Colbyn's Exam #2 Corrections
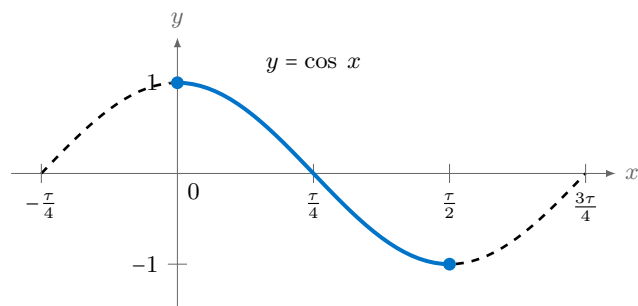
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Question #4B

I knew something was wrong here, this is why I started writing down the domain and ranges of the functions (in problems A to C).

In my case, I defaulted to thinking that $\left(f^{-1} \circ f\right)(x) = x$, which is only true for $0 \le x \le \pi$ (which is a really dumb mistake in hindsight).

Anyway the following is what I should have written:

Given $(\arccos \circ \cos)\left(\frac{7}{4}\pi\right)$:

$$
\begin{aligned}
\cos\left(\frac{7}{4}\pi\right) &= \cos\left(\frac{14}{8}\pi\right) \\
&= \cos\left(\pi + \frac{6}{8}\pi\right) \\
&= \cos\left(\pi\right)\cos\left(\frac{6}{8}\pi\right) - \sin\left(\pi\right)\sin\left(\frac{6}{8}\pi\right) \\
&= (-1)\left(-\frac{\sqrt{2}}{2}\right) - 0\sin\left(\frac{6}{8}\pi\right) \\
&= \frac{\sqrt{2}}{2} \\
&= \alpha
\end{aligned}
\tag{1}
$$

Therefore, we now consider $\arccos\left(\alpha\right)$, which is a special angle and therefore easy to identify (in my case I memorized the special angles in terms of fractions of a circle):

$$
\begin{aligned}
\arccos\left(\alpha\right) &= \arccos\left(\frac{\sqrt{2}}{2}\right) \\
&= \frac{1}{8}\tau \\
&= \frac{1}{4}\pi
\end{aligned}
\tag{2}
$$

Therefore:

$$
(\arccos \circ \cos)\left(\frac{7}{4}\pi\right) = \frac{1}{4}\pi
\tag{3}
$$

2

Regarding better studying habits

Overall, I should really do **all** of the practice problems (at least some from each topic). Although this applies more to my calculus class (e.g. I was pretty confident about a topic, and therefore I didn't bother to review the associated practice material, which has happened on two occasions, and in both, there was something weird that tripped me up.)

Miscellaneous

Regarding the remark that fractions are our friends, this was because it was evaluated VIA a calculator, which I punched in as fast as I could. What happened is that in a few places, I happen to use the same variable names as in the given formulas, and therefore got a few symbols mixed up (i.e. in one place, α should have been β and vice-versa.). After I realized my mistake, I recomputed the answers as fast as I could.