AE 434 Spacecraft Controls Final Project

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Abstract

This report presents the development and validation of a control system designed for spacecraft attitude manipulation, specifically focusing on detumbling and reorientation. Utilizing the Euler 3-1-3 rotation sequence and the spacecraft's inertia tensor, the control algorithms were implemented to achieve detumble and orientation. These control algorithms include state-space models, MATLAB's ODE-45 and other control engineering techniques. Simulation results confirmed the effectiveness of the control strategies, with the satellite reaching complete detumble and orientation maneuvers.

Nomenclature

ACS = Attitude Spacecraft Control
SISO = Single Input Single Output
MIMO = Multi-Input Multi-Output
ψ = Psi
ϕ = Phi
θ = Theta

1. Introduction

1.1 Background

Achieving control over a spacecraft's orientation and velocity is crucial to its success. Although even after the launch of the first satellite Sputnik in 1957, the realm of attitude controls was barren until around the 1970s [1]. Attitude determination solutions were found before this time, however, the complexity of these solutions left very few practical applications due computation limitations of the time. Due to this reason nearly all of the spacecraft at the time had little to no pointing requirements, using basic spacecraft spin or gravity stabilization as the solution such as Figure 1.

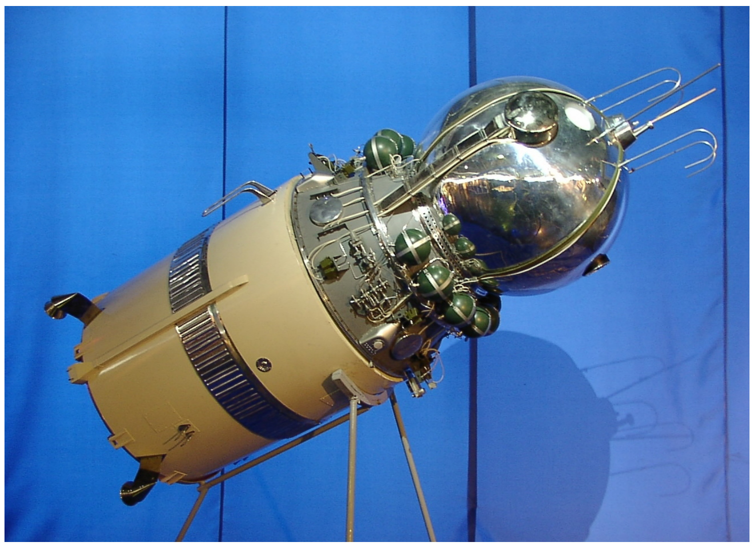


Figure 1: Vostok 1

Most modern spacecraft have some sort of ACS for pointing requirements, this ability is due to the advancement of technology as well as knowledge of controls science. These requirements can vary drastically for each spacecraft depending on budget, mission objectives, lifespan, or many other specific mission considerations. As technology has advanced, ACS has become a key asset to precision orientation with not just spacecraft but things like airplanes or even autonomous vehicles.

1.2 Theory

There are a variety of spacecraft control methods, however, they all fall under passive or active methods. One of the simplest being spin stabilization, using the rigid bodies angular momentum.

The rotational motion of a rigid body in space can be described by Euler's equations of motion, which are fundamental to understanding the dynamics of rotating systems such as spacecraft. These equations link the body's angular momentum to its angular velocity and the applied torques, accounting for the distribution of mass within the body—characterized by its moments of inertia.

For a body with principal moments of inertia about its principal axes, and with angular velocity components ω, For a body with principal moments of inertia I_{11}, I_{22}, I_{33} about its principal axes, and with angular velocity components ω_1, ω_2, ω_3 vary along these axes, Euler's equations are expressed as:

$$I = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix} = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 50 \end{bmatrix} \tag{1}$$

$$\omega_1 = \dot{\theta} cos\psi + \dot{\phi} sin\theta sin\psi \tag{2}$$

$$\omega_2 = \dot{\phi} sin\theta cos\psi - \dot{\theta} sin\psi \tag{3}$$

$$\omega_3 = \dot{\psi} + \dot{\phi} cos\theta \tag{4}$$

$$I_{11}\dot{\omega}_1 + (I_{33} - I_{22})\, \omega_2\omega_3 = M_1, \tag{5}$$

$$I_{22}\, \dot{\omega}_2 + (I_{11} - I_{33})\omega_3\omega_1 = M_2, \tag{6}$$

$$I_{33}\, \dot{\omega}_3 + (I_{22} - I_{11})\omega_1\, \omega_2 = M_3 \tag{7}$$

In the context of a spacecraft, these equations are pivotal in the design and implementation of attitude control systems. They calculate the required torques to achieve desired changes in the spacecraft's orientation, which is crucial for tasks such as maneuvering and stabilization. It can be assumed that the spacecraft has a constant angular velocity component about one of its principal axes, typically the axis associated with the spin or roll motion. In our equations, ω_3 = n which is a constant value n. This simplification reflects a that spacecraft has a steady spin about the I_{33} axis. Under this assumption, the equations can be further simplified to express the angular acceleration.

$$\dot{\omega}_1 = \frac{(I_{22} - I_{33})\omega_2\omega_3}{I_{11}} + \frac{M_1}{I_{11}} \tag{8}$$

$$\dot{\omega}_2 = \frac{(I_{33} - I_{11})\omega_3\omega_1}{I_{22}} + \frac{M_2}{I_{22}} \tag{9}$$

$$\dot{\omega}_3 = \frac{(I_{11} - I_{22})\omega_1\omega_2}{I_{33}} + \frac{M_3}{I_{33}} \tag{10}$$

These simplified expressions assist in analyzing the spacecraft's behavior when it is primarily controlled by the steady spin along one axis, and they aid in formulating control strategies that ensure the vehicle remains on its intended orientation path while in orbit. By solving these equations, either analytically or numerically, the evolution of the spacecraft's rotational state can be predicted over time, given initial conditions and a set of control torques. This predictive capability is essential for mission planning and real-time operations of spacecraft.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{11}$$

$$\dot{y}(t) = Cx(t) + Du(t) \tag{12}$$

u(t): inputs
x(t): states
y(t): outputs
A: state matrix
B: input matrix
C: output matrix
D: direct transition

Classical control theory's use of the Laplace domain and transfer functions serves SISO systems however fails for anything more than one. Modern control theory utilizes equation 11 and 12 to create a MIMO system which is a model a physical system as a set of input, output and state variables related by first order differential equations otherwise known as state-space representation.

2. Methodology

2.1 State-Space Equation

1. To begin, a State-Space representation was made, using the initial conditions given. These conditions included inertia tensors of the satellite and the initial orientation and angular velocities. Four matrices are used to create the state space representation. First: The "A Matrix". This matrix is known as the state matrix and expresses the current state of the controlled object in whatever variable desired. In this case, the A Matrix is given as:

$$A = \begin{bmatrix} 0 & \frac{(I_{22} - I_{33})\omega_3}{2 * I_{11}} & \frac{(I_{22} - I_{33})\omega_2}{2 * I_{11}} \\ \frac{(I_{33} - I_{11})\omega_3}{2 * I_{22}} & 0 & \frac{(I_{33} - I_{11})\omega_1}{2 * I_{22}} \\ \frac{(I_{11} - I_{22})\omega_2}{2 * I_{33}} & \frac{(I_{11} - I_{22})\omega_2}{2 * I_{33}} & 0 \end{bmatrix} \tag{13}$$

This form of the matrix is derived from the angular acceleration equation seen in Equation 4 through Equation 6. The A matrix is taken from the first term of the Euler equations with the angular velocity values being extracted. Note that there is an added coefficient of ½ for each state variable, this ensures when the A and x matrix are multiplied, they yield a result exactly equal to the Euler moment equations. Because angular velocities were extracted to form the A matrix, the entire A matrix must be multiplied by an angular velocity value to keep consistency. Therefore, the x matrix is represented by a column vector of every angular velocity as seen below.

$$x = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} \; x_i = \begin{bmatrix} .2 \\ .2 \\ 1 \end{bmatrix} \tag{14}$$

As mentioned in Equation 7, the state space format requires a Bu term. This Bu term will encapsulate the second term of Euler's equation. The B matrix is a constant and will be the inverse of the inertia tensors as seen below.

$$B = \begin{bmatrix} \frac{1}{I_{11}} & 0 & 0 \\ 0 & \frac{1}{I_{22}} & 0 \\ 0 & 0 & \frac{1}{I_{33}} \end{bmatrix} \tag{15}$$

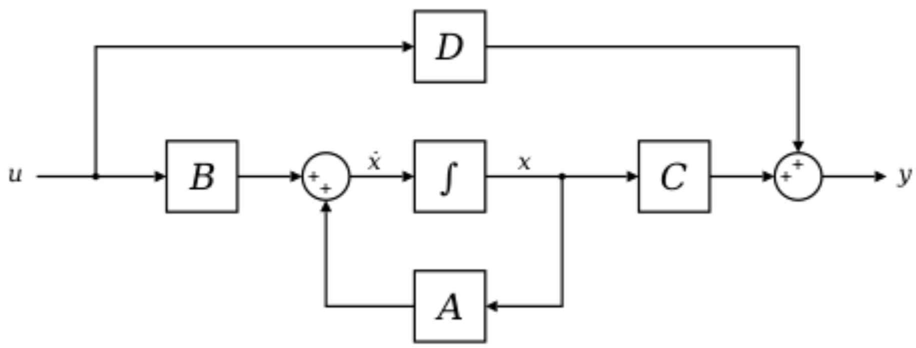
The second variable in the Bu term, the u vector, represents control inputs. In Euler's equation this would be the moment values applied to the spacecraft. These desired moments will be represented by a state outputted by ODE 45, in this case the angular velocities multiplied with the gain values.

$$u = \begin{bmatrix} -K * \omega_1 \\ -K * \omega_2 \\ -K * \omega_3 \end{bmatrix} \tag{16}$$

After determining these values, a state space model can be created as seen in Equation 17. This model represents the angular acceleration of the spacecraft in terms of one set of variables, the angular acceleration vector. This state space equation establishes the relationship between all variables and therefore, the motion of the system. ODE 45 can then take this relationship and iterate through a range of time values. By finding the angular acceleration steps between each time step, all the integral variables can be determined such as angular velocity or angular displacement.

$$\begin{bmatrix}\dot{\omega_1}\\ \dot{\omega_2}\\ \dot{\omega_3}\end{bmatrix} = \begin{bmatrix} 0 & \frac{(I_{22}-I_{33})\omega_3}{2*I_{11}} & \frac{(I_{22}-I_{33})\omega_2}{2*I_{11}} \\ \frac{(I_{33}-I_{11})\omega_3}{2*I_{22}} & 0 & \frac{(I_{33}-I_{11})\omega_1}{2*I_{22}} \\ \frac{(I_{11}-I_{22})\omega_2}{2*I_{33}} & \frac{(I_{11}-I_{22})\omega_2}{2*I_{33}} & 0 \end{bmatrix} \begin{bmatrix}\omega_1\\ \omega_2\\ \omega_3\end{bmatrix} + \begin{bmatrix} \frac{1}{I_{11}} & 0 & 0 \\ 0 & \frac{1}{I_{22}} & 0 \\ 0 & 0 & \frac{1}{I_{33}} \end{bmatrix} \begin{bmatrix} -K*\omega_1 \\ -K*\omega_2 \\ -K*\omega_3 \end{bmatrix} \tag{17}$$

The initial values for angular velocities are given through the initial conditions provided for the project as seen in Equation 14. Using these velocity values and the inertia tensor applied in Equation 1, an initial angular acceleration for each axis can be created.



For all the spacecraft control loops, the function seen above is used as a basic format. The A and B matrices are determined as mentioned previously. The feedforward matrix is unused, and the C matrix is implicit to the ODE function. For both the angular velocity control and angle control, the u input is the previous time step parameters multiplied by the selected gain. The y output values are the values output by the function for each timestep. For each timestep of ODE 45, the functions previous output values become the new input values and the cycle repeats until the desired outputs are achieved.

3.2 Selecting Gain Values

1. Although the state-space model represents the motion of our spacecraft, there are still values needed to input into the model to control the movement of the spacecraft. This is done using a gain value in the u matrix of the state space model. This K or gain value is found using the A and B matrices along with some desired inputs. These inputs mainly revolve around 2 factors, a settling criterion and settling time. The settling time and criterion are chosen arbitrarily. For controlling the angular velocities, a settling time of 25 seconds was chosen and a settling criterion of 2%. A 2% settling criterion has the equation form as seen below.

$$t_s = \frac{4}{\xi \omega_n} \tag{18}$$

From this equation, with the desired settling time, an equation for the product of the damping ratio and natural frequency can be found. To determine the damping ratio of the system, a natural frequency for each system needs to be found. The natural frequency can be found by evaluating the state space matrix at the initial conditions. After finding the state at initial conditions, this state space representation is translated to a transfer function and evaluated by the *damp()* function. This function returns the natural frequency of the un-compensated transfer function. This natural frequency can be implemented with the desired settling time to determine the damping ratio. Once natural frequency and damping ratio are attained, the following equation can be used to determine pole locations.

$$\lambda_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} \tag{19}$$

After determining desired pole locations, the gain can now be found using the *place()* function. This function takes the A and B matrix from the state space representation along with the desired pole location to output a gain which will attain the settling time and damping desired by the user. After outputting the K values from the *place()* function, the gains can be imported into the u matrix for control.

3.3 Single Axis Rotations

1. For controlling rotation about each axis, a different control method is necessary than for controlling the angular accelerations alone. Although the A, x, B, and u matrices are kept the same, the main change is in the output of the rotation functions. Equations relating the Euler angles with the angular velocities around the body frame can be used to continuously output the Euler rates to determine the Euler angles from ODE 45. The equations are as follows:

$$\dot{\Phi} = \frac{\cos(\Psi)\,\omega_2 + \sin(\Psi)\,\omega_1}{\sin\,(\Theta)} \tag{20}$$

$$\dot{\Theta} = \cos(\Psi)\,\omega_1 - \sin(\Psi)\,\omega_2 \tag{21}$$

$$\dot{\Psi} = \omega_3 - \left[\frac{\cos(\Psi)\,\omega_2 + \sin(\Psi)\,\omega_1}{\sin(\Theta)}\right] * \cos\,(\Theta) \tag{22}$$

These equations ae used in the first two controllers to track the Euler angles, however, in this stage of control these angles will be altered with one outside term to aid in steering the spacecraft. To reach the desired angles, tracking error controls can be implemented. To establish a method of error tracking, the difference between the present Euler angles and the desired Euler angles will be found. Throughout the use of ODE 45, this difference will be continuously determined with each time step. This error essentially becomes the angular velocity which then results in movements of the Euler angle. While the error is at its greatest, the angular velocity is at its greatest and while the error is at its least value, the angular velocity is at its least, resulting in an exponential decay towards the desired value. The equations governing this relationship can be seen below as an example for the phi rotation:

$$\Phi_{error} = \Phi_{desired} - \Phi \tag{23}$$

$$\dot{\Phi} = \dot{\Psi} + \Phi_{error} \tag{24}$$

The angle difference is combined with the existing Euler rate equations to continuously change the Euler rates until the desired Euler angle is achieved. Although this step of the process also uses the A, x, B, and u matrices, because the angular velocity values are near zero, all of these matrices also approach zero and thus do not affect the output Euler angles.

4. Results

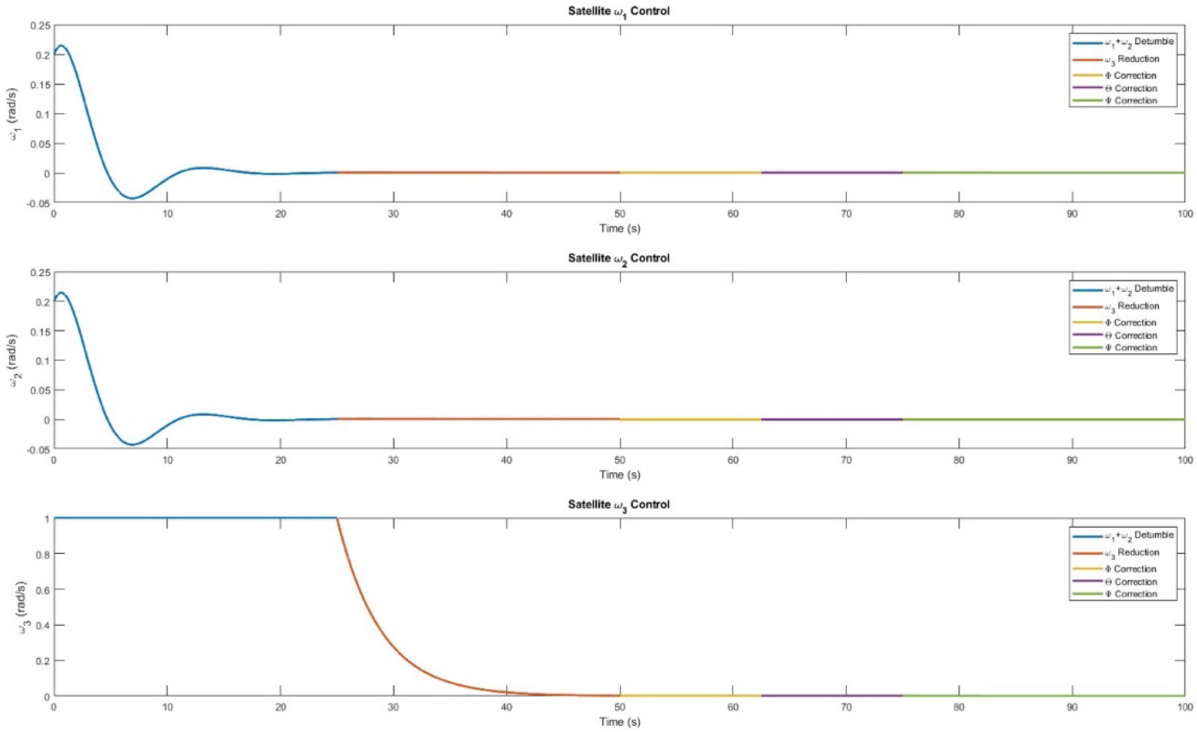


Figure 2: Angular velocity during detumble

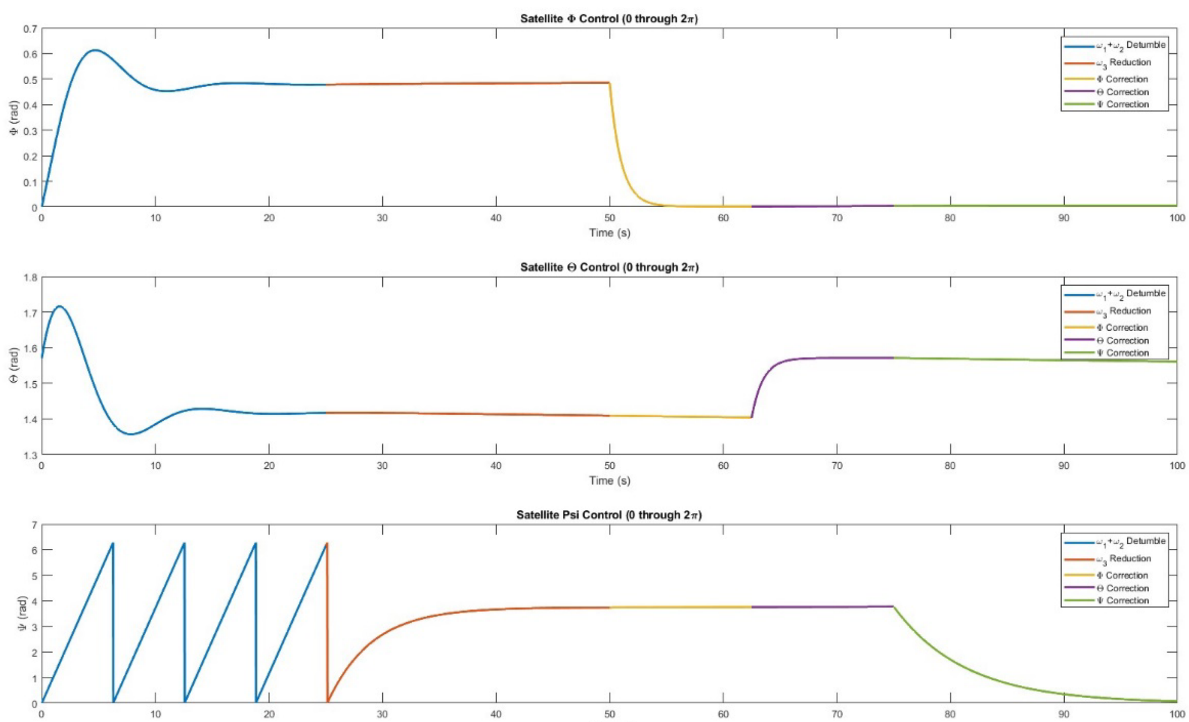


Figure 3: Single Axis Orientation

5. Discussion

Looking at Figure 1 and 2, these display angular velocity and orientation respectively during detumble. The first controllers (blue stage) goal is to drive both ω_1 and ω_2 to zero. While another controller (orange stage) is required to drive ω_3 to zero. As seen in the figures, these controllers were a success with blue stage controller reaching zero while ω_3 remained one. Following with the orange stage approaching zero keeping ω_1 and ω_2 near zero, once this has happened the system has become stable about the body axes.

The next problem was to orientate the spacecraft, using single axis rotations. The Euler angles can now be controlled, in the order of phi (yellow stage), theta (purple) and finally psi (green). Using the final values from the last ODE controller for the next controller, the angles can be continuously tracked from one phase to another. As seen in Figure 3, first the phi angle is driven towards zero as seen in yellow. After establishing the phi angle, the theta angle is next shifted to a value of $\frac{\pi}{2}$ in the purple section. Finally, the psi angle is shifted towards zero. It should be noted that the angles are varied from 0 to 2π, the psi value should be around 30 due to a constant increase in the psi angle for the first 25 seconds. The final conditions for the satellite are zero for every angular velocity value, an angle of zero for phi and psi, and finally an angle of $\frac{\pi}{2}$ for theta. All these stages can be seen in the attached video.

6. Conclusion

From the end points of the graphs, it is shown that detumble and orientation of the spacecraft were a success. In the presented example, the rapid settling time for a satellite was chosen due to computation time. However, a more realistic approach would have an extended detumble and orientation period to a time of ten minutes or more depending on several factors of the satellite. This approach would aim to mitigate rapid changes of angular velocity, thereby creating less load on the spacecraft. Resulting in enhanced accuracy for orientation, and overall, more reliable spacecraft performance.

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Yousuf – Introduction: Theory

Colby Davis – Matlab, 3D Video, Introduction, Discussion, Conclusion

Connor – Matlab, Discussion, Methodology

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Appendix