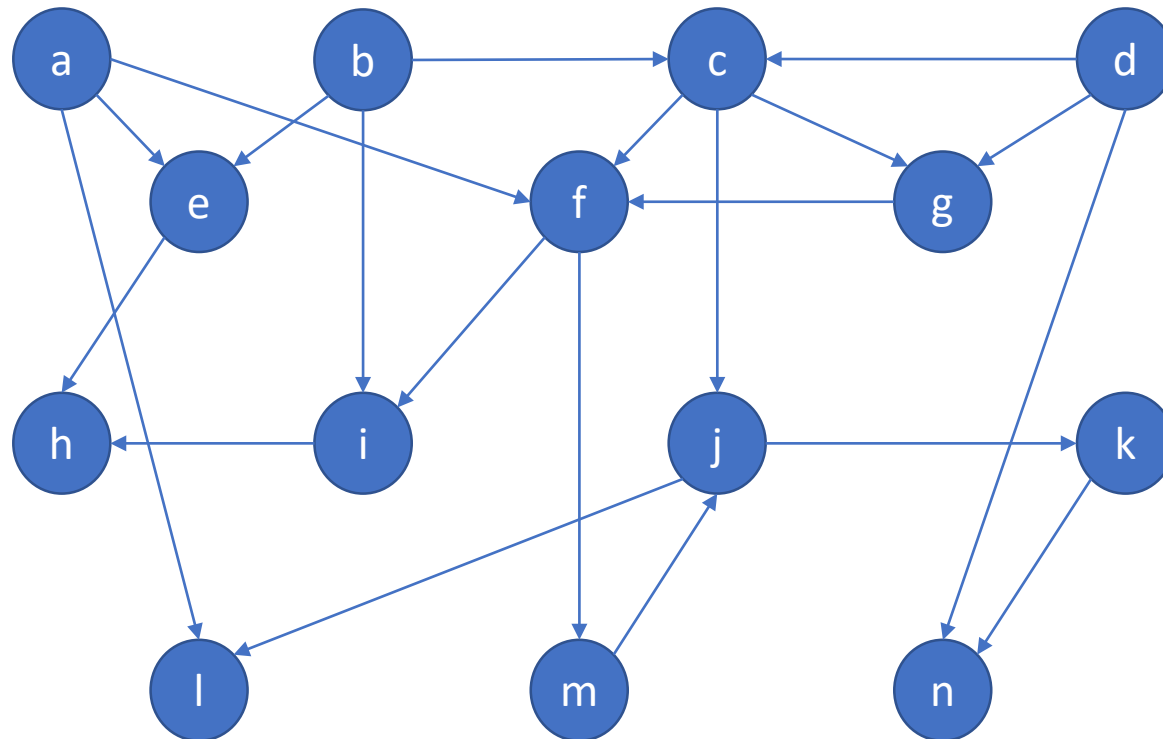


COMP 3711
Tutorial 8
Topological-Sort

Problem 1

Give a topological ordering of the following graph.



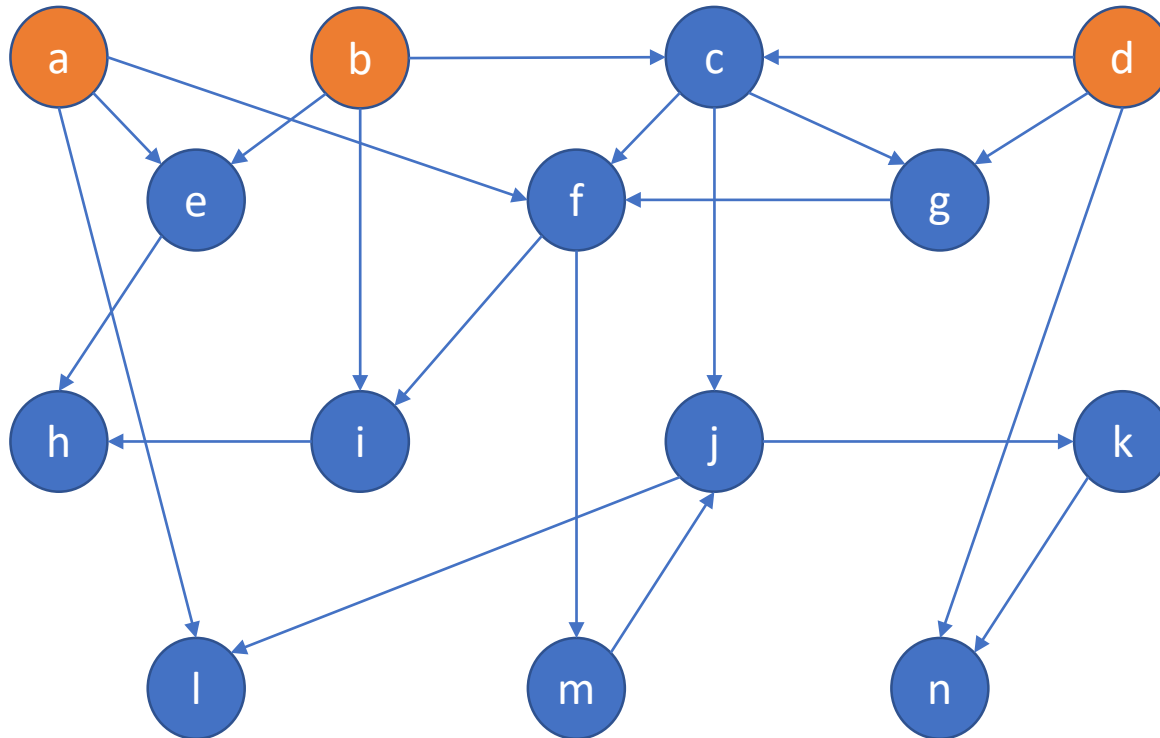
Problem 1

```
TopologicalSort(G){
    Q = empty; //initialize Queue
    For each u in V do //n times
        in_deg[u] = 0; //initialize the in-degree array
    For each u in V do //n times
        for each v in Adj(u) do //O(out_deg(u))
            in_deg[v] = in_deg[v]+1; //increase in-degree by one for each incoming edge
    For each u in V do //n times
        if(in_deg[u]==0)
            Enqueue(Q,u);

    While (Q is not empty) do //n times
        u = Dequeue(Q);
        Output u;
        for each v in Adj(u) do //O(out_deg(u))
            in_deg[v] = in_deg[v] - 1;
            if (in_deg[v]==0)
                Enqueue(Q,v);
}
```

Solution

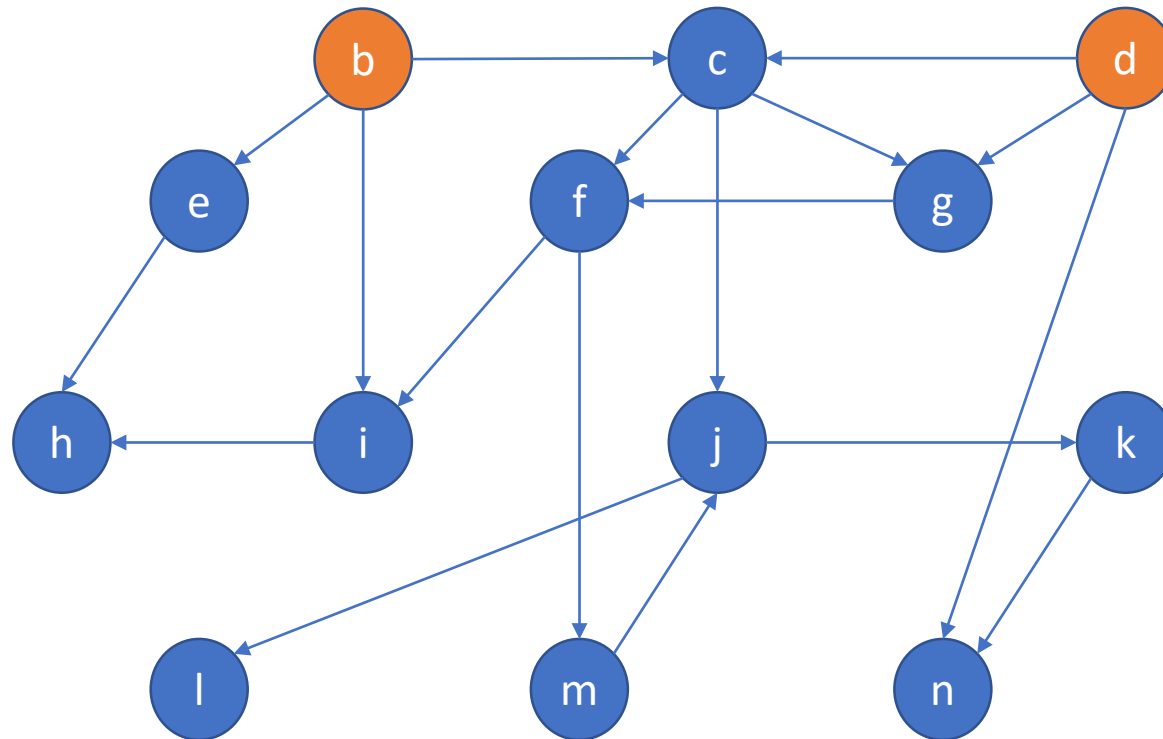
After initialization, 3 vertices are found to have in-degree 0 and are put on Q.



$Q = \{a, b, d\}$

Output=[]

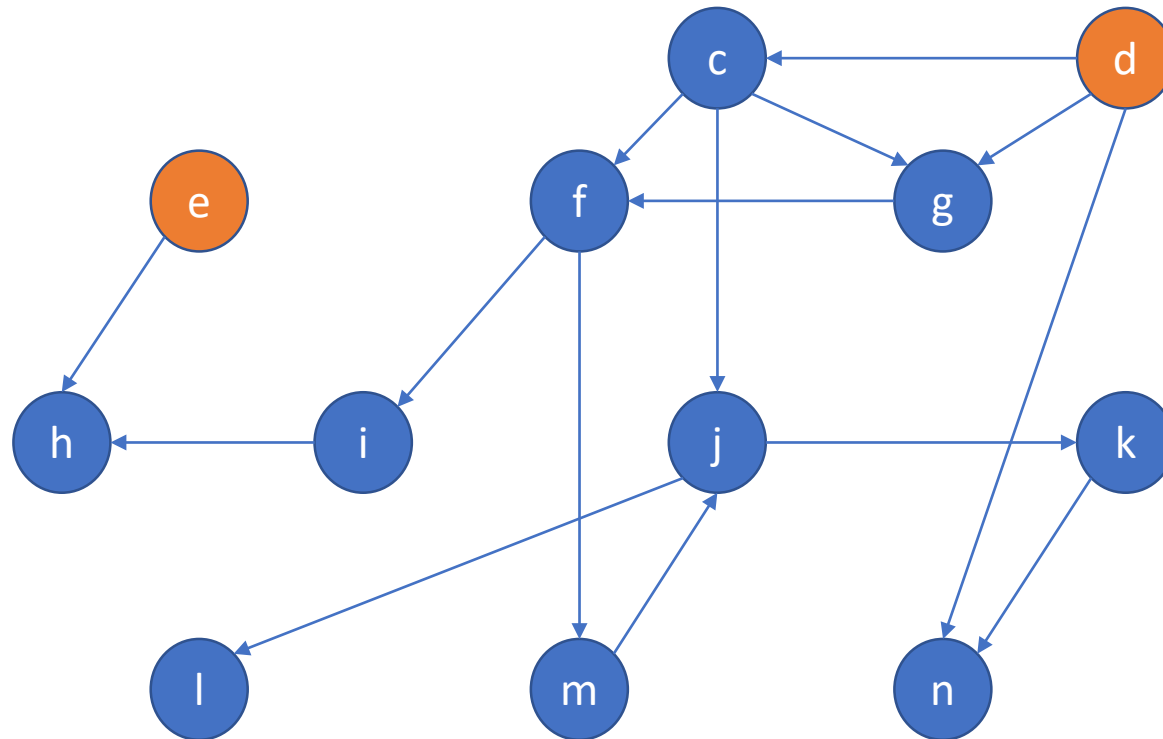
Solution



$Q=\{b,d\}$

Output=[a]

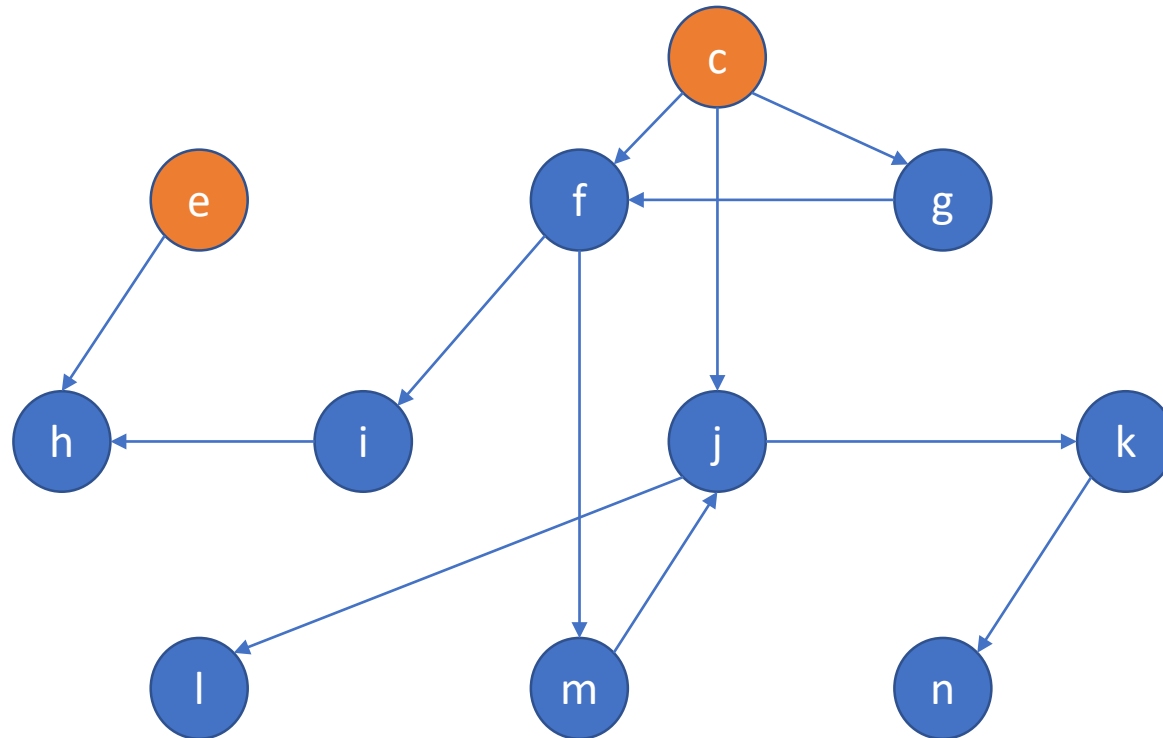
Solution



$Q=\{d,e\}$

Output=[a,b]

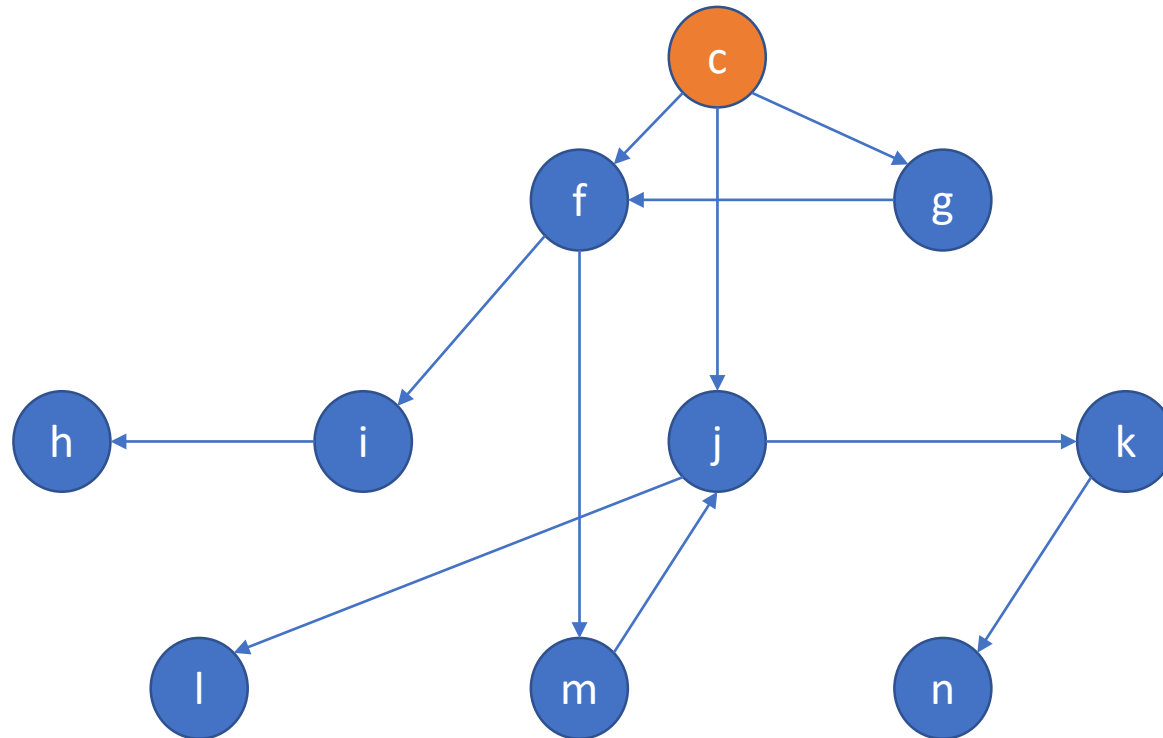
Solution



$Q=\{c,e\}$

Output=[a,b,d]

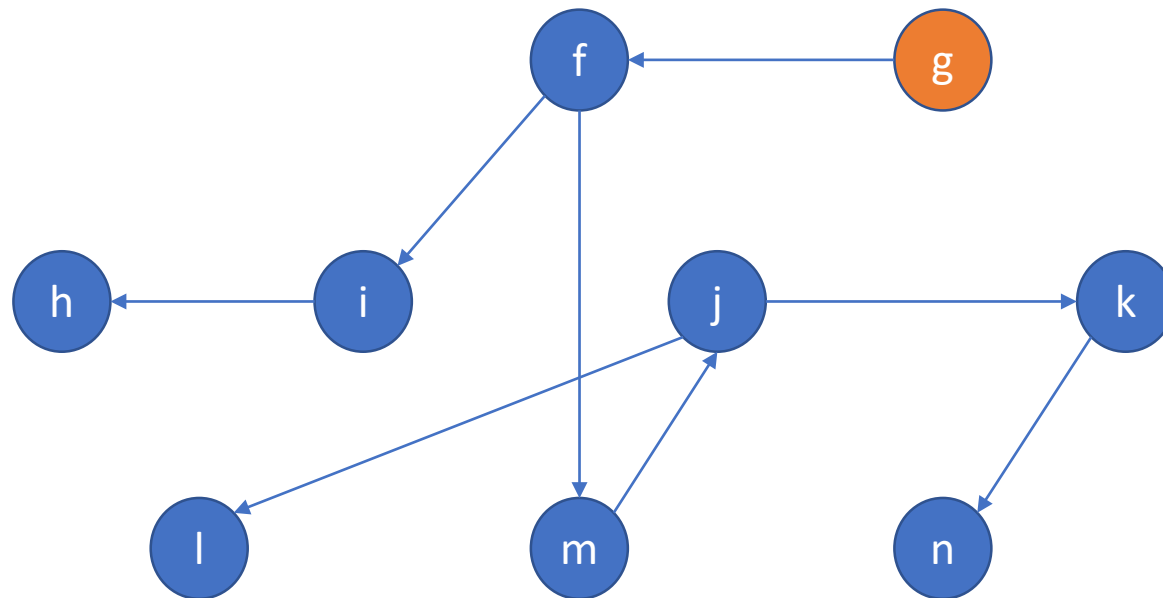
Solution



$Q=\{c\}$

Output=[a,b,d,e]

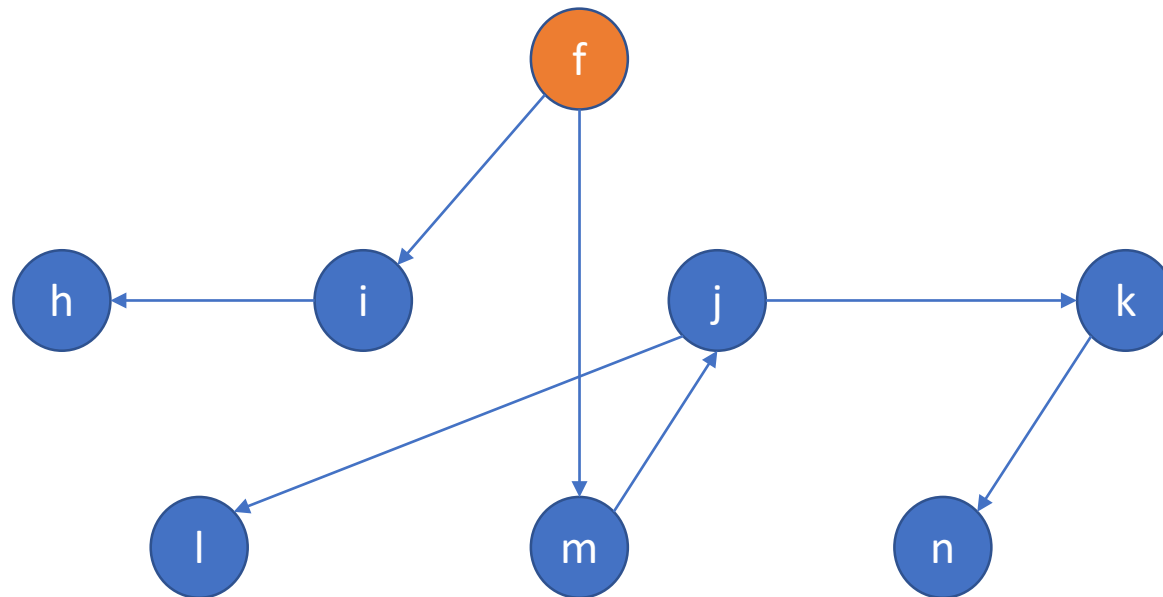
Solution



$Q=\{g\}$

Output=[a,b,d,e,c]

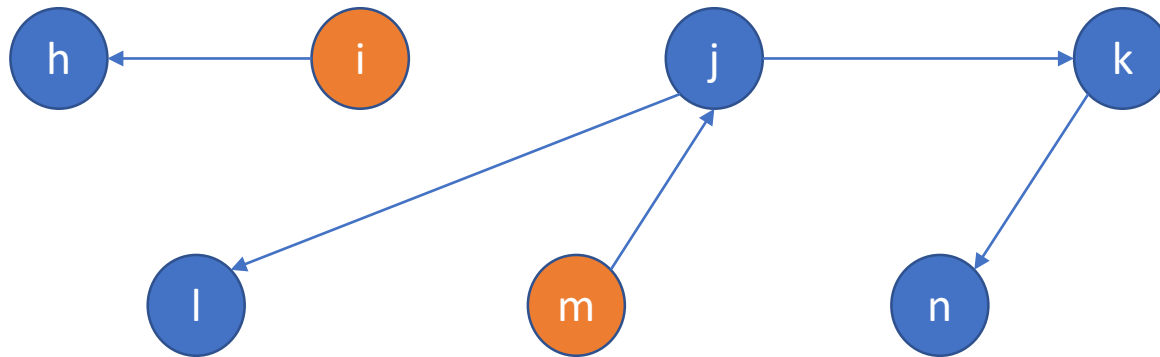
Solution



$Q=\{f\}$

Output=[a,b,d,e,c,g]

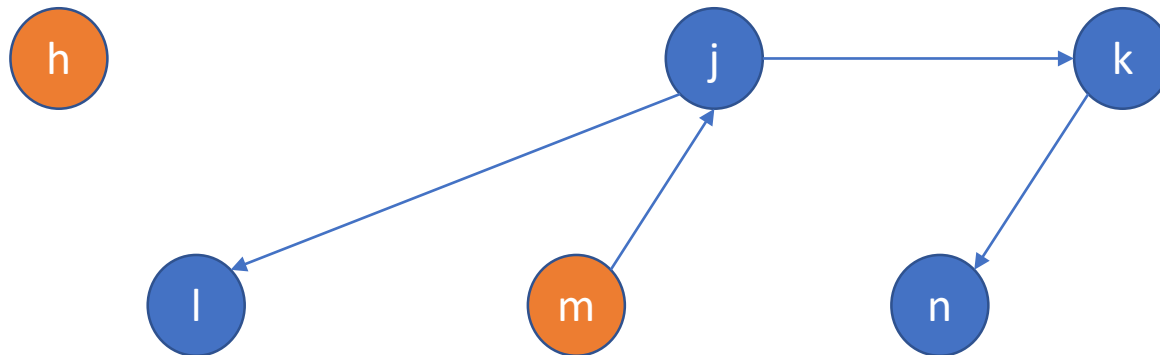
Solution



$Q=\{i,m\}$

Output=[a,b,d,e,c,g,f]

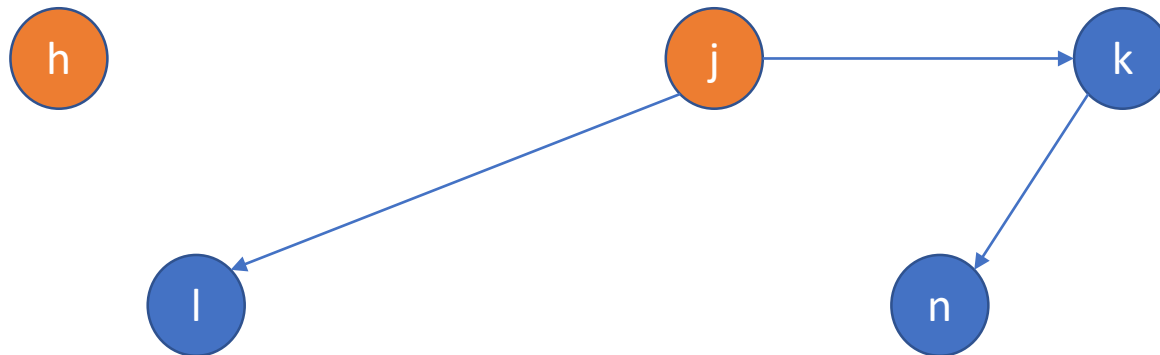
Solution



$Q=\{h,m\}$

Output=[a,b,d,e,c,g,f,i]

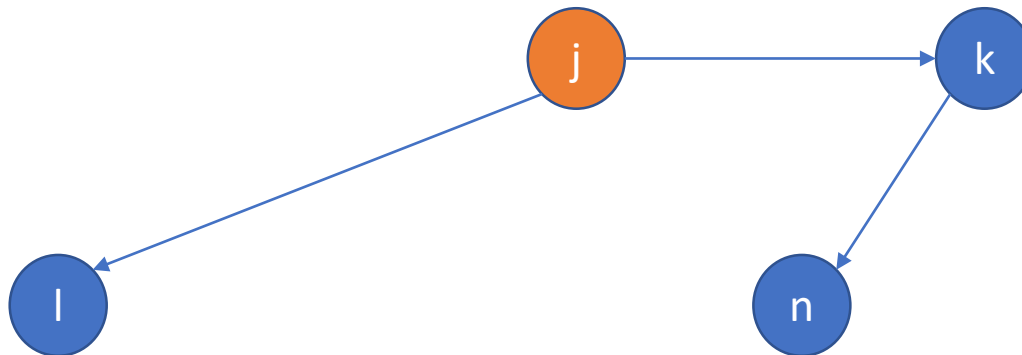
Solution



$Q=\{h,j\}$

Output=[a,b,d,e,c,g,f,l,m]

Solution



$Q=\{j\}$

Output=[a,b,d,e,c,g,f,i,m,h]

Solution



$Q=\{l,k\}$

Output=[a,b,d,e,c,g,f,i,m,h,j]

Solution

$Q=\{l,n\}$

Output=[a,b,d,e,c,g,f,i,m,h,j,k]



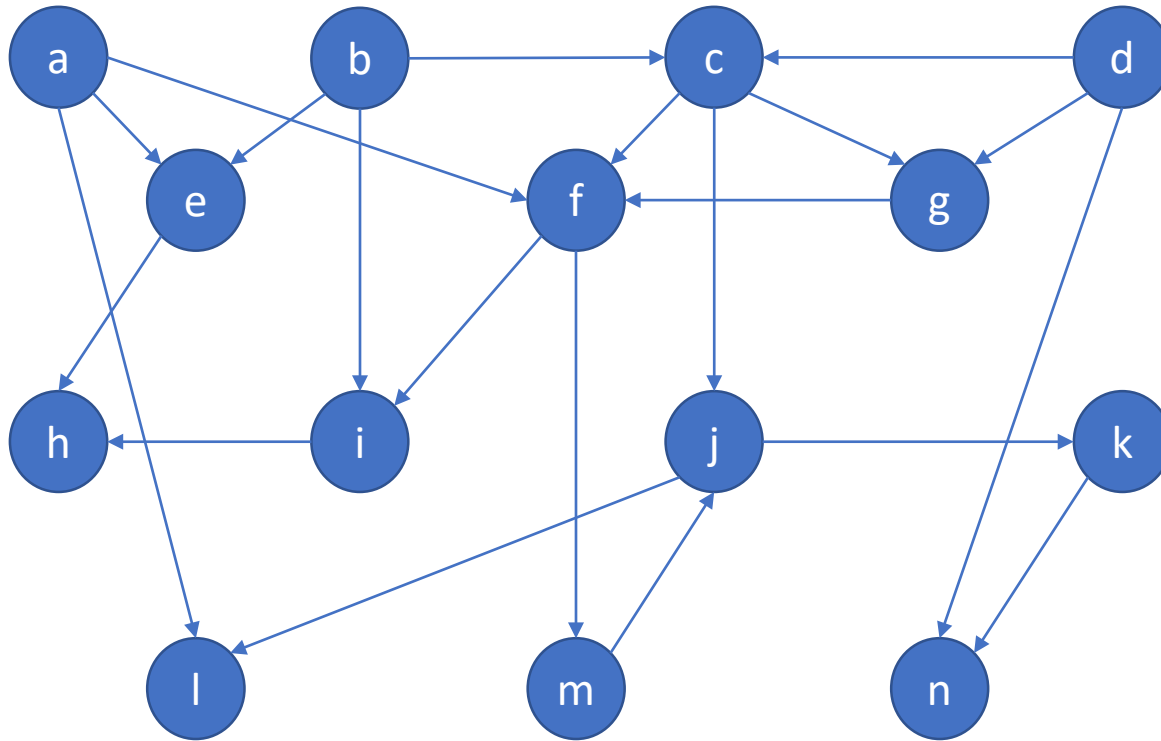
Solution

$Q=\{n\}$

Output=[a,b,d,e,c,g,f,i,m,h,j,k,l]



Solution



$Q=\{\}$

Output=[a,b,d,e,c,g,f,i,m,h,j,k,l,n]

Output is a topological ordering of the original input graph

Note: the enqueue operation here added items to front of queue.
It would have still worked if we had added items to tail of queue as we did in class.