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MATH 332

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Exercise 22.22. Find a polynomial of degree > 0 in $\mathbb{Z}_4[x]$ that is a unit. $\Diamond$

Proof. Let $\phi(x) = (2x+1)$. In $\mathbb{Z}_4$, then $\phi(x) \cdot \phi(x) = (2x+1)^2 = 4x^2 + 4x + 1 \equiv 1 (\text{mod } 4)$. Therefore $\phi(x)$ is a unit with degree > 0. $\square$

Excercise 22.24. If D is an integral domain, then $D[x]$ is an integral domain. $\Diamond$

Proof. Let $f, g \in D[x]$ such that $f(x) = a_0 + a_1 x + \ldots + a_n x^n$ is a polynomial of degree n and $g(x) = b_0 + b_1 x + \ldots + b_m x^m$ is a polynomial of degree m. Let $f(x) \neq 0$ and $g(x) \neq 0$. Then $f(x) \cdot g(x) = a_0 b_0 + (a_0 b_1 + a_1 b_0)x + \ldots + a_n b_m x^{n+m}$.

Suppose $f(x)g(x) = 0$. Then all coefficients a_i, b_j for $0 \leq i \leq n$ and $0 \leq j \leq m$ must be 0 because D is an integral domain and therefore has no zero divisors. Then, because D has no zero divisors, then $a_n = 0$ or $b_m = 0$, and either the degree of $f \neq n$ or the degree of $g \neq m$. Hence $D[x]$ is an integral domain. $\square$