Linear Algebra - Assignment 10

Colby Rush

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1 Problem 1

1. It's a simple matter of translating the c values of each part, yielding $\begin{pmatrix} 2 & 2 \\ -1 & 1 \end{pmatrix}$

2. Again, translate the c values of each part, yielding $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

2 Problem 2

1. We simply combine the vectors v_1 and v_2 and multiply it against A, finding $u_1 = \begin{pmatrix} 10 \\ 32 \end{pmatrix}$ and $u_2 = \begin{pmatrix} 3 \\ 10 \end{pmatrix}$

3 Problem 3

1. We set the equation as $\begin{pmatrix} 3 & 2 \\ 4 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} 3-\lambda & 2 \\ 4 & 1-\lambda \end{pmatrix}$

   Then we find the determinant of that, which is $\lambda^2 - 4\lambda + 5$. We can then factor it into $(\lambda + 5)(\lambda - 1)$, which gives us eigenvalues of $5, -1$.

   From this, we can set the system of equations for value 5 equal to:
   $-2x_1 + 2x_2 = 0$
   $4x_1 + -4x_2 = 0$
   or
   $\begin{pmatrix} -2 & 2 & 0 \\ 4 & -4 & 0 \end{pmatrix}$
   It's obvious to see the eigenspace is $c\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ for value 5.
   Using the same method, we reach a space of
   $4x_1 + 2x_2 = 0$
   $4x_1 + 2x_2 = 0$
   Again, obvious to see, after setting x_1 equal to -t, that the eigenvector for value -1 is equal to $c\begin{pmatrix} -1 \\ 2 \end{pmatrix}$

2. Using the same methods above, we find the determinant of $(-\lambda-1)(\lambda^2-1)$, giving eigenvalues of $-1, 1$ and eigenvectors $c\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ for value -1, and $c\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ for value 1.

3. Using the same methods above, we find the determinant of $-\lambda^3+3\lambda^2-2\lambda$, giving eigenvalues of $2, 1, 0$ and eigenvectors $c\begin{pmatrix} 7 \\ 3 \\ 1 \end{pmatrix}$ for value 2, and $c\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ for value 1, and $c\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

4. Using the same methods above, we find the determinant of $(2-\lambda)(3-\lambda)(\lambda^2-3\lambda+2)$, giving eigenvalues of $3, 2, 1$ and eigenvectors $c\begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}$ for value 3, and $c\begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$ for value 2, and $c\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ for value 1.

4 Problem 4

1. To find the value of k, we first do the normal process of multiplying by the identity λ matrix: $\begin{pmatrix} -1 & k \\ 4 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = \begin{pmatrix} -1-\lambda & k \\ 4 & 3-\lambda \end{pmatrix}$, then we find the determinant:
$(-\lambda-1)(-\lambda+3)-4k = \lambda^2-2\lambda-3-4k$. Since we want at least one eigenvalue of 5, we just need to find a value k that replaces at least one of the pairs with $(\lambda-5)$. Since 7 + -5 still equals -2 (middle term), we need to solve for 4k = 32, which is 8. A k value of 8 will yield the matrix with a eigenvalue of 5.