NAME ________________________ STUDENT ID # ______________________

AMS210.01 SAMPLE Final Exam

April 25th, 2018

Show all work to receive full credit.

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1) Given

$$\underset{\sim}{A} = \begin{bmatrix} 1 & -3 \\ -3 & 1 \end{bmatrix} \qquad \underset{\sim}{b} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

a) $\det\left(\underset{\sim}{A}\right)$

b) $\underset{\sim}{A}^{-1}$

c) Solve for $\underset{\sim}{x}$ in $\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$, using $\underset{\sim}{A}^{-1}$

d) Find the condition number of $\underset{\sim}{A}$ using the sum norm

e) Determine the eigenvalues and eigenvectors of $\underset{\sim}{A}$

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2) Fit this data with a regression model of the form $\hat{y} = qx + r$

Hours spent studying	1	2	3	4	5	6
Hours required to finish test	9	7	6	4	2	1

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3) The following model for learning a concept over a set of lessons identifies four states of learning: I = ignorance, E = exploratory thinking, S = superficial understanding and M = mastery. If now in state I, after one lesion you have 50% chance of still being in I and 50% chance of being in E. If now in state E, you have 25% chance of being in I, 50% in E, and 25% in S. If now in state S, you have 25% chance of being in E, 50% in S, and 25% in M. If in M, you always stay in M.

a) Write out the transition matrix for this Markov chain with the absorbing states listed first.

b) Compute the fundamental matrix $\underset{\sim}{N}$.

c) What is the expected number of rounds until mastery if currently in the state of ignorance?

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4) The following problem is about cell growth. Suppose there are three types of cells: young, midlife and old. Midlife cells create one young cell each period. Old cells create one young cell each period. Every young cell splits into two midlife cells each period. Every midlife cell splits into two old cells each period. Warning: to solve the growth rate and long-term distribution, you will need an understanding of complex numbers.

a) Produce the Leslie matrix representing this type of cell growth.

b) Find the long-term growth rate.

c) What is the long-term distribution?

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5)

$$\underset{\sim}{A}=\begin{bmatrix}1 & 0 & 1\\ -2 & 1 & 1\\ 3 & -1 & 0\end{bmatrix}$$

a) Find the basis for the range of $\underset{\sim}{A}$.

b) Find the basis for the Null space of $\underset{\sim}{A}$.

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6)

$$\underset{\sim}{B}=\begin{bmatrix}1 & 0\\ -2 & 1\\ 1 & 3\end{bmatrix} \qquad \underset{\sim}{x}=\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad \underset{\sim}{c}=\begin{bmatrix}20\\ -10\\ 0\end{bmatrix}$$

a) Calculate the pseudoinverse $\underset{\sim}{B}^{+}$.

b) Use $\underset{\sim}{B}^{+}$ to find an approximation of $\underset{\sim}{x}=\begin{bmatrix}x_1\\ x_2\end{bmatrix}$ for $\underset{\sim}{B}\,\underset{\sim}{x}=\underset{\sim}{c}$, call it $\underset{\sim}{x}^{*}$.

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