

AMS210.01 SAMPLE MIDTERM 2

March 29th, 2018

Show all work to receive full credit.

1) Given

$$\tilde{A} = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$$

- a) Determine the eigenvalues of the matrix.
- b) Determine the eigenvectors of the matrix.

2) Given

$$\tilde{A} = \begin{bmatrix} -1 & -1 & 1 \\ 1 & -2 & 3 \\ 2 & 2 & -4 \end{bmatrix} \quad \tilde{b} = \begin{bmatrix} 2 \\ 5 \\ -4 \end{bmatrix}$$

- a) Solve for \tilde{A}^{-1} using Elimination by Pivoting.
- b) Solve for \tilde{L} and \tilde{U} , using \tilde{LU} decomposition on \tilde{A} .
- c) Solve for $\det(\tilde{A})$.

3)

$$-9x_1 + 4x_2 - 3x_3 = 10$$

$$4x_1 + 8x_2 - 3x_3 = 30$$

$$-3x_1 + 3x_2 + 10x_3 = 20$$

- a) Rearrange the system so that it is of the form $\tilde{x}' = \tilde{D}\tilde{x}' + \tilde{b}$ such that $\|\tilde{D}\| < 1$.

- b) Solve for $\tilde{x}^{(2)}$. Let $\tilde{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

4) Given

$$\tilde{A} = \begin{bmatrix} 1/3 & 1/4 & 1/5 \\ 1/4 & 1/5 & 1/6 \\ 1/5 & 1/6 & 1/7 \end{bmatrix}$$

Find the condition number of the matrix \tilde{A} using the sum norm.