ECE 7011 Computational Electromagnetics
Spring 2020

Homework 1

January 21; due date: February 4th

Problem 1

In this problem you will solve Laplace equation:

$$\frac{\partial^2}{\partial x^2}\phi + \frac{\partial^2}{\partial y^2}\phi = 0$$

in a two-dimensional square domain $0 \le x, y \le \pi/2$ using Jacobi and Gauss-Seidel methods. Assume boundary conditions: $\phi = \sin(2x)$ for $y = \pi/2$ and $\phi = 0$ for $x = 0$, $x = \pi/2$, and $y = 0$. As initial guess for the iteration, you can assume $\phi^0(x, y) = 0$.

(a) Use three regular grids with (10,10), (20,20) and (30,30) grid points. For each grid, compare the error of the numerical solution at each iteration against the exact solution for this problem given by

$$\phi_e(x, y) = \frac{\sin(2x)\sinh(2y)}{\sinh(\pi)}.$$

Define the error measure

$$\delta = \sum_{i,j} \frac{\left|\phi_{i,j}^{num} - \phi_e(x_i, y_j)\right|^2}{N}$$

where the sum runs over the grid points and N is the total number of grid points.

(b) The above is a *global* error measure. Also investigate the behavior of the *local* error by plotting the function $\left|\phi_{i,j}^{num} - \phi_e(x_i, y_j)\right|^2$ for all nodes (i, j) at a few different iterations of your choice.

(c) Compare the results for different grids and the convergence rate of Jacobi versus Gauss-Seidel. Discuss your results.

(d) Change the initial guess to $\phi^0(x, y) = \frac{\sinh(2y)}{\sinh(\pi)}$ and compare the results against the previous zero-field initial guess.

Problem 2

The advection equation

$$\frac{\partial}{\partial t}\phi + u\frac{\partial}{\partial x}\phi = 0$$

is an hyperbolic equation (assume $u > 0$). Using forward-differencing for the time derivative and central-differencing for the spatial derivative, the advection equation can be easily discretized as

$$\phi_i^{n+1} = \phi_i^n - r(\phi_{i+1}^n - \phi_{i-1}^n) \qquad \text{where } r = (u\Delta t)/(2\Delta x).$$

(a) Assuming the solution domain as the unit interval $0 \le x \le 1$, and choosing a grid with 11 nodes, $i = 0,1,\ldots,10$, with:

- homogeneous Dirichlet boundary conditions: $\phi_0^n = \phi_{10}^n = 0$,
- initial conditions $\phi_5^0 = 10$, and $\phi_i^0 = 0$ for all other i .

Show that the above scheme goes unstable for $r = 0.1$ and $r = 10$ (indeed, this scheme is unstable for any choice of r).

(b) An alternative discretization is

$$\phi_i^{n+1} = \frac{1}{2}(\phi_{i+1}^n + \phi_{i-1}^n) - r(\phi_{i+1}^n - \phi_{i-1}^n).$$

Using the same boundary conditions and initial conditions, study the numerical solution for $r = 0.1$ and $r = 10$. Discuss your results.

Problem 3

Use Taylor series to show that the truncation error for the Yee's FDTD method in a one-dimensional staggered grid as discussed in class is $O(\Delta t^2)$ in time and $O(\Delta x^2)$ in space.