Project Carbon Cycle

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Objective

- Simulate carbon moving through the forest ecosystem

Glossary

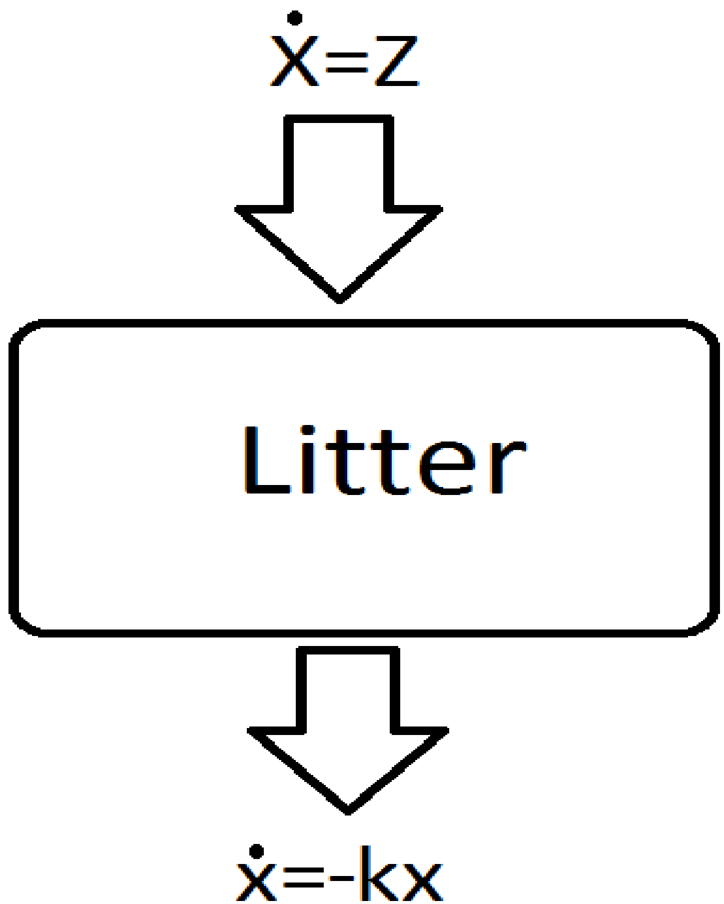
- Litter
- Humus
- Stable humus charcoal

Glossary

- Humification
- Carbonization

Simple Litter Model

- Calculate the amount of carbon trapped in litter at any given time



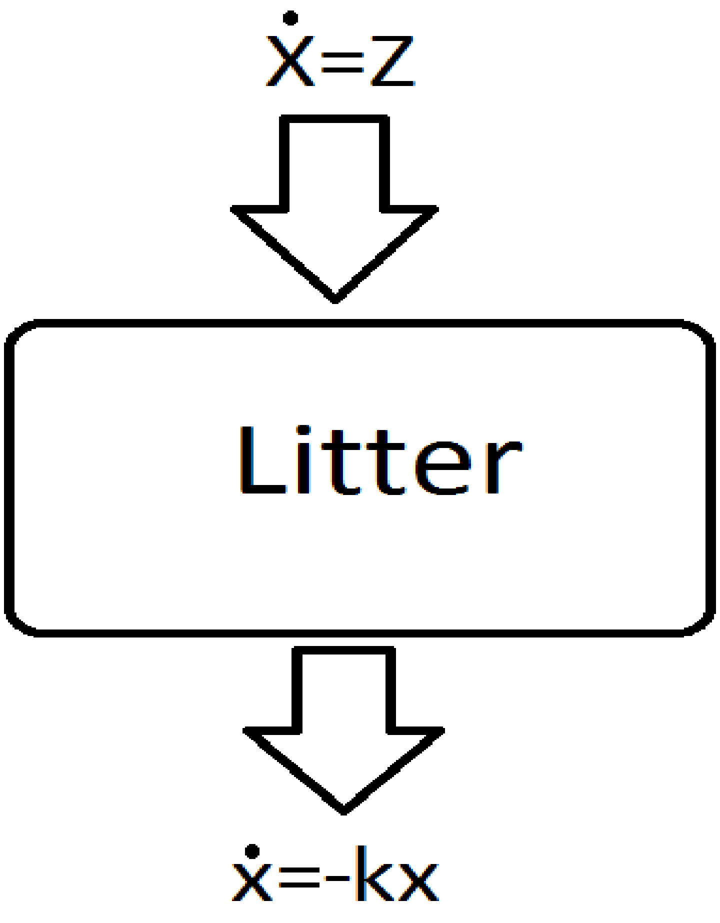
Simple Litter Model

- Begin with zero litter (after a ground fire)
- Leaves & branches fall from trees at a constant rate, creating litter
- Carbon exits litter by respiration and humification as litter is composted

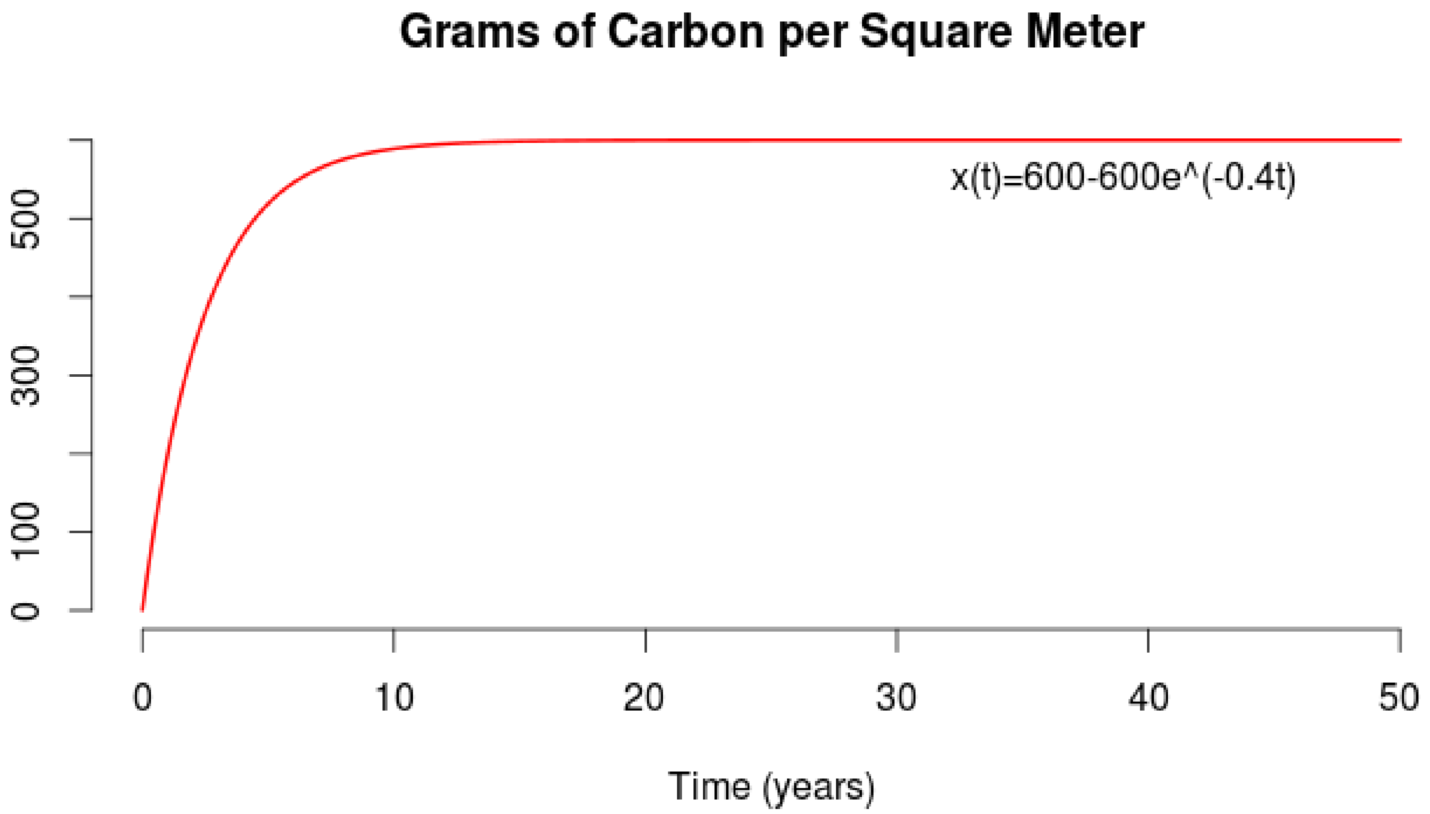
Simple Litter Model

- Model: one differential equation

$x'(t) = z - k*x(t)$



$x(t) = 600 - 600*\exp(-0.4t)$

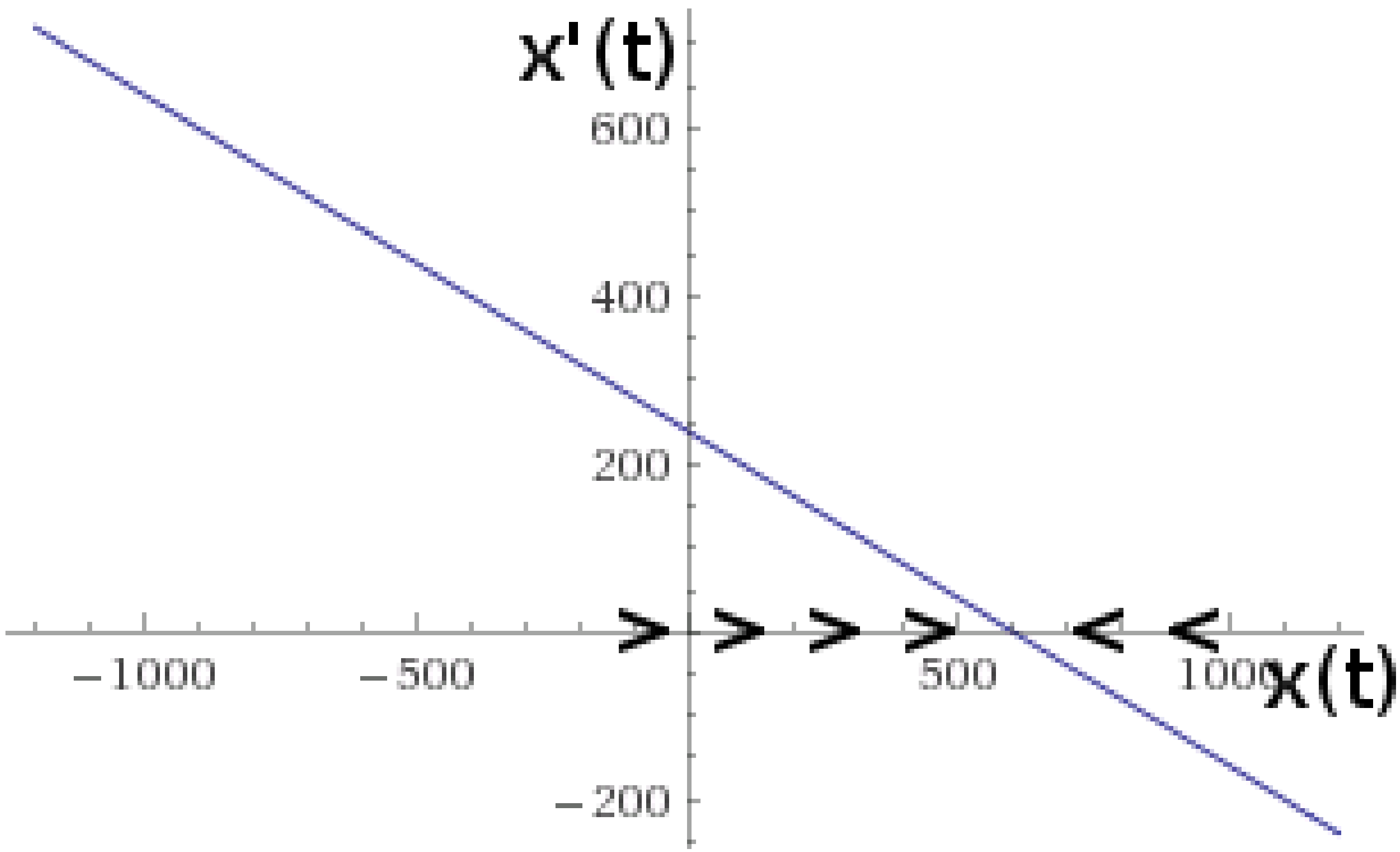


Steady States

- Assume z = 240, k = 0.4
- Equilibrium occurs when x'(t) = 0, so

  x'(t) = 240 -0.4*x(t) <=> x'(t) = -0.4(x(t) - 600)

Graphical Stability Analysis



Simple Litter Model

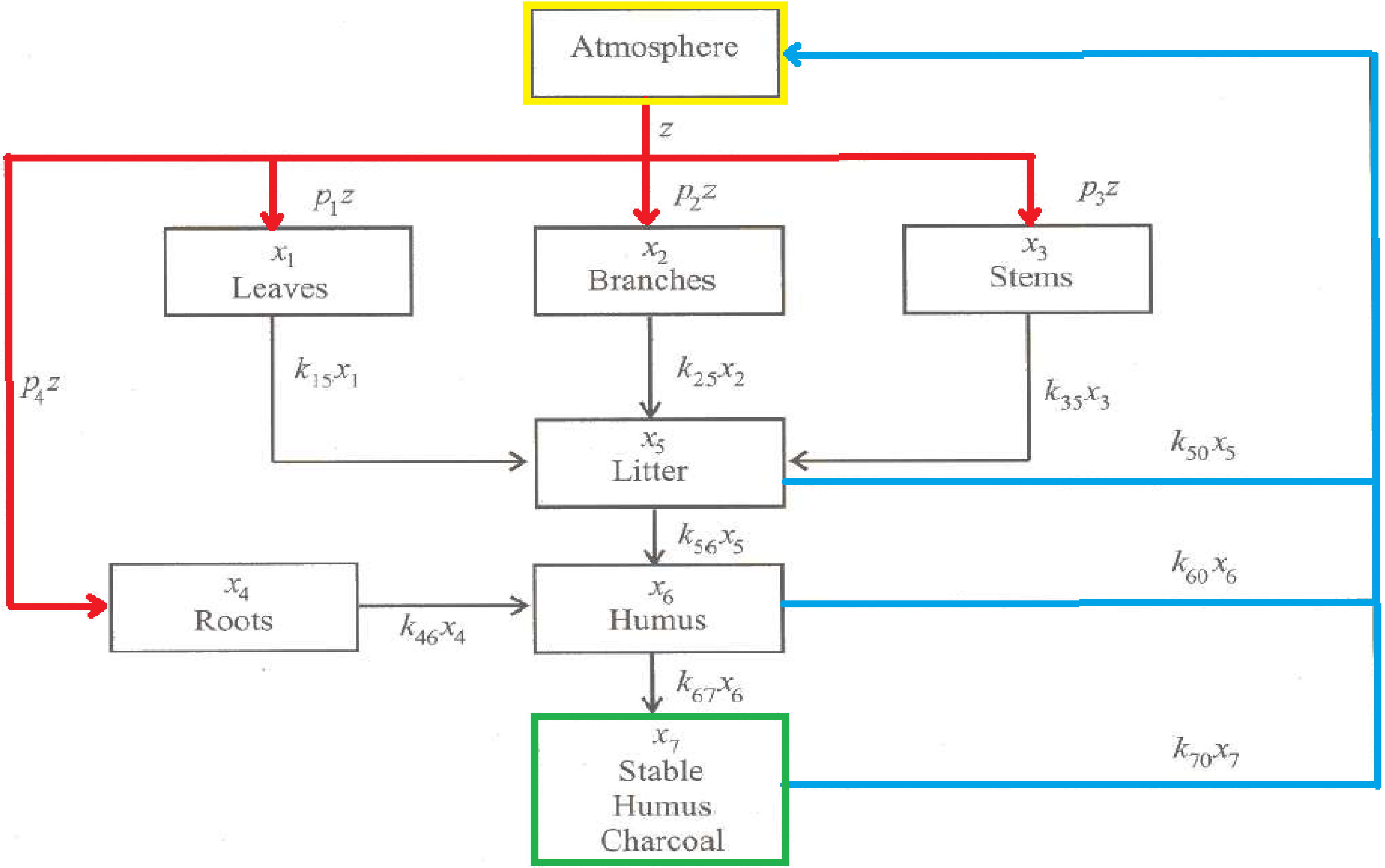
- Model: one differential equation

  $x'(t) = 240 – 0.4*x(t)$, and $x(0) = 0$

- Specific Solution

  $x(t) = 600 - 600*\exp(-0.4t)$

The Carbon Cycle



Given Data

		Tropical forest	Temperate forest	Grass-land	Agri-cultural area	Human area	Tundra and semi-desert
Z	Carbon entering System z (Gt C/yr)	27.8	8.7	10.7	7.5	0.2	2.1
	Partition coefficients						
$\mathbf{p_1}$	p_1 (Leaves)	0.3	0.3	0.6	0.8	0.3	0.5
$\mathbf{p_2}$	p_2 (Branches)	0.2	0.2	0.0	0.0	0.2	0.1
$\mathbf{p_3}$	p_3 (Stems)	0.3	0.3	0.0	0.0	0.3	0.1
$\mathbf{p_4}$	p_4 (Roots)	0.2	0.2	0.4	0.2	0.2	0.3
	Flows						
$\mathbf{k_{15}}$	Leaves to litter	1.0	0.5	1.0	1.0	1.0	1.0
$\mathbf{k_{25}}$	Branches to litter	0.1	0.1	0.1	0.1	0.1	0.1
$\mathbf{k_{35}}$	Stems to litter	0.033	0.0166	0.02	0.02	0.02	0.02
$\mathbf{k_{46}}$	Roots to humus	0.1	0.1	1.0	1.0	0.1	0.5
$\mathbf{k_{56}+k_{50}}$	Leaving litter	1.0	0.5	0.5	1.0	0.5	0.5
$\mathbf{k_{67}+k_{60}}$	Leaving humus	0.1	0.02	0.025	0.04	0.02	0.02
$\mathbf{k_{70}}$	Charcoal to atmosphere	0.002	0.002	0.002	0.002	0.002	0.002
h	Humification h	0.4	0.6	0.6	0.2	0.5	0.6
c	Carbonization c	0.05	0.05	0.05	0.05	0.05	0.05
A	Areas ($10^{12} m^2$)	36.1	17.0	18.8	17.4	2.0	29.7

D.E. Set Up

- $X'_1(t) \;=\; p_1 z \;-\; k_{15} X_1$

Leaves Atm. To Litter

- $X'_2(t) \;=\; p_2 z \;-\; k_{25} X_2$

Branches Atm. To Litter

- $X'_3(t) \;=\; p_3 z \;-\; k_{35} X_3$

Stems Atm. To Litter

- $X'_4(t) \;=\; p_4 z \;-\; k_{46} X_4$

Roots Atm. To Litter

D.E. Set Up

- $X'_1(t) = p_1 z - k_{15} X_1$

Leaves Atm. To Litter

- $X'_2(t) = p_2 z - k_{25} X_2$

Branches Atm. To Litter

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Stems Atm. To Litter

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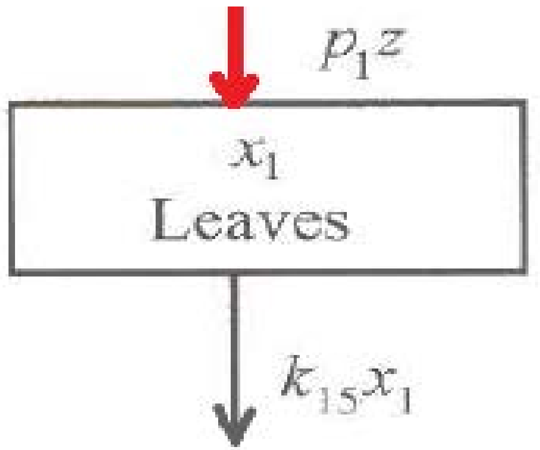
Roots Atm. To Litter



D.E. Set Up

- $X'_1(t) = p_1 z - k_{15} X_1$

Leaves Atm. To Litter

- **$X'_2(t) = p_2 z - k_{25} X_2$**

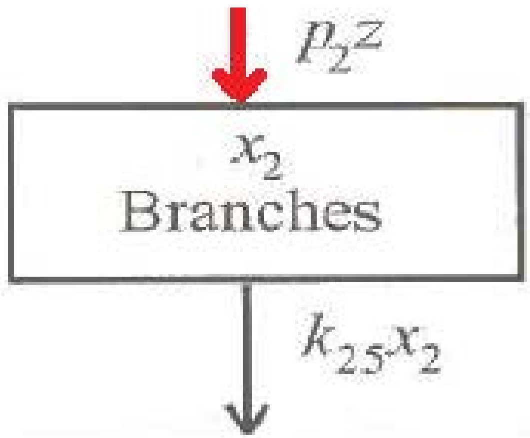
Branches Atm. To Litter

- $X'_3(t) = p_3 z - k_{35} X_3$

Stems Atm. To Litter

- $X'_4(t) = p_4 z - k_{46} X_4$

Roots Atm. To Litter



D.E. Set Up

- $X'_1(t) = p_1 z - k_{15} X_1$

Leaves Atm. To Litter

- $X'_2(t) = p_2 z - k_{25} X_2$

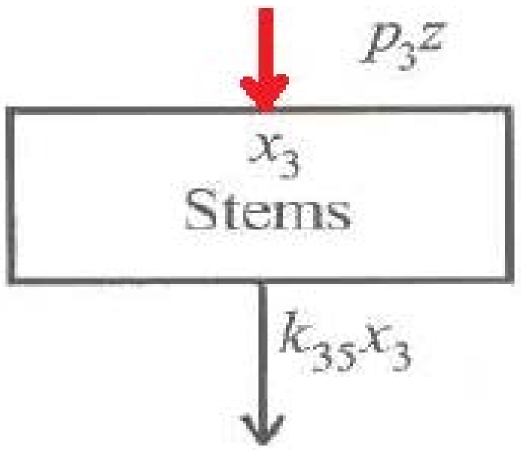
Branches Atm. To Litter

- $\mathbf{X'_3(t) = p_3 z - k_{35} X_3}$

Stems Atm. To Litter

- $X'_4(t) = p_4 z - k_{46} X_4$

Roots Atm. To Litter



D.E. Set Up

$$X'_5(t) = k_{15}X_1 + k_{25}X_2 + k_{35}X_3 - k_{56}X_5 - k_{50}X_5$$

Litter Leaves Branches Stems Humus Atm.

$$X'_6(t) = k_{46}X_4 + k_{56}X_5 - k_{67}X_6 - k_{60}X_6$$

Humus Roots Litter Charcoal Atm.

$$X'_7(t) = k_{67}X_6 - k_{70}X_7$$

Charcoal Humus Atm.

D.E. Set Up

$$X'_5(t) = k_{15}X_1 + k_{25}X_2 + k_{35}X_3 - k_{56}X_5 - k_{50}X_5$$

Litter Leaves Branches Stems Humus Atm.

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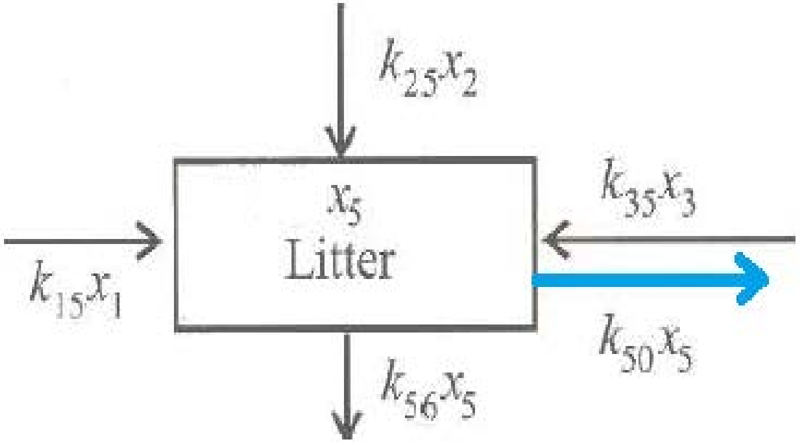
Charcoal Humus Atm.



Matrix Representation of DE

$$A = \begin{bmatrix} -k_{15} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k_{25} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_{35} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -k_{46} & 0 & 0 & 0 \\ k_{15} & k_{25} & k_{35} & 0 & -k_{50} - k_{56} & 0 & 0 \\ 0 & 0 & 0 & k_{46} & k_{56} & -k_{60} - k_{67} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{67} & -k_{70} \end{bmatrix}$$

$$b = \begin{bmatrix} p_1 z \\ p_2 z \\ p_3 z \\ p_4 z \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boxed{X' = AX + b}$$

$$X' = AX + b$$

- Diagonal of A

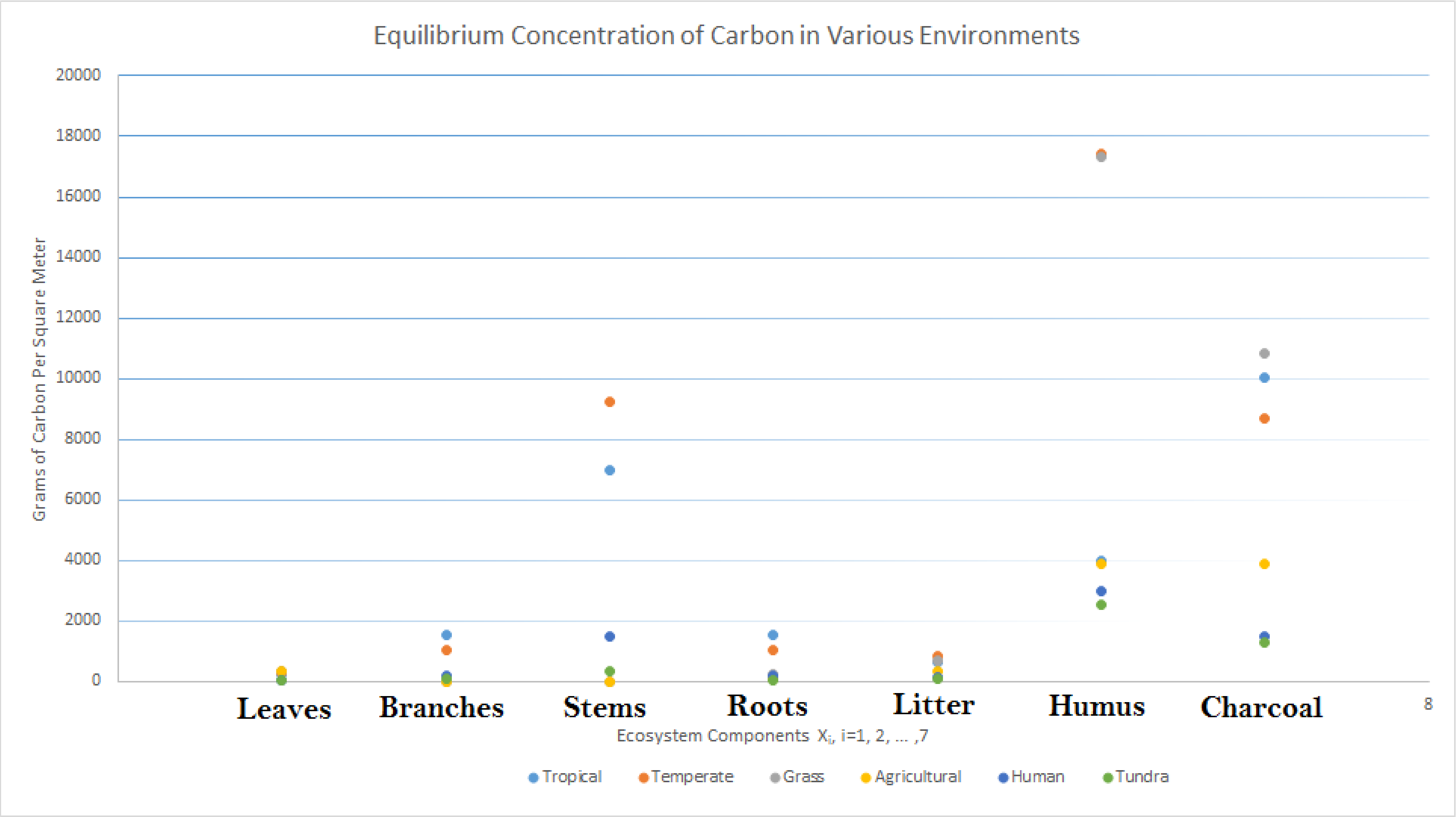
  Multiset of Eiganvalues

- Fixed Points: $X' = 0$

  $$0 = AX + b$$

  $$AX = -b$$

Fixed Point of The Ecosystems



Solution

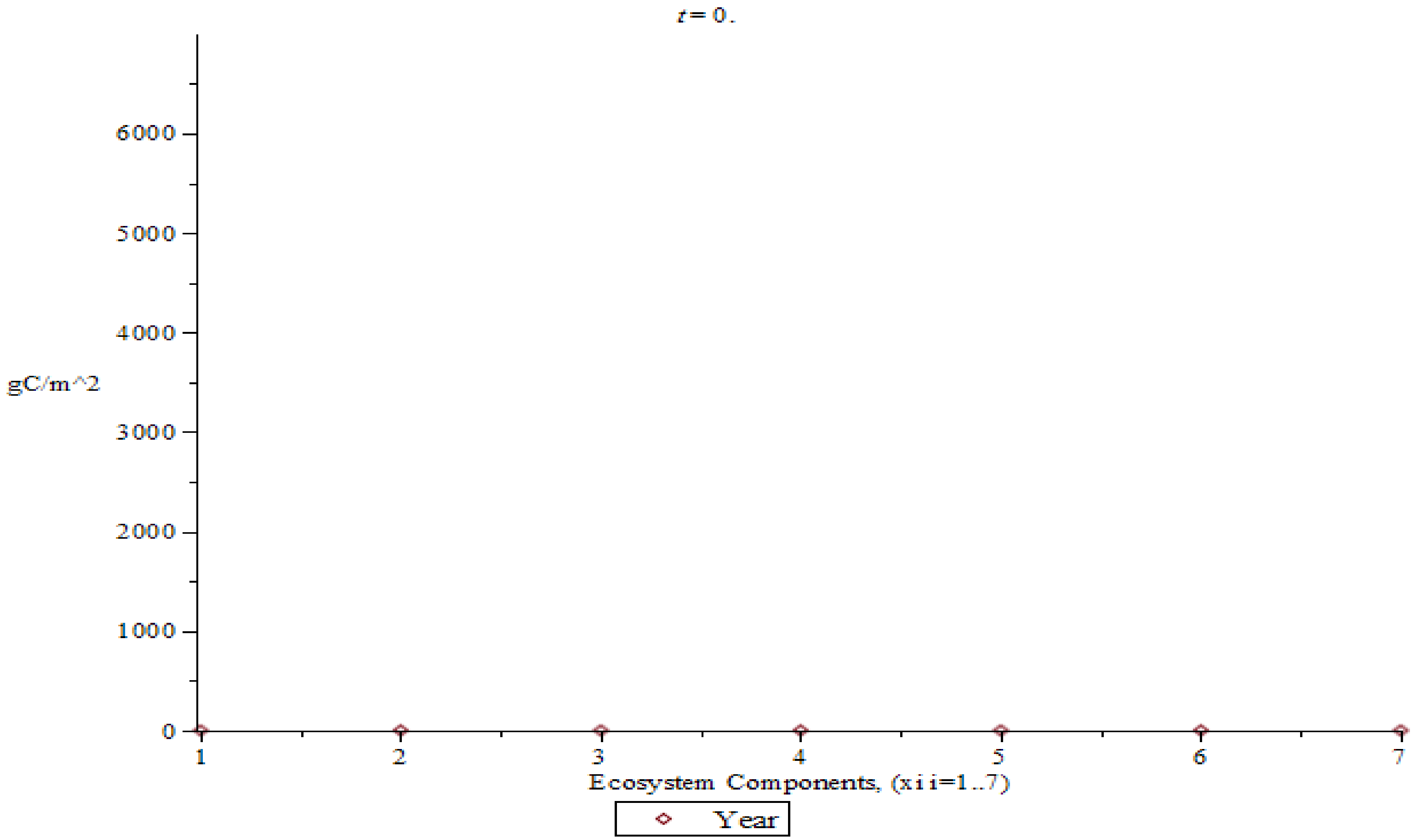
- Analytical
  - Maple
  - Matrix Exponential
    - $X' = AX+b$
    - $X = c_1 e^{At} - b/A$
- Numerical
  - Matlab
  - Euler Method

$$X(n+1) = X(n) + \delta t\, X'(n)$$

Ecosystems

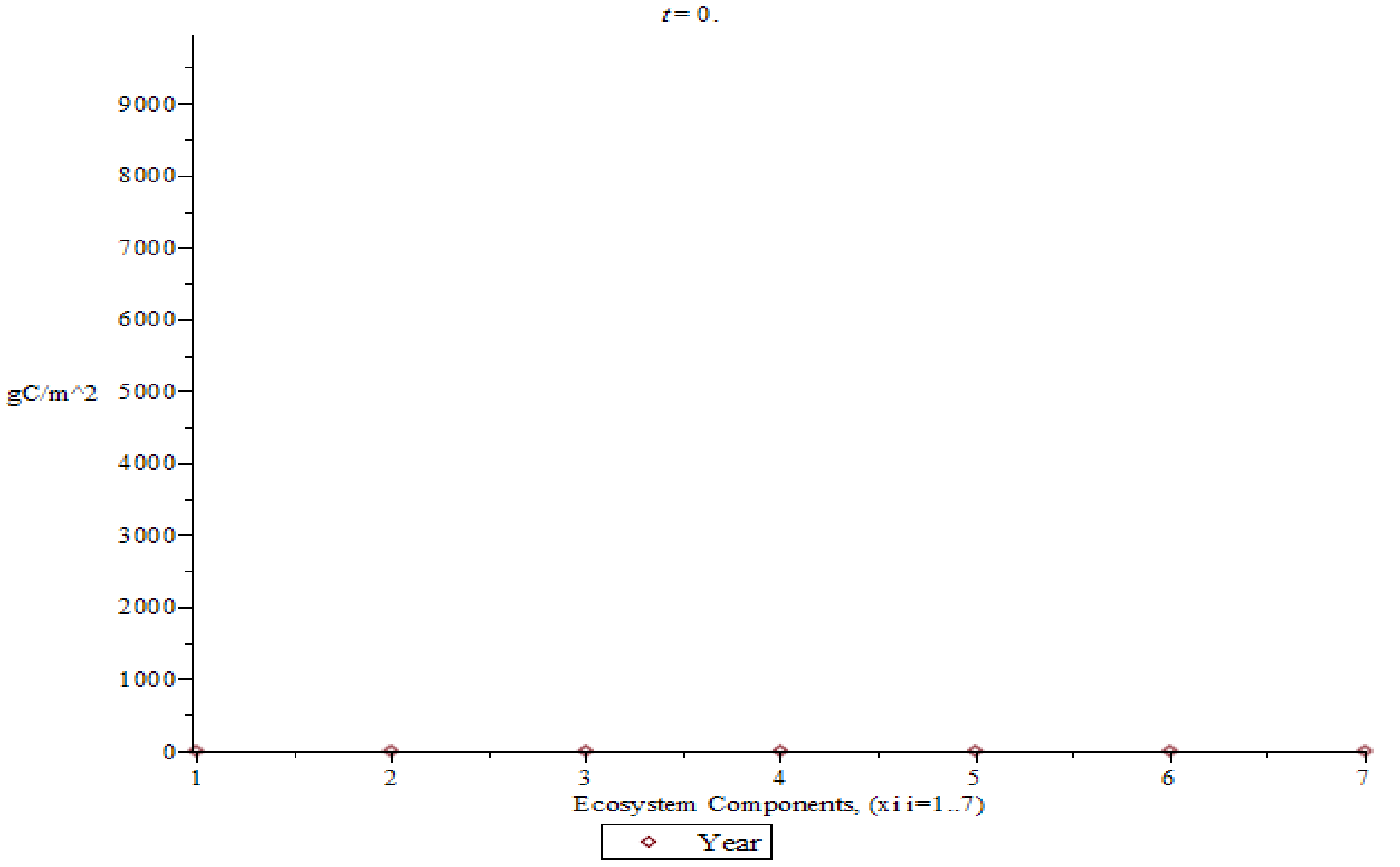
1. Tropical Forests
2. Grasslands
3. Agricultural Lands

Tropical Forest: First 225 Years



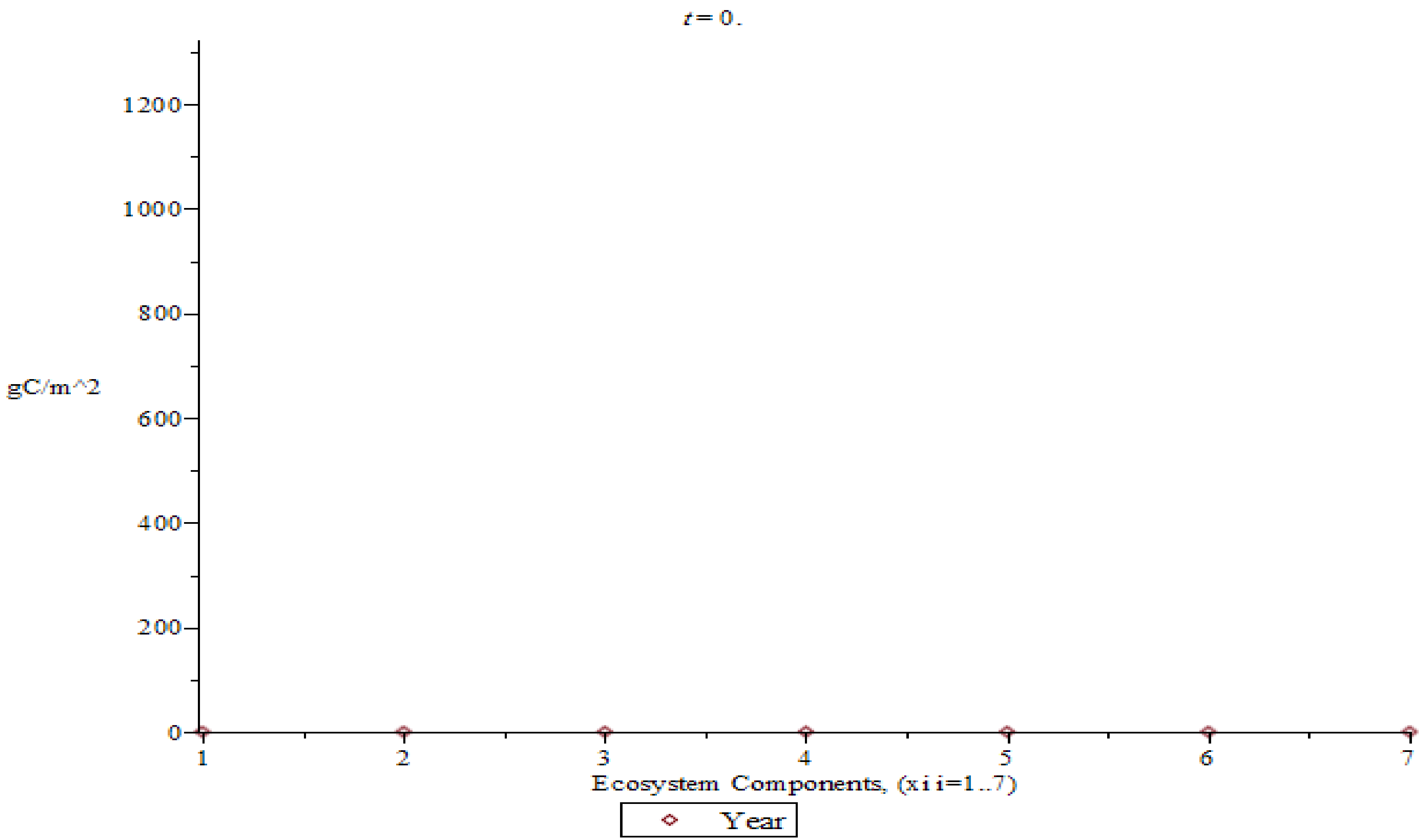
Leaves 1
Branches 2
Stems 3
Roots 4
Litter 5
Humus 6
Charcoal 7

Tropical Forest: First 2500 Years



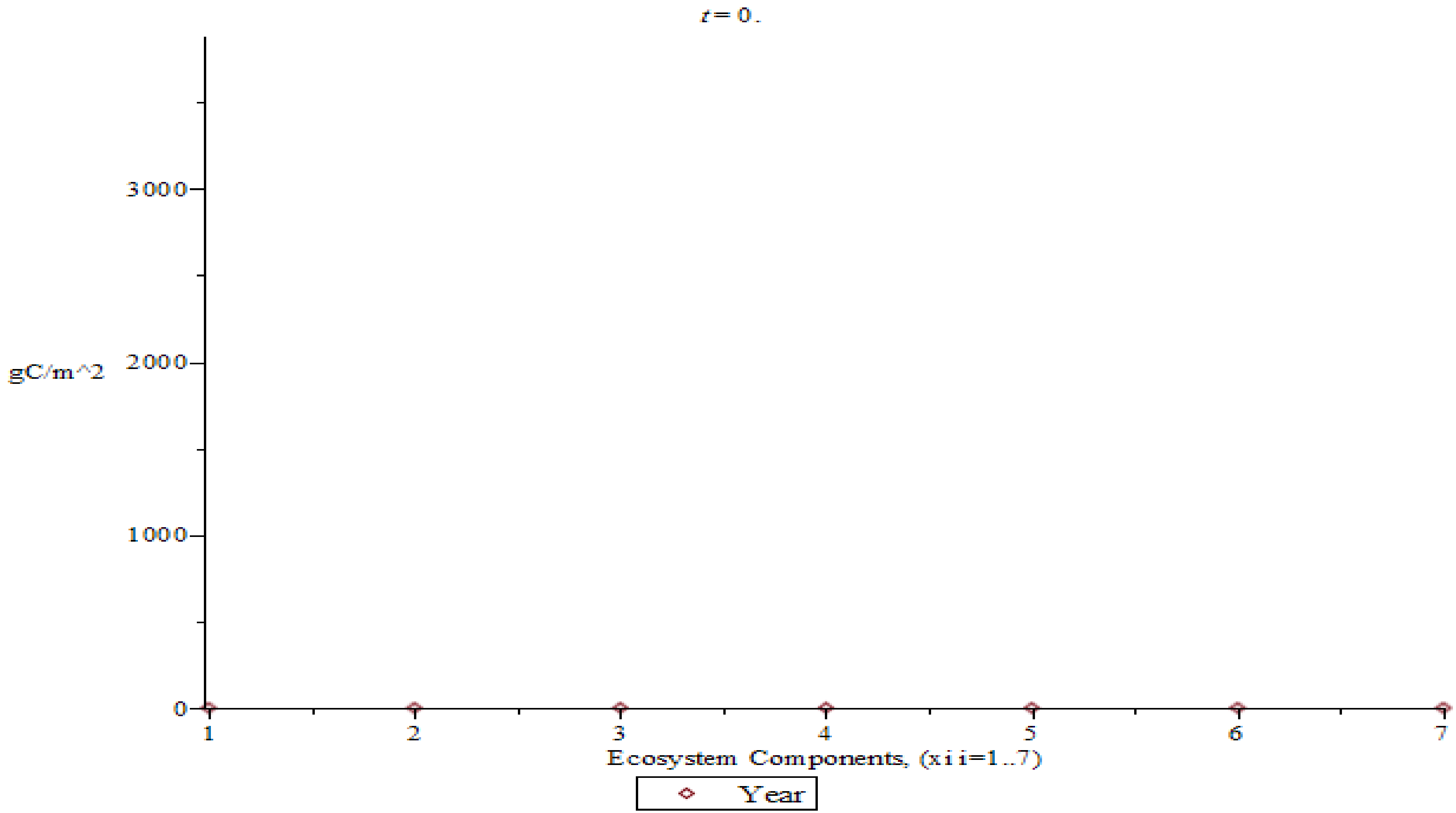
Leaves	Branches	Stems	Roots	Litter	Humus	Charcoal
1	2	3	4	5	6	7

Grassland: First 5 Years



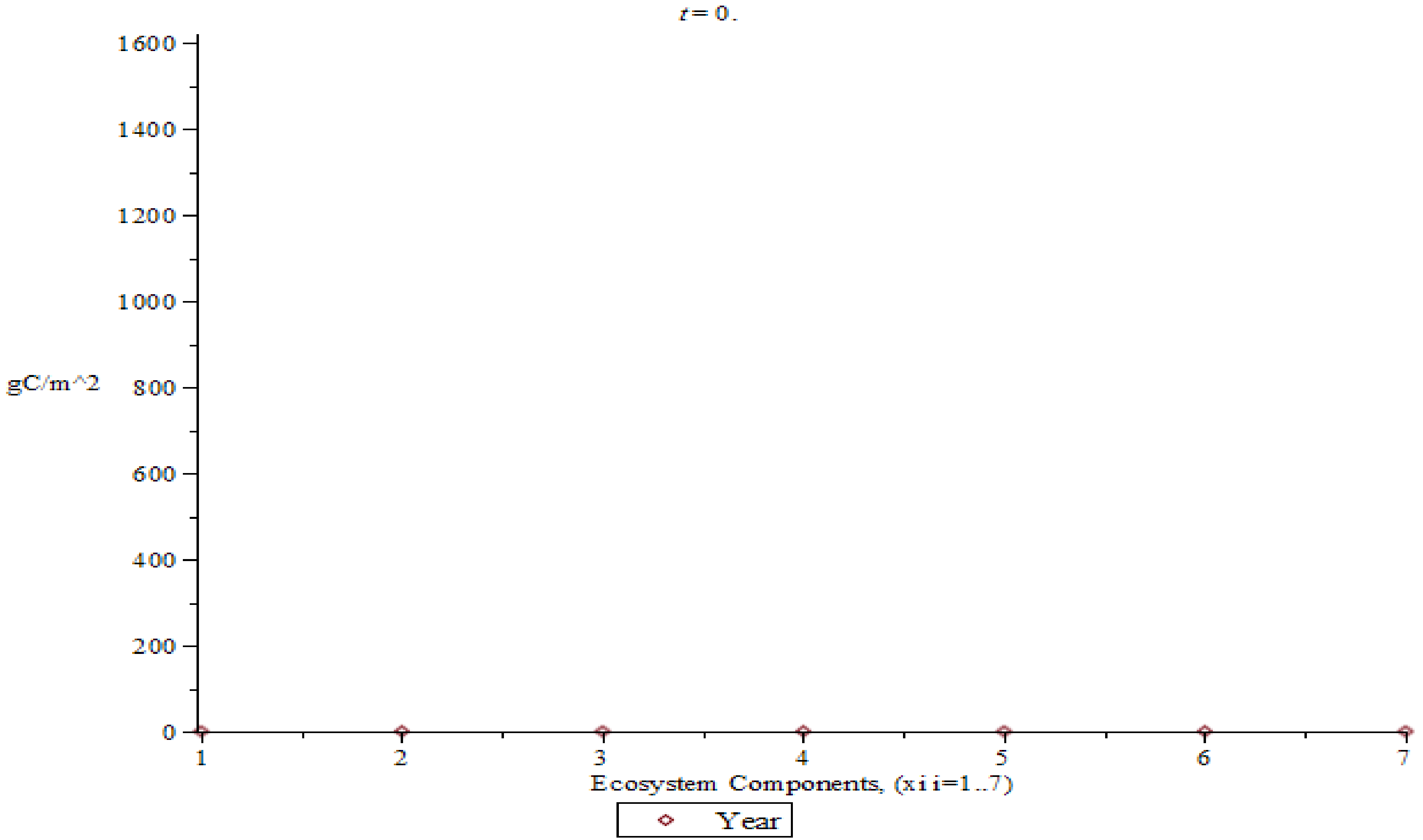
Leaves 1 Branches 2 Stems 3 Roots 4 Litter 5 Humus 6 Charcoal 7

Grasslands: First 3000 Years



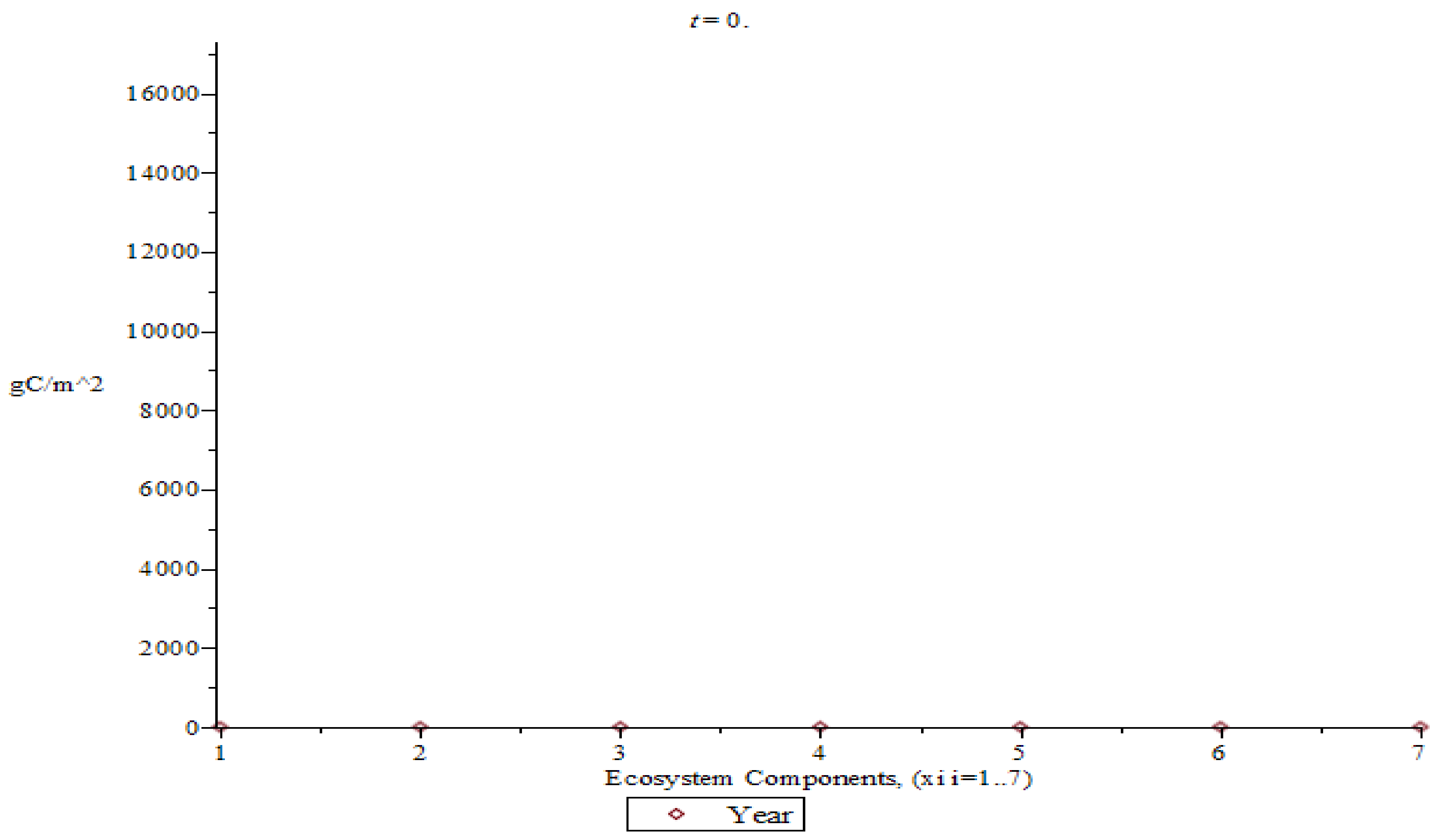
Leaves	Branches	Stems	Roots	Litter	Humus	Charcoal
1	2	3	4	5	6	7

Agricultural Land: First 15 Years



Leaves 1 Branches 2 Stems 3 Roots 4 Litter 5 Humus 6 Charcoal 7

Agricultural Land: First 3000 Years



Leaves	Branches	Stems	Roots	Litter	Humus	Charcoal
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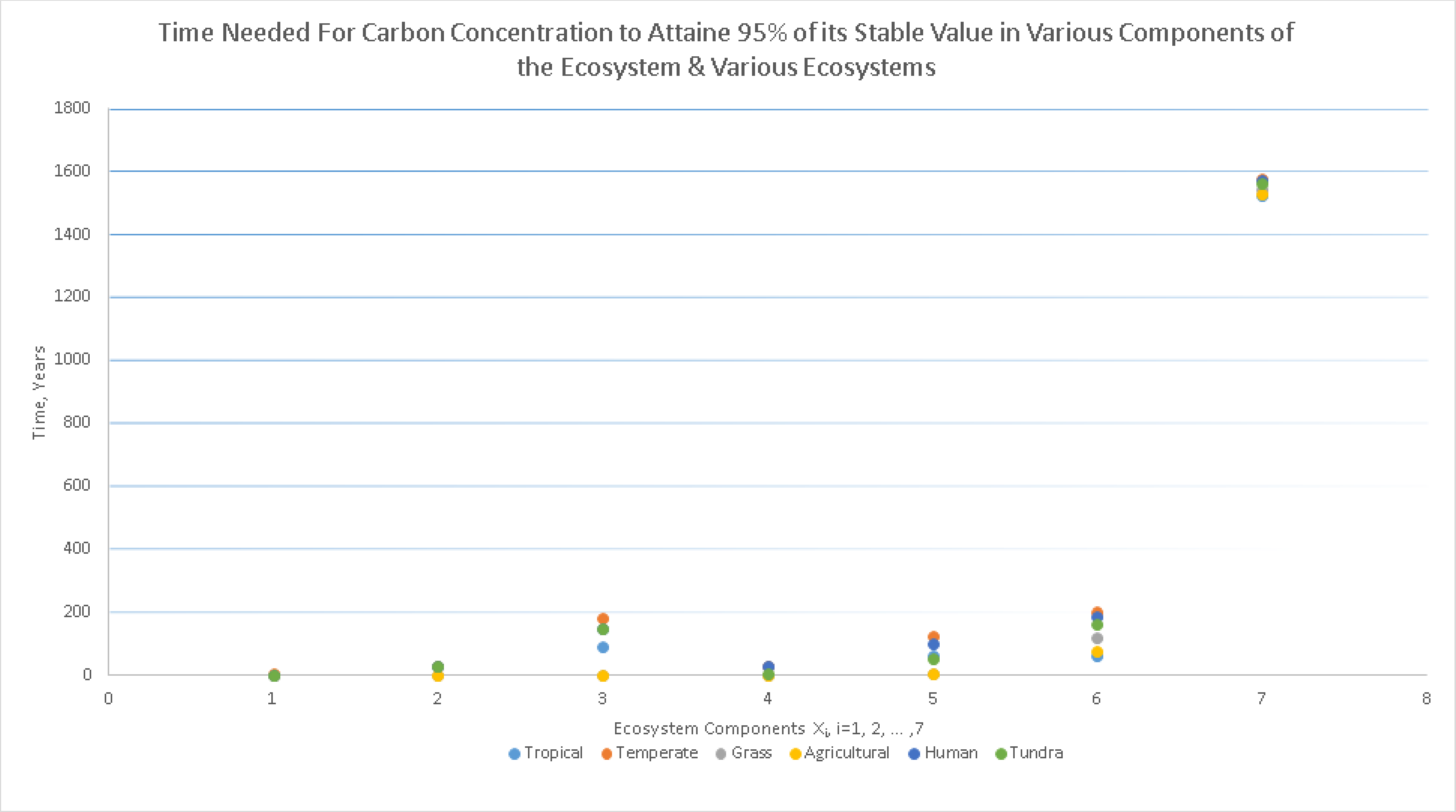
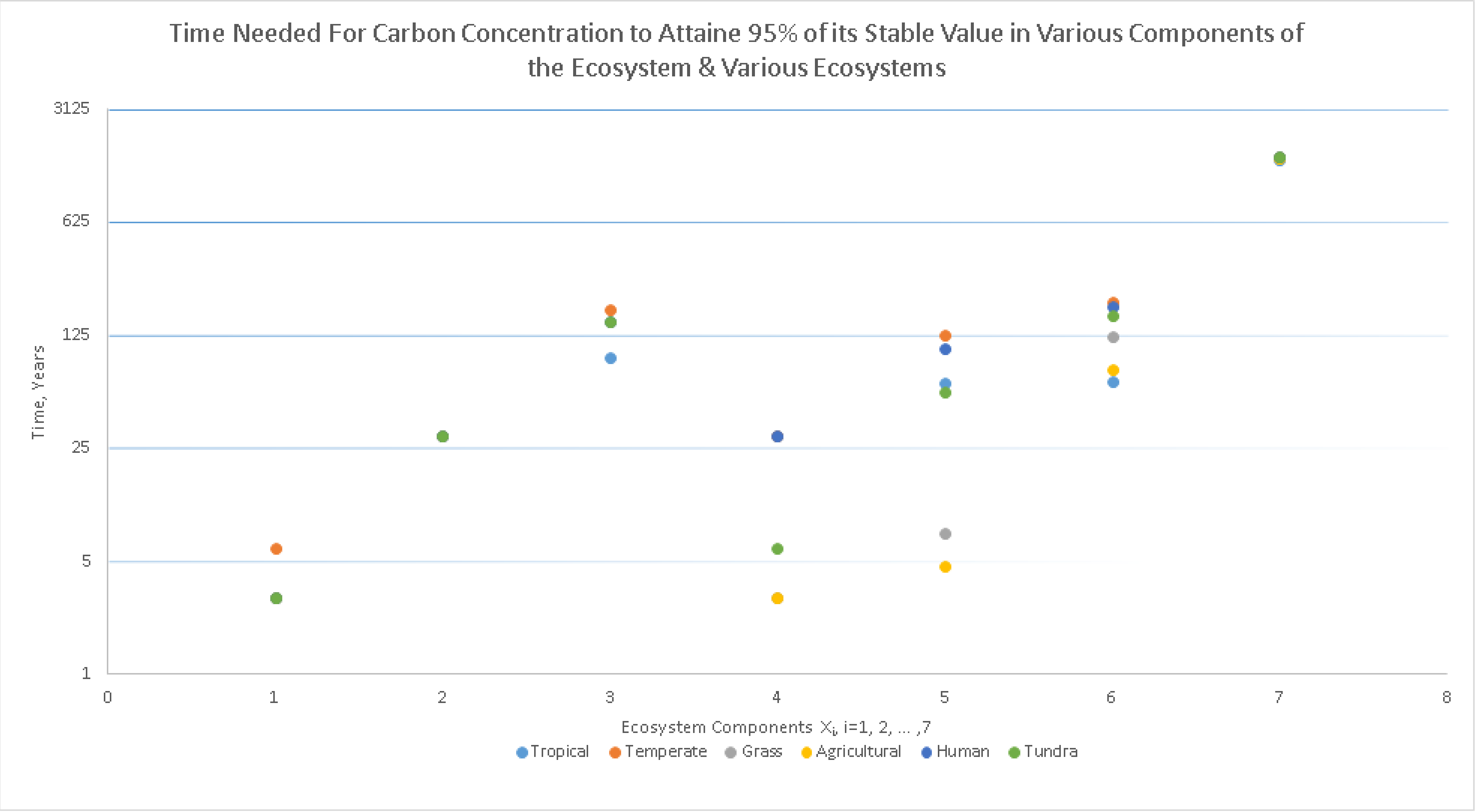
Time To Attain 95% of Fixed Point Concentration



Time To Attain 95% of Fixed Point Concentration



Analytical Stability Analysis

- After Converting the system into its Matrix form, the stability was found from the matrix's Eigenvalues

- Since all of the Eigenvalues were negative, it can be show that ANY fixed point is stable

- Since these Eigenvalues are linear, it can be shown that they are also not sensitive to change
- This means that the only way for instability in the system to occur, is for the flows in one subsection to reverse

Criticisms of the Model

- Like any mathematical model, there are errors made when creating a model for a system
- The main contributor for these errors in general is a lack of model variables

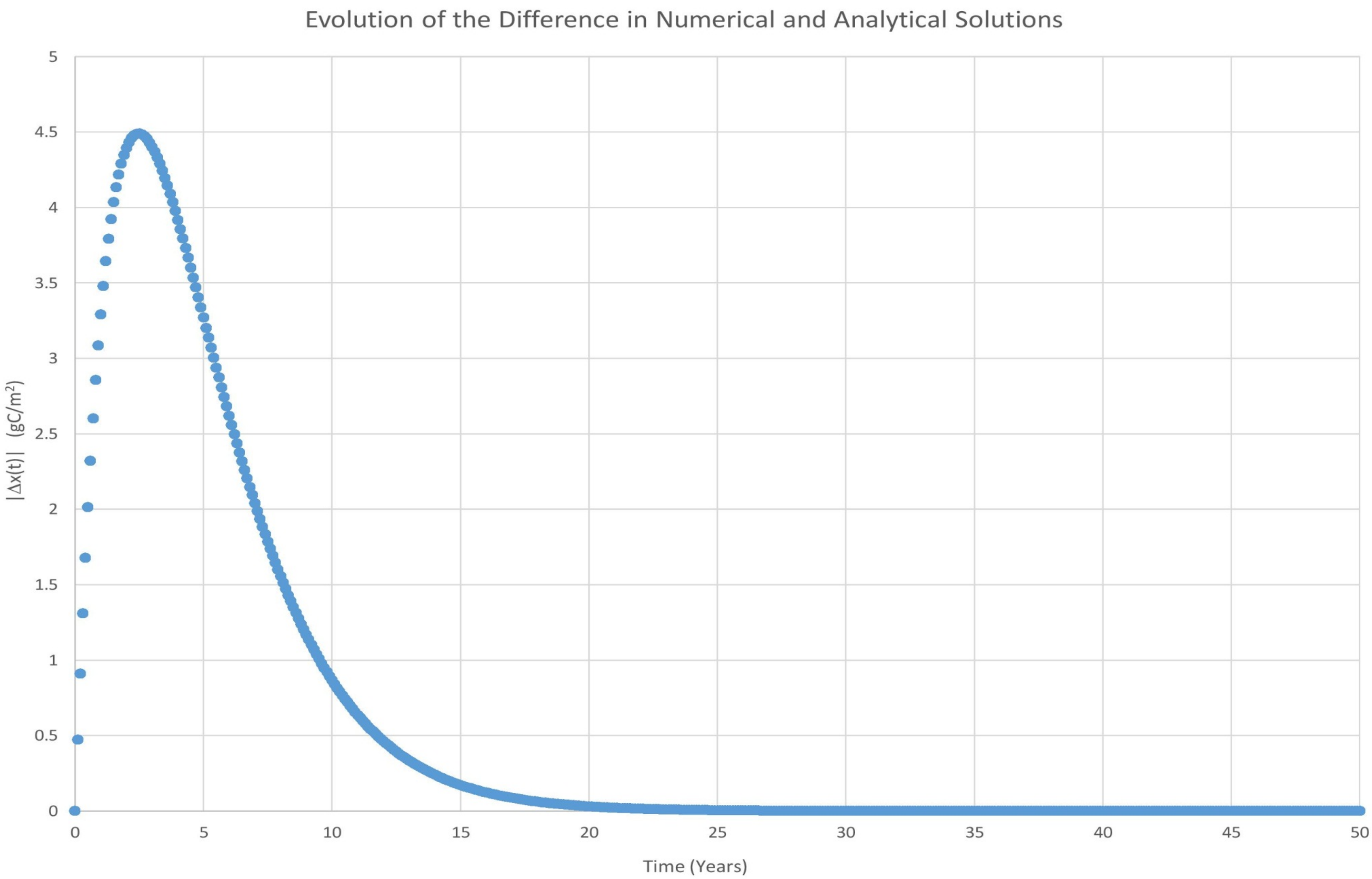
- For this model, the equations had not taken into account that the number of trees change with respect to how much atmospheric carbon dioxide there is

- Furthermore, this system is considered in isolation
- There are different types of trees in a given ecosystem and thus the values for the parameters for each part of the tree are actually functions of how many of each tree there are

Thanks for Your Attention

Questions

Backups: Numerical Error Propagation



Backups: Equilibrium Density of Carbon

	Tropical	Temperat	Grass	Agricultur	Human	Tundra
X_1	231	307	341	345	30	35
X_2	1540	1024	0	0	200	71
X_3	7001	9249	0	0	1500	354
X_4	1540	1024	228	86	200	42
X_5	616	819	683	345	160	99
X_6	4004	17400	17302	3879	3000	2545
X_7	10011	8700	10814	3879	1500	1273

Backups: 95% Time

	Tropical	Temperate	Grass	Agricultural	Human	Tundra
X_1	3	6	3	3	3	3
X_2	30	30	0	0	30	30
X_3	90.8	180.5	0	0	149.8	149.8
X_4	30	30	3	3	30	6
X_5	62.4	123.4	7.4	4.7	102.8	55.2
X_6	64.1	199.7	121.8	76.4	187.5	165
X_7	1521.6	1574.9	1541.5	1525	1570.8	1558.4