Colbyn's Exam #1 Corrections

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Question #1

Question: Consider the angle θ in with measure $-495°$.

The problem with this question is that I was trying to be smart by simplifying $-495°$ to $225°$, so thereafter all of my subsequent work was using an angle co-terminal to $-495°$. In this context, my mistake was equating co-terminal angles as being the same, but this only applies to the output of periodic functions where co-terminal angles **map to the same value**, but the arguments themselves represent different measures.

Question #1 (A)

Using the following relation:

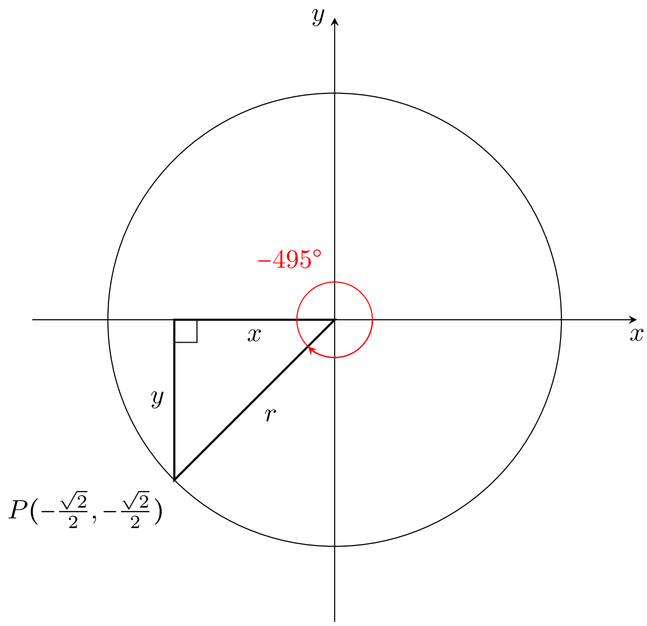
$$x^\circ = \frac{x}{360}\tau \text{ rad} \tag{1}$$

I can express $-495°$ in terms of turns (or τ) like so:

$$\begin{aligned} -495^\circ &= \frac{-495}{360}\tau \text{ rad} \\ &= -\frac{11}{8}\tau \text{ rad} \\ &= -1\frac{3}{8}\,\tau \end{aligned} \tag{2}$$

Since this angle is expressed in terms of turns, we have a intuitive idea about how the graph the given angle. I.e. because $-\frac{3}{8}$ can be considered a ratio of a circle, which is easy to picture, in the same manner that $\frac{3}{4}$ of a circle is easy to imagine, compared to e.g. $\frac{3}{2}$ half circles.

Therefore, I know that this angle makes one full revolution, and $-\frac{3}{8}$ of a revolution (going clockwise), which results in the following figure correctly drawn in the context of a $-495°$ angle:



Question #1 (B)

Using the following relation:

$$x° = \frac{x}{360}(2\pi) \text{ rad} \tag{3}$$

I can **correctly** express $-495°$ degrees in terms of radians like so:

$$\begin{aligned} -495° &= \frac{-495}{360}(2\pi) \text{ rad} \\ &= -\frac{11}{8}(2\pi) \text{ rad} \\ &= -\frac{11}{4}\pi \text{ rad} \end{aligned} \tag{4}$$

Answer: $-\frac{11}{4}\pi$ rad

Question #1 (C)

Question #3

This should have been correct, what happens is my brain frequently gets things mixed up, i.e. there was a disconnect between my internal thoughts and what manifested on paper. I know that $\sec(x)$ is the reciprocal of $\cos(x)$, just as $\csc(x)$ is the reciprocal of $\sin(x)$, and that $\cot(x)$ is the reciprocal of $\tan(x)$, and furthermore in the context of the unit circle, that $\sin(\theta)$ represents the y value, and that $\cos(\theta)$ represents the x value.

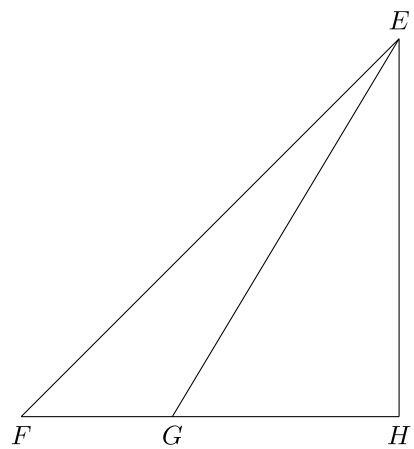
Therefore, given the some $P(-20, 21)$ for θ:

$$\begin{aligned} r &= \sqrt{x^2 + y^2} = 29 \\ \sin(\theta) &= \frac{y}{r} = \frac{21}{29} \\ \sec(\theta) &= \frac{r}{x} = \frac{29}{-20} = -\frac{29}{20} \end{aligned} \tag{5}$$

Answer: $\sin(\theta) = \frac{21}{29}$, $\sec(\theta) = -\frac{29}{20}$

Question #4

This was one of those questions where I wonder if I'm just dumb, or if it's legitimately a hard problem... Either way something I would have failed at regardless.



Information given:

- $\overline{FG} = 600\text{m}$
- $\measuredangle EFH = 1.91°$
- $\measuredangle EGH = 2.67°$

Solution:

$$
\begin{aligned}
\tan(2.67°) &= \frac{\overline{EH}}{x} \\
x &= \frac{\overline{EH}}{\tan(2.67°)} \\
\tan(1.91°) &= \frac{\overline{EH}}{600 + x} \\
(600 + x)(\tan(1.91°)) &= (600 + x)\frac{\overline{EH}}{600 + x} \\
(600 + x)(\tan(1.91°)) &= \overline{EH} \\
600 \cdot \tan(1.91°) + x \cdot \tan(1.91°) &= \overline{EH} \\
600 \cdot \tan(1.91°) &= \overline{EH} - \overline{EH}\frac{\tan(1.91°)}{\tan(2.67°)} \\
600 \cdot \tan(1.91°) &= \overline{EH}(1 - \frac{\tan(1.91°)}{\tan(2.67°)}) \\
\frac{600 \cdot \tan(1.91°)}{1 - \frac{\tan(1.91°)}{\tan(2.67°)}} &= \overline{EH} \approx 70
\end{aligned} \tag{6}
$$

During the exam I was stuck on the gap between G and H, in hindsight it all makes sense, i.e. just solve for $\overline{GH}$ using an unknown value for $\overline{EH}$, since we can factor this quantity out in the ensuing expression for $\tan(1.91°)$. Although this realization occurred after spending an hour or so on the problem, I suppose failing this question was inevitable.

Answer: $\overline{EH} \approx 70$ m

Question #5

Question #6

Miscellaneous

Location: `https://github.com/colbyn/exam-1-extra-corrections`.