

Problem 1

1.

D2

0	1	7	4	11	9
5	0	12	3	23	4
2	3	0	$\infty$	3	2
3	-2	22	0	2	1
4	2	2	4	0	4
8	3	3	5	1	0

D4

0	1	7	4	6	5
5	0	7	3	5	4
2	3	0	6	3	2
3	-2	5	0	2	1
4	2	2	4	0	4
5	3	3	5	1	0

D8

0	1	7	4	6	5
5	0	7	3	5	4
2	3	0	6	3	2
3	-2	4	0	2	1
4	2	2	4	0	4
5	3	3	5	1	0

2.

D1

0	1	7	$\infty$	$\infty$	10
5	0	12	3	$\infty$	15
2	3	0	$\infty$	$\infty$	2
$\infty$	-2	$\infty$	0	20	1
$\infty$	$\infty$	2	4	0	$\infty$
$\infty$	3	$\infty$	$\infty$	1	0

D2

0	1	7	4	$\infty$	10
5	0	12	3	$\infty$	15
2	3	0	6	$\infty$	2
3	-2	10	1	20	1
$\infty$	$\infty$	2	4	0	$\infty$
8	3	15	6	1	0

D3

0	1	7	4	$\infty$	9
5	0	12	3	$\infty$	14
2	3	0	6	$\infty$	2
3	-2	10	1	20	1
4	5	2	4	0	4
8	3	15	6	1	0

D4

0	1	7	4	24	5
5	0	12	3	23	4
2	3	0	6	26	2
3	-2	10	1	20	1
4	5	2	4	0	4
8	3	15	6	1	0

D5

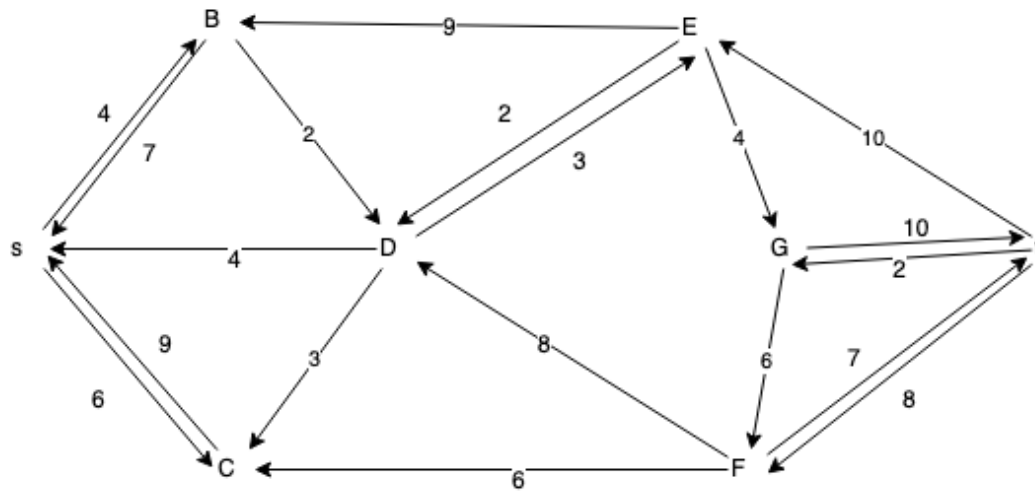
0	1	7	4	24	5
5	0	12	3	23	4
2	3	0	6	26	2
3	-2	10	1	20	1
4	5	2	4	0	4
5	3	3	5	1	0

D6

0	1	7	4	6	5
5	0	7	3	5	4
2	3	0	6	8	2
3	-2	4	1	2	1
4	5	2	4	0	4
5	3	3	5	1	0

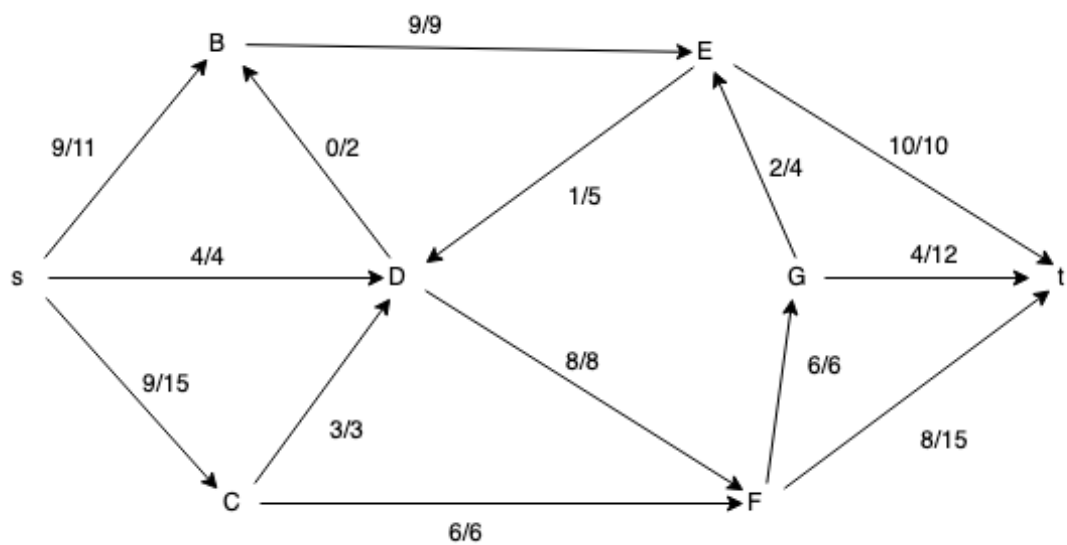
Problem 2

a)



b)  $p = \{s, B, D, E, G, t\}$   
 $cr(p) = 2$

c)



d)  $\min \text{ cut} = \{ \{s, B, C\}, \{D, E, F, G, t\} \} = 22$   
By max-flow-min-cut theorem, max flow = 22

Problem 3

- a) Let  $T'_1$  and  $T'_2$  be the MST of  $G$  that remove edge  $e = (u, v)$  after increasing the weight, there will be two cases:
1.  $e = (u, v)$  is still the lightest edge  
this mean  $T' = T$
  2. there is  $e' = (u', v')$  that lighter than  $e = (u, v)$   
this mean  $e' = (u', v')$  is added to connect  $T'_1$  and  $T'_2$ , and therefore  $T$  and  $T'$  differ by at most one edge.
- b) Assume we know the weight of edge  $e = (u, v)$  has been change to new weight.  
Go through the MST and separate the nodes into two sets( $T_1, T_2$ ) by removing edge  $e = (u, v)$  [ $O(V)$ ]  
Go through all the edges and find those that can connect  $T_1$  and  $T_2$  as  $E_{\text{connect}}$  [ $O(E)$ ]  
Look for the min cost edge in  $E_{\text{connect}}$ , and connect  $T_1$  and  $T_2$ . [ $O(1)$ ]