Homework #5

Due Date: 11:59pm, 11/17/2017

Reminder:

Please show all steps and do not just give the final answers. Material is based on Chapter 6, 7, and 8 from Ahn. Please upload your homework to Blackboard and you have two attempts to upload correctly for each homework.

1. If the variance of a normal population is 3, what is the 95th percentile of the variance of a random sample of size 15?

a) Find the percentile using the distribution table. Hint, use the distribution.

b) Find the percentile using R.

a) From table, we know that

where n=15 and =3, plug in the numbers.

P() =0.05 P(>5.0753) =0.05 P(<5.0753) =0.95

Therefore, the 95th percentile is 5.0753.

b) >3*qchisq(0.95, 14)/14

[1] 5.075312

2. In a scientific study, a statistical test yielded a p-value of 0.067. Which of the following is the correct decision? Explain your answer.

a) Reject H_0 at $\alpha = 0.05$ and reject it for $\alpha = 0.01$.

b) Reject H_0 at $\alpha = 0.05$ and but not for $\alpha = 0.01$.

c) Reject H_0 at $\alpha = 0.01$ and but not for $\alpha = 0.05$.

d) Do not reject H_0 at $\alpha = 0.05$ and do not reject it for $\alpha = 0.01$.

Answer d is correct. The p-value 0.067 is greater than 0.05 and 0.01, therefore, we do not reject H_0 at both $\alpha = 0.05$ and $\alpha = 0.01$.

3. A printer company claims that the mean warm-up time of a certain brand of printer is 15 seconds. An engineer of another company is conducting a statistical test to show this is an underestimate.

a) State the testing hypothesis.

b) The test yielded a p-value of 0.035. What would be the decision of the test if α = 0.05?

c) Suppose a further study establishes that the true mean warm-up time is 14 seconds. Did the engineer make the correct decision? If not, what type of error did he/she make?

a) H_0: (is also correct); H_1:

b) The p-value 0.035 is less than 0.05, so we reject the null hypothesis H_0.

c) The engineer did not make the correct decision and a type I error occurred. The true mean (14 seconds) is less than the population mean (15 seconds), which means the null hypothesis holds. This incorrect rejection of a true null hypothesis (false positive) lead to a type I error.

4. The incubation period of Middle East respiratory syndrome is known to have a normal distribution with a mean of 8 days and a standard deviation of 3 days. Suppose a group of researchers claimed that the true mean is shorter than 8 days. A test is conducted using a random sample of 20 patients. It is known that σ = 3.

a) Formulate hypotheses for this test.

b) Consider a rejection region { ≤ 6}. Suppose the test failed to reject H0. If the true mean incubation period is 5 days, what is the Type II error probability?

a) H_0: ; H_1:

b) Let X be the incubation period of Middle East Respiratory Syndrome, then . The type II error probability is $\beta(5)$ = P(do not reject H0 when =5) = P(X> 6) = P(Z>(6−5)/SQRT(0.45)) = 1−Φ(1.49) = 1−0.9319 = 0.0681.

5. The shelf life of a certain ointment is known to have a normal distribution. A sample of size 120 tubes of ointment gives = 36.1 and s = 3.7.

a) Construct a 95% confidence interval of the population mean shelf life.

b) Suppose a researcher believed that before the experiment σ = 4. What would be the required sample size to estimate the population mean to be within 0.5 months with 99% confidence?

(a) n =120 is large. $\bar{x}$=36.1, s =3.7

$\bar{x} \pm z_{\alpha/2}\, s$/sqrt($n$)=$\bar{x} \pm z_{0.025}\, s$/SQRT($n$)=

36.1±(1.96)(3.7/SQRT(120)) = 36.1 ±0.66

A 95% confidence interval is (35.44, 36.76).

(b) $n = (z_{\alpha/2}\, \sigma/E)^2 = (z_{0.005}\, \sigma/E)^2$=(2.58·4/0.5)^2 = 426.01

The required sample size is n=427.

6. The foot size of each of 16 men was measured, resulting in the sample mean of 27.32

cm. Assume that the distribution of foot sizes is normal with σ = 1.2 cm.

a) Test if the population mean of men's foot sizes is 28.0 cm using α = 0.01.

b) If α = 0.01 is used, what is the probability of a type II error when the population mean is 27.0 cm?

c) Find the sample size required to ensure that the type II error probability $\beta(27)$ = 0.1 when α = 0.01.

(a) H_0: $\mu=28$ versus H_1: $\mu\neq28$; $\alpha = 0.01$

The test statistic is $Z= (\bar{X}-\mu_0)/(\sigma/\text{SQRT}(n))$

and the rejection region is $|z|>z\alpha/2=z_{0.005} = 2.58$.

$Z= (\bar{X}-\mu_0)/(\sigma/\text{SQRT}(n)) = (27.32-28)/(1.2/\text{sqrt}(16)) = -2.27$

Since $|z|=2.27<2.58$, do not reject $H0$.

(b) $\beta(27) = \Phi(z_{.005} + (\mu0-\mu')/(\sigma/\text{SQRT}(n))) - \Phi(-z_{.005} + (\mu0-\mu')/(\sigma/\text{sqrt}(n)))$

$= \Phi(5.91) - \Phi(0.75) = 1 - 0.7734 = 0.2266$

(c) $n = (\sigma(z_{\alpha/2} + z_\beta)/(\mu_0-\mu'))^2 = (\sigma(z_{0.005} + z_{0.1})/(\mu_0-\mu'))^2$

$= (1.2(2.58+1.28)/(28-27))^2 = 21.46$

The required sample size is 22.

7. A computer company claims that the batteries in its laptops last 4 hours on average. A consumer report firm gathered a sample of 16 batteries and conducted tests on this claim. The sample mean was 3 hours 50 minutes, and the sample standard deviation was 20 minutes. Assume that the battery time distribution as normal.

a) Test if the average battery time is shorter than 4 hours at α = 0.05.

b) Construct a 95% confidence interval of the mean battery time.

c) If you were to test H0: μ =240 minutes vs. H1: $\mu \neq$ 240 minutes, what would you conclude from your result in part (b)?

d) Suppose that a further study establishes that, in fact, e population mean is 4 hours. Did the test in part (c) make a correct decision? If not, what type of error did it make?

(a) H_0: $\mu \geq 240$, H_1: $\mu < 240$ and $\alpha = 0.05$

The test statistic is $T = (\bar{X} - \mu_0)/(S/\text{SQRT}(n))$.
The rejection region is $t \leq -t_{\alpha,-1} = -t_{0.05,15} = -1.753$.
$t = (230-240)/(20/\text{SQRT}(16)) = -2 < -1.753$
Reject H_0. The p-value $= P(T<-2) = 0.032$

(b) $\bar{x} \pm t_{\alpha/2,n-1}(s/\text{sqrt}(n)) = 230 \pm 2.131(20/\text{SQRT}(16))$
$= 230 \pm 10.7$
A 95% confidence interval is (219.3, 240.7).

(c) Since 240 is included in the confidence interval, the mean battery time is not significantly different from 240 at level α=0.05, thus we do not reject H_0.

(d) The decision made in part (c) is correct.

8. BMI is obtained as weight (in kg) divided by the square of height (in M^2). Adults with BMI over 25 are considered overweight. A trainer at a health club measured the BMI of people who registered for his program this week. Assume that the population is normal. The numbers are given below.

29.4 24.2 25.6 23.6 23.0 22.4 27.4 27.8

a) Construct a 95% confidence interval for the mean BMI.
b) To find if newly registered people for the program are overweight on average, conduct an appropriate test using α = 0.05.
c) Suppose that a further study establishes that, in fact, the population BMI is 25.5. What did the test in part (b) lead to? Was it a correct decision? If not, what type of error did this test make?

(a) n=8 is small. $\bar{x}$=25.425, and s=2.542

$\bar{x} \pm t_{\alpha/2,-1}(s/\text{SQRT}(n)) = 25.425 \pm 2.365\,(2.542/\text{SQRT}(8)) = 25.425 \pm 2.126$

A 95% confidence interval is (23.30, 27.55).

(b) H_0: $\mu \leq 25$, H_1: $\mu > 25$ and $\alpha = 0.05$
The test statistic is $T = (\bar{X} - \mu_0)/(S/\text{SQRT}(n))$
The rejection region is $t \geq t_{\alpha,-1} = t_{0.05,7} = 1.895$.
$t = (25.425-25)/(2.542/\text{SQRT}(8) = 0.473 < 1.895$
Do not reject H_0. The p-value $= (T > 0.473) = 0.325$

(c) Type II error