Lecture 21

Plan: 1) finish arborescence
2) matroid union

Spanning tree game:

Given graph G, players alternate:

1)

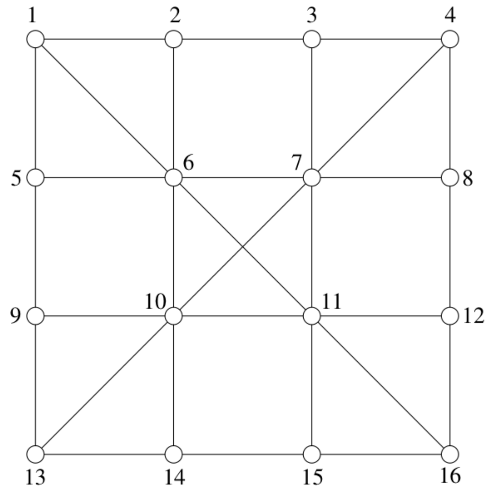
2)

P1 wins if

e.g. P1 win:

P1

P2



Recall:

P2 wins if A)

(

.)

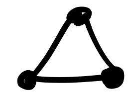
P1 wins if B)

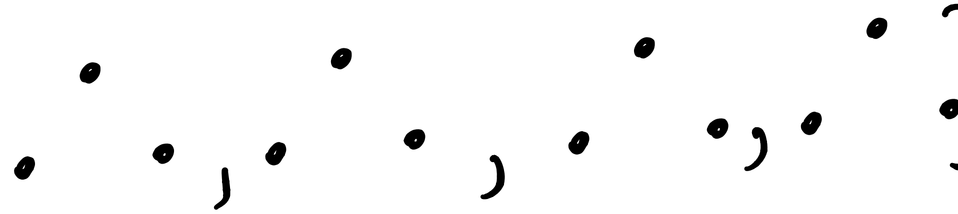
Today: with matroid union, show

Matroid Union:

Let $M = (E, \mathcal{I})$ matroid.

Recall dual matroid

E.g. If $M = M_G$ for $G =$ ,

$I^* = \{$  $\}$

i.e.

Theorem The dual matroid M^*

is

Proof Use

Fact: Can define a matroid using

I.e.

R1)

R2)

Then $M = (E, \mathcal{I})$

$\mathcal{I} =$

is

Thus

A)

B)

$\square$

e.g. disjoint spanning trees:

G has 2 disjoint spanning trees $\Leftarrow$

and

moreover,

Theorem: G has two disjoint spanning trees $\Leftrightarrow$

Proof Assume

- We only show $\Leftarrow$;

Plan: use Minimax theorem for

- Let

- G has 2 edge disjt. spanning trees $\iff$

- $r_M(F) =$

<u>min-max:</u>

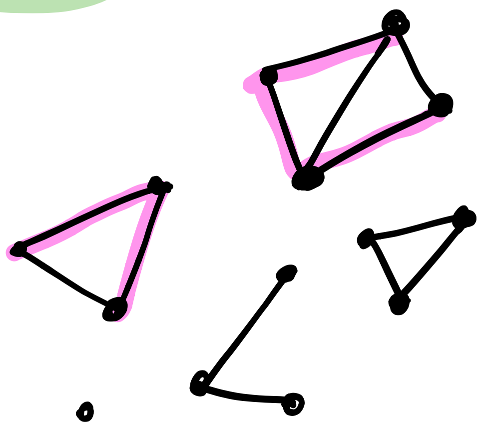
- <u>Matroid Intersection theorem:</u>

- <u>Recall</u>:

i.e.

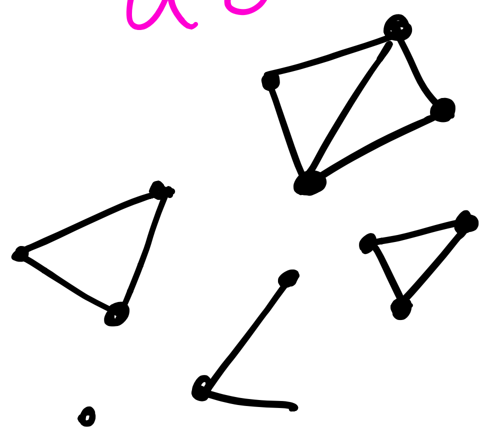
e.g.

U not closed



E

U closed



E

$\Rightarrow \star =$

$=$

$=$

$=$

$\Rightarrow$

$\Rightarrow$ $\square$.

(General) matroid union

- Let $M_1 = (E, \mathcal{I}_1)$, $M_2 = (E, \mathcal{I}_2)$ matroids.

Def. The <u>matroid union</u>

<u>Careful</u>:

<u>**Theorem**</u>: $M_1 \cup M_2$ is

Proof

Part 1 : It's a matroid.

- Let

  and

- Need to show

- Assume

• Since

$($ $)$

$\Rightarrow$

•

$($ $).$

$\Rightarrow$

End part 1 $\triangle$

Part 2: Rank function.

$r_{M_1 \cup M_2}(S) =$

- $\leq$ clear;

  $|S| =$

  $\leq$

- For $\geq$, use

- First

- Let

  $\Rightarrow$

- May assume

  ( ).

  $\Rightarrow$

- Then (i.e.)

because

- I.e.
- matroid intersection theorem :

$r_{M_1 \cup M_2}(E) =$

$\geq$

$=$

$=$

$=$

end part 2 Δ.