

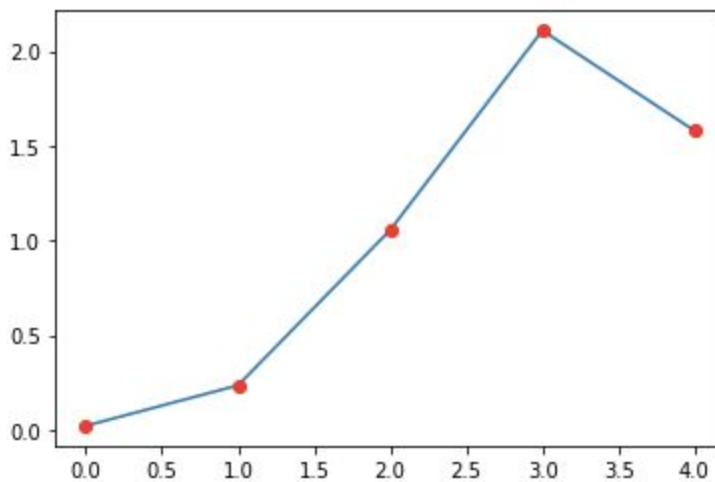
Cole Constantino  
Cxc820  
HW5

1) a.

$$\begin{aligned} p(y|\theta, n) &= \binom{n}{y} \theta^y (1-\theta)^{n-y} \\ \int_0^1 p(y|\theta, n) d\theta &= \frac{1}{n+1} \quad f_y(y|\theta, n) = P_\theta(\theta) \\ &\Rightarrow \binom{n}{y} \theta^y (1-\theta)^{n-y} \\ \int_0^1 f_y(x|\theta, n) P(\theta) d\theta &= \int_0^1 \binom{n}{y} \theta^y (1-\theta)^{n-y} d\theta \\ &\Rightarrow \boxed{\frac{1}{n+1}} \checkmark \end{aligned}$$

b. (matplotlib)

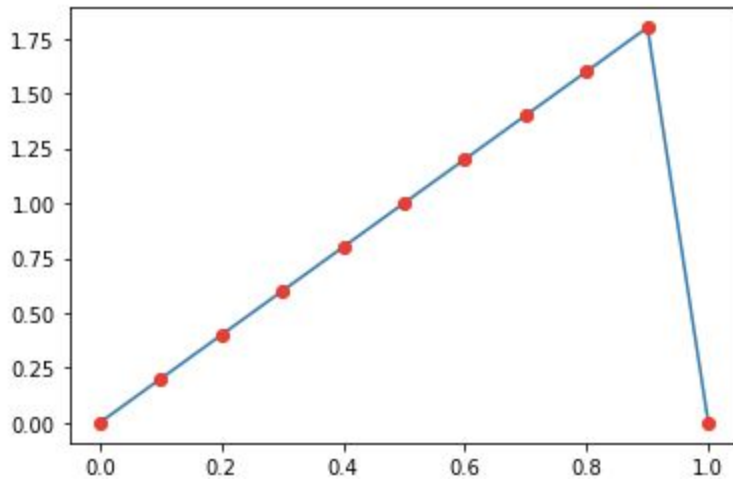
```
In [16]: runfile('/Users/coleconstantino/.spyder-py3/ter  
coleconstantino/.spyder-py3')
```



c.

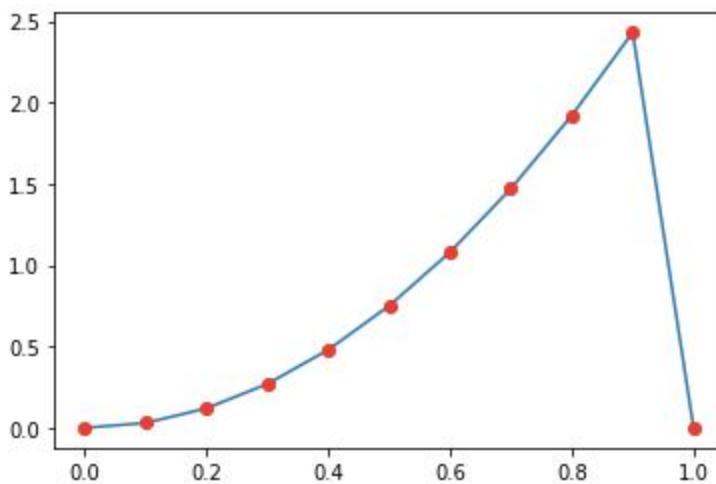
(n,heads)  $\Rightarrow$  (1,1)

```
In [18]: runfile('/Users/coleconstantino/.spyder-py3/t  
coleconstantino/.spyder-py3')
```



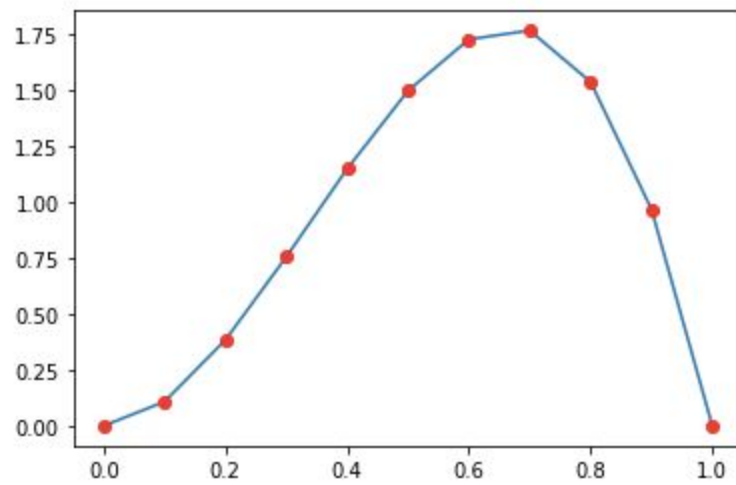
(n,heads)  $\Rightarrow$  (2,2)

```
In [19]: runfile('/Users/coleconstantino/.spyder-py3/t  
coleconstantino/.spyder-py3')
```



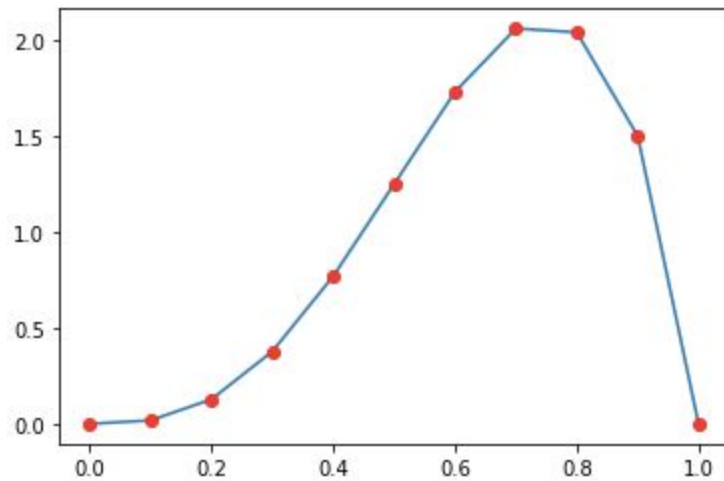
$(n, \text{heads}) \Rightarrow (3, 2)$

```
coleconstantino/.spyder-py3')
```



$(n, \text{heads}) \Rightarrow (3, 3)$

```
in [49]: runfile('/user/coleconstantino/.spyder-py3/coleconstantino/.spyder-py3')
```



2) [Java Code will be attached in the zip file]

a.

For this problem I was able to figure out a way to get all the posterior values via JAVA and copy and pasted it to matplotlib in python. [I did this because I am very experienced in Java, and not so much python]

```
Welcome to DrJava. Working directory is /Users/coleconstantino/Downloads
> run Question2A
h1 = [0.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
h2 = [0.2, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
h3 = [0.4, 0.0035, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
h4 = [0.2, 0.1009, 0.0063, 4.0E-4, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
h5 = [0.1, 0.8956, 0.9937, 0.9996, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0]
```

Paste to python and plot:

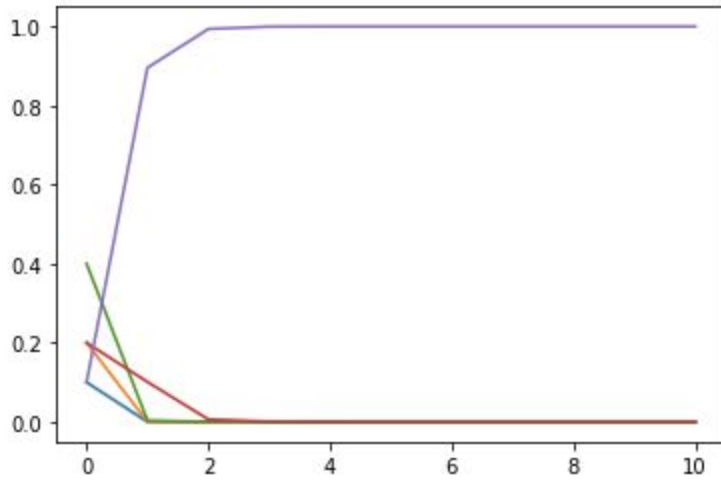
```
import matplotlib.pyplot as plt

h1 = [0.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
h2 = [0.2, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
h3 = [0.4, 0.0035, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
h4 = [0.2, 0.1009, 0.0063, 4.0E-4, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
h5 = [0.1, 0.8956, 0.9937, 0.9996, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0]

plt.plot(h1)
plt.plot(h2)
plt.plot(h3)
plt.plot(h4)
plt.plot(h5)
plt.show
```

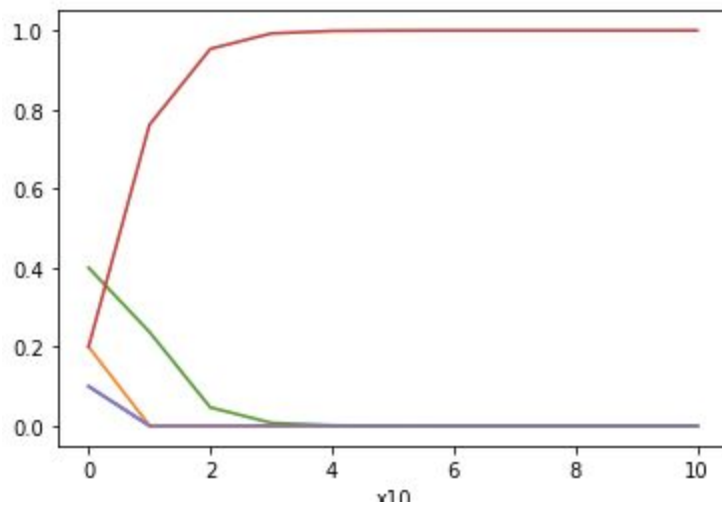
[h1]

```
In [8]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```



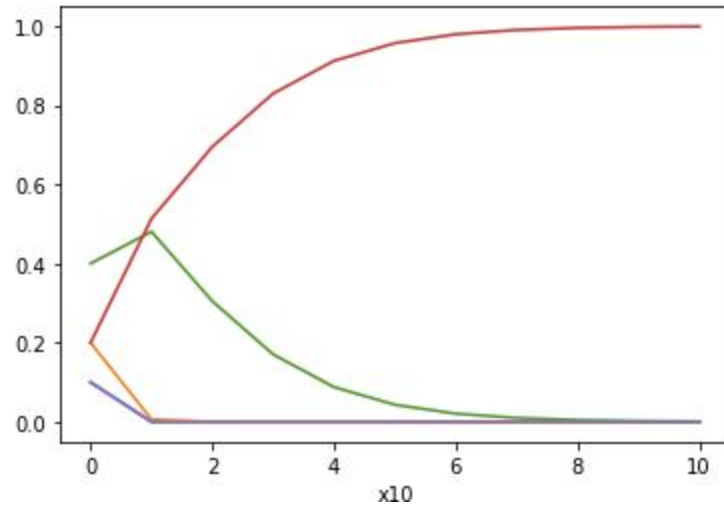
[h2]

```
In [19]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```



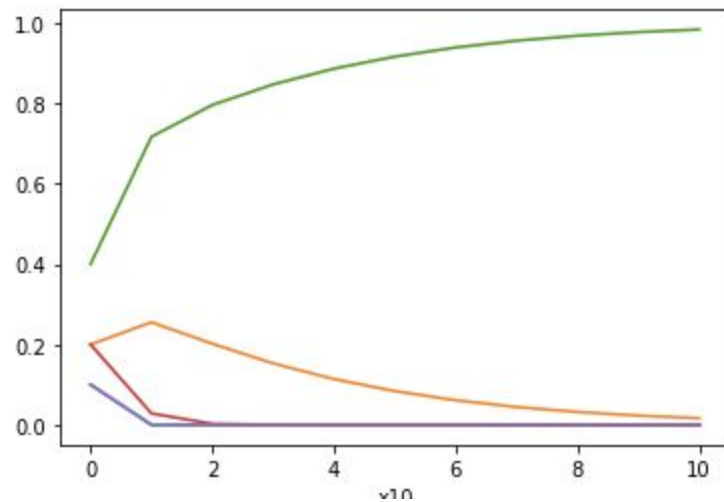
[h3]

```
in [20]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```



[h4]

```
in [21]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```



\*And just like magic, you can see how the algorithm learns which hypothesis it is as n increases

b.

Find  $N$  so that

$$\Rightarrow P(h_i | d_1, \dots, d_n) > .9$$
$$\Rightarrow P(h_i | d_1, \dots, d_n) = \alpha P(d | h_i) P(h_i)$$
$$= \alpha \prod_j P(d_j | h_i) P(h_i)$$

$$= \prod_j P(d_j | h_i) > \frac{.9}{\alpha P(h_i)}$$

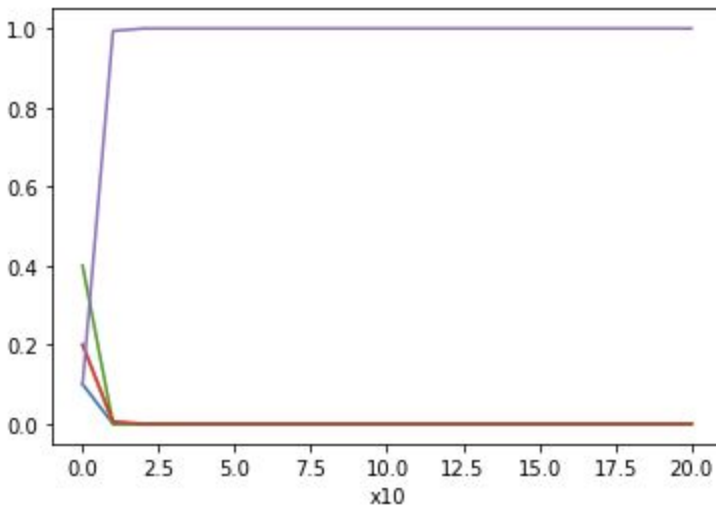
$\leftarrow$  min argument

C. While part a) was very useful and very cool, you can see that the distributions are not as uniform as possible. A big reason for this is the sample size of probabilities. If I increase the

datasets and remove my decimal formatting parameters it will make these graphs a lot more precise.[Below are my plots]

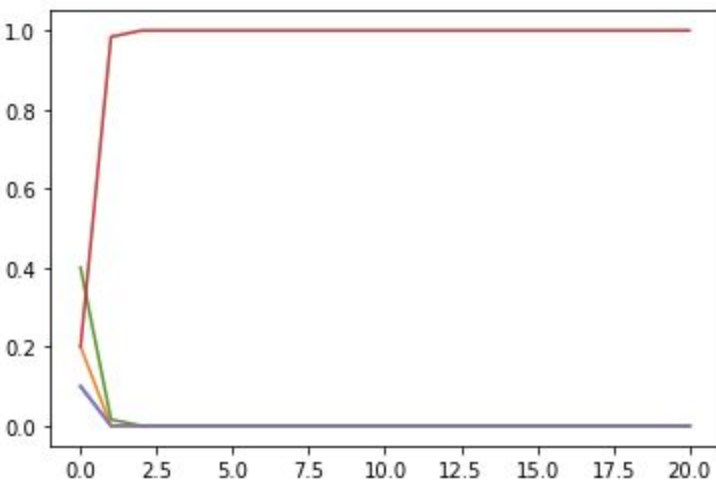
[h1]

```
In [24]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```



[h2]

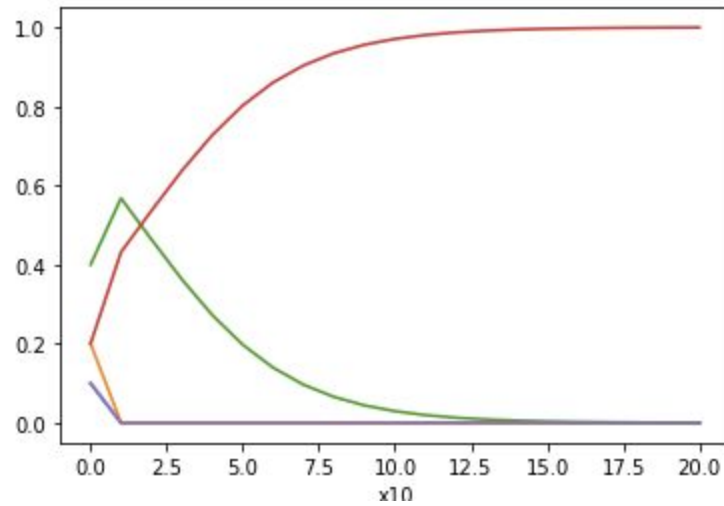
```
In [25]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```





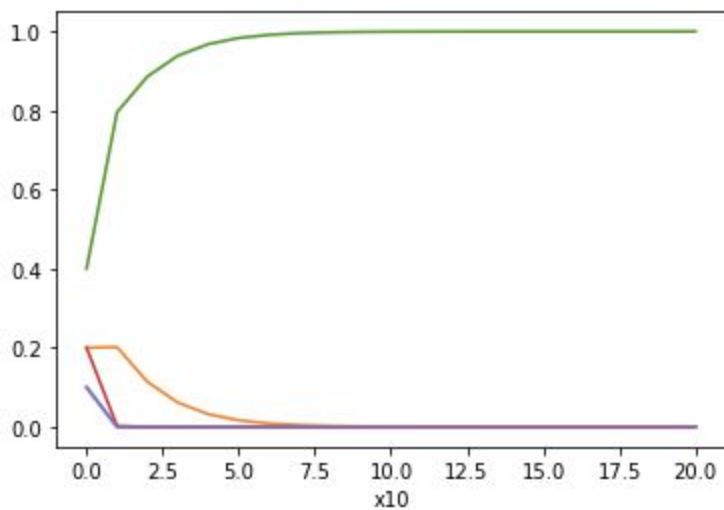
[h3]

```
In [26]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```



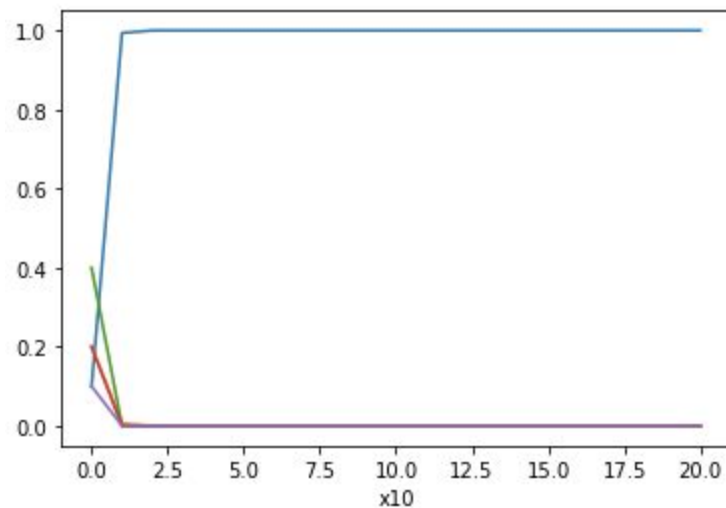
[h4]

```
In [23]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```



[h5]

```
In [27]: runfile('/Users/coleconstantino/.spyder-  
py3/temp.py', wdir='/Users/coleconstantino/.spyder-  
py3')
```



3)

$$D = \sum_{n=1}^N \sum_{k=1}^K r_{n,k} \|\mathbf{x}_n - \boldsymbol{\mu}_k\|^2$$

$\mu_1, \dots, \mu_K$  centers       $\mathcal{D} = \mathbf{x}_1, \dots, \mathbf{x}_n$

$\|\mathbf{x}_i - \mu_j\|^2$  Euclid Distance

assign  $\mathbf{x}_i$  to nearest  $\mu_j$        $\begin{cases} 1 = \text{min}_j \text{ argument } \|\mathbf{x}_i - \mu_j\|^2 \\ 0 = \text{else} \end{cases}$

Gradient       $\|\mathbf{x}_i - \mu_j\|^2 = (\mathbf{x}_i - \mu_j)^T (\mathbf{x}_i - \mu_j)$

$\nabla \sum 2i_j \nabla (\mathbf{x}_i - \mu_j)^T (\mathbf{x}_i - \mu_j)$

$\Rightarrow \nabla (\mathbf{x}_i^T \mathbf{x}_i - 2\mu_j^T \mathbf{x}_i + \mu_j^T \mu_j) = -2\mathbf{x}_i + 2\mu_j$

$= \langle -2\mathbf{x}_i, 2\mu_j \rangle$