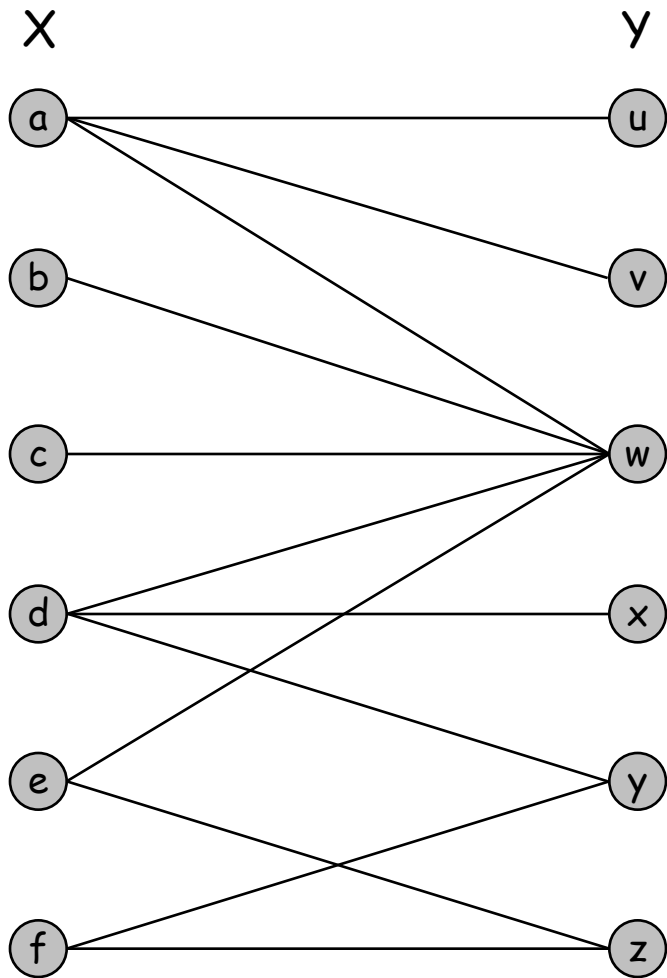


Bipartite Matching using Maximum Flow Algorithm

Version of April 25, 2019

Bipartite Matching Example



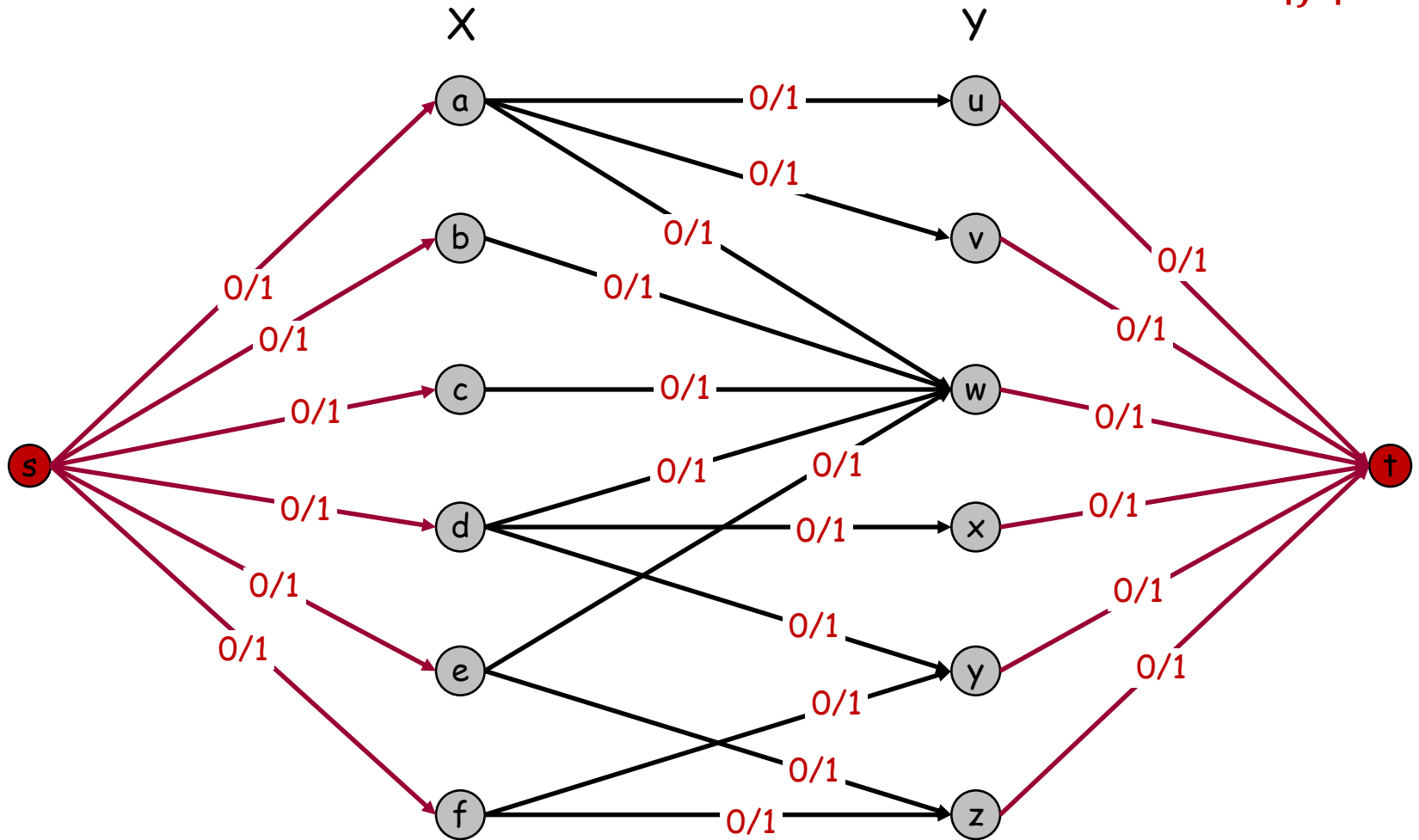
Find a
Maximum Bipartite Matching
in the graph using the
Max-Flow Method
taught in class

Recall: A matching can be
represented as an integral flow in
the corresponding flow graph.

Idea: Find
Maximum Bipartite Matching
by finding **Max-Integral Flow**
using Ford-Fulkerson augmenting
path algorithm.

Corresponding Flow Graph

Flow $|f| = 0$

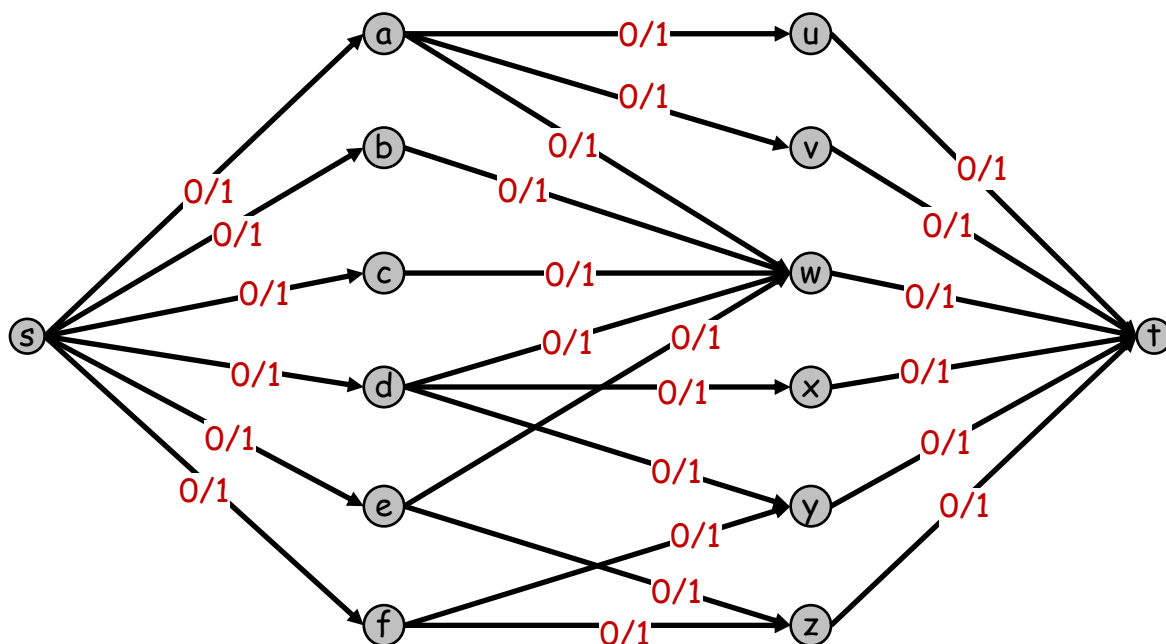


Matchings and their Corresponding Integral Flows

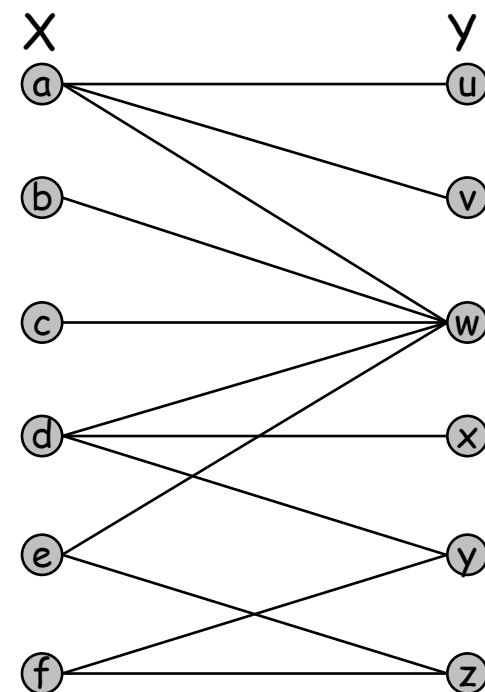
Following pages show matchings of size 0, 1, 2, 3, 4, 5 and their corresponding flows

A Flow and its Corresponding Matching

Flow $|f| = 0$

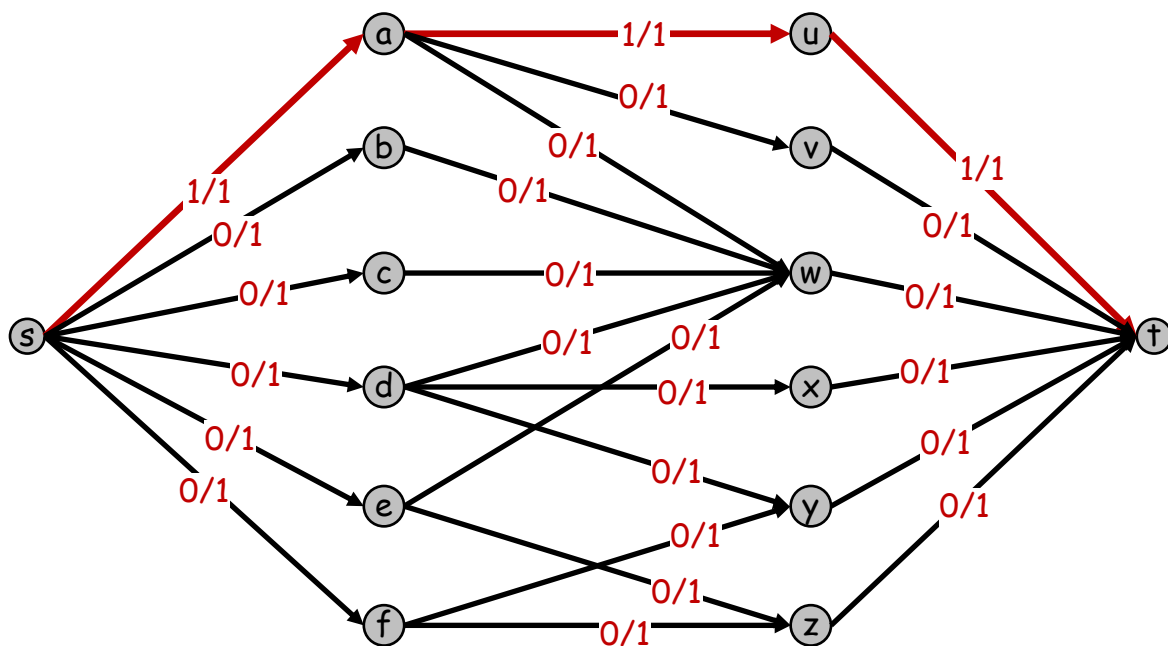


$|\text{Matching}| = 0$

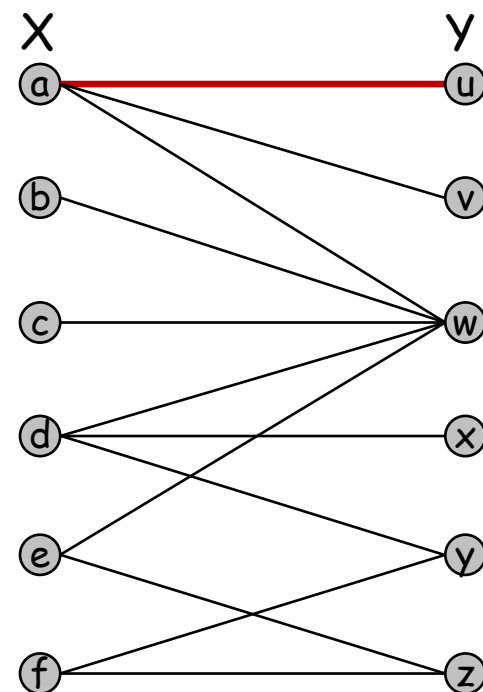


A Flow and its Corresponding Matching

Flow $|f| = 1$

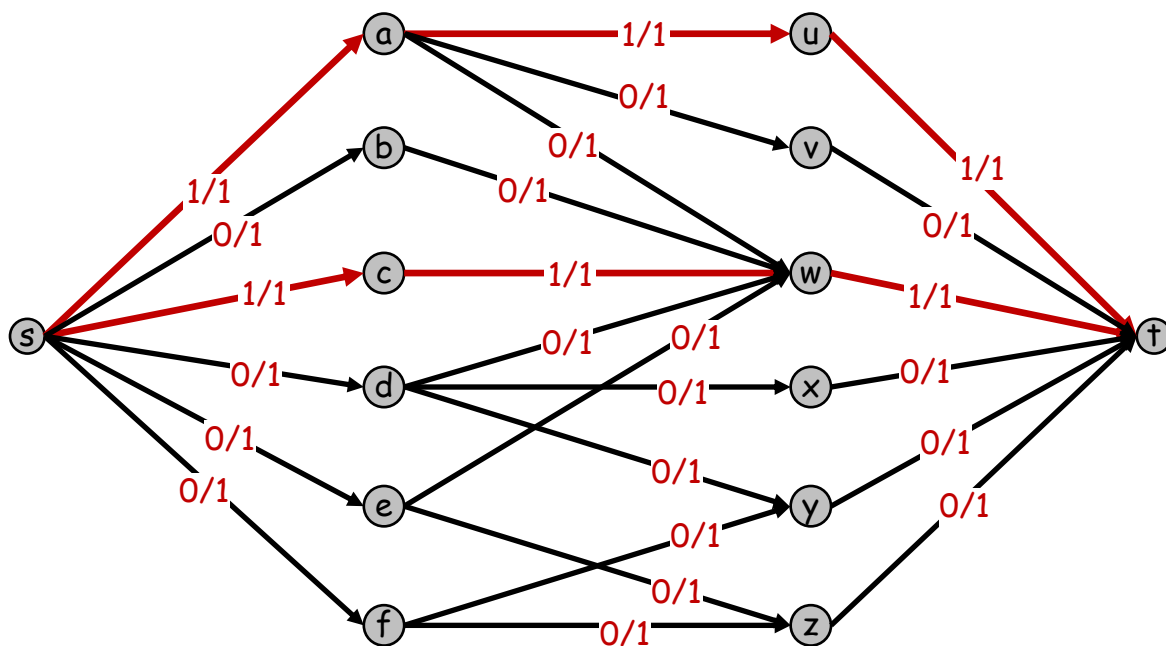


$|\text{Matching}| = 1$

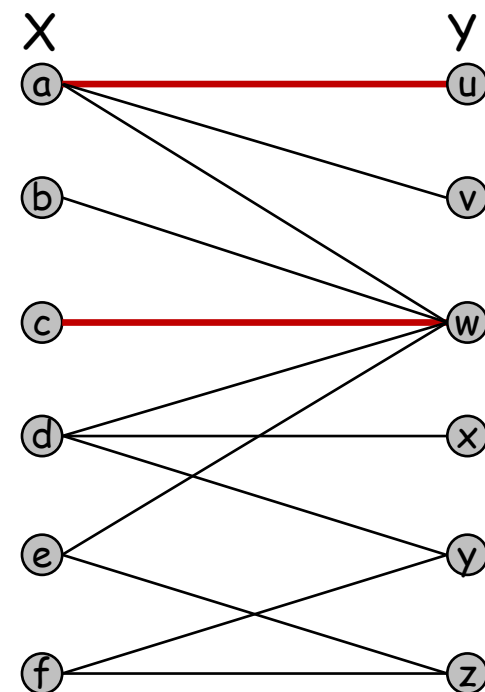


A Flow and its Corresponding Matching

Flow $|f| = 2$

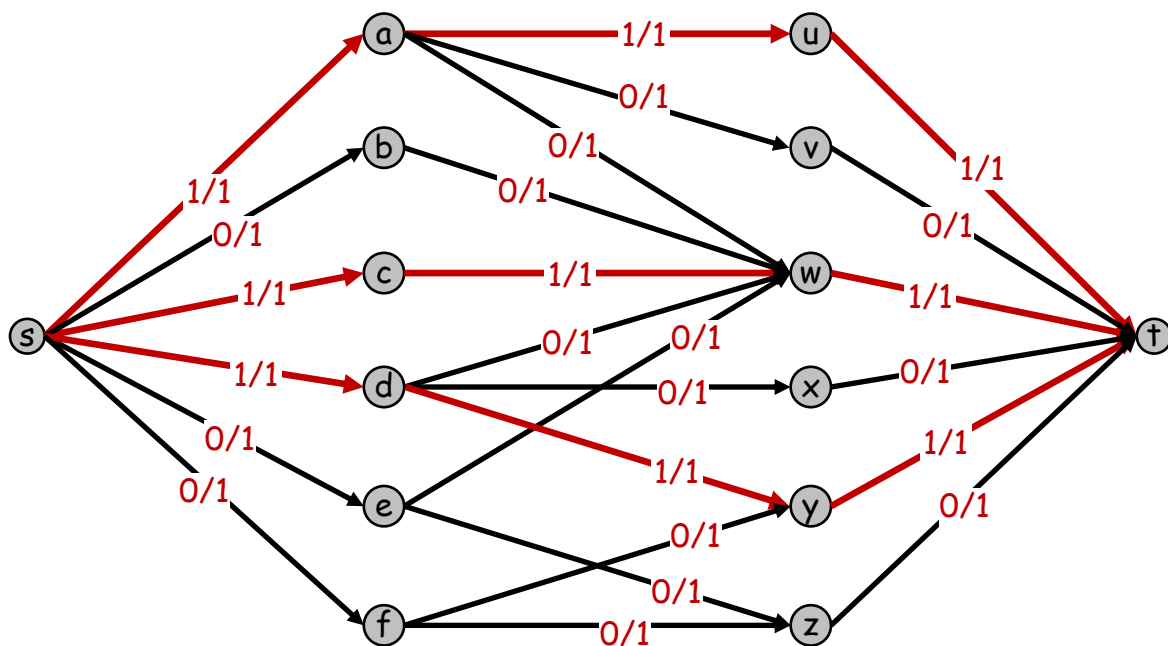


$|\text{Matching}| = 2$

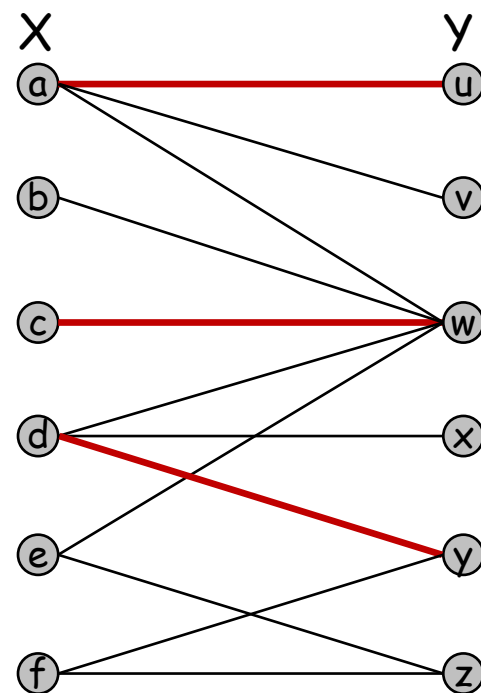


A Flow and its Corresponding Matching

Flow $|f| = 3$

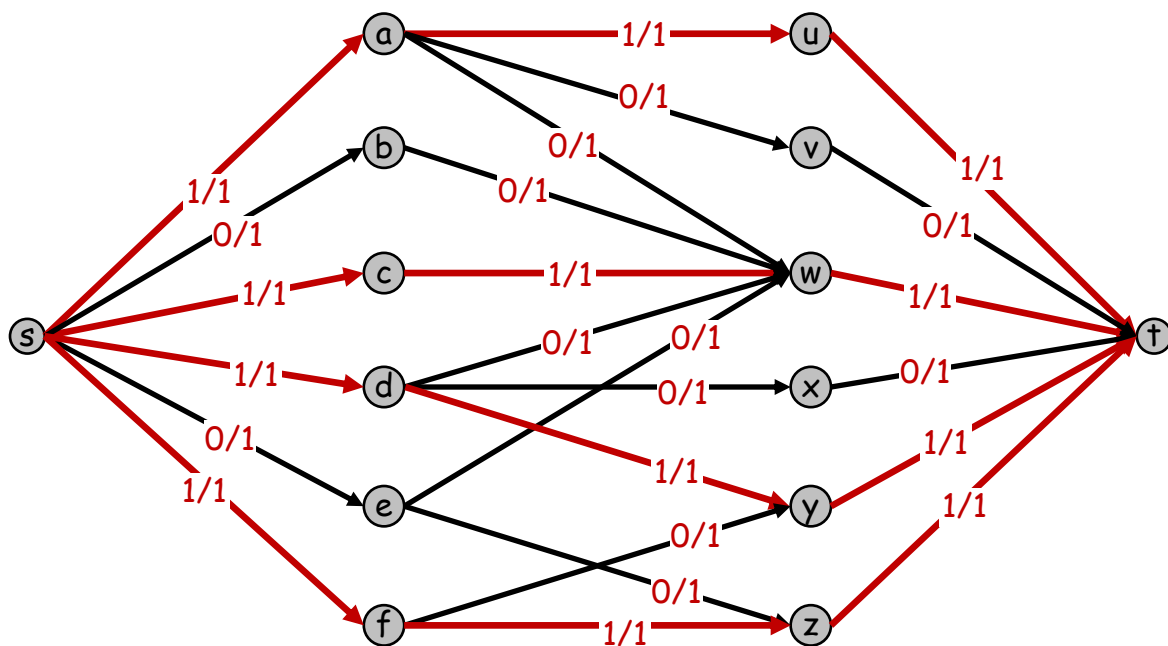


$|\text{Matching}| = 3$

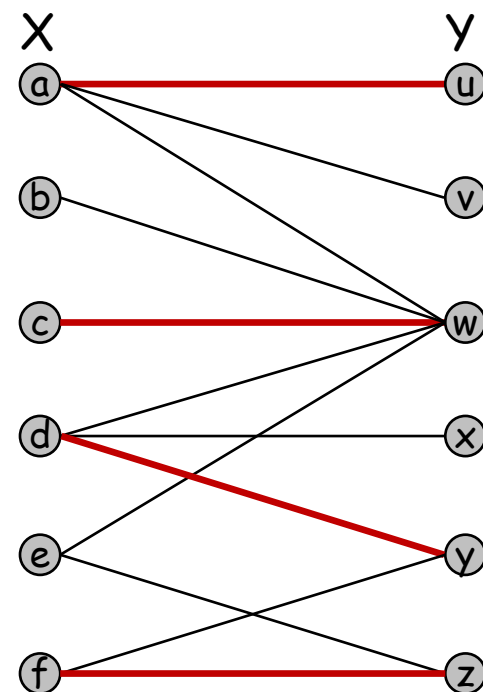


A Flow and its Corresponding Matching

Flow $|f| = 4$

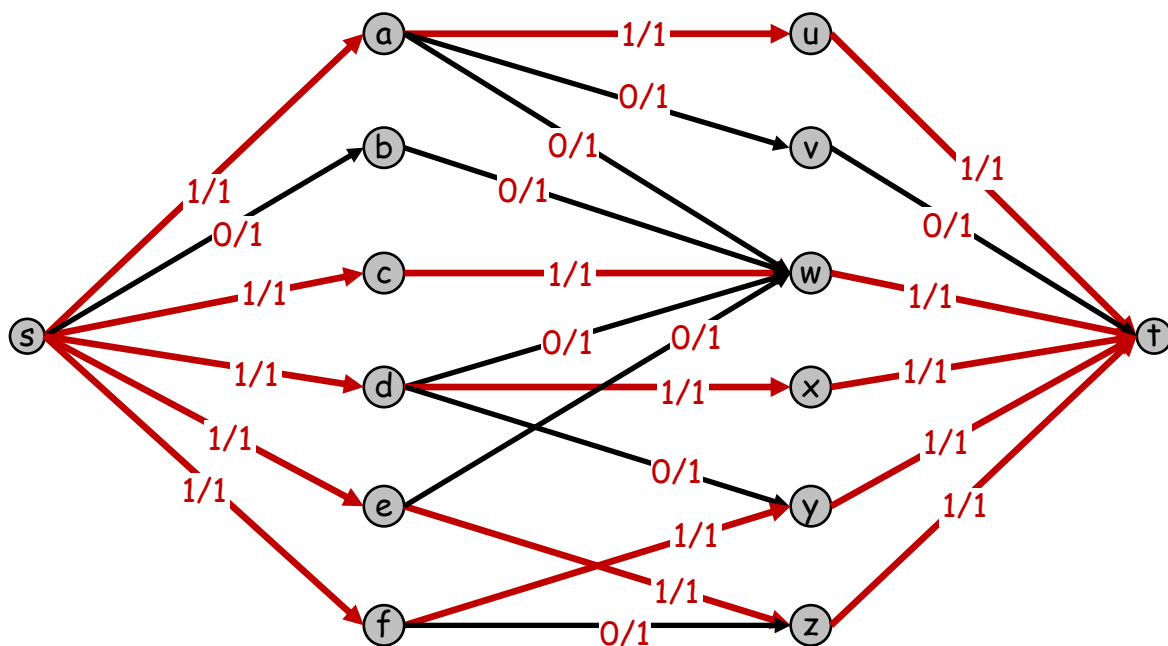


$|\text{Matching}| = 4$

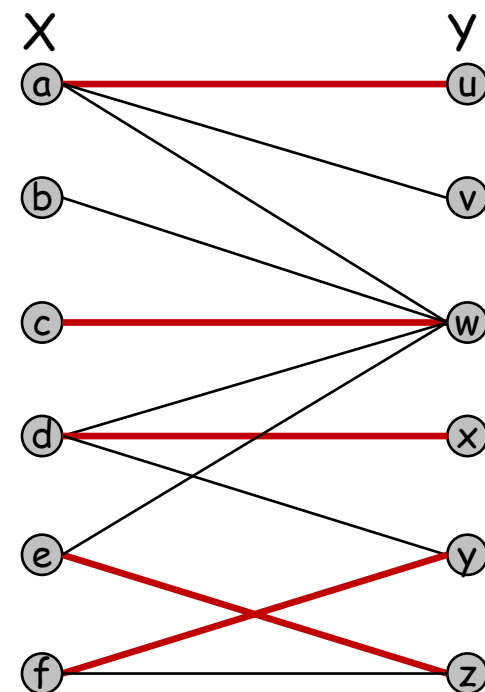


A Flow and its Corresponding Matching

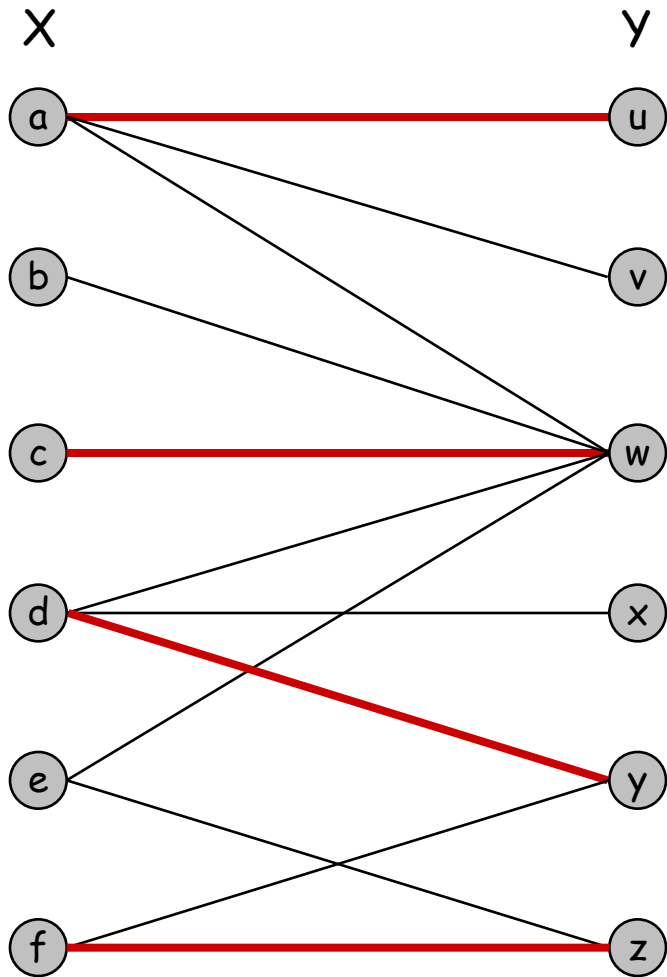
Flow $|f| = 5$



$|\text{Matching}| = 5$



Bipartite Matching Example



This matching has **size 4**.

Its corresponding flow has $|f| = 4$

The Ford-Fulkerson algorithm will find an **augmenting path** to increase the flow to $|f| = 5$.

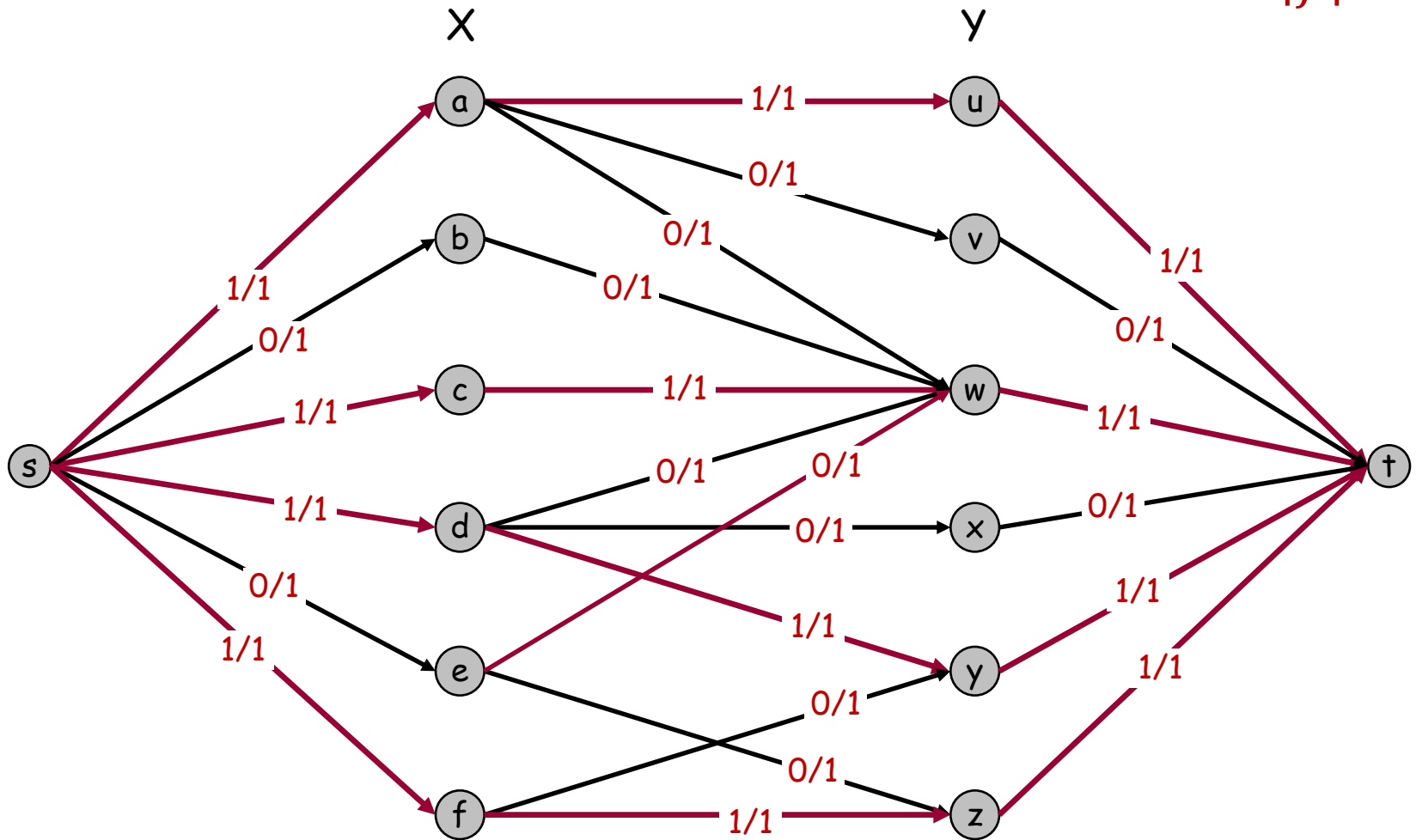
This will correspond to a new matching of **size 5**. We can show that $|f| = 5$ is a maximum flow which will imply that the matching of **size 5** is maximum as well

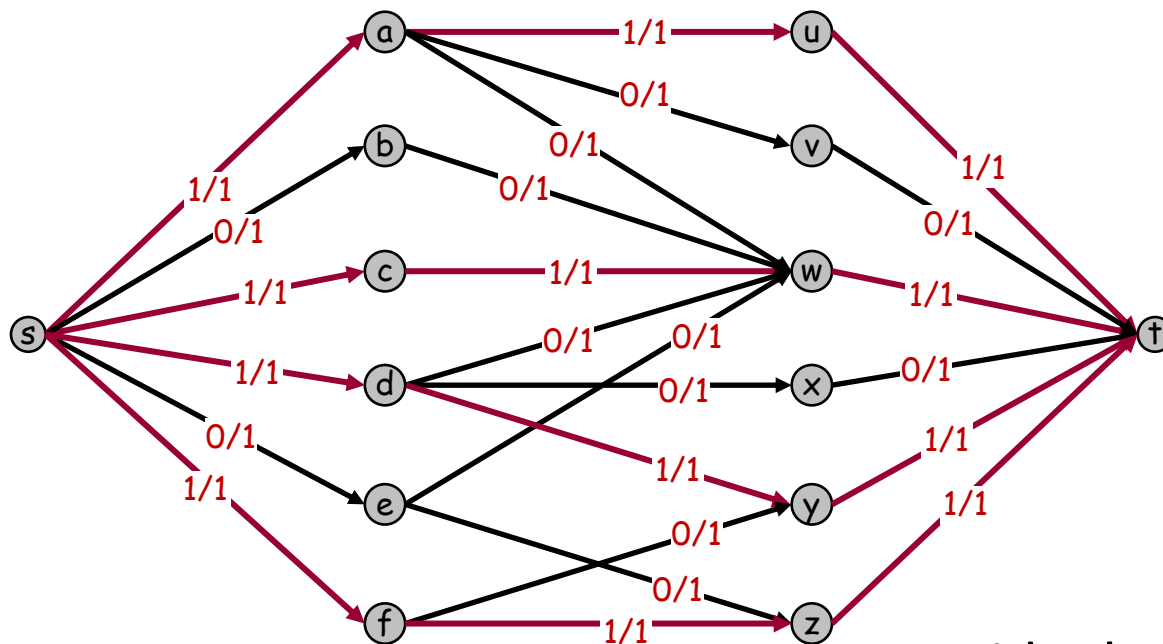
Example on previous page shows that **moving from 4 to 5** required **deleting 2 old matching edges and adding in 3 new ones**.

How was augmenting path technique able to do this? Details follow....

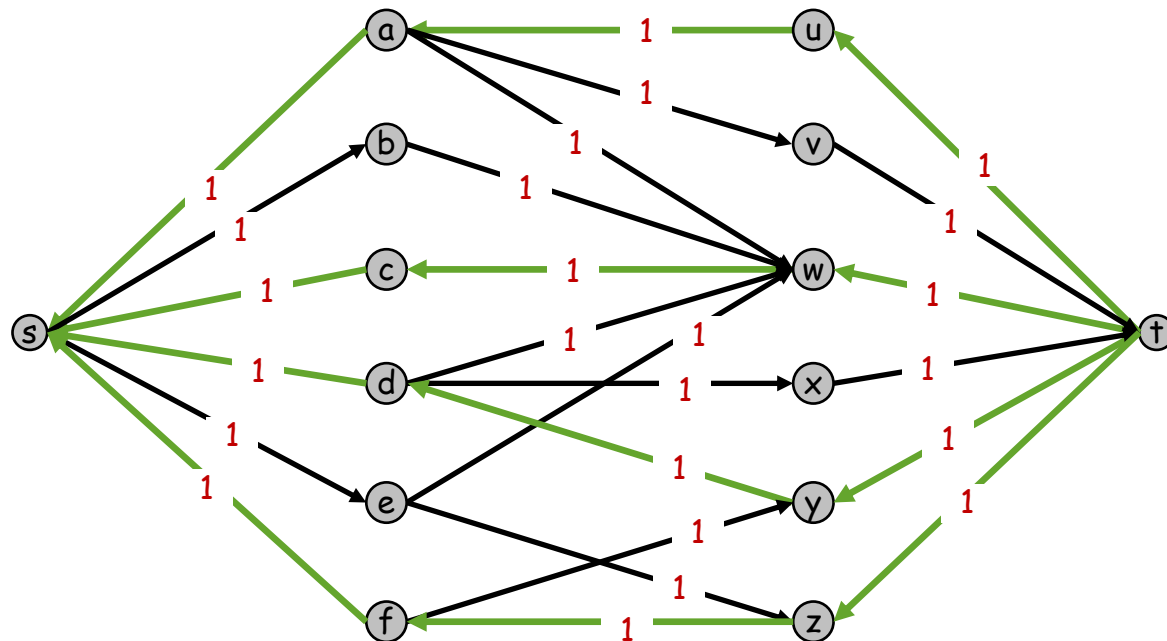
Flow Graph Corresponding to the Matching Result

Flow $|f| = 4$





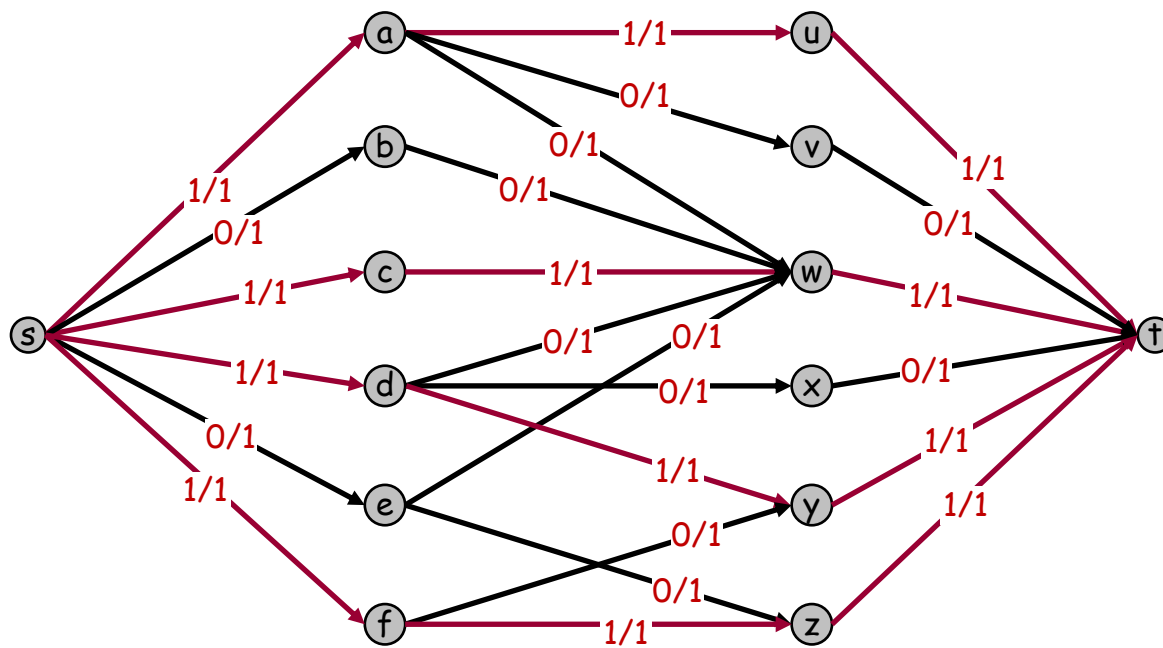
Residual Network



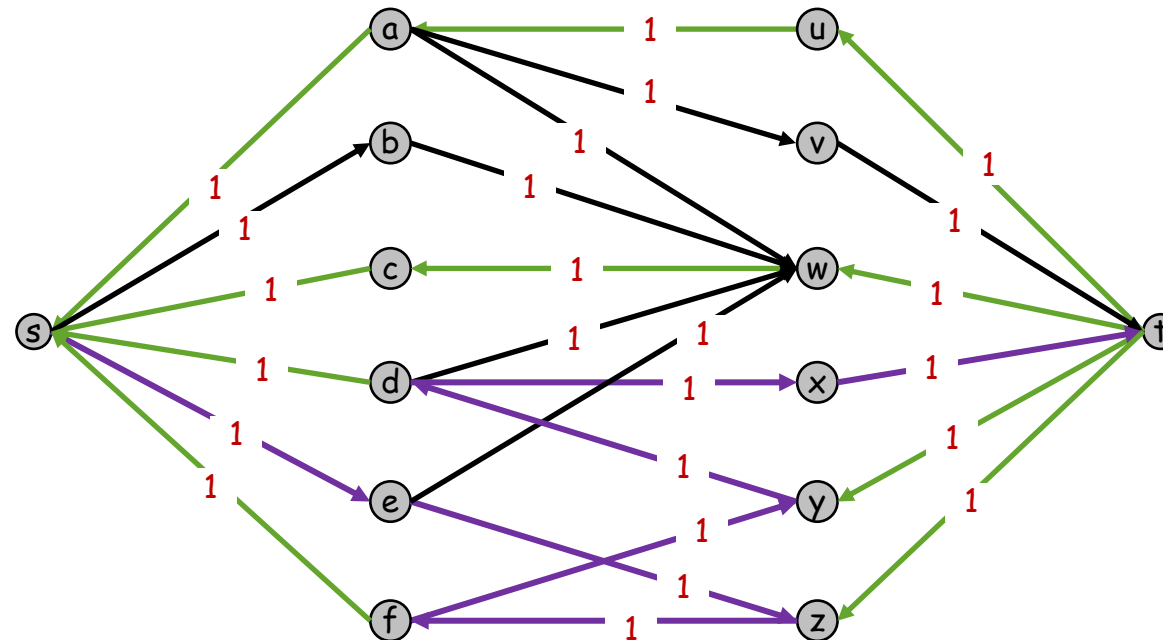
Because all original edges have capacity 1 and all flows are 0/1, every original edge is either in residual graph or FLIPS Direction in residual graph.

Edges from matching (and edges connecting them to s/t) are the ones that flip direction

Green edges will denote flipped direction

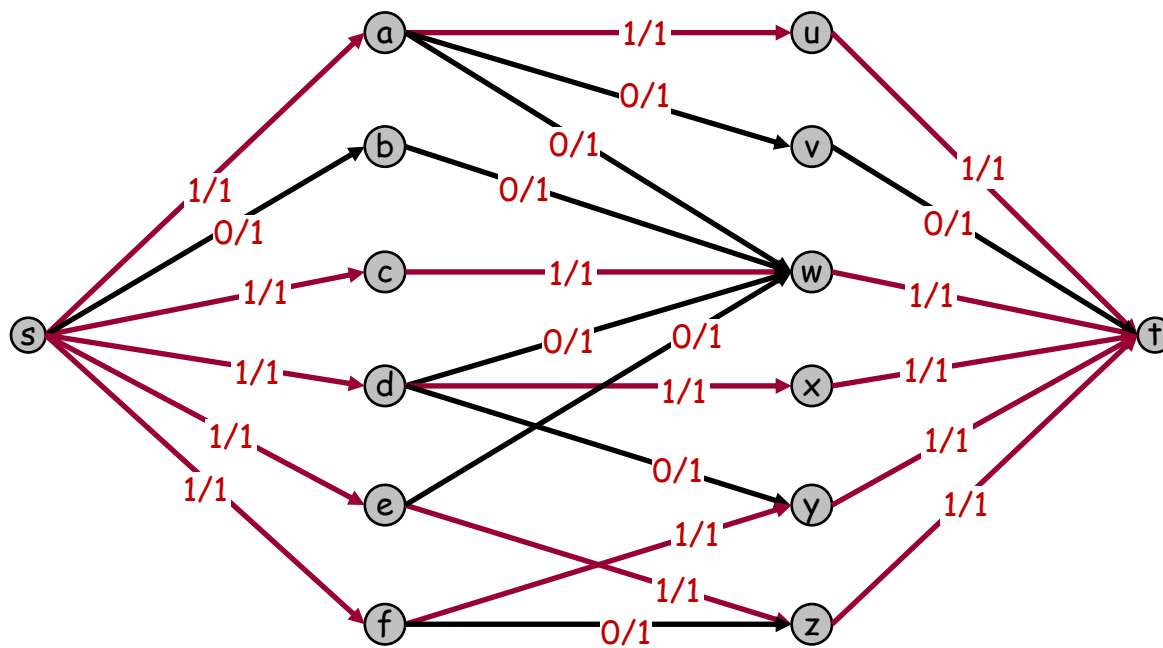


Flow $|f| = 4$

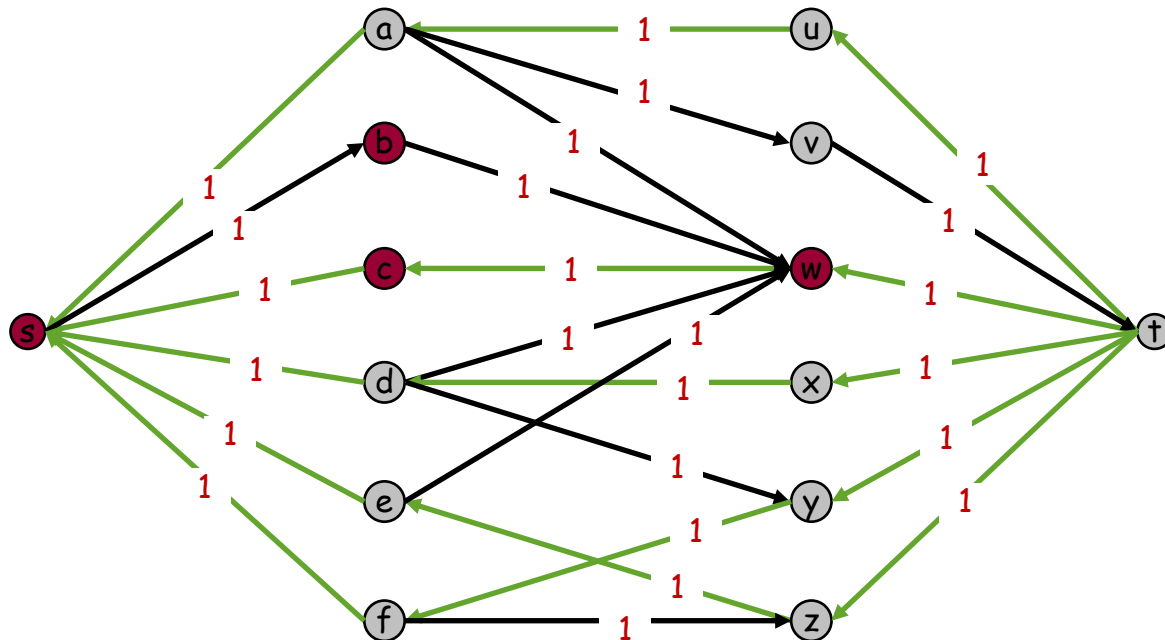


Residual Network
& Augmenting Path

This augmenting path (in purple) will delete (the flipped) edges from matching and add (forward) edges to matching



New flow after adding augmenting path has $|f| = 5$

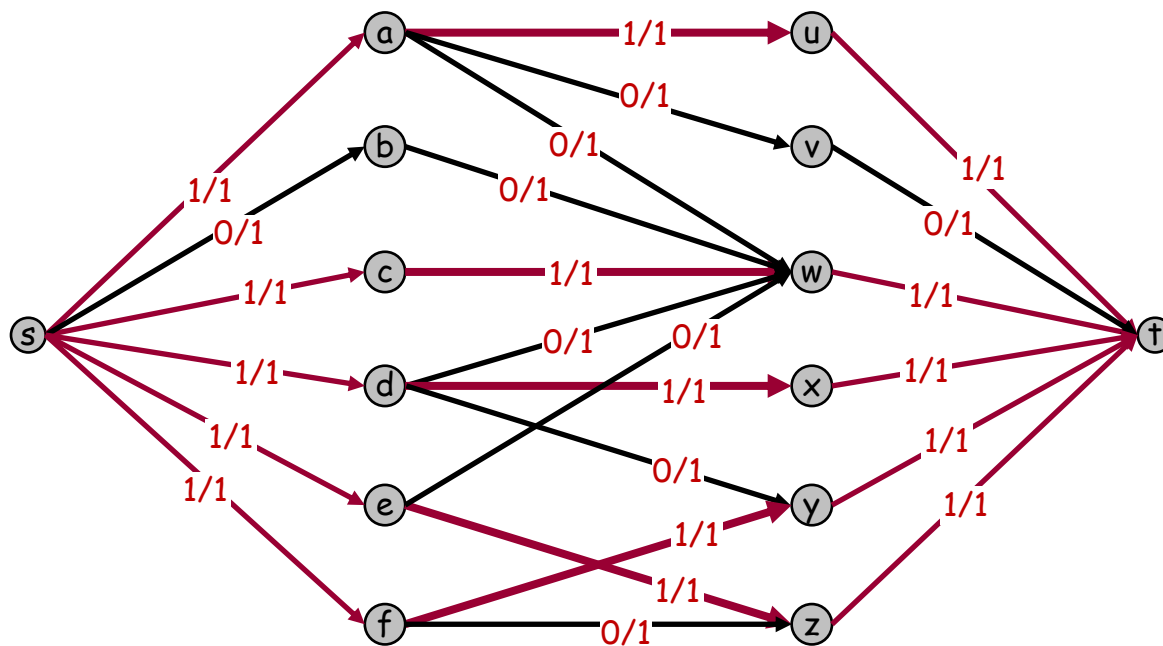


Residual Network

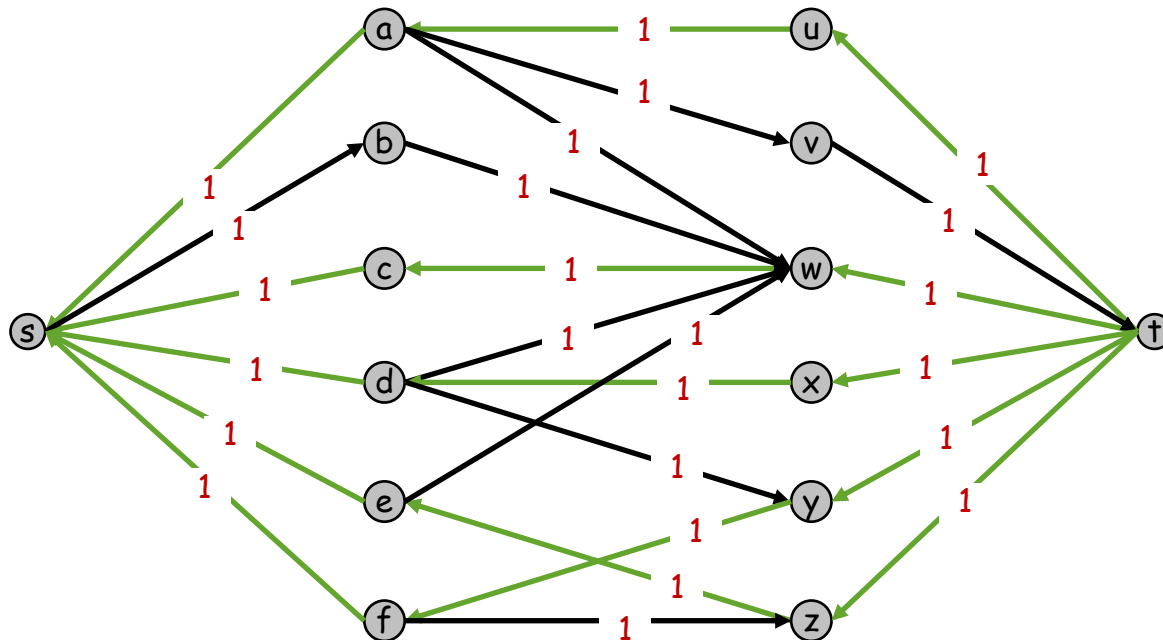
No augmenting path exists.

Associated cut has $S = \{s, b, c, w\}$.

This is a Max-Flow.



New flow after adding augmenting path has $|f| = 5$



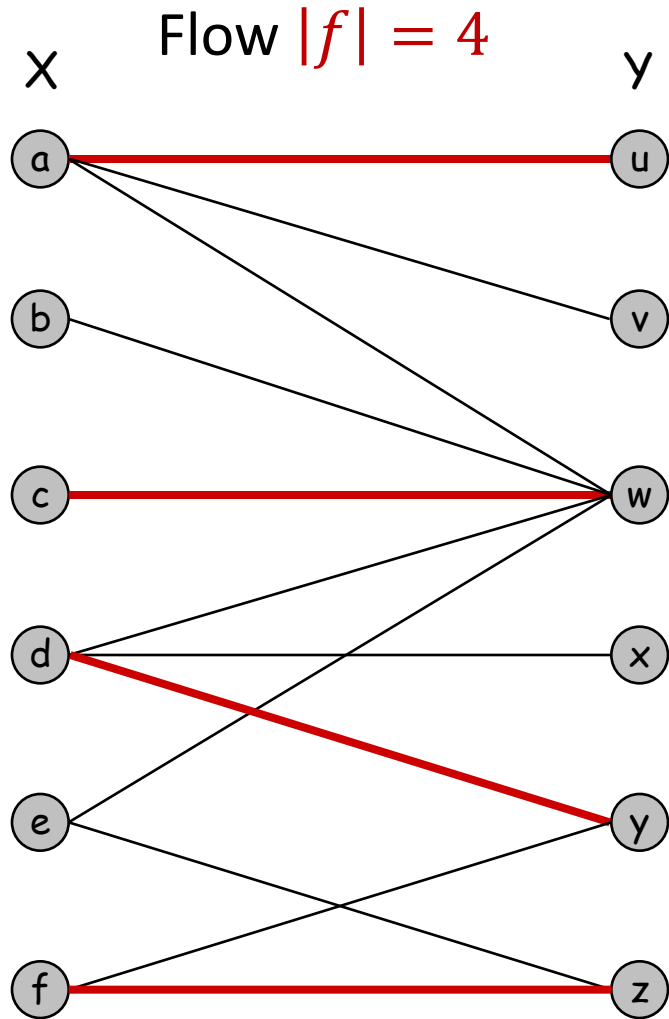
Residual Network

No augmenting path exists.

This is a Max-Flow.

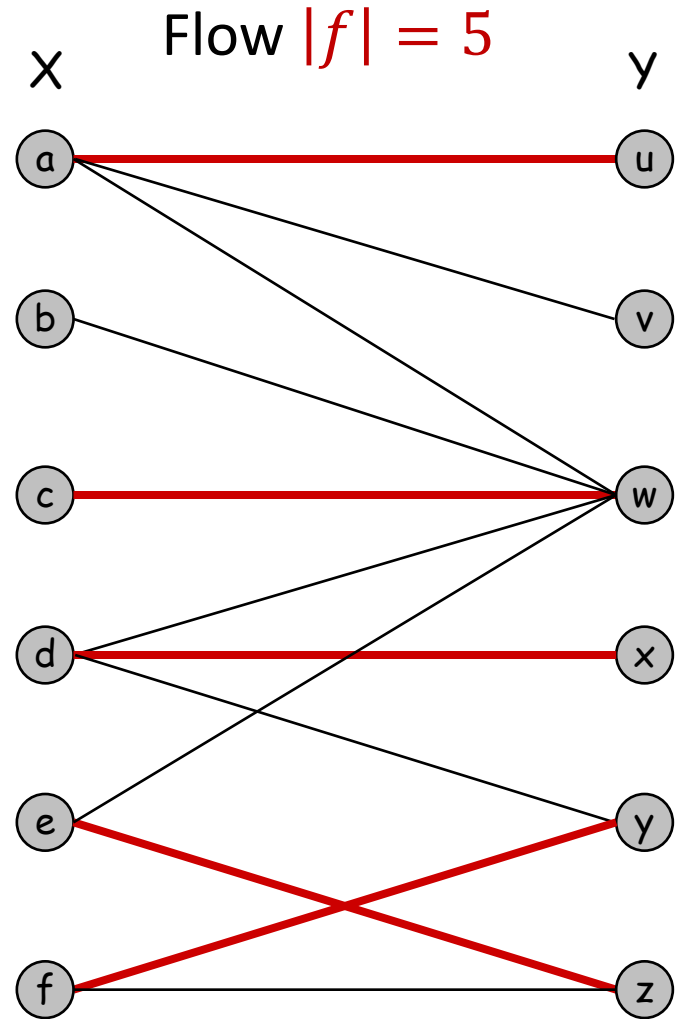
Heavy red edges are the corresponding maximum matching of size 5.

Bipartite Matching Example



Matching

$a-u, c-w, d-y, f-z$



Matching

$a-u, c-w, d-x, e-z, f-y$