ELEC412 Cheat Sheet

Chapter 3: PN Junctions

3.2 - Concept of PN Junction

Energy Band Diagram for a PN Junction

Built in potential: $\Phi_{bi} = kTln(\frac{N_A N_D}{n_i^2})$ **(1)**

I-V Characteristics of a PN Junction

$I = I_0 exp(V_a/V_t)$ **(2)**
Dynamic resistance: $R_{dyn} = \frac{d(V_a)}{dI} = \frac{V_t}{I}$ **(3)**
IV characteristics of pn junc. w/ rev. current:
$I = I_0 exp(V_a/V_t) - I_r$ **(4)**
If $I_r = I_0$ then $I = I_0[exp(V_a/V_t) - 1]$ - ideal diode eqn.**(5)**
Hole diffusion current flowing into the n-side in a p^+n junc.:
$I_p(x) = qD_p[d(p_n(x))/dx]A_{cs}$ **(6)**
$p_n(x) = p'exp(-x/L_p) + p_{n0}$ **(7)**
$d(p_n(x))/dx = -(p_n(x) - p_{n0})/L_p$ **(8)**
$I_p(x) = -qD_p(p_n(x) - p_{n0})A_{cs}/L_p$ **(9)**
$p' = p_{n0}[exp(qV_a/kT) - 1]$ **(10)**
$I_p = [qD_p n_i^2 A_{cs}/(N_D L_p)][exp(qV_a/kT) - 1]$ **(11)**
$I = I_p + I_n = I_0[exp(V_a/V_t)] - 1$ **(12)**
where $I_0 = qD_p n_i^2 A_{cs}/(N_D L_p) + qD_n n_i^2 A_{cs}/(N_A L_p)$
$I_G = qn_i W_j A_{cs}/(2\tau_0)$ **(13)**
$I = I_0 exp[(V_a/\eta_I V_t) - 1] - I_G$ **(14)**

Space-Charge and Charge Storage Effects

$C_{jun} = \varepsilon_s A_{cs}/W_j$ **(15)**
$W_j = \sqrt{2\varepsilon_s(V_{bi} - V_a)/(qN_{eff})}$ **(16)**
$W_j = [12\varepsilon_s(V_{bi} - V_a)/(qa)]^{1/3}$ **(17)**
$C_{diff} = [qA_{cs}(L_p p_{n0} + L_n p_{p0})/2V_t]exp(V_0/V_t)$ **(18)**
$C_{diff} \approx I_0 \tau_p/(2V_t)$ **(19)**

Tunnel PN Junctions

$D_{12} = D_b exp(-2d_b\sqrt{2m_e^*(\Phi_B - E)}/h')$ **(20)**
$I_{tun} = qA_{cs}\int_{E_c}^{\acute{E}}[f_t S_1 D_{12} S_2]dE$ **(21)**

Junction Breakdown

$V_{br} = \epsilon_s \acute{E}_{cr}^2/(2qN_{eff})$ **(22)**
$d(j_n)/dx - (\alpha_i - \beta_i)j_n = -(\alpha_i - \beta_i)j_T + \alpha_i j_T$ **(23)**
If $\alpha_i = \beta_i$ then $M_n = j_n(L)/j_n(0) = 1/[1 - \int_0^L(\alpha_i)dx]$ **(24)**
$V_{br} \approx 4E_g/q$ **(25)**

Noise in PN Junctions

$\hat{i}^2 = 2q(I_d + 2I_0)\delta f$ **(26)**

Heterojunctions

$\Delta E_c = \xi_1 - \xi_2$ **(27)**
$\phi_{bi} = E_{g1} - \Delta E_n - \Delta E_p + \Delta E_c$ **(28)**
$W_j = W_n + W + p$ **(29)**
$W_n = \sqrt{2\epsilon_1\epsilon_2 N_A V_{bi}/[qN_D(\epsilon_2 N_D + \epsilon_1 N_A)]}$
$W_p = \sqrt{2\epsilon_1\epsilon_2 N_D V_{bi}/[qN_A(\epsilon_2 N_D + \epsilon_1 N_A)]}$
$C_{jun} = \sqrt{\epsilon_1\epsilon_2 N_D N_A/[2(\epsilon_2 N_D + \epsilon_1 N_A)V_{bi}]}$ **(30)**
$I = I_0(1 - V_a/V_{bi})[exp(V_a/V_t) - 1]$ **(31)**

3.3 - Schottky Junction

Schottky Junction at Equilibrium

$f_{vx} = \sqrt{[m_e^*/(2\pi\Phi_t)]exp[-m_e^* v_x^2/(2\Phi_t)]}$ **(32)**
$\Phi_t = kT$
$\langle v_x \rangle = \int_0^\infty[v_x f_{vx}]dv_x = \sqrt{\Phi_t/(2\pi m_e^*)}$ **(33)**
$I = -qA_{cs}\int_0^\infty[v_x \partial n/\partial E]dE$ **(34)**
$I_{sm} = A^*T^2 A_{cs}exp[(-\Phi_{bi} - E_n)/\Phi_t]$ **(35)**
$I_{ms} = -A^*T^2 A_{cs}exp[-\Phi_B/\Phi_t]$ **(36)**
At equilib., no net current flowing $I_{sm} = -I_{ms}$
give $\Phi_B = \Phi_{bi} + E_n$ **(37)**

Schottky Junction under Bias

$\Phi_{bi} = \Phi_B - E_n - qV_a$ **(38)**
Total fwd-bias current $I = I_{sm} + I_{ms}$ **(39)**
$= A^*T^2 A_{cs}exp(-\Phi_B/\Phi_t)[exp(V_a/V_t) - 1]$
$= I_0[exp(V_a/V_t) - 1]$
where $I_0(= A^*T^2 A_{cs}exp(-\Phi_B/\Phi_t))$ is the saturation current
$I_{tun} = I_f exp(\Phi_B/E_\infty)$ **(40)**
$E_\infty = qh/4\pi\sqrt{N_D/(\epsilon_s m_e^*)}$

Nonideal Schottky Junctions

Schottky barrier height $\Phi_B = E_g - \Phi_0$ **(41)**
$\Delta\Phi_B = q\sqrt{qE'/(4\pi\epsilon_s)}$ **(42)**
$I_p = (qD_p p_{n0} A_{cs}/L_p)[exp(V_a/V_t - 1)]$ **(43)**
$\gamma^* = I_p/I$ **(44)**
$I = A^*T^2 A_{cs}exp(-\Phi_B/\Phi_t)[exp(V_a/(\eta_I V_t)) - 1]$ **(45)**

Capacitance Effect and Equivalent-Circuit Model of a Schottky Junction

$C_{jun} = A_{cs}\sqrt{qN_D\epsilon_s}$ **(46)**

Modification of the Barrier Height

$\Phi_B^* = \Phi_B - q/\epsilon_s\sqrt{n_1 a'/4\pi}$ **(47)**
$\Delta d = [a'p_1 - (W - a')n_2]/p_1$ **(48)**
$\Phi_B^* = \Phi_B + q^2 p_1 \Delta d^2/2\epsilon_s$ **(49)**

3.4 - Metal-Semiconductor Contact

$R_c = \sqrt{R_{sh} r_c}/d_c coth[L\sqrt{R_{sh}/r_c}]$ **(50)**
$R_{sh} = r_{sheet} L_{sh}/d_c$ **(51)**
$R = R_c + R_{sh} + R_{sp}$ **(52)**
$r_c \approx exp[2V_{bn}\sqrt{\epsilon_s m_e^*/N_D}/(h')]$ **(53)**

3.5 - MIS Junction and Field-Effect Properties

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Glossary of Variables

Φ_{bi} is the build-in potential
$V_{bi}(= \Phi_{bi}/q)$ is the build-in voltage in V
k(8.62 x $10^{-5} ev/K$): Boltzmann constant
T: absolute temp. in Kelvin
n_i is the intrinsic carrier density of the semiconductor $/m^3$
N_A is the acceptor density $/m^3$
N_D is the donor density $/m^3$
V_a is the applied voltage and is a fixed voltage in V
I_0 is the saturation current in A
$V_t(= kT/q)$ is the thermal voltage (0.026V unless said otherwise)
R_{dyn} is the dynamic resistance (Ω)
I_r is the reverse current (or leakage current)
I_p is the hole diffusion current flowing into the n side in a p^+n
q (1.602 x 10^{-19} C) is the electron charge in C
D_p is the hole diffusivity in m^2/s
D_n is the electron diffusivity in m^2/s
A_{cs} is the cross section of the pn junc in m^2
$d(p_n(x))/dx$ is the hole density grad. in the n-side in $/m^4$
p' is the excess hole density at x=0 in $/m^3$
$p_{n0}(= n_i^2/N_D)$ is the equilib. hole density in the n-side in $/m^3$
$p_{p0}(= n_i^2/N_A)$
L_p is the diffusion length of the holes in m
L_n is the diffusion length of the electrons in m
I_G thermal generation current
τ_0 is the generation lifetime of the carriers in s
τ_p is the lifetime of the holes in s
W_j is the width of the junc. region in m
η_I is the ideality factor, takes into account the recomb. effect
ε_s is the semiconductor permittivity in F/m
C_{jun} is the junction capacitance in F
$N_{eff} = N_A N_D/(N_A + N_D)$
a is the gradient of the dopant density in $/m^4$
V_0 is the forward-bias voltage in V
D_{12} the transmission coefficient (prob. that tunnelling event will occur)
D_b is a constant
m_e^* is the effective mass of the electrons in kg
d_b is the barrier width in m
$h'(= 1.05$ x $10^{-34} J \cdot s)$ is Planck's constant divided by 2π
I_{tun} is the tunneling current in A
$S_1 and S_2$ are the respective densities of states of E. bands in the two sides of the PN junction in $/m^3$
f_t is a tunneling freq. parameter in $m^4/s \cdot eV$
E_c is the E. at the conduction band edge in eV
$\acute{E}$ is the upper E. limit for tunneling in eV
$\acute{E}_{cr}^2$ is the critical field
V_{br} is the breakdown voltage in V
j_n is the electron current density in A/m^2
j_p is the hole current density in A/m^2
$j_T(= j_n + j_p)$ is the total current density in A/m^2
α_i is the electron ionization rate in /m
β_i is the hole ionization rate in /m
L is the length of the ionization region in m
M_n is the electron multiplication factor
E_g is the E. gap of the semiconductor in eV
$\hat{i}^2$ is the shot noise (generated when carriers cross a barrier) in A^2
I_d is the diode current in A
I_0 is the saturation current in A

δf is the freq. range in Hz
ξ_1 and ξ_2 are the respective electron affinities in the two semiconductors
E_{g1} is the E. gap of the narrow-gap p-type semiconductor in eV
ΔE_n is the E. difference between the conduction band edge and the Fermi level in the wide-gap semiconductor in eV
ΔE_p is the E. difference between the valence band edge and the Fermi level in the narrow-gap semiconductor in eV
ϵ_1 and ϵ_2 are the respective permittivities of the p-type and n-type semiconductors in F/m
v_x is the velocity of the electrons in the +x direction in m/s
m_e^* is the effective mass of the electrons in kg
$\langle v_x \rangle$ is the average velocity of electrons in m/s
f_{vx} is the velocity distribution of the electrons
$\partial n/\partial E$ is the rate at which the electron density changes with E. band in $/m^3 \cdot eV$
I_{sm} is the current of electrons moving from the semiconductor into the metal (pos. value)
$A^*(= 4\pi q m_e^* k^2/h^3 = 1.2$ x $10^6 A/m^2 \cdot K^2$ is called the Richardson constant
E_n is the E. diff. between the semiconductor Fermi lvl and the conduction band edge in eV
Φ_B is the barrier height in eV
E_∞ is a parameter dependent on the dopant density in J
I_f is a pre-exponential constant that depends on the field-emission process in A
Φ_0 is the location of the surface Fermi level in eV
E' is the electric field in V/m
$\Delta\Phi_B$ is the resulting lowering in the barrier height in eV
- usually quite small ($\approx 0.01eV$)
I_p is the minority carrier current in A
γ^* is the minority carrier injection ratio
I is the total current flowing across the Schottky junction in A
a' is the dopant layer thickness in m
n_1 is the interface dopant density in $/m^3$
n_2 is the substrate donor density in $/m^3$
W is the depletion-layer width in m Δd is the distance away from the interface in m
Φ_B^* is the effective barrier height in m
p_1 is the surface dopant density in $/m^3$
p_2 is the surface dopant density in $/m^3$
R_c is the contact resistance in Ω
R_{sh} is the shunt resistance in Ω
r_c is the specific contact resistance in $\Omega \cdot m^2$
d_c is the width of the contact in m
L is the contact length in m r_{sheet} is the semiconductor sheet resistance in Ω/square
L_{sh} is the length of the shunt path in m