Estimation of the effect of climate on infectious disesaes

Table of contents

1 Motivating Question

If climate change (through some sort of variable such as temperature or precipitation) is affecting the number of cases of infectious disease, it is an outstanding question how strong this effect must be to identify it from some background autocorrelated value. It is worth asking how correlated a confounding background process can be before we cannot recover a value of interest.

2 Data & model

Assuming we have some data on observed cases of a given infectious disease. The relationship between those observed cases and actual cases is a state process with some observation error, ϵ_o. Cases themselves are now given as a state space model where the number of cases at time $t+1$ are driven by the effect of both temperature variance (consistent through time) and mean temperature (increasing through time), as well as an unobserved driver that is correlated through time with the mean temperature. We can simplify this for a first pass and assume that there's no error in measurement. With this, then we say:

Let:

- Y_t represent the observed number of cases at time t.
- $X_{\mu,t}$ represent the mean temperature at time t, and let $X_{\sigma,t}$ represent the temperature variance, assumed constant over time.
- U_t represent the unobserved driver correlated with mean temperature $X_{\mu,t}$.

2.1 Model Equation

We can consider the model of observed cases where the number of observed cases at some future timepoint $t+1$ is given as

$$Y_{t+1} = \beta_0 + \beta_1 X_{\mu,t} + \beta_2 X_{\sigma,t} + U_t \tag{1}$$

2.2 Likelihood for Y_{t+1}

Assume the response Y_{t+1}follows a Negative Binomial distribution with mean $\lambda_t$and dispersion ϕ:

$$Y_{t+1} \sim \mathrm{NegBin}(\lambda_t, \phi), \quad \lambda_t = \exp(\beta_0 + \beta_1 X_{\mu,t} + \beta_2 X_{\sigma,t} + U_t). \tag{2}$$

2.3 Unobserved Process U_t

The unobserved process $U_t$is modeled as a latent variable, for example, using a Gaussian random walk or autoregressive process:

$$U_t \sim \mathcal{N}(\rho U_{t-1}, \tau^2), \tag{3}$$

where ρ controls the correlation structure, and τ^2 is the process variance.

2.4 Prior Distributions

Assign priors to the parameters:

$$\beta_0, \beta_1, \beta_2 \sim \mathcal{N}(0, 10), \quad \phi \sim \mathrm{Gamma}(1, 1), \tag{4}$$

and

$$\rho \sim \mathcal{Beta}(2, 2), \quad \tau^2 \sim \text{Inverse-Gamma}(2, 1). \tag{5}$$

2.5 Full Model

The full model is given by:

$$
\begin{aligned}
Y_{t+1} &\sim \mathrm{NegBin}(\lambda_t, \phi), \\
\lambda_t &= \exp(\beta_0 + \beta_1 X_{\mu,t} + \beta_2 X_{\sigma,t} + U_t), \\
U_t &\sim \mathcal{N}(\rho U_{t-1}, \tau^2), \quad \text{for } t = 2, \dots, T, \\
U_1 &\sim \mathcal{N}(0, \tau^2).
\end{aligned}
$$