① Big-O :-

$\Rightarrow$ The function that needs to be analysed is $T(x)$.

$T(x) \geq 0 \quad , x \geq 0$

We say,

$T(x) = O(f(x))$ if

$T(x) \leq a.f(x)$

for all $a > 0$, and $x > b$
(b = any constant)

e.g. Let,

$T(x) = x^2 + 10x + 20$

$= (x+5)^2 - 5$

Let $a = 2$

$T(x) \leq 2(x+5)^2 - 5$

now, $a > 0$ and $x > 5$

$\therefore T(x) = O(x+5)^2 \simeq O(x^2)$

$\therefore$ ~~$T(x) = O(x^2)$~~

$$\boxed{T(x) = O(x^2)}$$

Big Omega (Ω)

We say,

$$T(x) = \Omega(f(x)) \text{ if}$$

$$T(x) \geq a \cdot f(x) \text{ where}$$

$$a > 0 \quad , \quad x > b$$

eg. Let,

$$T(x) = x^2 + 10x + 20$$

$$= (x+5)^2 - 25$$

Let $a = 1$, $b = -25$

$T(x) \geq (x+5)^2 - 25$ if $x > -25$

$$T(x) = \Omega(x^2)$$

⟹ Big Theta (Θ).

When we say that particular function $T(x)$'s running time is $\Theta(n)$, we're saying that once n get large enough, the running time is at least $k_1 n$ and at the most $k_2 n$ for constants k_1 & k_2.

e.g. Let $T(x) = x^2 + 10x + 20$

if x gets too large,

$$K_1 x^2 \leq T(x) \leq K_2 x^2$$

$\therefore \quad T(x) = \Theta(x^2)$

also if we observe big O & big Ω of this function we see both of them are of the power x^2

$\therefore \quad T(x) = \Theta(x^2)$