

# Colbyn's Exam #1 Corrections

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September 28, 2020

## Question #1

**Question:** Consider the angle  $\theta$  in with measure  $-495^\circ$ .

The problem with this question is that I was trying to be smart by simplifying  $-495^\circ$  to  $225^\circ$ , so thereafter all of my subsequent work was using an angle co-terminal to  $-495^\circ$ . In this context, my mistake was equating co-terminal angles as being the same, but this only applies to the output of periodic functions where co-terminal angles **map to the same value**, but the arguments themselves represent different measures.

## Question #1 (A)

Using the following relation:

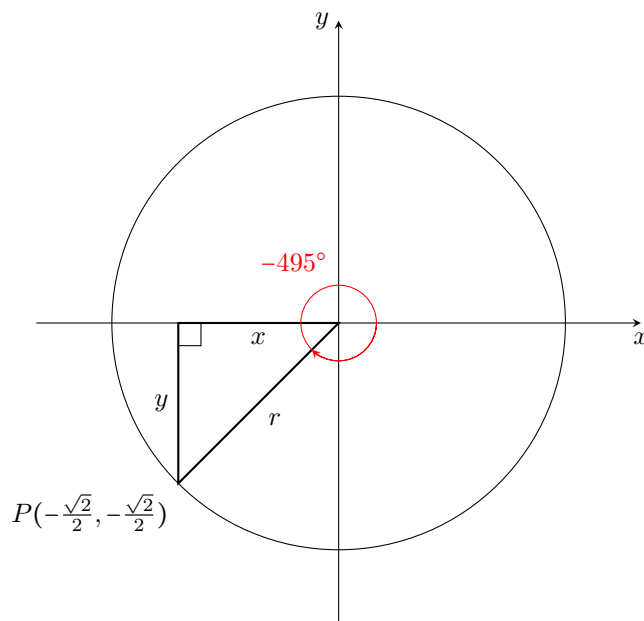
$$x^\circ = \frac{x}{360} \tau \text{ rad} \quad (1)$$

I can express  $-495^\circ$  in terms of turns (or  $\tau$ ) like so:

$$\begin{aligned} -495^\circ &= \frac{-495}{360} \tau \text{ rad} \\ &= -\frac{11}{8} \tau \text{ rad} \\ &= -1\frac{3}{8} \tau \end{aligned} \quad (2)$$

Since this angle is expressed in terms of turns, we have a intuitive idea about how the graph the given angle. I.e. because  $-\frac{3}{8}$  can be considered a ratio of a circle, which is easy to picture, in the same manner that  $\frac{3}{4}$  of a circle is easy to imagine, compared to e.g.  $\frac{3}{2}$  half circles.

Therefore, I know that this angle makes one full revolution, and  $-\frac{3}{8}$  of a revolution (going clockwise), which results in the following figure correctly drawn in the context of a  $-495^\circ$  angle:



### Question #1 (B)

Using the following relation:

$$x^\circ = \frac{x}{360}(2\pi) \text{ rad} \quad (3)$$

I can **correctly** express  $-495^\circ$  in terms of radians like so:

$$\begin{aligned} -495^\circ &= \frac{-495}{360}(2\pi) \text{ rad} \\ &= -\frac{11}{8}(2\pi) \text{ rad} \\ &= -\frac{11}{4}\pi \text{ rad} \end{aligned} \quad (4)$$

**Answer:**  $-\frac{11}{4}\pi \text{ rad}$

### Question #1 (C)

### Question #3

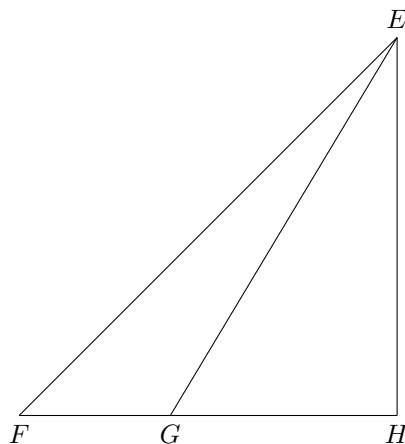
This should have been correct, what happens is my brain frequently gets things mixed up, i.e. there was a disconnect between my internal thoughts and what manifested on paper. I know that  $\sec(x)$  is the reciprocal of  $\cos(x)$ , just as  $\csc(x)$  is the reciprocal of  $\sin(x)$ , and that  $\cot(x)$  is the reciprocal of  $\tan(x)$ , and furthermore in the context of the unit circle, that  $\sin(\theta)$  represents the y value, and that  $\cos(\theta)$  represents the x value.

Therefore, given the some  $P(-20, 21)$  for  $\theta$ :

$$\begin{aligned} r &= \sqrt{x^2 + y^2} = 29 \\ \sin(\theta) &= \frac{y}{r} = \frac{21}{29} \\ \sec(\theta) &= \frac{r}{x} = \frac{29}{-20} = -\frac{29}{20} \end{aligned} \tag{5}$$

**Answer:**  $\sin(\theta) = \frac{21}{29}$ ,  $\sec(\theta) = -\frac{29}{20}$

### Question #4



Information given:

- $\overline{FG} = 600\text{m}$
- $\angle EFH = 1.91^\circ$
- $\angle EGH = 2.67^\circ$

During the exam I was stuck on the gap between  $G$  and  $H$ , in hindsight it all makes sense, i.e. just solve for  $\overline{GH}$  using an unknown value for  $\overline{EH}$ , since we can factor this quantity out in the ensuing expression for  $\tan(1.91^\circ)$ . Although this realization occurred after spending an hour or so on the problem, I suppose failing this question was inevitable.

Solution:

$$\begin{aligned}
\tan(2.67^\circ) &= \frac{\overline{EH}}{x} \\
x &= \frac{\overline{EH}}{\tan(2.67^\circ)} \\
\tan(1.91^\circ) &= \frac{\overline{EH}}{600 + x} \\
(600 + x)(\tan(1.91^\circ)) &= (600 + x) \frac{\overline{EH}}{600 + x} \\
(600 + x)(\tan(1.91^\circ)) &= \overline{EH} \\
600 \cdot \tan(1.91^\circ) + x \cdot \tan(1.91^\circ) &= \overline{EH} \\
600 \cdot \tan(1.91^\circ) &= \overline{EH} - \overline{EH} \frac{\tan(1.91^\circ)}{\tan(2.67^\circ)} \\
600 \cdot \tan(1.91^\circ) &= \overline{EH} \left(1 - \frac{\tan(1.91^\circ)}{\tan(2.67^\circ)}\right) \\
\frac{600 \cdot \tan(1.91^\circ)}{1 - \frac{\tan(1.91^\circ)}{\tan(2.67^\circ)}} &= \overline{EH} \approx 70
\end{aligned} \tag{6}$$

Why did it not occur to me to use  $600 + x$ ? After thinking about it, I was time constrained, so instead of working through something without certainty, I was trying to find predefined solutions to predefined problems. Whereas if I started incrementally,  $A$  may have lead to  $B$ , which may have lead to an obvious answer.

**Answer:**  $\overline{EH} \approx 70$  m

## Question #5

### Question #5 (A)

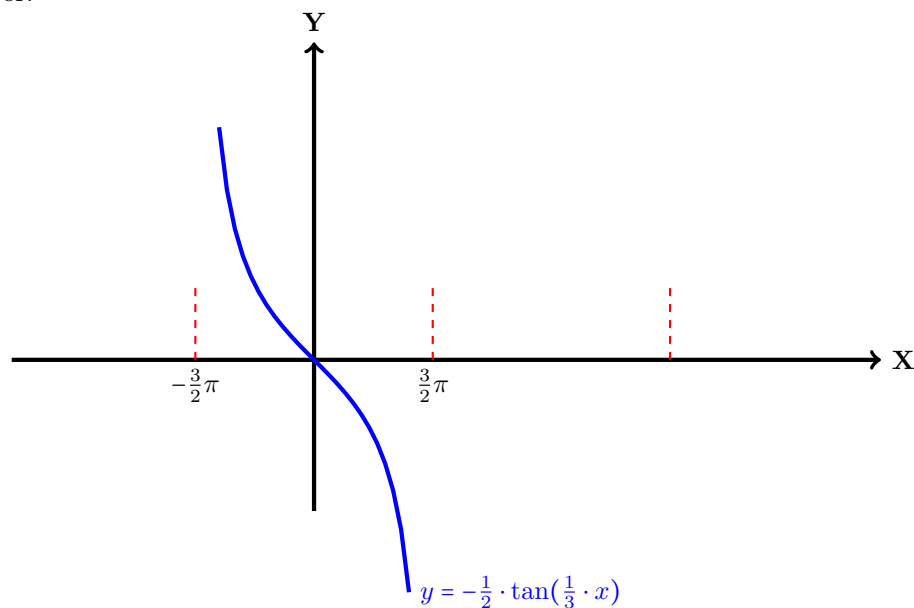
Again, this should have been correct, I made a dumb arithmetic error, I was rushing through my brain short circuited. Anyway the **correct** period for this function is:

$$\frac{\pi}{\frac{1}{3}} = \pi \cdot \frac{3}{1} = 3\pi \tag{7}$$

**Answer:**  $3\pi$

### Question #5 (B)

By default, the period of the tangent function is  $\pi$ , and it's easiest to graph a single period within the asymptotes, such as from  $-\frac{1}{2}\pi$  to  $\frac{1}{2}\pi$ , which is what I did, but didn't update the labels with the halved period, which is definitely an error.



### Question #6

This question should have been perfect, but my brain short circuited with the period, again. Anyway the **correct** period for this function is:

$$\frac{2\pi}{\frac{1}{3}} = 2\pi \cdot \frac{3}{1} = 6\pi \quad (8)$$

### Question #7

It's weird that I correctly computed the period for this function (which was more difficult), I even recomputed such in the same manner as the above two problems, yet without error. How does this happen? It's like I thought  $2 + 2 = 5$ , it makes no sense...

$$\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi \quad (9)$$

## Miscellaneous

Location: <https://github.com/colbyn/exam-1-extra-corrections>.