

1. Suppose that $y_p(t)$ is a particular solution of

$$y'(t) + y(t) = q(t) \quad (1)$$

where $q(t)$ is some polynomial of t . If $y_p(0) = 1$ find the solution of (1) that satisfies the initial condition $y(0) = 0$. Your answer should be in terms of $y_p(t)$ and a specific exponential function.

2. Consider the following *linear inhomogeneous* equation

$$y' - \tan(x)y = 1. \quad (2)$$

- (a) Why is it linear? Why is it inhomogeneous?
 - (b) Find a basic solution $y_h(x)$ of the associated *homogeneous* equation of (2). Hint: to find the anti-derivative of $\tan(x)$ recall $\tan(x) = \frac{\sin(x)}{\cos(x)}$ and try subbing $u = \cos(x)$.
 - (c) We now use *variation of parameters*. Sub $y(x) = u(x)y_h(x)$ and find a differential equation for $u(x)$.
 - (d) Solve for $u(x)$. It should contain an unknown constant. Use it to construct the general solution of (2).
3. (Tricky) In this problem we will find the family of curves orthogonal to the family of ellipses given by $x^2/2 + y^2 = C$
- (a) To get started let $y = y(x)$, and differentiate the equation of the ellipse with respect to x .
 - (b) Solve this equation for $y'(x)$ as a function of y and x .
 - (c) Wait! If we solve this differential equation for y , we'll just get back the family of ellipses. Instead we want the family of curves *orthogonal* to the ellipses. Since $y'(x)$ is the slope of the tangent line what should we do to the equation we just derived to make it orthogonal?
 - (d) Solve this new differential equation.
4. Find the general solution to $y' - t2y = 0$. What about $y' - (n+1)t^n y = 0$?