Name: ________________________________ **Student#:** _________________________ **Date:** _________________

COMP 2711: Discrete Mathematical Tools for Computer Science

In Class Exercise #5

1. **Which of the following statements (in which Z^+ stands for the positive integers and Z stands for all integers) is true and which is false? Don't forget to explain why.**

   a) $\forall z \in Z^+ (z^2 + 6z + 10 > 20)$
   b) $\forall z \in Z, (z^2 - z \geq 0)$
   c) $\exists z \in Z^+, (z - z^2 > 0)$
   d) $\exists z \in Z, (z^2 - z = 6)$

   Answer:
   a) False, because $1^2 + 6 \cdot 10 = 17$.
   b) True, because the graph of $y = z^2 - z$ is concave up
   and the y-coordinate is 0 at $z = 0$ and $z = 1$

   OR:
   Consider the following cases:
   1) $z < 0$: true, because $z^2 - z \geq 0 \Rightarrow z^2 \geq z$
   The LHS is positive, the RHS is negative.
   2) $z = 0$: true by substitution $0^2 - 0 \geq 0$.
   3) $z > 0$: true, because $z^2 - z \geq 0 \Rightarrow z^2 \geq z$,
   so we get $z \geq 1$ which holds in this case.

   c) False, because the graph of $y = z - z^2$ is concave down
   and the y-coordinate is 0 at $z = 1$.

   OR:
   Indeed, we will show that no such z exists.
   To prove the negation of an existential quantifier
   one proves the universal of the negation
   (see Lecture Slides).
   So we want to prove $\forall z \in Z^+, (z - z^2 \leq 0)$.
   Multiplying the inequality by -1 gives us $(z^2 - z \geq 0)$,
   which was already proved in part (b) (for an even bigger universe).

   d) True, because $(-2)^2 - (-2) = 6$.