AMS 310 More Test 2 Practice Problems **Fred Rispoli**

Solutions

1. Let X be a random variable with pdf f(x) = kx, 0 < x < 4.

a) Find the value of k. show all steps.

$\int_0^4 kx\, dx$ = 1 implies k = $\frac{1}{8}$

b) Find the cdf of X

F(x) = $\int_0^x \frac{1}{8} t\, dt$ = $\frac{1}{16}x^2$

c) Find P(X < 1 or X > 3)

P(X < 1 or X > 3) = F(1) + (1 – F(3)) = $\frac{1}{16} + \left(1 - \frac{3^2}{16}\right)$ = $\frac{8}{16} = 0.5$

d) Find E(X)

E(X) = $\int_0^4 \frac{1}{8}x^2 dx$ = $\frac{64}{24}$ = $\frac{8}{3}$ = 2.67

2. The time it takes a mechanic to change oil has an exponential distribution with mean 20.

a) Set up an integral to find P(15 < X < 25), then evaluate the integral.

P(15 < X < 25) = $\int_{15}^{25} \frac{1}{20} e^{-x/20} dx$ = $e^{-5/4} - e^{-3/4}$ = 0.18

b) Find the 40th percentile

Solve F(X) = 0.4 1 - $e^{-x/20}$ = 0.4 x ≈ 10.2

3. The random variables X and Y have the following joint probability distribution.

		X		
	f(x,y)	1	2	3
Y	1	0.10	0.30	0.20
	2	0.20	0.15	0.05

a) Find $P(X+Y>3)$.

$P(X+Y>3)$ = 0.20 + 0.15 + 0.05 = <u>0.40</u>

b) Find the marginal probability distributions $f_1(x)$ and $f_2(y)$.

X	1	2	3
f_1(X)	0.3	0.45	0.25

Y	1	2
f_2(y)	0.6	0.4

c) Determine if X and Y are independent. Justify your answer.

Not independent $\quad$ f(1,1) = 0.10, but $f_1(1)f_2(1)$ = 0.3(0.6) = 0.18

4. Suppose the random variables X and Y have joint pdf as follows:

$$f(x,y) = \begin{cases} cx+1 & x,y \geq 0 \ and \ x+y<1 \\ 0 & otherwise \end{cases}$$

a) Find the constant c

$\int_0^1 \int_0^{1-x} cx+1\, dy\, dx = \int_0^1 cx(1-x)+(1-x)\, dx = \frac{c}{2}-\frac{c}{3}+1-\frac{1}{2} = 1$. This implies that <u>c = 3</u>

b) Find the marginal pdf f_1(x) of X

$\int_0^{1-x} 3x+1\, dy = [3xy+y]_{y=0}^{y=1-x} = -3x^2+2x+1 = (3x+1)(1-x),\ 0 \leq x \leq 1$

c) Find $P(X<\frac{1}{2}, Y<\frac{1}{2})$

$= \int_0^{0.5} \int_0^{0.5} (3x+1) dy\, dx = \int_0^{0.5} (1.5x+0.5)\, dx = 0.4375$

5. Let the random variable have joint pdf as follows:

$$f(x, y) = \frac{3}{4}(x^2+3y^2) \qquad 0<x<1,\ 0<y<1$$

a) Find the marginal densities of X and Y

$f_1(x) = \int_0^1 f(x,y)dy = \frac{3}{4}(x^2+1)$, for $0 < x < 1$

$f_2(y) = \int_0^1 f(x,y)dx = \frac{1}{4}(1+9y^2)$, for $0 < y < 1$

b) Determine the conditional pdf's $f_1(x \mid y)$ and $f_2(y \mid x)$

$f_1(x \mid y) = \frac{f(x,y)}{f_2(y)} = \frac{\frac{3}{4}(x^2+3y^2)}{\frac{1}{4}(1+9y^2)} = \frac{3x^2+9y^2}{1+9y^2}$

$f_2(y \mid x) = \frac{f(x,y)}{f_1(x)} = \frac{\frac{3}{4}(x^2+3y^2)}{\frac{3}{4}(x^2+1)} = \frac{x^2+3y^2}{x^2+1}$

c) Find E(X) and E(Y)

$\mathrm{E}(\mathrm{X}) = \int_0^1 x f_1(x)dx = \frac{9}{16}$

$\mathrm{E}(\mathrm{Y}) = \int_0^1 y f_2(y)dy = \frac{11}{16}$

6. If the variance of a normal population is 9, what is the probability that the variance of a random sample of size 12 exceeds 16.102?

a) Find the probability using the distribution table.

b) Not on test but just for fun, find the probability using R or Excel

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2} = \frac{11S^2}{9} \text{ is distributed as } \chi^2 \text{ with 11 degrees of freedom.}$$

(a)

$$P(S^2 > 16.102) = P\left(\frac{11S^2}{9} > \frac{11(16.102)}{9}\right) = P(\chi^2 > 19.68) = 0.05$$

(b)

>1-pchisq(19.68, 11) = 0.05