

ROCKET TEAM PROJECT

TEAM #1

DESTINATION: Mercury

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## I. INTRODUCTION

Named after the Roman god of speed, Mercury's tight orbit around the sun makes it an incredibly challenging mission target. The extreme  $\Delta V$  requirements required to reach Mercury require a number of gravity assists. Two missions were used as templates for this report, and both made use of a significant number of assists from the inner planets. These were ESA and JAXA's *BepiColumbo* and NASA's *Messenger*, both of which entered orbits around the sun-blasted planet.

This analysis compares three families of propulsion (solid, liquid, and hybrid) for transit to the planet Mercury. The overall mission aims to deliver a 5,000 kg payload from LEO (200km) to a 100 km altitude circular orbit about Mercury's surface.

## II. TRAJECTORY

A Hohmann transfer to Mercury is extremely energy-intensive due to the inclined, tight orbit of Mercury and the extreme depths of the Sun's gravity well. A conventional transfer would require  $V_{\infty}$ s of 6.43 km/s and 7.92 km/s for Earth departure and Mercury arrival, respectively. This corresponds to  $4.964\text{km/s} + 6.536 = 11.5\text{ km/s}$  in the best case (and these values were taken from the lowest possible departure and arrival values independently; the true lowest for a Hohmann transfer is likely to be higher).

Designing a "brute-force" rocket to bring such a large payload on such a high-energy trajectory would be prohibitively expensive and infeasible for a number of technical reasons. Instead, a gravity assist trajectory was chosen. Due to the extreme difficulty in computing and evaluating these trajectories, a simple Earth-Venus-Venus-Mercury trajectory was chosen. After a departure burn from Earth of approximately 3750 m/s, three deep space maneuvers are performed along with two Venus flybys (totaling approximately 1250m/s). The Mercury orbit injection is performed in two parts to maximize Oberth and allow staging: a 3650 m/s elliptical orbit insertion followed by an 800 m/s circularization burn.

The trajectory was found using a publicly available tool<sup>[8]</sup>. The tool performed nonlinear optimization using differential evolution and identified a trajectory with a total  $\Delta V$  requirement of 9449 m/s, which is printed in Table 1. Patched conics and impulsive maneuvers were assumed, although the trajectory given should still suffice as a ballpark estimate. The maximum burn times were empirically selected to help select required TWRs for the various stages; they are a rough approximation of when it would be appropriate to assume impulsive burns.

*Table 1: Required maneuvers for double gravity-assist trajectory. Full maneuver data, including velocities in the prograde, normal, and binormal directions, is included in the appendix.*

Time and Date	Maneuver	Maneuver Total $\Delta V$	Max Burn Time
10/29/2037	Earth Escape	3754.4 m/s	10 min
11/17/2037	Earth-Venus DSM	25.7 m/s	60 min
7/5/2038	Venus-Venus DSM	255.4 m/s	60 min
1/27/2039	Venus-Mercury DSM	965.1 m/s	60 min
3/13/2039	Mercury Entry to Elliptical	3648.5 m/s	20 min
3/18/2039	Mercury Circularization	800.0 m/s	10 min

An attempt was made to verify the trajectory in STK using the Astrogator toolkit and Heliocentric (n-body) propagator. The trajectory required significant correction due to the n-body approximation, and while it *did* eventually result in a Mercury encounter, the required orbital insertion  $\Delta V$  was anomalously high. The extremely sensitive nature of the n-body gravity assist calculation made optimization in STK — *required* to get a reasonable  $\Delta V$  value — impossible, especially since the experimenters did not have access to the STK SNOPT optimizer and were forced to use gradient descent (which struggled with the nonlinear and extremely stiff problem).

Nonetheless, potentially feasible trajectories were obtained; screenshots from the process are visible in Figures 1 and 2.

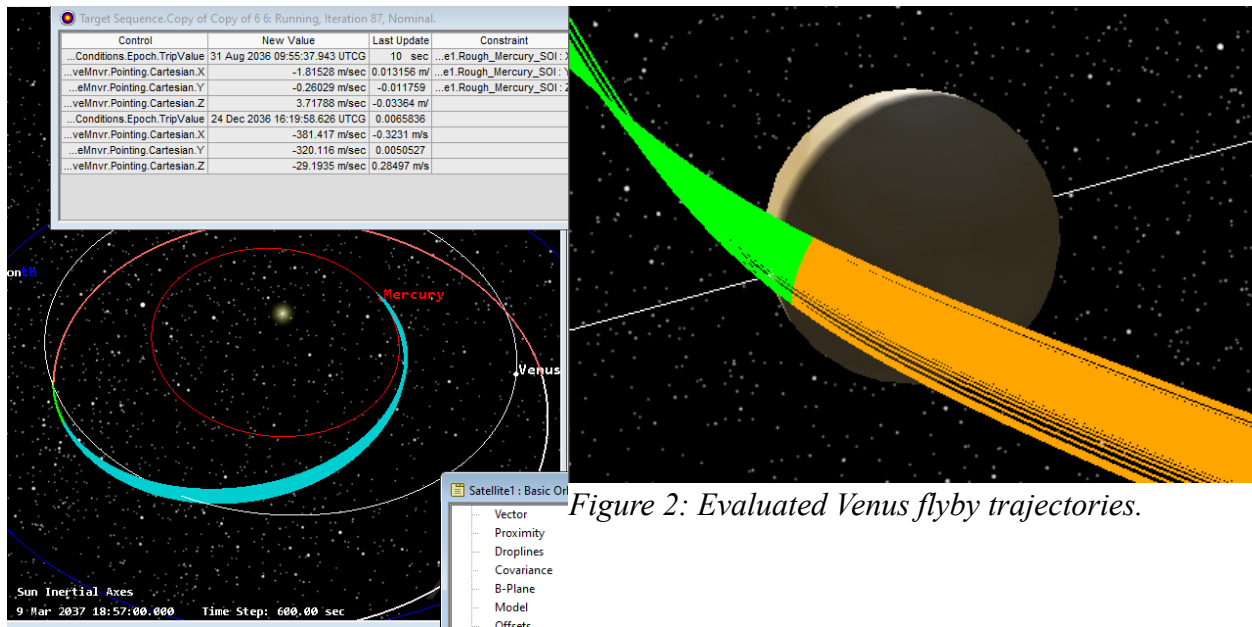


Figure 1: Optimization of the final Venus-Mercury trajectory. The struggling numerical optimizer is visible in the top of the screenshot.

Figure 2: Evaluated Venus flyby trajectories.

The total time from Earth departure burn to Mercury circularization burn was 1 year, 4 months, and 20 days. The travel distance could only be computed by detailed numerical integration of the trajectory, but a rough approximation (transit time \* Venus' average orbital velocity) gives a distance of  $35.02 \text{ km/s} * 1\text{y}4\text{m}20\text{d} = 10.25 \text{ AU}$ .

### III. LIQUID ROCKET MISSION

#### *i. General Information*

Due to the high  $\Delta V$  requirements of this mission, liquid rockets offer an extremely promising propulsion system. The small deep space maneuvers require low thrust and restartability, something offered primarily by liquid rockets. The high overall  $\Delta V$  demands excellent specific impulse, which is another strong suit of liquid rocket motors.

To minimize total weight, a hydrolox first stage was chosen for the Earth departure burn. This stage could fire soon after orbital insertion, minimizing boiloff losses and insulation requirements. Additionally, it offers the best specific impulse out of the "reasonable" liquid propellants. (Exotic combinations – like H/Li/F – were neglected due to their inimical relationship with living creatures and rocket engine components). However, for the remainder of the mission – especially so close to the Sun – a storable propellant was chosen. MMH and  $N_2O_4$  were chosen as the propellants due to their high specific impulse (relative to other storable propellants), hypergolic behavior (allowing for easy restartability), long life, similarity to previous missions (such as MESSENGER), and simple storability. UDMH and straight hydrazine offer similar performance, but CEA analysis suggested that MMH was the best option.

#### *ii. Staging*

A four-stage rocket was selected. The distribution of impulse among stages was not optimal, but was instead selected to avoid staging in the middle of any burn. The first stage was a hydrolox one intended for the Earth departure burn, and the remainder stages were hypergolic MMH-NTO for deep space maneuvering and Mercury orbital insertion.

The stage and maneuver breakdown is visible in

Stage	Maneuvers	Stage $\Delta V$	Propulsion/TWR <sub>i</sub>
1	Earth Ejection	3754.4 m/s	H <sub>2</sub> /LOX; 0.435
2	All three (3) DSMs	1246.1m/s	MMH/NTO, 0.023
3	Mercury Elliptical Insertion	3648.5m/s	MMH/NTO, 0.372
4	Mercury Circularization	800m/s	MMH/NTO, 0.121

Initial TWRs were kept low for each stage to save on engine mass; they were calculated from acceptable total stage burn times of ten minutes. (Except for the DSM stage, which had a total allowable burn time of 90 minutes). Stage 3 has a much lower TWR<sub>i</sub> due to its higher mass ratio; mid-burn, the rocket weighs comparatively less than Stage 1 would.

#### *iii. First Stage LH<sub>2</sub>/LOX Engine*

The first-stage engines produce a thrust of approximately 380 kN. A similar real-life engine would be the RL-10B, an expander cycle vacuum engine that produces 110kN of thrust at a chamber

pressure of 44 bar, O/F ratio of 5.88, an area ratio of 250:1, and TWR of 37.<sup>[9]</sup> This engine, and its variants, were taken as a starting point of the vacuum engine for the departure stage. (Practically speaking, three RL-10Bs would serve as an *excellent* propulsion system for this rocket.)

An area ratio of 250:1 and a chamber pressure of 50 bar were selected for the sake of analysis. CEA was used to determine the optimum O/F ratio and specific impulse. Due to the relatively long nozzle, the Isp was taken to be 75% of the way between frozen and equilibrium values.

The ideal vacuum Isp was found to occur at an O/F ratio of 4.92, with a vacuum equilibrium value of 469.8 s and frozen value of 454.5 s. By the above metric, the Isp of the engine would be 466 s, reasonable for an engine of this type. (Most vacuum hydrolox engines produce 450-470 s of specific impulse.)

CEA helpfully outputs a sonic velocity at the throat of  $1577.1 \text{ m s}^{-1}$  and a density of  $1.3361 \text{ kg/m}^3$  (assuming equilibrium composition.) These can be used to quickly and accurately calculate the mass flow rate for a given throat area. Given a target thrust of 385 kN, the required total mass flow rate is  $84.25 \text{ kg/s}$ , corresponding to a throat area of  $0.0400 \text{ m}^2$ . The weight of the engine, given a TWR of similar engines of 40, is expected to be around approximately 980 kg.

#### iv. Upper stage MMH/NTO Engines

All of the upper-stage engines produce a low enough thrust ( $<100 \text{ kN}$ ) to be expander or blowdown engines, and thus a similar design was chosen for all of them. The AMBR 556 N engine from Aerojet was used for comparison purposes: it has an area ratio of 400:1, Isp of 329 s, and TWR of 11.6. The chamber pressure was approximately 10 bar. The AJ-10 is another good engine for comparison, with an Isp of 316 s, TWR of 23, an expansion ratio of 55:1, and a chamber pressure of 16.6 bar. [10]

CEA computations were performed with a chamber pressure of 15 bar and area ratio of 400:1. A first pass to identify good O/F ratios was performed with values of  $\Phi$  between 0.8 and 3.0. An arbitrary weighting of 50% equilibrium, 50% frozen was used to estimate the final Isp. The ideal  $\Phi$  was determined to be near 1.1, and a closer pass was performed from 1 to 1.2. The ideal Isp was obtained with an O/F of 2.1337,  $\Phi$  of 1.17, and was approximately 347.8s. For equilibrium combustion, throat density was  $.77256 \text{ kg m}^{-3}$ , and velocity was  $1127.2 \text{ m/s}$ , corresponding to a mass flow of  $870.8 \text{ kg s}^{-1} \text{ m}^{-2}$ . The calculated Isp was anomalously high compared to similar existing engines; likely, these small engines have significant losses not accounted for by CEA.

A TWR of 12 was assumed for all three upper stage engines. As with the hydrolox engine, the parameters from CEA were used to calculate the throat area. Tabulated values for engine thrusts, masses, and throat areas are visible in Table 2. The similar mass and thrust of the stages 2 and 4 engines is a convenient coincidence; while stage 2 had a much higher mass, the low demands of deep space maneuvers required a much lower thrust.

#### v. Liquid Engine Overview

*Table 2: Summary of liquid engines used on this rocket.*

Stage	Propellant	Engine Isp	Thrust	Engine Mass	Throat Area	Exit Diameter	TWR	Burnout
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1	LH2/LOX	466.0 s	385 kN	981.5 kg	400. cm <sup>2</sup>	3.57 m	1.01
2	MMH/NTO	347.8 s	7.9 kN	67.1 kg	26.6 cm <sup>2</sup>	1.16 m	0.033
3	MMH/NTO	347.8 s	88 kN	747 kg	296. cm <sup>2</sup>	3.88 m	1.11
4	MMH/NTO	347.8 s	7.9 kN	67.1 kg	26.6 cm <sup>2</sup>	1.16 m	0.154

Values from CEA computations for each engine variant are summarized below. Stages 2-4 are all similar engines, and are grouped together below.

*Table 3: Summary data from CEA for the liquid stages. Data listed are from equilibrium conditions.*

Stage	C*	C <sub>T</sub>	Isp	T <sub>02</sub>	T <sub>f</sub>	p <sub>02</sub>
1	2372.8 m/s	1.9417	466.0 s	3238.2 K	610 K	50 bar
2-4	1722.5 m/s	2.0315	347.8 s	3226.4 K	745 K	15 bar

The final temperature for both stages is still fairly large, and additional efficiency could be extracted from an even larger nozzle. However, extraordinarily large nozzles begin to create mass penalties and manufacturing difficulties, so a constrained area ratio similar to existing engines was selected for both stages. Ideally, a model would be produced for the weight of the nozzle, and the entire rocket optimized to minimize stage weight.

The mass breakdown of the stages are in Table 4. For stages 1-3,  $m_{st}/m_p$  was assumed to be 0.03, where  $m_{st}$  is the structural mass of the tank. This was assumed to be 0.04 for the hydrolox tank; this value was taken from the Space Shuttle External Tank. Additionally, a flat structural mass of 0.01  $m_{ss}/m_0$  was assumed to account for miscellaneous stage hardware. For stage 1, this value was chosen to be 0.02 to include additional telemetry and control hardware apart from the payload. Engine mass was based on thrust and approximate TWR of similar real-world engines (as described above.)

*Table 4: Mass budget for each stage. Values of  $\lambda$  given consider the previous stage part of the payload. 2*

	Stage 1	Stage 2	Stage 3	Stage 4
Fuel Mass	8352.9 kg	3415.5 kg	4932.5 kg	442.1 kg
Oxidizer Mass	41096.2 kg	7287.5 kg	10524.4 kg	943.3 kg
Propellant Mass	49449 kg	10703 kg	15457 kg	1386 kg
Tank Mass	1978.0 kg	321.1 kg	463.7 kg	41.6 kg
Misc. Structural	882.6 kg	349.7 kg	235.3 kg	132.5 kg
Engine Mass	981.5 kg	67.1 kg	747.8 kg	67.1 kg
Total Mass	88263 kg	34971 kg	23530 kg	6627 kg
$\lambda$	0.6562	2.0567	0.3920	3.0736
$\epsilon$	0.547	0.0590	0.0433	0.112

Propellant densities of 0.0710, 1.141, .875, and 1.442 g/mL were assumed for LH2, LOX, MMH, and NTO, respectively. (Corresponding to saturated liquids and liquids at 20°C). The diameter of

each stage was selected to accommodate the engine bell of the stage immediately before it; this is to allow for reasonably shaped interstage fairings.

Tanks were chosen to be either cylindrical or capsule-shaped, depending on the required propellant volume and stage diameter. (The exceptionally wide stage 2, sized to accommodate the Stage 3 bell, required cylindrical tanks.) An additional allowance equal to the diameter of the engine bell \* 3 was added to accommodate engines and bulkheads; realistically, expandable nozzles could be used to save height.

Given these parameters, tank sizes for each stage could be calculated:

*Table 5: Geometric data for the liquid-propellant rocket.*

	<b>Stage 1</b>	<b>Stage 2</b>	<b>Stage 3</b>	<b>Stage 4</b>
Fuel Volume	117646 L	3903 L	5637 L	505 L
Oxidizer Volume	36018 L	5054 L	7299 L	654 L
Diameter	4 m	4 m	1.5 m	1.5 m
Tank Geometry	Capsule	Cylindrical	Capsule	Cylinder
Fuel Tank Height	10.70 m	1.644 m	3.690 m	0.286 m
Oxidizer Tank Height	4.200 m	1.736 m	4.630 m	0.370 m
Bulkhead/Engine Allowance	10.7 m	3.48 m	11.6 m	3.48 m
Stage Height	25.6 m	6.86 m	19.9 m	4.14 m

Assuming the 5000 kg payload is cylindrical, with a density of  $0.4 \text{ kg L}^{-1}$  and diameter of 1.5 m, the payload should have a height of approximately 7.07 m. This brings the total height of the liquid-propellant spacecraft to 56.5 m.

Ultimately, the liquid propellant rocket serves as a reasonable, albeit expensive, solution for this problem. Its total dry mass of roughly 12 tons could be lifted by many commercial launchers, and with a total mass of less than 90 ton, it would make an excellent payload for NASA's SLS rocket.

Many of the engines used in this design were bespoke, but many commercially available engines – such as the AJ10 and the RS25 – suit the needs of this mission quite well. The relatively large total height of 56.5m was mostly due to the enormous engine bells of the selected engines, which are probably in hindsight oversized (and would benefit from an optimization pass.) If the area ratios of the engines are maintained, expanding bells or multiple smaller engines could be used to keep the dimensions more reasonable.

#### IV. SOLID ROCKET MISSION

##### i. General Information

Initially, five stages were determined to be used for the mission to Mercury. However, an additional stage between the Venus-Mercury DSM and Mercury circularization was added to enter Mercury's sphere of influence. Similar to the liquid rocket mission, the chamber pressure was chosen to be a lower-than-normal value because the trajectory started from LEO.

The Space Shuttle Solid Rocket Booster (SRB) was used as a reference for this mission analysis. Similar propellant grains were assumed for this mission for ease of calculation. That said, the SRB Isp of 242s was input into the Google Docs spreadsheet for further calculations [1]. Additionally, the tank and pump structural masses were assumed to be negligible. Similarly, a relatively low chamber pressure of 10 atm was chosen because the mission starts from LEO.

##### ii. CEA Analysis

Using an Excel spreadsheet, the oxidizer to fuel ratio was calculated to be 1.298 with a gas constant of  $198.5 \text{ J/kg}^{-1}\text{K}^{-1}$  for the mixture. The outer diameter and throat diameter were chosen to be 148 and 54 inches, respectively [2]. With a chosen phi of 1.54, the results of the analysis can be seen tabulated in Table 6. This was chosen using engineering judgement based on previous examples where the propellant was rich.

Table 6: Solid Propellant Mixture Analysis

Species	Mass [kg]	n [kmol]	X	Mw [kg/kmol]	Y	Mwm [kg/kmol]	Cp [J/kmol*K]	Cpm [J/kmol*K]
CO <sub>2</sub>	26.4	0.6	0.1875	44	0.197	8.250	8.39e2	1.65e2
H <sub>2</sub> O	10.8	0.6	0.1875	18	0.081	3.375	4.18e3	3.37e2
N <sub>2</sub>	2.8	0.1	0.0313	28	0.021	0.875	1.04e3	21.7
Al <sub>2</sub> O <sub>3</sub>	40	0.4	0.125	100	0.299	12.50	880	2.63e2
HCl	43.2	1.2	0.375	36	0.322	13.5	4.04e3	1.30e3
Al	5.4	0.2	0.063	27	0.040	1.688	900	36.3
C <sub>4</sub> H <sub>6</sub>	5.4	0.1	0.0313	54	0.040	1.688	79.81	3.22
TOTAL	134	3.2	1	---	1	41.88	---	2.13e3

Similar to the Isp, the SRB area ratio was a design constraint input into CEA along with the oxidizer to fuel ratio and combustion chamber pressure. This analysis utilized an equilibrium flow. The CEA results are tabulated in Table 7. Here, the CEA analysis was used as a framework for the chosen fuel and oxidizer. Because the Isp for each grain was assumed to be the same

Table 7: Equilibrium Flow CEA Analysis

Exit Pressure, Pe [bar]	0.2342
Exit Temperature, Te [K]	1683.2
Exit Mach Number, Me	2.854
Characteristic Velocity, C* [ms <sup>-1</sup> ]	1506.0
Thrust Coefficient, C <sub>t</sub>	1.5830
Equivalent Velocity, Ueq [ms <sup>-1</sup> ]	2384.0
Specific Impulse, Isp [s]	243.02



Because CEA depends on the propellant chemical composition, the values presented in Table XX will be the same throughout the rocket. Similarly, with the same propellant throughout the entire rocket, to produce different amounts of thrust, the mass flow rate had to be different from grain to grain. With the mass flow rates tabulated in Table 8, the assumed grain geometries are listed as well.

Table 8: Thrust and Mass Flow Rate

Stage	Thrust [N]	Equivalent Velocity, $U_{eq}$ [m/s]	Mass Flow Rate $\times 10^3$ [kg/s]	$\varepsilon$	$\lambda$	Propellant Mass [kg]
1	2575569.2	2384.0	1.0804	0.10127	0.1402	651161.1
2	2443.8	2384.0	0.0010	0.393	32.3	1853.5
3	15980.0	2384.0	0.0067	0.153	5.88	12120.3
4	37893.3	2384.0	0.0159	0.114	1.59	28740.8
5	174875.7	2384.0	0.0734	0.105	0.571	29475.0
6	49011.4	2384.0	0.0206	0.103	0.362	12391.2

Comparing the CEA analysis Isp value of 243.02s with the referenced SRB Isp of 242s, the conditions were chosen for the mission were determined to be acceptable. However, the chosen fuel and oxidizer remained consistent throughout the entire rocket. Since different thrusts were calculated for each grain, the mass flow rates were different throughout the rocket. Using the equations from Chapter 12, the burnable areas were calculated for each grain.

### iii. Fuel and Oxidizer

The propellant chosen consisted of 5% aluminum powder, 12% HTPB, and 83% ammonium perchlorate. This formula is based on the formulation used in the Embry-Riddle Future Space Explorers and Developers Society (ERFSEDS) mixing standard operating procedure [3]. However, it is important to note this mission utilized a simplified version of this formula in an attempt to reduce complexity. According to recent density calculations, the propellant density for this mission was the approximate value of ERFSEDS' most recent mix of  $1440.85 \text{ kgm}^{-3}$ .

Using the ERFSEDS' calculated density, the propellant volume and length of each grain was calculated using a MATLAB script and tabulated in Table 9. It is important to note that since the first, fifth, and sixth stage require the highest  $\Delta u$ , the corresponding lengths make up the bulk of the rocket. When calculating the volume and length of each stage, the exit diameter was assumed to be the outer grain diameter with the throat diameter corresponding to the inner diameter of the grain.

Using a MATLAB script and Figure 12.15 from the textbook, the regression rate was calculated to equal  $6.52\text{e-}4 \text{ ms}^{-1}$  [4].

*Table 9: Stage Propellant Volume and Length*

Stage	Volume [m <sup>3</sup> ]	Length [m]	$\Delta u$ [m/s]	Burn time [s]	Geometry
1	451.9	46.97	3683.6	600	BATES
2	1.3	0.13	43.7	1800	Rod and Tube
3	8.4	0.87	311.5	1800	Rod and Tube
4	19.9	2.07	990.7	1800	Rod and Tube
5	20.5	2.13	2000.0	400	BATES
6	8.6	0.89	2549.5	600	BATES

Similar to the liquid mission, the Earth escape and Mercury circularization burn times were assumed to be 600s with an assumed DSM burn time of 1800s, or 30 minutes. The DSM burn time was referenced from NASA's Juno spacecraft as it conducted a second DSM [5]. The geometries for each stage were determined based on the role in the complete trajectory: BATES grains were chosen for Earth escape and Mercury entry/circularization because this geometry provided a progressive increase in thrust whereas the rod and tube were chosen for neutral thrust DSM roles.

Using the Tianwen-1 mission as a reference, the payload volume was assumed to be 14.43 m<sup>3</sup>, and the half angle of the payload stage was assumed to be 30° [6]. Using these, the length of the payload stage was calculated to be 0.814 m contributing to 1.51% of the 53.9 m rocket.

## V. HYBRID ROCKET MISSION

### i. General Information

Similar to the liquid rocket design, four stages are used for the hybrid rocket engine. The four stages are as follows:

*Table 10: Stage Summary - Hybrid Rocket*

Stage	Role	$\Delta u$ [m/s]	Burn Time [s]	Isp [s]	Propellant & Oxidizer
1	Earth Departure	3754.4	600	280	HTPB + LOx
2	Misc. DSM	1246.1	4648.4	247	HTPB + N <sub>2</sub> O
3	Mercury Entry Burn	3648.5	1200	247	HTPB + N <sub>2</sub> O
4	Mercury Circularization	800	600	247	HTPB + N <sub>2</sub> O

The propellant & oxidizer combinations mentioned above were chosen due to existing thorough research and available theoretical values. LOx is only used for the first stage due to its cryogenic nature causing storability issues going towards Mercury.

### ii. CEA Analysis

*Table 11: Rocket Mass - Hybrid Rocket*

	Stage 1	Stage 2	Stage 3	Stage
Propellant Mass	353021 kg	35279 kg	35280.4 kg	2049.5 kg
Tank Mass	1978 kg	721.1 kg	620 kg	120 kg
Misc. Structural	2900 kg	3029 kg	3699.7 kg	236.8 kg
Engine Mass	30079 kg	1050 kg	842.8 kg	434.6 kg
Total Mass	387978 kg	40080 kg	40422 kg	2846.3 kg

The optimum values of Isp and O/F were chosen for the initial design of each stage at a chamber pressure ( $p_0$ ) of 35 bar and exit pressure ( $p_e$ ) of 1 bar. The corresponding calculated chamber and propellant properties are calculated below:

*Table 12: Hybrid stage coefficients*

Stage	$A_e/A^*$	$C^*$ [m/s]	$C_t$	$U_{eq}$ [m/s]	T (N)
1	90	1820	1.87	3354.5	1615580.5
2	50	1604	1.78	2575.2	18384.1
3	50	1604	1.78	2575.2	76573.8
4	50	1604	1.78	2575.2	8273.8

The structural and payload coefficients were estimated to be 0.01 and 0.0863 respectively. These result in a propellant mass of 428286.6 kg and total mass of 473724.4 kg for a rocket with a total thrust of 1718812.3 N.

### iii. Nozzle Dimensions

The throat diameter was set to be 1m which leads to a throat area of  $0.785m^2$ . Upon iterating, 5 ports of equal diameter of 0.33m each (at the throat) were chosen for the final design. The density ( $\rho_f$ ) HTPB was used as  $920 \text{ kg/m}^3$ . To obtain the set Isp and calculated thrust the following dimensions were chosen:

Table 13: Hybrid stage physical dimensions

Stage	Exit Area [ $m^2$ ]	Length, L [m]
1	70.65	28.6
2	39.25	4.2
3	39.25	12.3
4	39.25	1.5

This resulted in a hybrid rocket of length 46.6m

## VI. CONCLUSION

Ultimately, with significant gravity assists, a payload as large as 5000 kg can be successfully delivered to a low hermeocentric orbit. The still-enormous mission  $\Delta V$  requirements of approximately 9500 m/s demand high specific impulse; for this reason, both the solid rocket and hybrid rocket solutions had *tremendous* total masses (over 650 tons and 474 tons, respectively). The relatively lithe 90-ton of the liquid rocket is still significant, a heavy-lift vehicle such as SLS would be required to loft it to orbit.

Generally, the rockets presented in this report are significantly under-analyzed and under-optimized. The gravity assist trajectory chosen was compromised significantly by limited computation and software resources, and further iteration is required to bring engine sizes and area ratios in line with more reasonable values. However, the values presented in this report should make the feasibility of various propulsion schemes clear.

## VII. REFERENCES

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## VIII. APPENDIX

## i. Miscellaneous Tables

Table 11: Solid Propellant Mixture Analysis

Species	Mass [kg]	n [kmol]	X	Mw [kg/kmol ]	Y	Mwm [kg/kmol ]	Cp [J/kmol*K ]	Cpm [J/kmol*K ]
CO <sub>2</sub>	26.4	0.6	0.1875	44	0.197	8.250	8.39e2	1.65e2
H <sub>2</sub> O	10.8	0.6	0.1875	18	0.081	3.375	4.18e3	3.37e2
N <sub>2</sub>	2.8	0.1	0.0313	28	0.021	0.875	1.04e3	21.7
Al <sub>2</sub> O <sub>3</sub>	40	0.4	0.125	100	0.299	12.50	880	2.63e2
HCl	43.2	1.2	0.375	36	0.322	13.5	4.04e3	1.30e3
Al	5.4	0.2	0.063	27	0.040	1.688	900	36.3
C <sub>4</sub> H <sub>6</sub>	5.4	0.1	0.0313	54	0.040	1.688	79.81	3.22
TOTAL	134	3.2	1	---	1	41.88	---	2.13e3

Table 12: Equilibrium Flow CEA Analysis

Exit Pressure, Pe [bar]	0.2342
Exit Temperature, Te [K]	1683.2
Exit Mach Number, Me	2.854
Characteristic Velocity, C* [ms <sup>-1</sup> ]	1506.0
Thrust Coefficient, C <sub>t</sub>	1.5830
Equivalent Velocity, Ueq [m/s]	2384.0
Specific Impulse, Isp [s]	243.02

Table 13: Thrust and Mass Flow Rate

Stage	Thrust [N]	Equivalent Velocity, Ueq [m/s]	Mass Flow Rate x 10 <sup>3</sup> [kgs <sup>-1</sup> ]
1	2575569.2	2384.0	1.0804
2	2443.8	2384.0	0.0010
3	15980.0	2384.0	0.0067
4	37893.3	2384.0	0.0159
5	174875.7	2384.0	0.0734
6	49011.4	2384.0	0.0206

*Table 14: Stage Propellant Volume and Length*

Stage	Volume [m <sup>3</sup> ]	Length [m]	$\Delta u$ [m/s]	Burn time [s]
1	451.9284	46.9714	3683.6	600
2	1.2864	0.1337	43.7	1800
3	8.4119	0.8743	311.5	1800
4	19.9471	2.0732	990.7	1800
5	20.4567	2.1262	2000	400
6	8.5999	0.8938	2549.5	600

ii. *Full Trajectory Optimizer Output*

Sequence:	Earth-Venus-Venus-Mercury
Departure:	Year 2037 - Day 302 - 09:55:27
Arrival:	Year 2039 - Day 72 - 01:21:30
Total $\Delta V$ :	9449.0 m/s
Steps:	
Earth escape:	
Date:	Year 2037 - Day 302 - 09:55:27 UT T+ 0y - 0d - 00:00:00 MET
Ejection angle:	51.2°
$\Delta V$ :	3754.4 m/s
Prograde:	3754.4
Normal:	0.0
Binormal:	0.0
Earth-Venus DSM:	
Date:	Year 2037 - Day 321 - 15:05:11 UT T+ 0y - 19d - 06:09:44 MET
$\Delta V$ :	25.7 m/s
Prograde:	-15.4
Normal:	2.7
Binormal:	-20.4
Flyby around Venus:	
SOI enter date:	Year 2038 - Day 116 - 03:20:40 UT T+ 0y - 178d - 17:25:13 MET
SOI exit date:	Year 2038 - Day 118 - 10:03:45 UT T+ 0y - 181d - 00:08:18 MET
Periapsis altitude:	31819 km
Inclination:	41°
Venus-Venus DSM:	
Date:	Year 2038 - Day 186 - 04:29:13 UT T+ 0y - 248d - 18:33:46 MET
$\Delta V$ :	255.3 m/s
Prograde:	-253.4
Normal:	28.5
Binormal:	-13.6
Flyby around Venus:	
SOI enter date:	Year 2038 - Day 343 - 22:17:05 UT T+ 1y - 41d - 13:21:38 MET
SOI exit date:	Year 2038 - Day 346 - 00:36:23 UT T+ 1y - 43d - 15:40:56 MET
Periapsis altitude:	145 km
Inclination:	25°
Venus-Mercury DSM:	
Date:	Year 2039 - Day 27 - 10:56:37 UT T+ 1y - 90d - 02:01:010 MET
$\Delta V$ :	964.6 m/s
Prograde:	-955.2



Normal:	137.7
Binormal:	2.5
Mercury circularization:	
Date:	Year 2039 - Day 72 - 01:21:30 UT
	T+ 1y - 134d - 16:26:03 MET
$\Delta V$ :	4448.5 m/s
Prograde:	-4448.5
Normal:	0.0
Binormal:	0.0

### iii. CEA Output, MMH/NTO

THEORETICAL ROCKET PERFORMANCE ASSUMING EQUILIBRIUM

COMPOSITION DURING EXPANSION FROM INFINITE AREA COMBUSTOR

Pin = 217.6 PSIA

CASE = \_\_\_\_\_

	REACTANT	WT FRACTION (SEE NOTE)	ENERGY KJ/KG-MOL	TEMP K
FUEL	CH6N2 (L), MMH	1.0000000	54200.000	298.150
OXIDANT	N2O4 (L)	1.0000000	-17549.000	298.150

O/F= 2.13368 %FUEL= 31.911363 R,EQ.RATIO= 1.170000 PHI,EQ.RATIO= 1.170000

	CHAMBER	THROAT	EXIT
Pinf/P	1.0000	1.7300	9261.03
P, BAR	15.000	8.6708	0.00162
T, K	3226.44	3063.85	744.92
RHO, KG/CU M	1.2533 0	7.7256-1	6.2216-4
H, KJ/KG	245.55	-389.75	-5876.85
U, KJ/KG	-951.26	-1512.10	-6137.19
G, KJ/KG	-38083.2	-36787.1	-14726.3
S, KJ/(KG) (K)	11.8796	11.8796	11.8796
M, (1/n)	22.415	22.697	23.791
(dLV/dLP) t	-1.02747	-1.02283	-1.00000
(dLV/dLT) p	1.5410	1.4754	1.0000
Cp, KJ/(KG) (K)	6.0593	5.7158	1.6209
GAMMAS	1.1337	1.1321	1.2749
SON VEL, M/SEC	1164.8	1127.2	576.1
MACH NUMBER	0.000	1.000	6.074

PERFORMANCE PARAMETERS

Ae/At	1.0000	400.00
CSTAR, M/SEC	1722.5	1722.5
CF	0.6544	2.0315
Ivac, M/SEC	2122.9	3573.7
Isp, M/SEC	1127.2	3499.3

MASS FRACTIONS

*CO	0.11893	0.11144	0.00857
*CO2	0.11796	0.12974	0.29136
*H	0.00095	0.00075	0.00000
HNO	0.00001	0.00000	0.00000
HO2	0.00004	0.00003	0.00000
*H2	0.00655	0.00601	0.00953
H2O	0.28969	0.29976	0.28921
H2O2	0.00001	0.00000	0.00000

*NO	0.01324	0.00993	0.00000
NO2	0.00001	0.00001	0.00000
*N2	0.39514	0.39669	0.40133
*O	0.00499	0.00351	0.00000
*OH	0.03323	0.02668	0.00000
*O2	0.01924	0.01544	0.00000

\* THERMODYNAMIC PROPERTIES FITTED TO 20000.K

NOTE. WEIGHT FRACTION OF FUEL IN TOTAL FUELS AND OF OXIDANT IN TOTAL OXIDANTS

#### THEORETICAL ROCKET PERFORMANCE ASSUMING FROZEN COMPOSITION

Pin = 217.6 PSIA

CASE = \_\_\_\_\_

	REACTANT	WT FRACTION (SEE NOTE)	ENERGY KJ/KG-MOL	TEMP K
FUEL	CH6N2 (L),MMH	1.000000	54200.000	298.150
OXIDANT	N2O4 (L)	1.000000	-17549.000	298.150

O/F= 2.13368 %FUEL= 31.911363 R,EQ.RATIO= 1.170000 PHI,EQ.RATIO= 1.170000

	CHAMBER	THROAT	EXIT
Pinf/P	1.0000	1.7877	14592.7
P, BAR	15.000	8.3907	0.00103
T, K	3226.44	2900.74	396.23
RHO, KG/CU M	1.2533 0	7.7981-1	6.9937-4
H, KJ/KG	245.55	-414.06	-4879.95
U, KJ/KG	-951.26	-1490.06	-5026.93
G, KJ/KG	-38083.2	-34873.6	-9586.99
S, KJ/(KG) (K)	11.8796	11.8796	11.8796
M, (1/n)	22.415	22.415	22.415
Cp, KJ/(KG) (K)	2.0376	2.0119	1.4082
GAMMAS	1.2226	1.2260	1.3576
SON VEL,M/SEC	1209.6	1148.6	446.7
MACH NUMBER	0.000	1.000	7.168

#### PERFORMANCE PARAMETERS

Ae/At	1.0000	400.00
CSTAR, M/SEC	1674.7	1674.7
CF	0.6858	1.9118
Ivac, M/SEC	2085.4	3247.6
Isp, M/SEC	1148.6	3201.7

#### MASS FRACTIONS

*CO	0.11893	*CO2	0.11796	*H	0.00095
HNO	0.00001	HO2	0.00004	*H2	0.00655
H2O	0.28969	H2O2	0.00001	*NO	0.01324
NO2	0.00001	*N2	0.39514	*O	0.00499
*OH	0.03323	*O2	0.01924		

\* THERMODYNAMIC PROPERTIES FITTED TO 20000.K

NOTE. WEIGHT FRACTION OF FUEL IN TOTAL FUELS AND OF OXIDANT IN TOTAL OXIDANTS

*iv. CEA Output, LH2/LOX*

THEORETICAL ROCKET PERFORMANCE ASSUMING EQUILIBRIUM

COMPOSITION DURING EXPANSION FROM INFINITE AREA COMBUSTOR

Pin = 725.2 PSIA

CASE = \_\_\_\_\_

	REACTANT	WT FRACTION (SEE NOTE)	ENERGY KJ/KG-MOL	TEMP K
FUEL	H2 (L)	1.000000	-9012.000	20.270
OXIDANT	O2 (L)	1.000000	-12979.000	90.170

O/F= 4.92000 %FUEL= 16.891892 R,EQ.RATIO= 1.613147 PHI,EQ.RATIO= 1.613147

	CHAMBER	THROAT	EXIT
Pinf/P	1.0000	1.7491	6427.17
P, BAR	50.000	28.586	0.00778
T, K	3238.20	3022.13	610.35
RHO, KG/CU M	2.1640 0	1.3361 0	1.8295-3
H, KJ/KG	-1092.25	-2335.90	-11706.2
U, KJ/KG	-3402.76	-4475.37	-12131.4
G, KJ/KG	-65120.6	-62091.9	-23774.5
S, KJ/(KG) (K)	19.7728	19.7728	19.7728

M, (1/n)	11.653	11.745	11.934
(dLV/dLP)t	-1.01233	-1.00820	-1.00000
(dLV/dLT)p	1.2309	1.1634	1.0000
Cp, KJ/(KG) (K)	7.3073	6.4723	2.8271
GAMMas	1.1569	1.1626	1.3270
SON VEL,M/SEC	1634.9	1577.1	751.2
MACH NUMBER	0.000	1.000	6.133

PERFORMANCE PARAMETERS

Ae/At	1.0000	250.00
CSTAR, M/SEC	2372.8	2372.8
CF	0.6647	1.9417
Ivac, M/SEC	2933.7	4699.7
Isp, M/SEC	1577.1	4607.4

MASS FRACTIONS

*H	0.00231	0.00165	0.00000
HO2	0.00001	0.00000	0.00000
*H2	0.06373	0.06366	0.06421
H2O	0.90542	0.91720	0.93579
H2O2	0.00001	0.00000	0.00000
*O	0.00099	0.00045	0.00000
*OH	0.02630	0.01645	0.00000
*O2	0.00123	0.00058	0.00000

\* THERMODYNAMIC PROPERTIES FITTED TO 20000.K

NOTE. WEIGHT FRACTION OF FUEL IN TOTAL FUELS AND OF OXIDANT IN TOTAL OXIDANTS

THEORETICAL ROCKET PERFORMANCE ASSUMING FROZEN COMPOSITION

Pin = 725.2 PSIA

CASE = \_\_\_\_\_

	REACTANT	WT FRACTION (SEE NOTE)	ENERGY KJ/KG-MOL	TEMP K
FUEL	H2 (L)	1.0000000	-9012.000	20.270
OXIDANT	O2 (L)	1.0000000	-12979.000	90.170

O/F= 4.92000 %FUEL= 16.891892 R,EQ.RATIO= 1.613147 PHI,EQ.RATIO= 1.613147

	CHAMBER	THROAT	EXIT
Pinf/P	1.0000	1.7776	7301.02
P, BAR	50.000	28.127	0.00685
T, K	3238.20	2933.61	500.24
RHO, KG/CU M	2.1640 0	1.3438 0	1.9187-3
H, KJ/KG	-1092.25	-2358.09	-11026.2
U, KJ/KG	-3402.76	-4451.27	-11383.1
G, KJ/KG	-65120.6	-60363.8	-20917.3
S, KJ/(KG) (K)	19.7728	19.7728	19.7728
M, (1/n)	11.653	11.653	11.653
Cp, KJ/(KG) (K)	4.1902	4.1194	2.7912
GAMMA <sub>s</sub>	1.2052	1.2095	1.3434
SON VEL, M/SEC	1668.7	1591.1	692.5
MACH NUMBER	0.000	1.000	6.437

## PERFORMANCE PARAMETERS

Ae/At	1.0000	250.00
CSTAR, M/SEC	2338.5	2338.5
CF	0.6804	1.9060
Ivac, M/SEC	2906.7	4537.4
Isp, M/SEC	1591.1	4457.3

## MASS FRACTIONS

*H	0.00231	HO2	0.00001	*H2	0.06373
H2O	0.90542	H2O2	0.00001	*O	0.00099
*OH	0.02630	*O2	0.00123		

\* THERMODYNAMIC PROPERTIES FITTED TO 20000.K

NOTE. WEIGHT FRACTION OF FUEL IN TOTAL FUELS AND OF OXIDANT IN TOTAL OXIDANTS