

COMP 3711 – Spring 2019
Tutorial 3b

1. Consider the HIRE-ASSISTANT algorithm described in the lecture notes.

Assume that the candidates are presented in a random order.

The analysis in the lecture notes calculated the *Expected* number of hires. For this problem calculate:

- (a) the probability that you hire exactly one person.
- (b) the probability that you hire exactly n people.

2. Use indicator random variables to solve the following, known as the **hat-check problem**. Each of n customers gives a hat to a hat-check person at a restaurant.

The hat-check person gives the hats back to the customers in a random order.

What is the expected number of customers who get back their own hat?

An equivalent question is to suppose that there are n students in a class who have just submitted their homework. The teacher gives the homeworks back to the students in a random order and asks the students to mark the homework they have been handed. What is the expected number of students who have been asked to mark their own homework?

3. Let $A[1..n]$ be an array of n distinct numbers. In class we said that if $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an **inversion** of A .

Suppose that the elements of A form a uniform random permutation of $\langle 1, 2, \dots, n \rangle$.

Use indicator random variables to compute the expected number of inversions.

4. (a) Illustrate how Mergesort would work on input $[1, 2, 3, 4, 5, 6, 7, 8, 9]$.
(b) Illustrate how Mergesort would work on input $[9, 8, 7, 6, 5, 4, 3, 2, 1]$.
(c) Illustrate how Quicksort would work on input $[1, 2, 3, 4, 5, 6, 7, 8]$.
Assume that the last item in the subarray is always chosen as the pivot.
(d) Illustrate how Quicksort would work on input $[8, 7, 6, 5, 4, 3, 2, 1]$.
Assume that the last item in the subarray is always chosen as the pivot.