AMS 310 Homework #6 Solutions

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12.5 points per problem

1.(12.5) A test is conducted to compare the tread wear of certain type of tires on highways paved with asphalt and highways paved with concrete. Road tests were conducted with the same type of tires on two types of highways. Summary data on the mileage of the tires up to a certain level of wear are given below.

	Sample Size	Sample Mean	Sample Standard Dev.
Asphalt	35	29,700	9,700
Concrete	35	25,500	7,800

Does this information suggest that tires wear faster on concrete-paved highways than asphalt paved highways? Test using α = 0.01.

$m = n = 35$ are large.

$H_0: \mu_1 = \mu_2, H_1: \mu_1 > \mu_2$

$\alpha = 0.1$

The test statistic is

$$Z = \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}}.$$

The rejection region is $z \geq z_\alpha = z_{0.01} = 1.28$.

$$z = \frac{29700 - 25500}{\sqrt{\frac{(9700)^2}{35} + \frac{(7800)^2}{35}}} = 1.996 > 1.28$$

Reject H_0.

There is sufficient evidence that mean drying time of the paints made by Company A is shorter.

p-value $= P(Z > 1.996) = 1 - \Phi(1.996) = 1 - 0.977 = 0.023$

2. In a study on the healing process of bone fractures, researchers observed the time required to completely heal a bone fracture. Ten rats were randomly divided into two groups of five. No treatment was done to one group and a medical treatment was given to the other group. The time spent (in days) to completely heal the fracture for each rat is given below:

Control: 30, 22, 28, 35, 45
Treatment: 33, 40, 24, 25, 24

a) (6.5) Test to see if the mean duration of healing can be reduced by the medical treatment at $\alpha = 0.01$.
b) (6) Construct a 99% confidence interval for the difference of the means of the two groups.

$m = n = 5$ are small. $\bar{x} = 32,\ s_1 = 8.63;\ \bar{y} = 29.2, s_2 = 7.12$

$H_0: \mu_1 = \mu_2, H_1: \mu_1 > \mu_2$

$\alpha = 0.1$

$$s_p^2 = \frac{(m-1)s_1^2 + (n-1)s_2^2}{m+n-2} = \frac{4(8.63)^2 + 4(7.12)^2}{5+5-2} = 62.59$$

The test statistic is

$$T = \frac{\bar{X} - \bar{Y}}{S_p\sqrt{\frac{1}{m} + \frac{1}{n}}}.$$

The rejection region is $t \geq t_{\alpha,m+n-2} = t_{0.1,8} = 1.397$.

$$t = \frac{32 - 29.2}{\sqrt{62.59}\sqrt{\frac{1}{5} + \frac{1}{5}}} = 0.560 < 1.397$$

Do not reject H_0.

p-value= $P(T > 0.56) = 0.2954$

3. (12.5) The following table shows summary data on mercury concentration in salmons (in ppm) from two different areas of the Pacific Ocean.

Area 1: m = 15, $\bar{x}$ = 0.0860, s_1 = 0.0032
Area 2: n = 15, $\bar{y}$ = 0.0884, s_1 = 0.0028

Do the data suggest that the mercury concentration is higher in salmons from Area 2? Assume equal variance and use $\alpha = 0.05$.

Note that $m = n = 15$ are small.
H_0: $\mu_1 = \mu_2$, H_1: $\mu_1 < \mu_2$ and $\alpha = 0.05$

$s_p = [(m-1)s_1^2+(n-1)s_2^2]/(m+n-2)$
$= [14(0.0032)^2+14(0.0028)^2]/(15+15-2) = 0.00000904$

The test statistic is $T = (\bar{X}-\bar{Y})\ /Sp\text{SQRT}(1/m+1/n)$.

The rejection region is $t \leq -t_{\alpha,+n-2} = -t_{0.05,28} = -1.701$.

$t = (0.0860-0.0884)/\ (\text{SQRT}(0.00000904)\ \text{SQRT}(1/15+1/15)\ ) = -2.186 < -1.701$

Reject H_0 $\quad$ p-value = $\text{P}(T < -2.186) = 0.0187$

4. Independent random samples are selected from two populations. The summary statistics are given below. Assume unequal variances for the questions below.

m = 5	$\bar{x}$ = 12.7	s_1 = 3.2
n = 7	$\bar{y}$ = 9.9	s_2 = 2.1

a) (6) Construct a 95% confidence interval for the difference of the means.
b) (6.5) Test if the means of the two populations are different based on the result from part (a).

a) Both m=5, n=7 are small.

Using the formula for ν we get ν = 6.43 → ν=6

$\bar{x}-\bar{y} \pm t_{0.025,6}\,\text{SQRT}(s_1^2/m+s_2^2/n)$
$= (12.7 - 9.9) \pm 2.447\text{SQRT}((3.2)^2/5+(2.1)^2/7) = 2.8 \pm 4.0$

Thus, a 95% CI is $(-1.2, 6.8)$.

(b) Do not reject H_0 because 0 is included in the confidence interval.

5. (12.5) Within a school district, students were randomly assigned to one of two Math teachers - Mrs. Smith and Mrs. Jones. After the assignment, Mrs. Smith had 30 students, and Mrs. Jones had 25 students.

At the end of the year, each class took the same standardized test. Mrs. Smith's students had an average test score of 78, with a standard deviation of 10; and Mrs. Jones' students had an average test score of 85, with a standard deviation of 15.

Test the hypothesis that Mrs. Smith and Mrs. Jones are equally effective teachers. Use a 0.10 level of significance. (Assume that student performance is approximately normal.)

Notice that m = 30, but n = 25 which is small. We assume equal variance.

H_0: $\mu_1 = \mu_2$ vs. H_1: $\mu_1 \neq \mu_2$ and $\alpha = 0.10$

$s_p^2 = [(m-1)s_1^2+(n-1)s_2^2]/(m+n-2) = 156.604$

The test statistic is $T = (\bar{X}-\bar{Y})\ /Sp\text{SQRT}(1/m+1/n)$.

The rejection region is $|t| \geq t_{\alpha/2,+m+n-2} = t_{0.05,53} = 1.701$

$t = (78-85)/\ (\text{SQRT}(156.604)\ \text{SQRT}(1/30+1/25)\ ) = -2.066 < -1.701$
Reject H_0. P-value $0.02 <$ p-value $< .05 < 0.10$

The data indicates that there is a significant difference at the $\alpha = 0.10$ level.

6. A manufacturer of furniture claims that the company's new model of bookshelves is easier to assemble than the old model. To test the claim, 7 people were assigned to assemble bookshelves from each of the two models. Here are the times (in minutes) they needed to assemble the bookshelves.

	Person						
	1	2	3	4	5	6	7
Old model	17	22	20	14	15	21	24
New model	15	19	18	13	15	19	23

a) (4) Construct a 95% confidence interval for the differences.
b) (4) Test if the mean assembly time for the new model is shorter than that of the old model using $\alpha = 0.05$.
c) (4.5) Now consider the data as two independent samples and conduct the test as a two sample test. Assume equal variances. Is the decision reversed?

	Person						
	1	2	3	4	5	6	7
Old model	17	22	20	14	15	21	24
New model	15	19	18	13	15	19	23
Difference	2	3	2	1	0	2	1

(a)

$\bar{d} = 1.57,\ s_d = 0.976$

$$\bar{d} \pm t_{\frac{\alpha}{2},n-1}\frac{s_d}{\sqrt{n}} = \bar{d} \pm t_{0.025,6}\frac{s_d}{\sqrt{6}} = 1.57 \pm 2.447\frac{0.976}{\sqrt{7}} = 1.57 \pm 0.90$$

A 95% confidence interval is $(0.67, 2.47)$.

(b) $H_0\colon \delta = 0,\ H_1\colon \delta > 0$

$\alpha = 0.05$

The test statistic is

$$T = \frac{\bar{D}}{S_d/\sqrt{n}}.$$

The rejection region is $t \geq t_{\alpha,n-1} = t_{0.05,6} = 1.943$.

$$t = \frac{1.57}{0.976/\sqrt{7}} = 4.256 > 1.943$$

Reject H_0.

p-value$= P(T > 4.256) = 0.0027$

(c) $m = n = 7,\ \bar{x} = 19,\ s_1 = 3.74;\ \ \bar{y} = 17.4, s_2 = 3.36$

$H_0\colon \mu_1 - \mu_2 = 0,\ \ H_1\colon \mu_1 - \mu_2 > 0$

$$s_p^2 = \frac{(m-1)s_1^2 + (n-1)s_2^2}{m+n-2} = \frac{6(3.74)^2 + 6(3.36)^2}{7+7-2} = 12.64$$

The test statistic is

(d) $$T = \frac{\bar{X}-\bar{Y}}{S_p\sqrt{\frac{1}{m}+\frac{1}{n}}}.$$

The rejection region is $t \geq t_{\alpha,m+n-2} = t_{0.05,12} = 1.782$.

$$t = \frac{19-17.4}{\sqrt{12.64}\sqrt{\frac{1}{7}+\frac{1}{7}}} = 0.8419 < 1.782$$

Do not reject H_0 .

p-value$= P(T > 0.8419) = 0.2082$

The decision is reversed because the within subject variation is ignored in part (c).

7. Public surveys were conducted on environmental issues in 2015 and 2016. One of the questions on the survey was: "How serious do you think the atmospheric contamination by exhaust gas is?" In 2015, 420 out of 1090 people surveyed said it was serious, and in 2016, 1063 out of 2,600 people surveyed said it is serious.

a) (9.5) Find a 95% confidence interval for the difference between the two proportions.

b) (3) Is there a significant difference at $\alpha = 0.05$?

a) $m = 1090,\ x = 420, n = 2600, y = 1063$. Thus, $\hat{p}_1 = \frac{x}{m} = \frac{420}{1090} = 0.385$ and $\hat{p}_2 = \frac{y}{n} = \frac{1063}{2600} = 0.409$.

$$\hat{p}_1 - \hat{p}_2 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{m} + \frac{\hat{p}_2(1-\hat{p}_2)}{n}}$$

$$= 0.385 - 0.409 \pm z_{0.05} \sqrt{\frac{(0.385)(0.615)}{1090} + \frac{(0.409)(0.591)}{2600}}$$

$$= -0.024 \pm 1.645 \cdot 0.0176 = -0.024 \pm 0.029$$

Therefore, a 90% confidence interval for $p_1 - p_2$ is $(-0.053, -0.005)$.

b) There is a significant difference because 0 is not contained in the confidence interval.

8. In a US senatorial election, 9 voters were randomly chosen from those who voted for a candidate from a conservative party, and 9 were chosen from those who voted for a liberal candidate. Their ages are given below.

Conservative: 51, 76, 62, 55, 39, 43, 46, 49, 56

Liberal: 44, 62, 60, 51, 35, 41, 39, 39, 36

a) (9.5) Test for equal variances between the two groups using $\alpha = 0.05$.

b)(3) Confirm your answer using software, either R or Excel. Give the results.

(a) $m = n = 9,\ s_1^2 = 123.5, s_2^2 = 102.4$

$H_0: \sigma_1^2 = \sigma_2^2, H_1: \sigma_1^2 \neq \sigma_2^2$

$\alpha = 0.05$

The test statistic is

$$F = \frac{S_1^2}{S_2^2}.$$

(b)

The rejection region is $f \geq F_{\frac{\alpha}{2},m-1,n-1} = F_{0.025,8,8} = 4.433$ or $f \leq \frac{1}{F_{\frac{\alpha}{2},n-1,m-1}} = \frac{1}{F_{0.025,8,8}} = 0.226$.

$$f = \frac{123.5}{102.4} = 1.206$$

Because $0.226 < 1.206 < 4.433$, we do not reject H_0. There is not enough evidence that the variances are different.

p-value$= 2P(T > 1.206) = 2(0.399) = 0.798$

b)

F-Test Two-Sample for Variances (Using Excel)

	Conservative	*Liberal*
Mean	53	45.22
Variance	123.5	102.44
Observations	9	9
df	8	8
F	1.21	
P(F<=f) one-tail	0.40	

Notice F = 1.21 (consistent with 1.206 above)

Notice the p-value = 2(0.40) ≈ 0.798 (given above)