AMS 310 Practice Test 3 Solutions **Fred Rispoli**

1. Assume a population has a distribution with a mean of 100 and a standard deviation of 10. For a sample of size 50, find the following.

a) P(99 < $\bar{X}$ < 102)

$\phi(\frac{102-100}{10/\sqrt{50}}) - \phi(\frac{99-100}{10/\sqrt{50}}) = \phi(1.41) - \phi(-0.71) = 0.9207 - 0.2389 \approx .68$

b) P($\bar{X}$ > 97)

$1 - \phi(\frac{97-100}{10/\sqrt{50}}) = 1 - \phi(-2.12) = 1 - 0.0170 = 0.983$

c) The 70th percentile of $\bar{X}$.

The 70h percentile occurs when z = .52, so solve $.52 = \frac{\bar{x}-100}{10/\sqrt{50}}$ this gives $\bar{X} = 100.74$

2. The weight of adult bottlenose dolphins was found to follow a normal distribution with a mean of 550 pounds and a standard deviation of 50 pounds.

a) What percentage of bottlenose dolphins weigh from 400 to 600 pounds?

$P(400 < X < 600) = P(\frac{400-550}{50} < Z < \frac{600-550}{50}) = P(-3 < Z < 1) = \phi(1) - \phi(-3) = 0.8413 - 0.0014 \approx 0.84$

b) If $\bar{X}$ represents the mean weight of a random sample of 9 adult dolphins, what is

P(500 < $\bar{X}$ < 580)?

$P(500 < \bar{X} < 580) = P(\frac{500-550}{50/\sqrt{9}} < Z < \frac{580-550}{50/\sqrt{9}}) = P(-3 < Z < 1.8) = \phi(1.8) - \phi(-3) = 0.9641 - 0.0014 \approx$ .96

c) In a random sample of 9 adult bottlenose dolphins, what is the probability that 5 of them are heavier than 560 pounds?

Set p = probability that one dolphin weighs more than 560

$P(X > 560) = P(z > \frac{560-550}{50} = P(Z > .2) = 1 - \phi(0.2)$

$= 1 - .5793 = .4207$

Set Y = # of dolphins that are heavier than 560. Y is a binomial rv

$P(Y = 5) = \binom{9}{5}(.4207)^5(1-.4207)^4 = 0.187$

3. The survival rate of a certain type of cancer using an existing medication is known to be 40%. A pharmaceutical company claims that the survival rate of a new drug is higher. The new drug is given to 25 patients to test this claim. Let X be the number of cures out of the 25 patients. Suppose the rejection region is $\{X \geq 13\}$.

a) State the testing hypotheses.

H_0: $p \leq 0.40$ $\qquad$ H_1: $p > 0.40$, claim

b) Determine the type of error that can occur when the true survival rate is 35%. Find the error probability.

First notice that when $p = 0.35$, H_0 is true. So a Type 1 error may occur.

P(Type 1 Error) = P($X \geq 13$ given $p = 0.35$) = $\sum_{x=13}^{25} bin(25, 0.35)$ = 1 - $\sum_{x=0}^{12} bin(25, .35)$

Use the normal approximation to the binomial

$\mu = np = 25(.35) = 8.75$ $\qquad$ $\sigma = \sqrt{np(1-p)} = 2.38$ $\qquad$ $z = (12.5 - 8.75)/2.38 = 1.58$

P(Type 1 Error) = 1 - $\sum_{x=0}^{12} bin(25, .35)$ = $P(Z > 1.58) = 1 - \phi(1.58) \approx 0.06$

c) Determine the type of error that can occur when the true survival rate is 55%. Find the error probability.

First note that if $p = 0.55$ then H_0 is false.

P(Type 2 Error) = P($X \leq 12$ given $p = 0.55$) = $\sum_{x=0}^{12} bin(25, .55)$

Use the normal approximation to the binomial

Now $\mu = np = 25(0.55) = 13.75$ and

$\sigma = \sqrt{np(1-p)} = 2.49$ $\qquad$ $z = (12.5 - 13.75)/2.49 = -0.50$

$\sum_{x=0}^{12} bin(25, 0.55) = P(Z \leq -0.50) = \phi(-0.50) \approx 0.31$

4. Match each item in the left column with the correct item in the right column

p-value = 0.02 ____c____	a. do not reject H_0 at $\alpha = 0.10$
p-value = 0.07 ____b____	b. reject H_0 at $\alpha = 0.10$, but not at 0.05
p-value = 0.30 ____a____	c. reject H_0 at $\alpha = 0.05$, but not at 0.01
p-value = 0.0006 ____d____	d. reject H_0 at $\alpha = 0.01$

5. a) A study of 49 bowlers showed that their average score was 186. The standard deviation of the population is 6, and the scores are normally distributed. Construct a 95% confidence interval for the mean score of all bowlers. Show all work involved.

$$(\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}})$$

$186 \pm 1.96(\frac{6}{\sqrt{49}}) = 186 \pm 1.68$ this gives (184.32, 187.68) as the 95% confidence interval

b) A study of 9 bowlers showed that their average score was 186. The standard deviation of the sample is 6, and the scores are normally distributed. Construct a 95% confidence interval for the mean score of all bowlers. Show all work involved.

$$(\bar{x} - t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}, \bar{x} + t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}) \qquad \text{df} = 8 \qquad \frac{\alpha}{2} = 0.025$$

$186 \pm 2.306(\frac{6}{\sqrt{9}}) = 186 \pm 4.612$ this gives (181.388, 190.612)

6. a) Suppose that 67 murders were studied and it was found that 10 were committed by women. Construct a 90% confidence interval for the true proportion of murders in New York committed by women. Show all work involved.

$$(\hat{p} - z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}) \qquad \hat{p} = \frac{10}{67} = 0.149 \qquad \frac{\alpha}{2} = 0.05$$

$0.149 \pm 1.645\sqrt{\frac{(.149)(.851)}{67}}$ this gives (0.077, 0.221)

b) Experts claim that at most 10% of murders in New York are committed by women. Is there enough evidence at $\alpha = 0.10$ to reject the claim if in a sample of 67 murders, 10 were committed by women? Use the 5 step method.

H_0: $p \le 0.10$ (claim) $\qquad$ H_1: $p > 0.10$

$\alpha = 0.10$ $\qquad$ $\hat{p} = \frac{10}{67} = 0.149$

Test Statistic $z = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$ $\qquad$ Critical Value $z = 1.28$

Test Value $z = 1.335$ which is greater than 1.28, so reject H_0

The data does not support the claim at $\alpha = -0.10$,

p-value = $P(Z > 1.335) = 1 - P(Z < 1.335) = 0.091$

c) Suppose that the true percentage of murders committed by women is 20%. What type of error, if any does this test involve? Explain.

If $p = 0.20$, then H_0 is false. There is no error in this case

7. A manufacturer claims that the standard deviation of the drying time of a certain type of paint **is** 18 minutes. A sample of five test panels produced a standard deviation of 21 minutes. Test the claim at $\alpha = 0.05$. Use the 5 step method.

1. H_0: $\sigma = 18$, (claim) $\quad$ H_1: $\sigma \neq 18$

2. $\alpha = 0.05$ $\quad$ df = 4

3. Test Statistic $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$ $\quad$ Critical Values: 0.4844 and 11.143

4. Test Value $\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(4)21^2}{18^2} = 5.44$ $\quad$ Do not reject H_0

5. The data supports the claim at $\alpha = 0.05$, $\quad$ $2(0.10) < \text{p-value} < 2(0.25)$

8. A factory manager wants to determine the average time it takes to finish a certain process by workers, and he wants to test it with randomly selected workers. He wants to be able to assert with 95% confidence that the mean of his sample is off by at most 1 minute. If the population standard deviation is 3.2 minutes, how large of a sample must he take?

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left[\frac{(1.96)(3.2)}{1}\right]^2 = 39.3 \text{ round up to } 40$$

9. A cell phone store has sold 150 phones of brand A and had 14 of them returned as defective. Additionally it has sold 125 phones of Brand B and had 15 of them returned as defective. Is there statistical evidence that Brand A has a smaller chance of being returned than Brand B? Use $\alpha = 0.05$ to test the claim.

$\widehat{p_A} = 0.093 = \frac{14}{150}$ $\quad$ $\widehat{p_B} = 0.120 = \frac{15}{125}$ $\quad$ $\hat{p} = \frac{x+y}{m+n} = \frac{29}{275} = 0.1055$

1. H_0: $p_A \geq p_B$ $\quad$ H_1: $p_A < p_B$ (claim)

2. $\alpha = 0.05$

3. Test Statistic $Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{m}+\frac{1}{n}\right)}}$ $\quad$ Critical Value $Z = -1.645$

4. $Z = \frac{0.093-0.120}{\sqrt{(0.1055)(0.8945)\left(\frac{1}{150}+\frac{1}{125}\right)}} = -0.726$, do not reject H_0

5. The data does not support the claim at $\alpha = 0.05$, p-value = $\phi(-0.726) = 0.234$

10. To test whether a college course is working a pre- and post-test is arranged for the students. The results are given below. Compare the scores with a t-test as indicated. Use α = 0.05 to test the claim.

a) Assuming the scores are randomly selected from the two groups. Assume equal variances.

b) Assume that the scores are pairs of scores for ten students.

Test				**Scores**						
Pre-test	**77**	**56**	**64**	**60**	**57**	**53**	**72**	**62**	**65**	**66**
Post-test	**88**	**74**	**83**	**68**	**58**	**50**	**67**	**64**	**74**	**60**

a) $H_0: \mu_{pre} = \mu_{post}$ $H_1: \mu_{pre} \neq \mu_{post}$

$\alpha = 0.05$ df = 10 + 10 – 2 = 18

Test statistic t = $\frac{\overline{(x}-\bar{y})-\Delta}{s_p\sqrt{\frac{1}{m}+\frac{1}{n}},}$ where $s_p^2 = \frac{(m-1)s_1^2+(n-1)s_2^2}{m+n-2}$ Critical Values: t = ± 2.101

Test Value t = -1.25 $s_p^2 = 9.67^2$

Do not reject H_0, there is no significant difference in scores at $\alpha = 0.05$,

(2)(.10) < p-value < (2)(.25)

0.2 < p < 0.5 Not required but included just as a check: P-Value = 0.228

b) Here we develop the test as a paired data t test

Test				Scores						
Pre-test	77	56	64	60	57	53	72	62	65	66
Post-test	88	74	83	68	58	50	67	64	74	60
Difference	-11	-18	-19	-8	-1	3	5	-2	-9	6

$\widehat{D} = -5.40$ $s_d = 9.03$ n = 10

1. $H_0: \widehat{D} = 0$ $H_1: \widehat{D} \neq 0$

2. $\alpha = 0.05$ df = 9

3. Test Statistic: $T = \frac{\bar{D}}{s_d/\sqrt{n}}$ Critical Values, use df = 9 and $\alpha/2$ to get ± 2.262

4. Test Value = -1.89 Do not reject H_0

5. There is no significant difference in scores at $\alpha = 0.05$, $0.025(2) < \text{p-value} < 0.05(2)$

11. A study has been done to test the effect of classical music on the brain. Twenty sixth-grade students were randomly divided into two groups of 10 students, and the same math test was given to these students. The first group of students listened to classical music for 20 minutes right before taking the test, and the other group took the test without listening to the music.

The scores are given below.

Group with music: 91, 77, 58, 89, 83, 78, 74, 81, 91 and 88	$\bar{x} = 81$	$s_1 = 10.1$
Group without music: 81, 65, 69, 69, 67, 61, 67, 87, 64, 81	$\bar{y} = 71.1$	$s_2 = 8.7$

Assume equal variance for performing the following calculations.

a) Construct a 95% confidence interval for the mean difference.

b) Test if the students who listened to classical music before the test scored higher than the other group of students using $\alpha = 0.05$.

c) Conduct the same test as in part (b) using $\alpha = 0.01$.

First note that both $m = n = 10$ are small.
$\bar{x} = 81$, $s_1 = 10.1$; and $\bar{y} = 71.1$, $s_2 = 8.7$

(a)

$(s_p)^2 \quad = (m-1)s_1^2 + (n-1)\, s_2^2/(m+n-2)$

$= (9(10.1)^2+9(8.7)^2\,)/(10 + 10 - 2) = 88.85$

Since $t_{\alpha/2,\, m+n-2} = t_{0.025,18} = 2.101$,

$\bar{x} - \bar{y} \pm t_{\alpha/2,+n-2}\, s_p\, \text{SQRT}(1/m+1/n) = 9.9 \pm 8.86$.

Thus, a 95% confidence interval for $\mu_1 - \mu_2$ is (1.04, 18.76).

(b)

H_0: $\mu 1 = \mu 2$, H_1: $\mu\, 1 > \mu 2$

$\alpha = 0.05$

The test statistic is $T = (\bar{X} - \bar{Y})\,/(Sp\text{SQRT}(1/m+1/n))$

The rejection region is $t \geq t_{\alpha,+n-2} = t_{0.05,18} = 1.734$.

t = (81−71.1)/(SQRT(88.85)SQRT(1/10+1/10)) = 2.349 > 1.734

Reject H_0.

p-value = (T > 2.349) = 0.015

(c) Do not reject at $\alpha = 0.01$ because the p-value of 0.015 is larger than 0.01.

12. A survey on computers requiring repairs within two years was conducted. 21 out of 200 computers from company A, and 37 out of 200 computers from company B required repairs. Do these data show that computers from company A are more reliable than computers from company B? Test using $\alpha = 0.01$.

Sample Proportions $\widehat{p_1} = 0.105$ $\widehat{p_2} = 0.185$ Pooled proportion: 0.145

1. Hypotheses: H_0: $p_1 = p_2$ H_1: $p_1 < p_2$ (claim)

2. $\alpha = 0.01$

3. Test Statistic: $Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{m}+\frac{1}{n}\right)}}$ Rejection Region: -2.33

4. Value of Test Statistic: z = -2.27 Fail to reject H_0

5. The data does not support the claim that computers from A are more reliable at $\alpha = 0.01$.

P-value: 0.0116

13. In animal cancer research the potential of human drugs and other substances are studied. Four hundred ppm of benzidine dihydrochloride is given to each of the male and female mice from a certain strain. In one of these experiments, tumors were found in 54 out of 484 male mice, and 127 out of 429 female mice.

a) Find a 99% confidence interval for the difference between the tumor rate of male mice and female mice.

b) Test if the tumor rate of female mice is higher than the tumor rate of male mice. Use α = 0.01 to test the claim.

(a) $m = 484, x = 54, n = 429, y = 127.$

Thus, $\hat{p}_1 = \frac{x}{m} = \frac{54}{484} = 0.112$ and $\hat{p}_2 = \frac{y}{n} = \frac{127}{429} = 0.296.$

$$\hat{p}_1 - \hat{p}_2 \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{m} + \frac{\hat{p}_2(1-\hat{p}_2)}{n}}$$

$$= 0.112 - 0.296 \pm z_{0.005}\sqrt{\frac{(0.112)(0.888)}{484} + \frac{(0.296)(0.704)}{429}}$$

$$= -0.184 \pm 2.58 \cdot 0.026 = -0.184 \pm 0.068$$

Therefore, a 99% confidence interval for $p_1 - p_2$ is $(-0.252, -0.116)$.

(b) 1. $H_0: p_1 = p_2, \quad H_1: p_1 < p_2$

2. $\alpha = 0.01$

3. The test statistic is

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{m} + \frac{1}{n}\right)}}.$$

$$\hat{p} = \frac{x+y}{m+n} = \frac{54+127}{484+429} = \frac{181}{913} = 0.198$$

The rejection region is $Z \le -z_\alpha = -z_{0.01} = -2.33.$

4.

$$z = \frac{0.112 - 0.296}{\sqrt{(0.198)(0.802)\left(\frac{1}{484} + \frac{1}{429}\right)}} = -6.96 < -2.33$$

Reject H_0.

5. There is enough evidence that the tumor rate of female mice is higher than that of male mice. p-value$= P(|Z| > 4.56) = 2\Phi(-4.56) \approx 0$