§6.6 Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx} (\sin^{-1} x) = \frac{1}{\sqrt{1 - x^2}} \frac{d}{dx} (\csc^{-1} x) = -\frac{1}{x\sqrt{x^2 + 1}}$$

$$\frac{d}{dx}(\cos^{-1}x) = \frac{1}{-\sqrt{1-x^2}} \frac{d}{dx}(\sec^{-1}x) = -\frac{1}{x\sqrt{x^2-1}}$$
(1)

$$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2} \frac{d}{dx}(\cot^{-1}x) = -\frac{1}{1+x^2}$$

§6.6 Integrals of Inverse Trigonometric Functions

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$$

$$\int \frac{1}{x^2+1} dx = \tan^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \frac{1}{a} \tan^{-1} \left(\frac{x}{a}\right) + C$$
(2)

§6.7 Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \operatorname{sech} x = \frac{1}{\cosh x} \tag{3}$$

$$tanh x = \frac{\sinh x}{\cosh x} \quad \coth x = \frac{\cosh x}{\sinh x}$$

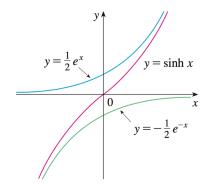


FIGURE 1 $y = \sinh x = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

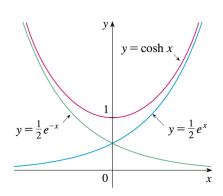


FIGURE 2 $y = \cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

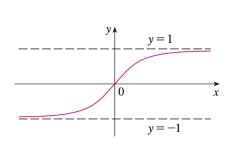


FIGURE 3 $y = \tanh x$

§6.7 Hyperbolic Function Identities

$$\sinh(-x) = -\sinh(x) \qquad \cosh(-x) = \cosh(x)$$

$$\cosh^2 x - \sinh^2 x = 1 \qquad 1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$$
(4)

$$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y \tag{5}$$

§6.7 Hyperbolic Function Derivatives

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sinh x\right) = \cosh x \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\operatorname{csch}x\right) = -\operatorname{csch}x\operatorname{coth}x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\cosh x\right) = \sinh x \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\operatorname{sech}x\right) = -\operatorname{sech}x\tanh x$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh x\right) = \sec^2 x \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\coth x\right) = -\operatorname{csch}^2 x$$

$$(6)$$

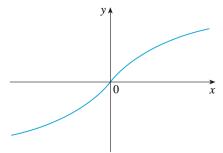
§6.7 Inverse Hyperbolic Functions

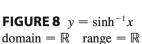
$$y = \sinh^{-1} x \iff \sinh y = x$$
for all $y \ge 0$ $y = \cosh^{-1} x \iff \cosh y = x$

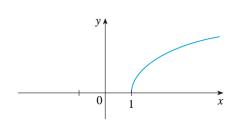
$$y = \tanh^{-1} x \iff \tanh y = x$$

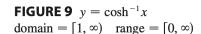
$$\begin{cases} \sinh^{-1} x = \ln \left(x + \sqrt{x^2 + 1} \right) & x \in \mathbb{R} \\ \cosh^{-1} x = \ln \left(x + \sqrt{x^2 - 1} \right) & x \ge 1 \\ \tanh^{-1} x = \frac{1}{2} \ln \left(\frac{x + 1}{1 - x} \right) & -1 \le x < 1 \end{cases}$$

$$(7)$$









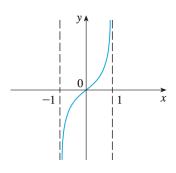


FIGURE 10 $y = \tanh^{-1} x$ domain = (-1, 1) range = \mathbb{R}

§6.7 Inverse Hyperbolic Function Derivatives

$$\frac{d}{dx} \left(\sinh^{-1} x \right) = \frac{1}{\sqrt{1+x^2}} \quad \frac{d}{dx} \left(\operatorname{csch}^{-1} x \right) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$\frac{d}{dx} \left(\cosh^{-1} x \right) = \frac{1}{\sqrt{x^2-1}} \quad \frac{d}{dx} \left(\operatorname{sech}^{-1} x \right) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx} \left(\tanh^{-1} x \right) = \frac{1}{1-x^2} \quad \frac{d}{dx} \left(\coth^{-1} x \right) = \frac{1}{1-x^2}$$
(8)