§6.6 Derivatives of Inverse Trigonometric Functions

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1} x\right) &= \frac{1}{\sqrt{1-x^2}} & \frac{\mathrm{d}}{\mathrm{d}x}\left(\csc^{-1} x\right) &= -\frac{1}{x\sqrt{x^2+1}} \\
\frac{\mathrm{d}}{\mathrm{d}x}\left(\cos^{-1} x\right) &= \frac{1}{-\sqrt{1-x^2}} & \frac{\mathrm{d}}{\mathrm{d}x}\left(\sec^{-1} x\right) &= -\frac{1}{x\sqrt{x^2-1}} \\
\frac{\mathrm{d}}{\mathrm{d}x}\left(\tan^{-1} x\right) &= \frac{1}{1+x^2} & \frac{\mathrm{d}}{\mathrm{d}x}\left(\cot^{-1} x\right) &= -\frac{1}{1+x^2}
\end{aligned} \tag{1}$$

§6.6 Integrals of Inverse Trigonometric Functions

$$\begin{gathered}
\int \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \sin^{-1} x + C \\
\int \frac{1}{x^2+1}\mathrm{d}x = \tan^{-1} x + C \\
\int \frac{1}{\sqrt{x^2+a^2}}\mathrm{d}x = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C
\end{gathered} \tag{2}$$

§6.7 Hyperbolic Functions

$$\begin{aligned}
\sinh x &= \frac{e^x - e^{-x}}{2} & \operatorname{csch} x &= \frac{1}{\sinh x} \\
\cosh x &= \frac{e^x + e^{-x}}{2} & \operatorname{sech} x &= \frac{1}{\cosh x} \\
\tanh x &= \frac{\sinh x}{\cosh x} & \coth x &= \frac{\cosh x}{\sinh x}
\end{aligned} \tag{3}$$

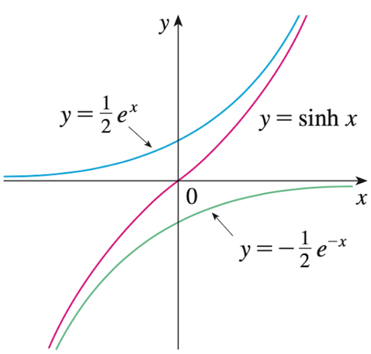

FIGURE 1
$y = \sinh x = \frac{1}{2}e^x - \frac{1}{2}e^{-x}$

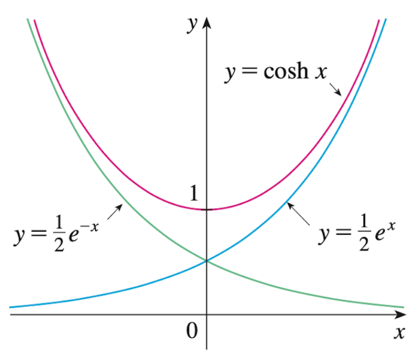

FIGURE 2
$y = \cosh x = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$

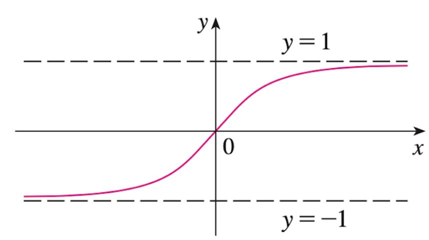

FIGURE 3
$y = \tanh x$

§6.7 Hyperbolic Function Identities

$$\begin{aligned}
\sinh(-x) &= -\sinh(x) & \cosh(-x) &= \cosh(x) \\
\cosh^2 x - \sinh^2 x &= 1 & 1 - \tanh^2 x &= \operatorname{sech}^2 x
\end{aligned} \tag{4}$$

$$\begin{gathered}
\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y \\
\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y
\end{gathered} \tag{5}$$

§6.7 Hyperbolic Function Derivatives

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}x}(\sinh x) = \cosh x \quad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{csch} x) = -\operatorname{csch} x \coth x \\
&\frac{\mathrm{d}}{\mathrm{d}x}(\cosh x) = \sinh x \quad \frac{\mathrm{d}}{\mathrm{d}x}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x \\
&\frac{\mathrm{d}}{\mathrm{d}x}(\tanh x) = \sec^2 x \quad \frac{\mathrm{d}}{\mathrm{d}x}(\coth x) = -\operatorname{csch}^2 x
\end{aligned}
\tag{6}
$$

§6.7 Inverse Hyperbolic Functions

$$
\begin{array}{rl}
& y = \sinh^{-1} x \iff \sinh y = x \\
\text{for all } y \geq 0 & y = \cosh^{-1} x \iff \cosh y = x \\
& y = \tanh^{-1} x \iff \tanh y = x
\end{array}
\quad
\begin{cases}
\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right) & x \in \mathbb{R} \\
\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right) & x \geq 1 \\
\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{x+1}{1-x}\right) & -1 \leq x < 1
\end{cases}
\tag{7}
$$

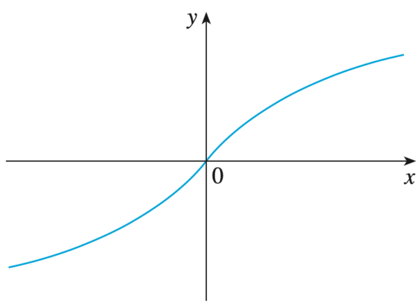

FIGURE 8 $y = \sinh^{-1} x$
domain $= \mathbb{R}$ range $= \mathbb{R}$

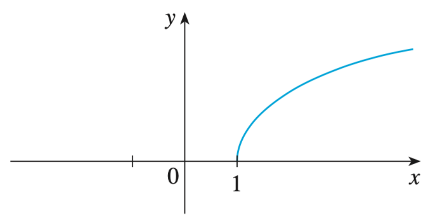

FIGURE 9 $y = \cosh^{-1} x$
domain $= [1, \infty)$ range $= [0, \infty)$

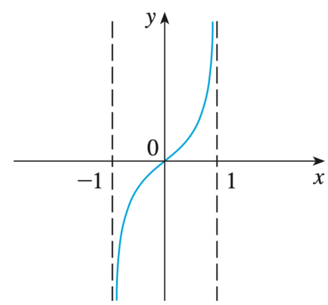

FIGURE 10 $y = \tanh^{-1} x$
domain $= (-1, 1)$ range $= \mathbb{R}$

§6.7 Inverse Hyperbolic Function Derivatives

$$
\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}x}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{1+x^2}} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\operatorname{csch}^{-1} x\right) = -\frac{1}{|x|\sqrt{x^2+1}} \\
&\frac{\mathrm{d}}{\mathrm{d}x}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\operatorname{sech}^{-1} x\right) = -\frac{1}{x\sqrt{1-x^2}} \\
&\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2} \quad \frac{\mathrm{d}}{\mathrm{d}x}\left(\coth^{-1} x\right) = \frac{1}{1-x^2}
\end{aligned}
\tag{8}
$$