1) $z_j^l = \sum_k a_j^{l-1} w_{kj} + b_j^l$

(a) $a_j^l = f(z_j^l) \qquad \frac{da_j^l}{dz_j^l} = f'(z_j^l)$

$$\Delta_i^L = \frac{dC}{dz_j^L} = \frac{dC}{da_j^L}\frac{da_j^L}{dz_j^L} = \frac{dC}{a_j^L} f'(z_j^L)$$

for $l < L$

$$\Delta_j^l = \frac{dC}{dz_j^l} = \sum_k \frac{dC}{dz_k^{l+1}} \frac{dz_k^{l+1}}{da_j^l} \frac{da_j^l}{dz_j^l}$$

$$= \sum_k \Delta_k^{l+1} \left[w^{(l+1)^T} \right]_{kj} f'(z_j^l)$$

(b) $$\frac{dC}{dw_{ij}^l} = \frac{dC}{dz_j^l}\frac{dz_j^l}{dw_{ij}^l} = \Delta_j^l \, a_i^{l-1} \frac{dw_{ij}}{dw_{ij}} = \Delta_j^l a_i^{l-1}$$

$$\frac{dC}{db_i^l} = \frac{dC}{dz_j^l}\frac{dz_j^l}{db_j^l} = \Delta_j^l$$

2) $a_j^l = z_j^l$

$$\Delta_j^L = \frac{dC}{dz_j^L} = \frac{dC}{da_j^L}\frac{da_j^L}{dz_j^L} = \frac{dC}{da_j^L}$$

$l < L$

$$\Delta_j^l = \frac{dC}{dz_j^l} = \sum_k \frac{dC}{dz_k^{l+1}} \frac{dz_k^{l+1}}{da_j^l} \frac{da_j^l}{dz_j^l} = \sum_k \Delta_k^{l+1} \left(w^{l+1} \right)_{kj}$$

$$\frac{dC}{dw_{ij}^l} = a_j^{l-1} \Delta_j^l \qquad \frac{dC}{db_j^l} = \Delta_j^l$$