COMP3711: Design and Analysis of Algorithms

Tutorial 11 AVL Question

HKUST

Part 1: AVL Trees

Construct an AVL tree by inserting the items 134625 in that order.

Next construct another AVL tree on those items by inserting in the order 123456.

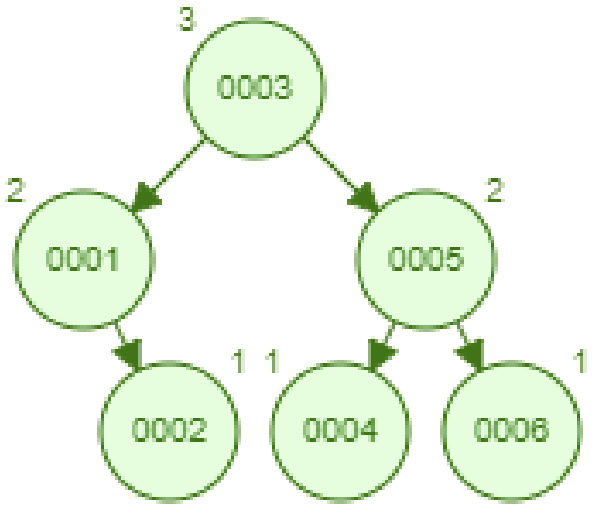
Do they have the same height?

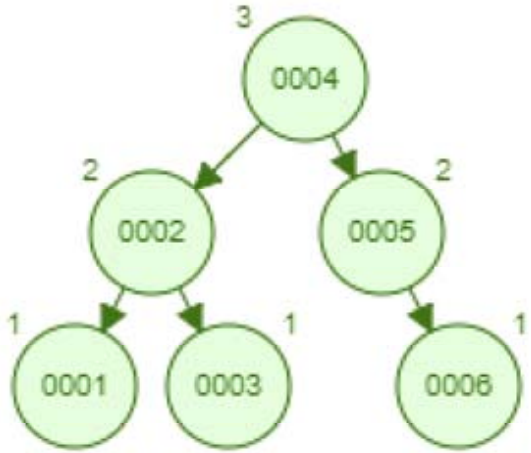
Now construct an AVL tree by inserting the items 5362471 in that order and another by inserting 4261357 in that order.

Do those two trees have the same height?

Part 1: AVL Trees

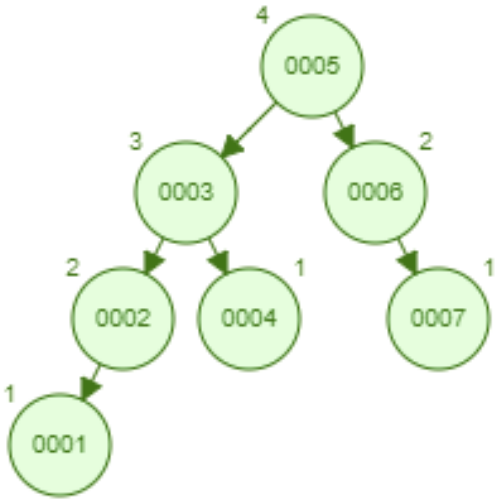
The two trees below are the ones built for 134625 and then 123456. They do have the same height.

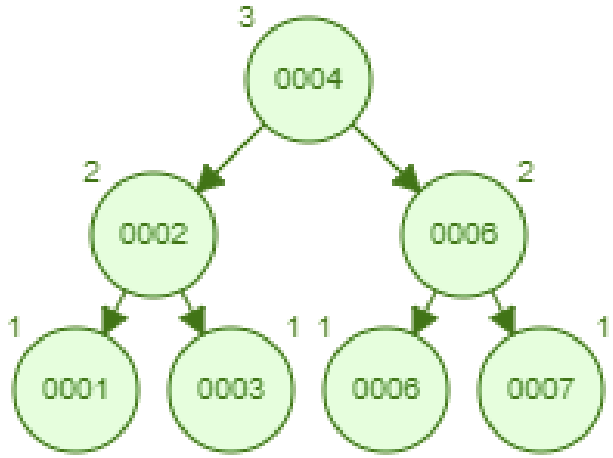




Part 1: AVL Trees

The two further trees below are the ones built for 5362471 and then 4261357. They do NOT have the same height.





You can construct these trees using the web site pointed to by the class lecture note page).

Part 2: AVL Trees

What are the minimum and maximum heights for an AVL tree with 88 nodes labelled $1, 2, 3 \ldots, 88$?

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A binary tree of height h can hold at most $2^{h+1} - 1$ nodes (this is when it is full).

The minimum height for ANY binary tree with n nodes is therefore $h = \lceil \log_2 n + 1 \rceil - 1$ which is the height of a complete tree with n nodes (all levels full except for possibly the bottom one).

Such a tree is an AVL tree as well.
So, this is the minimum for AVL trees too.

In our case minimum $h = \lceil \log_2 89 \rceil - 1 = 6$.

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In class we learned that the minimum number of nodes in an AVL tree of height 8 is 88:
Thus, an AVL tree of height 8 with 88 nodes exists.

Again from class, we saw that the minimum number of nodes in an AVL tree of height 9 is $88 + 54 + 1 > 88$ and larger heights require even more nodes.
Thus the max height $= 8$.