SOLVING IVP'S USING GEN + PARTICULAR

1) Note that $Ce^{-t^2/2}$

is general soln of

$\underline{A}$ $\quad y' + ty = 0.$ (check).

Suppose $y_0(t)$ solves

$\underline{B}$ $\quad y' + ty = \cos(t).$

Find $\underline{\text{all}}$ solutions of E.

$\underline{\text{Answer}}$: Adding any general solution to a particular solution is another particular soln.

So the family

$y(t) = Ce^{-t^2/2}$.

$y_0(t) + Ce^{\cdots}$.

for $C \in \mathbb{R}$. solves E.

why is it everything? Imagine another particular solution $y_1(t)$:

it $y_1(t) - y_0(t)$ solves $\underline{A}$!

so $y_1(t) - y_0(t) = Ce^{-t^2/2}$ for some C,

thus y_1 is in the family. ✓

2) Sep of vars

Sep of vars to find genl soln of

$\dot{y} = 1 + y^2$.

a) write as $\dot{y} = h(y) g(t)$.

$\frac{dy}{dt} = (1+y^2)\, 1$

$$dt = (1+y^2)$$

$\downarrow$ fxn of y $\quad \uparrow$ fxn of t

b) do questionable thing:

$$\frac{1}{1+y^2}\,dy = dt$$

c) integrate $\displaystyle\int \frac{1}{1+y^2}\,dy = \int dt$

$$\arctan(y) = t + C$$

d) check for divisions by zero:

$1+y^2$ never zero ✓

1 never 0. ✓

3) VAR. OF PARAMS.

for solving inhomog. linear PDE.

Ex: solve

$$y' - t^2 y = t.$$

a) First solve homog version:

$$y' - t^2 y = 0.$$

separation of vars: $\frac{dy}{dt} = t^2 y$; $\frac{dy}{y} = t^2 dt$

$\ln|y| = \frac{t^3}{3} + C$

$$\boxed{y = Ce^{t^3/3}.}$$

or, knowing that it's linear, soln is $Ce^{\int_0^t u^2 du} = Ce^{t^3/3}$

antideriv of t^2.

b) Now try variation of params:

Guess $y(t) = u(t)e^{t^3/3}$.

differentiate,

$$(u(t)e^{t^3/3})' - t^2 u(t) e^{t^3/3} = t$$

or $u(t)\cancel{t^2 e^{t^3/3}} + u'(t)e^{t^3/3} - \cancel{t^2 u(t)e^{t^3/3}} = t$

$$u'(t) = te^{-t^3/3}.$$

Integrate this... how? No closed form...

$$u(t) = \int_c^t ue^{-u^3/3}\,du.$$

$$\boxed{e^{t^3/3}\int_c^t ue^{-u^3/3}\,du}$$