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Homework 1

1) In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$).

   Answer:

2) Show that, if c is a positive real number, then $g(n) = 1 + c + c^2 + \ldots + c^n$ is:

   (a) $\Theta(1)$ if $c < 1$
   (b) $\Theta(1)$ if $c = 1$
   (c) $\Theta(1)$ if $c > 1$

   Answer:
   If c = 1, $g(n) = 1 + 1 + \ldots + 1 = n + 1 = \Theta(n)$. Otherwise:

   $$g(n) = \frac{c^{n+1} - 1}{c - 1} = \frac{1 - c^{n+1}}{1 - c}$$

   If $c < 1$, then $1 - c < 1 - c^{n+1} < 1$. So, $1 < g(n) < \frac{1}{1-c}$. Thus, $g(n) = \Theta(1)$

   If $c > 1$, then $c^{n+1} > c^{n+1} - 1 > c^n$. So, $\frac{c^n}{1-c} < g(n) < \frac{c}{1-c} * c^n$. Thus, $g(n) = \Theta(c^n)$

3) Determine the number of Paths of length 2 in a complete graph of n nodes. Give your answer in Big-O notation as a function of n.

   Answer: