Real Analysis

Midterm-ustc-2016-version2

October 13, 2025

Problem 1.

Let $\{f_n\}$ be a sequence of measurable functions on $\mathbb{R}$. For any $x \in \mathbb{R}$, if the sequence $\{f_n(x)\}$ diverges, then x is called a **divergence point** of f_n. Prove that the set of divergence points of f_n is measurable.

Problem 2.

Let $E_1 \subseteq E_2 \subseteq \cdots \subseteq \mathbb{R}$, and $E := \bigcup_{k=1}^{\infty} E_k$ be a **bounded set**. Prove that:

$$m^*(E) = \lim_{k\to\infty} m^*(E_k).$$

Problem 3.

Let $\{f_n\}$ converge to f a.e. on $[0,1]$, and for some $r > 0$ and any n, we have $\int_0^1 |f_n|^r dx \le M < \infty$. Prove that for any $p \in (0,r)$, $||f_n - f||_p \to 0$.

Problem 4.

Let $\{f_n\}$ and f be measurable functions, and assume that:

$$\forall \epsilon > 0, \lim_{n \to \infty} m\{x : |f_n(x) - f(x)| \geq \epsilon\} = 0.$$

Prove that $\{f_n\}$ has a subsequence that converges to f a.e.

Problem 5.

State **Egorov's Theorem**, and explain why it is not true when $m(E) = \infty$.