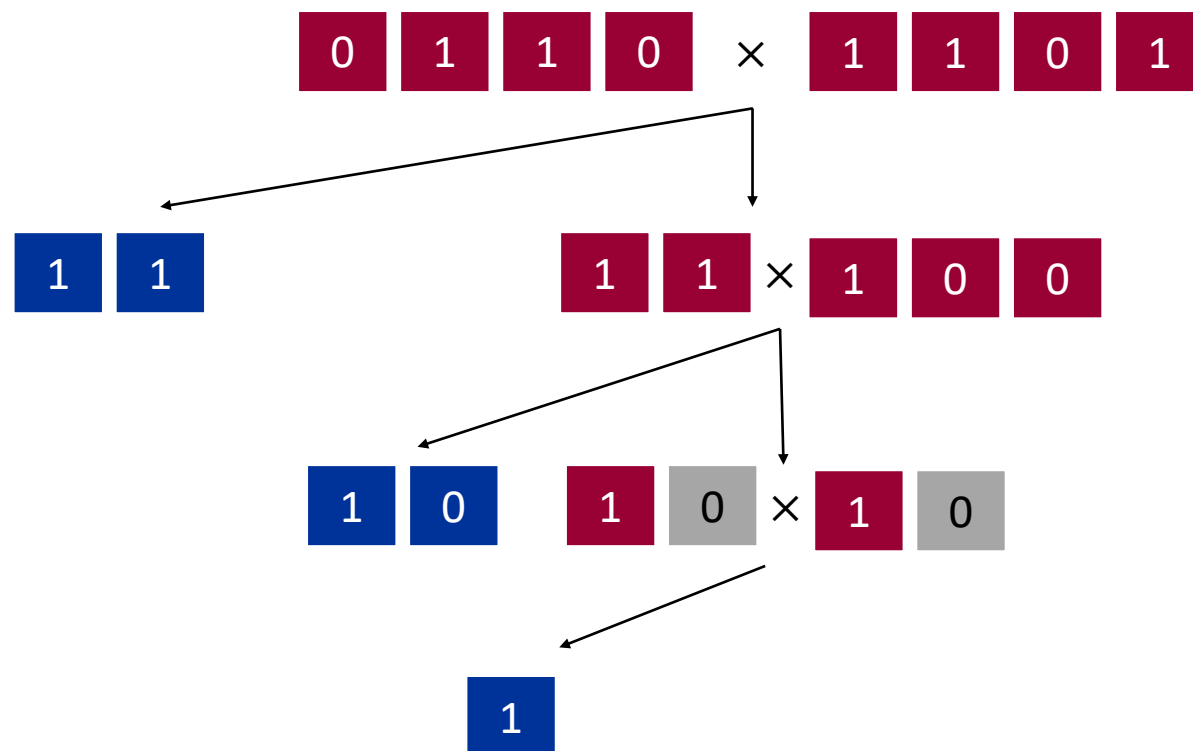
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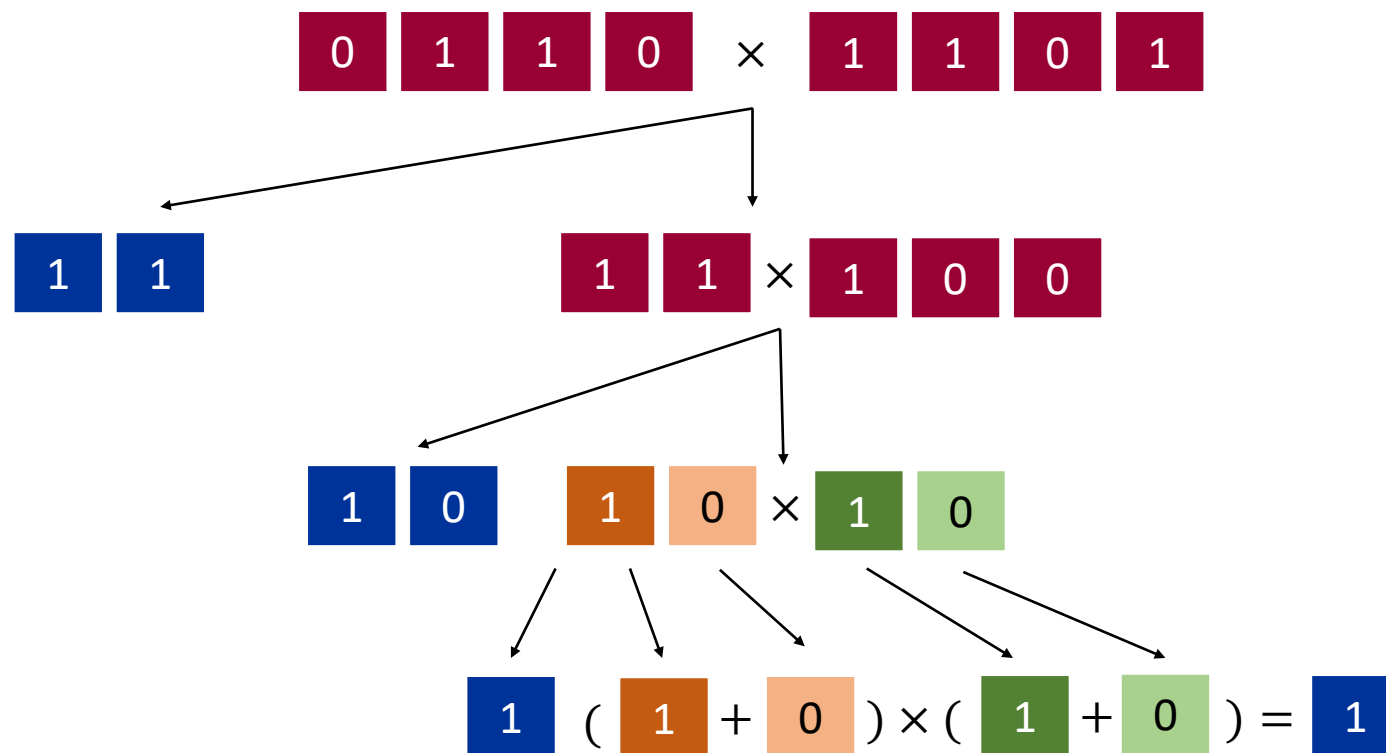
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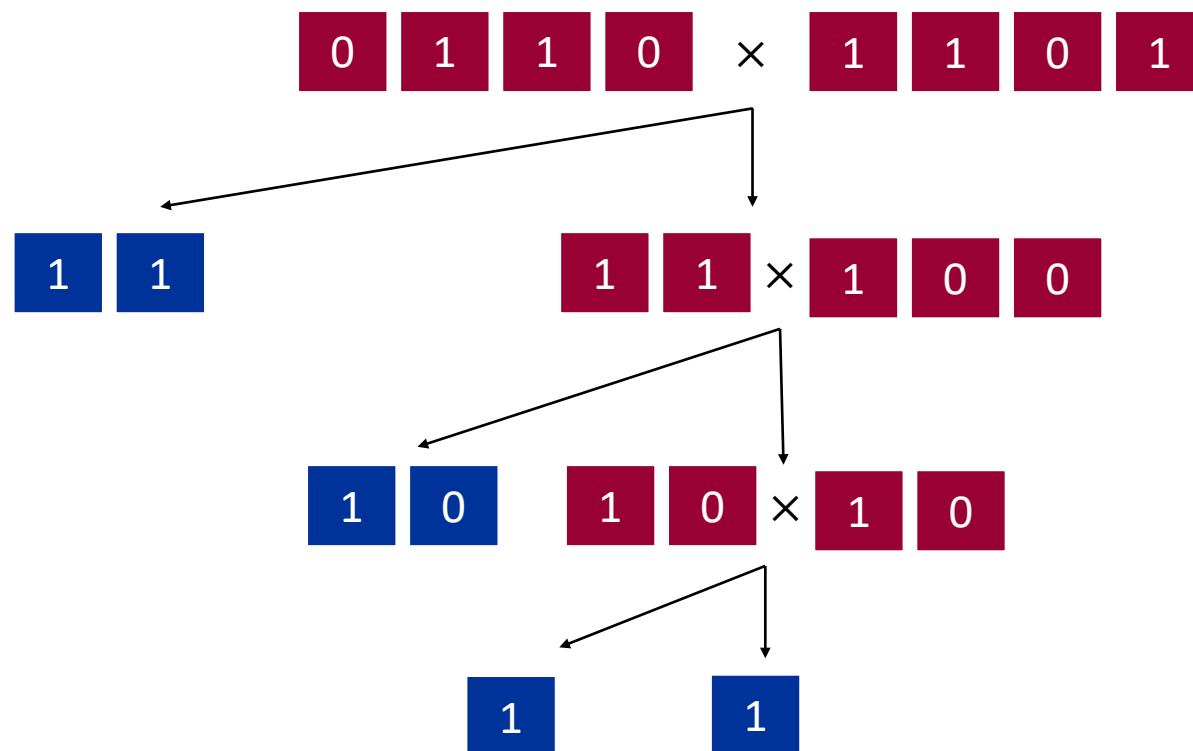
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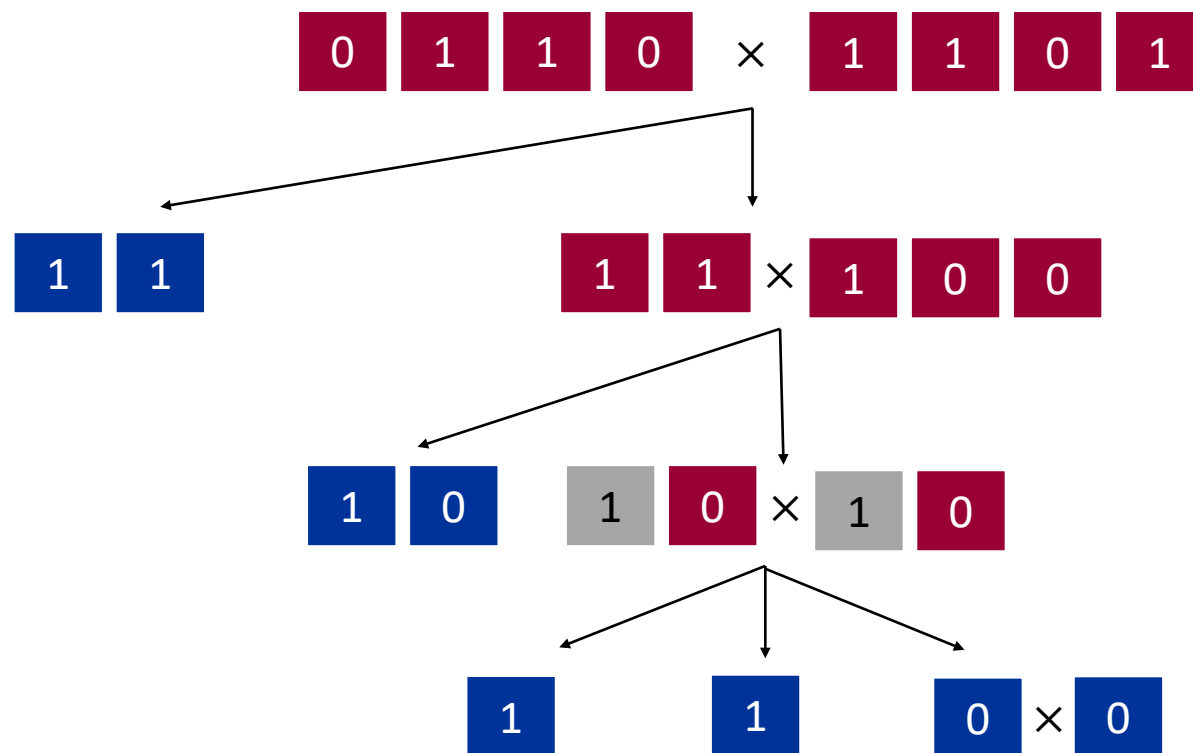


# Karatsuba Multiplication: Example



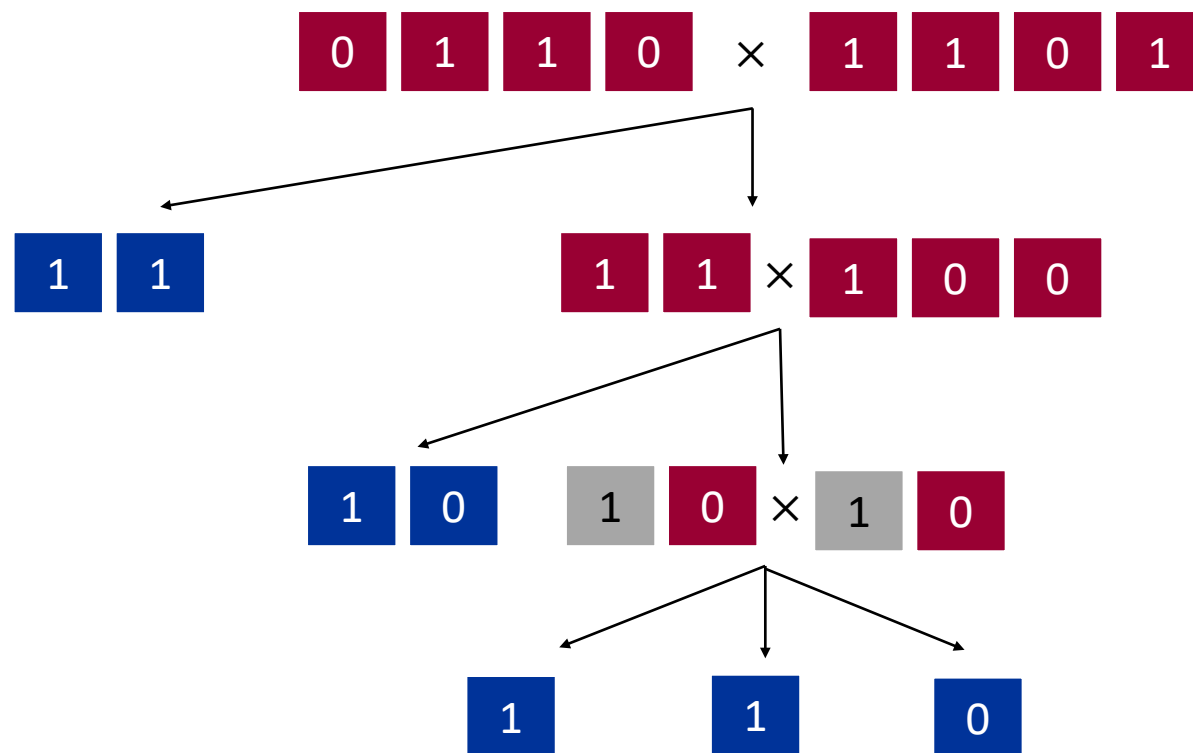
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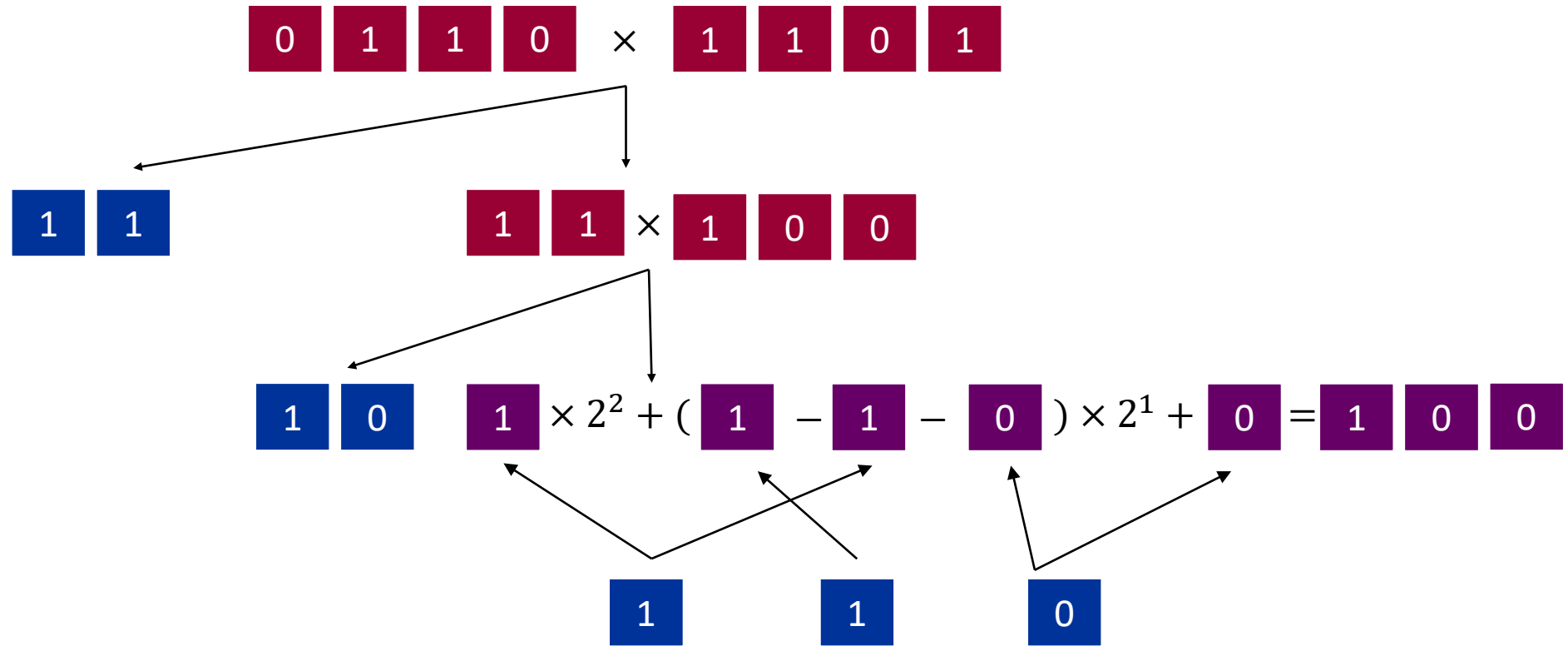
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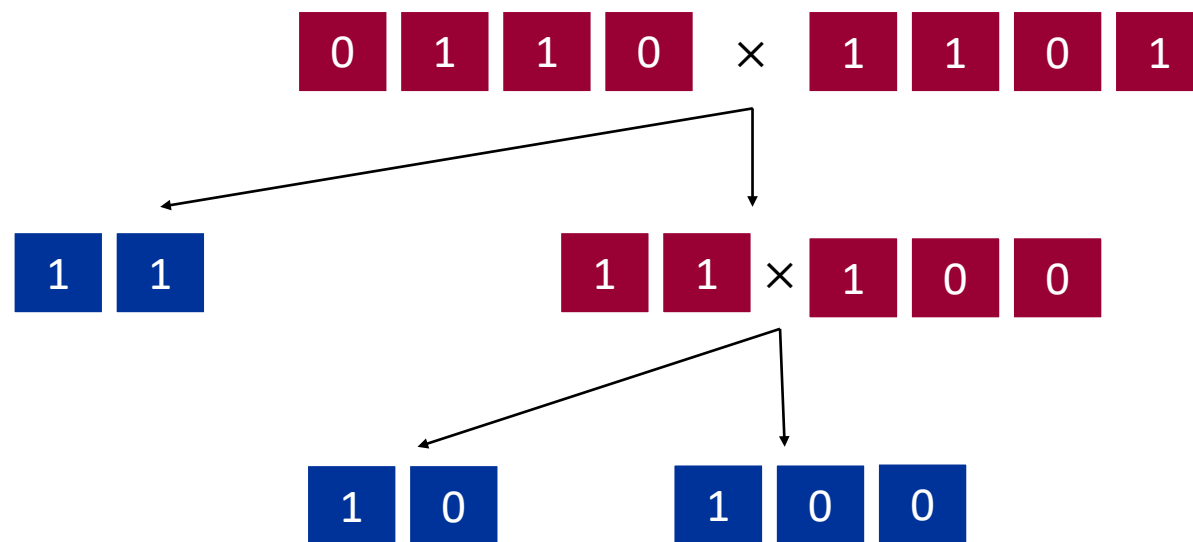
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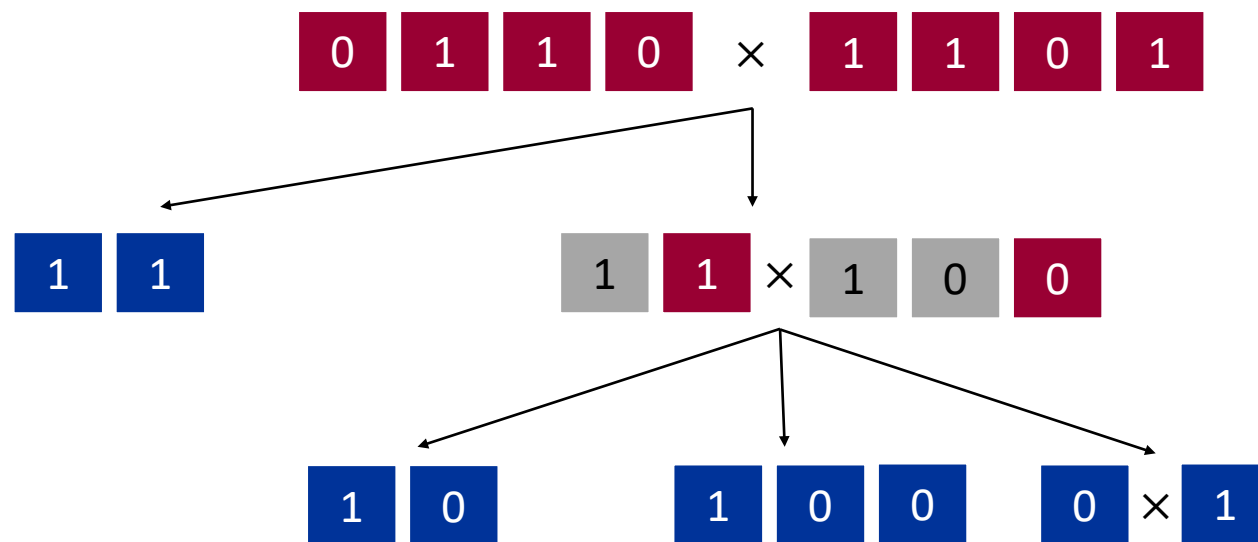
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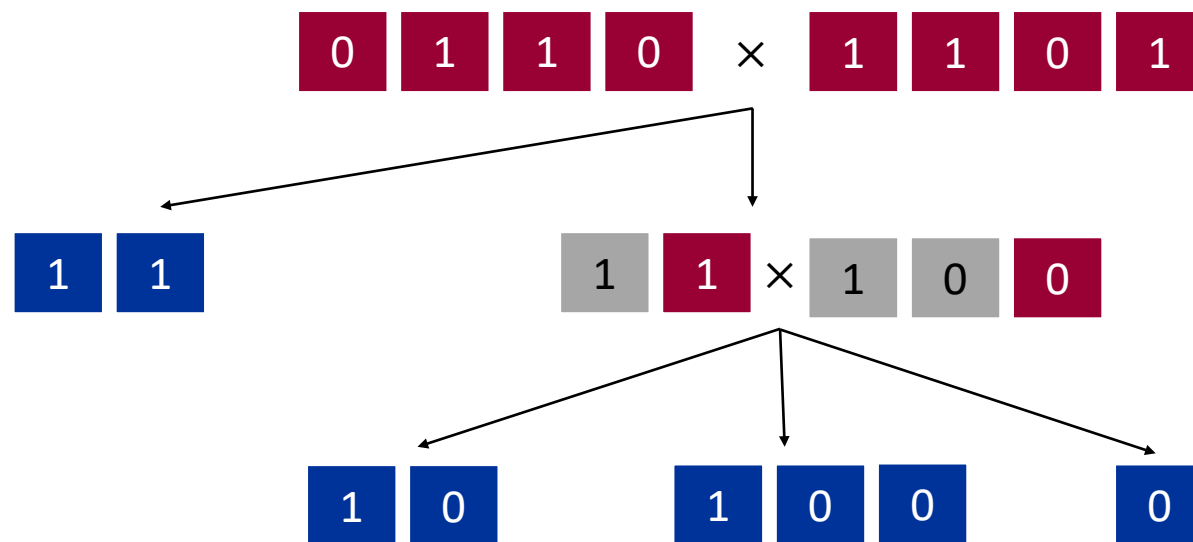
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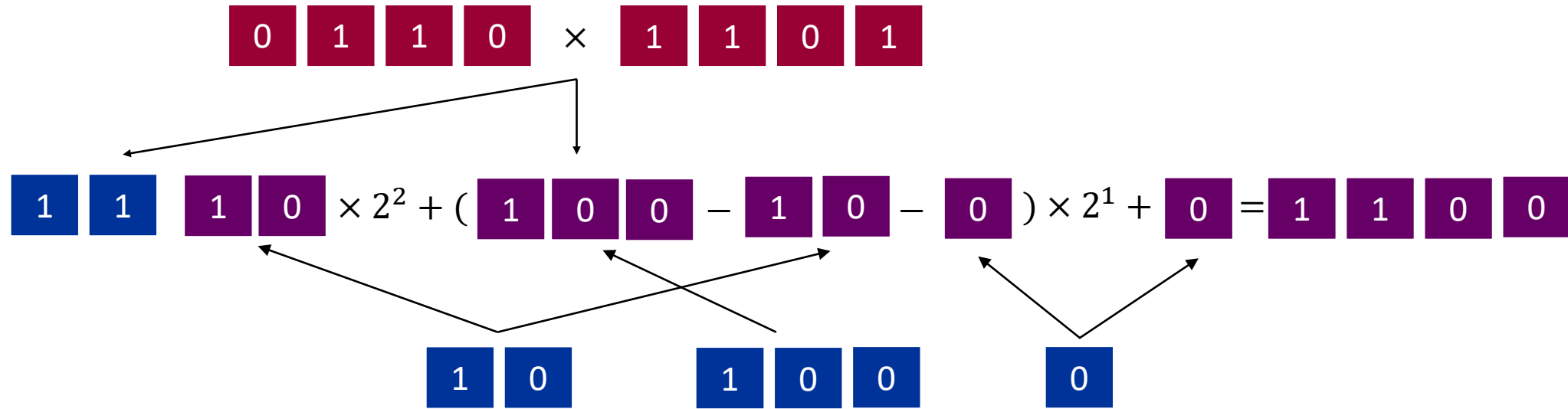
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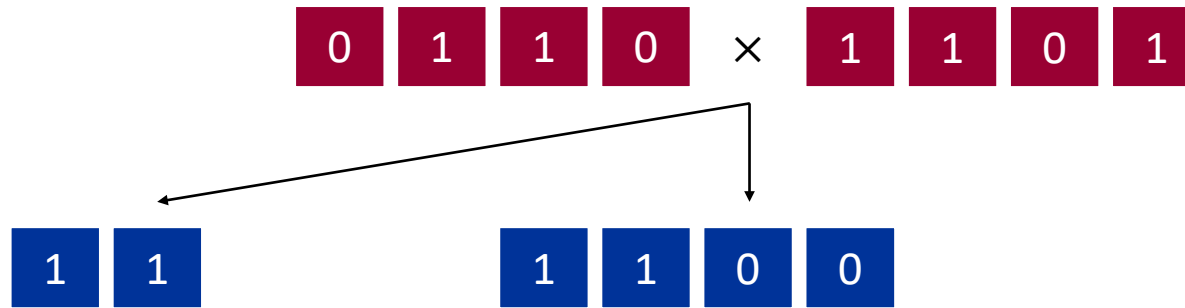
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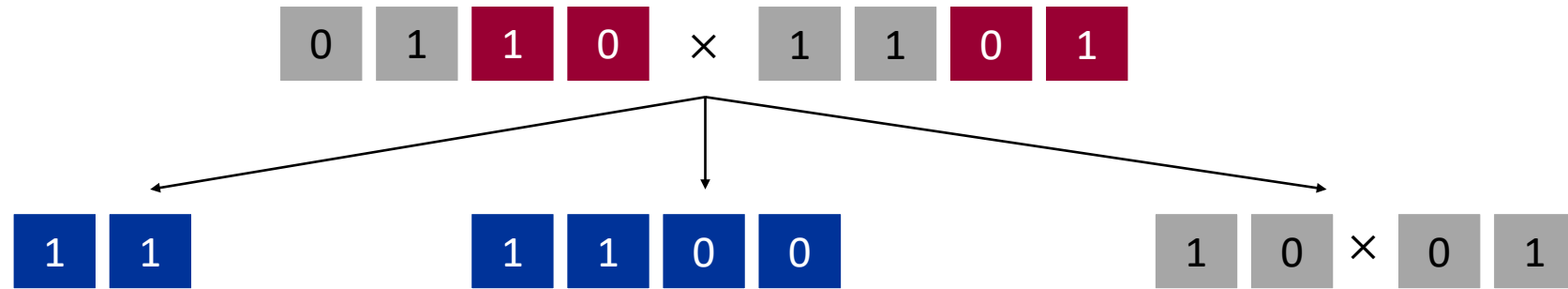


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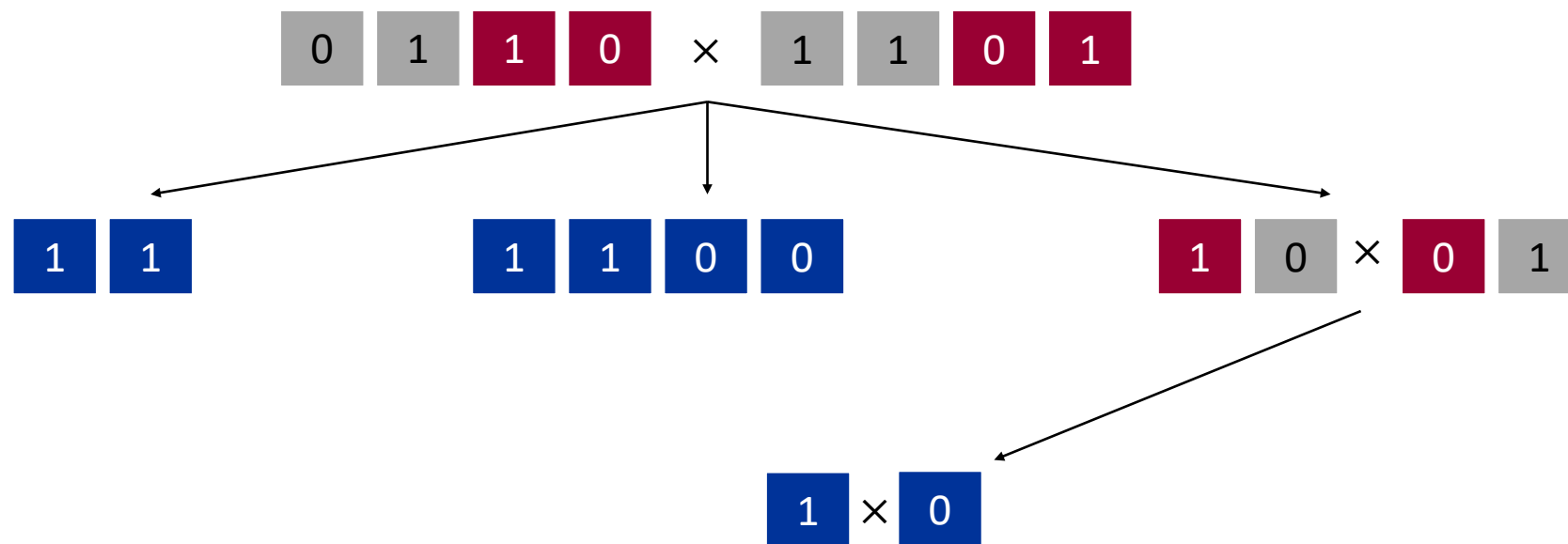
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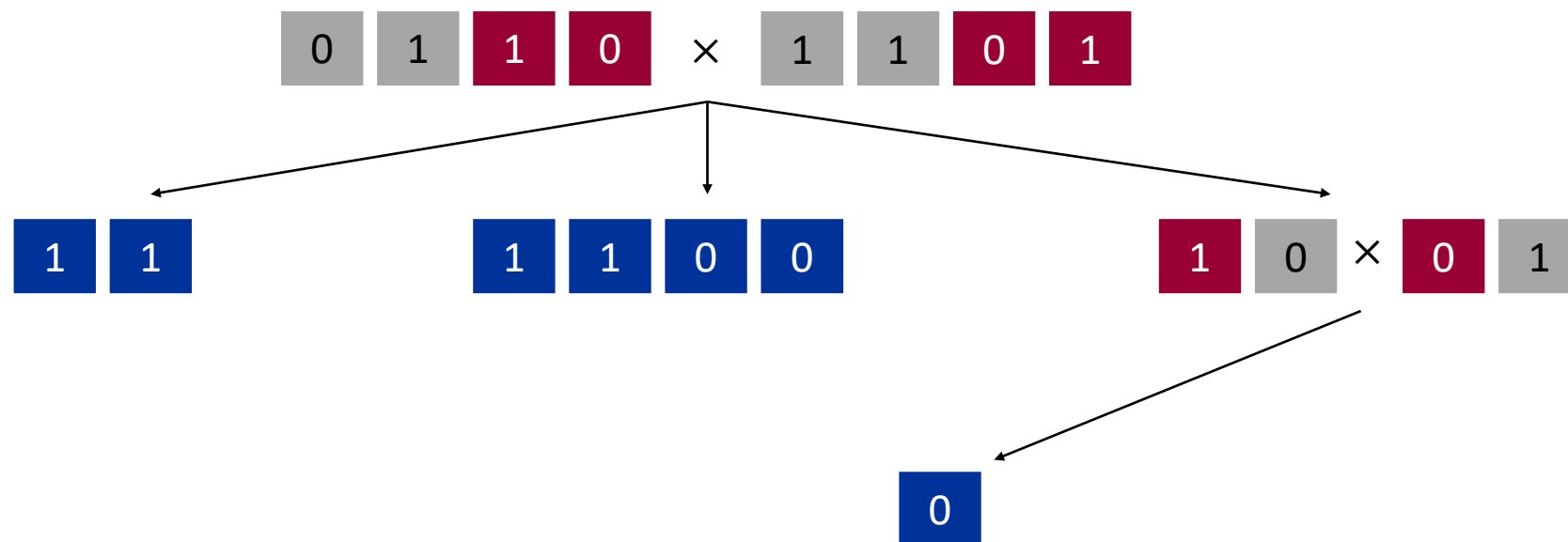
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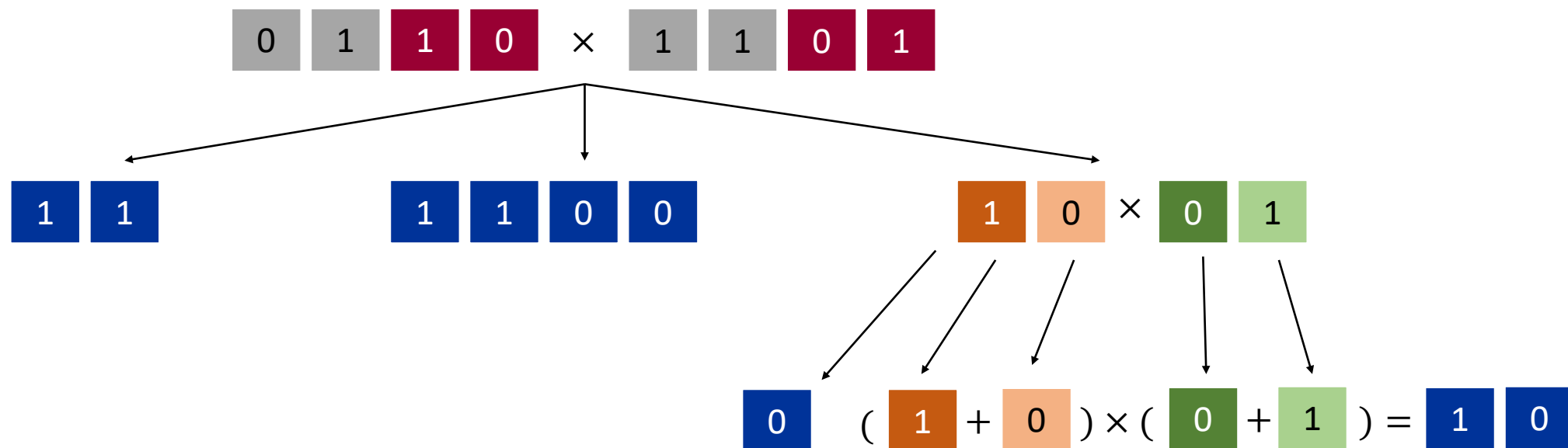
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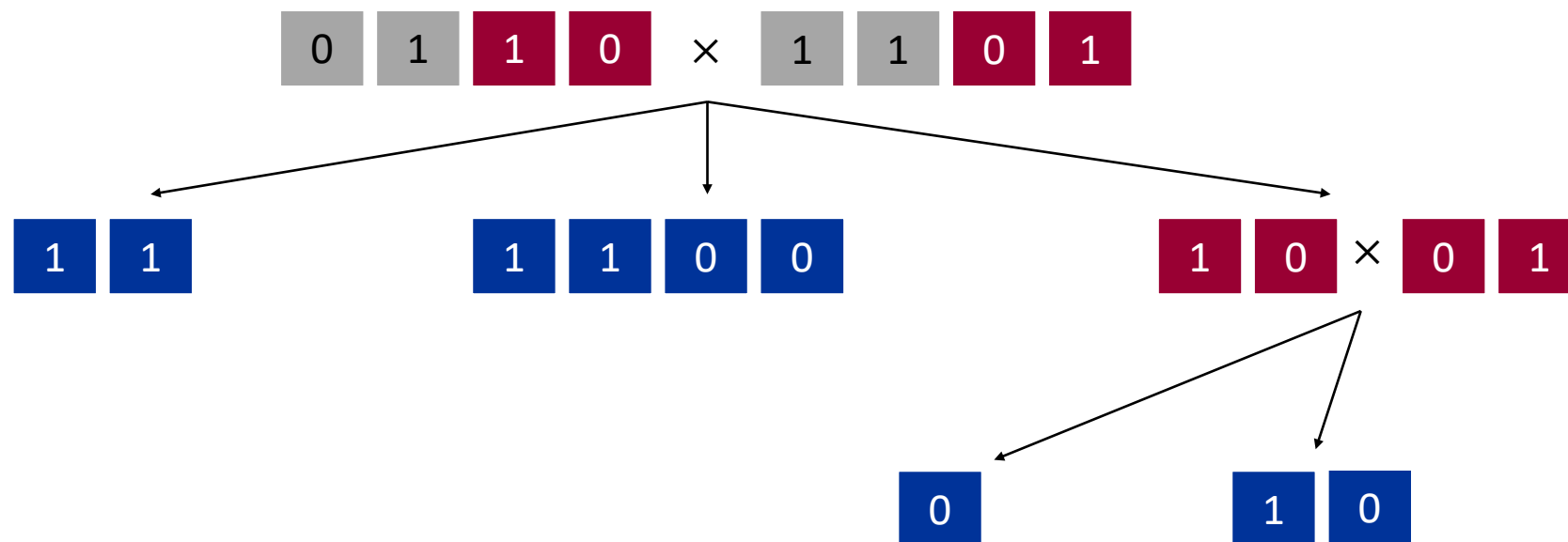
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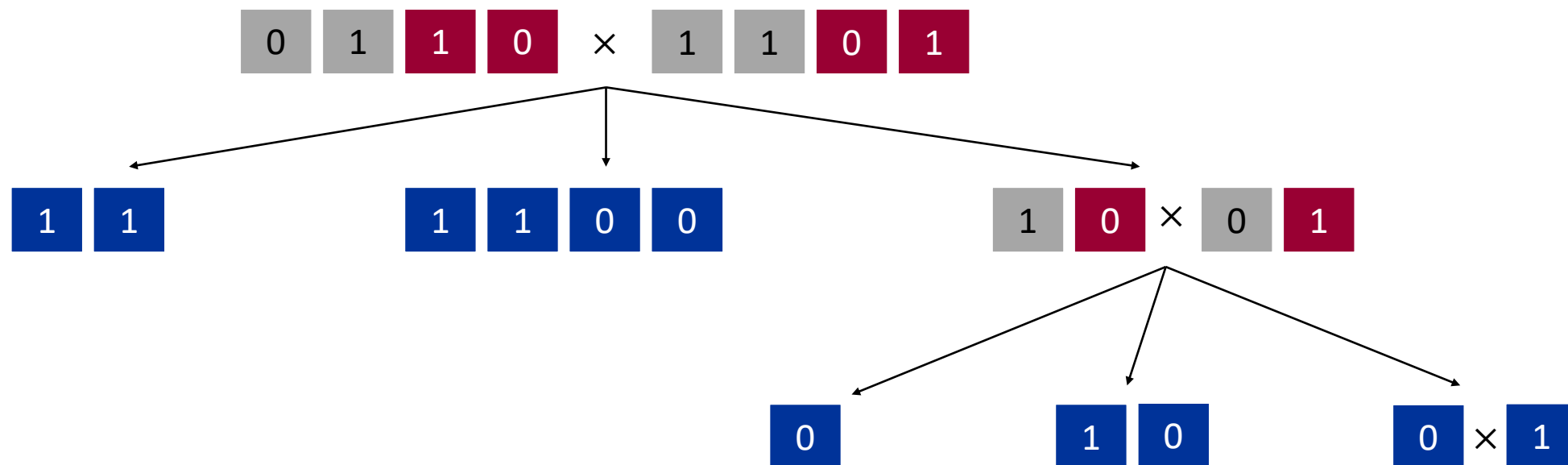
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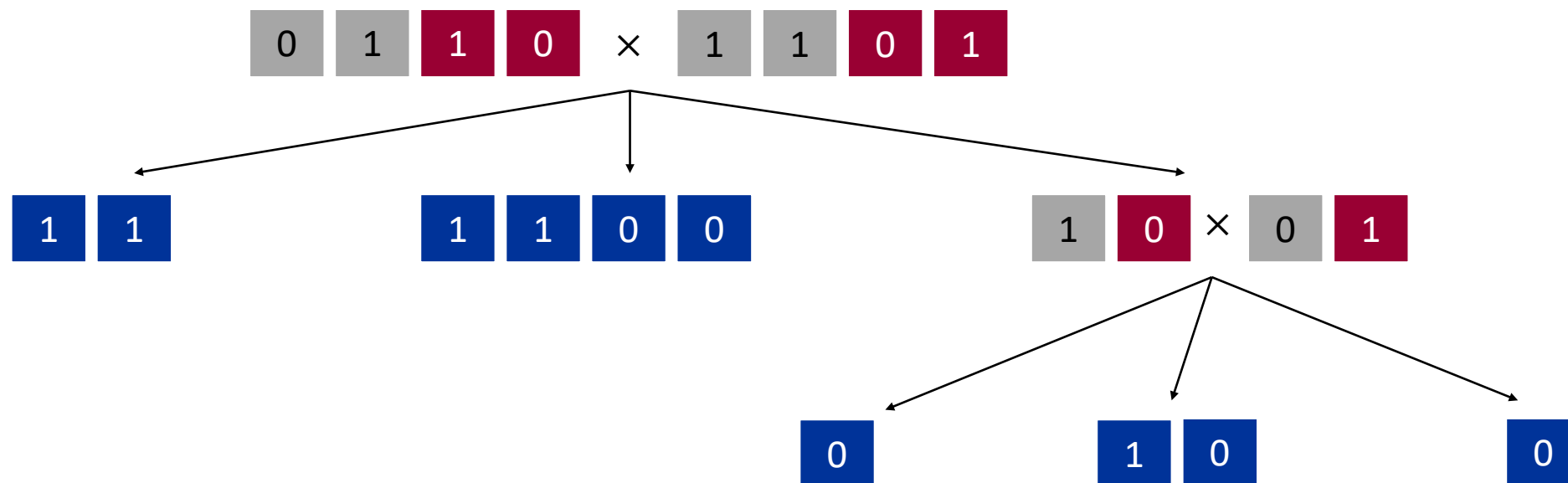
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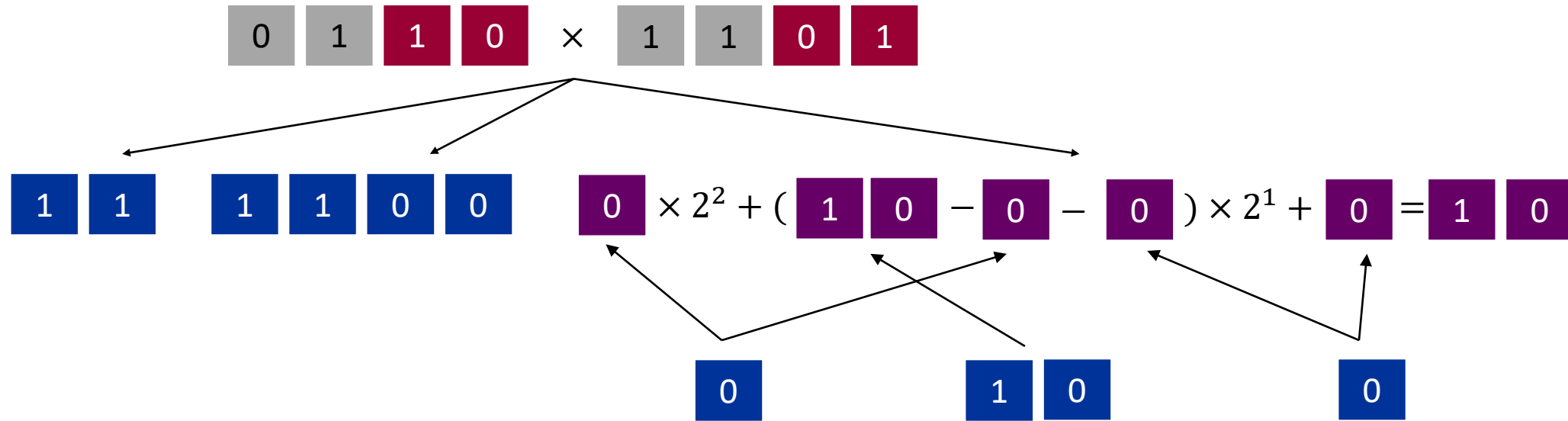
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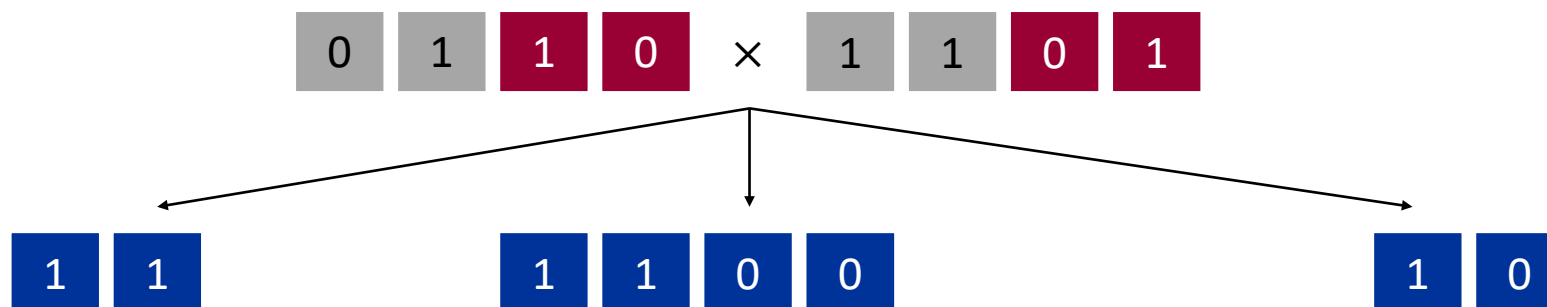


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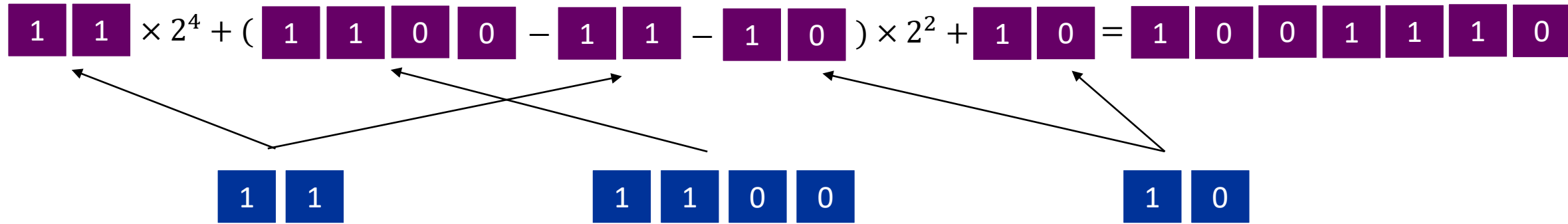
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