Multicriteria Methods and Logic Aggregation in Suitability Maps

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In this paper, we identify and describe fundamental logic properties of multicriteria methods for land-use suitability analysis and the design of suitability maps. The existing multicriteria methods can be evaluated from the standpoint of their ability to support the necessary logic properties that affect the expressive power of evaluation methods. The paper investigates and compares simple additive scoring, multiattribute value technique, multiattribute utility technique, analytic hierarchy process, ordered weighted average, outranking methods, and logic scoring of preference (LSP). We introduce canonical forms of logic aggregation in suitability maps and show how to use canonical aggregation structures to design LSP suitability maps that evaluate distributions of points of interests (POIs) in urban areas.

1. INTRODUCTION

This paper has four main goals. The first goal is to identify logic properties that affect the expressive power of multicriteria decision methods (MCDM) that can be used to create suitability maps of a specific geographic region. The second goal is to investigate land-use suitability assessment methods that are based on simple additive scoring (SAS), multiattribute value technique (MAVT), multiattribute utility technique (MAUT), analytic hierarchy process (AHP), ordered weighted average (OWA), outranking methods (ELECTRE and PROMETHEE), and logic scoring of preference (LSP) in view of satisfaction of the identified logic properties. The third goal is to identify and evaluate canonical forms of logic aggregation structures that can be used in the design of suitability maps. The fourth goal is to show a method and a tool for making realistic and useful LSP suitability maps.

It is important to note that MCDM cover an area that is much wider than the decision problems we study in this paper. We investigate only the cases where a

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decision issue has progressed to the point where stakeholders are ready to take an action, where multiple alternatives are identified as physical locations in a geographic (or urban) area, and these alternatives must be evaluated in detail. In addition, we are interested only in cases beyond additive models, i.e., situations where decision makers (DMs) need and are able to specify logic conditions that evaluation criteria must satisfy to reflect stakeholder requirements. From the standpoint of genuine MCDM (e.g., Ref. 1), this is a rather specific situation because MCDM frequently investigate cases where DMs have difficulties in understanding what they really want to do and the decision-aiding procedure is used as a mechanism for developing preferences and value judgments.

In the evaluation of geographic locations, we usually deal with very large number of alternatives (locations) that include both dominated and nondominated (Pareto optimal) cases.

We assume that the analyzed geographic region is divided into square cells (typical size 5–100 m) and that each cell with central coordinates X, Y is characterized by an array of suitability attribute values $a_1(X, Y), \ldots, a_n(X, Y)$, $n \geq 1$.[2] The attributes are defined as quantitative parameters that affect the suitability of a cell for some specific land use (e.g., housing, recreation, agriculture, industrial development). The set of attributes must be complete, i.e., it must include *all* relevant components. Generally, the attributes must be justifiable and not redundant with each other.

The purpose of MCDM is to provide a criterion function $\sigma : \mathbb{R}^n \to [0, 1]$ for computing an overall degree of suitability $S(X, Y) = \sigma(a_1(X, Y), \ldots, a_n(X, Y))$ that reflects the suitability of location X, Y for a specific land use. The overall suitability is a matter of degree: $0 \leq S(X, Y) \leq 1$. As in all soft computing models, here 0 denotes a completely unsuitable location, and 1 denotes the highest level of suitability. A suitability map is defined as a distribution of the overall suitability $S(X, Y)$ for a specific geographic region.[3] In many cases, however, we are not interested in computing the whole suitability map, but only the suitability in a few selected locations (e.g., in a real-estate database, only for the coordinates of houses that are for sale). So, in this paper, we assume that all suitability scores hold for a specific location, and for readability, the coordinates X and Y can sometimes be omitted.

Nowadays, suitability maps are assumed to be dynamically created using data from geographic information system (GIS) databases in a way illustrated in Figure 1. The multicriteria decision model must be interfaced both with the user and with the GIS database. An attribute ETL interface [e.g., the Google maps Application Programming Interface (API)[4]] is necessary to extract, transform, and load the set of cell attribute values from the GIS database. The multicriteria decision model is used to implement the criterion function σ and to generate the overall suitability score $S(X, Y)$. The user input includes a specification of desired suitability attributes and parameters of the decision model. A suitability criterion interface is necessary for accepting user input, presenting numerical results and for rendering the resulting suitability map.

Suitability maps introduced in GIS literature use a variety of decision models.[2,5–11] The emphasis of such efforts is primarily (and naturally) on the

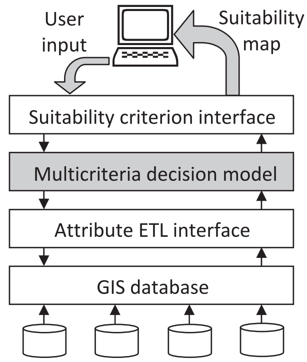

Figure 1. Dynamic generation of suitability maps using GIS.

selection of attributes and the use of suitability maps. So far, the GIS literature avoided problems of evaluating the credibility of MCDM used for the development of suitability maps.

Except for interfacing with GIS components, the land-use suitability assessment problems do not differ from evaluation problems in other areas. All multicriteria decision models are ultimately models of human decision making in the area of evaluation of complex alternatives. Therefore, the credibility of MCDM used in GIS context depends on their ability to express observable properties of human evaluation logic.[12] The initial goal of this paper is to evaluate the expressive power of GIS-related MCDM based on the level of their ability to support the concepts of human evaluation logic.

The remainder of this paper is structured as follows. In Section 2, we investigate necessary logic properties of GIS-related MCDM. In Section 3, we compare the logic properties of the selected decision methods. In Section 4, we present canonical forms of logic aggregation structures that are necessary for building evaluation criteria. In Section 5, we introduce LSP/POI suitability maps. Section 6 summarizes our conclusions.

2. FUNDAMENTAL PROPERTIES OF MCDM

The properties of human evaluation logic are easily observable, and they are also observable in the context of land-use evaluation. Our goal is to first identify properties that are frequently encountered in all worth assessment problems. The most important identified properties include the following:

1. Ability to combine any number of attributes.
2. Ability to combine objective and subjective inputs.
3. Ability to combine absolute and relative criteria.
4. Flexible adjustment of relative importance of attributes.
5. Modeling of simultaneity requirements.

   a. Modeling of soft simultaneity.
   b. Modeling of hard simultaneity.
6. Modeling of replaceability requirements.
   a. Modeling of soft replaceability.
   b. Modeling of hard replaceability.
7. Modeling of balanced simultaneity/replaceability.
8. Modeling of mandatory, desired, and optional requirements.
9. Modeling of sufficient, desired, and optional requirements.
10. Ability to express suitability as an aggregate of usefulness and affordability (separation of the usefulness analysis and the cost analysis).

2.1. Ability to Combine Any Number of Attributes

We assume that the analyzed geographic region is divided into cells and that each cell is characterized by attributes $a_1(X, Y), \dots, a_n(X, Y),\ \ a_i(X, Y) \in \mathbb{R},\ \ i = 1, \dots, n, n \geq 1$. In special cases, $n = 1$. For example, the acoustic pollution (noise) maps presented in Ref. 8 use in each cell a single scalar value, the level of noise. However, in all cases of more complex suitability maps, we have $n > 1$. For example, the housing suitability maps discussed in Ref. 2 use $n = 11$ attributes. If we want to develop maps of complex suitability indicators that depend on a variety of inputs, it is reasonable to expect that the number of input attributes can be large (e.g., up to 100 or more). So, in a general case, MCDM used to compute land-use suitability must be able to support any number of inputs.

It should be noted that this requirement indirectly transforms into a requirement to use nonlinear MCDM models. Indeed, in the case of linear weighted aggregation models (i.e., all methods that use SAS), the overall suitability $S(X, Y)$ is a dot product of a vector of n suitability components $s = [s_1(X, Y), \dots, s_n(X, Y)]$ (one component for each attribute) and a corresponding vector of relative weights $W = [W_1, \dots, W_n]$: $S(X, Y) = \sum_{i=1}^{n} W_i S_i(X, Y)$, where $0 \leq s_i(X, Y) \leq 1, 0 < W_i < 1$, $i = 1, \dots, n$, and $W_1 + \cdots + W_n = 1$.

Unfortunately, in all linear models, a large value of n automatically yields a low significance of individual components. For example, if $n = 100$, the average significance (the ability to change the overall suitability S) of each input is only 1%, and in the case of unequal weights, some inputs will have a significance considerably below 1%, which means that such inputs may (and should) be neglected. This is not acceptable since credible decision models must be equally applicable for any number of inputs.

2.2. Ability to Combine Objective and Subjective Inputs

Some input attributes are objectively measurable values (e.g., distances, slopes, altitudes, temperatures). Other inputs are subjective and must be assessed by experts (e.g., aesthetic quality of a home or an area, the quality of local educational and/or medical institutions, the quality of public transportation, restaurants, parks). MCDM for GIS applications must be able to efficiently combine and aggregate objective and subjective inputs.

2.3. Ability to Combine Absolute and Relative Criteria

Each suitability attribute must satisfy specific requirements. The requirements can be defined as elementary criterion functions $g_i : \mathbb{R} \rightarrow [0, 1]$, $i = 1, 2, \ldots, n$. By definition, the attribute suitability $s_i(X, Y) = g_i(a_i(X, Y))$ is the degree of satisfaction of the attribute requirements. The suitability of an attribute is a component of the overall suitability of the evaluated location X, Y.

Suppose that a DM needs a suitability map for housing in a rural area, where it is possible to buy land and build a house. Let the attribute $a_i(X, Y) = t$ denote the traveling time between location $L = (X, Y)$ and the closest elementary school (or hospital, or airport, or any other point of interest). An absolute criterion is a criterion that evaluates location L regardless of other competitive locations. For example, a DM may specify the absolute requirements as threshold times t_{min} and t_{max} so that all times $t \geq t_{max}$ are considered to be unacceptable, and all times $t \leq t_{min}$ are considered to be perfectly acceptable. Then, the elementary criterion function might be defined as follows: $g_i(t) = \max(0, \min(1, (t_{max} - t)/(t_{max} - t_{min})))$. So, $g_i(t)$ specifies the attribute suitability for any value of t. If the DM wants to compare locations L_1 and L_2, with respect to the access to school, then the comparison can be based on $g_i(t_1)$ and $g_i(t_2)$. If t_{min} and t_{max} are well justifiable values, then $g_i(t_1)$ and $g_i(t_2)$ are very credible results.

Another approach is to define a relative criterion. In the case of k locations, an example of such a criterion could be $g_i(t) = t_{min}/t$, $t_{min} = \min(t_1, t_2, \ldots, t_k)$. The relative criterion evaluates relationships between competitors regardless of the DM's actual needs: The closest location is considered perfectly suitable ($g_i(t_{min}) = 1$), and a location where $t = 2t_{min}$ gets $g_i(t) = 0.5$. Obviously, the use of such criteria can frequently be questionable, e.g., if only two locations are available, then the location that is 2 min from the closest school is likely to be equally attractive as the location that is 1 min from the school, but the suitability level $g_i(t) = t_{min}/t = 0.5$ does not reflect this fact. Similarly, locations that are 1 h and 2 h from the closest school might be equally unacceptable, but the "best case" t =1 h would still yield the suitability level of 1, which is inappropriate. For some other values, e.g., $t_{min} = 7$ min, the presented relative criterion might be appropriate.

It is highly unlikely that the DM knows what is better, but does not know what is good. In other words, if the DM can specify a justifiable relative criterion, it is very likely that the DM can also specify a justifiable absolute criterion. Consequently, absolute attribute criteria are much more frequent (and more desirable) than relative criteria, but generally both types of criteria are needed for building evaluation decision models.

Absolute criteria can be used to evaluate a single alternative (e.g., a single location), whereas relative criteria can be applied only if we have multiple alternatives. Relative criteria are not acceptable as a general approach to evaluation; they are appropriate only in situations where desired values of attributes are unknown. Such situations are infrequent, but possible, and MCDM must be able to combine and aggregate absolute and relative attribute criteria, e.g., every homebuyer can specify the minimum acceptable home area as well as the area that completely satisfies homebuyer's requirements. In the presence of such a clear absolute criterion, it would be meaningless to make decisions based on relative area ratios of two or more homes. However, the suitability of extended surroundings (as a mean value of distances from various less important and sometimes unexpected POIs) cannot be easily defined and calibrated in advance, and if it is taken into account, a relative criterion might be appropriate.

2.4. Flexible Adjustment of the Relative Importance of Attributes

In a general case, multiple attributes are not equally important. The relative importance of an attribute is usually expressed using multiplicative or implicative weights.[13] The relative importance has two roles in suitability criteria: It defines the level of contribution of an attribute to the overall suitability, and it defines compensatory properties between attributes. The overall suitability S in a given location (X, Y) is a soft computing logic function of n attribute suitability degrees $(s_1, \ldots, s_n)$: $\lambda : [0, 1]^n \to [0, 1]$. If $S = \lambda(s_1, \ldots, s_j, \ldots, s_k, \ldots, s_n)$ and the suitability s_j is more important than the suitability s_k, then $\partial S/\partial s_j \geq \partial S/\partial s_k$. Under the same assumptions (s_j is more important than s_k, and some compensation between s_j and s_k is possible), we also have $\lambda(s_1, \ldots, s_j, \ldots, s_k, \ldots, s_n) = \lambda(s_1, \ldots, s_j - p, \ldots, s_k + q, \ldots, s_n)$, $p < q$. In other words, a suitability decrement p of an attribute can be compensated by the suitability increment q of a less important attribute, but the increment q must be greater than the decrement p. These properties are essential in human reasoning and must be supported by MCDM.

2.5. Modeling of Simultaneity Requirements

The function $\lambda : [0, 1]^n \to [0, 1]$ that aggregates all attribute suitability degrees and computes the overall suitability degree is essentially a logic function. In human decision making, the aggregation of suitability degrees is usually a stepwise process, where small groups of related suitability degrees are aggregated and replaced by an aggregated suitability degree. The process of stepwise aggregation terminates when the suitability degrees of subsystems at the highest level are aggregated yielding the overall suitability degree S as a compound function of n attribute suitability degrees.

Let us investigate an aggregation step, where a DM aggregates $m > 1$ suitability degrees using an aggregation function $\mu : [0, 1]^m \to [0, 1]$. In human decision making, $\mu(s_1, \ldots, s_m)$ is very frequently a model of simultaneity; e.g., a homebuyer regularly prefers locations that are both close to schools for children *and* to jobs for parents. Such a simultaneity requirement can be modeled using some form of partial conjunction,[13,14] i.e., μ is expected to have an adjustable degree of similarity with conjunction $s_1 \wedge \cdots \wedge s_m$. A normalized degree of proximity between $\mu(s_1, \ldots, s_m)$ and $s_1 \wedge \cdots \wedge s_m$ is called the local andness[12] and can be defined by $\alpha = [(s_1 \vee \cdots \vee s_m) - \mu(s_1, \ldots, s_m)]/[(s_1 \vee \cdots \vee s_m) - (s_1 \wedge \cdots \wedge s_m)]$. Since $(s_1 \wedge \cdots \wedge s_m) \leq \mu(s_1, \ldots, s_m) \leq (s_1 \vee \cdots \vee s_m)$, it follows that $0 \leq \alpha \leq 1$. The aggregator $\mu(s_1, \ldots, s_m)$ is a model of simultaneity if $0.5 < \alpha \leq 1$ and (assuming equal importance of inputs, and inputs that are not all identical) $(s_1 \wedge \cdots \wedge s_m) \leq \mu(s_1, \ldots, s_m) < (s_1 + \cdots + s_m)/m$. This form of andness depends on input degrees of suitability and humans usually think globally using an average andness, e.g., $\bar{\alpha} = \int_{[0,1]^m} \alpha(s_1, \ldots, s_m) ds_1 \ldots ds_m, 0 \leq \bar{\alpha} \leq 1$. If a DM needs a high simultaneity (e.g., $\bar{\alpha} > 0.7$), this usually means that all inputs must be at least partially satisfied. In other words, if $\exists i \in \{1, \ldots, m\}, \quad s_i = 0$ then $\mu(s_1, \ldots, s_m) = 0$, and it is mandatory to satisfy all inputs. This kind of simultaneity is called a *hard simultaneity*, and the corresponding family of aggregators is called the *hard partial conjunction* (HPC)[14]. The average andness of HPC is usually between 2/3 and 1. In other cases, a DM may need a *soft simultaneity* (and a *soft partial conjunction* (*SPC*)), where the average andness is typically between 1/2 and 2/3, and if $\exists i \in \{1, \ldots, m\}$, $s_i > 0$, then $\mu(s_1, \ldots, s_m) > 0$. Supporting hard and soft models of simultaneity is an important requirement that MCDM must satisfy.

2.6. Modeling of Replaceability Requirements

Replaceability requirements are symmetrical and complementary to simultaneity requirements. Replaceability means that a high suitability in a group of attributes can be achieved using any one of the attributes (i.e., they can replace each other); e.g., a home location can be considered suitable for recreational activities if it is close to a lake *or* close to ski terrains. The intensity of replaceability can be determined using the average orness indicator that is the complement of andness: $\bar{\omega} = 1 - \bar{\alpha}$. Replaceability aggregators satisfy the conditions $0.5 < \bar{\omega} \leq 1$ and $(s_1 + \cdots + s_m)/m < \mu(s_1, \ldots, s_m) \leq (s_1 \vee \cdots \vee s_m)$. High orness means low andness and vice versa. Similarly to the case of simultaneity, a high level of replaceability (e.g., $\bar{\omega} > 0.7$) may be combined with the requirement for *hard replaceability*, where $\exists i \in \{1, \ldots, m\}$, $s_i = 1$ implies that $\mu(s_1, \ldots, s_m) = 1$. *Soft replaceability* is any form of replaceability that does not satisfy the hard replaceability requirements. In the case of soft replaceability, we have that $\exists i \in \{1, \ldots, m\}, s_i < 1$ implies $\mu(s_1, \ldots, s_m) < 1$. HPD and SPD aggregators are usually made as De Morgan duals of HPC and SPC[14] (see Section 4). No MCDM can model observable properties of human evaluation reasoning unless it supports HPD, SPD, HPC, and SPC with continuously adjustable degree of andness/orness.

2.7. Modeling of Balanced Simultaneity/Replaceability

If the simultaneity and replaceability are balanced, then $\alpha = \omega = 0.5$. In the case of two variables, from definition $\alpha = [(s_1 \vee s_2) - \mu(s_1, s_2)]/[(s_1 \vee s_2) - (s_1 \wedge s_2)] = 0.5$ it follows that $\mu(s_1, s_2) = ((s_1 \vee s_2) + (s_1 \wedge s_2))/2 = (s_1 + s_2)/2$. This result indicates that the arithmetic mean is a soft computing logic function that combines simultaneity and replaceability requirements in a balanced way: the DM would like that all attributes are simultaneously satisfied, but at the same time she or he accepts that any attribute can compensate any other attribute. It is important to understand that the arithmetic mean represents a model of this specific logic condition and nothing more. MCDM that use the arithmetic mean are acceptable only in cases where the DM can justify the use of this specific logic condition.

According to Malczewski,[6] "GIS implementations of the weighted summation procedures are often used without full understanding of the assumptions underlying this approach." Indeed, the arithmetic mean is the ultimate compromise between the contrasting requirements of simultaneity and replaceability, and its unjustified use in MCDM is regularly an oversimplification. Since human evaluation reasoning is provably based on adjustable levels of andness and orness, the same form of oversimplification is obtained whenever decision models apply any form of fixed andness/orness (including, e.g., the geometric mean,[15–17] or multilinear utility functions used for aggregation of utility independent components).[1,18,19]

2.8. Modeling of Mandatory, Desired, and Optional Requirements

Using models of simultaneity, replaceability, and negation ($x \mapsto 1 - x$), it is possible to create a variety of compound soft computing logic functions that precisely reflect the needs of a DM. A compound aggregator that is most frequent in human evaluation reasoning is used to combine mandatory and nonmandatory attributes. Most frequently, there are one or more mandatory attributes and one or more nonmandatory attributes; e.g., a DM may reject home locations that do not have good ground transportation, but accept locations that are far from an international airport. In such cases, the ground transportation is a mandatory requirement, and the vicinity of an international airport is desired, but not mandatory. Optional attributes are also nonmandatory and have lower significance than desired attributes. To model such requirements, we need aggregators $\mu(s_{man}, s_{des}, s_{op})$ that satisfy the condition that $s_{man} = 0$, $s_{des} > 0$, $s_{op} > 0$ implies $\mu(s_{man}, s_{des}, s_{op}) = 0$, and if a compensation between s_{des} and s_{op} is possible, then $\mu(s_{man}, s_{des}, s_{op}) = \mu(s_{man}, s_{des} - p, s_{op} + q)$, $s_{man} > 0$, $p < q$. Optional attributes can be omitted, and in such cases, we use only mandatory and desired inputs. An aggregator with these properties is the partial absorption function,[12,20,21] which is obtained as a combination of SPD and HPC. This additionally supports the need for adjustable simultaneity and replaceability models.

2.9. Modeling of Sufficient, Desired, and Optional Requirements

Sufficient, desired, and optional requirements are a disjunctive counterpart of mandatory, desired, and optional requirements. If the sufficient input is completely satisfied, then the desired and optional inputs have no effect. If the sufficient attribute has low or even zero suitability degree, this can be partially compensated by the desired and optional attributes. The corresponding aggregators $\mu(s_{suf}, s_{des}, s_{op})$ satisfy the condition that $s_{suf} = 1$, $s_{des} < 1$, $s_{op} < 1$ implies $\mu(s_{suf}, s_{des}, s_{op}) = 1$, and if a compensation between s_{des} and s_{op} is possible, then $\mu(s_{suf}, s_{des}, s_{op}) = \mu(s_{suf}, s_{des} - p, s_{op} + q)$, $s_{suf} < 1$, $p < q$.

2.10. Ability to Express Suitability as an Aggregate of Usefulness and Affordability

Land use is regularly related to a variety of costs (e.g., the cost of land, the cost of building infrastructure and objects, the cost of financing). The overall suitability depends on two simultaneous requirements: finding locations that are very useful for a specific purpose and at the same time inexpensive or affordable. A justifiable way to solve that problem is to define *usefulness* (convenience) as a nonfinancial part of the overall suitability, and *affordability* as an overall result of cost analysis (an aggregate of cost components only). Then, the overall suitability can be conveniently expressed as an aggregate of usefulness and affordability. This approach is an alternative or a complement to the use of bipolar models.[22–26]

Separation of cost and usefulness attributes reflects human reasoning, where the overall cost is compared with the corresponding overall usefulness. Of course, there are cases, where the overall suitability does not depend on cost. In such cases, the overall suitability reduces to usefulness.

The presented list of 10 fundamental properties is not intended and/or proved to be necessary and sufficient in all cases. However, the presented conditions are relevant for many land-use decision problems and show that logic aggregation of attribute suitability is a frequently needed property. A more detailed analysis of mathematical conditions can be found in Ref. 12, and sample applications of logic aggregation can be found in Refs. 2, 12, and 27.

The necessary properties of MCDM do not change if the attributes of a cell are functions of time, or functions of the values of attributes in other cells.

3. PROPERTIES OF GIS-RELATED MCDM

Several GIS-related MCDM approaches have been presented in the literature.[5,6,28] Among the decision methods used in these approaches, we selected a representative set that consists of the following techniques: *SAS* (a.k.a. simple additive weighting (SAW)[16,29–31] *MAVT*,[18,19,32] *MAUT*,[18,19,33,34] *AHP*,[17,35,36] *OWA*,[37,38] *outranking methods*,[16,39–41] and *LSP*.[2,12]

The 10 fundamental features presented in the preceding section can be used to investigate the selected decision methods in view of their appropriateness for land-use evaluation and suitability map construction.

3.1. Simple Additive Scoring

The SAS technique[28–31] is the oldest of all techniques. Its popularity is primarily based on simplicity. SAS uses the concept of a weighted average in which weights are used to denote the relative importance of suitability attributes. A DM directly assigns a weight w_i to each suitability attribute a_i, $i = 1, \ldots, n$. These assigned weights are usually rescaled to normalized weights W_i, $i = 1, \ldots, n$, such that $\sum_{i=1}^{n} W_i = 1$. The overall score $S(x, y)$, representing the overall degree of suitability of each cell X, Y, is then computed by

- determining the n suitability components (scores in an arbitrary range) $s_1(X, Y), \ldots, s_n(X, Y)$ that are obtained from the evaluation of the n attributes for the cell;
- multiplying each suitability score $s_i(X, Y)$, $i = 1, \ldots, n$, with the (normalized or not normalized) weight W_i of its corresponding attribute;
- summing the products over all attributes, i.e., $S(X, Y) = \sum_{i=1}^{n} W_i S_i(X, Y)$.

Therefore, SAS uses a simple linear weighted aggregation model. Simple geometric mean scoring[15] $\log S(X, Y) = \sum_{i=1}^{n} W_i \log S_i(X, Y)$ is similar to SAS because it uses a simple weighted average with fixed andness.

3.2. Multiattribute Value Technique

In MAVT,[18,19,32] suitability attributes $a_i(X, Y)$, $i = 1, \ldots, n$ are evaluated using value functions that aim to mathematically represent human judgments. A single-attribute value function translates the performance of the allowed alternative attribute values into a value score, which represents the degree to which a decision objective is achieved. As such, a value function v_i associates a number (or "value") $v_i(a_i)$ with each allowed alternative value of attribute a_i in such a way that a preference order on the alternatives is consistent with the DM's value judgments.

For aggregation, more complex multiple-attribute value functions are used. The most commonly used function is the SAW function $S(X, Y) = \sum_{i=1}^{n} W_i v_i(a_i(X, Y))$ where W_i is the associated weight of suitability attribute a_i, and $v_i(a_i(X, Y))$ is the value score of the suitability attribute value $a_i(X, Y)$ (of cell X, Y). This approach is valid if suitability attributes are preferentially independent.

The weights W_i are scaling constants that have to be derived with reference to the attribute ranges. These need to be elicited through questions, which capture acceptability of trade-offs (e.g., "how many units of one suitability attribute are worth how many units of another suitability attribute?"). Weights must sum up to 1, i.e., $\sum_{i=1}^{n} W_i = 1$.

As such, the MAVT approach is similar to the SAS method, except that the scores $s_i(X, Y)$ are replaced by values $v_i(a_i(X, Y))$ that are obtained with the value functions v_i, $i = 1, \ldots, n$. Similarly to SAS, MAVT also belongs to fixed-andness methods.

3.3. Multiattribute Utility Technique

MAUT[18,33] is used and treated separately from MAVT when "risks" or "uncertainties" have a significant role in the definition and assessment of allowed alternatives. The attitude of the DM toward risk is incorporated into the assessment of a single-attribute utility function u_i, which is obtained through utility analysis and translates the allowed values of a suitability attribute a_i into "utility units." A "utility unit" is a relative value between 0 and 1 (where 0 and 1 denote the worst and best values, respectively). The concept of a utility function is inherently probabilistic in nature.

Aggregation is done by an overall, multiple-attribute utility function. An SAW function is most commonly used. In such a case, $S(X, Y) = \sum_{i=1}^{n} W_i u_i(a_i(X, Y))$.

Suitability attributes must be preferentially independent.[18] The weights W_i, $i = 1, \ldots, n$ have to sum up to 1, i.e., $\sum_{i=1}^{n} W_i = 1$.

From an aggregation point of view, the MAUT approach is similar to the MAVT and SAS approaches. Even in cases, where the aggregation is based on slightly more complex aggregators, e.g., $1 + kS(X, Y) = \prod_{i=1}^{n} [1 + kw_i u_i(a_i(X, Y))]$, where weights w_i and the constant k are selected so that the maximum utilities (i.e., 1) produce the maximum suitability ($S(X, Y) = 1$), this approach provides only a fixed andness.

3.4. Analytic Hierarchy Process

Cognitive psychology has found that people have limited ability for comparisons of alternatives based on large number of attributes. The highest accuracy is obtained when comparison includes only two components. To maximize the accuracy of evaluation, AHP[17,35,36] uses a structured pairwise comparison approach. The main steps in AHP can be summarized as follows:

- Model the problem as a hierarchy containing the decision goal, the alternatives for reaching it, and the suitability attributes for evaluating the alternatives.
- Establish priorities (normalized weights) among the elements of the hierarchy by making a series of judgments based on pairwise comparisons of the elements.
- Synthesize these judgments to yield a set of overall weights for the hierarchy. This is done by means of a sequence of multiplications of the matrices of relative weights at each level of the hierarchy.
- Check the consistency of the judgments.
- Come to a final decision based on the results of this process.

The root of the aggregation tree is the overall suitability, and all other nodes are hierarchically structured simpler attributes subdivided into subattributes. The suitability in any node of the aggregation tree is computed by an additive weighting function with normalized weights and the computation of the overall suitability model consists of multiple SAS models. Consequently, the andness of aggregators is not adjustable.

3.5. Ordered Weighted Averaging

In an OWA approach,[38] the DM specifies the decision-relevant suitability attributes to be used as evaluation criteria, identifies preferred criteria values on a qualitative scale, and defines the relative importance of each criterion by assigning weights. The weighted criterion values are then combined using an OWA aggregation operator,[37] resulting in an evaluation score for each cell. OWA allows the DM to specify a decision strategy that reflects his/her decision-related preferences.

The OWA operators[37] provide a parameterized class of mean-type aggregation operators. Many notable mean operators such as the maximum, arithmetic mean, median, and minimum are members of this class. OWA operators allow to model linguistically expressed aggregation instructions.

An OWA operator of dimension n is a mapping function $F : [0, 1]^n \rightarrow [0, 1]$ that has an associated collection of weights $W = [W_1, \ldots, W_n] \in [0, 1]^n$, for which it holds that $\sum_{i=1}^{n} W_i = 1$, and with $F(s_1, \ldots, s_n) = \sum_{i=1}^{n} W_i s_i'$ where s_i' is the ith largest value of the s_i.

By choosing different weights W, different aggregation operators can be implemented. The OWA operator is a nonlinear operator as a result of the process of determining the values s_i'. The standard OWA aggregators provide models of adjustable andness and orness for the SPC and the SPD.

3.6. Outranking Methods

Outranking methods, such as variants of ELECTRE and PROMETHEE,[16,40,41] are used in the areas of GIS and environmental planning.[10,39] The basic idea of these methods is to use strictly relative criteria that express a range from indifference to strong preference of one alternative over another alternative, separately for all individual attributes. The attribute preferences are then averaged using the arithmetic mean to generate the credibility of the outranking relation of two alternatives (PROMETHEE). In such a case, no input is mandatory. The overall degree of outranking of two alternatives can also be computed using a product (ELECTRE III), making all attributes mandatory.

3.7. Logic Scoring of Preference

With the LSP method,[2,12] suitability maps are created with the following main steps:

1. *Creation of an attribute tree:* This tree contains a hierarchical structure of all attributes that affect the overall suitability according to requirements of the DM.
2. *Definition of elementary criteria:* The DM has to provide an elementary criterion for each attribute involved in the decision process. These criteria will be evaluated during suitability map construction. For each analyzed cell X, Y, the evaluation of each attribute $a_i(X, Y) \in \mathbb{R}, i = 1, \ldots, n$, will result in an elementary satisfaction degree (elementary suitability score) $s_i(X, Y) \in [0, 1]$.
3. *Creation of the aggregation structure:* For each analyzed cell, all associated elementary satisfaction degrees must be aggregated to produce the overall suitability. Therefore, the DM has to create an aggregation structure, which adequately reflects his/her domain knowledge and reasoning.
4. *Computation of the overall suitability degree:* Once the attribute tree, the elementary criteria, and the aggregation structure are available, the suitability map construction can start. The elementary criteria are evaluated and their resulting elementary satisfaction degrees can be aggregated to compute the overall satisfaction degree (i.e., the overall suitability) of each analyzed cell.

Aggregation in LSP is done via a logic aggregation structure consisting of simple and compound LSP aggregators. The DM can use them to construct an easily understandable aggregation schema, which is consistent with observable properties of human reasoning in the area of evaluation.

LSP aggregators are graded continuous logic functions based on a superposition of the fundamental *generalized conjunction/disjunction* (GCD) function, symbolically denoted $S = W_1 s_1 \diamond \cdots \diamond W_n s_n$, or, in the case of equal weights, $S = s_1 \diamond \cdots \diamond s_n$. Among many GCD implementations,[13,14] the most frequently used is the weighted power mean function $W_1 s_1 \diamond \cdots \diamond W_n s_n = (W_1 s_1^r + \cdots + W_n s_n^r)^{1/r}$, where $r \in [-\infty, +\infty]$ and $0 < W_i < 1, i = 1, \ldots, n$, such that $\sum_{i=1}^{n} W_i = 1$. The parameter r determines the logic behavior of the function and permits continuous transition from conjunction to disjunction and modeling of soft and hard variants of partial conjunction/disjunction. The weights reflect semantic aspects of the criterion function. The GCD aggregators can be used to construct more complex, compound operators like the conjunctive partial absorption (CPA) and disjunctive partial absorption (DPA),[20,21] which can be used to aggregate mandatory/sufficient and desired/optional inputs.

For suitability map construction, the overall suitability degree of each analyzed cell X, Y must be computed. This is done in two steps: First, the elementary satisfaction degrees $s_1(X, Y), s_2(X, Y), \ldots, s_n(X, Y)$ are aggregated using the logic aggregation structure. This generates the overall satisfaction degree $s(X, Y)$ of the cell. Second, cost (if applicable) is taken into account. Cost is dealt with separately. This better reflects human reasoning and allows for more efficient cost/satisfaction studies. In a general case, the cost can be a function of the analyzed cells. For each cell X, Y, the cost function returns the associated cost $c(X, Y)$ of the cell. If the importance of high satisfaction of criteria is the same as the importance of low cost, then the overall suitability degree $S(X, Y)$ of the cell can be computed by $S(X, Y) = s(X, Y)/c(X, Y)$. Alternatively, more flexible definitions of $S(X, Y)$ can be found in Ref. 2.

3.8. Comparison of MCDM for the Design of Suitability Maps

The presented seven evaluation methods have been applied in evaluation of land use and in other space management decision problems. In the case of suitability maps, successful methods must provide high expressive power in the area of modeling sophisticated logic conditions that characterize human reasoning. All methods that use fixed andness/orness can create serious errors in suitability maps, and the remaining sections of this paper provide a proof for this claim. In addition, a clear definition of attribute criteria, models of hard and soft partial conjunction/disjunction, and partial absorption are indispensable components for building justifiable suitability maps. These properties are provided only by the LSP method. Therefore, our goal is now to show the process of creating and using the LSP suitability maps. The fundamental step in that direction is a study of canonical forms of logic aggregation structures.

4. CANONICAL FORMS OF LOGIC AGGREGATION STRUCTURES

Logic aggregation of suitability (or degrees of membership) is the most important step in designing suitability maps. This step determines the expressive power of suitability criteria and the quality and usability of suitability maps. Generally,

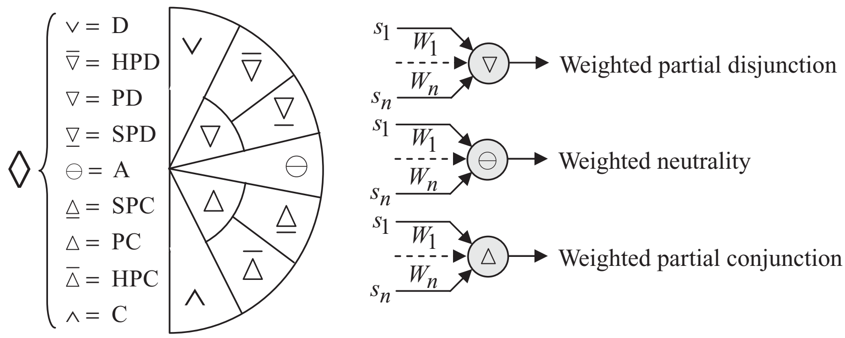

Figure 2. Symbols and graphic notation used for basic types of GCD.

the logic aggregation is based on superposition of basic GCD functions and (less frequently) negation.

In evaluation logic, there are seven basic GCD aggregator types[14]: conjunction (C), HPC, SPC, neutrality (A), SPD, hard partial disjunction (HPD), and disjunction (D). For these aggregators, we use symbols and graphic notation shown in Figure 2.

In this paper, we will assume that these aggregators are modeled using weighted power means as follows:

$$
\begin{aligned}
&W_1 s_1 \Delta \cdots \Delta W_n s_n = \left(W_1 s_1^r + \cdots + W_n s_n^r\right)^{1/r}, \quad -\infty < r < 1\\
&W_1 s_1 \bar{\Delta} \cdots \bar{\Delta} W_n s_n = \left(W_1 s_1^r + \cdots + W_n s_n^r\right)^{1/r}, \quad -\infty < r \le 0\\
&W_1 s_1 \underline{\Delta} \cdots \underline{\Delta} W_n s_n = \left(W_1 s_1^r + \cdots + W_n s_n^r\right)^{1/r}, \quad 0 < r \le 1\\
&W_1 s_1 \nabla \cdots \nabla W_n s_n = 1 - W_1(1 - s_1) \bar{\Delta} \cdots \bar{\Delta} W_n(1 - s_n)\\
&W_1 s_1 \underline{\nabla} \cdots \underline{\nabla} W_n s_n = 1 - W_1(1 - s_1) \underline{\Delta} \cdots \underline{\Delta} W_n(1 - s_n)\\
&W_1 s_1 \bar{\nabla} \cdots \bar{\nabla} W_n s_n = 1 - W_1(1 - s_1) \bar{\Delta} \cdots \bar{\Delta} W_n(1 - s_n)\\
&W_1 s_1 \ominus \cdots \ominus W_n s_n = W_1 s_1 + \cdots + W_n s_n\\
&s_1 \wedge \cdots \wedge s_n = \left(W_1 s_1^r + \cdots + W_n s_n^r\right)^{1/r}, \quad r \to -\infty\\
&s_1 \vee \cdots \vee s_n = \left(W_1 s_1^r + \cdots + W_n s_n^r\right)^{1/r}, \quad r \to +\infty
\end{aligned}
$$

Using these models, we provide the following fundamental properties of the HPC, SPC, HPD, and SPD aggregators:

$$
\begin{aligned}
&W_1 s_1 \bar{\Delta} \cdots \bar{\Delta} W_n s_n = 0, \quad s_i = 0, i \in \{1, \ldots, n\}\\
&W_1 s_1 \underline{\Delta} \cdots \underline{\Delta} W_n s_n > 0, \quad s_i > 0, i \in \{1, \ldots, n\}\\
&W_1 s_1 \bar{\nabla} \cdots \bar{\nabla} W_n s_n = 1, \quad s_i = 1, i \in \{1, \ldots, n\}\\
&W_1 s_1 \underline{\nabla} \cdots \underline{\nabla} W_n s_n < 1, \quad s_i < 1, i \in \{1, \ldots, n\}
\end{aligned}
$$

In the above definitions, the partial conjunction and the partial disjunction satisfy De Morgan symmetry. If that is not necessary, then it is possible to use SPD defined as $W_1 s_1 \underline{\nabla} \cdots \underline{\nabla} W_n s_n = (W_1 s_1^r + \cdots + W_n s_n^r)^{1/r}, \quad 1 < r < +\infty$.

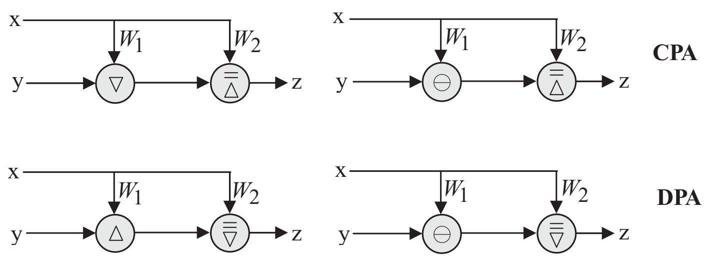

Figure 3. Two usual versions of CPA and DPA.

The most important compound aggregators are the CPA and the DPA aggregators.[12,20] These aggregators can be implemented in variety of ways, and two simplest examples are shown in Figure 3.

The CPA $z = x \,\underline{\triangleright}\, y$ aggregates a mandatory input x and a nonmandatory (desired/optional) input y as follows:

$$x \,\underline{\triangleright}\, y = W_2 x \bar{\bar{\Delta}} (1 - W_2)[(1 - W_1)x \tilde{\nabla} W_1 y], \quad \bar{\bar{\Delta}} \in \{\wedge, \bar{\Delta}\}, \quad \tilde{\nabla} \in \{\vee, \Delta, \ominus\}.$$

In an extreme case, where $\bar{\bar{\Delta}} = \wedge, \quad \tilde{\nabla} = \vee$, we have

$$x \,\underline{\triangleright}\, y = W_2 x \wedge (1 - W_2)[(1 - W_1)x \vee W_1 y] = x \wedge (x \vee y) = x,$$

i.e., the partial absorption becomes the total absorption of the nonmandatory input y. The fundamental properties of CPA are

$$\begin{array}{ll} 0 \,\underline{\triangleright}\, y = 0, & 0 \le y \le 1 \\ 0 < x \,\underline{\triangleright}\, 0 < x, & 0 < x \le 1 \\ x < x \,\underline{\triangleright}\, 1 < 1, & 0 < x < 1 \end{array}$$

Therefore, we can define the penalty P and reward R as follows:

$$\begin{array}{lll} x \,\underline{\triangleright}\, 0 = x - P, & 0 < x \le 1, & P > 0 \\ x \,\underline{\triangleright}\, 1 = x + R, & 0 < x < 1, & R > 0 \end{array}$$

The parameters of the CPA and DPA (weights and exponents of the weighted power mean) can be computed from the desired values of the mean penalty and the mean reward.[20]

The DPA $z = x \bar{\triangleright} y$ aggregates a sufficient input x and a desired/optional input y. The usual implementation is $x \bar{\triangleright} y = W_2 x \bar{\bar{\nabla}} (1 - W_2)[(1 - W_1)x \tilde{\Delta} W_1 y]$,

$\bar{\bar{\nabla}} \in \{\vee, \bar{\nabla}\}$, $\tilde{\Delta} \in \{\wedge, \Delta, \ominus\}$, yielding the following properties:

$$\begin{array}{ll} 1 \bar{\triangleright} y = 1, & 0 \le y \le 1 \\ 0 < x \bar{\triangleright} 0 < x, & 0 < x < 1 \\ x < x \bar{\triangleright} 1 < 1, & 0 < x < 1 \end{array}$$

Consequently, the penalty and reward concepts and the method for calculating weights and the weighted power mean exponent remain the same as in the case of CPA. In an extreme case $\bar{\bar{\nabla}} = \vee$, $\tilde{\Delta} = \wedge$ and $x \bar{\triangleright} y = x \vee (x \wedge y) = x$ (total absorption). Another case of the total absorption is obtained if $\bar{\bar{\nabla}} = \vee$, $\tilde{\Delta} = \ominus$, $x \ge y$: $x \bar{\triangleright} y = x \vee [(1 - W_1)x + W_1 y] = x$.

Logic aggregation structures usually consist of combining the GCD, strong negation ($x \mapsto 1 - x$), CPA and DPA and can have arbitrary configuration and parameters. However, some aggregation structures have regular and recognizable forms and are frequently encountered in evaluation models. These structures can be called *canonical aggregation structures* (CAS) and include the following five basic cases:

1. conjunctive CAS with increasing andness,
2. disjunctive CAS with increasing orness,
3. aggregated mandatory/optional and sufficient/optional CAS,
4. distributed mandatory/optional and sufficient/optional CAS, and
5. mandatory/desired/optional and sufficient/desired/optional CAS.

4.1. Conjunctive CAS with Increasing Andness

The structure of conjunctive CAS with increasing andness is shown in Figure 4. For each aggregator, we can define its *shadow* as the set of input attributes that affect the output of the aggregator. If the total number of input attributes is n, then the shadow of an aggregator can have the size from 2 (for a leaf aggregator in the initial aggregation layer) to n (for the root aggregator, which generates the overall suitability). The main idea of this CAS is that with increasing degree of aggregation, the shadow of aggregators increases, including more and more of input attributes, and the importance of the covered area must also increase. In other words, the andness of an aggregator is proportional to the size of its shadow. Both the shadow and the andness attain the maximum value (not necessarily the pure conjunction) for the root aggregator. At some point, the shadow reaches the level where the aggregators must become HPC because it is not acceptable that a significant segment of inputs has zero suitability. Therefore, the CAS with strictly increasing andness makes mandatory all terminal results of the SPC layer. However, in this CAS no individual input attribute can be mandatory, and the input aggregation layer usually starts with aggregators that have a medium andness (either neutrality or SPC).

In many cases, it is necessary to provide mandatory and nonmandatory (desired or optional) inputs. An input attribute becomes mandatory if the path through the aggregation tree that connects the attribute and the root of the tree is going only through HPC aggregators. Such a version of conjunctive CAS is shown in Figure 5.

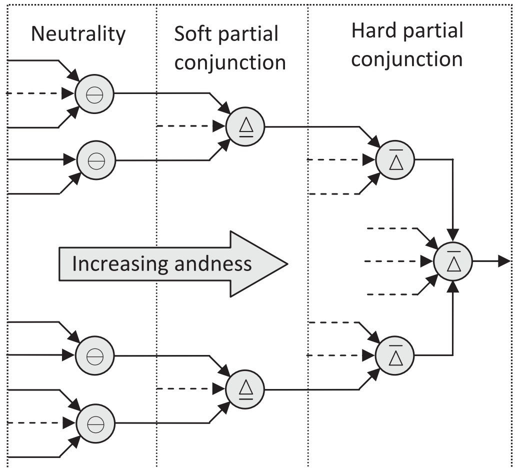

Figure 4. Conjunctive CAS with increasing andness and nonmandatory inputs.

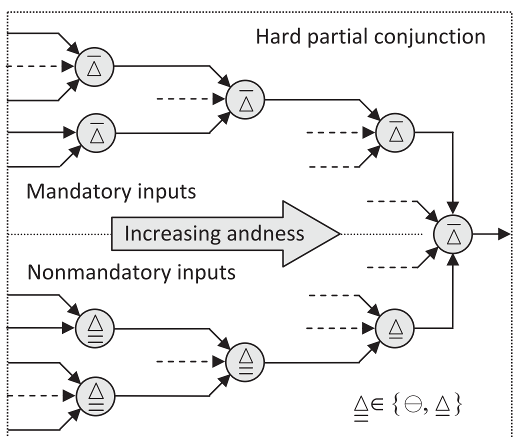

Figure 5. Conjunctive CAS with increasing andness and separated mandatory and nonmandatory inputs.

The mandatory input attributes are separated in a group that forms an HPC subtree, and the nonmandatory inputs are separated in a group that forms an SPC subtree.

The conjunctive structures shown in Figures 4 and 5 are applicable in many cases, where the aggregation structures reflect graded simultaneity requirements. This happens mostly in cases where suitability maps aggregate heterogeneous requirements that can be grouped into mandatory requirements and desired (but not mandatory) requirements. It should be noted, however, that the CAS shown in Figure 5 requires that at least one of nonmandatory inputs must be properly satisfied because the terminal node of the nonmandatory subtree is an input in an HPC

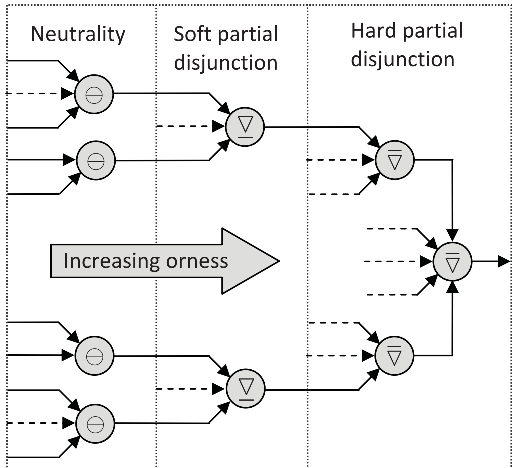

Figure 6. Disjunctive CAS with increasing orness.

aggregator. In other words, for the CAS in Figure 5, the group of nonmandatory inputs is collectively mandatory and this sometimes may be unacceptable. For complex aggregation structures, that property is not a problem because there are always satisfied nonmandatory requirements. If that is not the case (as in the case of criteria with a small number of inputs), it is advisable to use one of mandatory/optional aggregation structures.

4.2. Disjunctive CAS with Increasing Orness

The disjunctive CAS with increasing orness is a structure symmetric to the conjunctive CAS with increasing andness. The version shown in Figure 6 is based on the concept that the orness of an aggregator should be proportional to the size of its shadow. No individual input attribute is considered sufficient to replace all other attributes. At some level of the size of shadow, the aggregators become HPD. If we want to have sufficient input aggregators, then we can use the CAS shown in Figure 7. Similarly to the conjunctive case shown in Figure 4, the CAS shown in Figure 6 has the property that the group of nonsufficient inputs is collectively sufficient because the terminal aggregator of the nonsufficient subtree is an input in an HPD aggregator. This property is sometimes desirable. In all other cases, this problem can be solved by feeding the output of the nonsufficient subtree into an optional/desired input of DPA aggregators.

4.3. Aggregated Mandatory/Optional and Sufficient/Optional CAS

Separation of mandatory and nonmandatory inputs can be guaranteed if we use the CAS shown in Figure 8. This CAS is particularly suitable for relatively small

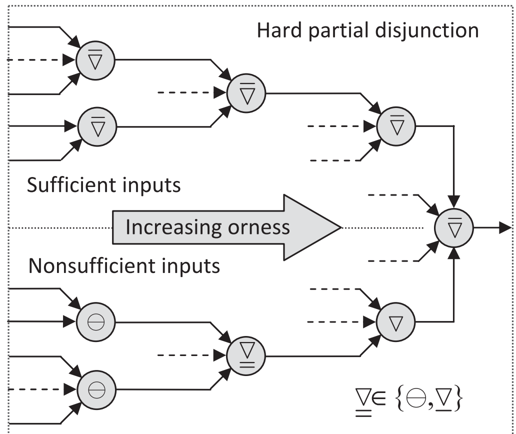

Figure 7. Disjunctive CAS with increasing orness and separated sufficient and non-sufficient inputs.

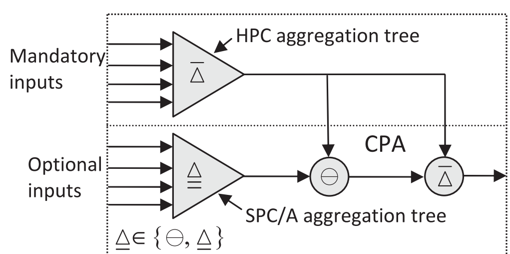

Figure 8. Aggregated mandatory/optional CAS.

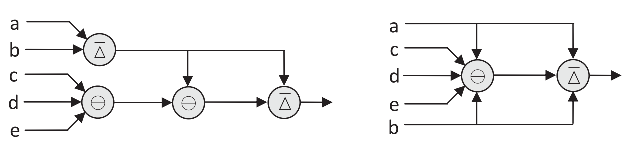

Figure 9. An example of transfiguration of a simple aggregated mandatory/optional CAS.

number of inputs, where all mandatory inputs can be grouped and all optional inputs can be grouped based on their mandatory/nonmandatory status. The mandatory inputs are aggregated using an HPC aggregation tree, and the optional inputs are aggregated using a tree based on SPC and neutrality. In particularly simple special cases, where the number of inputs is small (exemplified in Figure 9), the aggregation can be done directly on an expanded CPA aggregator.

Aggregated mandatory/optional CAS is not suitable in cases where grouping of all mandatory, and all optional inputs may be inappropriate because of heterogeneity of members in each group. In such cases, we can use distributed mandatory/optional CAS.

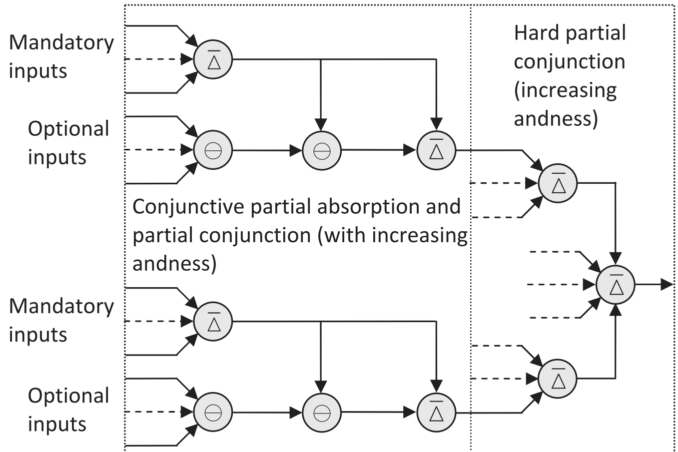

Figure 10. Distributed mandatory/optional CAS.

If the HPC aggregators ($\bar{\Delta}$) in Figures 8 or 9 are replaced by the HPD aggregators ($\bar{\nabla}$), we get the aggregated sufficient/optional CAS that combines a group of sufficient inputs and a group of optional inputs. The main property of the resulting CAS is that the optional inputs can significantly compensate deficiencies of sufficient inputs.

4.4. Distributed Mandatory/Optional and Sufficient/Optional CAS

Distributed mandatory/optional CAS is exemplified in Figure 10. This CAS can have any number of mandatory and nonmandatory inputs that are grouped according to similarity or connectivity of attributes in each group. It is based on multiple aggregated mandatory/optional structures that are frequently combined with SPC and HPC aggregators that typically have increasing andness. The resulting CAS supports an overall simultaneity combined with any number of strictly mandatory and strictly nonmandatory inputs.

If the HPC aggregators ($\bar{\Delta}$) in Figure 10 are replaced by the HPD aggregators ($\bar{\nabla}$), we get the distributed sufficient/optional CAS that combines any number of strictly sufficient inputs and any number of strictly optional inputs. In addition to DPA aggregators, such CAS usually includes SPD and HPD aggregators that support the increasing orness. As in all sufficient/optional structures, the optional/desired inputs can significantly compensate deficiencies of sufficient inputs.

4.5. Mandatory/Desired/Optional and Sufficient/Desired/Optional CAS

Mandatory/optional and sufficient/optional structures can be nested. The common form of such CAS based on nesting $t = (x \trianglerighteq y) \trianglerighteq z$ is shown in Figure 11. The inputs x, y, and z can be interpreted as mandatory, desired, and optional inputs,

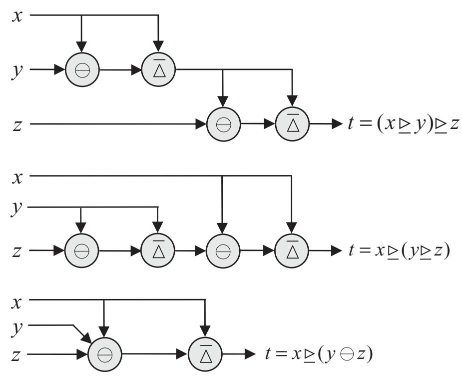

Figure 11. Nested CPA aggregators and their CPA approximation.

respectively. Both desired and optional inputs are nonmandatory, and penalty/reward effects of desired input are assumed to be greater than the effects of the optional input. Similar effects can be achieved with nesting of the form $t = x \trianglerighteq (y \trianglerighteq z)$ as well as with the approximation $t = x \trianglerighteq (y \ominus z)$ that uses a single CPA without nesting. In all cases, x is the mandatory input. The effects of desired and optional inputs y and z can be adjusted by selecting appropriate weights. It is important to mention that the impact of the desired and optional inputs can be low, but it must not be negligible because there is no reason for using negligible attributes. Generally, it is important to check the impact of the optional input z, in particular for the aggregator $t = x \trianglerighteq (y \trianglerighteq z)$, which uses two stages of attenuation of the impact of z.

Similar to other canonical structures that use CPA, we can replace the HPC aggregators by the HPD aggregators to get nested or approximate sufficient/desired/optional CAS. All nested CPA and DPA structures can be expanded by using appropriate aggregation trees to generate the inputs x, y, and z, similarly, as in the case of aggregated mandatory/optional and sufficient/optional CAS (Figure 8).

4.6. Decreasing Andness and Decreasing Orness CAS

Decreasing andness and decreasing orness forms are symmetric to increasing andness and increasing orness forms, but their applicability is rather low. They deserve to be discussed mostly to help avoiding their inappropriate use.

The simplest example of CAS, where the andness of the partial conjunction aggregator is a decreasing function of the size of its shadow is shown in Figure 12. Both p and q are functions of mandatory inputs. So, all inputs must be satisfied to have $p > 0$ and $q > 0$. This implies that both p and q are rather significant. However, the aggregation of p and q is based on an aggregator that is close to neutrality (andness $\alpha \approx 1/2$). That indicates a modest importance of both p and q because only one of them is sufficient to produce the output $t > 0$. Of course, this inconsistence in the interpretation of significance of p and q is unacceptable unless there is a strong and detailed justification. Similar request for detailed justification

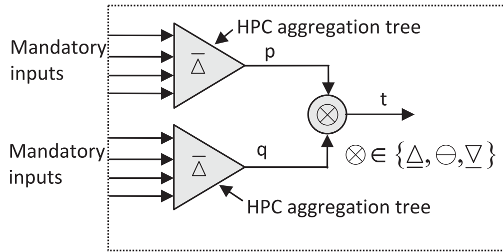

Figure 12. Andness as a decreasing function of the shadow of an aggregator.

holds also in the case of decreasing orness that can be obtained if HPC aggregators in Figure 12 are replaced by HPD aggregators.

5. LOGIC AGGREGATION IN SUITABILITY MAPS

In this section, we present the CAS-based logic aggregation in designing and implementing LSP suitability maps. Our goal is to present LSP maps that can be developed and used by nonprofessional users to evaluate the suitability of a location of home or apartment in urban areas. The decision about the suitability of a given location depends on the easy access to POIs that are important for a specific DM. Typical POIs that are taken into account include stores, restaurants, schools, parks, public transportation stations, etc. In each point, the DM can determine the overall suitability based on proximity to selected POIs. The distribution of such suitability values is the POI's suitability map.

The majority of modern online maps (e.g., Google maps,[42] GPS maps, and others) include a variety of POIs. In particular, Google offers several APIs for accessing the database containing a wide variety of POIs.[4] We will use Google maps and Google API to develop LSP/POI suitability maps that are displayed as a (transparent, semitransparent, or nontransparent) layer on top of Google maps.

5.1. Selecting Elementary Attributes

The first step in building an LSP criterion function consists of selecting elementary input attributes. In the case of POI-based suitability maps, the attributes are distances from specific categories of POIs. There are various ways to evaluate categories of POIs (e.g., restaurants, schools, stores).[27,43] For simplicity, in this paper, we use attributes that are defined as distances from the closest POI of specific type and select their groups based on the desired type of suitability map. A menu for selecting the type of LSP/POI map is shown in Figure 13.

Each type of LSP/POI map is defined using a set of representative POIs that describe the needs of specific category of users. For example, the "Students" category is described using the following 12 POIs: bookstore, bus station, coffee shop,

Figure 13. A menu for selecting the type of LSP/POI suitability map.

college, deli, fast food restaurant, grocery store, gym, library, swimming pool, tennis court, and train station. The "Retirement" category is characterized by 16 POIs: bus station, church, coffee shop, farmer's market, garden center, grocery store, hospital, library, medical center, park, pharmacy, restaurant, taxi service, tea house, train station, and travel agency. For the first 10 categories, we use predefined but editable representative sets of POIs. In the last category denoted as "make your own map," the user is offered a list of 94 types of POIs to select appropriate subset and use it in creating a desires specific LSP/POI map.

5.2. Canonical Forms of Elementary Criteria

The second step in building an LSP criterion function consists of building elementary criteria for the evaluation of input attributes. Since our attributes are distances from an evaluated location to selected POIs, it is convenient to differentiate three characteristic cases:

1. preferred large distances,
2. preferred small distances, and
3. preferred range of distances.

The first case occurs in situations, where the DM wants to avoid vicinity of undesirable objects. Indeed, some DMs would not like to live next door to a funeral home, cemetery, airport, hospital, or a very popular restaurant. The second case corresponds to situations, where proximity to selected POIs is desirable. That might be the case of proximity to parks, grocery stores, or children day care centers. In the third case, the DM wants to avoid both the excessive proximity and the excessive distance. Such criterion is frequently used for restaurants and schools: Excessive

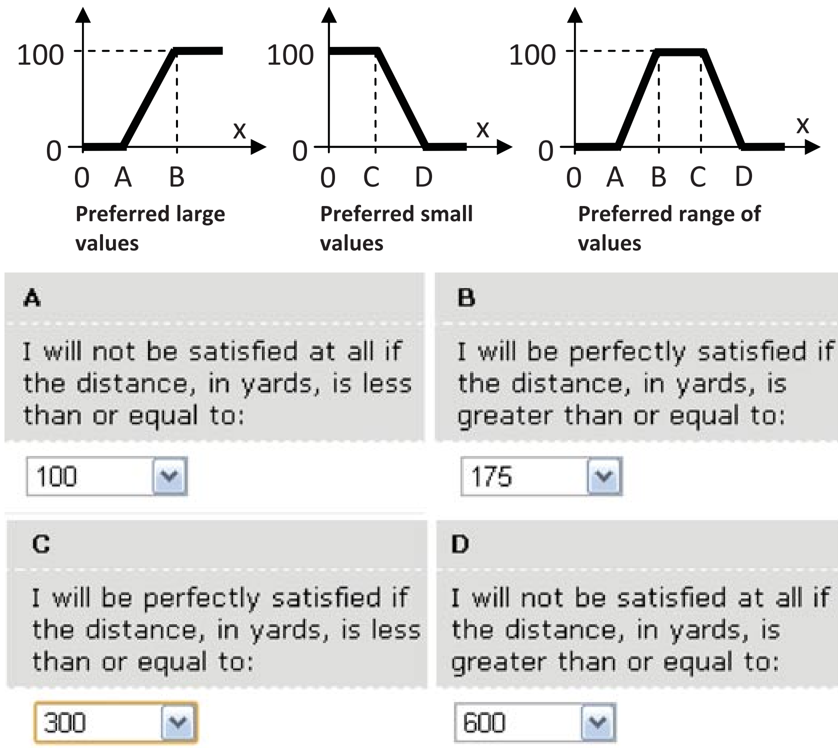

Figure 14. Canonical forms of elementary criteria and menus for selecting their parameters (A, B, C, and D).

proximity is generally inconvenient (noise, too much traffic, etc.), but a walking distance is highly desirable, yielding a preferred range of distances.

The presented three characteristic cases yield three canonical forms of elementary criteria shown in Figure 14. These forms are convenient for nonprofessional DMs because of simplicity in selecting parameters A, B, C, and D. The selection of parameters is based on four verbalized menus shown in Figure 14. The users of the menus can select three types of inputs: A, B only (A < B), or C, D only (C < D), or all four parameters A, B, C, and D (A < B < C < D).

5.3. The Logic Aggregation Structure

For each input attribute, the corresponding elementary attribute criterion returns the elementary suitability, which is interpreted either as a degree of truth of the statement claiming that the attribute perfectly satisfies user requirements, or as a degree of membership in a fuzzy set of perfect locations. Each elementary suitability belongs to [0,1], and the next step is to create a logic aggregation structure

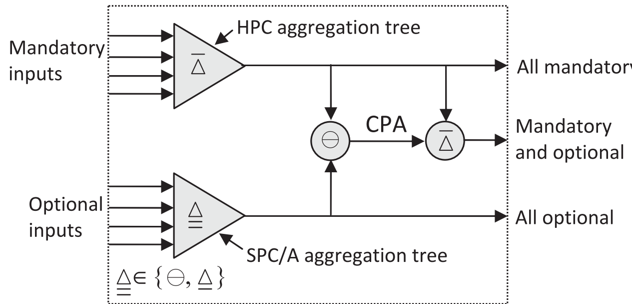

Figure 15. The aggregation structure for LSP/POI suitability maps.

that aggregates all elementary attribute suitability values and computes the overall suitability.

The selection of the logic aggregation structure can be based on CAS presented in Section 4. Taking into account that the number of input attributes in LSP/POI maps is small (regularly less than 20), some POIs can be specified as mandatory and some POIs are optional, the maps must be produced and used by nonprofessional DMs, the aggregation structure should not be too complex, and the high precision of results is not critical, it is appropriate to select the aggregated mandatory/optional CAS, modified as shown in Figure 15. In addition to regular (mandatory and optional) output, this aggregation structure also provides two special cases, where all inputs are mandatory, or all inputs are optional.

The selected aggregation structure is supported by the menus presented in Figure 16. In the first step, users are requested to select the type of attributes: each attribute can be mandatory, or optional, or not included in the LSP/POI map (the example in Figure 16 uses three mandatory and seven optional attributes). Mandatory attributes are aggregated using an HPC aggregation tree, and optional attributes are aggregated using an SPC tree. The results of the two preliminary aggregations are then combined using the CPA aggregator.

Each input in the LSP/POI model can have nine verbalized levels of relative importance (from 1 = lowest to 9 = highest), as illustrated in Figure 16. The selected levels are then used to compute relative weights of attributes, e.g., if the mandatory attributes have the importance levels 8, 7, and 8, they will be aggregated using normalized weights 8/23, 7/23, and 8/23.

Nonprofessional users cannot perform fine-tuning of properties of the CPA, HPC, and SPC aggregators. However, it is convenient to offer users to select between three levels of severity of requirements (relaxed, standard, and strict) shown in Figure 16 and Table I. As the requirements move from *relaxed* to *strict*, the penalty of CPA increases, the reward decreases, and the andness of HPC and SPC aggregators increases.

Our implementation of LSP/POI suitability maps[44] is based on Google maps, and the suitability map is presented as either gray or color-coded layer with adjustable transparency on top of a Google map of specific area. The default area is

Table I. Three selectable levels of requirements.

	Requirements		
Aggregated mandatory/optional CAS parameters (%)	Relaxed	Standard	Strict
Mean CPA penalty	15	30	45
Mean CPA reward	30	25	20
HPC andness	67	77	87
SPC andness	38	50	62

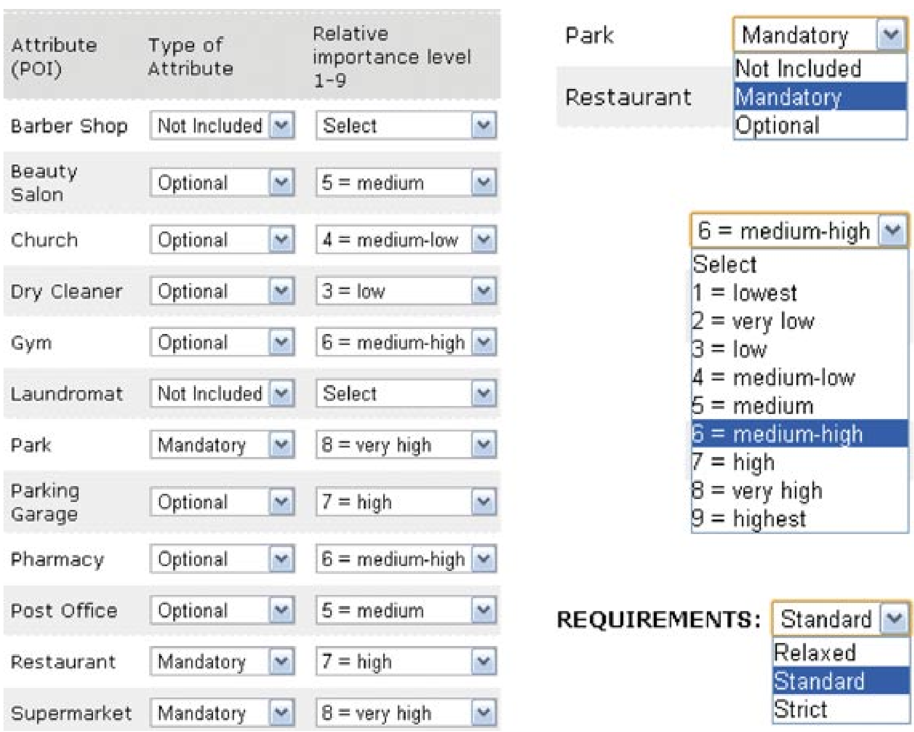

Figure 16. Menus for selecting the type and parameters of the aggregation structure.

a 2-mile square divided into $35 \times 35 = 1225$ cells, where in each cell we compute suitability according to a selected LSP/POI criterion. A fragment of suitability map with percentage suitability shown in each cell is shown in Figure 17.

5.4. Implementing LSP/POI Suitability Maps

The parameters of LSP/POI suitability maps are selected according to menus shown in Figures 13–16. For example, the criterion for suitability of an area for a young family home includes the following 18 POI categories: airport access,

Figure 17. A fragment of a 75% transparent LSP/POI map on top of a Google map with POIs.

bank, cemetery, church, day care service, day care centers, dry cleaner, elementary school, farmer's market, funeral home, grocery store, gym, high school, hospital, kindergarten, middle school, park, and restaurants. For each category, the user may decide to include or not to include the category. Each included POI category can be either mandatory or optional.

The role of logic aggregation is visible in Figure 18. The maps show the suitability of location for a young family home in the Sunset area of San Francisco. Dark areas denote low suitability, and light areas denote high suitability. For better visibility, the maps are gray and nontransparent. The first three maps show the effects of increasing andness (relaxed, standard, and strict criterion without mandatory requirements). The fourth map shows the case where the proximity to day care, elementary school, and kindergarten are mandatory.

The availability of HPC and the use of CPA that aggregates mandatory and optional attributes is indispensable for making correct suitability maps. Indeed, the four cases shown in Figure 18 demonstrate that if mandatory requirements are not properly modeled, some of completely unsuitable (black) areas could be interpreted as suitable (partially acceptable, coded as gray), yielding wrong decisions, inconsistent with expectations of DM. Furthermore, the map substantially changes when another set of attributes is selected as mandatory. Such effects are visible in Figure 19 (compared to Figure 18).

5.5. Fuzzy and Logic Interpretation of Suitability Maps

Fuzziness and partial truth belong to the same family of concepts. Both of them belong to soft computing (computing with variables that are a matter of degree). In

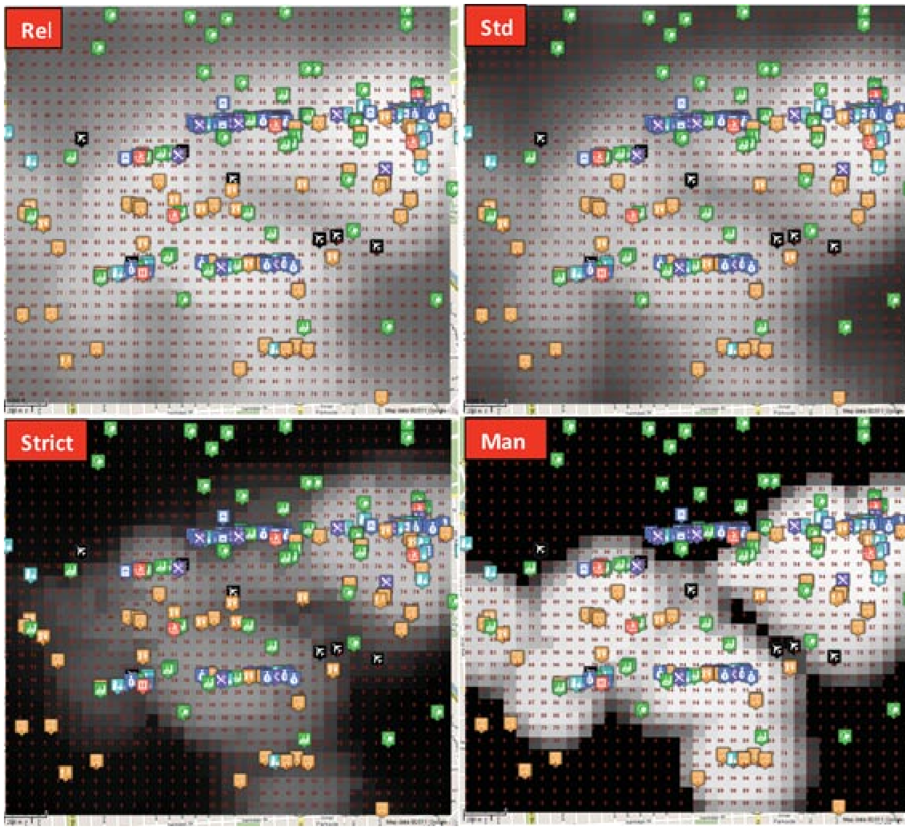

Figure 18. Young family home location suitability maps (relaxed, standard, strict, and mandatory requirements).

the area of suitability analysis, the fuzzy interpretation and the logic interpretation of evaluation results are equivalent. Indeed, elementary criteria can be interpreted as membership functions of input variables (degree of membership in a set of systems that completely satisfy user requirements)[45]. Similarly, the overall suitability can be interpreted as the compound degree of membership in the set of systems that completely satisfy all user requirements.

On the other hand, the same concepts can be interpreted as logic concepts. Each elementary suitability score represents a degree of truth of the statement claiming that the specific input completely satisfies user needs. The overall suitability is the degree of truth of the statement claiming that an evaluated system as a whole completely satisfies all user requirements. Therefore, the suitability maps can have equivalent logic and fuzzy interpretation, and both interpretations are equally acceptable.

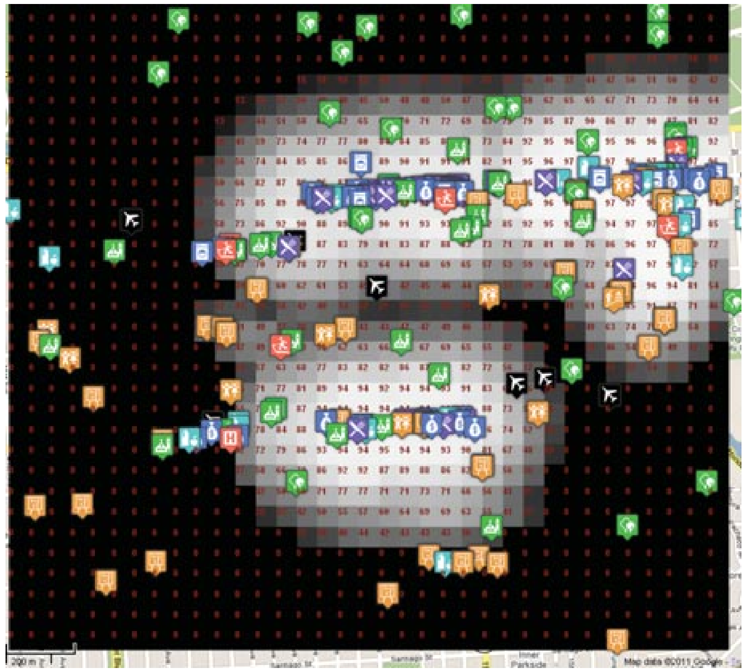

Figure 19. Young family home location suitability map in the case of mandatory proximity to restaurants and grocery stores.

6. CONCLUSIONS

All MCDMs should be models of human decision making and the only way to prove their credibility is to show their proximity to human evaluation logic. Our analysis shows that the majority of existing MCDM are not derived with an explicit goal to model observable properties of human reasoning. Indeed, human reasoning is restricted to neither the use of arithmetic mean nor relative criteria only. Humans use a spectrum of absolute and relative elementary criteria and a spectrum of soft computing logic aggregators, such as SPC and HPC, and SPD and HPD, CPA and DPA, as well as other compound aggregators.

Decision methods that are used in space management and land-use evaluation problems cannot be randomly selected, without appropriate justification. The justification for using a specific evaluation method in a GIS environment should be based on investigating the capability of the method to support features that are proved to characterize human decision making. Many oversimplifications that are reported as frequent in the GIS literature[6] (particularly those based on simple additive models) are based on the fact that they are "easy to understand and intuitively appealing." Unfortunately, that is not enough. Before using mathematical models, it is first necessary to prove that they are appropriate, and in the case of suitability maps that means the satisfaction of a list of logic requirements.

LSP is a decision method developed with the goal to support logic operators that model observable properties of human evaluation reasoning. Consequently, it is realistic to expect that the LSP method can provide highly accurate and justifiable models for GIS applications, such as land-use evaluation, suitability maps, and natural resources planning. Our case study of suitability maps that analyzes the suitability of a region with respect to POIs clearly illustrates the features and advantages of the LSP method for generating suitability maps.

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