Random Number Generators

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January 7, 2026

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When implementing these algorithms in practice, how might we generate *quality* random numbers?

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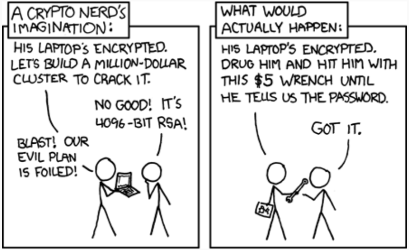
- *Yes- a system is only as strong as its weakest link!*
- If an adversary could recreate our process for generating random numbers, it stands to reason they can find our private keys
- This completely sidesteps the challenge of breaking a cryptographic algorithm!



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  - *"...none of these random number engines are cryptographically secure..."*
- How about Java's `Random` class?
  - *"Instances of java.util.Random are not cryptographically secure..."*

What exactly makes these standard generators inadequate for our purposes?

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Before discussing this, it's worthwhile to discuss exactly what we want in a random number generator. We will focus on the probability distribution U_N, the uniform distribution with support $\{0, 1, 2, \ldots, N-1\}$.

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- The distribution of values outputted by G should be "indistinguishable" from U_N
  - Determined by statistical tests (more on this later)
- If an adversary observes a sequence of values $\{x_i\}$, it should be challenging for them to predict future values *or* recover past values
  - (e.g. 0,1,2,0,1,2,... is uniformly distributed, but easy to predict)

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where the initial value X_0 is referred to as the seed. For example, if $c = 0$ and $m = 7$, we have the sequences:

$$(5, 3, 6, 5, 3 \dots) \qquad (X_0 = 5, a = 2)$$

$$(5, 1, 3, 2, 6, 4, 5 \dots) \qquad (X_0 = 5, a = 3)$$

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We would like a period as long as possible. By the Pigeonhole Principle, the period of an LCG is at most m. As we saw on the previous slide, the value of the period is sensitive to the parameters. We'll attempt to classify the best choices of parameters for maximizing period length.

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Note that if m is prime, we attain a maximal period of $m - 1$ when taking a to be a generator of $(\mathbb{Z}/m\mathbb{Z})^*$.

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$$\begin{aligned} X_{n+1} &= a \cdot \left(a^n \cdot X_0 + c \cdot \frac{a^n - 1}{a - 1} \right) + c \\ &= a^{n+1} \cdot X_0 + \left(\frac{a - 1}{a - 1} + \frac{a^{n+1} - a}{a - 1} \right) \cdot c \\ &= a^{n+1} \cdot X_0 + \left(\frac{a^{n+1} - 1}{a - 1} \right) \cdot c \end{aligned}$$

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Suppose $X_k = X_0$. Using the previous lemma:

$$a^k \cdot X_0 + c \cdot \frac{a^k - 1}{a - 1} \equiv X_0 \bmod m$$
$$(a^k - 1)\left(X_0 + \frac{c}{a-1}\right) \equiv 0 \bmod m$$

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Since $(\mathbb{Z}/m\mathbb{Z})^*$ is an integral domain, we must have $a^k \equiv 1 \bmod m$, giving us a period of $\mathrm{ord}(a)$. □

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An LCG has full period m if:

- *$gcd(m, c)$=1*
- *$a - 1$ is divisible by all prime factors of m*
- *If $m \equiv 0 \bmod 4$, then $a \equiv 1 \bmod 4$.*

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If we follow these criteria, we can create a sequence with a period as long as we like. The question is: how secure is this?

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$$\mathbb{P}(\text{fail to find } m) \leq \prod_{(u,v) \in D} \mathbb{P}(\gcd(u, v) \neq m) \leq \left(1 - \frac{6}{\pi^2}\right)^{\binom{n-3}{2}}$$

which vanishes as $n \to \infty$

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Our system is now *completely compromised*, as an attacker can both recover previous values and predict future ones!

Case Study: randq1

- Many programming languages use some form of LCG for generating random integers
- A popular choice is `randq1`, with the following parameters:
  - $m = 2^{32}, a = 1664525, c = 1013904223$
- Running the previous algorithm has a near perfect chance at recovering m, regardless of starting seed

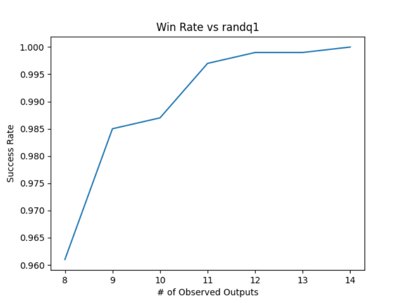

Figure 1: Average Win rate against `randq1`

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As the name suggests, the twister relies on Mersenne primes (i.e. primes of the form $2^n - 1$. The most common variant is MT19937, which generates a random unsigned 32 bit integer

MT19937, At a Glance

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    - Perform bitwise operations with “magic numbers”
    - These constants are experimentally chosen to pass statistical randomness tests

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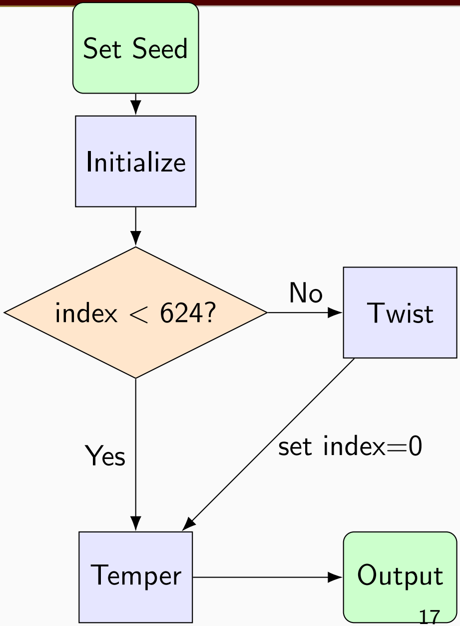
   - Once all 624 elements have been used, we must *twist* the state vector to get new numbers
   - Essentially amounts to a matrix multiplication in $\mathbb{F}_2$, along with more bit operations



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- Passes a large amount of statistical randomness tests

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- The temper and twist steps can be phrased as linear transformations in $\mathbb{F}_2^{32}$
  - *These matrices are invertible!*
- If our opponent calculates the inverse matrices associated with the transformations (which are the same for all instances of MT19937), they can recover the state vector

As with our LCG, we lack both forwards and backwards security, a massive problem for security

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If pure mathematical techniques can't cut it, where might we get a source of "true randomness"?

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The answer lies, not in mathematics, but in the physical world. Many processes exhibit random (or at the very least hard to predict) behavior. Examples include:

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- Electronic hardware
  - Clock jitter, OS interrupts, disk reads
- You
  - Mouse clicks, keyboard timing, cursor velocity

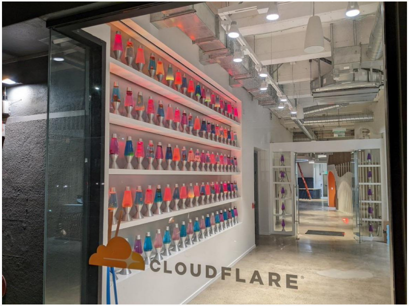

Figure 2: Cloudflare's "lamp wall" used in generating SSL keys

TRNGs

Generators that rely on these phenomena are referred to as true random number generators (TRNGS)

- All TRNGs rely on a quality source of entropy
- The key is that the underlying phenomenon should be impossible to predict, even if your adversary knows what the source is

We'll discuss how your computer (probably) generates random numbers for cryptographic applications.

Case Study: `/dev/random`

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3. Number Generation
    - Initialize a good PRNG with a portion of the entropy pool
    - Return the number(s) and decrease the amount of entropy

Statistical Testing

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- Generate 10^6 numbers in $\{0, 1, \ldots, 2^{32} - 1\}$
- Assume that G gives a uniform distribution as null hypothesis
- Experimental evidence fails to reject H_0 at standard $p < 0.05$
  - But very sensitive to starting seed

Generator	χ^2	p
randq	237.3	0.7791
MT19937	245.2	0.6584
urandom	245.5	0.6530

Table 1: χ^2 test with $\nu = 255$ degrees of freedom

The NIST Standard

Of course, there are more tests than just goodness-of-fit. NIST has a battery of statistical tests that are used in evaluating RNGs. Some notes about our three candidates:

- LCG (185/187)
  - Abject failure in the spectral test
  - Marsaglia showed that outputs of an LCG do not uniformly populate the unit cube, but instead lie on hyperplanes
- Both MT19937 and `/dev/random` pass all statistical tests
  - `/dev/random` has slightly more uniform p values

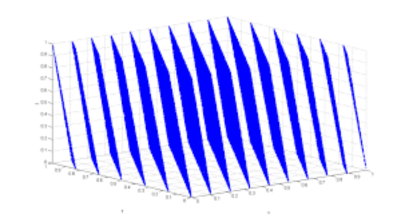

Figure 3: Spectral test on IBM's RANDU