Math 375: Homework 7&8

Due on Tue, 21 Oct 2014 12:30 pm

Prof. Korotkevich 12:30pm

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Homework 7

Problem 1

Section 2.5, Computer Problem 5(a), p. 116 of Sauer:

Use Gauss-Sidel Method to solve the following sparse system for $n = 100$ within 6 correct decimal places (forward error in the infinity norm):

$$\begin{bmatrix} 3 & -1 & & & \\ -1 & 3 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 3 & -1 \\ & & & -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ \\ \vdots \\ \\ x_n \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ \vdots \\ 1 \\ 2 \end{bmatrix}$$

Solution:

See: problem71.m [13] and gausssidel.m [14]. From Matlab: problem71 output

```
>> problem71
 n: 100, steps: 21, backerror: 9.559033e-07
```

Problem 2

Exercise 2.5.2(c), p. 116 of Sauer:

Rearrange the equations to form a strictly diagonally dominant system. Apply two steps of the Jacobi and Gauss-Sidel Methods from starting vector $[0, \ldots, 0]$

Solution:

$$\begin{aligned} u + 4v \quad &+0w = 5 \\ 0u + v \quad &+2w = 2 \\ 4u + 0v \quad &+3w = 0 \end{aligned}$$

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 2 \\ 4 & 0 & 3 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}$$

Which is not strictly diagonally dominant, to make it so, rearrange rows and columns:

$$\begin{bmatrix} 4 & 1 & 0 \\ 0 & 4 & 3 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} v \\ u \\ w \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix}$$

 Problem 2 continued on next page...

Using the Jacobi Method, let: $\left[\begin{bmatrix} v_0 \\ u_0 \\ w_0 \end{bmatrix}\right] = \left[\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right], x_k = \left[\begin{bmatrix} v_k \\ u_k \\ w_k \end{bmatrix}\right]$ and D, L, U be the diagonal, lower diagonal and upper diagonal matrices respectively and $k = 0, 1, 2$.

$$\begin{bmatrix} v_{k+1} \\ u_{k+1} \\ w_{k+1} \end{bmatrix} = D^{-1}(b - (L+U)x_k)$$

$$\begin{bmatrix} v_{k+1} \\ u_{k+1} \\ w_{k+1} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} - \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \right) x_k \right)$$

$$\begin{bmatrix} v_1 \\ u_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right) = \begin{bmatrix} \frac{5}{4} \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} v_2 \\ u_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{5}{4} \\ 0 \\ 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \\ \frac{5}{4} \end{bmatrix} \right) = \begin{bmatrix} \frac{5}{4} \\ -\frac{3}{4} \\ -\frac{1}{8} \end{bmatrix}$$

$$\begin{bmatrix} v_3 \\ u_3 \\ w_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 3 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{5}{4} \\ -\frac{3}{4} \\ -\frac{1}{8} \end{bmatrix} \right)$$

$$= \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \left(\begin{bmatrix} 5 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} -\frac{3}{4} \\ -\frac{3}{8} \\ \frac{5}{4} \end{bmatrix} \right) = \underline{\underline{\begin{bmatrix} \frac{23}{16} \\ \frac{3}{32} \\ -\frac{1}{8} \end{bmatrix}}}$$

Using Gauss-Sidel Method, with the same start as above, but with $x_{k+1} = D^{-1}(b - Ux_k - Lx_{k+1})$ and multiplying D^{-1} through:

$$\begin{bmatrix} v_{k+1} \\ u_{k+1} \\ w_{k+1} \end{bmatrix} = \begin{bmatrix} \frac{5-u_k-0w_k}{4} \\ \frac{0-0v_{k+1}-3w_k}{4} \\ \frac{2-v_{k+1}-0u_{k+1}}{2} \end{bmatrix} = \begin{bmatrix} \frac{5-0-0}{4} \\ \frac{0-0\cdot\frac{5}{4}-0\cdot 0}{4} \\ \frac{2-\frac{5}{4}-0\cdot 0}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{4} \\ 0 \\ \frac{3}{8} \end{bmatrix}$$

$$\begin{bmatrix} v_2 \\ u_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} \frac{5-0-0w_k}{4} \\ \frac{0-0v_{k+1}-\frac{9}{8}}{4} \\ \frac{2-\frac{5}{4}-0u_{k+1}}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{4} \\ -\frac{9}{32} \\ \frac{3}{8} \end{bmatrix}$$

$$\begin{bmatrix} v_3 \\ u_3 \\ w_3 \end{bmatrix} = \begin{bmatrix} \frac{5-(-\frac{9}{32})-0w_k}{4} \\ \frac{0-0v_{k+1}-\frac{9}{8}}{4} \\ \frac{2-\frac{169}{128}-0u_{k+1}}{2} \end{bmatrix} = \underline{\underline{\begin{bmatrix} \frac{169}{128} \\ \frac{9}{32} \\ \frac{87}{256} \end{bmatrix}}}$$

Problem 3

How many floating point operations does it take to compute

(1) the product $A\vec{x}$, where A is the sparse $n \times n$ matrix from Problem 1?

$\boxed{\text{Solution:}}$

Given that the sparse matrix only requires that the values that are not zero be visited this leaves the number of operations to be on the order of $O(n(n-1)(n-1))$ or $O(3n-2)$. For the 500×500 sparse matrix, this is then $\underline{\underline{\approx 1498}}$ floating point operations (assuming the multiply-add's are fused).

(2) the product $B\vec{x}$, where B is a full $n \times n$ matrix?

$\boxed{\text{Solution:}}$

Again, assuming the multiply-add's are fused, then the total number of floating point operations is on the order of $O(\frac{n(n+1)}{2})$, the number of operations is this $\underline{\underline{\approx 125250}}$.

Problem 4

This problem concerns polynomial interpolation based on expansion

$$p(x) = c_1 + c_2x + c_3x^2 + \cdots + c_nx^{n-1}$$

in the monomial basis in tandem with the Vandermonde approach.

(a) Assume that the data $\{(x_k, y_k) : k = 1, \ldots, n\}$ is given, where the nodes x_k are distinct. Derive the linear system $V\vec{c} = \vec{y}$ that uniquely determines the coefficients c_k.

$\boxed{\text{Solution:}}$

Matrix multiplication is defined as the following:

$$A\vec{c} = \vec{y}$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} a_{11}c_1 + a_{12}c_2 + \cdots + a_{1n}c_n \\ a_{21}c_1 + a_{22}c_2 + \cdots + a_{2n}c_n \\ \vdots \\ a_{m1}c_1 + a_{m2}c_2 + \cdots + a_{mn}c_n \end{bmatrix}$$

This form illustrates that the vector $\vec{c}$ can represent the coefficients associated with the monomial $p(x)$ by carefully choosing the matrix V, with the number of columns V_n the same as c_n rows of $\vec{c}$ and the number of rows in V equal to the number of rows in $\vec{y}$ or V_m the same as y_n:

$$V\vec{c} = \vec{y}$$

$$\begin{bmatrix} x^{k-1} & x^k & \cdots & x^{k+n-1} \\ x^{k-1} & x^k & \cdots & x^{k+n-1} \\ \vdots & \vdots & \ddots & \vdots \\ x^{k-1} & x^k & \cdots & x^{k+n-1} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} x^{k-1}c_1 + x^kc_2 + \cdots + x^{k+n-1}c_n \\ x^{k-1}c_1 + x^kc_2 + \cdots + x^{k+n-1}c_n \\ \vdots \\ x^{k-1}c_1 + x^kc_2 + \cdots + x^{k+n-1}c_n \end{bmatrix}$$

Problem 4 continued on next page…

By setting $k = 1$, the following linear system arises, matching the original monomial $p(x)$ for the linear system of results in $\vec{y}$

$$V\vec{c} = \vec{y}$$

$$\begin{bmatrix} x^0 & x^1 & \cdots & x^{n-1} \\ x^0 & x^1 & \cdots & x^{n-1} \\ \vdots & \vdots & \ddots & \vdots \\ x^0 & x^1 & \cdots & x^{n-1} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} x^0c_1 + x^1c_2 + \cdots + x^{n-1}c_n \\ x^0c_1 + x^1c_2 + \cdots + x^{n-1}c_n \\ \vdots \\ x^0c_1 + x^1c_2 + \cdots + x^{n-1}c_n \end{bmatrix}$$

For $\vec{y}$ to be a basis, requires that the system $V\vec{c}$ be linearly independent, that is:

$$c_1 + x^1c_2 + \cdots + x^n = 0$$

and

$$c_1 = c_2 = \cdots = c_n = 0$$

(with non-zero roots, if there is a zero root, this is simply the zero-vector, which by definition of vector spaces, must be included anyway).

This form of V is a *Vandermonde* matrix, and it's determinant is defined as:

$$\det(V) = \prod_{1 \leq i < j \leq n} (x_j - x_i)$$

and if $x_j \neq x_i$ then the determinant will be non-zero and it shows that V must be unique. Given these properties, then the matrix V must be invertible. Which will then uniquely define the vector $\vec{c}$ as:

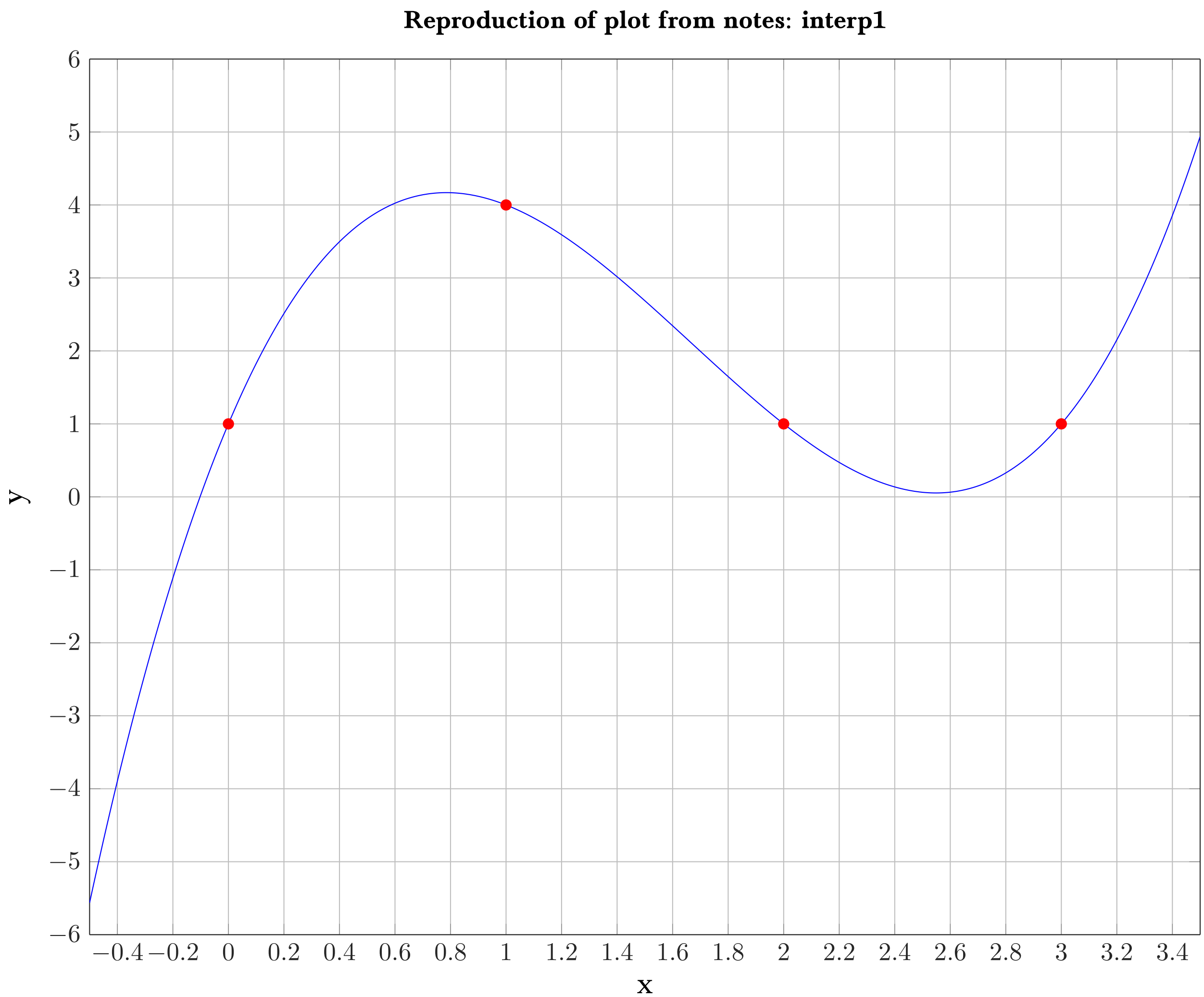
$$\vec{c} = V^{-1}\vec{y}$$

□

(b) Write a Matlab function (calling sequence `function c = interpvandmon(x,y)`) that returns the vector of expansion coefficients $\vec{c} = (c_1, \ldots, c_n)^T$ defining the interpolant, given input vectors $\vec{x} = (x_1, \ldots, x_n)^T$ and $\vec{y} = (y_1, \ldots, y_n)^T$. Use your function to reproduce the plot given in the notes for the data set $D_4 = \{(0,1); (1,4); (2,1); (3,1)\}$.

Solution:

See problem4b.m [15] and interpvandmon.m [15].



Homework 8

Problem 1

Excercises 3.1.1(c) and 3.1.2(c), p.149 of Sauer

3.1.1(c) Use Lagrange interpolation to find a polynomial that passes through the points (0,-2),(2,1),(4,4)

Solution:

With 3 points, the Lagrange interpolating polynomial is:

$$\begin{aligned}
P_2(x) &= y_1\frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + y_2\frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} + y_3\frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}\\
P(x) &= (-2)\frac{(x-2)(x-4)}{(0-2)(0-4)} + 1\frac{(x-0)(x-4)}{(2-0)(2-4)} + 4\frac{(x-0)(x-2)}{(4-0)(4-2)}\\
&= \frac{(-2)(x-2)(x-4)}{8} + \frac{x(x-4)}{-4} + \frac{4x(x-2)}{8}\\
&= \frac{-(x-2)(x-4)}{4} + \frac{-x(x-4)}{4} + \frac{2x(x-2)}{4}\\
&= \frac{-x^2+6x-8-x^2+4x+2x^2-4x}{4}\\
&= \frac{6x-8}{4} = \underline{\underline{\frac{3x-4}{2}}}
\end{aligned}$$

3.1.2(c) Use Newton's divided differences to find the interpolating polynomials of the points (0,-2),(2,1),(4,4) and verify against Lagrange interpolating polynomial

Solution:

$$\begin{aligned}
P(x) &= f[x_1] + f[x_1\ x_2](x-x_1) + f[x_1\ x_2\ x_3](x-x_1)(x-x_2)\\
f[x_1\ x_2] &= \frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{3}{2}\\
f[x_2\ x_3] &= \frac{f(x_3)-f(x_2)}{x_3-x_2} = \frac{3}{2}\\
f[x_1\ x_2\ x_3] &= \frac{\frac{3}{2}-\frac{3}{2}}{4-2} = 0\\
P(x) &= -2 + \frac{3}{2}(x-0) + 0\\
&= -2 + \frac{3x}{2} = \underline{\underline{\frac{3x-4}{2}}}\square
\end{aligned}$$

Problem 2

Use the `interpnewt` and `hornernewt(c,x,z)`

Solution:

Not entirely sure what to do here... TODO..

Problem 3

Use your routines from Problem 2 to interpolate the data sets $\{(x_j, y_j) : j = 1, \ldots, n\}$, where $y_j = f(x_j)$ with $f(x) = 1/(x^2 + 1)$, in each case, return 3 plots

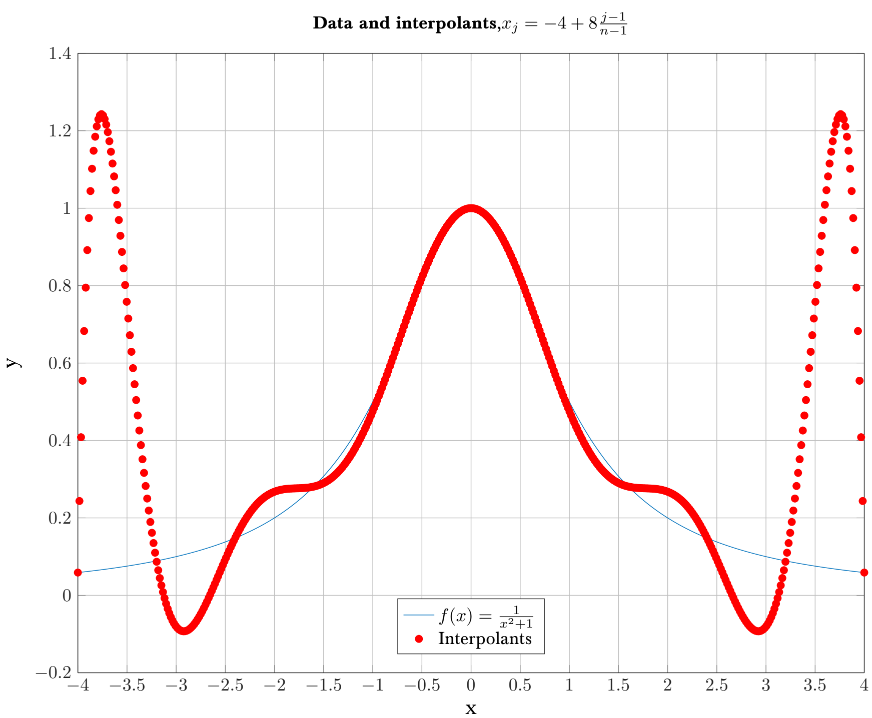
- Data and interpolant on [-4,4]. Interpolants should be plotted on a dense collection of points, say 500 equispaced. Include plot markers.
- The error $e(x) = f(x) - p(x)$ for $x \in [-4, 4]$
- The function $g(x) = \frac{1}{n!} \prod_{k=1}^{n} (x - x_k)$ for $x \in [-4, 4]$

Discuss results

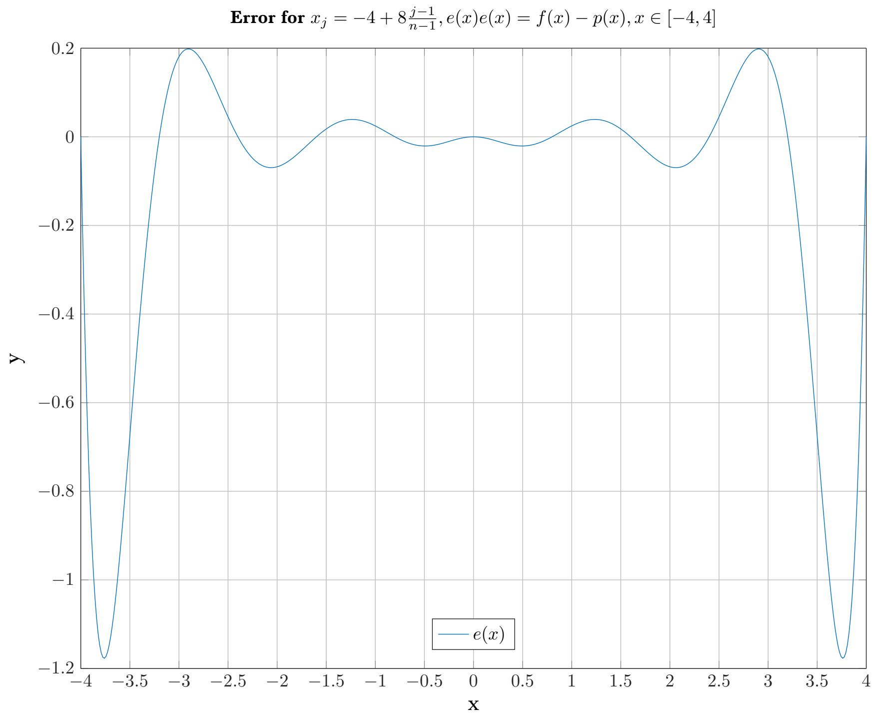
(a) $x_j = -4 + 8\frac{j-1}{n-1}, n = 11$

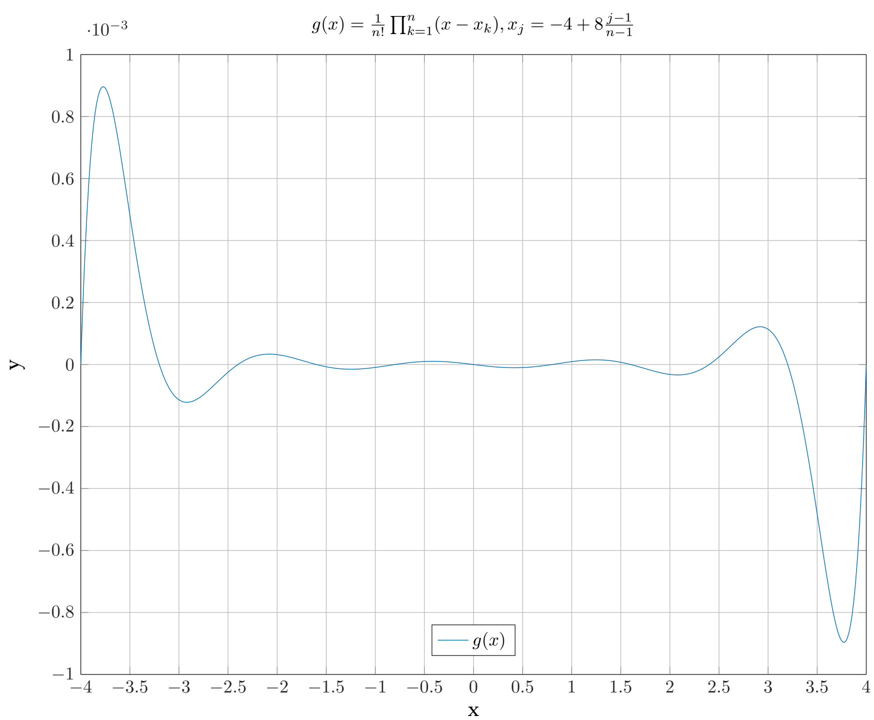
Solution:

See: problem83.m [15] and problem83gx.m [17].



Given the equations, this shows the effect of the starting positions of interpolation points as defined by the x_j function and the method having to create larger coefficients to get from the start point to the actual function. These start points are quite far from the actual function toward the edges and the polynomial generated is trying to force the final points through [-4,4].



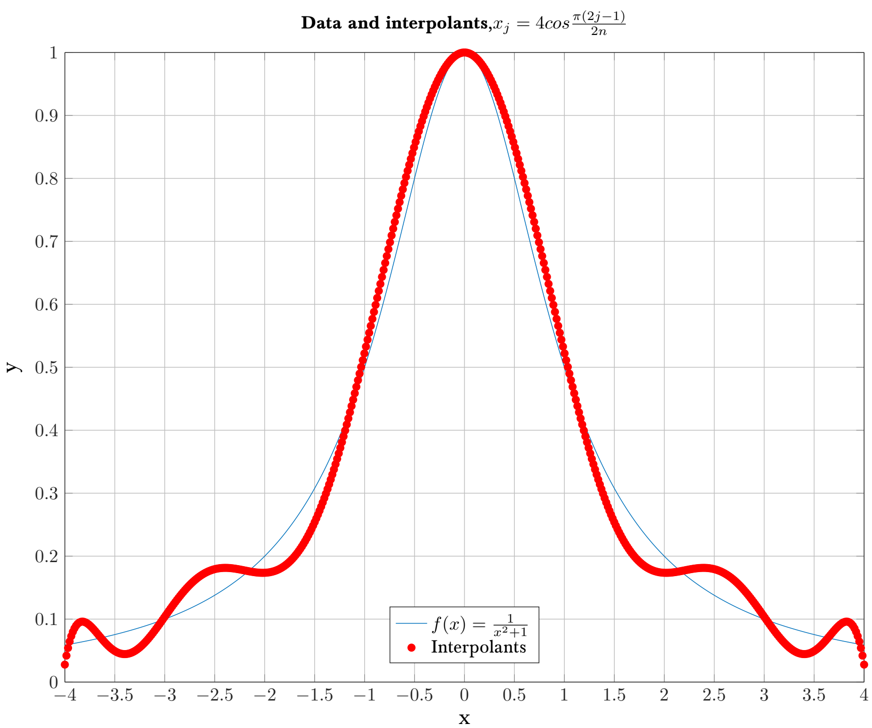


These plots more directly illustrate the point of the starting positions being far from the original function. The order of magnitude of the $e(x)$ and $g(x)$ is 10 times greater than the following plots.

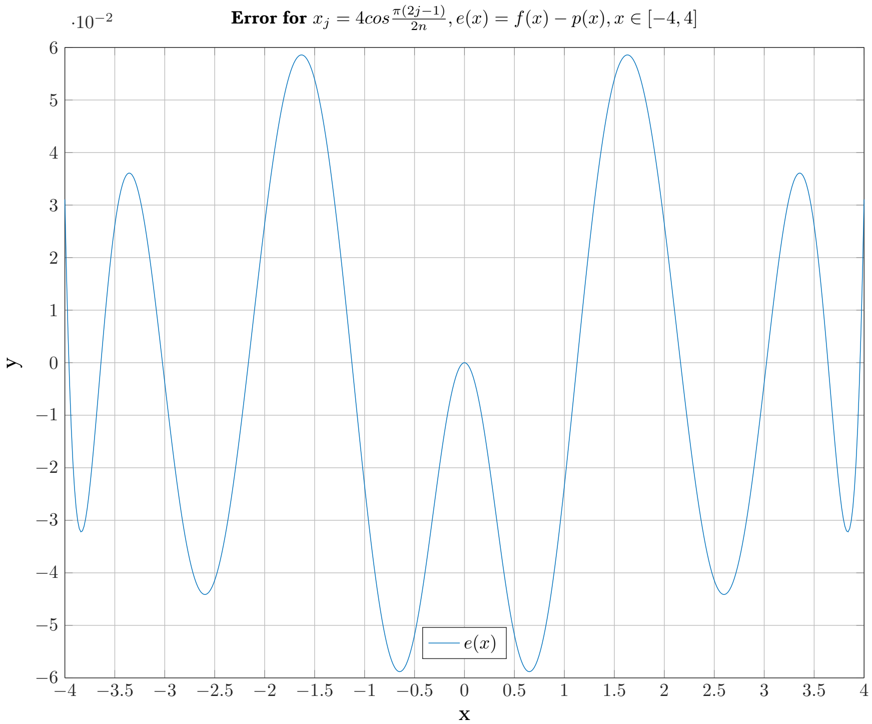
(b) $x_j = -4cos\frac{\pi(2j-1)}{2n}, n = 11$

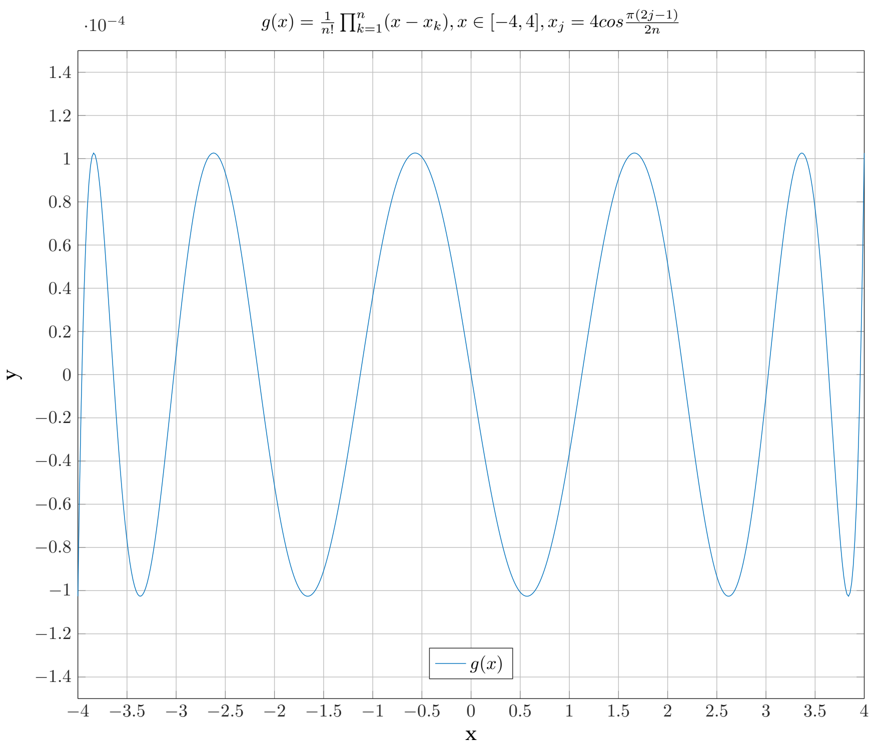
Solution:

See: problem83.m [15] and problem83gx.m [17].



In this case, the starting positions are nearer the actual function, keeping the coefficient generation more settled.





Again, notice that the order of magnitude is 10 times less than the previous plot. Also note that the sinusolidal nature of the starting positions helps with keeping the interpolation settling around the actual function behavior.

Problem 3 continued on next page. . .

Problem 4

Exercise 3.2.4, p.156 of Sauer: Consider $f(x) = \frac{1}{x+5}$ with interpolation nodes $x_j = 0, 2, 4, 6, 8, 10$. Find upper bound for the interpolation error at:

(a) $x = 1$

Solution:

Interpolation error: $\frac{f^{n+1}(\xi)}{(n+1)!} \prod_{j=0}^{n}(x - x_j)$, given 6 nodes, $n = 6$, $f^{(6)} = \frac{720}{(x+5)^7}$, then:

$$\begin{aligned}
e(x) &= \frac{720(x-0)(x-2)(x-4)(x-6)(x-8)(x-10)}{720(x+5)^7} \\
&= \frac{x(x-2)(x-4)(x-6)(x-8)(x-10)}{(x+5)^7} \\
&\text{let} x = 1, \\
e(1) &= \frac{(-1)(-3)(-5)(-7)(-9)}{6^7} \underline{\underline{\approx -0.003375771604938}}
\end{aligned}$$

(b) $x = 5$

Solution:

Following the work in part (a):

$$\begin{aligned}
&\text{let} x = 5, \\
e(5) &= \frac{5(3)(1)(-1)(-3)(-5)}{10^7} \underline{\underline{= -2.250 \times 10^{-05}}}
\end{aligned}$$

Appendix

problem71.m

```
%
% Homework 7
% MATH 375 - Korotkevich
%  problem 1
%
% Compute solution via Gauss-Sidel,
% prints number of interations taken to solve and
% backerror
%

clearvars;

n = 100;

% Setup matrices:
%  3 -1
% -1  3 -1
%  0 -1  3 -1 ...
%  ...
e = ones(n,1);
A = spdiags( [ -e e.*3 -e ], -1:1, n, n );
% b [ 2 1 ... 1 2 ]'
b = ones(n,1);
b(1) = 2;
b(n) = 2;
%%%

[x,s,b] = gausssidel( A, b, 1e-6 );

disp( sprintf(' n: %d, steps: %d, backerror: %1.7g', n, s, b) );
```

gausssidel.m

```
% Function guasssidel
% MATH375 - Prof Korotkevich
%  Homework 7
%
% A - sparse, tridiagonally dom matrix
% b - rhs solution vector
% t - tolerence
% x - lhs solution vector
% steps - number of interations taken
% backerr - back error in inf norm
function [ x,steps,backerr ] = gausssidel( A, b, t )

if ( length(b) ~= length(A) )
  error( 'dimension mismatch' );
end

n = length(b);

x = zeros(n,1);
nx = zeros(n,1);

L = tril(A,-1);
U = triu(A,1);
D = diag(A);
Dinv = 1.0 ./ D;

steps = 1;
maxc = 1000;
rerr = Inf;

while ( (rerr > t) && (steps < maxc) )

  for k=1:n
    Lxk = L(k,1:k-1)*nx(1:k-1);
    Uxk = U(k,k+1:end)*x(k+1:end);

    if ( k == 1 )
      Lxk = 0;
    elseif ( k == n )
      Uxk = 0;
    end

    nx(k) = Dinv(k)*( b(k) - Uxk - Lxk );
  end

  rerr = abs(norm(nx-x)/(norm(nx)));
  x = nx;
  steps = steps + 1;
end

if ( steps == maxc )
  error( sprintf('stopping after %d iterations',steps) );
end

% calculate error
errs = zeros(n,1);
cor = zeros(n,1);
%diffs = zeros(n,1);

for j=1:n
  for k=1:n
    cor(j) = cor(j) + A(j,k);
    errs(j) = errs(j) + A(j,k)*x(j);
  end
  diffs(j) = abs(errs(j) - cor(j));
end

backerr = max(diffs,[],2);


end
```

 [Appendix] continued on next page...

problem4b.m

```
%
% Homework 7
% MATH 375 - Korotkevich
% Colby Gutierrez-Kraybill
%  problem 4b
%
% Plot usage of interpvandmon
%

clearvars;
clf;
hold off;

D4 = [ [0 1]; [1 4]; [2 1]; [3 1] ];

x = D4(:,1);
y = D4(:,2);
c = interpvandmon( x, y );
xr = [-0.5:0.01:3.5];

h = plot(xr, polyval(fliplr(c'), xr), '-b', x, y, 'or' );
grid on;
set(h(2),'MarkerEdgeColor','none','MarkerFaceColor','r');
xlabel('x');
ylabel('y');
title('Reproduction of plot from notes: interp1');
cleanfigure;
matlab2tikz('problem4bplot.tex','showInfo',...
  false, 'extraAxisOptions',['xlabel style={font={\large}},' ...
  'ylabel style={font={\large}}']);
```

interpvandmon.m

```
%
% Homework 7
% MATH 375 - Korotkevich
% Colby Gutierrez-Kraybill
%  problem 4b
%
% Compute coefficients from Vandermonde matrix
% x - vector of length n with values for [x_1 ... x_n]^T (roots)
% y - vector of length n with values for [y_1 ... y_n]^T
% c - founding coefficients of [c_1 ... c_n]^T
function [ c ] = interpvandmon( x, y )

polypow = repmat([0:1:length(x)-1],length(x),1);
A = repmat( x, 1, length(x) ).^polypow;
c = A \ y;

end
```

problem83.m

```
%
% Homework 8
% MATH 375 - Korotkevich
% Colby Gutierrez-Kraybill
%  problem 3
%

clearvars;
clf;

fx = @(x) 1./((x.^2)+1);

n = 11;
jr = [1:n];

```

[Appendix] continued on next page...

```
xf1 = @(j,n) -4+(8*((j-1)/(n-1)));
xj1 = xf1(jr,n);
yj1 = fx(xj1);

xf2 = @(j,n) 4*cos((pi*((2*j)-1))/(2*n));
xj2 = xf2(jr,n);
yj2 = fx(xj2);
pr = linspace(-4,4,500);
fxpr = fx(pr);

c1 = interpnewt( xj1, yj1 );
c2 = interpnewt( xj2, yj2 );

p1 = hornernewt( c1, xj1, pr );
p2 = hornernewt( c2, xj2, pr );

% 1) Data and Interp
f=1;
figure(f);
plot(pr, fxpr, '-', pr, p1, 'r.' );
grid on;
l(f) = legend('$f(x)=\frac{1}{x^2+1}$','Interpolants','Location','South');
set(l(f),'Interpreter','Latex');
title( 'Data and interpolants,$x_j=-4+8\frac{j-1}{n-1}$',...
  'Interpreter', 'Latex');
xlabel('x');
ylabel('y');
cleanfigure;
matlab2tikz('p31f.tex','showInfo',...
  false, 'extraAxisOptions',['xlabel style={font={\large}},' ...
  'ylabel style={font={\large}}']);

% 1) Error
f=f+1;
figure(f);
plot( pr, (fxpr - p1) );
grid on;
l(f) = legend('$e(x)$','Location','South');
set(l(f),'Interpreter','Latex');
title( 'Error for $x_j=-4+8\frac{j-1}{n-1},e(x)e(x)=f(x)-p(x), x\in [-4,4]$',...
  'Interpreter', 'Latex' );
xlabel('x');
ylabel('y');
cleanfigure;
matlab2tikz('p31e.tex','showInfo',...
  false, 'extraAxisOptions',['xlabel style={font={\large}},' ...
  'ylabel style={font={\large}}']);

% g(x)
f=f+1;
figure(f);
plot( pr, problem83gx(pr,xj1) );
grid on;
l(f) = legend('$g(x)$',...
  'Location','South');
set(l(f),'Interpreter','Latex');
title( '$g(x)=\frac{1}{n!}\prod_{k=1}^{n}(x-x_k),x_j=-4+8\frac{j-1}{n-1}$', ...
  'Interpreter', 'Latex' );
xlabel('x');
ylabel('y');
cleanfigure;
matlab2tikz('p31g.tex','showInfo',...
  false, 'extraAxisOptions',['xlabel style={font={\large}},' ...
  'ylabel style={font={\large}}']);

% 2) Data and Interp
f=f+1;
figure(f);
plot(pr, fxpr, '-', pr, p2, 'r.' );
grid on;
l(f) = legend('$f(x)=\frac{1}{x^2+1}$','Interpolants','Location','South');
set(l(f),'Interpreter','Latex');
title( 'Data and interpolants,$x_j=4cos\frac{\pi(2j-1)}{2n}$',...
```

 [Appendix] continued on next page...

```
  'Interpreter', 'Latex' );
xlabel('x');
ylabel('y');
cleanfigure;
matlab2tikz('p32f.tex','showInfo',...
  false, 'extraAxisOptions',['xlabel style={font={\large}},' ...
  'ylabel style={font={\large}}']);

% 2) error
f=f+1;
figure(f);
plot( pr, (fxpr - p2) );
grid on;
l(f) = legend('$e(x)$','Location','South');
set(l(f),'Interpreter','Latex');
title( 'Error for $x_j=4cos\frac{\pi(2j-1)}{2n},e(x)=f(x)-p(x), x\in [-4,4]$', ...
  'Interpreter', 'Latex' );
xlabel('x');
ylabel('y');
matlab2tikz('p32e.tex','showInfo',...
  false, 'extraAxisOptions',['xlabel style={font={\large}},' ...
  'ylabel style={font={\large}}']);

% 2) g(x)
f=f+1;
figure(f);
plot( pr, problem83gx(pr,xj2) );
grid on;
l(f) = legend('$g(x)$','Location','South');
set(l(f),'Interpreter','Latex');
title( '$g(x)=\frac{1}{n!}\prod_{k=1}^{n}(x-x_k), x\in [-4,4],x_j=4cos\frac{\pi(2j-1)}{2n}$', ...
  'Interpreter', 'Latex' );
xlabel('x');
ylabel('y');
matlab2tikz('p32g.tex','showInfo',...
  false, 'extraAxisOptions',['xlabel style={font={\large}},' ...
  'ylabel style={font={\large}}']);
```

problem83gx.m

```
%
% Homework 8
% MATH 375 - Korotkevich
% Colby Gutierrez-Kraybill
%  problem 3
%
function [ g ] = problem83gx( x, xj )

n = length(xj);
nfinv = (1/factorial(n));
px = 1;

for k=1:n
  px = px.*(x - xj(k));
end

g = nfinv*px;

end
```