3406 HW 2 Cole Aulery

① a) $A = \begin{bmatrix} 1 & 2 & 4 \\ -2 & 3 & 1 \\ -4 & 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

a) $AB = \begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix} \cdot 2 + \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \cdot 2 + \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \cdot 3 = \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$

b) $AB = \begin{bmatrix} 1 \cdot 2 + 2 \cdot 2 + 4 \cdot 3 \\ -2 \cdot 2 + 3 \cdot 2 + 1 \cdot 3 \\ -4 \cdot 2 + 1 \cdot 2 + 2 \cdot 3 \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 0 \end{bmatrix}$

② a) $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + 0y \\ 0x + y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

b) $Q = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0x + y \\ x + 0y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}$

c) $R = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + 0y \\ -x + y \end{bmatrix} = \begin{bmatrix} x \\ y - x \end{bmatrix}$

③ $P = \begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix}$, $\begin{bmatrix} \cos\pi & -\sin\pi \\ \sin\pi & \cos\pi \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$

$= \begin{bmatrix} x\cos\pi - y\sin\pi \\ x\sin\pi + y\cos\pi \end{bmatrix}$, $x = 1$, $y = 0$

$= \begin{bmatrix} \cos\pi \\ \sin\pi \end{bmatrix}$

④ $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

⑤

(5) $2x-3y=3$
$4x-5y+z=7$
$2x-y-3z=5$

$$\rightarrow \begin{array}{c} \begin{array}{cccc} x & y & z & \end{array} \\ \left[\begin{array}{cccc} 2 & 3 & 0 & 3 \\ 4 & -5 & 1 & 7 \\ 2 & -1 & -3 & 5 \end{array}\right] \end{array} \begin{array}{l} \\ +(-2\cdot\text{first row}) \\ +\text{first row} \end{array}$$

$$\downarrow$$

$$\left[\begin{array}{cccc} 2 & 3 & 0 & 3 \\ 0 & -11 & 1 & 1 \\ 0 & -4 & -3 & 2 \end{array}\right] + \frac{4}{11}\cdot\text{2nd row}$$

$$\leftarrow \left[\begin{array}{cccc} 2 & 3 & 0 & 3 \\ 0 & -11 & 1 & 1 \\ 0 & 0 & -\frac{37}{11} & \frac{18}{11} \end{array}\right]$$

$-\frac{37}{11}z=\frac{18}{11}$

$-37z=18$

$\boxed{z=-\frac{18}{37}}$

$-11y-\frac{18}{37}=1$

$\boxed{y=-5/37}$

$2x-15/37=3$

$\boxed{x=\frac{63}{37}}$

(6) True; The triangle inequality states $\|v+w\| \leq |v|+|w|$ as an upper bound for $\|v+w\| \cdot \|v-w\|$ approaches the minimum, thus is conversly bounded by $\|v\|-\|w\|$.