

CS 188  
Fall 2018

Introduction to  
Artificial Intelligence

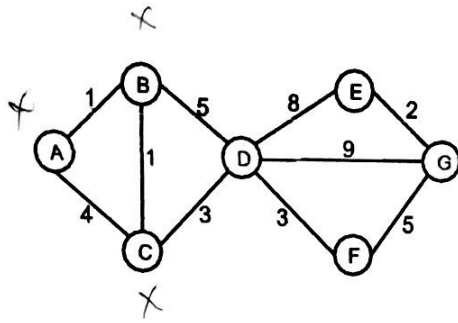
Written HW 1

**Due:** Tuesday 9/4/2018 at 11:59pm (submit via Gradescope)

**Policy:** Can be solved in groups (acknowledge collaborators) but must be written up individually

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Collaborators	None

# Q1. Search



Node	$h_1$	$h_2$
A	9.5	10
B	9	12
C	8	10
D	7	8
E	1.5	1
F	4	4.5
G	0	0

Consider the state space graph shown above. A is the start state and G is the goal state. The costs for each edge are shown on the graph. Each edge can be traversed in both directions. Note that the heuristic  $h_1$  is consistent but the heuristic  $h_2$  is not consistent.

## (a) Possible paths returned

For each of the following graph search strategies (do not answer for tree search), mark which, if any, of the listed paths it could return. Note that for some search strategies the specific path returned might depend on tie-breaking behavior. In any such cases, make sure to mark all paths that could be returned under some tie-breaking scheme.

Search Algorithm	A-B-D-G	A-C-D-G	A-B-C-D-F-G
Depth first search	X	X	X
Breadth first search	X	X	X
Uniform cost search			X
A* search with heuristic $h_1$	X		
A* search with heuristic $h_2$		X	

## (b) Heuristic function properties

Suppose you are completing the new heuristic function  $h_3$  shown below. All the values are fixed except  $h_3(B)$ .

Node	A	B	C	D	E	F	G
$h_3$	10	?	9	7	1.5	4.5	0

For each of the following conditions, write the set of values that are possible for  $h_3(B)$ . For example, to denote all non-negative numbers, write  $[0, \infty]$ , to denote the empty set, write  $\emptyset$ , and so on.

(i) What values of  $h_3(B)$  make  $h_3$  admissible?

$[0, 10]$

(ii) What values of  $h_3(B)$  make  $h_3$  consistent?

$[9, 10]$

(iii) What values of  $h_3(B)$  will cause A\* graph search to expand node A, then node C, then node B, then node D in order?

$[14, 24]$

Handwritten notes:  
 $(AC, 15)$   
 $(ACB, 15)$   
 $AB > 15$   
 $AB < ACD$   
 $ACD = 25$

## Q2. $n$ -pacmen search

Consider the problem of controlling  $n$  pacmen simultaneously. Several pacmen can be in the same square at the same time, and at each time step, each pacman moves by at most one unit vertically or horizontally (in other words, a pacman can stop, and also several pacmen can move simultaneously). The goal of the game is to have all the pacmen be at the same square in the minimum number of time steps. In this question, use the following notation: let  $M$  denote the number of squares in the maze that are not walls (i.e. the number of squares where pacmen can go);  $n$  the number of pacmen; and  $p_i = (x_i, y_i) : i = 1 \dots n$ , the position of pacman  $i$ . Assume that the maze is connected.

(a) What is the state space of this problem?

- The location of each Pacman  $p_i = (x_i, y_i)$ ,  $i = 1 \dots n$
- Actions: Move left, right, up, down or stop
- Successor: update location
- Goal test: is location of all Pacmen the same

(b) What is the size of the state space (not a bound, the exact size)?

World state:

Agent positions:  $M$

$$\text{Exact size} = M^n$$

(c) Give the tightest upper bound on the branching factor of this problem.

$$\text{b.f.} = 5^n$$

(d) Bound the number of nodes expanded by uniform cost tree search on this problem, as a function of  $n$  and  $M$ . Justify your answer.

Since all cost is the same for each step, this will run as BFS does. The tree has a max height of  $M$ . Thus, since the branching factor is  $5^n$ , the max nodes expanded  $5^{nM}$

(e) Which of the following heuristics are admissible? Which one(s), if any, are consistent? Circle the corresponding Roman numerals and briefly justify all your answers.

1. The number of (ordered) pairs  $(i, j)$  of pacmen with different coordinates:

$$h_1(p_1, \dots, p_n) = \sum_{i=1}^n \sum_{j=i+1}^n 1[p_i \neq p_j] \quad \text{where} \quad 1[p_i \neq p_j] = \begin{cases} 1 & \text{if } p_i \neq p_j \\ 0 & \text{otherwise} \end{cases}$$

(i) Consistent? (ii) Admissible?

cc) This is not admissible. Ex. If three pacmen were in different coordinates but all could move to the same spot in one move the heuristic would overestimate. The heuristic is therefore inconsistent for the same reason i.e. actual arc cost > arc cost

$$h_2((x_1, y_1), \dots, (x_n, y_n)) = \frac{1}{2} \max \{ \max_{i,j} |x_i - x_j|, \max_{i,j} |y_i - y_j| \}$$

(i) Consistent? (ii) Admissible?

cc) Consistent. The heuristic arc cost from one space to another will be less than the actual arc cost.

cc) Admissible. The cost of the heuristic is less than the cost to the goal and it is not negative.