Lecture 23

Plan: 1) finish stating ellipsoid
2) Analyze ellipsoid
3) Apply to LP. ← Next time

Analysis of ellipsoid

Recall main lemma:

Volume Lemma: Let E' be ellipsoid after E in the algorithm.

Then:

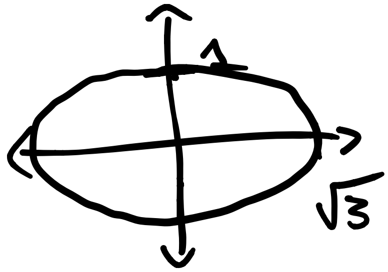
Before proving, some preliminaries:

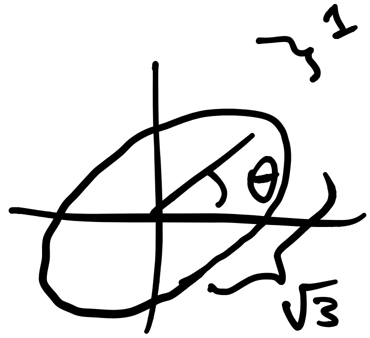
Def:

$E(c, A) := \{ \qquad \}.$

e.g. $A =$, $A' =$

$a = (0,0)$





Recall: Matrix $A \in \mathbb{R}^{n \times n}$

Equivalent conditions: Let A be a symetric matrix. Then:

A P.D.

$\iff$

$\iff$ $($ $)$

$\iff$

Facts about ellipsoids:

•

$$E(0, I) = \{ \quad \}.$$

Proof:

Let $A = B^T B$;

$$E(0, I) =$$

$$T : x \longmapsto y :=$$

$y^T y \le 1 \iff$

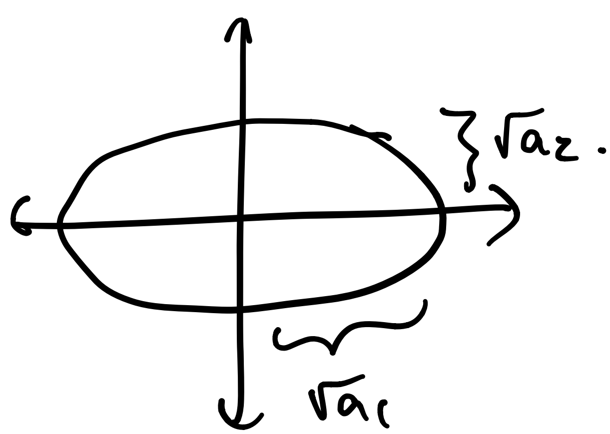
"

- **Volume:** if $A = \begin{bmatrix} a_1 & & 0 \\ & \ddots & \\ 0 & & a_n \end{bmatrix}$,

i.e.

then

$\text{vol}\ E(e, A) =$



(because $\text{vol}\ E(e, A) =$,

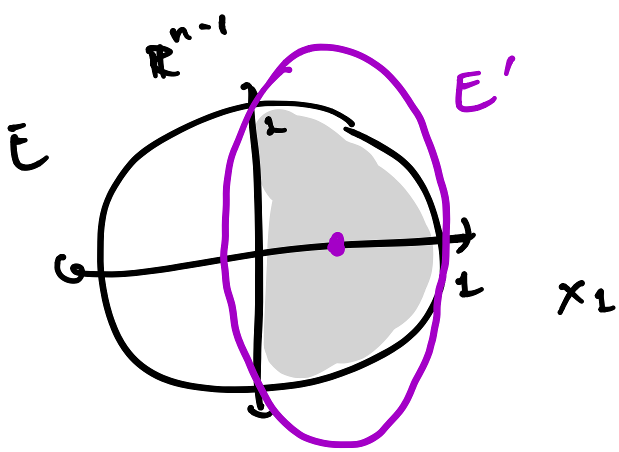
&   =

⇒   =

=   =

Proof of Volume Lemma:

- Begin with special case



- **Claim:** We can take

$E' = \{x : \quad \}$.

i.e. $E' = E(a, A)$ where $a =$

and $A =$

Proof of claim:

- Need to show

- Let $x \in$ Then

$=$

$=$

$=$

$\leq \qquad \leq \quad .$ $\qquad \square$

- Proof of volume lemma in this case:

$\mathrm{vol}\, E(c, A) = \qquad ;$

$=$

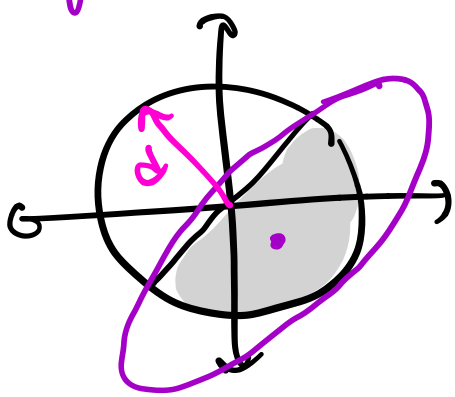
$= \qquad =$

$\leq$

$=$ $=$ . . $\square$

used inequality

- what if we have some other inequality ?



▷ can assume

▷ Figure out E' by ,

$\leq$.

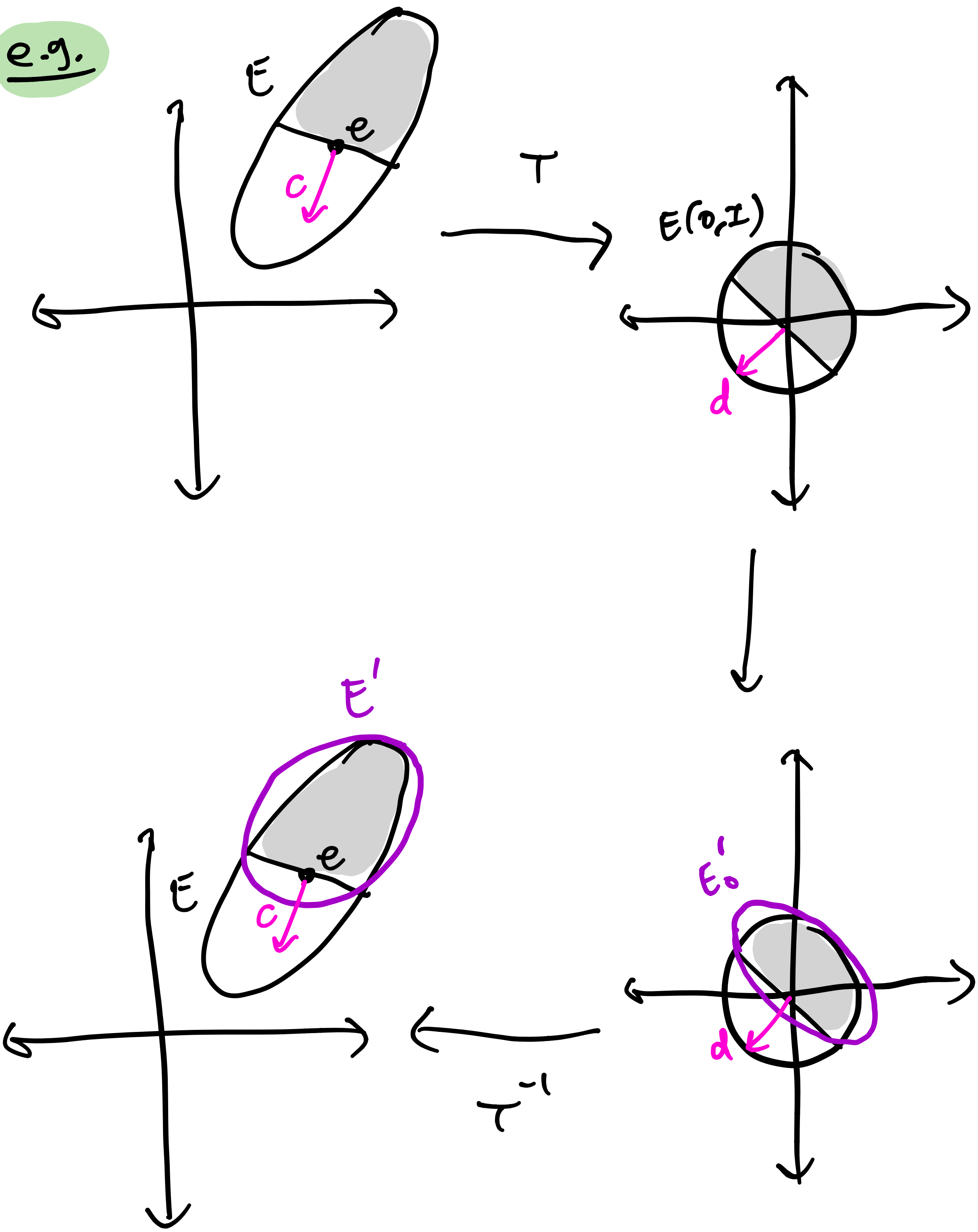
▷ End up with $E' =$,

$=$

• What if E not unit sphere? .

$$
\begin{tikzcd}
E \arrow[r] & {} \arrow[d] \\
E' & {} \arrow[l]
\end{tikzcd}
$$

e.g.



- Now

$$\frac{\mathrm{vol}\ E'}{\mathrm{vol}\ E} =$$

$$= \qquad\qquad \leq$$

Completes proof of volume lemma. $\square$.

How to compute E'?

-

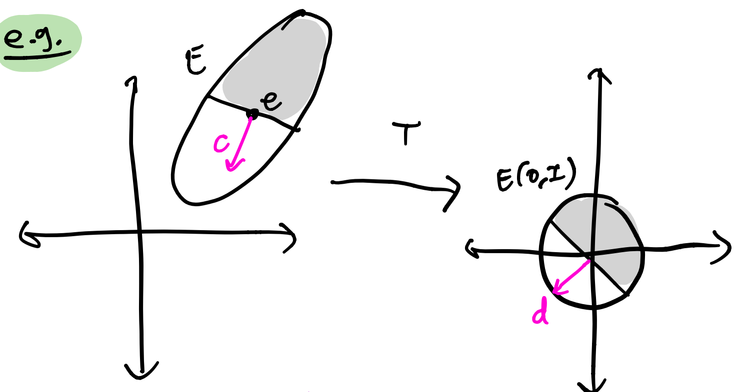
- Let's carefully compute

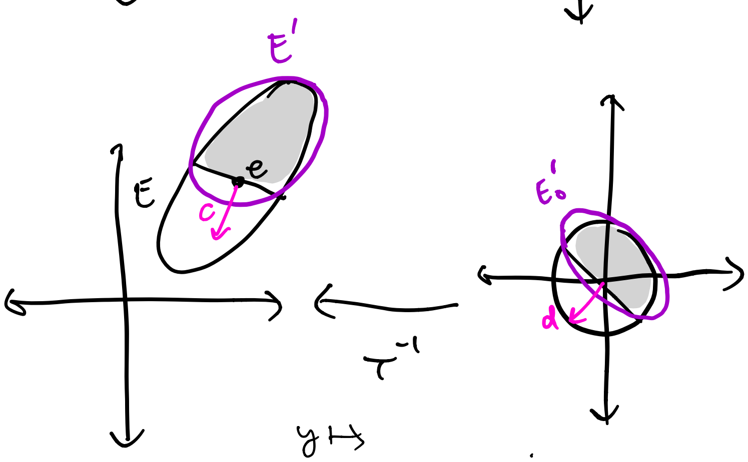
If $E = E(e, A)$, recall

$T : x \longmapsto y :=$

has

e.g.





- First find d. Under T,

$\{x : c^T x \le c^T e\} \xrightarrow[T]{} \{y :$ $\}$

$= \{y :$ $\} = \{y :$ $\}$,

for $d =$ $=$

- Recall that

$E_0' = E($ $,$ $)$

- Let $b =$ $=$;

Applying T^{-1} to E_0' yields

$$E' = E(\quad , \quad).$$

$$= E(\quad , \quad).$$

Ellipsoid (Concretely):

- Initialize $E =$
- while $c \notin P$:
  - ▷ Let
  - ▷ Let $b =$

▷ set $e \leftarrow$

▷ set $A \leftarrow$

.

Analysis summary:

After k iterations,

$\text{vol } E \leq$.

$\Rightarrow$

terminates in $\leq$

steps.

Linear programming: