Variation in thermal pressures and resource availability drives disease dynamics

Cole Brookson David Vasseur

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1 Model

Vinton and Vasseur (2022) provide a basic model for temperature-dependent consumer-resource dynamics with a chemostat model of resource, R, supply, given by

$$\frac{dR}{dt} = D(S - R) - f(R, T)C, \tag{1}$$

where there is an inflow density S and an outflow rate D and $f(R, T)$ i the functional response of R with respect to temperature T. The biomass change of the consumer C is given by

$$\frac{dC}{dt} = (1 - \delta)f(R, T) - m(T)C, \tag{2}$$

where $(1 - \delta)$ is the consumption efficiency (denoted as a fraction), and m is the rate of respiration. W assume a Boltzmann-Arrhenius relationship and approximate that function $m(T)$ with

$$m(T) = m_a e^{m_b T} + m_c, \tag{3}$$

with m_a, m_b and m_c all > 0. The functional response is a standard type II, with attack rate a, handling rate $(1/h)$, but re-state according to the Michaelis-Menten form of

$$f(R,T) = I_{max}(T) \times \frac{R}{R + R_{\text{half}}}, \tag{4}$$

and $I_{max}T$ is the maximum uptake rate which we state as equivalent to the handling rate $I_{max}T \equiv 1/h$, and then the half-saturation density is made equivalnt via $R_{\text{half}} \equiv \frac{1}{a \times h}$. Note that the resource saturation is reached by $R/(R_{\text{half}} + R)$, which is independent of I_{max}. Last,

$$I_{max}(T) = e^{-(T-T_I)^2 \gamma}, \tag{5}$$

is the equation governing the relationship with temperature, where T_I is the optimum temperature for consumption, and γ is the breadth of response.

1.1 Equilibrium Solutions

Vinton and Vasseur (2022) showed that there is a coexistence equilibrium solution where

$$R_e = \frac{mR_{half}}{(1-\delta)I_{max}(T) - m}, \tag{6}$$

and

$$C_e = \frac{D(S - R_e)(R_e - R_{half})}{I_{max}(T)R_e}, \tag{7}$$

Vinton, Anna C., and David A. Vasseur. 2022. "Resource Limitation Determines Realized Thermal Performance of Consumers in Trophodynamic Models." *Ecology Letters* 25 (10): 2142–55. https://doi.org/10.1111/ele.14086.