1. You are given n pairs of nuts and bolts

$$(N_1, B_1), (N_2, B_2), \ldots, (N_n, B_n).$$

Each pair is a different size than the others. Someone has unscrewed all of the nuts off of the bolts and mixed them up.

Problem: Match all nuts up with their corresponding bolts.

If we could separately

- (i) sort all the bolts by increasing size and then
- (ii) sort all the nuts by increasing thread size
- $\Rightarrow$  problem would be easily solvable in  $O(n \log n)$  time.

After sorting, just match them up in order from smallest to largest. The difficulty is that we can't do this because we can't compare the sizes of two nuts directly or the sizes of two bolts directly.

The only operation available is to try to screw a bolt B into a nut N and then, by seeing whether the bolt

- (a) goes loosely in,
- (b) perfectly fits or
- (c) can't go in at all,

decide whether their thread sizes satisfy

(a) 
$$B < N$$
, (b)  $B = N$  or (c)  $B > N$ .

Hint: Try to modify Quicksort.