§6.6 Derivatives of Inverse Trigonometric Functions

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\csc^{-1} x\right) = -\frac{1}{x\sqrt{x^2+1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos^{-1} x\right) = \frac{1}{-\sqrt{1-x^2}} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\sec^{-1} x\right) = -\frac{1}{x\sqrt{x^2-1}} \tag{1}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1} x\right) = \frac{1}{1+x^2} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\cot^{-1} x\right) = -\frac{1}{1+x^2}$$

§6.6 Integrals of Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1}\,\mathrm{d}x = \tan^{-1} x + C \tag{2}$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,\mathrm{d}x = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

§6.7 Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \operatorname{sech} x = \frac{1}{\cosh x} \tag{3}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \quad \coth x = \frac{\cosh x}{\sinh x}$$

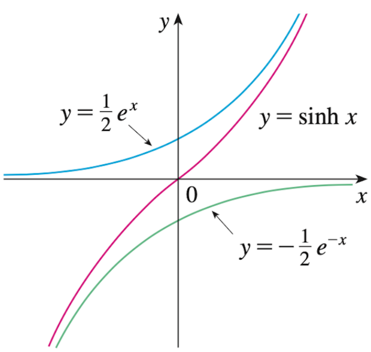

FIGURE 1
$y = \sinh x = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

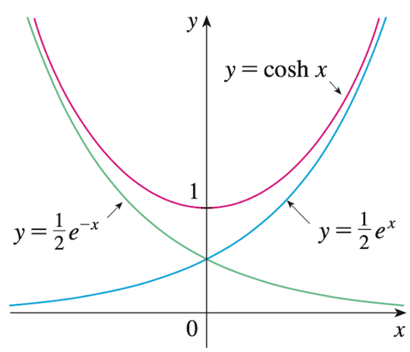

FIGURE 2
$y = \cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

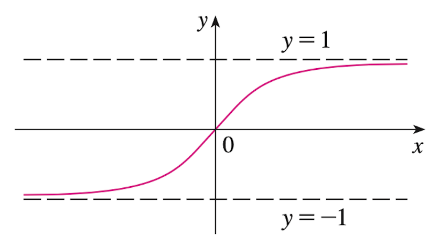

FIGURE 3
$y = \tanh x$

§6.7 Hyperbolic Function Identities

$$\sinh(-x) = -\sinh(x) \qquad \cosh(-x) = \cosh(x)$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x \tag{4}$$

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y \tag{5}$$

§6.7 Hyperbolic Function Derivatives

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sinh x) = \cosh x \quad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = \sinh x \quad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x \tag{6}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}(\tanh x) = \sec^2 x \quad \frac{\mathrm{d}}{\mathrm{d}x}(\coth x) = -\operatorname{csch}^2 x$$

§6.7 Inverse Hyperbolic Functions

$$\begin{array}{rl} & y = \sinh^{-1} x \iff \sinh y = x \\ \text{for all } y \geq 0 & y = \cosh^{-1} x \iff \cosh y = x \\ & y = \tanh^{-1} x \iff \tanh y = x \end{array} \quad \begin{cases} \sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right) & x \in \mathbb{R} \\ \cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) & x \geq 1 \\ \tanh^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{1-x}\right) & -1 \leq x < 1 \end{cases} \tag{7}$$

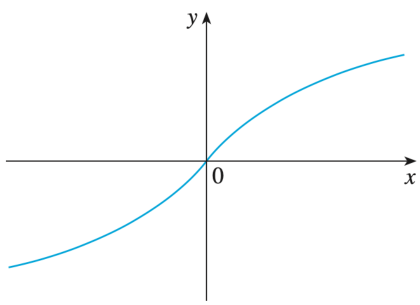

FIGURE 8 $y = \sinh^{-1}x$
domain $= \mathbb{R}$ range $= \mathbb{R}$

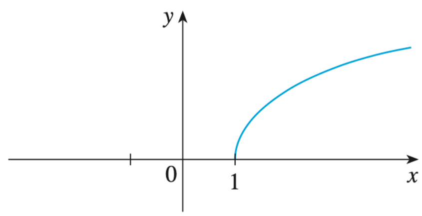

FIGURE 9 $y = \cosh^{-1}x$
domain $= [1, \infty)$ range $= [0, \infty)$

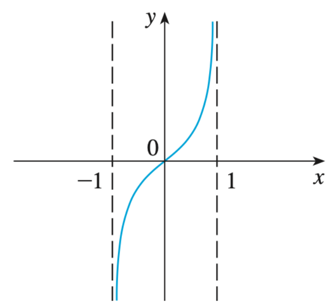

FIGURE 10 $y = \tanh^{-1}x$
domain $= (-1, 1)$ range $= \mathbb{R}$

§6.7 Inverse Hyperbolic Function Derivatives

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\operatorname{csch}^{-1} x\right) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\operatorname{sech}^{-1} x\right) = -\frac{1}{x\sqrt{1-x^2}} \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\coth^{-1} x\right) = \frac{1}{1-x^2}$$