Lecture 17

Matroid polytope!

1) finish algo proof (see lec16)
2) TU proof
3) Facets

Next time: Matroid intersect.

T.U. Proof

Recall matroid polytope

Want to show $P_M = P$ where

$P = \{$

(rank $_S$)

(nonnegativity) $\}$.

T.U. proof

- Note that $X = \{$ $\}$.

(rank $\{i\}$)

why? •

•

(rank $_S$).

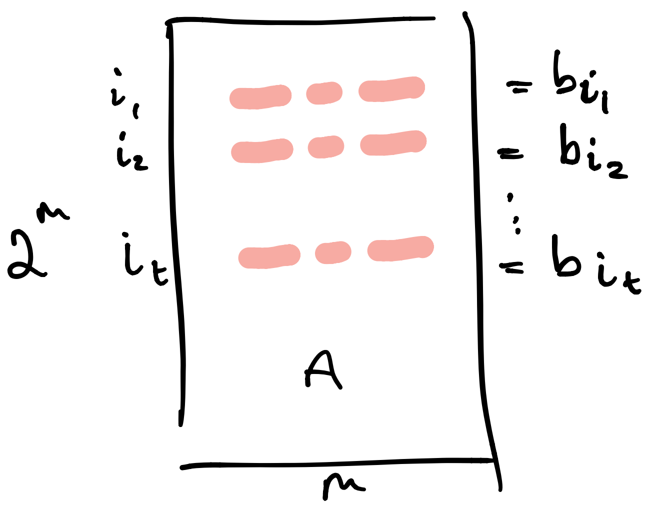
$\Rightarrow$

- If $P = \{ \qquad \}$,

  is A TU?

- Recall that vertices come from n tight constraints.

  ( ).



- Instead of showing A T.U.,

$\Rightarrow$

- In fact, submatrix A' will be even more special:

$\Longleftrightarrow$

,

$\Rightarrow$

$$A' = \begin{bmatrix} \\ \\ \\ \end{bmatrix}$$

( ).

-

-

Claim Let F be a face of P.

$F = \{ x \in \mathbb{R}^E :$

(tight chain)

$,$

$\}.$

(set $x_{E \setminus B} \to 0$)

•

Lemma: $\forall x \in P$, the tight constraints

$T := \{ S : \}$

are closed under $\cap$ and $\cup$.

i.e.

Proof of claim from lemma:

- From polyhedra, we know

$F = \{ x \in \mathbb{R}^E :$

$,$

$\}.$

i.e.

- Enough to show

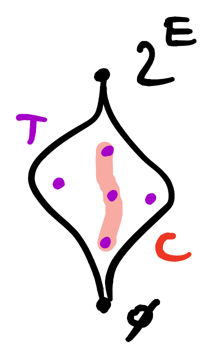
•

$:=$

$=$

$=:$

• To show,



• We claim

• Suppose

$\Rightarrow$

$($ $).$

$\Rightarrow$ The set

$V(S) = \{ \quad \}$

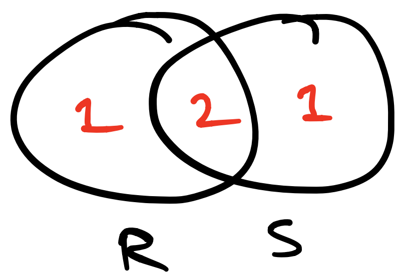
- Among all such S, take one with

$($ $).$

- Let

- **Lemma** ⇒

⇒



- Since

else

- Let

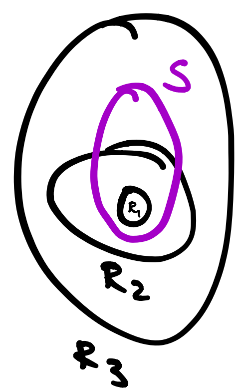
- But

  because

  (Exercise).

  and

  ( ).



$\Rightarrow$ $\square$

Corollary: Let x vertex of P.

$\Rightarrow$

(double check yourself).

Proof of lemma Want to show

$T := \{ \qquad \}$

closed under $\cap$ and $\cup$.

-

- $=$ (1)

  (2)

  $=$

  (3)

  $\leq$

  $\leq$ (4)

- (1) because
- (2) AKA
  holds because
- (3) because
- (4) is
- ★

$\Rightarrow$

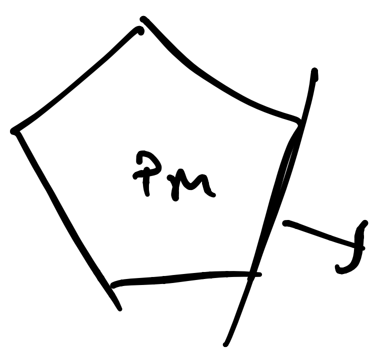
$\square$.

we'll skip facet proof; see pdf.

Facets of P_M

- which of the $2^{|E|}$ inequalities

define facets of P_M?



- For simplicity,

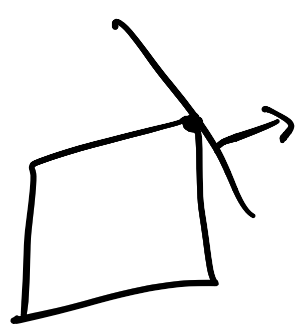
( ).

$\Rightarrow$

i.e.

- Rank constraints? $x(S) \le r(S)$.

  ▸ if S not closed,



- If S <u>seperable</u>, i.e.

$\Leftarrow$

- **Fact**: $S \rightsquigarrow$ facet $\Leftrightarrow$

<u>Proof omitted</u>

- **E.g.** <u>Graphic matroid</u> $M(G)$;

▷ **Exercise** $F \subseteq E$ inseparable $\iff$

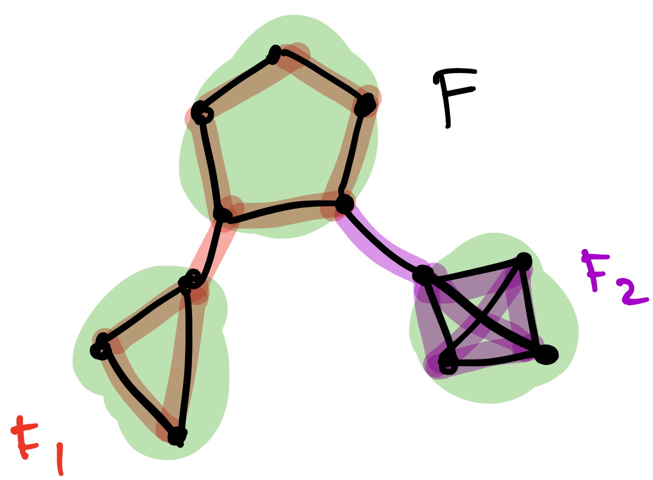
(V, F) is either

▷

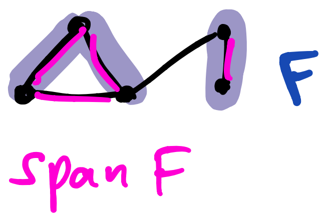
▷

E.g.

$|V| = 12$



▷ Span(F) =



▷ Thus F closed & inseparable

$\Leftrightarrow$

$\Rightarrow$ "Forest polytope" is minimally described by

$P = \{ x \in \mathbb{R}^E :$

$\}$.

"spanning tree" polytope:

$$P = \left\{ x \in \mathbb{R}^E : x(E) = |V| - 1 \right\},$$