CSE 150 – Foundations of Computer Science – Honors
Sample Exam Problems
Solutions
December 12, 2017

1. Consider the following binary relations on the integers: R_1 is the empty relation, R_2 is $\mathbf{Z} \times \mathbf{Z}$, and R_3 is the set of all pairs (i, j) such that $i * j \geq 1$. Indicate which of the following properties each relation satisfies.

	R_1	R_2	R_3
reflexive		x	
symmetric	x	x	x
antisymmetric	x		
transitive	x	x	x

   Note that R_3 does not contain the pair $(0, 0)$.

2. Let A be the set $\{1, 2, 3\}$.

   (a) Give a minimal binary relation R_1 on A that is transitive but not symmetric.

   Answer. The relation $R_1 = \{(1, 2)\}$ is a minimal binary relation on A that is transitive but not symmetric.

   (b) Give a minimal binary relation R_2 on A that is reflexive but neither symmetric nor transitive.

   Answer. The relation $R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$ is a minimal binary relation on A that is reflexive, but neither symmetric nor transitive.

3. Suppose R and S are antisymmetric binary relations an a set A. Does it follow that the union $R \cup S$ is also antisymmetric? Explain.

   Answer. Let R be the binary relation $\{(1, 2)\}$ on the set $A = \{1, 2\}$, and S be the binary relation $\{2, 1)\}$. Both R and S are anti-symmetric, but the union $R \cup S$ is not.

4. Define a binary relation R on the real numbers by : xRy iff $x^2 \leq y^2$. Is R a partial order?

   Answer. The relation R is transitive and reflexive; hence it is not a strict partial order. Since R is not anti-symmetric, it is not a weak partial order either. (Note that $-1\,R\,1$ and $1\,R\,-1$).

5. Let A be the set of all *infinite* binary sequences that contain only a *finite* number of 1's. Is the set A countable or uncountable? Explain.

   Answer. The (infinite) set A is countable, as $\{0,1\}^* surj A$ and the set $\{0,1\}^*$ of all finite-length binary strings is countable. A surjection $f : \{0,1\}^* \to A$ can be defined as follows: (i) $f(\lambda)$ is the infinite sequence of 0's only and (ii) for any binary string w that ends with a 1, $f(w)$ is the infinite binary sequence that begins with w and continues with an infinite sequence of 0's.

6. Give a recursive definition of a function that maps a non-empty list of integers $[a_1, a_2, \ldots, a_n]$ to the value $1 \cdot a_1 + 2 \cdot a_2 + \cdots + n \cdot a_n$; and maps the empty list to 0.

   You may use the standard arithmetic functions, the basic list functions, *cons*, *hd*, and *tl*, and the additional list functions, *length*, *concat*, and *reverse*, that were discussed in class. You may want to express the desired function in terms of a suitable auxiliary recursive function.

   Answer. We define the desired function G in terms of an auxiliary function H:

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   H(n,L) = if L=nil then 0
   else n*head(L) + H(n+1,tail(L))
   G(L) = H(1,L)
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