Lecture 22

plan: 1) Finish matroid union
2) Ellipsoid

Ellipsoid Algorithm

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Consequences

Given convex set $P \subseteq \mathbb{R}^n$,

- **Separation (SEP):**

  Given $y \in \mathbb{R}^n$, decide

- Optimization (OPT)

  Given vector $c \in \mathbb{R}^n$, find

Examples

- Linear programming:

- Matroid polytope:

$M = (E, \mathcal{I})$ matroid,

$P =$

Thm

$$\boxed{P = \{ x \in \mathbb{R}^E : \qquad \qquad \}.}$$

However,

- OPT for P

- Matroid intersection polytope:

• **Amazing Result:**

Theorem (Grötschel, Lovasz, Schrijver '81)

$\Longleftrightarrow$

Proof idea:

$\Rightarrow$ Ellipsoid algorithm:

- Actually,

MEM!

Thm (GLS '88):

Actually,

Proof: not covered.

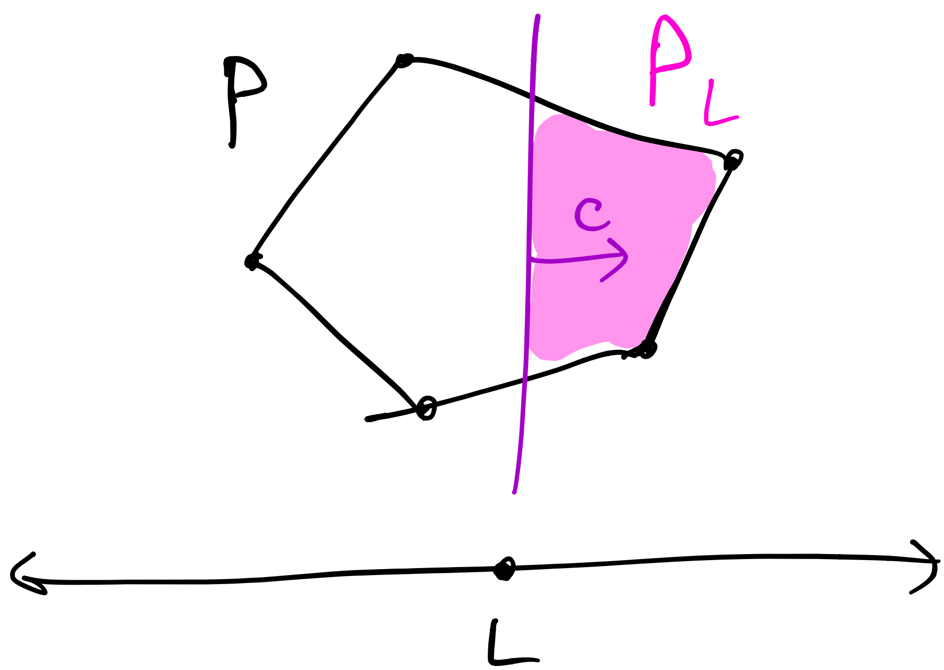
OPT vs. feasibility

- First we solve simpler problem:

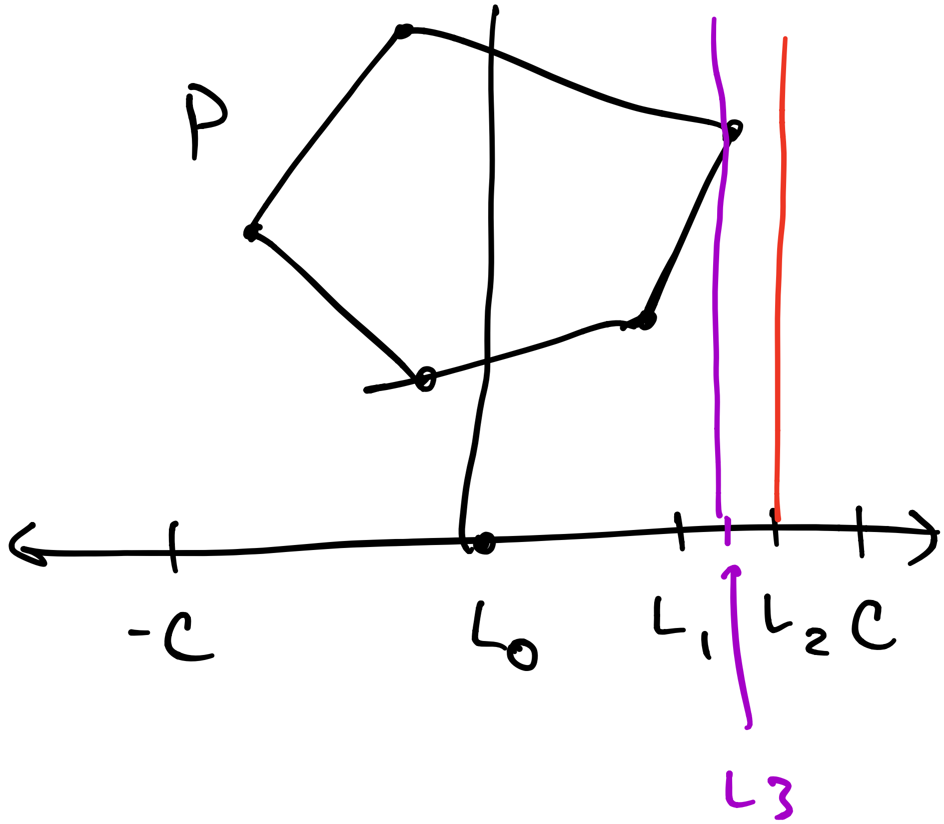
$\boxed{\textbf{FEAS:} \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad}$

- OPT reduces to FEAS:

binary search:



- Given a-priori bound



- optimizes

- for LP,

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(finally!)

The Algorithm

- Solves FEAS in time

- <u>ε, R dependence not a big deal:</u>
(& actually necessary).

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Algorithm idea:

- Set $E =$

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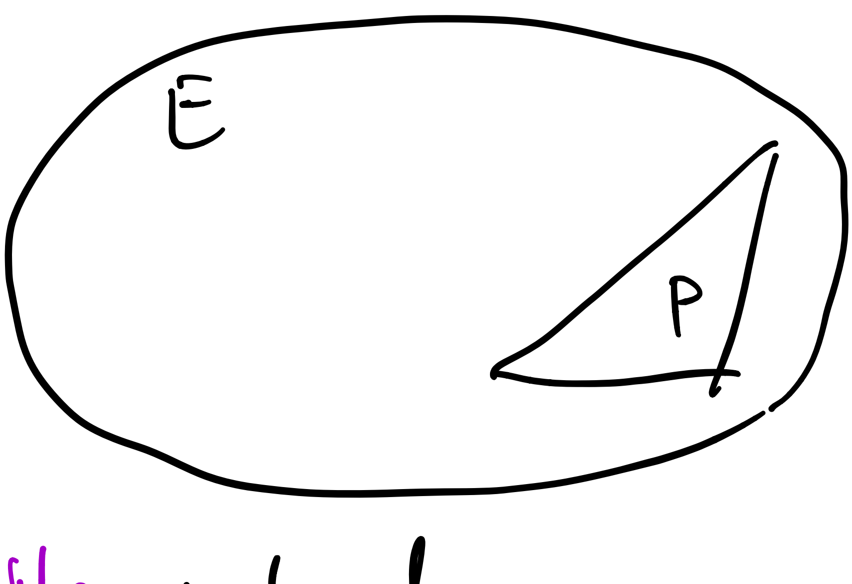

- While not done:

▷ Check

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▷ if $\varnothing$, return

▷ else ,

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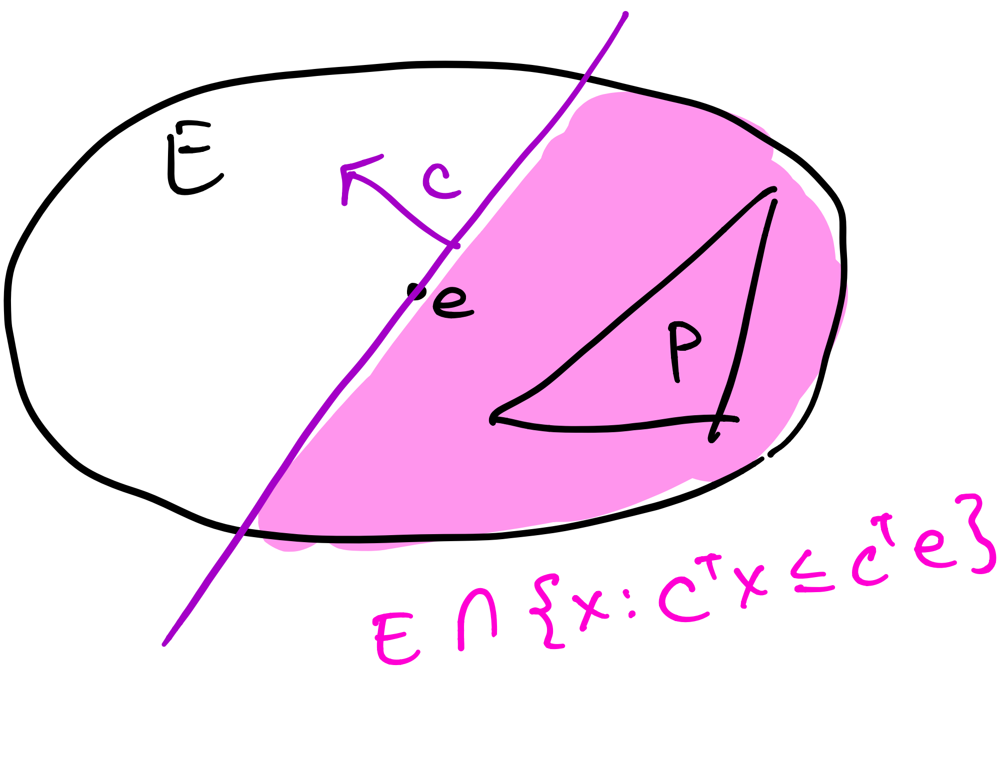
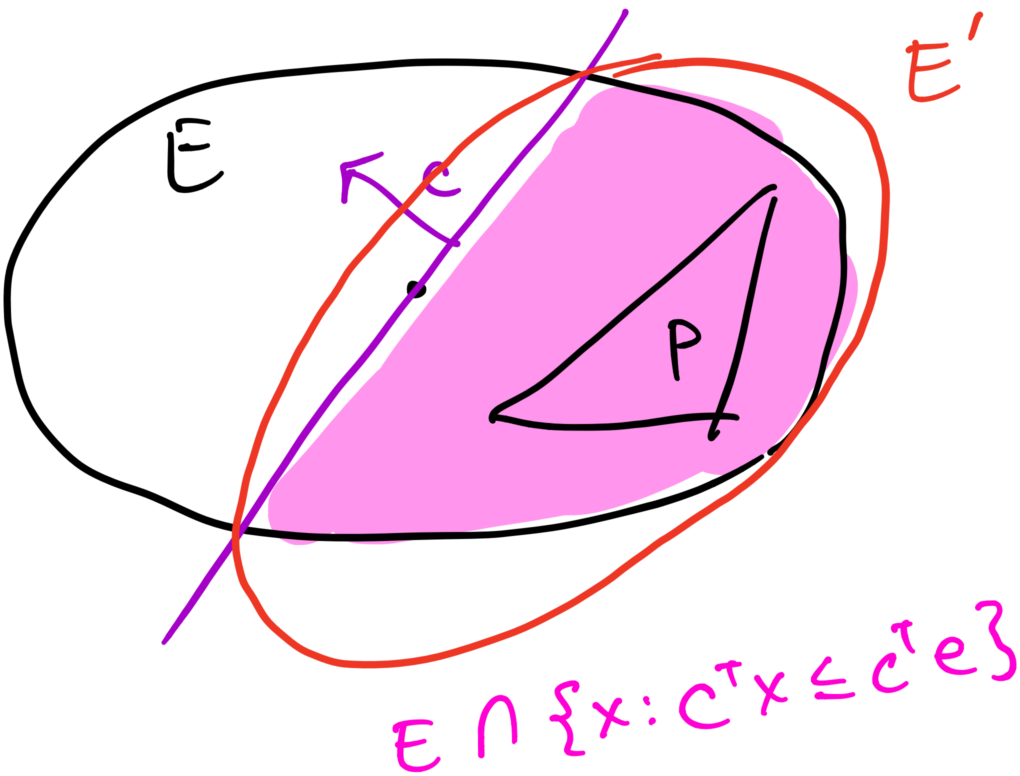


▷ Lot E'



(.)

▷ Set

Runtime:

- **Volume Lemma:**

$\leq$

- As

Issues:

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