Linear Algebra - Assignment 5

Colby Rush

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1 Problem 1

AB_1 $= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} a & 0 \\ c & 0 \end{pmatrix}$, B_1A $= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} =$
$\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix}$
AB_2 $= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & a \\ 0 & c \end{pmatrix}$, B_2A $= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} =$
$\begin{pmatrix} c & d \\ 0 & 0 \end{pmatrix}$
Since A, B_1, and B_2 commute, this means that the result of c and b for AB_1 equals 0, with a being any value. This also means that the result of c for in AB_2 equals zero, and that d can be any value. Thus, $a = d$ and $b = c = 0$.

2 Problem 2

All of the matricies except A^2 + 2AB + B^2 will equal $(A + B)^2$, because they all give the same result, where AB+BA will cancel itself out.

3 Problem 3

1. $A^2 = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} = \begin{pmatrix} cos^2\theta + -sin^2\theta & cos\theta - sin\theta + -sin\theta cos\theta \\ sin\theta cos\theta + cos\theta sin\theta & -sin^2\theta + cos^2\theta \end{pmatrix} =$
$\begin{pmatrix} cos2\theta & -sin2\theta \\ sin2\theta & cos2\theta \end{pmatrix}$, making the pattern clear that $A^k = \begin{pmatrix} cosk\theta & -sink\theta \\ sink\theta & cosk\theta \end{pmatrix}$
2.

4 Problem 4

1. $\begin{pmatrix} 5 & 7 \\ -3 & -6 \end{pmatrix}$ yields the row echelon form $\begin{pmatrix} -3 & -6 \\ 0 & -9 \end{pmatrix}$, giving a determinant of 27, meaning it is invertible.

2. $\begin{pmatrix} -4 & 6 \\ 6 & -9 \end{pmatrix}$ has no consistent row echelon form, meaning it's determinant is zero, so it is not invertible.

3. $\begin{pmatrix} 5 & 0 & 0 \\ -3 & -7 & 0 \\ 8 & 5 & -1 \end{pmatrix}$ yields the determinant of 35, meaning it is invertible.

4. $\begin{pmatrix} -7 & 0 & 4 \\ 3 & 0 & -1 \\ 2 & 0 & -9 \end{pmatrix}$ has no consistent row echelon form, meaning it's determinant is zero, so it is not invertible.