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Homework 1

1) In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$).

Answer:

(a) $f(n) = n - 100$ and $g(n) = n - 200$
Both are $O(n)$, so $f = \Theta(g)$

(b) $f(n) = n^{1/2}$ and $g(n) = n^{2/3}$
Since they're both power s of n, compare the powers. $\frac{1}{2} < \frac{2}{3}$ so $f = O(g)$

(c) $f(n) = 100n{+}\log\, n$ and $g(n) = n + (\log\, n)^2$
They are both $O(n)$ so $f = \Theta(g)$

(d) $f(n) = n\, \log\, n$ and $g(n) = 10n\, \log\, 10n$
They are both $O(\log\, n)$ so $f = \Theta(g)$

(e) $f(n) =\log\, 2n$ and $g(n) =\log\, 3n$
They are both $O(\log\, n)$ so $f = \Theta(g)$

(f) $f(n) = 10\, \log\, n$ and $g(n) =\log\, (n^2)$
Both are $O(\log\, n)$ so $f = \Theta(g)$

(g) $f(n) = n^{1.01}$ and $g(n) = nlog^2n$
If both sides are divided by n we then need to compare $n^{0.01}$ and log^2n.
Ultimately, the power function wins, so $f = \Omega(g)$

(h) $f(n) = \frac{n^2}{logn}$ and $g(n) = n(\log\, n)^2$
Divide both sides by $\frac{n}{logn}$ and we will only need to compare n and $(logn)^3$.
The result is $f = \Omega(g)$

(i) $f(n) = n^0.1$ and $g(n) = (\log\, n)^{10}$
Similar to problems g and h, $f = \Omega(g)$

(j)
(k)
(l)
(m)
(n)
(o)
(p)
(q)

2) Show that, if c is a positive real number, then $g(n) = 1 + c + c^2 + \ldots + c^n$ is:

(a) $\Theta(1)$ if $c < 1$
(b) $\Theta(1)$ if $c = 1$

(c) $\Theta(1)$ if $c > 1$

Answer:
If c = 1, $g(n) = 1 + 1 + ... + 1 = n + 1 = \Theta(n)$. Otherwise:

$$g(n) = \frac{c^{n+1}-1}{c-1} = \frac{1-c^{n+1}}{1-c}$$

If $c < 1$, then $1 - c < 1 - c^{n+1} < 1$. So, $1 < g(n) < \frac{1}{1-c}$. Thus, $g(n) = \Theta(1)$

If $c > 1$, then $c^{n+1} > c^{n+1} - 1 > c^n$. So, $\frac{c^n}{1-c} < g(n) < \frac{c}{1-c} * c^n$.
Thus, $g(n) = \Theta(c^n)$

3) Determine the number of paths of length 2 in a complete graph of n nodes. Give your answer in Big-O notation as a function of n.

Answer: