

## §6.6 Derivatives of Inverse Trigonometric Functions

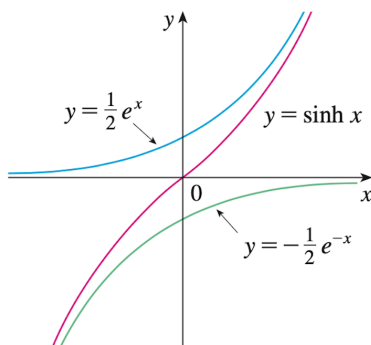
$$\begin{aligned}\frac{d}{dx}(\sin^{-1} x) &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx}(\csc^{-1} x) &= -\frac{1}{x\sqrt{x^2+1}} \\ \frac{d}{dx}(\cos^{-1} x) &= \frac{1}{-\sqrt{1-x^2}} & \frac{d}{dx}(\sec^{-1} x) &= -\frac{1}{x\sqrt{x^2-1}} \\ \frac{d}{dx}(\tan^{-1} x) &= \frac{1}{1+x^2} & \frac{d}{dx}(\cot^{-1} x) &= -\frac{1}{1+x^2}\end{aligned}\quad (1)$$

## §6.6 Integrals of Inverse Trigonometric Functions

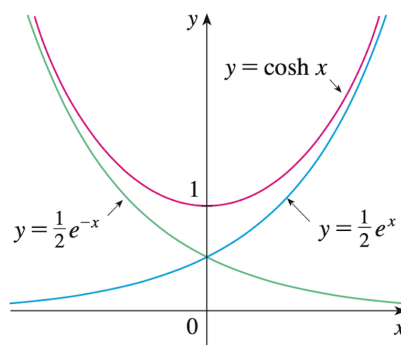
$$\begin{aligned}\int \frac{1}{\sqrt{1-x^2}} dx &= \sin^{-1} x + C \\ \int \frac{1}{x^2+1} dx &= \tan^{-1} x + C \\ \int \frac{1}{\sqrt{x^2+a^2}} dx &= \frac{1}{a} \tan^{-1} \left( \frac{x}{a} \right) + C\end{aligned}\quad (2)$$

## §6.7 Hyperbolic Functions

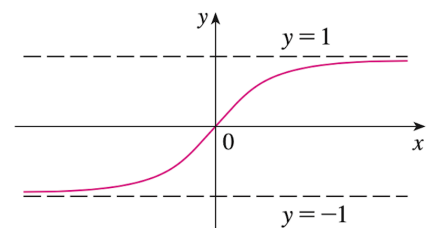
$$\begin{aligned}\sinh x &= \frac{e^x - e^{-x}}{2} & \operatorname{csch} x &= \frac{1}{\sinh x} \\ \cosh x &= \frac{e^x + e^{-x}}{2} & \operatorname{sech} x &= \frac{1}{\cosh x} \\ \tanh x &= \frac{\sinh x}{\cosh x} & \coth x &= \frac{\cosh x}{\sinh x}\end{aligned}\quad (3)$$



**FIGURE 1**  
 $y = \sinh x = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$



**FIGURE 2**  
 $y = \cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$



**FIGURE 3**  
 $y = \tanh x$

## §6.7 Hyperbolic Function Identities

$$\begin{aligned}\sinh(-x) &= -\sinh(x) & \cosh(-x) &= \cosh(x) \\ \cosh^2 x - \sinh^2 x &= 1 & 1 - \tanh^2 x &= \operatorname{sech}^2 x \\ \sinh(x+y) &= \sinh x \cosh y + \cosh x \sinh y \\ \cosh(x+y) &= \cosh x \cosh y + \sinh x \sinh y\end{aligned}\quad (4)$$

$$(5)$$

## §6.7 Hyperbolic Function Derivatives

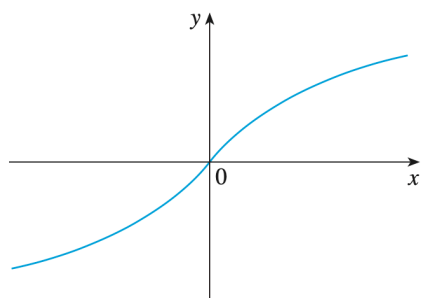
$$\frac{d}{dx} (\sinh x) = \cosh x \quad \frac{d}{dx} (\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$\frac{d}{dx} (\cosh x) = \sinh x \quad \frac{d}{dx} (\operatorname{sech} x) = -\operatorname{sech} x \tanh x \quad (6)$$

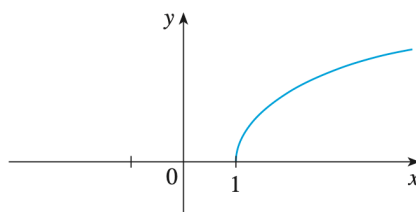
$$\frac{d}{dx} (\tanh x) = \operatorname{sech}^2 x \quad \frac{d}{dx} (\coth x) = -\operatorname{csch}^2 x$$

## §6.7 Inverse Hyperbolic Functions

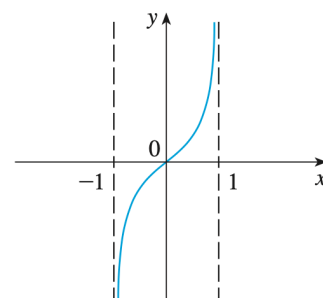
$$\begin{array}{l} y = \sinh^{-1} x \iff \sinh y = x \\ \text{for all } y \geq 0 \quad y = \cosh^{-1} x \iff \cosh y = x \\ y = \tanh^{-1} x \iff \tanh y = x \end{array} \quad \left\{ \begin{array}{l} \sinh^{-1} x = \ln(x + \sqrt{x^2 + 1}) \quad x \in \mathbb{R} \\ \cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) \quad x \geq 1 \\ \tanh^{-1} x = \frac{1}{2} \ln\left(\frac{x+1}{1-x}\right) \quad -1 < x < 1 \end{array} \right. \quad (7)$$



**FIGURE 8**  $y = \sinh^{-1} x$   
domain =  $\mathbb{R}$  range =  $\mathbb{R}$



**FIGURE 9**  $y = \cosh^{-1} x$   
domain =  $[1, \infty)$  range =  $[0, \infty)$



**FIGURE 10**  $y = \tanh^{-1} x$   
domain =  $(-1, 1)$  range =  $\mathbb{R}$

## §6.7 Inverse Hyperbolic Function Derivatives

$$\frac{d}{dx} (\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \quad \frac{d}{dx} (\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dx} (\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}} \quad \frac{d}{dx} (\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}} \quad (8)$$

$$\frac{d}{dx} (\tanh^{-1} x) = \frac{1}{1-x^2} \quad \frac{d}{dx} (\coth^{-1} x) = \frac{1}{1-x^2}$$