Cahier de calcul

— réponses —

Margarita philosophica (La perle philosophique), Gregor REISCH (1508)

Cette gravure, extraite d'un manuel d'université de l'époque, représente Arithmetica, allégorie des mathématiques, arbitrant une compétition entre Boèce, qui utilise les chiffres indo-arabes, et Pythagore, qui utilise un boulier.

Page web du *Cahier de calcul*,
dernières versions

Ce cahier de calcul a été écrit collectivement.

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Le pictogramme 🕐 de l'horloge a été créé par Ralf Schmitzer (The Noun Project).

L'illustration de la couverture vient de Wikimedia.

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Sommaire

Fiche nº 1. Fractions

Réponses

1.1 a) . . . . $\dfrac{4}{5}$

1.1 b) . . . . 2^5

1.1 c) 3

1.1 d) . . . . $-2 \times 3^{3k-2}$

1.2 a) . . . . $\dfrac{1}{6}$

1.2 b) . . . . $\dfrac{7}{15}$

1.2 c) 9

1.2 d) . . . . $\dfrac{1}{9}$

1.3 a) 247

1.3 b) . . . . $\dfrac{203}{24}$

1.3 c) . . . . $\dfrac{-10}{3}$

1.3 d) . . . . $1\,000$

1.4 . . . . $\dfrac{16}{35}$

1.5 a) . . . . $4\,022$

1.5 b) . . . . $\dfrac{1}{2}$

1.5 c) 1

1.5 d) 2

1.6 a) . . . . $\dfrac{-1}{n(n+1)^2}$

1.6 b) . . . . $-\dfrac{ab}{a-b}$

1.6 c) . . . . $\dfrac{3}{2}n$

1.7 . . . . $\dfrac{n^3+n}{n+1}$

1.8 a) . . . . $4+\dfrac{5}{6}$

1.8 b) . . . . $1+\dfrac{1}{k-1}$

1.8 c) . . . . $3+\dfrac{5}{x-2}$

1.9 . . . . $2t$

1.10 a) . . . . $\dfrac{3}{5} > \dfrac{5}{9}$

1.10 b) . . . . $\dfrac{12}{11} > \dfrac{10}{12}$

1.10 c) . . . . $\dfrac{125}{25} = \dfrac{105}{21}$

1.11 . . . . Non

1.12 . . . . $A > B$

Fiche nº 2. Puissances

Réponses

2.1 a) .... 10^8

2.1 b) .... 10^{15}

2.1 c) .... 10^2

2.1 d) .... 10^{-2}

2.1 e) .... 10^4

2.1 f) .... 10^{-8}

2.2 a) .... 15^4

2.2 b) .... 5^{-6}

2.2 c) .... 2^7

2.2 d) .... 7^{-2}

2.2 e) .... 3^5

2.2 f) .... 3^{28}

2.3 a) .... $2^{-4} \times 3^{-1}$

2.3 b) .... $2^{21} \times 3$

2.3 c) 2

2.3 d) .... $2^{38} \times 3^{26}$

2.4 a) 8

2.4 b) 11

2.4 c) .... 3^{10}

2.4 d) .... $2^6 \times 5$

2.5 a) .... $\dfrac{x}{x+1}$

2.5 b) .... $\dfrac{1}{x-2}$

2.5 c) .... $\dfrac{2x}{x+1}$

2.5 d) .... $\dfrac{2}{x-2}$

Fiche nº 3. Calcul littéral

3.1 a) .... $8x^3 - 6x^2 + \dfrac{3}{2}x - \dfrac{1}{8}$

3.1 b) .... $x^5 - 2x^4 + x^3 - x^2 + 2x - 1$

3.1 c) .... $x^5 - x^3 + x^2 - 1$

3.1 d) .... $x^5 + 2x^4 + x^3 - x^2 - 2x - 1$

3.1 e) .... $x^5 - x^3 - x^2 + 1$

3.1 f) .... $x^4 + x^2 + 1$

3.2 a) .... $-2 + 12x - 17x^2 + 8x^3 - 3x^4$

3.2 b) .... $-28 + 21x$

3.2 c) .... $2 + x^3 - x^4 - x^5$

3.2 d) .... $-1 - 3x - 3x^2 + x^3$

3.2 e) .... $1 + x^4$

3.2 f) .... $1 + 2x + 3x^2 + 2x^3 + x^4$

3.3 a) .... $-6(6x + 7)$

3.3 b) .... $4(5x + 4)(-5x + 1)$

3.3 c) .... $2(3x - 4)(10x + 3)$

3.3 d) .... $-8(x + 1)(x + 16)$

3.4 a) .... $(x - 1)^2$

3.4 b) .... $(x + 2)^2$

3.4 c) .... $(x + 1)(x + 2)$

3.4 d) .... $3\left(x + \dfrac{7 - \sqrt{37}}{6}\right)\left(x + \dfrac{7 + \sqrt{37}}{6}\right)$

3.4 e) .... $2\left(x + \dfrac{3 - \sqrt{233}}{4}\right)\left(x + \dfrac{3 + \sqrt{233}}{4}\right)$

3.4 f) .... $-5(x - 1)\left(x - \dfrac{1}{5}\right)$

3.5 a) .... $(x + y - z)(x + y + z)$

3.5 b) .... $3(14x + 3y)(-4x + y)$

3.5 c) .... $(x + 1)(y + 1)$

3.5 d) .... $(x - 1)(y - 1)$

3.5 e) .... $(x + y)(x + 1)^2$

3.5 f) .... $\left(a^2 + b^2\right)\left(y - 4x^2\right)\left(y + 4x^2\right)$

3.6 a) .... $(x - 1)(x + 1)\left(x^2 + 1\right)$

3.6 b) .... $-8\left(x^2 + 1\right)(x - 4)(x + 4)$

3.6 c) .... $\left(x^2 + x + 1\right)\left(x^2 - x + 1\right)$

3.6 d) .... $\left(a^2 + b^2\right)\left(c^2 + d^2\right)$

3.6 e) ...... $\left(a^2 + b^2 + c^2 + d^2\right)\left(p^2 + q^2 + r^2 + s^2\right)$

Fiche nº 4. Racines carrées

Réponses

4.1 a) . . . . $\boxed{5}$

4.1 b) . . . . $\boxed{\sqrt{3}-1}$

4.1 c) . . . . $\boxed{-\sqrt{3}+2}$

4.1 d) . . . . $\boxed{\sqrt{7}-2}$

4.1 e) . . . . $\boxed{\pi-3}$

4.1 f) . . . . $\boxed{|3-a|}$

4.2 a) . . . . $\boxed{20}$

4.2 b) . . . . $\boxed{9+4\sqrt{5}}$

4.2 c) . . . . $\boxed{1+\sqrt{3}}$

4.2 d) . . . . $\boxed{3+\sqrt{2}}$

4.2 e) . . . . $\boxed{12\sqrt{7}}$

4.2 f) . . . . $\boxed{12}$

4.2 g) . . . . $\boxed{9-\dfrac{10}{3}\sqrt{2}}$

4.2 h) . . . . $\boxed{10}$

4.3 a) . . . . $\boxed{2-\sqrt{2}-\sqrt{3}+\dfrac{1}{2}\sqrt{6}}$

4.3 b) . . . . $\boxed{3-2\sqrt{2}}$

4.3 c) . . . . $\boxed{1-\sqrt{10}+\sqrt{15}}$

4.3 d) . . . . $\boxed{\sqrt{15}+\sqrt{10}-\sqrt{6}-2}$

4.4 a) . . . . $\boxed{-(\sqrt{2}+\sqrt{3})}$

4.4 b) . . . . $\boxed{-\dfrac{3+\sqrt{2}+\sqrt{3}+\sqrt{6}}{2}}$

4.4 c) . . . . $\boxed{2\sqrt{2}}$

4.4 d) . . . . $\boxed{50-25\sqrt{3}}$

4.5 . . . . $\boxed{\dfrac{\sqrt{2}+2-\sqrt{6}}{4}}$

4.6 a) . . . . $\boxed{\dfrac{x}{\sqrt{x-1}}}$

4.6 b) . . . . $\boxed{x-\sqrt{x^2-1}}$

4.6 c) . . . . $\boxed{1+\sqrt{x-1}}$

4.6 d) . . . . $\boxed{\dfrac{1}{2}\dfrac{1}{x-1}}$

4.6 e) . . . . $\boxed{\dfrac{x(x-2)}{(x-1)\sqrt{x-1}}}$

4.6 f) . . . . $\boxed{-4(x-1)^2}$

4.7 a) . . . . $\boxed{\sqrt{2}}$

4.7 b) . . . . $\boxed{2\sqrt{2}}$

4.8 a) . . . . $\boxed{-11+5\sqrt{5}}$

4.8 b) . . . . $\boxed{1+\sqrt{2}}$

4.8 c) . . . . $\boxed{1+\sqrt{2}}$

4.8 d) . . . . $\boxed{\sqrt{3}}$

4.8 e) . . . . $\boxed{1+\sqrt{5}}$

4.8 f) . . . . $\boxed{\ln(1+\sqrt{2})}$

4.9 . . . . $\boxed{1}$

Fiche nº 5. Expressions algébriques

Réponses

5.1 a) . . . . $\boxed{7a^2 + 12a + 7}$

5.1 b) . . . . $\boxed{a^2 - 1}$

5.1 c) . . . . $\boxed{4a^2 - a - 3}$

5.1 d) . . . . $\boxed{-a^2 + 1}$

5.2 a) . . . . $\boxed{8 + 6i}$

5.2 b) . . . . $\boxed{8 - 6i}$

5.2 c) . . . . $\boxed{18 - 26i}$

5.2 d) . . . . $\boxed{-9 - 46i}$

5.3 a) . . . . $\boxed{39 - 18i}$

5.3 b) . . . . $\boxed{2197}$

5.3 c) . . . . $\boxed{-4 + 43i\sqrt{5}}$

5.3 d) . . . . $\boxed{1}$

5.4 a) . . . . $\boxed{3}$

5.4 b) . . . . $\boxed{1}$

5.4 c) . . . . $\boxed{1}$

5.4 d) . . . . $\boxed{0}$

5.4 e) . . . . $\boxed{-1}$

5.4 f) . . . . $\boxed{31}$

5.5 a) . . . . $\boxed{a^2 + 2}$

5.5 b) . . . . $\boxed{a^3 + 3a}$

5.5 c) . . . . $\boxed{a^4 + 4a^2 + 2}$

5.6 a) . . . . $\boxed{a^2 - 2b}$

5.6 b) . . . . $\boxed{ab - 3c}$

5.6 c) . . . . $\boxed{a^3 - 3ab + 3c}$

5.7 a) . . . . $\boxed{ab - c}$

5.7 b) . . . . $\boxed{ac}$

5.7 c) . . . . $\boxed{-2ac + b^2}$

5.8 a) . . . . $\boxed{a^2b - ac - 2b^2}$

5.8 b) . . . $\boxed{a^4 - 4a^2b + 4ac + 2b^2}$

5.8 c) . . . . $\boxed{0}$

5.8 d) . . . . $\boxed{1}$

5.8 e) . . . . $\boxed{a}$

Fiche nº 6. Équations du second degré

6.1 a) . . . . 3 et 3

6.1 b) . . . . $-1/3$ et $-1/3$

6.1 c) . . . . 2 et -6

6.1 d) . . . . 2 et 3

6.2 a) . . . . 0 donc 5

6.2 b) . . . . 0 donc $-3/2$

6.2 c) . . . . $\varnothing$

6.2 d) . . . . 1 donc -5

6.2 e) . . . . 1 donc $8/3$

6.2 f) . . . . -1 donc $-19/5$

6.3 a) . . . . 6 et 7

6.3 b) . . . . -3 et -5

6.3 c) . . . . -7 et -11

6.3 d) . . . . -3 et 11

6.3 e) . . . . a et b

6.3 f) . . . . $a-b$ et $a+b$

6.4 a) . . . . $2/3$

6.4 b) . . . . $-2/7$

6.4 c) . . . . $-1/m$

6.4 d) . . . . $2m/(m+3)$

6.5 a) . . . . 1 donc $(a-b)/(b-c)$

6.5 b) . . . . 1 donc $c(a-b)/(a(b-c))$

6.5 c) . . . . m donc $-(m+a+b)$

6.5 d) . . . . m donc $m(a-b)/(b-c)$

6.5 e) . . . . m donc ab/m

6.5 f) . . . . $a+b$ puis $2ab/(a+b)$.

6.6 a) . . . . $x^2-22x+117=0$

6.6 b) . . . . $x^2-6x-187=0$

6.6 c) . . . . $x^2-4x+1=0$

6.7 a) . . . . $x^2-2mx+3=0$

6.7 b) . . . . . $2x^2-(4m+1)x+(2m^2+m-15)=0$

6.7 c) . . . $m^2x^2+(m-2m^2)x+(m^2-m-2)=0$

6.8 a) . . . . $m=-3/4$ et $x=3/4$

6.8 b) . . . $m=-1$ et $x=-2$, ou $m=7$ et $x=2/3$

6.8 c) . . . . . . $m=1$ et $x=-1$, ou $m=-1$ et $x=1$

6.9 a) . . . . $a=2$ et $b=3$

6.9 b) . . . . $a=-2$ et $b=1$

6.9 c) . . . . $a=-3$ et $b=5$

6.9 d) . . . . $a=1/2$ et $b=8$

6.9 e) . . . . $a=1$ et $b=3\sqrt{7}$

6.10 a) . . . . $]-\infty,1]\cup[\sqrt{2},+\infty[$

6.10 b) . . . . $[-3,5]$

6.10 c) . . . . $]-\infty,-1]\cup[2/3,+\infty[$

6.10 d) . . . . $]-\infty,-1/2[\cup[4,+\infty[$

Fiche n⁰ 7. Exponentielle et logarithme

Réponses

7.1 a) .... $4\ln(2)$

7.1 b) .... $9\ln(2)$

7.1 c) .... $-3\ln(2)$

7.1 d) .... $\dfrac{1}{2}\ln(2)$

7.1 e) .... $3\ln(2)$

7.1 f) .... $2\ln(2) + 2\ln(3)$

7.2 a) .... $-\ln(3) - 2\ln(2)$

7.2 b) .... $2\ln(3) - 2\ln(2)$

7.2 c) .... $\ln(3) + 11\ln(2)$

7.2 d) .... $3\ln(5) + 2\ln(2)$

7.2 e) .... $-2\ln(5) + 4\ln 2$

7.2 f) .... $2\ln(5) - 2\ln 2$

7.3 .... $-2\ln(2) - 2\ln(5)$

7.4 a) .... $\dfrac{25}{8}\ln(\sqrt{2} - 1)$

7.4 b) .... $17 + 12\sqrt{2}$

7.4 c) 0

7.4 d) 0

7.5 a) 8

7.5 b) .... $\dfrac{1}{2}$

7.5 c) .... $\dfrac{1}{3}$

7.5 d) .... $\dfrac{1}{9}$

7.5 e) .... $-\dfrac{1}{2}$

7.5 f) .... $\dfrac{3}{2}$

7.6 a) .... -2

7.6 b) .... $\dfrac{1}{\ln 2}$

7.6 c) .... -17

7.6 d) 1

7.6 e) .... -1

7.6 f) .... e

7.7 a) .... impaire

7.7 b) .... impaire

7.7 c) .... impaire

7.7 d) .... impaire

7.8 a) .... $\mathbb{R}$

7.8 b) .... ok

7.8 c) 1

7.8 d) .... -1

7.9 a) .... $x + \ln(2)$

7.9 b) .... $\dfrac{e^x}{\sqrt{1+x}}$

7.9 c) .... $\ln|x-1|$

7.9 d) .... $-\dfrac{1}{1+x}$

7.9 e) .... $(1+x)^x$

7.10 a) .... $x \geqslant \dfrac{\ln 12 + 5}{3}$

7.10 b) .... $x \in [0,1]$

7.10 c) .... $x \geqslant \dfrac{2}{e}$

7.10 d) .... $x \geqslant -\dfrac{1}{12}$

7.11 a) .... $\varnothing$

7.11 b) .... $\dfrac{-13 - \sqrt{273}}{2}$

Fiche nº 8. Trigonométrie

Réponses

8.1 a)....................................... $\boxed{0}$

8.1 b)....................................... $\boxed{0}$

8.1 c)....................................... $\boxed{-\dfrac{1}{2}}$

8.1 d).................... $\boxed{-1-\sqrt{3}}$

8.2 a)....................................... $\boxed{0}$

8.2 b).................................... $\boxed{-\sin x}$

8.2 c).................................... $\boxed{2\cos x}$

8.2 d).................................... $\boxed{-2\cos x}$

8.3 a)................................. $\boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

8.3 b)................................. $\boxed{\dfrac{\sqrt{2}+\sqrt{6}}{4}}$

8.3 c)................................. $\boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

8.3 d)................................. $\boxed{2-\sqrt{3}}$

8.4 a).................................... $\boxed{-\sin x}$

8.4 b).................................... $\boxed{\dfrac{1}{\cos x}}$

8.5 a).................................... $\boxed{0}$

8.5 b)................... $\boxed{4\cos^3 x - 3\cos x}$

8.6 a)................................. $\boxed{\dfrac{\sqrt{2+\sqrt{2}}}{2}}$

8.6 b)................................. $\boxed{\dfrac{\sqrt{2-\sqrt{2}}}{2}}$

8.7 a).................................... $\boxed{\tan x}$

8.7 b).................................... $\boxed{2}$

8.7 c).................... $\boxed{8\cos^4 x - 8\cos^2 x + 1}$

8.8 a).................................... $\boxed{\{\pi/3, 5\pi/3\}}$

8.8 a).................................... $\boxed{\left\{-\dfrac{\pi}{3}, \dfrac{\pi}{3}\right\}}$

8.8 a)...... $\boxed{\left\{\dfrac{\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{-\dfrac{\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\}}$

8.8 b).................................... $\boxed{\{4\pi/3, 5\pi/3\}}$

8.8 b).................................... $\boxed{\left\{\dfrac{-2\pi}{3}, \dfrac{-\pi}{3}\right\}}$

8.8 b)..... $\boxed{\left\{\dfrac{4\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\dfrac{5\pi}{3} + 2k\pi, k \in \mathbb{Z}\right\}}$

8.8 c).................................... $\boxed{\{7\pi/6, 11\pi/6\}}$

8.8 c).................................... $\boxed{\left\{-\dfrac{5\pi}{6}, -\dfrac{\pi}{6}\right\}}$

8.8 c).... $\boxed{\left\{\dfrac{7\pi}{6} + 2k\pi, k \in \mathbb{Z}\right\} \cup \left\{\dfrac{11\pi}{6} + 2k\pi, k \in \mathbb{Z}\right\}}$

8.8 d).................................... $\boxed{\{3\pi/4, 7\pi/4\}}$

8.8 d).................................... $\boxed{\{-\pi/4, 3\pi/4\}}$

8.8 d).................................... $\boxed{\left\{\dfrac{3\pi}{4} + k\pi, k \in \mathbb{Z}\right\}}$

8.8 e).................................... $\boxed{\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\}}$

8.8 e).................................... $\boxed{\left\{-\dfrac{3\pi}{4}, -\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{3\pi}{4}\right\}}$

8.8 e).................................... $\boxed{\left\{\dfrac{\pi}{4} + k\dfrac{\pi}{2}, k \in \mathbb{Z}\right\}}$

8.8 f).................................... $\boxed{\left\{\dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}\right\}}$

8.8 f).................................... $\boxed{\left\{-\dfrac{5\pi}{6}, -\dfrac{\pi}{6}, \dfrac{\pi}{6}, \dfrac{5\pi}{6}\right\}}$

8.8 f)........ $\boxed{\left\{\dfrac{\pi}{6} + k\pi, k \in \mathbb{Z}\right\} \cup \left\{\dfrac{5\pi}{6} + k\pi, k \in \mathbb{Z}\right\}}$

8.8 g).................................... $\boxed{\left\{\dfrac{\pi}{12}, \dfrac{11\pi}{12}, \dfrac{13\pi}{12}, \dfrac{23\pi}{12}\right\}}$

8.8 g).................................... $\boxed{\left\{-\dfrac{11\pi}{12}, -\dfrac{\pi}{12}, \dfrac{\pi}{12}, \dfrac{11\pi}{12}\right\}}$

8.8 g) . . . . . . $\left\{\dfrac{\pi}{12}+k\pi, k\in\mathbb{Z}\right\}\cup\left\{\dfrac{11\pi}{12}+k\pi, k\in\mathbb{Z}\right\}$

8.8 h) . . . . $\{\pi/6, 5\pi/6, 3\pi/2\}$

8.8 h) . . . . $\{-\pi/2, \pi/6, 5\pi/6\}$

8.8 h) . . . . $\left\{\dfrac{\pi}{6}+k\dfrac{2\pi}{3}, k\in\mathbb{Z}\right\}$

8.8 i) . . . . $\{\pi/7, 13\pi/7\}$

8.8 i) . . . . $\left\{-\dfrac{\pi}{7}, \dfrac{\pi}{7}\right\}$

8.8 i) . . . . . . $\left\{\dfrac{\pi}{7}+2k\pi, k\in\mathbb{Z}\right\}\cup\left\{-\dfrac{\pi}{7}+2k\pi, k\in\mathbb{Z}\right\}$

8.8 j) . . . . $\{5\pi/14, 9\pi/14\}$

8.8 j) . . . . $\{5\pi/14, 9\pi/14\}$

8.8 j) . . . . . . $\left\{\dfrac{5\pi}{14}+2k\pi, k\in\mathbb{Z}\right\}\cup\left\{\dfrac{9\pi}{14}+2k\pi, k\in\mathbb{Z}\right\}$

8.9 a) . . . . $[0, 3\pi/4]\cup[5\pi/4, 2\pi]$

8.9 a) . . . . $[-3\pi/4, 3\pi/4]$

8.9 b) . . . . $[\pi/3, 5\pi/3]$

8.9 b) . . . . $\left[-\pi, -\dfrac{\pi}{3}\right]\cup\left[\dfrac{\pi}{3}, \pi\right]$

8.9 c) . . . . $\left[0, \dfrac{\pi}{6}\right]\cup\left[\dfrac{5\pi}{6}, 2\pi\right]$

8.9 c) . . . . $\left[-\pi, \dfrac{\pi}{6}\right]\cup\left[\dfrac{5\pi}{6}, \pi\right]$

8.9 d) . . . . $\left[0, \dfrac{\pi}{6}\right]\cup\left[\dfrac{5\pi}{6}, \dfrac{7\pi}{6}\right]\cup\left[\dfrac{11\pi}{6}, 2\pi\right]$

8.9 d) . . . . $\left[-\pi, -\dfrac{5\pi}{6}\right]\cup\left[-\dfrac{\pi}{6}, \dfrac{\pi}{6}\right]\cup\left[\dfrac{5\pi}{6}, \pi\right]$

8.9 e) . . . . $\left[\dfrac{\pi}{4}, \dfrac{\pi}{2}\right[\cup\left[\dfrac{5\pi}{4}, \dfrac{3\pi}{2}\right[$

8.9 e) . . . . $\left[-\dfrac{3\pi}{4}, -\dfrac{\pi}{2}\right[\cup\left[\dfrac{\pi}{4}, \dfrac{\pi}{2}\right[$

8.9 f) . . . . . $\left[\dfrac{\pi}{4}, \dfrac{\pi}{2}\right[\cup\left]\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right]\cup\left[\dfrac{5\pi}{4}, \dfrac{3\pi}{2}\right[\cup\left]\dfrac{3\pi}{2}, \dfrac{7\pi}{4}\right]$

8.9 f) . . . . $\left[-\dfrac{3\pi}{4}, -\dfrac{\pi}{2}\right[\cup\left]-\dfrac{\pi}{2}, -\dfrac{\pi}{4}\right]\cup\left[\dfrac{\pi}{4}, \dfrac{\pi}{2}\right[\cup\left]\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right]$

8.9 g) . . . . $\left[0, \dfrac{3\pi}{4}\right]\cup\left[\dfrac{7\pi}{4}, 2\pi\right]$

8.9 g) . . . . $[-\pi/4, 3\pi/4]$

8.9 h) . . . . $\left[0, \dfrac{3\pi}{8}\right]\cup\left[\dfrac{7\pi}{8}, \dfrac{11\pi}{8}\right]\cup\left[\dfrac{15\pi}{8}, 2\pi\right]$

8.9 h) . . . . $\left[-\pi, -\dfrac{5\pi}{8}\right]\cup\left[-\dfrac{\pi}{8}, \dfrac{3\pi}{8}\right]\cup\left[\dfrac{7\pi}{8}, \pi\right]$

Fiche nº 9. Dérivation

Réponses

9.1 a) .... $\boxed{6x^2 + 2x - 11}$

9.1 b) .... $\boxed{5x^4 - 6x^2 + 4x - 15}$

9.1 c) .... $\boxed{(2x^2 - 2x + 10)\exp(2x)}$

9.1 d) .... $\boxed{(6x-1)\ln(x-2) + \dfrac{3x^2 - x}{x-2}}$

9.2 a) .... $\boxed{5(x^2 - 5x)^4(2x-5)}$

9.2 b) .... $\boxed{4(2x^3 + 4x - 1)(3x^2 + 2)}$

9.2 c) .... $\boxed{8\cos^2(x) - 6\cos(x)\sin(x) - 4}$

9.2 d) ...... $\boxed{-3(3\cos(x) - \sin(x))^2(3\sin(x) + \cos(x))}$

9.3 a) .... $\boxed{\dfrac{2x}{x^2 + 1}}$

9.3 b) .... $\boxed{\dfrac{1}{x\ln(x)}}$

9.3 c) .... $\boxed{(-2x^2 + 3x + 1)\exp(x^2 + x)}$

9.3 d) .... $\boxed{6\cos(2x)\exp(3\sin(2x))}$

9.4 a) .... $\boxed{\dfrac{6x}{(x^2 + 1)^2}\cos\left(\dfrac{2x^2 - 1}{x^2 + 1}\right)}$

9.4 b) .... $\boxed{\dfrac{2x^2 + 2x - 8}{(x^2 + 4)^2}\sin\left(\dfrac{2x + 1}{x^2 + 4}\right)}$

9.4 c) .... $\boxed{\dfrac{\cos(x)}{2\sqrt{\sin(x)}}}$

9.4 d) .... $\boxed{\dfrac{\cos(\sqrt{x})}{2\sqrt{x}}}$

9.5 a) ...... $\boxed{\dfrac{(2x+3)(2\sin(x) + 3) - (x^2 + 3x) \times 2\cos(x)}{(2\sin(x) + 3)^2}}$

9.5 b) .... $\boxed{\dfrac{2 - 3x}{2\sqrt{x}(3x + 2)^2}}$

9.5 c) ...... $\boxed{-2\dfrac{(x^2 + 1)\sin(2x + 1) + x\cos(2x + 1)}{(x^2 + 1)^2}}$

9.5 d) .... $\boxed{\dfrac{(4x + 3)\ln(x) - 2x - 3}{(\ln(x))^2}}$

9.6 a) .... $\boxed{2x\sin\left(\dfrac{1}{x}\right) - \cos\left(\dfrac{1}{x}\right)}$

9.6 b) .... $\boxed{\dfrac{9}{(9 - x^2)\sqrt{9 - x^2}}}$

9.6 c) .... $\boxed{\dfrac{1}{1 - x^2}}$

9.6 d) .... $\boxed{\dfrac{x\cos(x) - \sin(x)}{x\sin(x)}}$

9.7 a) .... $\boxed{\dfrac{10x - 5}{(3 - x)^2(2 + x)^2}}$

9.7 b) .... $\boxed{\dfrac{2}{x + 1}\left(x + \dfrac{1 + \sqrt{3}}{2}\right)\left(x + \dfrac{1 - \sqrt{3}}{2}\right)}$

9.7 c) .... $\boxed{\dfrac{2x^2 + 2x + 5}{(x + 2)(x - 1)^2}}$

9.7 d) .... $\boxed{\dfrac{x^2}{(x + 1)^2}}$

9.7 e) .... $\boxed{\dfrac{2}{x(1 - \ln(x))^2}}$

Fiche nº 10. Primitives

10.1 a) .... $\boxed{\ln|t+1|}$

10.1 b) .... $\boxed{-\dfrac{3}{t+2}}$

10.1 c) .... $\boxed{-\dfrac{3}{2(t+2)^2}}$

10.1 d) .... $\boxed{-\dfrac{\cos(4t)}{4}}$

10.2 a) .... $\boxed{\dfrac{2}{3}(1+t)^{\frac{3}{2}} - \dfrac{3}{4}t^{\frac{4}{3}}}$

10.2 b) .... $\boxed{\dfrac{1}{2}e^{2t+1}}$

10.2 c) .... $\boxed{\dfrac{1}{2}\arcsin(2t)}$

10.2 d) .... $\boxed{\dfrac{1}{3}\arctan(3t)}$

10.3 a) .... $\boxed{\dfrac{2}{3}\ln|1+t^3|}$

10.3 b) .... $\boxed{\dfrac{1}{6}(1+2t^2)^{\frac{3}{2}}}$

10.3 c) .... $\boxed{-\sqrt{1-t^2}}$

10.3 d) .... $\boxed{\dfrac{3}{4}(1+7t^2)^{\frac{2}{3}}}$

10.3 e) .... $\boxed{\dfrac{1}{6}\ln(1+3t^2)}$

10.3 f) .... $\boxed{-\dfrac{1}{(1+3t^2)^2}}$

10.4 a) .... $\boxed{\dfrac{1}{4}\ln^4 t}$

10.4 b) .... $\boxed{2\sqrt{\ln t}}$

10.4 c) .... $\boxed{\dfrac{2}{(3-e^{2t})^2}}$

10.4 d) .... $\boxed{-\dfrac{2}{3t^{\frac{3}{2}}}}$

10.4 e) .... $\boxed{\ln|1-e^{-t}+e^t|}$

10.4 f) .... $\boxed{-e^{\frac{1}{t}}}$

10.5 a) .... $\boxed{-\dfrac{1}{3}\cos^3 t}$

10.5 b) .... $\boxed{e^{\sin t}}$

10.5 c) .... $\boxed{-\ln|\cos t|}$

10.5 d) .... $\boxed{-\ln|1-\sin t|}$

10.5 e) .... $\boxed{-2\cos\sqrt{t}}$

10.5 f) .... $\boxed{\dfrac{1}{\pi}\sin(\pi\ln t)}$

10.5 g) .... $\boxed{\tan t - t}$

10.5 h) .... $\boxed{\dfrac{1}{2}\tan^2 t + \ln|\cos t|}$

10.5 i) .... $\boxed{\dfrac{1}{4}\tan^4 t}$

10.5 j) .... $\boxed{2\sqrt{\tan t}}$

10.5 k) .... $\boxed{-\dfrac{1}{\tan t}}$

10.5 l) .... $\boxed{\dfrac{1}{2}\dfrac{1}{(1-\sin t)^2}}$

10.5 m) .... $\boxed{\dfrac{1}{2}\arctan(2t)}$

10.5 n) .... $\boxed{\arctan(e^t)}$

10.6 a) .... $\boxed{\dfrac{1}{2}(\arcsin(t))^2}$

10.6 b) .... $\boxed{\ln|\arcsin(t)|}$

10.7 a) .... $\boxed{\dfrac{t}{2} + \dfrac{\sin(2t)}{4}}$

10.7 b) .... $\boxed{-\dfrac{\cos(4t)}{8} - \dfrac{\cos(2t)}{4}}$

10.7 c) .... $\boxed{-\cos t + \dfrac{1}{3}\cos^3 t}$

10.7 d) . . . . $\boxed{\ln(1 + \sin^2 t)}$

10.7 e) . . . . $\boxed{\ln|\tan t|}$

10.7 f) . . . . $\boxed{-\cotan t + \tan t}$

10.7 g) . . . . $\boxed{\dfrac{1}{4}\ln|\tan 2t|}$

10.8 a) . . . . $\boxed{t + \ln|t| - \dfrac{1}{t}}$

10.8 b) . . . . $\boxed{\ln|t| - \dfrac{1}{2t^2}}$

10.8 c) . . . . $\boxed{t + \dfrac{t^3}{3} + \dfrac{t^5}{5}}$

10.8 d) . . . . $\boxed{t - \dfrac{t^2}{2} + \dfrac{t^3}{3}}$

10.8 e) . . . . $\boxed{t - 2\ln|t + 1|}$

10.8 f) . . . . $\boxed{t - \dfrac{t^2}{2} + \dfrac{t^3}{3} - \ln|t + 1|}$

10.8 g) . . . . $\boxed{\dfrac{1}{2}\ln(1 + t^2) - \arctan(t)}$

10.8 h) . . . . $\boxed{\ln|t + 1| + \dfrac{1}{t + 1}}$

10.9 a) . . . . $\boxed{2(t - 1) \text{ puis } \dfrac{1}{3}t^3 - t^2 + 5t}$

10.9 b) . . . . $\boxed{-\dfrac{1}{t^2}\left(\dfrac{2}{t} + 1\right) \text{ puis } -\dfrac{1}{t} + \ln|t|}$

10.9 c) . . . . $\boxed{\dfrac{1}{2\sqrt{t}} + \dfrac{3}{t^4} \text{ puis } \dfrac{2}{3}t^{\frac{3}{2}} + \dfrac{1}{2t^2}}$

10.9 d) . . . . $\boxed{-\dfrac{4}{t^5} - \dfrac{3}{2}\dfrac{1}{t^{5/2}} \text{ puis } -\dfrac{1}{3}\dfrac{1}{t^3} - \dfrac{2}{\sqrt{t}}}$

10.9 e) . . . . $\boxed{2e^{2t} - 3e^{-3t} \text{ puis } \dfrac{1}{2}e^{2t} - \dfrac{1}{3}e^{-3t}}$

10.9 f) . . . . $\boxed{3e^{3t-2} \text{ puis } \dfrac{1}{3}e^{3t-2}}$

10.9 g) . . . . . $\boxed{-\dfrac{t(t^3 + 2)}{(t - 1)^2(t^2 + t + 1)^2} \text{ puis } \dfrac{1}{3}\ln(|t^3 - 1|)}$

10.9 h) . . . $\boxed{-\dfrac{3t^2 - 2t - 3}{(t^2 + 1)^2} \text{ puis } \dfrac{3}{2}\ln(t^2 + 1) - \arctan(t)}$

10.9 i) . . . . $\boxed{\cos t(3\cos^2 t - 2) \text{ puis } -\dfrac{1}{3}\cos^3 t}$

10.9 j) . . . . $\boxed{-\dfrac{2t\sin\frac{1}{t} + \cos\frac{1}{t}}{t^4} \text{ puis } \cos\dfrac{1}{t}}$

10.9 k) . . . . $\boxed{\dfrac{2e^t}{(2 + e^t)^2} \text{ puis } \ln(2 + e^t)}$

10.9 l) . . . . . . $\boxed{\dfrac{2\cos t + 3}{(2 + 3\cos t)^2} \text{ puis } -\dfrac{1}{3}\ln|2 + 3\cos t|}$

10.9 m) . . . . $\boxed{\dfrac{1}{(1 - t^2)^{3/2}} \text{ puis } -\sqrt{1 - t^2}}$

10.9 n) . . . . . . $\boxed{2\dfrac{3\cos^2 t - 1}{(1 + \cos^2 t)^2} \text{ puis } -\ln(1 + \cos^2(t))}$

10.9 o) . . . . $\boxed{(1 - 2t^2)e^{-t^2} \text{ puis } -\dfrac{1}{2}e^{-t^2}}$

10.9 p) . . . . $\boxed{\dfrac{\ln t - 2}{t^2} \text{ puis } \ln t - \dfrac{1}{2}\ln^2 t}$

10.9 q) . . . . $\boxed{-\dfrac{1 + \ln t}{t^2 \ln^2 t} \text{ puis } \ln|\ln t|}$

10.9 r) . . . . $\boxed{\dfrac{\cos\ln t - \sin\ln t}{t^2} \text{ puis } -\cos(\ln t))}$

10.11 a) . . . . $\boxed{-\dfrac{e^t(e^{2t} - 1)}{(1 + e^{2t})^2}) \text{ puis } \arctan(e^t)}$

10.11 b) . . . . $\boxed{\sinh(t)^2 + \cosh^2(t) \text{ puis } \dfrac{1}{2}\sinh^2(t)}$

Fiche nº 11. Calcul d'intégrales

Réponses

11.1 a)........ $\boxed{\text{Positif}}$

11.1 b)...... $\boxed{\text{Négatif}}$

11.1 c)........ $\boxed{\text{Positif}}$

11.2 a)............ $\boxed{14}$

11.2 b) .... $\boxed{50}$

11.2 c) .... $\boxed{\dfrac{147}{2}}$

11.2 d) .... $\boxed{-54}$

11.2 e) .... $\boxed{0}$

11.2 f) .... $\boxed{\dfrac{5}{2}}$

11.3 a) .... $\boxed{8}$

11.3 b) .... $\boxed{-2}$

11.3 c) .... $\boxed{\dfrac{8}{3}}$

11.3 d) .... $\boxed{0}$

11.3 e) .... $\boxed{-\dfrac{1}{30}}$

11.3 f)........ $\boxed{-\dfrac{2}{101}}$

11.4 a) .... $\boxed{0}$

11.4 b) .... $\boxed{1}$

11.4 c) .... $\boxed{\dfrac{1}{2}}$

11.4 d) .... $\boxed{18}$

11.4 e) ...... $\boxed{e^2 - e^{-3}}$

11.4 f) .... $\boxed{-\ln 3}$

11.5 a) .... $\boxed{78}$

11.5 b) ..... $\boxed{2(e^3 - 1)}$

11.5 c) . $\boxed{\dfrac{1}{\pi}\ln\left(1 + \dfrac{\pi}{2}\right)}$

11.5 d) .... $\boxed{\dfrac{\sqrt{2}}{6}}$

11.5 e) .... $\boxed{6}$

11.5 f) ...... $\boxed{\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}}$

11.6 a) .... $\boxed{0}$

11.6 b) .... $\boxed{0}$

11.6 c) ..... $\boxed{\ln\left(\dfrac{2}{\sqrt{3}}\right)}$

11.6 d) .... $\boxed{-\dfrac{1}{384}}$

11.6 e) ... $\boxed{\dfrac{1}{2}\left(1 - \dfrac{1}{e}\right)}$

11.6 f) .... $\boxed{\dfrac{7}{48}}$

11.7 a) .... $\boxed{\dfrac{1}{2} - \dfrac{1}{e + 1}}$

11.7 b) .... $\boxed{\dfrac{17}{2}}$

11.7 c) .... $\boxed{e^2}$

11.7 d) .... $\boxed{3e - 4}$

11.7 e) .... $\boxed{-\dfrac{1}{3}}$

11.7 f) .... $\boxed{\dfrac{5}{8}}$

11.8 a) .... $\boxed{0}$

11.8 b) .... $\boxed{\dfrac{\pi}{4}}$

11.8 c) .... $\boxed{\dfrac{99}{\ln 10}}$

11.8 d) .... $\boxed{\dfrac{e - \frac{1}{e}}{2}}$

11.8 e) .... $\boxed{\dfrac{2}{3}}$

11.8 f) .... $\boxed{\dfrac{2\pi}{9}}$

Fiche nº 12. Intégration par parties

Réponses

12.1 a) . . . . $\boxed{\dfrac{\pi}{2} - 1}$

12.1 b) . . . . $\boxed{\dfrac{5}{2}\mathrm{ch}(2) - \dfrac{1}{2}\mathrm{sh}(2) - \dfrac{3}{2}}$

12.1 c) . . . . $\boxed{4}$

12.1 d) . . . . $\boxed{\dfrac{(\ln(2))^2 2^{\ln(2)} - 2\ln(2) - 2^{\ln(2)} + 2}{(\ln(2))^2}}$

12.1 e) . . . . $\boxed{1}$

12.1 f) . . . . $\boxed{2\ln(2) - \dfrac{3}{4}}$

12.1 g) . . . . $\boxed{\ln(2) - 2 + \dfrac{\pi}{2}}$

12.1 h) . . . . $\boxed{\dfrac{\pi}{4} - \dfrac{1}{2}}$

12.1 i) . . . . $\boxed{\dfrac{\pi}{12} + \dfrac{\sqrt{3}}{2} - 1}$

12.1 j) . . . . $\boxed{-\dfrac{2\sqrt{2}}{3} + \dfrac{4}{3}}$

12.1 k) . . . . $\boxed{\dfrac{4}{3}\sqrt{2}\ln(2) - \dfrac{8}{9}\sqrt{2} + \dfrac{4}{9}}$

12.1 l) . . . . $\boxed{\dfrac{\pi}{4} - \dfrac{1}{2}\ln(2) - \dfrac{\pi^2}{32}}$

12.2 a) . . . . $\boxed{\begin{cases} \mathbb{R} \to \mathbb{R} \\ x \mapsto (-x+2)\mathrm{e}^x \end{cases}}$

12.2 b) . . . . $\boxed{\begin{cases} \mathbb{R}_+^* \to \mathbb{R} \\ x \mapsto -\dfrac{1+\ln x}{x} \end{cases}}$

12.2 c) . . . . $\boxed{\begin{cases} \mathbb{R} \to \mathbb{R} \\ x \mapsto x\arctan(x) - \dfrac{1}{2}\ln(1+x^2) \end{cases}}$

12.2 d) . . . . $\boxed{\begin{cases} \mathbb{R} \to \mathbb{R} \\ x \mapsto x\mathrm{sh}(x) - \mathrm{ch}(x) \end{cases}}$

12.3 a) . . . . $\boxed{\dfrac{5}{2} - \mathrm{e}^2}$

12.3 b) . . . . $\boxed{\dfrac{\mathrm{e}^{\frac{\pi}{2}} + 1}{2}}$

12.4 a) . . . $\boxed{\begin{cases} \mathbb{R} \to \mathbb{R} \\ x \mapsto \dfrac{1}{2}(-\cos(x)\mathrm{sh}(x) + \sin(x)\mathrm{ch}(x)) \end{cases}}$

12.4 b) . . . . $\boxed{\begin{cases} \mathbb{R}_+^* \to \mathbb{R} \\ x \mapsto x\ln^2 x - 2x\ln x + 2x \end{cases}}$

12.4 c) . . . . . . $\boxed{\begin{cases} \mathbb{R}_+^* \to \mathbb{R} \\ x \mapsto x^3\left(\dfrac{1}{3}\ln^2 x - \dfrac{2}{9}\ln x + \dfrac{2}{27}\right) \end{cases}}$

12.4 d) . . $\boxed{\begin{cases}]-1,1[\to \mathbb{R} \\ x \mapsto \dfrac{1}{2}\mathrm{e}^{\arccos(x)}\left(x - \sqrt{1-x^2}\right) \end{cases}}$

Fiche nº 13. Changements de variable

Réponses

13.1 a) . . . . $\boxed{\dfrac{\pi}{2}}$

13.1 b) . . . . $\boxed{\dfrac{\pi}{6}}$

13.1 c) . . . . $\boxed{2\arctan(\mathrm{e}) - \dfrac{\pi}{2}}$

13.1 d) . . . . $\boxed{\dfrac{1}{4}}$

13.1 e) . . . . $\boxed{\dfrac{1}{12}}$

13.1 f) . . . . $\boxed{2\ln\left(\dfrac{3}{2}\right)}$

13.2 a) . . . . $\boxed{\dfrac{\pi}{3\sqrt{3}}}$

13.2 b) . . . . $\boxed{\dfrac{1}{2}\ln\left(\dfrac{2\mathrm{e}+1}{3}\right)}$

13.2 c) . . . . $\boxed{\dfrac{\pi}{2}}$

13.2 d) . . . . $\boxed{\dfrac{1}{4} + \dfrac{\pi}{8}}$

13.2 e) . . . . $\boxed{\dfrac{\pi}{12}}$

13.2 f) . . . . $\boxed{\dfrac{1}{2}\ln\dfrac{5}{2}}$

13.3 a) . . . . $\boxed{2\mathrm{e}^2}$

13.3 b) . . . . $\boxed{-2((\sqrt{3}-1)\ln(\sqrt{3}-1) - 4 + 2\sqrt{3}}$

13.4 a) . . . . $\boxed{\begin{cases} \left]0, \dfrac{\pi}{2}\right[& \to & \mathbb{R} \\ x & \mapsto & \tan x + \ln\tan(x) \end{cases}}$

13.4 b) . . . . $\boxed{\begin{cases} \mathbb{R} & \to & \mathbb{R} \\ x & \mapsto & \dfrac{x}{2} - \dfrac{\mathrm{e}^{-2x}}{4} \end{cases}}$

13.4 c) . . . . $\boxed{\begin{cases} \mathbb{R}_+^* & \to & \mathbb{R} \\ x & \mapsto & 2\arctan\left(\sqrt{\mathrm{e}^x - 1}\right) \end{cases}}$

13.4 d) . . . . $\boxed{\begin{cases} \mathbb{R}_+^* & \to & \mathbb{R} \\ x & \mapsto & \dfrac{3}{2}\ln(x^{\frac{2}{3}} + 1) \end{cases}}$

13.4 e) . . . . $\boxed{\begin{cases} \left]1, +\infty\right[& \to & \mathbb{R} \\ x & \mapsto & \arctan\sqrt{x^2 - 1} \end{cases}}$

Fiche nº 14. Intégration des fractions rationnelles

Réponses

14.1 a) .... $\boxed{\ln\left(\dfrac{3}{2}\right)}$

14.1 b) .... $\boxed{\dfrac{1}{2}\ln\left(\dfrac{5}{3}\right)}$

14.2 a) .... $\boxed{2\ln\dfrac{9}{10}}$

14.2 b) .... $\boxed{\ln(a+1)}$

14.3 a) ...... $\boxed{\dfrac{3}{2}+\ln(3)-\ln(2)}$

14.3 b) .... $\boxed{-\dfrac{1}{48}+\dfrac{51}{64}\ln\dfrac{21}{19}}$

14.4 a) .... $\boxed{\ln\left(\dfrac{7}{3}\right)}$

14.4 b) .... $\boxed{\ln\dfrac{33}{28}}$

14.5 a) .... $\boxed{\ln\left(2\sqrt{\sqrt{2}-1}\right)}$

14.5 b) .... $\boxed{\dfrac{1}{2a}\ln\left(\dfrac{a+1}{2}\right)}$

14.6 a) .... $\boxed{1\text{ et }2}$

14.6 b) ...... $\boxed{A=-1\text{ et }B=1}$

14.6 c) .... $\boxed{2\ln\dfrac{4}{3}}$

14.7 a) .... $\boxed{-\ln(3)}$

14.7 b) .... $\boxed{2\ln\dfrac{4}{3}}$

14.7 c) .... $\boxed{\dfrac{1}{2}\ln\dfrac{3}{2}}$

14.7 d) .... $\boxed{\dfrac{1}{4}\ln\dfrac{1}{5}}$

14.8 .... $\boxed{\dfrac{1}{2\sqrt{a}}\ln\left(\dfrac{\sqrt{a}-a}{a+\sqrt{a}}\right)}$

14.9 a) .... $\boxed{\dfrac{1}{a^2+x^2}}$

14.9 b) .... $\boxed{\dfrac{1}{a}\arctan\left(\dfrac{x}{a}\right)}$

14.10 a) .... $\boxed{\dfrac{\pi}{4}}$

14.10 b) .... $\boxed{\dfrac{\pi}{6\sqrt{3}}}$

14.10 c) .... $\boxed{\dfrac{\pi}{2\sqrt{2}}}$

14.11 a) .... $\boxed{\left(x+\dfrac{1}{2}\right)^2+\dfrac{3}{4}}$

14.11 b) ...... $\boxed{2\left(x-\dfrac{3}{4}\right)^2-\dfrac{1}{8}}$

14.11 c) .. $\boxed{\sqrt{2}\left(x+\tfrac{1}{4}\right)^2+\sqrt{2}\dfrac{15}{16}}$

14.11 d) ..... $\boxed{a\left(x+\dfrac{a}{2}\right)^2+\dfrac{3a^3}{4}}$

14.12 a) .... $\boxed{\dfrac{1}{2}}$

14.12 b) .... $\boxed{\dfrac{2\pi}{3\sqrt{3}}}$

14.12 c) .... $\boxed{\dfrac{2\pi}{3\sqrt{3}}}$

14.12 d) .... $\boxed{\ln(2)}$

14.13 a) .... $\boxed{\dfrac{\pi}{12}}$

14.13 b) .... $\boxed{\ln\left(\dfrac{a^2}{a^2-1}\right)}$

14.14 .... $\boxed{\dfrac{1}{3}\left(\ln(2)+\dfrac{\pi}{\sqrt{3}}\right)}$

Fiche nº 15. Systèmes linéaires

Réponses

15.1 a) .... $\{(3,1)\}$

15.1 b) .... $\{(7,2)\}$

15.1 c) .... $\left\{\left(\dfrac{1}{3},\dfrac{2}{3}\right)\right\}$

15.1 d) .... $\left\{\left(\dfrac{\sqrt{2}}{3},\dfrac{\sqrt{2}}{2}\right)\right\}$

15.2 a) .... $\left\{\left(1-\dfrac{a}{4},\dfrac{-1}{2}+\dfrac{3}{8}a\right)\right\}$

15.2 b) .... $(2,-3)$

15.2 c) .... $\left\{\left(\dfrac{1}{13}a+\dfrac{5}{13}a^2,\dfrac{2}{13}a-\dfrac{3}{13}a^2\right)\right\}$

15.2 d) .... $(a-2a^2,a+a^2)$

15.3 a) .... $\{(1+z,-z,z)\ ;\ z\in\mathbb{R}\}$

15.3 b) .... $\{(1,y,3+2y)\ ;\ y\in\mathbb{R}\}$

15.3 c) .... $\left\{\left(\dfrac{13}{6}-\dfrac{5}{3}z,-\dfrac{1}{3}+\dfrac{4}{3}z,z\right)\ ;\ z\in\mathbb{R}\right\}$

15.3 d) .... $\left\{\left(x,\dfrac{-5}{12}-\dfrac{3}{2}x,\dfrac{-25}{24}-\dfrac{7}{4}x\right)\ ;\ x\in\mathbb{R}\right\}$

15.4 a) .... $\{(2,-1,3)\}$

15.4 b) .... $\{(-1,4,2)\}$

15.4 c) .... $\varnothing$

15.4 d) .... $\left\{\left(-\dfrac{2}{7}-z,\dfrac{3}{7},z\right)\ ;\ z\in\mathbb{R}\right\}$

15.5 a) .... $\{(1,1/2,1/2)\}$

15.5 b) .... $\varnothing$

15.5 c) .... $\{(5z,1-4z,z)\ ;\ z\in\mathbb{R}\}$

15.5 d) .... $\left\{\left(1,\dfrac{1}{a+2},\dfrac{1}{a+2}\right)\right\}$

15.6 a) .... $\{(5,3,-1)\}$

15.6 b) .... $\varnothing$

15.6 c) .... $\left\{\left(\dfrac{a^2+a-1}{a^3-1}c,\dfrac{a^2-a+1}{a^3-1}c,\dfrac{-a^2+a+1}{a^3-1}c\right)\right\}$

15.7 a) .... $\{(0,0,0)\}$

15.7 b) .... $\{(x,y,-x-y)\ ;\ (x,y)\in\mathbb{R}^2\}$

15.7 c) .... $\{(x,x,x)\ ;\ x\in\mathbb{R}\}$

Fiche nº 16. Nombres complexes

Réponses

16.1 a) . . . . $\boxed{4 + 32\mathrm{i}}$

16.1 b) . . . . $\boxed{13 - \mathrm{i}}$

16.1 c) . . . . $\boxed{7 - 24\mathrm{i}}$

16.1 d) . . . . $\boxed{5}$

16.1 e) . . $\boxed{-119 + 120\mathrm{i}}$

16.1 f) . . . . . $\boxed{\dfrac{3}{10} + \dfrac{1}{10}\mathrm{i}}$

16.1 g) . . . . . $\boxed{\dfrac{4}{29} - \dfrac{19}{29}\mathrm{i}}$

16.1 h) . . . . . $\boxed{\dfrac{1}{2} - \dfrac{\sqrt{3}}{2}\mathrm{i}}$

16.2 a) . . . . $\boxed{12}$

16.2 b) . . . . $\boxed{8\mathrm{e}^{\mathrm{i}\pi}}$

16.2 c) . . . . $\boxed{\sqrt{3}\mathrm{e}^{\mathrm{i}\frac{\pi}{2}}}$

16.2 d) . . . . $\boxed{2\mathrm{e}^{-\mathrm{i}\frac{\pi}{2}}}$

16.2 e) . . . . $\boxed{2e^{\mathrm{i}\frac{8\pi}{5}}}$

16.2 f) . . . . . $\boxed{5\sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}}$

16.2 g) . . . . $\boxed{10\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}}$

16.2 h) $\boxed{2\cos\left(\dfrac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}}$

16.3 a) . . . . $\boxed{1}$

16.3 b) . . . $\boxed{\dfrac{1}{\sqrt{2}} + \mathrm{i}\dfrac{1}{\sqrt{2}}}$

16.3 c) . . $\boxed{-\dfrac{1}{\sqrt{2}} + \mathrm{i}\dfrac{1}{\sqrt{2}}}$

Fiche nº 17. Trigonométrie et nombres complexes

Réponses

17.1 a) .... $\boxed{2\cos\left(\dfrac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{\pi}{12}}}$

17.1 b) .... $\boxed{\left(-2\cos\left(\dfrac{7\pi}{12}\right)\right)\mathrm{e}^{-\mathrm{i}\frac{5\pi}{12}}}$

17.1 c) .... $\boxed{2\sin\left(\dfrac{\pi}{12}\right)\mathrm{e}^{\frac{-7\mathrm{i}\pi}{12}}}$

17.1 d) .... $\boxed{2\cos\left(\dfrac{5\pi}{12}\right)\mathrm{e}^{\frac{5\mathrm{i}\pi}{12}}}$

17.1 e) .... $\boxed{2\cos\left(\dfrac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{13\pi}{12}}}$

17.1 f) .... $\boxed{2\sin\left(\dfrac{\pi}{24}\right)\mathrm{e}^{-\mathrm{i}\frac{11\pi}{24}}}$

17.1 g) .... $\boxed{\dfrac{\cos\left(\frac{\pi}{12}\right)}{\sin\left(\frac{\pi}{24}\right)}\mathrm{e}^{\frac{13\mathrm{i}\pi}{24}}}$

17.1 h) .... $\boxed{2^{27}\cos^{27}\left(\dfrac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}}$

17.2 a) .... $\boxed{2\cos\left(\dfrac{\pi}{12}\right)\mathrm{e}^{\mathrm{i}\frac{5\pi}{12}}}$

17.2 b) .... $\boxed{2\sin\left(\dfrac{\pi}{12}\right)\mathrm{e}^{-\mathrm{i}\frac{\pi}{12}}}$

17.3 a) .... $\boxed{\dfrac{1}{4}\cos(3x)+\dfrac{3}{4}\cos(x)}$

17.3 b) .... $\boxed{-\dfrac{1}{4}\cos(4x)+\dfrac{1}{2}\cos(2x)-\dfrac{1}{4}}$

17.3 c) ... $\boxed{-\dfrac{1}{8}\cos(6x)+\dfrac{1}{4}\cos(4x)-\dfrac{3}{8}\cos(2x)+\dfrac{1}{4}}$

17.3 d) ... $\boxed{-\dfrac{\sin(9x)}{8}+\dfrac{3\sin(5x)}{8}-\dfrac{\sin(3x)}{8}-\dfrac{3\sin(x)}{8}}$

17.3 e) .... $\boxed{\dfrac{\cos(9x)}{8}+\dfrac{3\cos(5x)}{8}+\dfrac{\cos(3x)}{8}+\dfrac{3\cos(x)}{8}}$

17.3 f) .... $\boxed{-\dfrac{1}{4}\sin(11x)+\dfrac{1}{4}\sin(5x)+\dfrac{1}{2}\sin(3x)}$

17.4 a) .... $\boxed{4\cos^3(x)-3\cos(x)}$

17.4 b) .... $\boxed{4\cos^3(x)\sin(x)-4\cos(x)\sin^3(x)}$

17.5 a) .... $\boxed{2\cos(2x)\cos(x)}$

17.5 b) .... $\boxed{2\cos(4x)\sin(x)}$

17.5 c) .... $\boxed{2\sin(x)\sin(2x)}$

17.5 d) .... $\boxed{2\sin(4x)\cos(x)}$

17.6 a) .... $\boxed{\dfrac{\sin\left(\frac{3x}{2}\right)\sin(2x)}{\sin\left(\frac{x}{2}\right)}}$

17.6 b) .... $\boxed{\dfrac{\sin(8x)}{2\sin(x)}}$

17.6 c) .... $\boxed{0}$

17.7 a) .... $\boxed{\dfrac{\mathrm{e}^{\pi}+1}{2}}$

17.7 b) .... $\boxed{\dfrac{1}{5}(\mathrm{e}^{\pi}-2)}$

Fiche nº 18. Sommes et produits

18.1 a) .... $\boxed{n(n+2)}$

18.1 b) .... $\boxed{\dfrac{7(n+1)(n+4)}{2}}$

18.1 c) .... $\boxed{\dfrac{n(5n+1)}{2}}$

18.1 d) .... $\boxed{\dfrac{(n-2)(n-7)}{6}}$

18.2 a) .... $\boxed{\dfrac{n(n+1)(n+2)}{3}}$

18.2 b) ... $\boxed{n(n+1)(n^2+n+4)}$

18.2 c) .... $\boxed{\dfrac{9}{2}(3^{n-2}-1)}$

18.2 d) .... $\boxed{\dfrac{5^{n+1}-2^{n+1}}{3}}$

18.2 e) ... $\boxed{\dfrac{7}{6}(7^n-1)+n(n+4)}$

18.2 f) .... $\boxed{\dfrac{n+1}{2n}}$

18.3 a) .... $\boxed{2^{q-p+1}}$

18.3 b) .... $\boxed{3^{\frac{n(n+1)}{2}}}$

18.4 a) .... $\boxed{5^n(n!)^{\frac{3}{2}}}$

18.4 b) .... $\boxed{0}$

18.5 a) .... $\boxed{\dfrac{n(n+1)}{2}}$

18.5 b) .... $\boxed{0}$

18.5 c) ..... $\boxed{n2^{n+1}+2(1-2^n)}$

18.5 d) .... $\boxed{\dfrac{n^2(n+1)^2}{4}}$

18.6 a) .... $\boxed{(n+3)^3-2^3}$

18.6 b) .... $\boxed{\ln(n+1)}$

18.6 c) .... $\boxed{1-\dfrac{1}{(n+1)!}}$

18.6 d) .... $\boxed{(n+1)!-1}$

18.7 a) .... $\boxed{n+1}$

18.7 b) .... $\boxed{1-4n^2}$

18.7 c) .... $\boxed{\dfrac{1}{n}}$

18.7 d) .... $\boxed{\dfrac{n+1}{2n}}$

18.8 a) .... $\boxed{1-\dfrac{1}{n+1}}$

18.8 b) .... $\boxed{\dfrac{1}{2}-\dfrac{1}{n+3}}$

18.9 a) .... $\boxed{2n^2+n}$

18.9 b) .... $\boxed{\dfrac{n(3n+1)}{2}}$

18.10 a) .... $\boxed{\dfrac{n^2(n+1)}{2}}$

18.10 b) .... $\boxed{\dfrac{n(n+3)}{4}}$

18.10 c) .... $\boxed{\dfrac{n(n^2-1)}{2}}$

18.10 d) . $\boxed{\dfrac{n(n+1)(7n^2+13n+4)}{12}}$

18.10 e) .... $\boxed{\dfrac{n(n+1)}{2}\ln(n!)}$

18.10 f) ..... $\boxed{\dfrac{n(n+1)(4n-1)}{6}}$

Fiche nᵒ 19. Coefficients binomiaux

19.1 a) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{10\ 100}$

19.1 b) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{720}$

19.1 c) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{1}{30}}$

19.1 d) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{15}$

19.1 e) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{56}$

19.1 f) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{140}$

19.2 a) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{9!}{5!}}$

19.2 b) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{\dbinom{9}{4}}$

19.2 c) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{2^n \times n!}$

19.2 d) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{(2n+1)!}{2^n \times n!}}$

19.3 a) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{n(n-1)}{2}}$

19.3 b) $\ldots\ldots\ldots$ $\boxed{\dfrac{n(n-1)(n-2)}{6}}$

19.3 c) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{k+1}{n-k}}$

19.3 d) $\ldots\ldots\ldots\ldots$ $\boxed{(n+2)(n+1)}$

19.3 e) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{1}{(n+1)!}}$

19.3 f) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{n! \times (n-3)}{2^{2n+2}}}$

19.4 a) $\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{(n+1)^3}{n \times (n+2)!}}$

19.4 b) $\ldots\ldots$ $\boxed{\dfrac{3(3n+2)(3n+1)}{a^3(n+1)^2}}$

19.5 a) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{3^n}$

19.5 b) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{0}$

19.5 c) $\ldots\ldots\ldots\ldots\ldots\ldots$ $\boxed{6^n}$

19.5 d) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{12 \times 15^n}$

19.6 a) $\ldots\ldots\ldots$ $\boxed{2 \times \displaystyle\sum_{p=0}^{\lfloor \frac{n}{2} \rfloor} \binom{n}{2p}}$

19.6 b) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{2^{n-1}}$

19.7 a) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{2^n}$

19.7 b) $\ldots\ldots\ldots\ldots\ldots$ $\boxed{n2^{n-1}}$

19.7 c) $\ldots\ldots\ldots$ $\boxed{n(n+1)2^{n-2}}$

19.7 d) $\ldots\ldots\ldots\ldots$ $\boxed{\dfrac{2^{n+1}-1}{n+1}}$

19.8 a) $\ldots\ldots\ldots\ldots$ $\boxed{\dbinom{2n}{n}}$

19.8 b) $\ldots\ldots\ldots\ldots$ $\boxed{\displaystyle\sum_{k=0}^{n} \binom{n}{k}^2}$

19.8 c) $\ldots\ldots\ldots\ldots$ $\boxed{\dbinom{2n}{n}}$

Fiche nº 20. Manipulation des fonctions usuelles

Réponses

20.1 a) .... $\boxed{\dfrac{\pi}{6}}$

20.1 b) .... $\boxed{2}$

20.1 c) .... $\boxed{\dfrac{\pi}{4}}$

20.1 d) .... $\boxed{\dfrac{\pi}{6}}$

20.1 e) .... $\boxed{\dfrac{\pi}{4}}$

20.1 f) .... $\boxed{\dfrac{\pi}{3}}$

20.2 a) .... $\boxed{1}$

20.2 b) .... $\boxed{0}$

20.2 c) .... $\boxed{5/4}$

20.2 d) .... $\boxed{4/3}$

20.2 e) .... $\boxed{\dfrac{13}{12}}$

20.2 f) .... $\boxed{3/5}$

20.3 a) .... $\boxed{\operatorname{sh}(x+y)}$

20.3 b) .... $\boxed{\operatorname{ch}(x-y)}$

20.4 a) .... $\boxed{\dfrac{\ln(2)}{\ln(3)}}$

20.4 b) .... $\boxed{1}$

20.4 c) .... $\boxed{-\dfrac{\ln(3)}{\ln(2)}}$

20.4 d) .... $\boxed{\dfrac{\ln(4)}{\ln(20/3)}}$

20.5 a) .... $\boxed{\dfrac{\ln\left(\frac{\sqrt{17}-1}{2}\right)}{\ln(2)}}$

20.5 b) .... $\boxed{\left\{0,\frac{1}{2}\right\}}$

20.5 c) .... $\boxed{1-\dfrac{\ln(2)}{\ln(3)}}$

20.5 d) .... $\boxed{\dfrac{\ln\left(\frac{\sqrt{5}-1}{2}\right)}{\ln(3)}}$

20.6 a) .... $\boxed{1}$

20.6 b) .... $\boxed{0}$

20.6 c) .... $\boxed{\{2k\pi\ ;\ k\in\mathbb{Z}\}}$

20.6 d) .. $\boxed{\begin{array}{l}\left\{\frac{\pi}{3}+2k\pi\ ;\ k\in\mathbb{Z}\right\}\\ \cup\left\{\frac{2\pi}{3}+2k\pi\ ;\ k\in\mathbb{Z}\right\}\end{array}}$

20.6 e) . $\boxed{\begin{array}{l}\left\{\frac{1}{3}+2k\pi\ ;\ k\in\mathbb{Z}\right\}\\ \cup\left\{\pi-\frac{1}{3}+2k\pi\ ;\ k\in\mathbb{Z}\right\}\end{array}}$

20.6 f) .... $\boxed{1}$

20.7 a) . $\boxed{\{\ln(\sqrt{5}-2),\ln(\sqrt{5}+2)\}}$

20.7 b) .... $\boxed{\ln(1+\sqrt{2})}$

20.7 c) .... $\boxed{\dfrac{1}{2}\ln(2)}$

20.7 d) . $\boxed{[-\ln(4+\sqrt{15}),\ln(4+\sqrt{15})]}$

20.7 e) ..... $\boxed{\left[\ln(3+\sqrt{10}),+\infty\right[}$

20.7 f) .... $\boxed{\left]-\infty,\frac{1}{2}\ln(3)\right]}$

20.8 a) ... $\boxed{x\mapsto \ln(2)\times 2^x+2x}$

20.8 b) . $\boxed{x\mapsto\dfrac{15^x\ln(3/5)+3^x\ln(3)}{(5^x+1)^2}}$

20.8 c) ...... $\boxed{x\mapsto(\ln(x)+1)x^x}$

20.8 d) .. $\boxed{x\mapsto\dfrac{\pi}{2\sqrt{1-x^2}\arccos(x)^2}}$

20.9 a) .... $\boxed{x\mapsto 2x\dfrac{1}{\sqrt{1-x^4}}}$

20.9 b) ... $\boxed{x\mapsto\operatorname{ch}^2(x)+\operatorname{sh}^2(x)}$

20.9 c) .... $\boxed{x\mapsto\dfrac{1-\operatorname{th}^2(x)}{1+\operatorname{th}^2(x)}}$

20.9 d) .... $\boxed{x\mapsto\operatorname{sh}(x)\operatorname{ch}(\operatorname{ch}(x))}$

20.10 a) .... $\boxed{x\mapsto 0}$

20.10 b) .... $\boxed{x\mapsto 0}$

20.11 a) $\boxed{x\mapsto(\ln(x)+1)x^x e^{-x^{2x}}}$

20.11 b) . $\boxed{x\mapsto\dfrac{\operatorname{sh}(x)}{\operatorname{ch}(x)^2}\dfrac{1}{2\sqrt{\ln(\operatorname{ch}(x))}}}$

20.11 c) .... $\boxed{x\mapsto\arcsin(x)}$

20.11 d) .... $\boxed{x\mapsto\arctan(x)}$

Fiche nᵒ 21. Suites numériques

Réponses

21.1 a) . . . . $\dfrac{12}{5}$

21.1 b) 8

21.1 c) . . . . $\dfrac{(2n+5) \times 2^{n+3}}{5}$

21.1 d) . . . . . $\dfrac{3(2n+1) \times 2^{3n+2}}{5}$

21.2 a) . . . . $2n\ln(n)$

21.2 b) . . . . $4n\ln(2n)$

21.3 a) 13

21.3 b) 29

21.4 a) . . . . $2^{\frac{1}{8}}$

21.4 b) . . . . $2^{\frac{1}{64}}$

21.5 a) 2

21.5 b) 2

21.6 a) 21

21.6 b) . . . . $10\,000$

21.6 c) . . . . $2\,001$

21.6 d) . . . . $10\,201$

21.7 a) . . . . $\dfrac{17}{24}$

21.7 b) . . . . $\dfrac{1}{24}$

21.8 a) . . . . $\dfrac{3}{512}$

21.8 b) . . . . $\dfrac{3\,069}{512}$

21.8 c) . . . . $\dfrac{3}{1\,024}$

21.8 d) . . . . $\dfrac{6\,141}{1\,024}$

21.9 a) . . . . $\dfrac{\pi\sqrt{5}}{5}$

21.9 b) . . . . $\dfrac{11\sqrt{5}}{25}$

21.10 a) . . . . $3^n + (-2)^n$

21.10 b) 211

21.11 a) . . $\dfrac{(1+\sqrt{2})^n - (1-\sqrt{2})^n}{2}$

21.11 b) . . . . $2\sqrt{2}$

21.12 a) 257

21.12 b) . . . . $65\,537$

21.12 c) . . . . F_n

21.12 d) . . . . $F_{n+1} - 2$

21.12 e) . . . . $F_{n+1} + 2^{2^n+1}$

21.12 f) . . . . F_{n+2}

Fiche nᵒ 22. Développements limités

Réponses

22.1 a) .... $3x - x^2 + \dfrac{x^3}{2} - \dfrac{x^4}{2} + \underset{x \to 0}{\mathrm{o}}(x^4)$

22.1 b) .... $x - \dfrac{3}{2}x^2 + \dfrac{11}{6}x^3 - \dfrac{25}{12}x^4 + \underset{x \to 0}{\mathrm{o}}(x^4)$

22.1 c) .... $\dfrac{x^3}{2} - \dfrac{x^5}{24} + \underset{x \to 0}{\mathrm{o}}(x^6)$

22.1 d) .... $x + x^2 + \dfrac{x^3}{3} - \dfrac{x^5}{30} - \dfrac{x^6}{90} + \underset{x \to 0}{\mathrm{o}}(x^6)$

22.2 a) .... $\mathrm{e} - \dfrac{\mathrm{e}x}{2} + \dfrac{11\mathrm{e}x^2}{24} - \dfrac{7\mathrm{e}x^3}{16} + \dfrac{2\,447\mathrm{e}x^4}{5\,760} + \underset{x \to 0}{\mathrm{O}}(x^5)$

22.2 b) .... $1 - \dfrac{1}{4}x^2 - \dfrac{1}{96}x^4 - \dfrac{19}{5\,760}x^6 + \underset{x \to 0}{\mathrm{O}}(x^7)$

22.2 c) .... $\mathrm{e}\left(1 + \mathrm{i}x - x^2 - \dfrac{5}{6}\mathrm{i}x^3\right) + \underset{x \to 0}{\mathrm{o}}(x^3)$

22.2 d) .... $1 - x + \dfrac{3}{2}(x-1)^2 + \underset{x \to 1}{\mathrm{o}}\left((x-1)^2\right)$

22.3 a) .... $1 - \dfrac{3\pi^2}{8}\left(x - \dfrac{\pi}{3}\right)^2 + \underset{x \to \frac{\pi}{3}}{\mathrm{o}}\left(\left(x - \dfrac{\pi}{3}\right)^2\right)$

22.3 b) .... $1 + 2\left(x - \dfrac{\pi}{4}\right) + 2\left(x - \dfrac{\pi}{4}\right)^2 + \dfrac{8}{3}\left(x - \dfrac{\pi}{4}\right)^3 + \underset{x \to \frac{\pi}{4}}{\mathrm{O}}\left(\left(x - \dfrac{\pi}{4}\right)^4\right)$

22.3 c) .... $-1 + \dfrac{\pi^2}{8}\left(x - \dfrac{\pi}{2}\right)^4 - \dfrac{\pi^2}{48}\left(x - \dfrac{\pi}{2}\right)^6 + \underset{x \to \frac{\pi}{2}}{\mathrm{o}}\left(\left(x - \dfrac{\pi}{2}\right)^7\right)$

22.4 a) .... $-\dfrac{1}{2x} + \dfrac{1}{12} - \dfrac{1}{720}x^2 + \underset{x \to 0}{\mathrm{o}}(x^2)$

22.4 b) .... $\dfrac{1}{x^2} - \dfrac{1}{x^3} + \dfrac{5}{6x^4} - \dfrac{5}{6x^5} + \underset{x \to +\infty}{\mathrm{O}}\left(\dfrac{1}{x^6}\right)$

22.4 c) .... $-\ln(x) + 1 - \dfrac{1}{2x} + \dfrac{1}{3x^2} - \dfrac{1}{4x^3} + \underset{x \to +\infty}{\mathrm{o}}\left(\dfrac{1}{x^3}\right)$

22.4 d) .... $\mathrm{e}^{-\frac{1}{2}}\left(\mathrm{e}^x + \dfrac{\mathrm{e}^x}{3x} - \dfrac{7\mathrm{e}^x}{36x^2}\right) + \underset{x \to +\infty}{\mathrm{o}}\left(\dfrac{\mathrm{e}^x}{x^2}\right)$

Fiche nº 23. Arithmétique

Réponses

23.1 a) . . . . $(6, 7)$

23.1 b) . . . . $(-7, 2)$

23.1 c) . . . . $(-6, 7)$

23.1 d) . . . . $(7, 2)$

23.2 a) 20

23.2 b) 4

23.3 a) 2

23.3 b) 4

23.4 1

23.5 a) 154

23.5 b) . . . . $\dfrac{65}{18}$

23.5 c) . . . . $29\ 160$

23.5 d) . . . . . . $\dfrac{1}{29\ 160}$

23.6 a) . . . . $(216, 192)$

23.6 b) . . . . . . $(12, 30)$

23.7 a) . . . . $(-5, 2)$

23.7 b) . . $8 \ (\mathrm{mod}\ 13)$

23.7 c) . . $11 \ (\mathrm{mod}\ 13)$

23.8 a) 56

23.8 b) . . . .
$(3\ 331, 10\ 548)$

23.9 a) . . . $2 \times 3 \times 337$

23.9 b) . . . . . . 7×17^2

23.9 c) . . . . 43×47

23.9 d) $\boxed{\text{Il est premier}}$

23.10 a) 67

23.10 b) 7

23.11 a) 1

23.11 b) 1

23.11 c) 6

23.11 d) 5

23.11 e) 66

23.11 f) 2

Fiche nᵒ 24. Polynômes

Réponses

24.1 a) .... $\begin{array}{l} Q = X^2 + 2X + 1 \\ R = 2 \end{array}$

24.1 b) .... $\begin{array}{l} Q = X^2 - 4X + 7 \\ R = -3X - 8 \end{array}$

24.1 c) .... $\begin{array}{l} Q = X^2 - 1 \\ R = -X^2 + X + 1 \end{array}$

24.1 d) .... $\begin{array}{l} Q = 13X + \dfrac{25}{2} \\ R = \dfrac{1}{2}\left(29X^2 - 5X - 23\right) \end{array}$

24.2 a) .... $\boxed{R = 1}$

24.2 b) .... $\boxed{R = 0}$

24.2 c) .... $\boxed{R = -2nX + 4n - 1}$

24.2 d) .... $\boxed{R = X^2 + X - 1}$

24.3 a) .... $\boxed{R = 2X - 3}$

24.3 b) .... $\boxed{R = -2X^3 - 3X^2 + 1}$

24.3 c) .... $\boxed{R = -8X^3 + 21X^2 - 20X + 5}$

24.3 d) .... $\boxed{R = -29X^3 + 11X^2 + 2X - 1}$

24.4 a) .... $\boxed{R = -36X + 24}$

24.4 b) .... $\boxed{24 - 36\mathrm{i}}$

24.5 a) .... $\boxed{R = -108X - 150}$

24.5 b) .... $\boxed{-150 - 108\sqrt{2}}$

24.6 a) .... $\boxed{76 - 92\sqrt{2}}$

24.6 b) .... $\boxed{8 - 206\mathrm{i}}$

24.7 a) .... $\boxed{(X - 1)^2(X^2 + 1)}$

24.7 b) .... $\boxed{(X^2 - 2X + 2)(X^2 - 2X + 5)}$

24.7 c) .... $\boxed{(X - 1)^2(X^2 + 1)(X + 1)(X - 2)}$

Fiche nº 25. Décomposition en éléments simples

Réponses

25.1 a) . . . . $\boxed{X - 3 - \dfrac{1}{X} + \dfrac{1}{X+1} + \dfrac{7}{X+2}}$

25.1 b) . . . . $\boxed{1 - \dfrac{2}{X} + \dfrac{1}{2(X+1)} + \dfrac{3}{2(X-1)}}$

25.1 c) . . . . $\boxed{1 + \dfrac{\pi}{2(X-\pi)} - \dfrac{\pi}{2(X+\pi)}}$

25.2 a) . . . . $\boxed{\dfrac{\mathrm{e}-1}{(\mathrm{e}-2)(X+\mathrm{e})} + \dfrac{1}{(2-\mathrm{e})(X+2)}}$

25.2 b) . . . . $\boxed{\dfrac{3}{2(X-1)} - \dfrac{1+i}{4(X-i)} - \dfrac{1-i}{4(X+i)}}$

25.2 c) . $\boxed{1 - \dfrac{5}{(\sqrt{2}+\sqrt{3})(X+\sqrt{3})} - \dfrac{4}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-X)}}$

25.3 a) . . . . . $\boxed{\dfrac{-3}{X-2} + \dfrac{1}{X-3} + \dfrac{2}{X-1} + \dfrac{1}{(X-1)^2}}$

25.3 b) . . $\boxed{\dfrac{2}{X} + \dfrac{2}{X^2} - \dfrac{11}{4(X-1)} + \dfrac{3}{2(X-1)^2} + \dfrac{3}{4(X+1)}}$

25.3 c) . . . . $\boxed{\dfrac{1}{\pi^2 X} - \dfrac{1}{\pi^2(X+\pi)} - \dfrac{1+\pi}{\pi(X+\pi)^2}}$

25.3 d) . . . . . $\boxed{\frac{2}{X-i} + \frac{1}{(X-i)^2} - \frac{2}{X-(1+i)} + \frac{1}{(X-(1+i))^2}}$

25.4 a) . . . $\boxed{\dfrac{1}{X+1} - \dfrac{1}{2(X-1)} - \dfrac{1+3i}{4(X-i)} - \dfrac{1-3i}{4(X+i)}}$

25.4 b) . $\boxed{\dfrac{1}{2X} + \dfrac{5}{6(X+2)} + \dfrac{2}{3(X-1)} + \dfrac{1}{(X-1)^2}}$.

25.5 a) . . . . $\boxed{\dfrac{1}{2(n+1)} - \dfrac{1}{2n} + \dfrac{1}{4}}$

25.5 b) . . . . $\boxed{-\dfrac{2}{n+2} + \dfrac{1}{n} - \dfrac{1}{3}}$

25.6 a) . . . . $\boxed{\dfrac{2}{X+1} + \dfrac{1}{(X+1)^2} + \dfrac{1-2X}{X^2+1}}$

25.6 b) . . . . . $\boxed{\dfrac{1}{2(X-1)} - \dfrac{3}{2(X+1)} + \dfrac{X-1}{X^2+X+1}}$

25.7 a) . . . . $\boxed{1 - 2\ln(3)}$

25.7 b) . . . . $\boxed{-\dfrac{1}{2}\ln(3) + \dfrac{2}{3}\ln(2)}$

25.7 c) . . . . $\boxed{\dfrac{2}{3} - 4\ln(2) + 2\ln(3)}$

25.7 d) . . . . $\boxed{\dfrac{1}{18} - \dfrac{1}{9}\ln(5) + \dfrac{2}{9}\ln(2)}$

25.7 e) . . . . $\boxed{\dfrac{\pi}{8}}$

25.7 f) . . . . $\boxed{\dfrac{1}{2}\ln(2) - \dfrac{1}{4}\ln(3)}$

25.8 a) . . . . $\boxed{\dfrac{1}{2}\ln\left|\dfrac{x-1}{1+x}\right|}$

25.8 b) . . . . $\boxed{x \longmapsto \dfrac{1}{4(1-2x)^2}}$

25.8 c) . . . . $\boxed{\dfrac{1}{\sqrt{2}}\arctan\left(\dfrac{x}{\sqrt{2}}\right)}$

25.8 d) . . . . $\boxed{\dfrac{2}{\sqrt{3}}\arctan\left(\dfrac{2}{\sqrt{3}}X + \dfrac{1}{\sqrt{3}}\right)}$

25.8 e) . . . . $\boxed{2}$

25.8 f) . $\boxed{\frac{x^2}{2} + 2x + \frac{1}{6}\ln|x+1| - \frac{1}{2}\ln|x-1| + \frac{16}{3}\ln|x-2|}$

25.8 g) . . . .
$\boxed{x \longmapsto \frac{1}{6}\ln(x^2+2) - \frac{1}{3}\ln|x+1| + \frac{\sqrt{2}}{3}\arctan\left(\frac{x}{\sqrt{2}}\right)}$

25.8 h) . . . . $\boxed{x \longmapsto \dfrac{1}{2}\dfrac{2x-1}{x^2-1} + \dfrac{1}{2}\ln\left|\dfrac{1-x}{1+x}\right|}$

Fiche nᵒ 26. Calcul matriciel

Réponses

26.1 a) . . . . $\begin{pmatrix} 1 & -3 & -1 \\ 3 & 3 & 4 \\ 9 & -7 & 3 \end{pmatrix}$

26.1 b) . . . . $\begin{pmatrix} -2 & -6 & -5 \\ 15 & -1 & 11 \\ 18 & -26 & -1 \end{pmatrix}$

26.1 c) . . . . 17 (matrice 1×1)

26.1 d) . . . . $\begin{pmatrix} 1 & 7 & -2 \\ 2 & 14 & -4 \\ -1 & -7 & 2 \end{pmatrix}$

26.1 e) . . . . $\begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$

26.1 f) . . . . $\begin{pmatrix} -5 & 15 & 3 \end{pmatrix}$

26.1 g) . . . . $\begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$

26.1 h) . . . . $\begin{pmatrix} 5 & 3 & -1 & 1 \\ 4 & 3 & 1 & 2 \end{pmatrix}$

26.1 i) . . . . $\begin{pmatrix} 1 & 7 & -2 \\ 7 & 49 & -14 \\ -2 & -14 & 4 \end{pmatrix}$

26.2 a) . . . . $\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$

26.2 b) . . . . $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

26.2 c) . . . . $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$

26.2 d) . . . . $\begin{pmatrix} 4 & 5 \\ 0 & 9 \end{pmatrix}$

26.2 e) . . . . $\begin{pmatrix} 8 & 19 \\ 0 & 27 \end{pmatrix}$

26.2 f) . . . . $\begin{pmatrix} 2^k & 3^k - 2^k \\ 0 & 3^k \end{pmatrix}$

26.2 g) . . . . $\begin{pmatrix} \cos(2\theta) & -\sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix}$

26.2 h) . . . . $\begin{pmatrix} \cos(3\theta) & -\sin(3\theta) \\ \sin(3\theta) & \cos(3\theta) \end{pmatrix}$

26.2 i) . . . . $\begin{pmatrix} \cos(k\theta) & -\sin(k\theta) \\ \sin(k\theta) & \cos(k\theta) \end{pmatrix}$

26.2 j) . . . . $\begin{pmatrix} n & \cdots & n \\ \vdots & (n) & \vdots \\ n & \cdots & n \end{pmatrix}$

26.2 k) . . . . $\begin{pmatrix} n^2 & \cdots & n^2 \\ \vdots & (n^2) & \vdots \\ n^2 & \cdots & n^2 \end{pmatrix}$

26.2 l) . . . . $n^{k-1} D$

26.3 a) . . . . $2 \times 3^{j-i} \times 5^{i-1}$

26.3 b) . . . . $2^{i+1} 3^{j-i} (2^n - 1)$

26.3 c) . . . . $2 \times 3^{i+j} \left(1 - \left(\frac{2}{3} \right)^n \right)$

26.3 d) . . . . $\begin{pmatrix} i-1 \\ j \end{pmatrix} + \begin{pmatrix} i-1 \\ j-2 \end{pmatrix}$

26.4 a) . . . . $2^{i-j} \begin{pmatrix} i-1 \\ j-1 \end{pmatrix}$

26.4 b) . . . . $\begin{array}{c} (1 - \delta_{i,1})(\delta_{i-1,j+1} + \delta_{i,j}) \\ + (1 - \delta_{i,n})(\delta_{i,j} + \delta_{i+1,j-1}) \end{array}$

26.5 a) . . . . $\dfrac{1}{2(\pi - \mathrm{e})} \begin{pmatrix} 2 & -\mathrm{e} \\ -2 & \pi \end{pmatrix}$

26.5 b) . . . . $\dfrac{1}{3} \begin{pmatrix} 1 & -1 - 2\mathrm{i} \\ 1 & -1 + \mathrm{i} \end{pmatrix}$

26.5 c) . . . . $\dfrac{1}{2} \begin{pmatrix} 5 & 2 & -1 \\ 3 & 2 & -1 \\ -6 & -2 & 2 \end{pmatrix}$

26.5 d) . . . . $\dfrac{1}{4\pi}\begin{pmatrix} 0 & 4 & 0 \\ 0 & -2 & -2 \\ 2 & -1 & 1 \end{pmatrix}$

26.5 e) . . . . $\dfrac{1}{8}\begin{pmatrix} 8 & 4 & -2 \\ -16 & -6 & 7 \\ 0 & -2 & 1 \end{pmatrix}$

26.5 f) . . . . $\dfrac{1}{6}\begin{pmatrix} -2 & 2 & 2 \\ 1 & -1 & 2 \\ 4 & 2 & -4 \end{pmatrix}$

26.5 g) . . . . $\dfrac{1}{2}\begin{pmatrix} 4 & -2 & 2 & 0 \\ 8 & -6 & 4 & 2 \\ -7 & 5 & -3 & -1 \\ -5 & 3 & -1 & -1 \end{pmatrix}$

26.5 h) . . . . $\boxed{\text{Non inversible !}}$

26.5 i) . . . . $\dfrac{1}{2}\begin{pmatrix} 0 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$

26.6 a) . . . . $\boxed{\lambda \neq 1}$

26.6 b) . . . . $\dfrac{1}{1-\lambda}\begin{pmatrix} -4 & -1 & 3 \\ 2\lambda+2 & \lambda & -2\lambda-1 \\ \lambda-1 & 0 & 1-\lambda \end{pmatrix}$

26.6 c) . . . . $\boxed{\lambda \neq 1}$

26.6 d) . . . . . $\dfrac{1}{1-\lambda}\begin{pmatrix} -1-\lambda+\lambda^2 & 1-\lambda & 2-\lambda \\ 1 & 0 & -1 \\ 1-\lambda^2 & \lambda-1 & \lambda-1 \end{pmatrix}$

Fiche nº 27. Algèbre linéaire

Réponses

27.1 a) . . . . $(3, -1)$

27.1 b) . . . . $(-1, 3)$

27.1 c) . . . . $(9/11, 2/11)$

27.1 d) . . . . $(-2, 4/5, 11/5)$

27.1 e) . . . . $(-1, 1/2, 1/2)$

27.1 f) . . . . $(0, 2, 4, 1)$

27.1 g) . . . . $(1/2, -\sqrt{3}/2)$

27.2 a) 2

27.2 b) 1

27.2 c) 1

27.2 d) 2

27.2 e) 2

27.2 f) 1

27.3 a) 2

27.3 b) 2

27.3 c) 3

27.3 d) 4

27.4 a) . . . . $\begin{pmatrix} 1 & 1 \\ 3 & -5 \end{pmatrix}$

27.4 b) . . . . $\begin{pmatrix} -5 & 3 \\ 1 & 1 \end{pmatrix}$

27.4 c) . . . . $\dfrac{1}{2}\begin{pmatrix} -19 & -43 \\ 9 & 21 \end{pmatrix}$

27.4 d) . . . . $\begin{pmatrix} 1 & 0 & 1 \\ 3 & -1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$

27.4 e) . . . . $\begin{pmatrix} 1 & 2 & 4 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$

27.5 a) . . . . $\begin{pmatrix} -1 & -1 & 1 \\ 4 & 15 & 0 \end{pmatrix}$

27.5 b) . . . . $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Fiche nº 28. Équations différentielles

Réponses

28.1 a) . . . . $\boxed{x \longmapsto 56\mathrm{e}^{12x}}$

28.1 b) . . . . $\boxed{x \longmapsto 6\mathrm{e}^x - 1}$

28.1 c) . . . . $\boxed{x \longmapsto \dfrac{8\mathrm{e}^{3x} - 5}{3}}$

28.1 d) . . . . $\boxed{x \longmapsto 9\mathrm{e}^{2x} - 6}$

28.2 a) . . . . $\boxed{x \longmapsto \mathrm{e}^{(6-x)/5}}$

28.2 b) . . . . $\boxed{x \longmapsto 1 - 2\mathrm{e}^{-2x/7+2}}$

28.2 c) . . . . $\boxed{x \longmapsto \left(\dfrac{6}{\sqrt{5}} + \pi\right)\mathrm{e}^{\sqrt{5}x} - \dfrac{6}{\sqrt{5}}}$

28.2 d) . . . . $\boxed{x \longmapsto \left(12 + \dfrac{2\mathrm{e}}{\pi}\right)\mathrm{e}^{\pi x - \pi^2} - \dfrac{2\mathrm{e}}{\pi}}$

28.3 a) . . . . $\boxed{x \longmapsto \mathrm{e}^{2x}}$

28.3 b) . . . . $\boxed{x \longmapsto \mathrm{e}^x}$

28.4 a) . . . . $\boxed{x \longmapsto 2\mathrm{e}^{2x} - \mathrm{e}^x}$

28.4 b) . . . . $\boxed{x \longmapsto (2 - 3\mathrm{i})\mathrm{e}^x + (3\mathrm{i} - 1)\mathrm{e}^{2x}}$

28.5 a) . . . . $\boxed{x \longmapsto \mathrm{e}^x}$

28.5 b) . . . . $\boxed{x \longmapsto 7\mathrm{e}^{-x} - 5\mathrm{e}^{-2x}}$

28.5 c) . . . . $\boxed{x \longmapsto \dfrac{4}{3}\mathrm{e}^x - \dfrac{1}{3}\mathrm{e}^{-2x}}$

28.5 d) . . . . $\boxed{x \longmapsto (2 - x)\mathrm{e}^x}$

28.5 e) . . . . $\boxed{x \longmapsto (2 - x)\mathrm{e}^{2-2x}}$

28.6 a) . . . . $\boxed{x \longmapsto \cos x + 2\sin x}$

28.6 b) . . . . $\boxed{x \longmapsto \mathrm{e}^{-x/2}\left(\cos\dfrac{\sqrt{3}x}{2} - \dfrac{1}{\sqrt{3}}\sin\dfrac{\sqrt{3}x}{2}\right)}$

28.6 c) . . . . $\boxed{x \longmapsto \mathrm{e}^{-x}\sin(x)}$

28.6 d) . . . . $\boxed{x \longmapsto \mathrm{e}^x\left(\dfrac{-1+\mathrm{i}}{2}\mathrm{e}^{2\mathrm{i}x} + \dfrac{1+\mathrm{i}}{2}\mathrm{e}^{-2\mathrm{i}x}\right)}$

Fiche nº 29. Séries numériques

Réponses

29.1 a) ... $\boxed{\text{Divergente}}$

29.1 b) .... $\boxed{2}$

29.1 c) .... $\boxed{2 + \sqrt{2}}$

29.1 d) .... $\boxed{\dfrac{1}{2 \times 3^9}}$

29.2 a) .... $\boxed{\text{e}}$

29.2 b) .... $\boxed{\text{e}^2 - 3}$

29.2 c) .... $\boxed{\text{e}^{\frac{1}{2}}}$

29.3 a) .... $\boxed{\dfrac{\pi^2}{6}}$

29.3 b) ... $\boxed{\text{Divergente}}$

29.3 c) ... $\boxed{\text{Divergente}}$

29.4 a) .... $\boxed{\dfrac{1}{12}}$

29.4 b) .... $\boxed{\dfrac{\text{e}}{\text{e} - 1}}$

29.4 c) .... $\boxed{\dfrac{1 - 7\text{i}}{350}}$

29.4 d) ... $\boxed{\dfrac{-2 - 5\sqrt{2}i}{54}}$

29.5 a) .... $\boxed{1}$

29.5 b) .... $\boxed{\dfrac{1}{4}}$

29.5 c) .... $\boxed{\ln(2)}$

29.5 d) .... $\boxed{\dfrac{\pi}{4}}$

29.6 a) ... $\boxed{\text{Divergente}}$

29.6 b) .... $\boxed{4}$

29.7 a) .... $\boxed{2}$

29.7 b) .... $\boxed{\dfrac{11}{4}}$

29.7 c) .... $\boxed{16}$

29.7 d) ..... $\boxed{\dfrac{2\text{e}^3}{(\text{e} - 1)^3}}$

Fiche nº 30. Produits scalaires

Réponses

30.1 a) .... $\boxed{4\ln(2) - 2}$

30.1 b) .... $\boxed{\dfrac{7}{12}}$

30.1 c) .... $\boxed{2\sin(1) + \cos(1) - 1}$

30.1 d) .... $\boxed{\dfrac{1}{2}(e^2 - 1)}$

30.2 a) .... $\boxed{11}$

30.2 b) .... $\boxed{10}$

30.2 c) .... $\boxed{0}$

30.3 a) .... $\boxed{\dfrac{1}{6\sqrt{5}}}$

30.3 b) .... $\boxed{\dfrac{1}{5\sqrt{3}}}$

30.3 c) .... $\boxed{\dfrac{1}{3}}$

30.4 a) .... $\boxed{\left(1, 2\sqrt{3}(X - \tfrac{1}{2}))\right)}$

30.4 b) .... $\boxed{\left(\sqrt{3}X, \sqrt{\dfrac{15}{43}}(4X^2 - 9X + 4)\right)}$

30.5 a) .... $\boxed{\dfrac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}}$

30.5 b) .... $\boxed{\dfrac{1}{5}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 4 \end{pmatrix}}$

30.5 c) .... $\boxed{\dfrac{1}{11}\begin{pmatrix} 9 & -6 & 2 \\ -6 & -7 & 6 \\ 2 & 6 & 9 \end{pmatrix}}$

Fiche nº 31. Autour du groupe symétrique

Réponses

31.1 a).. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 1 & 3 & 2 & 6 & 5 \end{pmatrix}$

31.1 b).. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 6 & 5 & 1 & 3 & 4 \end{pmatrix}$

31.1 c).. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 3 & 2 & 5 & 1 \end{pmatrix}$

31.1 d).. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 6 & 5 & 3 & 4 \end{pmatrix}$

31.1 e).. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 5 & 4 & 2 & 3 \end{pmatrix}$

31.1 f).. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 1 & 5 & 4 \end{pmatrix}$

31.2 a)................... $(a\ b)$

31.2 b)................ $(c\ b\ a)$

31.2 c)............ $(7\ 2\ 5\ 3\ 1)$

31.2 d)................ $(a\ c\ b)$

31.2 e).............. $(2\ 1\ 5\ 4)$

31.2 f)............ $(1\ 2\ 7\ 5\ 3)$

31.3 a) $(1\ 7\ 4)(2\ 6\ 8\ 10)(3\ 9\ 5)$

31.3 b).. $(1\ 3\ 10\ 6\ 4)(5\ 7)(8\ 9)$

31.3 c)....... $(1\ 7)(2\ 4\ 3\ 5\ 8)$

31.3 d)............. $(1\ 2)(3\ 4)$

31.3 e).......... $(1\ 4\ 6\ 2\ 3\ 5)$

31.4 a)........... $(1\ 4\ 2)(5\ 6)$

31.4 b)..................... id

31.4 c)............ $(1\ 2\ 6\ 5\ 3)$

31.4 d)....... $(1\ 6\ 7\ 4)(2\ 5\ 3)$

31.5 a).................... -1

31.5 b).................... 1

31.5 c).................... 1

31.5 d)................... -1

31.5 e)................... -1

31.5 f).................... 1

31.6 a)................... -1

31.6 b).................... 1

31.6 c).................... 1

31.6 d).................... 1

Fiche nº 32. Déterminants

Réponses

32.1 a) . . . . $\boxed{-2a^2}$

32.1 b) . . . . $\boxed{6}$

32.1 c) . . . . $\boxed{-5 + 6\text{i}}$

32.1 d) . . . . $\boxed{20}$

32.2 a) . . . . $\boxed{-2}$

32.2 b) . . . . $\boxed{9\ln(2)}$

32.2 c) . . . . $\boxed{227/336}$

32.2 d) . . . . $\boxed{3\,919}$

32.2 e) . . . . $\boxed{7\sqrt{2} + 13}$

32.3 a) . . . . $\boxed{0}$

32.3 b) . . . . $\boxed{-40}$

32.3 c) . . . . $\boxed{0}$

32.4 a) . . . . $\boxed{-4}$

32.4 b) . . . . $\boxed{6\text{i} - 12}$

32.4 c) . . . . $\boxed{4/375}$

32.5 a) . . . . $\boxed{x^3 + y^3 + z^3 - 3xyz}$

32.5 b) . . . . $\boxed{-6\ln^3(a)}$

32.5 c) . . . $\boxed{(y - x)(z - y)(z - x)}$

32.5 d) . . . . $\boxed{0}$

Fiche nº 33. Fonctions de deux variables

33.1 a) . . . . $\boxed{\{(x,y) \in \mathbb{R}^2 \mid x - 1 \leqslant y \leqslant x + 1\}}$

33.1 b) . . . . $\boxed{]0, +\infty[\times [0, +\infty[}$

33.1 c) . . . . $\boxed{\{(x,y) \in \mathbb{R}^2 \mid y \geqslant 0\} \backslash \{(0,0)\}}$

33.1 d) . . . . $\boxed{\varnothing}$

33.2 a) . . . . $\boxed{\dfrac{\partial f}{\partial x}(x,y) = 2x + y \text{ et } \dfrac{\partial f}{\partial y}(x,y) = 5y^4 + x}$

33.2 b) . . . . $\boxed{\dfrac{\partial f}{\partial x}(x,y) = 2y\cos(2xy - y) \text{ et } \dfrac{\partial f}{\partial y}(x,y) = (2x - 1)\cos(2xy - y)}$

33.2 c) . . . . $\boxed{\dfrac{\partial f}{\partial x}(x,y) = (2xy, 2x) \text{ et } \dfrac{\partial f}{\partial y}(x,y) = (x^2, -2y)}$

33.2 d) . . . . $\boxed{\dfrac{\partial f}{\partial x}(x,y) = \dfrac{2}{1 + (2x + y)^2} \text{ et } \dfrac{\partial f}{\partial y}(x,y) = \dfrac{1}{1 + (2x + y)^2}}$

33.3 a) . . . . $\boxed{\dfrac{\partial f}{\partial x}(x,y) = -\sin(x - y) \text{ et } \dfrac{\partial f}{\partial y}(x,y) = \sin(x - y)}$

33.3 b) . . . . $\boxed{\dfrac{\partial f}{\partial x}(x,y) = \cos(\mathrm{e}^{xy}) - xy\sin(\mathrm{e}^{xy})\,\mathrm{e}^{xy} \text{ et } \dfrac{\partial f}{\partial y}(x,y) = -x^2\sin(\mathrm{e}^{xy})\,\mathrm{e}^{xy}}$

33.3 c) . . . . $\boxed{\dfrac{\partial f}{\partial x}(x,y) = y\,x^{y-1} \text{ et } \dfrac{\partial f}{\partial y}(x,y) = x^y \ln x}$

33.3 d) . . . . $\boxed{\begin{aligned} &\frac{\partial f}{\partial x}(x,y) = \begin{cases} \dfrac{y^2(y^2 - x^2)}{(x^2 + y^2)^2} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{sinon} \end{cases} \\ &\text{et } \frac{\partial f}{\partial y}(x,y) = \begin{cases} \dfrac{2x^3 y}{(x^2 + y^2)^2} & \text{si } (x,y) \neq (0,0) \\ 0 & \text{sinon} \end{cases} \end{aligned}}$

33.4 a) . . . . $\boxed{\sin(2t)}$

33.4 b) . . . . $\boxed{\dfrac{2\mathrm{e}^{4t} + \mathrm{e}^{-2t}}{\sqrt{\mathrm{e}^{4t} - \mathrm{e}^{-2t}}}}$

33.4 c) . . . . $\boxed{-72\cos(4t) - 46\sin(4t)}$

33.5 a) . . . . $\boxed{\dfrac{\partial(f \circ \varphi)}{\partial u}(u,v) = \dfrac{1}{2}\dfrac{\partial f}{\partial x}\left(\dfrac{u+v}{2}, \dfrac{v-u}{2c}\right) - \dfrac{1}{2c}\dfrac{\partial f}{\partial y}\left(\dfrac{u+v}{2}, \dfrac{v-u}{2c}\right)}$

33.5 a) . . . . $\boxed{\dfrac{\partial(f \circ \varphi)}{\partial v}(u,v) = \dfrac{1}{2}\dfrac{\partial f}{\partial x}\left(\dfrac{u+v}{2}, \dfrac{v-u}{2c}\right) + \dfrac{1}{2c}\dfrac{\partial f}{\partial y}\left(\dfrac{u+v}{2}, \dfrac{v-u}{2c}\right)}$

33.5 b) .... $\boxed{\dfrac{\partial(f \circ \varphi)}{\partial r}(r,\theta) = \cos\theta \dfrac{\partial f}{\partial x}(r\cos\theta, r\sin\theta) + \sin\theta \dfrac{\partial f}{\partial y}(r\cos\theta, r\sin\theta)}$

33.5 b) .... $\boxed{\dfrac{\partial(f \circ \varphi)}{\partial \theta}(r,\theta) = -r\sin\theta \dfrac{\partial f}{\partial x}(r\cos\theta, r\sin\theta) + r\cos\theta \dfrac{\partial f}{\partial y}(r\cos\theta, r\sin\theta)}$