Colin Casazza

Kendall Jordan

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SDSU CS556

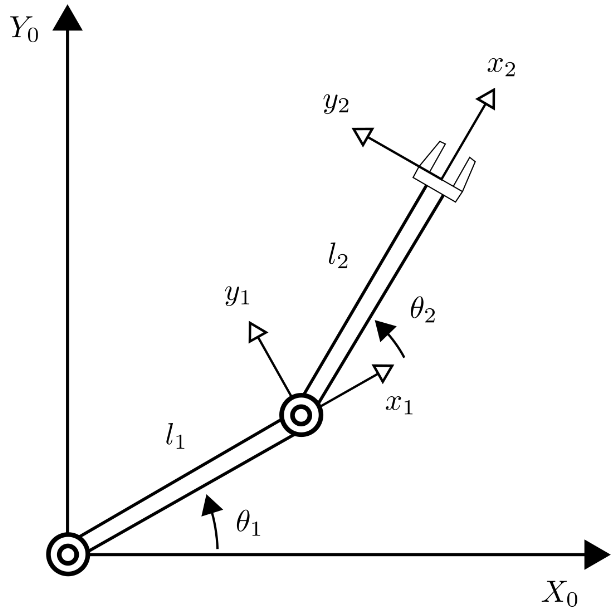
Assignment 3, Problem 2

The purpose of this problem is to determine the inverse kinematics of a manipulator with the method of Decomposition and Approximation

In [1]:
```
# packages
import math
import random
import matplotlib
import tinyik
import numpy as np
import sympy as sp
import pandas as pd
from sympy.solvers import solve
from sympy.physics.mechanics import dynamicsymbols
from sympy.physics.vector import init_vprinting
import plotly.express as px
import plotly.graph_objects as go
from sklearn.cluster import KMeans
from sklearn.linear_model import LinearRegression
from itertools import combinations

# configure pretty latex like printing
init_vprinting(use_latex="mathjax", pretty_print=False)

# number of random input angles
# to fuzz into the forward kinematics
# equations for approximation
NUMBER_TRIALS = 50000

# link lengths (in centimeters)
L1_LENGTH = 100
L2_LENGTH = 100

# the resolution for our
# our grid space approximation
NUMBER_ROWS = 50
NUMBER_COLS = 50
```

In [2]:
```python
# here's an image of the robot arm in question
from IPython.display import Image

Image("fig/img1.png", width=300)
```

Out[2]:



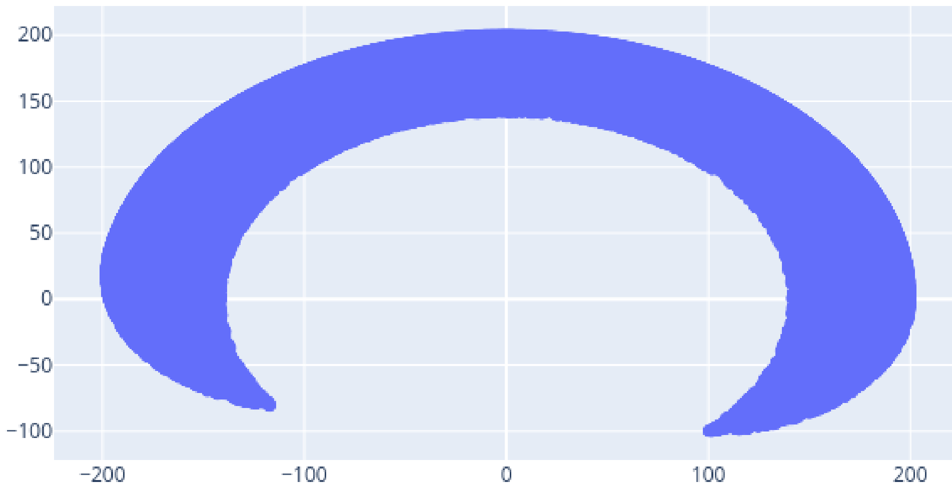
In [3]:
```
"""
Derive the forward kinematics equations to find the
corresponding x,y coordnates based on given (θ1, θ2)
pair for the 2PR manipulator
"""

# declare the symbolic variables
# we'll need to do linear algebra
theta1, theta2, l1, l2, theta, alpha, a, d = dynamicsymbols(
    "theta1 theta2 l1 l2 theta alpha a d"
)

# rotation matrix based off
# of our dh table
rotation_matrix = sp.Matrix(
    [
        [sp.cos(theta), -sp.sin(theta) * sp.cos(alpha), sp.sin(theta)
* sp.sin(alpha)],
        [sp.sin(theta), sp.cos(theta) * sp.cos(alpha), -sp.cos(theta)
* sp.sin(alpha)],
        [0, sp.sin(alpha), sp.cos(alpha)],
    ]
)

# x,y,z transformation matrix
transformation_matrix = sp.Matrix([a * sp.cos(theta), a * sp.sin(thet
a), d])

# the definition for the last row
# in a homogenous transformation matrix
last_row = sp.Matrix([[0, 0, 0, 1]])

# combine the rotation, transformation and last row
# to get homogenous transformation matrix
t = sp.Matrix.vstack(sp.Matrix.hstack(rotation_matrix, transformation
_matrix), last_row)

# given the homogenous transformation matrix
# we need to find the transformation from
# frame 0 to frame 2
t_02 = t.subs({alpha: 0, a: l1, theta: theta1, d: 0}) * t.subs(
    {alpha: 0, a: l2, theta: theta2, d: 0}
)

t_02.simplify()
display(t_02)
```

$$\begin{bmatrix} \cos(\theta_1 + \theta_2) & -\sin(\theta_1 + \theta_2) & 0 & l_1 \cos(\theta_1) + l_2 \cos(\theta_1 + \theta_2) \\ \sin(\theta_1 + \theta_2) & \cos(\theta_1 + \theta_2) & 0 & l_1 \sin(\theta_1) + l_2 \sin(\theta_1 + \theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In [4]:
```
# give our transformation matrix
# we can derive forward kinematics
# for example
x_position = t_02[0, 3]
y_position = t_02[1, 3]
fk_x = sp.lambdify((l1, l2, theta1, theta2), x_position, "numpy")
fk_y = sp.lambdify((l1, l2, theta1, theta2), y_position, "numpy")

def RR_forward_kinematics(theta1, theta2):
    """
        inverse kinematics for our 2PR robot
        given joint angles theta1 and theta2
        returns the x and y positions of the
        end effector
    """
    return (
        fk_x(L1_LENGTH, L2_LENGTH, theta1, theta2),
        fk_y(L1_LENGTH, L2_LENGTH, theta1, theta2),
    )


def RR_inverse_kinematics(x, y):
    """
        forward kinematics for our 2PR robot
        given a desired x any y coordinate
        returns both joint angles for the arm

        https://pythonrobotics.readthedocs.io/en/latest/modules/arm_n
avigation.html
    """
    try:
        # calculate both solutions for
        # theta 2
        a = x ** 2 + y ** 2 - L1_LENGTH ** 2 - L2_LENGTH ** 2
        b = 2 * L1_LENGTH * L2_LENGTH
        theta2_positive = math.acos((a / b))
        theta2_negative = -1 * theta2_positive

        # calculate both solutions for theta2
        theta1_positive = math.atan2(y, x) - math.atan2(
            L1_LENGTH * math.sin(theta2_positive),
            L1_LENGTH + L2_LENGTH * math.cos(theta2_positive),
        )
        theta1_negative = math.atan2(y, x) - math.atan2(
            L1_LENGTH * math.sin(theta2_negative),
            L1_LENGTH + L2_LENGTH * math.cos(theta2_negative),
        )
        return [(theta1_positive, theta2_positive), (theta1_negative,
theta2_negative)]

    # an error is thrown when this function trys to
    # calculate the inverse kinematics for a position
    # outside the workspace of the 2PR arm
    except ValueError as e:
        return None
```

In [5]:

```
"""
Generate NUMBER_TRIALS random (θ1, θ2) pairs,
and  use the forward kinematics equations to find the
corresponding  points in workspace. Keeps a record of
points in each cell.
"""

# random angle inputs
# for theta1 and theta2

inputs = [
    (np.deg2rad(random.uniform(0, 170)), np.deg2rad(random.uniform(-9
0, 90)))
    for x in range(NUMBER_TRIALS)
]

# map the forward kinematics
# equations onto the input set
outputs = [
    {
        "angles": (i[0], i[1]),
        "fk": RR_forward_kinematics(i[0], i[1])
    } for i in inputs
]

# plot the fuzzed forward
# kinematics to show our workspace
x_cord = list(map(lambda e: e["fk"][0], outputs))
y_cord = list(map(lambda e: e["fk"][1], outputs))

workspace_plot = go.Figure(
    data=go.Scattergl(
        x = x_cord,
        y = y_cord,
        mode='markers'
    )
)

workspace_plot.show(renderer="png")
```



In [6]:

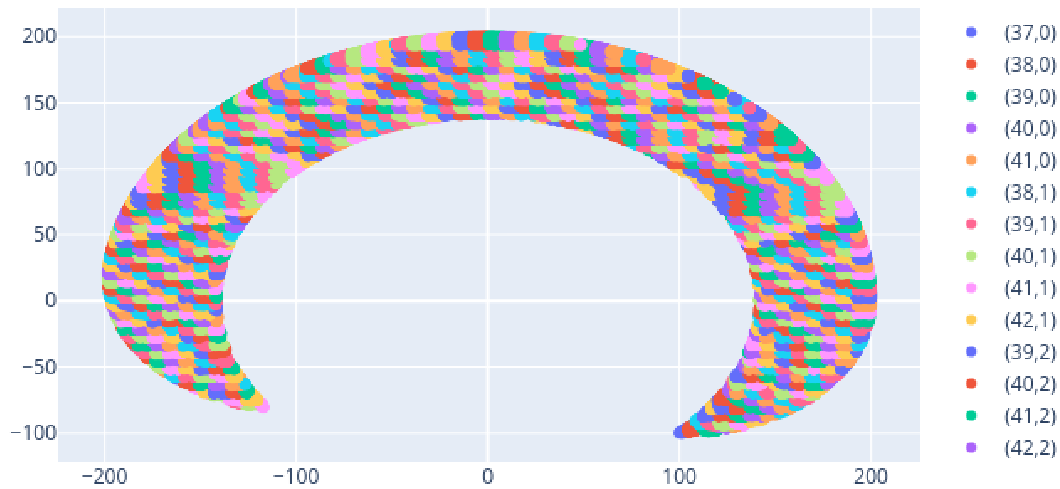
```
# helper variables for
# our workspace frame
x_min = math.floor(min(x_cord))
y_min = math.floor(min(y_cord))
x_max = math.ceil(max(x_cord))
y_max = math.ceil(max(y_cord))
workspace_width = x_max - x_min
workspace_height = y_max - y_min

# helper variables for our
# grid representation
cell_width = workspace_width / NUMBER_COLS
grid_width = math.ceil(workspace_width / cell_width)
cell_height = workspace_height / NUMBER_ROWS
grid_height = math.ceil(workspace_height / cell_height)

# helper function to map x,y position
# vector to matching grid coordinates
def get_grid_coordinates(x, y):
    _x = math.floor(((x - x_min) / workspace_width) * grid_width)
    _y = math.floor(((y - y_min) / workspace_height) * grid_height)
    return _x, _y
```

In [7]:

```
"""
Divide the joint and workspace in into square cells.

You will need to decide on the size of the cells in the
joint space and work space, for example x=y=10 cm.
"""
# first set up a grid of points
# so we can determine how to group
# results from forward kinematics
grid_points = [[[] for x in range(grid_width)] for y in range(grid_he
ight)]

# for each generated forward kinematics
# output coordinate, we need to find it's
# position in the grid and put in the grid
# array we made previously
for output in outputs:
    x_cord, y_cord = get_grid_coordinates(output["fk"][0], output["f
k"][1])
    grid_points[y_cord][x_cord].append(
        {
            "angles": (output["angles"][0], output["angles"][1]),
            "fk": (output["fk"][0], output["fk"][1]),
        }
    )
```

In [8]:

```
# some helper objects for the chart
# we're about to make, dictionary of
# colors we can randomly select from
hex_colors_dic = {}
rgb_colors_dic = {}
hex_colors_only = []

for name, hex in matplotlib.colors.cnames.items():
    hex_colors_only.append(hex)
    hex_colors_dic[name] = hex
    rgb_colors_dic[name] = matplotlib.colors.to_rgb(hex)

"""
show how we've split the workspace into grid
sections. each colored, square cluster
of points share the same grid coordinates
"""
grid_plot = go.Figure()
for y_index, row in enumerate(grid_points):
    for x_index, col in enumerate(row):
        if len(col) == 0:
            continue
        x = list(map(lambda x: x["fk"][0], col))
        y = list(map(lambda y: y["fk"][1], col))
        color = random.choice(hex_colors_only)
        grid_plot.add_trace(
            go.Scattergl(
                x=x,
                y=y,
                mode="markers",
                fillcolor=color,
                name="(" + str(x_index) + "," + str(y_index) + ")",
            )
        )

# show the plot
grid_plot.show(renderer="png")
```


200
150
100
50
0
−50
−100
−200
−100
0
100
200
(37,0)
(38,0)
(39,0)
(40,0)
(41,0)
(38,1)
(39,1)
(40,1)
(41,1)
(42,1)
(39,2)
(40,2)
(41,2)
(42,2)

In [9]:
```python
"""
Using k-mean clustering, divide the points
in each cell into two clusters, representing
the two solutions.
"""

# now lets turn the output grid
# into a data frame for easier
# analysis of our approximation
grid_df = pd.DataFrame(e for e in grid_points)

# helper function to find
# k_means clusters for each
# cell of the dataframe
# and split coordinates in cell
# into groups based on k_means cluster
def k_means_split(cell):

    # if the cell doesn't have at
    # least 2 points, we can't
    # do a kmeans cluster
    if len(cell) < 2:
        return None

    # make x and y lists and
    # pair them for kmeans inputs
    x = [ e['angles'][0] for e in cell ]
    y = [ e['angles'][1] for e in cell ]
    x_y_pairs = list(zip(x, y))

    # create the kmeans model
    kmeans = KMeans(n_clusters=2)
    cluster_indexes = kmeans.fit_predict(x_y_pairs)

    # group the points by cluster
    return [
        [e for i, e in enumerate(cell) if cluster_indexes[i] == 0],
        [e for i, e in enumerate(cell) if cluster_indexes[i] == 1],
    ]

# apply the function to split
# each cell into two clusters
cluster_grid_df = grid_df.applymap(k_means_split)
```

In [10]:
```
"""
In each cell and each cluster in the cell,
approximate the inverse kinematics by
linear relationship of the form

x=ai1+bi2+ci;    x=di1+ei2+fi;

where i=1,2, ..., n are the cell numbers,
and the coefficients ai to fi are found
using regression (curve fitting).

Store the coefficients. This completes
the decomposition and approximation.
"""

# now that we have our grid of
# clustered cells, we can do a
# linear regression on each
# cluster in all the cells
def lin_reg(cell):
    # if there aren't any points
    # in the cell, do nothing
    if cell == None:
        return None

    # for each cluster in the cell
    # do a linear regression for the
    # cluster's set of points
    def _lin_reg_helper(cluster):
        x_list = list(map(lambda e: e["fk"][0], cluster))
        y_list = list(map(lambda e: e["fk"][1], cluster))
        x_y_pairs = list(zip(x_list, y_list))
        t1_list = list(map(lambda e: e["angles"][0], cluster))
        t2_list = list(map(lambda e: e["angles"][1], cluster))
        t1_t2_pairs = list(zip(t1_list, t2_list))
        return LinearRegression().fit(x_y_pairs, t1_t2_pairs)

    return [_lin_reg_helper(cluster) for cluster in cell]


lin_reg_cluster_grid = cluster_grid_df.applymap(lin_reg)
```

In [11]:
```
"""
Given a set of 100 position (x,y) points
find the corresponding two solution (θ1, θ2) pairs
for each position.
"""

trial_errors = []
predicted_solutions = []
inputs = []

while len(trial_errors) < 1000:
    # generate some random input positions
    # NOTE: these aren't guaranteed to be within
    # the workspace of the robot.
    px, py = random.uniform(x_min, x_max), random.uniform(y_min, y_ma
x)

    # get the grid coordinates
    # for the regression lookup
    x, y = get_grid_coordinates(px, py)
    ik = RR_inverse_kinematics(px, py)

    # don't do anything if there's nothing in the cell
    # or if the inverse kinematics returns None
    if lin_reg_cluster_grid[x][y] is None or ik is None:
        continue

    # save the inputs and actual
    # inverse kinematics
    inputs.append((px,py))

    # get the inverse kinematic solutions
    # based of the cluster linear regressions
    c1, c2 = lin_reg_cluster_grid[x][y]

    # using the two linear regression objects
    # for each cell, pridict the two inverse
    # kinematic solutions for the given coordinates
    solution_1 = c1.predict([[px, py]])[-1]
    solution_2 = c2.predict([[px, py]])[-1]

    # given the predictetd solutions, see where
    # the arm would go if we used these input angles
    solution1_fk = RR_forward_kinematics(solution_1[0], solution_1[1
])
    solution2_fk = RR_forward_kinematics(solution_2[0], solution_2[1
])

    # helper function for finding the
    # distance between two numbers when
    # we don't know which is larger
    def distance(a, b):
        if (a < 0) and (b < 0) or (a > 0) and (b > 0):
            return abs(abs(a) - abs(b))
        if (a < 0) and (b > 0) or (a > 0) and (b < 0):
            return abs(abs(a) + abs(b))
```

```
    # determine the total error in our approximation
    # by finding the distance between the actual point and
    # the two points predicted by linear approximation
    solution_1_error = abs(math.hypot(px - solution1_fk[0], py - solu
tion1_fk[1]))
    solution_2_error = abs(math.hypot(px - solution2_fk[0], py - solu
tion2_fk[1]))

    if(solution_1_error <= solution_2_error):
        predicted_solutions.append(solution1_fk)
        trial_errors.append(solution_1_error)
    else:
        predicted_solutions.append(solution2_fk)
        trial_errors.append(solution_2_error)

# take the average of all of the
# total errors for each trial
average_error = sum(trial_errors) / len(trial_errors)
print("average error: ", average_error, " cm")
```

```
average error:  0.11455529528597302  cm
```

In [12]:
```
"""
here's a chart showing the inputs to
out test above on the x y plane

each datapoint's size and color
coresponds to the amount of error encountered
during our approximation step, where
smaller, darker datapoints are more accurate
and larger lighter datapoints have more error

Note: sizes of data points in scatter plot
are not to scale in terms of error
"""

x_inputs = list(map(lambda x: x[0], inputs))
y_inputs = list(map(lambda x: x[1], inputs))
normalized_error = [ (x+5) for x in trial_errors]

error_chart = go.Figure(
    data=go.Scattergl(
        x = x_inputs,
        y = y_inputs,
        mode='markers',
        marker=dict(
            size=normalized_error,
            color=trial_errors,
            colorscale='Viridis'
        )
    )
)
error_chart.show(renderer="png")
```