Colbyn's Exam #1 Corrections

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Question #1

Question: Consider the angle θ in with measure $-495°$.

The problem with this question is that I was trying to be smart by simplifying $-495°$ to $225°$, so thereafter all of my subsequent work was using an angle co-terminal to $-495°$. In this context, my mistake was equating co-terminal angles as being the same, but this only applies to the output of periodic functions where co-terminal angles **map to the same value**, but the arguments themselves represent different measures.

Question #1 (A)

Using the following relation:

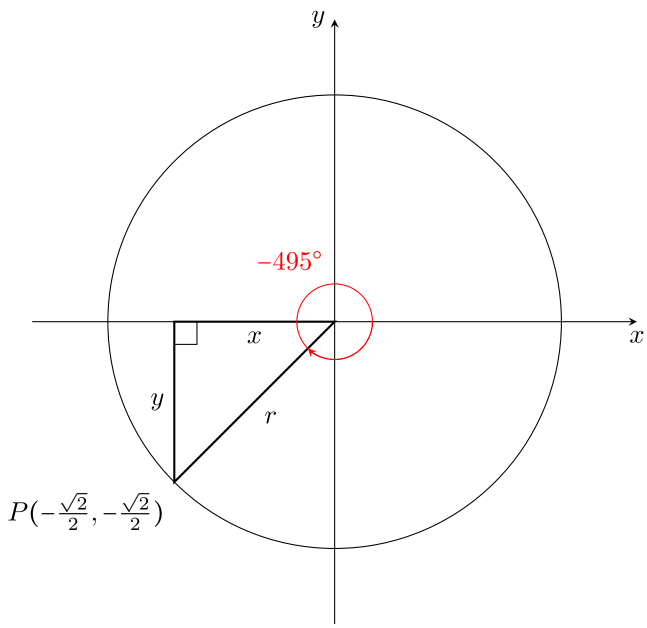
$$x° = \frac{x}{360}\tau \text{ rad} \tag{1}$$

I can express $-495°$ in terms of turns (or τ) like so:

$$\begin{aligned} -495° &= \frac{-495}{360}\tau \text{ rad} \\ &= -\frac{11}{8}\tau \text{ rad} \\ &= -1\frac{3}{8}\,\tau \end{aligned} \tag{2}$$

Since this angle is expressed in terms of turns, we have a intuitive idea about how the graph the given angle. I.e. because $-\frac{3}{8}$ can be considered a ratio of a circle, which is easy to picture, in the same manner that $\frac{3}{4}$ of a circle is easy to imagine, compared to e.g. $\frac{3}{2}$ half circles.

Therefore, I know that this angle makes one full revolution, and $-\frac{3}{8}$ of a revolution (going clockwise), which results in the following figure correctly drawn in the context of a $-495°$ angle:



Question #1 (B)

Using the following relation:

$$x° = \frac{x}{360}(2\pi)\text{ rad} \tag{3}$$

I can **correctly** express $-495°$ in terms of radians like so:

$$\begin{aligned} -495° &= \frac{-495}{360}(2\pi)\text{ rad} \\ &= -\frac{11}{8}(2\pi)\text{ rad} \\ &= -\frac{11}{4}\pi\text{ rad} \end{aligned} \tag{4}$$

Answer: $-\frac{11}{4}\pi$ rad

Question #1 (C)

Question #3

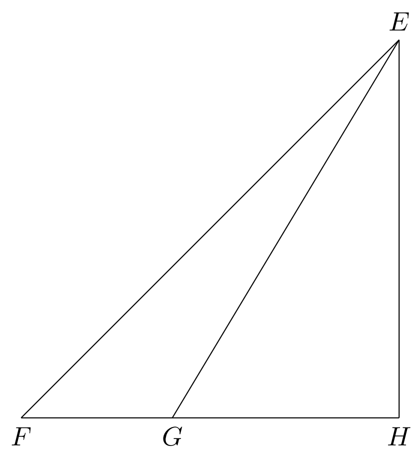
This should have been correct, what happens is my brain frequently gets things mixed up, i.e. there was a disconnect between my internal thoughts and what manifested on paper. I know that $\sec(x)$ is the reciprocal of $\cos(x)$, just as $\csc(x)$ is the reciprocal of $\sin(x)$, and that $\cot(x)$ is the reciprocal of $\tan(x)$, and furthermore in the context of the unit circle, that $\sin(\theta)$ represents the y value, and that $\cos(\theta)$ represents the x value.

Therefore, given the some $P(-20, 21)$ for θ:

$$\begin{aligned} r &= \sqrt{x^2 + y^2} = 29 \\ \sin(\theta) &= \frac{y}{r} = \frac{21}{29} \\ \sec(\theta) &= \frac{r}{x} = \frac{29}{-20} = -\frac{29}{20} \end{aligned} \tag{5}$$

Answer: $\sin(\theta) = \frac{21}{29}$, $\sec(\theta) = -\frac{29}{20}$

Question #4



Information given:

- $\overline{FG}$ = 600m
- $\measuredangle EFH$ = 1.91°
- $\measuredangle EGH$ = 2.67°

During the exam I was stuck on the gap between G and H, in hindsight it all makes sense, i.e. just solve for $\overline{GH}$ using an unknown value for $\overline{EH}$, since we can factor this quantity out in the ensuing expression for $\tan(1.91°)$. Although this realization occurred after spending an hour or so on the problem, I suppose failing this question was inevitable.

Solution:

$$
\begin{aligned}
\tan(2.67°) &= \frac{\overline{EH}}{x} \\
x &= \frac{\overline{EH}}{\tan(2.67°)} \\
\tan(1.91°) &= \frac{\overline{EH}}{600 + x} \\
(600 + x)(\tan(1.91°)) &= (600 + x)\frac{\overline{EH}}{600 + x} \\
(600 + x)(\tan(1.91°)) &= \overline{EH} \\
600 \cdot \tan(1.91°) + x \cdot \tan(1.91°) &= \overline{EH} \\
600 \cdot \tan(1.91°) &= \overline{EH} - \overline{EH}\frac{\tan(1.91°)}{\tan(2.67°)} \\
600 \cdot \tan(1.91°) &= \overline{EH}(1 - \frac{\tan(1.91°)}{\tan(2.67°)}) \\
\frac{600 \cdot \tan(1.91°)}{1 - \frac{\tan(1.91°)}{\tan(2.67°)}} &= \overline{EH} \approx 70
\end{aligned}
\tag{6}
$$

Why did it not occur to me to use $600 + x$? After thinking about it, I was time constrained, so instead of working though something without certainty, I was trying to find predefined solutions to predefined problems. Whereas if I started incrementally, A may have lead to B, which may have lead to an obvious answer.

Answer: $\overline{EH} \approx 70$ m

Question #5

Question #5 (A)

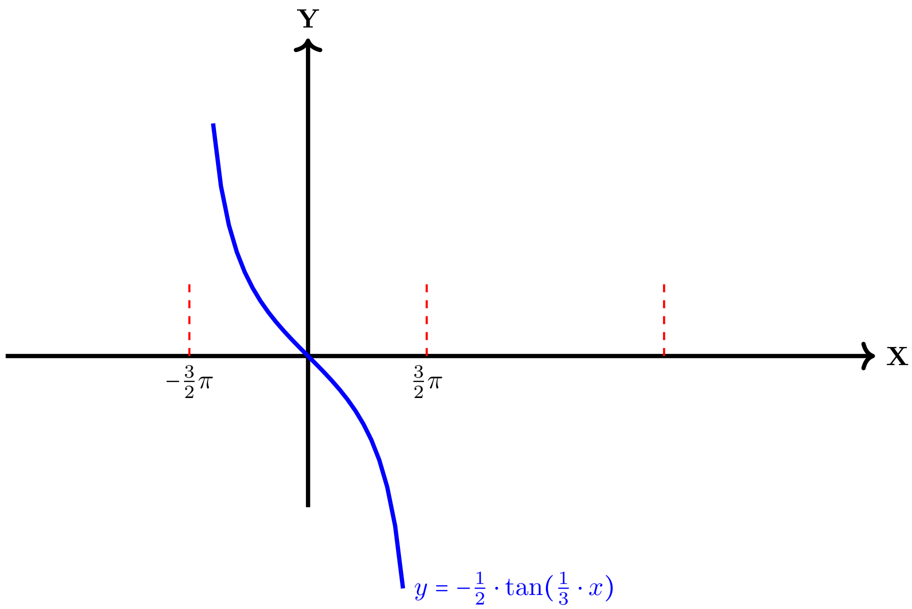
Again, this should have been correct, I made a dumb arithmetic error, I was rushing through my brain short circuited. Anyway the **correct** period for this function is:

$$
\frac{\pi}{\frac{1}{3}} = \pi \cdot \frac{3}{1} = 3\pi \tag{7}
$$

Answer: 3π

Question #5 (B)

By default, the period of the tangent function is π, and it's easiest to graph a single period within the asymptotes, such as from $-\frac{1}{2}\pi$ to $\frac{1}{2}\pi$, which is what I did, but didn't update the labels with the halved period, which is definitely an error.



Question #6

This question should have been perfect, but my brain short circuited with the period, again. Anyway the **correct** period for this function is:

$$\frac{2\pi}{\frac{1}{3}} = 2\pi \cdot \frac{3}{1} = 6\pi \tag{8}$$

Question #7

It's weird that I correctly computed the period for this function (which was more difficult), I even recomputed such in the same manner as the above two problems, yet without error. How does this happen? It's like I thought $2 + 2 = 5$, it makes no sense...

$$\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi \tag{9}$$

Miscellaneous

Location: https://github.com/colbyn/exam-1-extra-corrections.