Power Series

1. Radius of Convergence and Domain of Convergence

1. $\sum_{n=1}^{\infty}(1+\frac{1}{n})^{n^2}x^n$
2. $\sum_{n=0}^{\infty}\frac{1^n+2^n+\cdots+k^n}{n^2}x^n$, where k >1, $k\in N$
3. $\sum_{n=0}^{\infty}\frac{3^{-\sqrt{n}}}{\sqrt{1+n^2}}x^n$
4. $\sum_{n=1}^{\infty}sin\frac{1}{3n}(x^2+x+1)^n$
5. $\sum_{n=1}^{\infty}(1+\frac{1}{n})^{-n^2}e^{-nx}$
6. $\sum_{n=1}^{\infty}\sin\frac{\pi}{2^n}(\frac{1}{x})^n$
7. $\sum_{n=1}^{\infty}n^{n^2}x^{n^2}$

2. Power Series $\iff$ Elementary Form

Important Facts for Power Series:

1. The Convergence Domain of Power Series is (x_0-R,x_0+R) , where R is determined by Cauchy Hadmard Formula. Moreover, it is *absolutely and uniformly* convergent.
2. If Power Series converges at the end point of the interval ($\text{WLOG,converges at } x_0+R$), we conclude that the function

$$S(x)=\sum_{n=0}^{\infty}a_n(x-x_0)^n,\quad x\in(x_0-R,x_0+R]$$

   is continuous at $x_0+R,\quad ,i.e,\quad \lim_{x\to x_0+R}S(x)=S(x_0+R)$. This fact is useful when we want to calculate the values of a number series. (Remark: The "exact value" of a number series is actually evaluated by a series that converges to the "exact value")

Express the following power series as Elementary Function on their domain

1. $\sum_{n=1}^{\infty}nx^n$
2. $\sum_{n=1}^{\infty}\frac{2n+1}{2^{n+1}}x^{2n}$
3. $\sum_{n=1}^{\infty}(-1)^n n^2x^n$
4. $\sum_{n=0}^{\infty}\frac{1}{2n+1}x^{2n+1}$
5. $\sum_{n=2}^{\infty}(-1)^n\frac{x^n}{n(n-1)}$

6. $\sum_{n=2}^{\infty}(-1)^n \dfrac{x^{n+1}}{n^2-1}$

Find the values of the following series:

1. $1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots$
2. $1 - \dfrac{1}{3} + \dfrac{1}{5} - \dfrac{1}{7} + \cdots$
3. $\dfrac{1}{2} - \dfrac{1}{5} + \dfrac{1}{8} - \dfrac{1}{11} + \cdots$