ECE 7011 Computational Electromagnetics Spring 2020

February 4, 2020

Homework 2

Due date: Feb. 18

Problem 1

Write a FDTD code in a staggered grid for one-dimensional Maxwell's equations:

$$\frac{\partial}{\partial t} E_x = -\frac{1}{\varepsilon_0} \frac{\partial}{\partial z} H_y$$

$$\frac{\partial}{\partial t} H_y = -\frac{1}{\mu_0} \frac{\partial}{\partial z} E_x$$

in the domain $0 \le z \le 0.5$ m and $t \ge 0$ s, with $\Delta z = 10^{-3}$ m and $c_f = 1/2$ (Courant number), so that $\Delta t = 1.67$ ps. Use homogeneous Dirichlet boundary conditions for the electric field and discretize the domain with 501 nodes (for the electric field, including the two end points), and 500 half-nodes (for the magnetic field).

(a) Let the initial conditions be

$$E_i^0 = \exp\left[-\left(\frac{i - i_c}{i_w}\right)^2\right] \qquad H_{i+\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{\eta_0} \exp\left[-\left(\frac{i + \frac{1}{2} - i_c - \frac{c_f}{2}}{i_w}\right)^2\right]$$

with $i_c = 80$, $i_w = 4$, and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$. Run the FDTD algorithm for 700 time steps and plot the electric field distribution as a function of position along the grid for time step numbers $n = 0$, 100, 200,…,700. Compare your numerical solution against the exact solution:

$$E_x(z,t) = \exp\left[-\left(\frac{z - z_c - ct}{w}\right)^2\right] \text{ with } z_c = i_c \Delta z,\ w = i_w \Delta z, \text{ and } c = 1\Big/\sqrt{\mu_0 \varepsilon_0}.$$

(b) Change the initial condition parameter $i_w = 30$, while keeping all other parameters the same, and discuss the effects on the numerical results and numerical error.

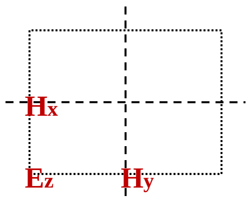
(c) Use $i_w = 4$ back again and plot the results, but now with initial conditions

$$E_i^0 = \exp\left[-\left(\frac{i - i_c}{i_w}\right)^2\right] \qquad H_{i+\frac{1}{2}}^{\frac{1}{2}} = 0$$

and discuss your results

Problem 2

Write a FDTD code to simulate the TM fields in a two-dimensional rectangular cavity with dimensions 100 mm along *x* and 50 mm along *y*. Use 100 cells (101 nodes) along both *x* (*i*=0,…,100) and *y* (*j*=0,…,100). The TM fields have E_z, H_x, H_y components. Set E_z on integer grid points and H_x, H_y at half grid points along the opposite coordinates, as indicated below.



The cavity has perfectly electrical conducting walls. For the initial conditions, set random values for the electric field at all points (try both a zero-mean and a non-zero mean uniform distribution), and zero values for the magnetic field. Use $c_f = 0.8$ and run the code for 8192 time steps. Sample the resulting electric field at node (*i*,*j*)=(13,29) and take the FFT of this time-domain signal. Plot the magnitude-squared of the FFT in the range from 0 to 10 GHz and interpret your results.

Problem 3

Study the stability of the following FD two schemes for the solution of the one-dimensional diffusion equation:

$$\frac{\phi_{n+1}^{l} - 2\phi_{n}^{l} + \phi_{n-1}^{l}}{\Delta x^2} = \begin{cases} \dfrac{\mu\sigma}{\Delta t}\left(\phi_n^{l+1} - \phi_n^{l}\right) \\ \dfrac{\mu\sigma}{\Delta t}\left(\phi_n^{l} - \phi_n^{l-1}\right) \end{cases}$$

Classify these schemes as conditionally stable (under which condition?) or unstable.

Problem 4

Using the dispersion relation on a two-dimensional FDTD grid

$$\left[\frac{1}{c\Delta t}\sin\left(\frac{\omega\Delta t}{2}\right)\right]^2 = \left[\frac{1}{\Delta x}\sin\left(\frac{\tilde{k}_x\Delta x}{2}\right)\right]^2 + \left[\frac{1}{\Delta y}\sin\left(\frac{\tilde{k}_y\Delta y}{2}\right)\right]^2$$

and assuming $\Delta x = \Delta y$ and $c_f = 1$, plot $\tilde{v}/c$ as a function of the azimuth angle ϕ, for $k\Delta x = 2\pi/20,\ 2\pi/10,\ 2\pi/5,\ 2\pi/2$. Interpret your results.

Note: $\tilde{k}_x = \tilde{k}\cos\phi$ $\quad$ $\tilde{k}_y = \tilde{k}\sin\phi$ $\quad$ $\tilde{v} = \omega/\tilde{k}$