300 Quals Guide

June 23, 2025

1 Exponential Families

Fact 1 ($E[T(X)$)
$= \partial_{\eta_i} A(\eta)]$ asdf

2 James-Stein Estimator

Setup: $\mathcal{N}(\mu, \sigma^2)$ with σ^2 known.

Theorem 2 (SURE - Stein Unbiased Risk Estimator)
Letting $\hat{\mu}(x) = x + g(x)$ for $g : \mathbb{R}^p \to \mathbb{R}^p$ almost differentiable, and assume that $\mathbf{E}[\sum_{i=1}^p |\partial_i g_i(X)|] < \infty$. Then:

$$\mathbf{E}_\mu[\|\hat{\mu}(X) - \mu\|^2] = p\sigma^2 + \mathbf{E}\left[\|g(X)\|^2 + 2\sigma^2 \sum_{i=1}^p \partial_i g_i(X)\right].$$

Proved using integration by Parts – see Lec 17 300c.

Fact 3 (UMVU is not admissible in $\mathcal{N}(\mu, 1)$ model)
Because James Stein renders X inadmissible.

J-S estimator is given by:

$$\hat{\mu}^{JS}(X) = \left(1 - \frac{\sigma^2(p-2)}{\|X\|_2^2}\right) X,$$

biased towards the origin. Prove that it has better risk by SURE estimator.

3 Bayes Estimators

Fact 4 (Unique Bayes is Admissible)
Idea is that if $R(\hat{\theta}', \theta) \leqslant R(\hat{\theta}, \theta)$ for all θ, it would then be Bayes.
provided the prior isn't super weird (eg continuous dist with an atom)

Fact 5 (Constant risk Bayes is minimax)
If not, some other estimator would render Bayes inadmissible, which would make that estimator the Bayes estimator.

Fact 6 (Bayes is not UMVU if $r_\Lambda < \infty$)
Under square error loss, Bayes estimators are biased

4 UMVU

5 MRE

Fact 7 (UMVU is MRE if UMVU is location equivariant)
(In a location model) Is this just with square error loss?

6 Minimax