Complex Networks Exercise 1 - Network Models ¶

Cole MacLean - April 3, 2016

Part 1 - Erdös-Rényi Networks

In this section, a function is defined to take as inputs the number of nodes and probably of 2 nodes being linked to generate an ER random graph. Histograms are then plotted for various combinations of node count and probability of linkage to analyse the characteristics of ER random networks.

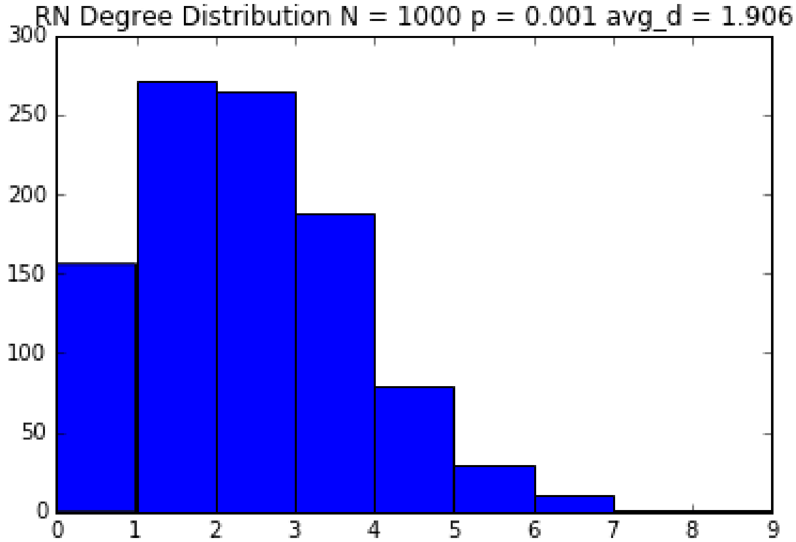
In [1]:

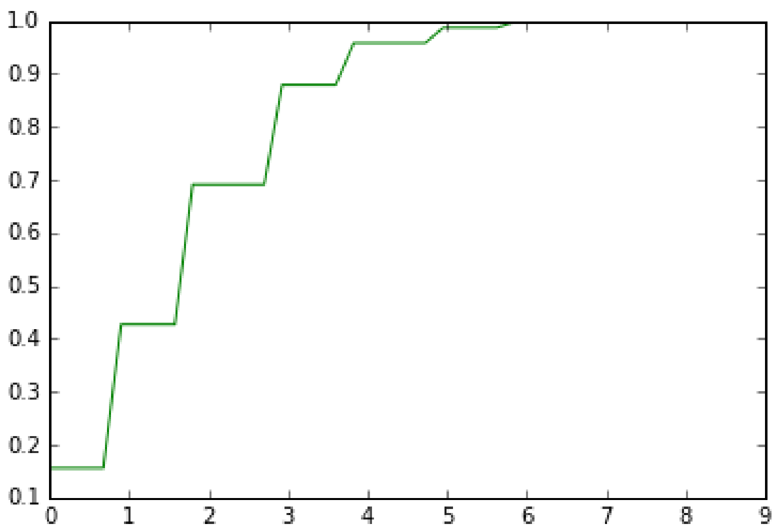
```
import random
import numpy as np
import matplotlib.pyplot as plt
import powerlaw as pl
import warnings
warnings.filterwarnings('ignore')
%matplotlib inline

def rand_ER_network(node_count,link_prob):
    network = ({node:[link for link in range(node_count) #generator that ran
                if random.random() <= link_prob]         #directed network
                for node in range(node_count)})
    for node,links in network.items(): #search over network to develop non-d
        for link in links:             #to the current node if not already c
            if node not in network[link]:
                network[link].append(node)
    return network
```

In [25]:

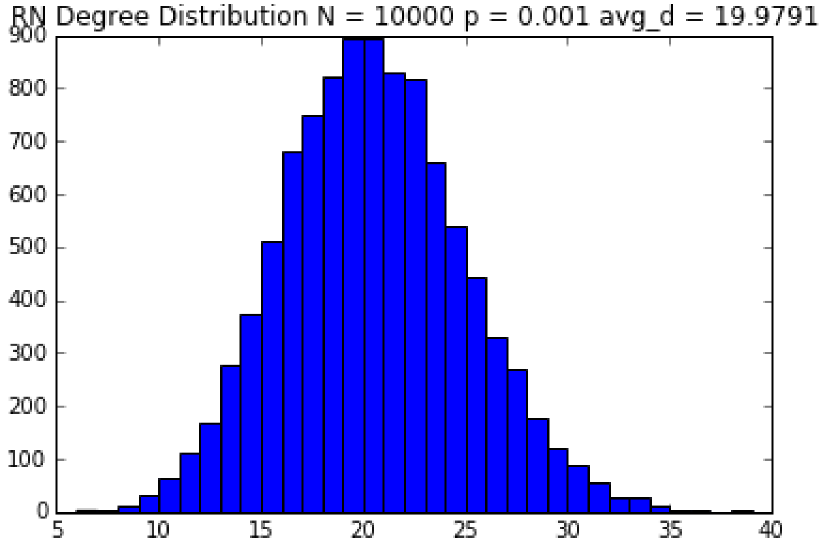
```
def plot_distributions(node_count,link_prob):
    network = rand_ER_network(node_count,link_prob)
    degrees = [len(network[i]) for i in network.keys()] #Extract node degree
    plt.hist(degrees, bins=range(min(degrees), max(degrees) + 1, 1))
    plt.title("RN Degree Distribution N = " + str(node_count) + " p = " + st
    plt.show() #plot degree distribution
    values, base = np.histogram(degrees, bins=40)
    cumulative = np.cumsum(values)/node_count
    plt.plot(base[:-1], cumulative, c='green')
    plt.show() #plot cummulative distribution
    return degrees
```

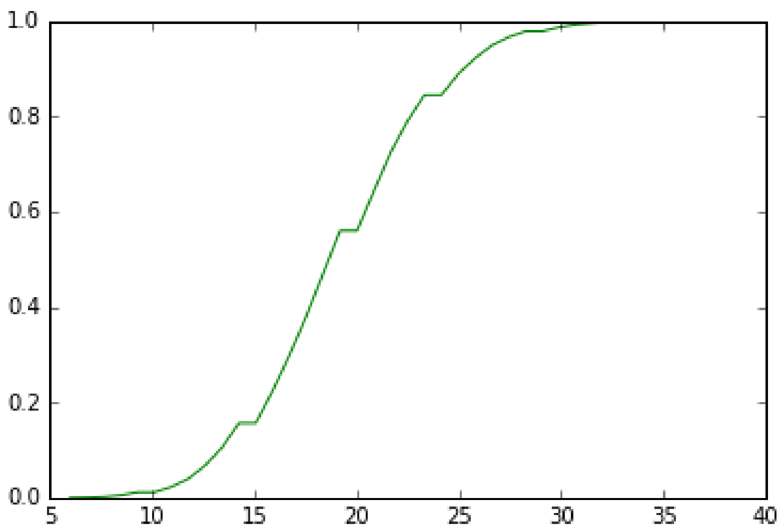
In [37]:
```
degrees = plot_distributions(1000,0.001)
```



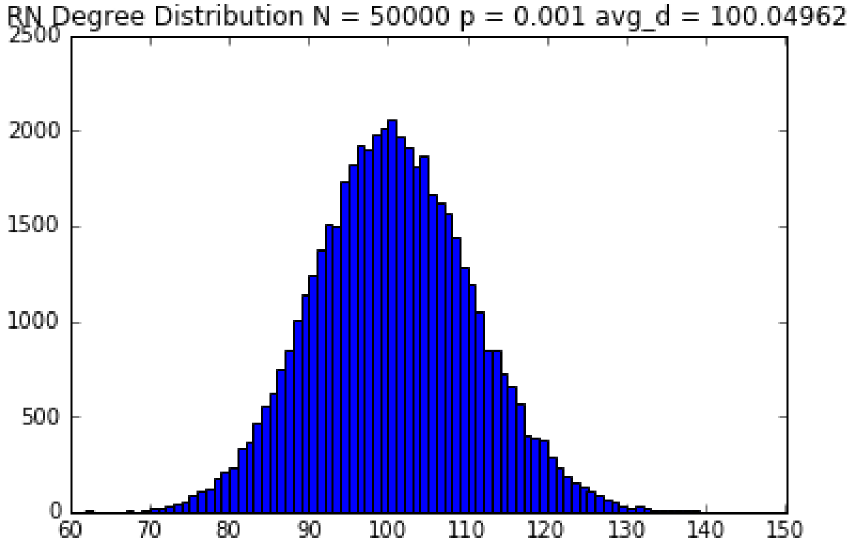


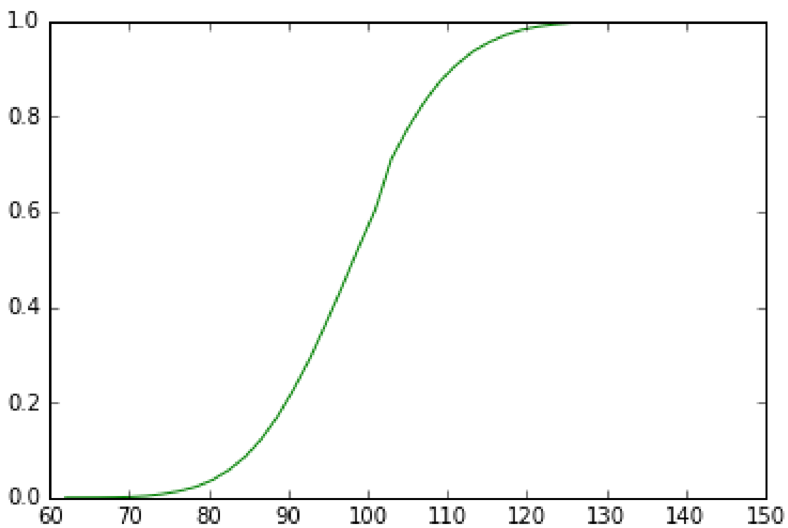
In [38]:
```
degrees = plot_distributions(10000,0.001)
```



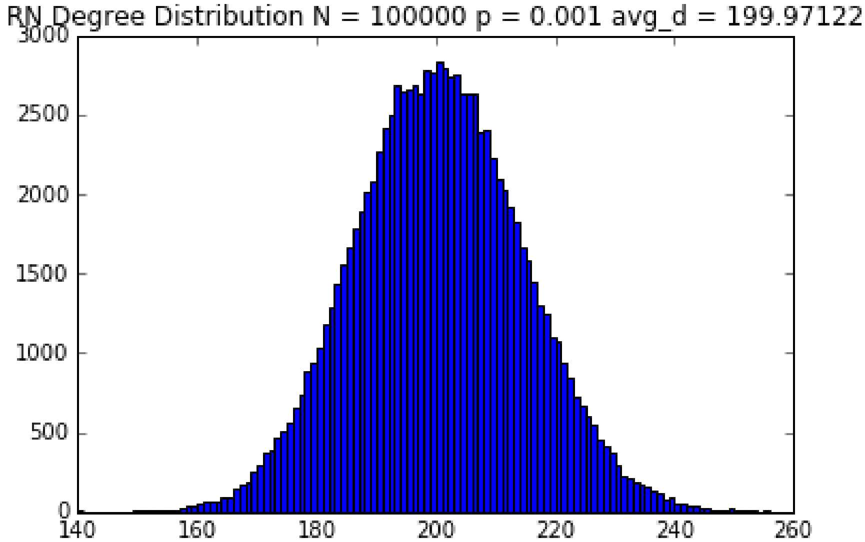


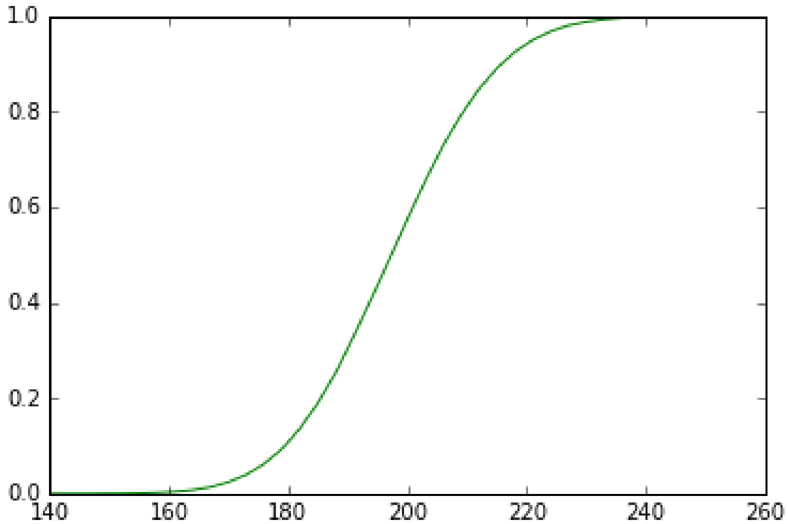
In [39]:
```
degrees = plot_distributions(50000,0.001)
```



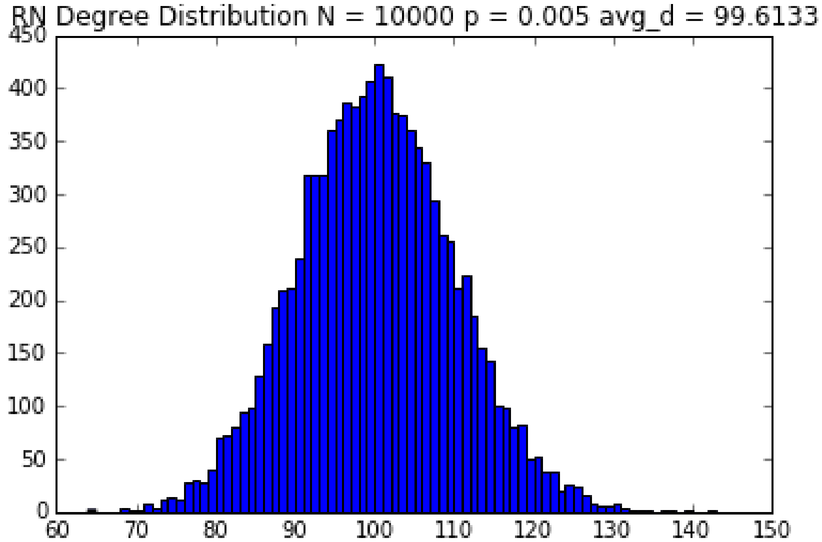


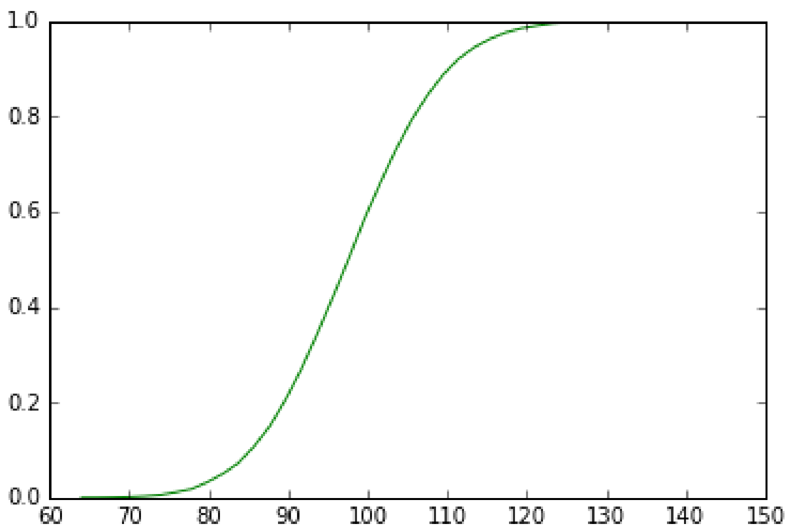
In [40]:
```
degrees = plot_distributions(100000,0.001)
```



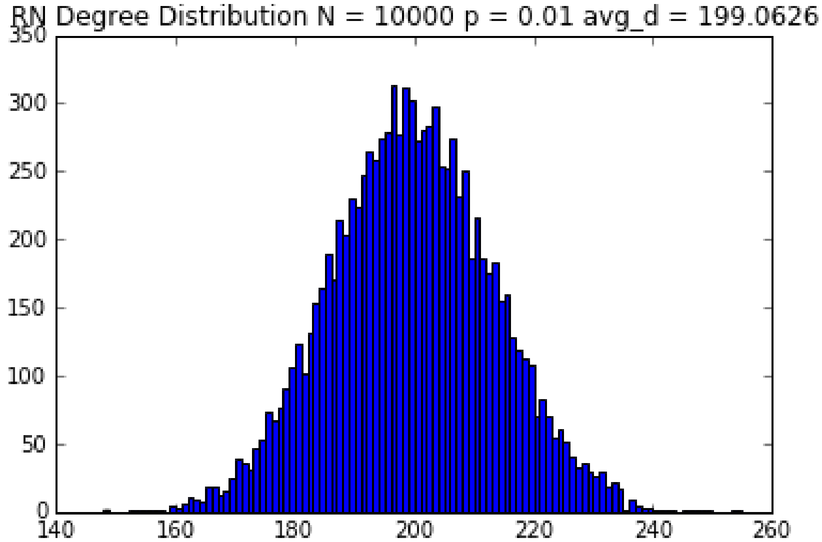


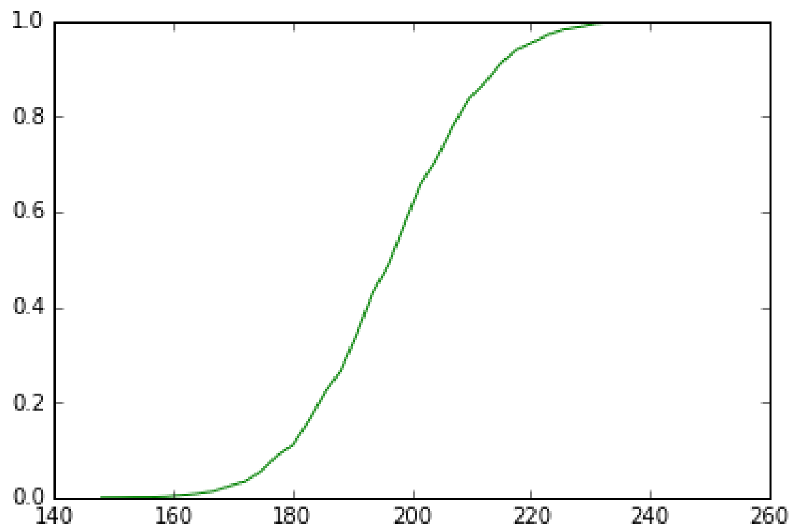
In [41]:
```
degrees = plot_distributions(10000,0.005)
```



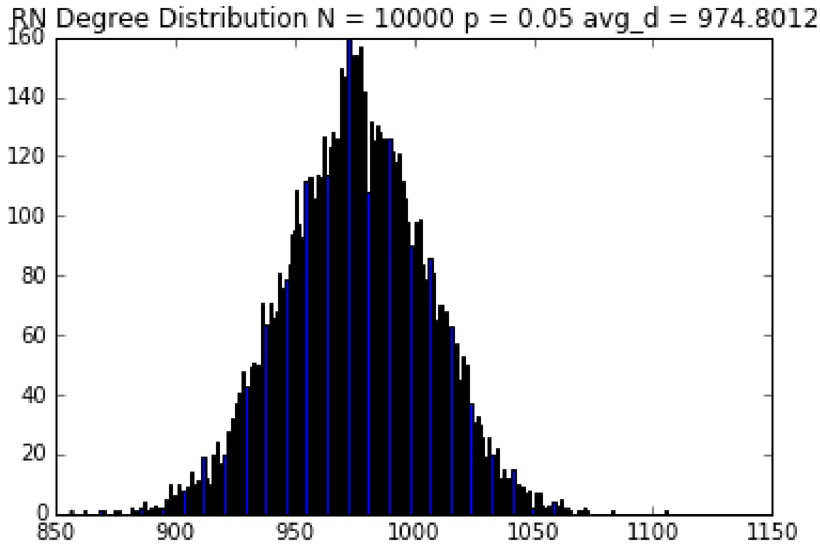


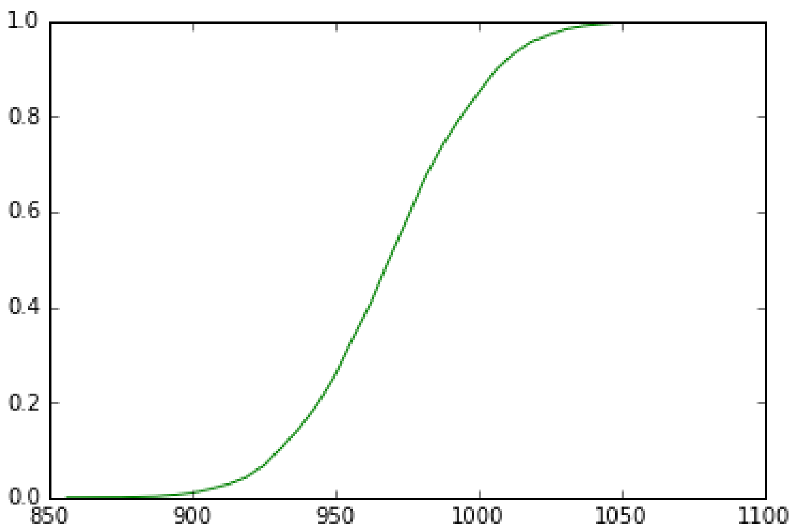
In [42]:
```
degrees = plot_distributions(10000,0.01)
```





In [43]:
```
degrees = plot_distributions(10000,0.05)
```





From the plots, we can see that the ER random networks tend to normality, and converge to Poisson distributions for larger and larger N.

Part 2 - Barabási–Albert (BA) Preferential Attachment Model

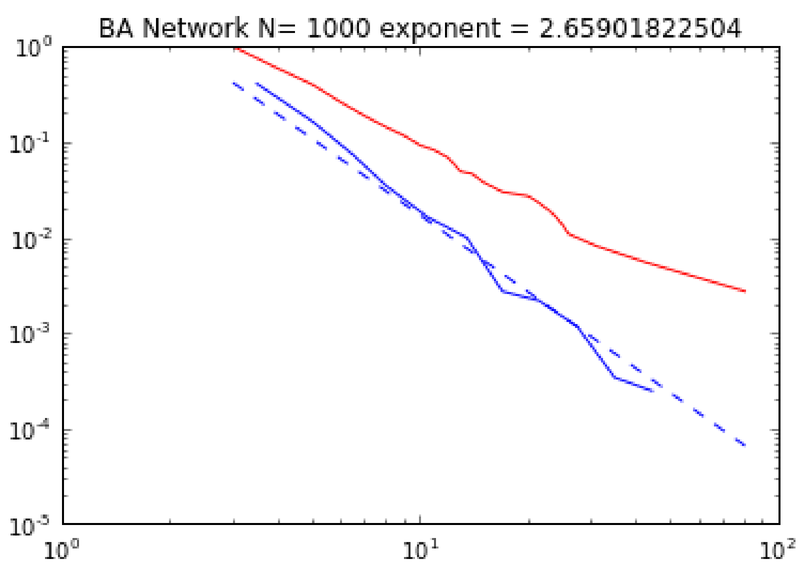
In this section, the Barabási–Albert method of using growth and preferential attachment to model complex networks is analyzed

In [2]:
```
def BA_network(start_node_count,start_link_prob,tot_nodes):
    network = rand_ER_network(start_node_count,start_link_prob)#Random ER ne
    for new_node in range(start_node_count,tot_nodes):#growth iterator, addi
        network[new_node] = [] #add new node to network
        tot_network_degrees = sum([len(network[i]) for i in network.keys()])
        for node,links in network.items():#preferential attachement iterator
            if tot_network_degrees:#because original network initiated with
                link_prob = len(links)/tot_network_degrees#of network with s
            else:
                link_prob = start_link_prob#use original start_link_prob if
            if random.random() <= link_prob:#connect nodes non-directionally
                network[new_node].append(node)
                network[node].append(new_node)
    return network
```

In [58]:
```
import powerlaw as pl
def plot_log_distributions(node_count,link_prob,tot_nodes):
    network = BA_network(node_count,link_prob,tot_nodes)
    degrees = [len(network[i])+1 for i in network.keys()] #Extract node degr
    fig = plt.figure()
    ax = fig.add_subplot(111)
    fit = pl.Fit(np.array(degrees),discrete=True, fit_method='Likelihood')
    fit.power_law.plot_pdf( color= 'b',linestyle='--',label='fit ccdf',ax=ax
    fit.plot_pdf( color= 'b',ax=ax)
    ax.set_title('BA Network N= '+ str(tot_nodes) + ' exponent = ' + str(fit
    fit.plot_ccdf( color= 'r',ax=ax)
    return degrees
```

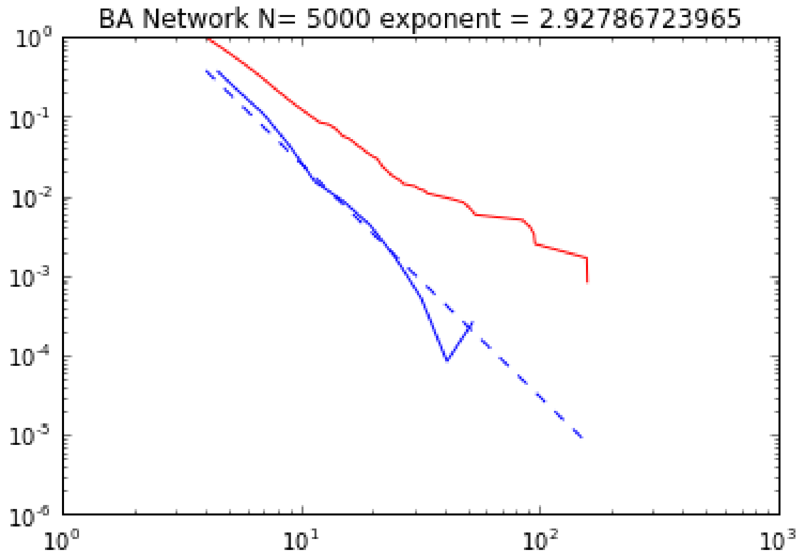
In [54]:
```
degrees = plot_log_distributions(2,0.5,1000)
```

```
Calculating best minimal value for power law fit
```



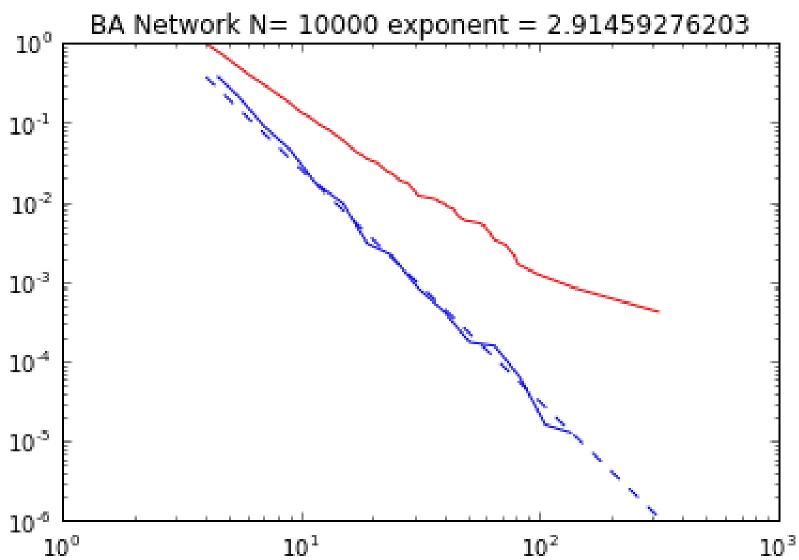
In [55]:
```
degrees = plot_log_distributions(2,0.5,5000)
```

Calculating best minimal value for power law fit



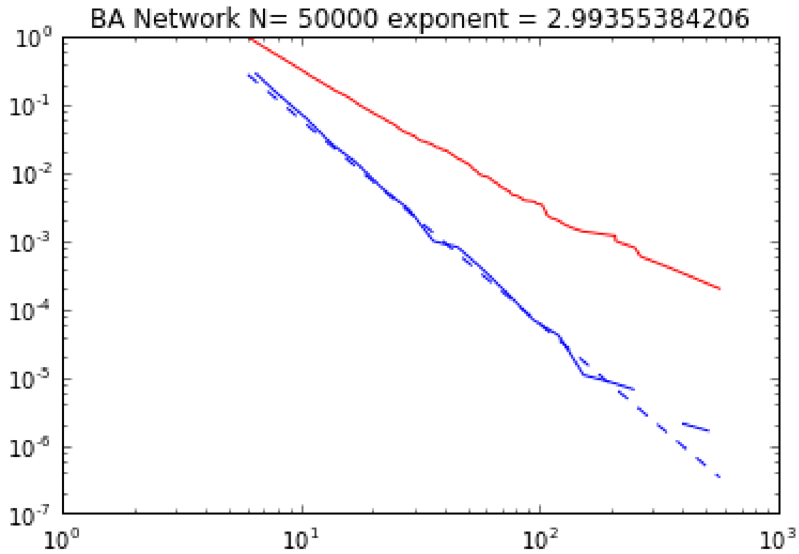
In [56]:
```
degrees = plot_log_distributions(2,0.5,10000)
```

Calculating best minimal value for power law fit



In [57]:
```
degrees = plot_log_distributions(2,0.5,50000)
```

```
Calculating best minimal value for power law fit
```



Using pythons powerlaw library to compute the pdf and ccdf of various sized BA networks, we can see that the exponent of the power law distributions converge to 3.0 for large values of N.