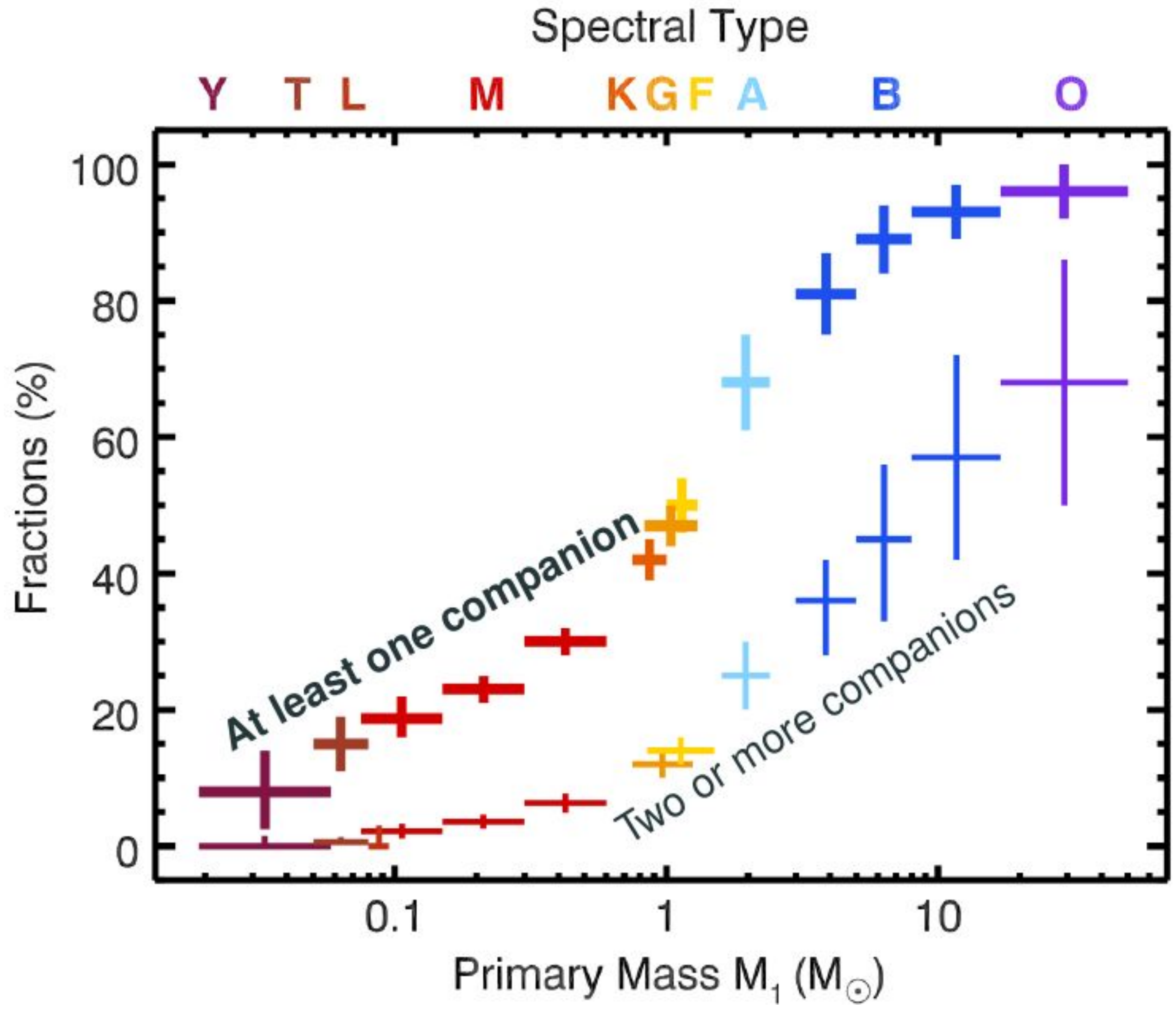
One star, two star, red star, blue star

Image: Casey Reed

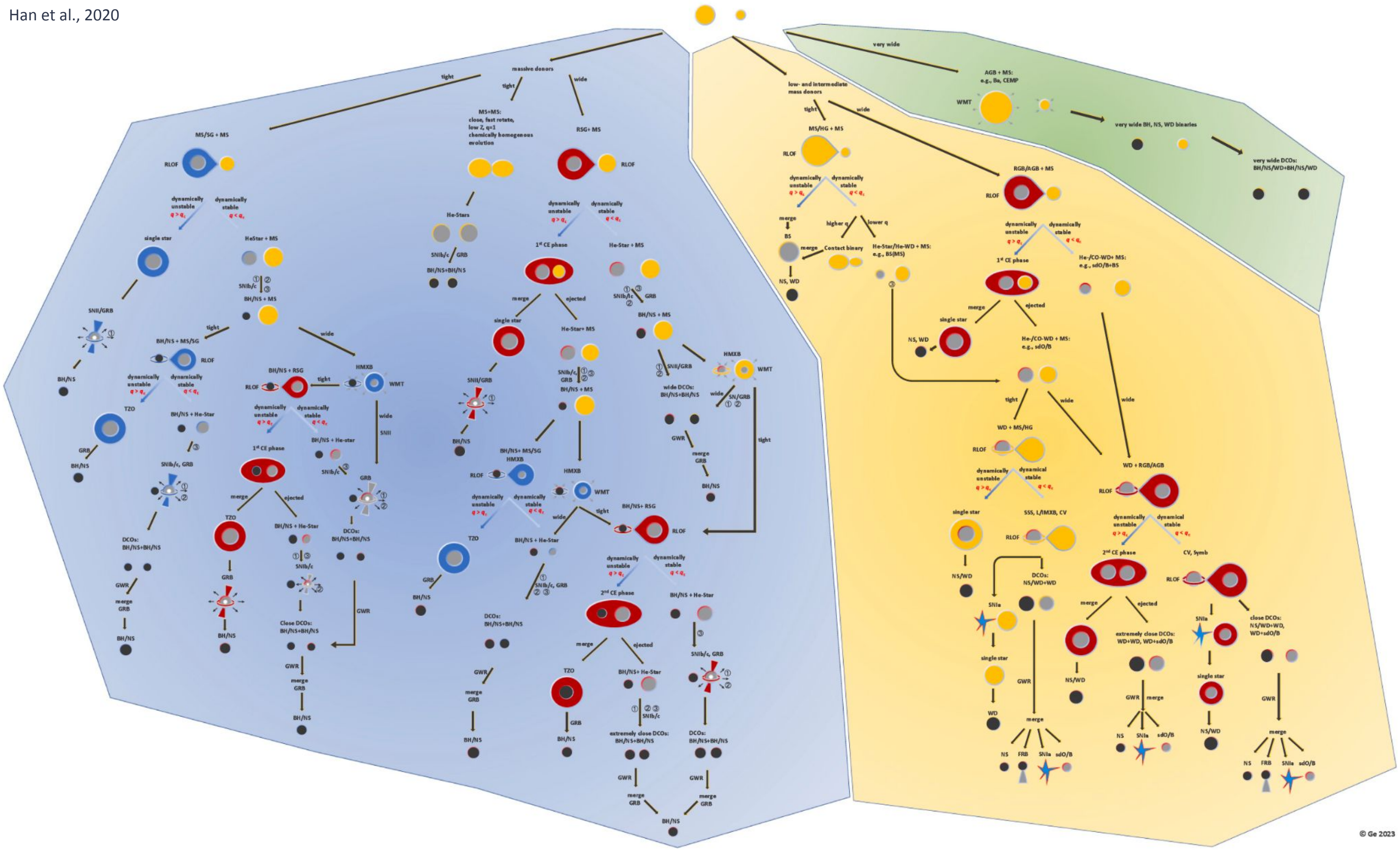
Part I
Cole Johnston | MPA
11/10/2024



Oeffener et al., 2023

binary evolution

Han et al., 2020



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Aims

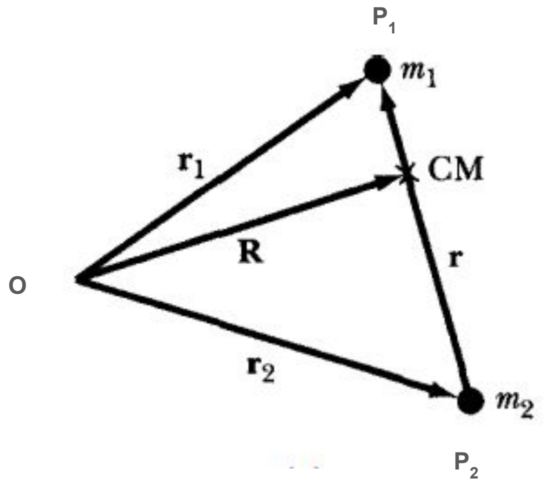
Part I: Orbits and Observations

- Describe binary orbits
- Understand observational techniques
- What do we get from different observations?

Part II: Examples

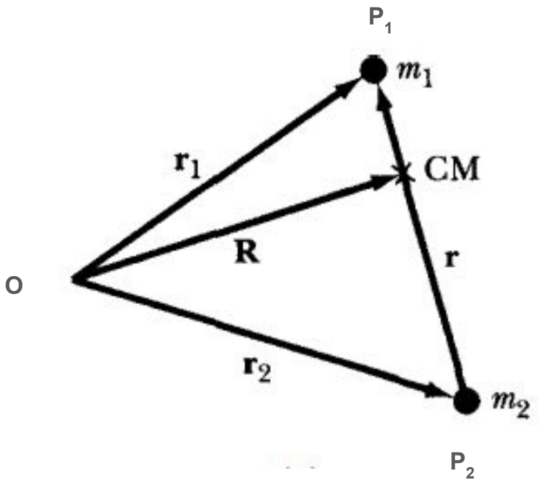
- Spectroscopy
  - Instrumental considerations
  - T_{eff} , logg, vsini, macro-turbulence
  - SB1
  - SB2 → Disentangling

How do we describe a binary: the two body problem



- Two stars of mass $m_{1,2}$ at positions $P_{1,2}$

How do we describe a binary: the two body problem



- Two stars of mass $m_{1,2}$ at positions $P_{1,2}$

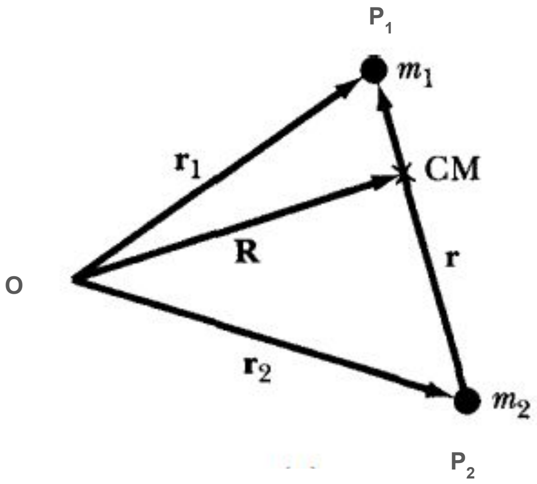
$$\overrightarrow{OP_1} = \overrightarrow{r_1} \;\; ; \;\; \overrightarrow{OP_2} = \overrightarrow{r_2}$$

$$\overrightarrow{OC} = \overrightarrow{R}$$

$$\overrightarrow{P_2P_1} = \overrightarrow{r} = \overrightarrow{r_1} - \overrightarrow{r_2}$$

$$\overrightarrow{F_1} = -\overrightarrow{F_2}$$

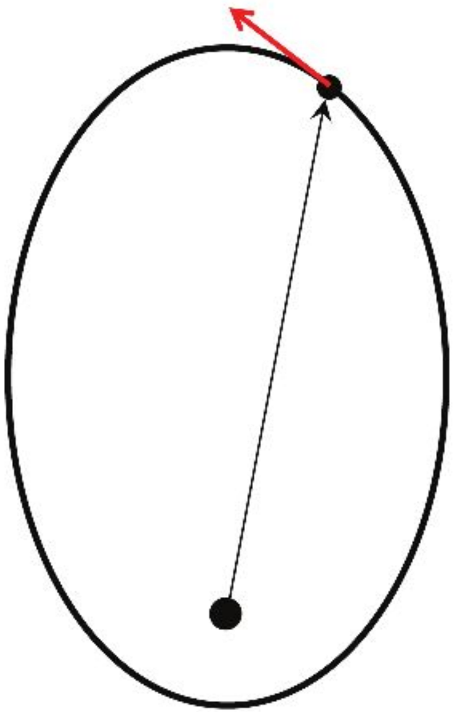
How do we describe a binary: the two body problem



$$\vec{F_1} = -\vec{F_2} \qquad |F| = \frac{Gm_1m_2}{r^2}$$

$$m_1\ddot{r_1} = -\frac{Gm_1m_2}{r^2}(\hat{r}) \;\; ; \;\; m_2\ddot{r_2} = -\frac{Gm_1m_2}{r^2}(-\hat{r})$$

How do we describe a binary: the relative orbit



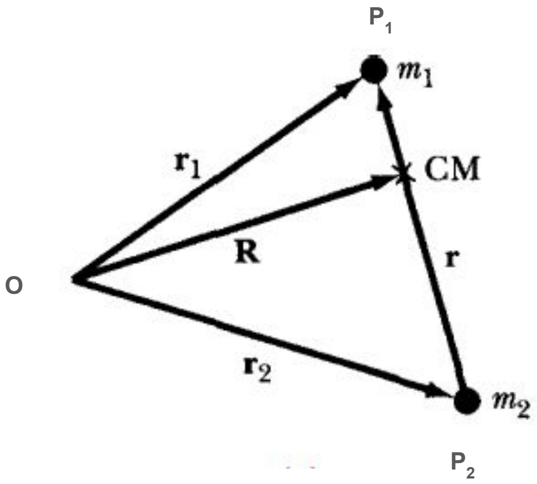
$$m_1\ddot{r}_1 = -\frac{Gm_1m_2}{r^2}(\hat{r}) \;\; ; \;\; m_2\ddot{r}_2 = -\frac{Gm_1m_2}{r^2}(-\hat{r})$$

$$\ddot{\vec{r}} = \ddot{\vec{r_1}} - \ddot{\vec{r_2}}$$

$$\ddot{\vec{r}} = -\frac{Gm_2}{r^2}(\hat{r}) - \frac{Gm_1}{r^2}(\hat{r}) = \frac{-G(m_1+m_2)}{r^2}(\hat{r})$$

$$\mu = \frac{m_1m_2}{(m_1+m_2)} \;\; ; \;\; \mu\ddot{\vec{r}} = \frac{Gm_1m_2}{r^2}\hat{r}$$

How do we describe a binary: the barycentric orbit



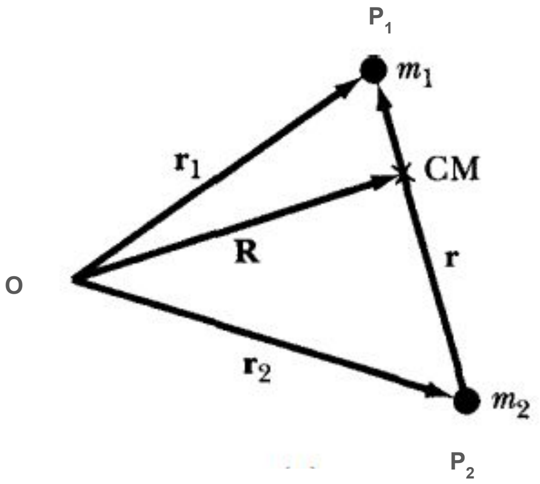
$$\overrightarrow{CP_1} = \overrightarrow{R_1} \; ; \; \overrightarrow{CP_2} = \overrightarrow{R_2}$$

$$CoM \rightarrow m_1\overrightarrow{r_1} + m_2\overrightarrow{r_2} = (m_1 + m_2)\,\overrightarrow{R}$$

$$m_1\overrightarrow{R_1} + m_2\overrightarrow{R_2} = 0$$

$$\overrightarrow{r} = -\frac{m_1 + m_2}{m_1}\overrightarrow{R_2} \qquad \overrightarrow{r} = \frac{m_1 + m_2}{m_2}\overrightarrow{R_1}$$

How do we describe a binary: the barycentric orbit

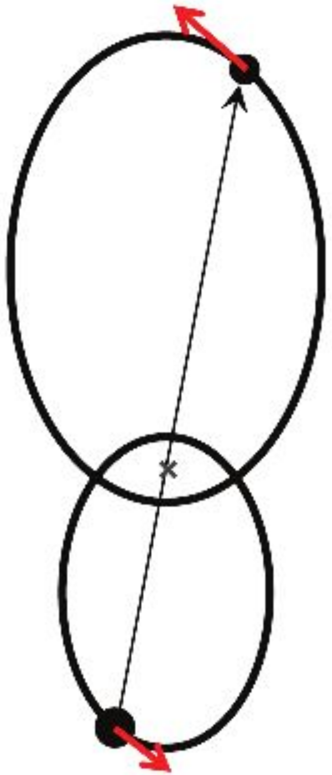


$$\ddot{\vec{r_1}} = 0 + \ddot{\vec{R_1}} \;\; ; \;\; \ddot{\vec{r_2}} = 0 + \ddot{\vec{R_2}}$$

$$\ddot{\vec{R_1}} = -\frac{Gm_2}{r^3}(\vec{R_1} - \vec{R_2}) \quad \ddot{\vec{R_2}} = -\frac{Gm_1}{r^3}(\vec{R_2} - \vec{R_1})$$

Remember our values for r!

How do we describe a binary: the barycentric orbit

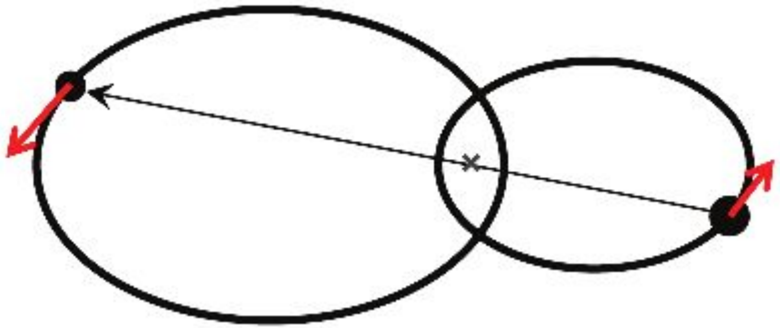


$$\ddot{\vec{r_1}} = 0 + \ddot{\vec{R_1}} \quad ; \quad \ddot{\vec{r_2}} = 0 + \ddot{\vec{R_2}}$$

$$\ddot{\vec{R_1}} = -\frac{Gm_2}{r^3}(\vec{R_1} - \vec{R_2}) \quad \ddot{\vec{R_2}} = -\frac{Gm_1}{r^3}(\vec{R_2} - \vec{R_1})$$

$$\ddot{\vec{R_1}} = -\frac{Gm_2^3}{(m_1 + m_2)^2}\frac{\vec{R_1}}{\vec{R_1}^3} \quad \ddot{\vec{R_2}} = -\frac{Gm_1^3}{(m_1 + m_2)^2}\frac{\vec{R_2}}{\vec{R_2}^3}$$

How do we describe a binary



Relative orbit

$$a = a \;;\; M = G\,(m_1 + m_2)$$

Barycentric orbit
Star 1

$$a = a_1 \;;\; M = \frac{G m_2^3}{(m_1 + m_2)}$$

Barycentric orbit
Star 2

$$a = a_2 \;;\; M = \frac{G m_1^3}{(m_1 + m_2)}$$

How do we describe a binary

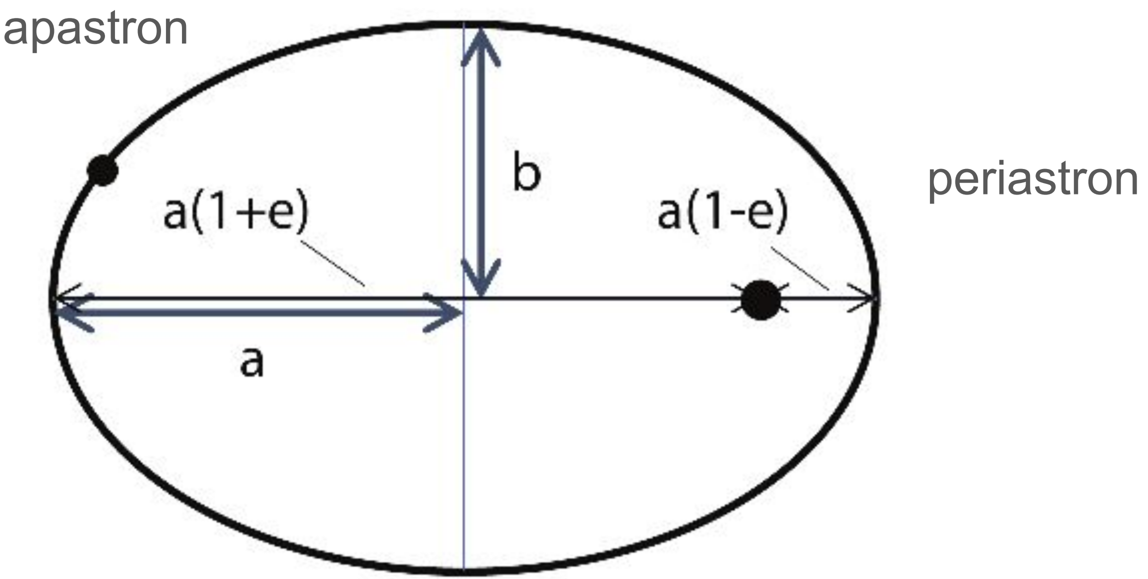
What do we need to know about a binary to describe its orbit?

How do we describe a binary

What do we need to know about a binary to describe its orbit?

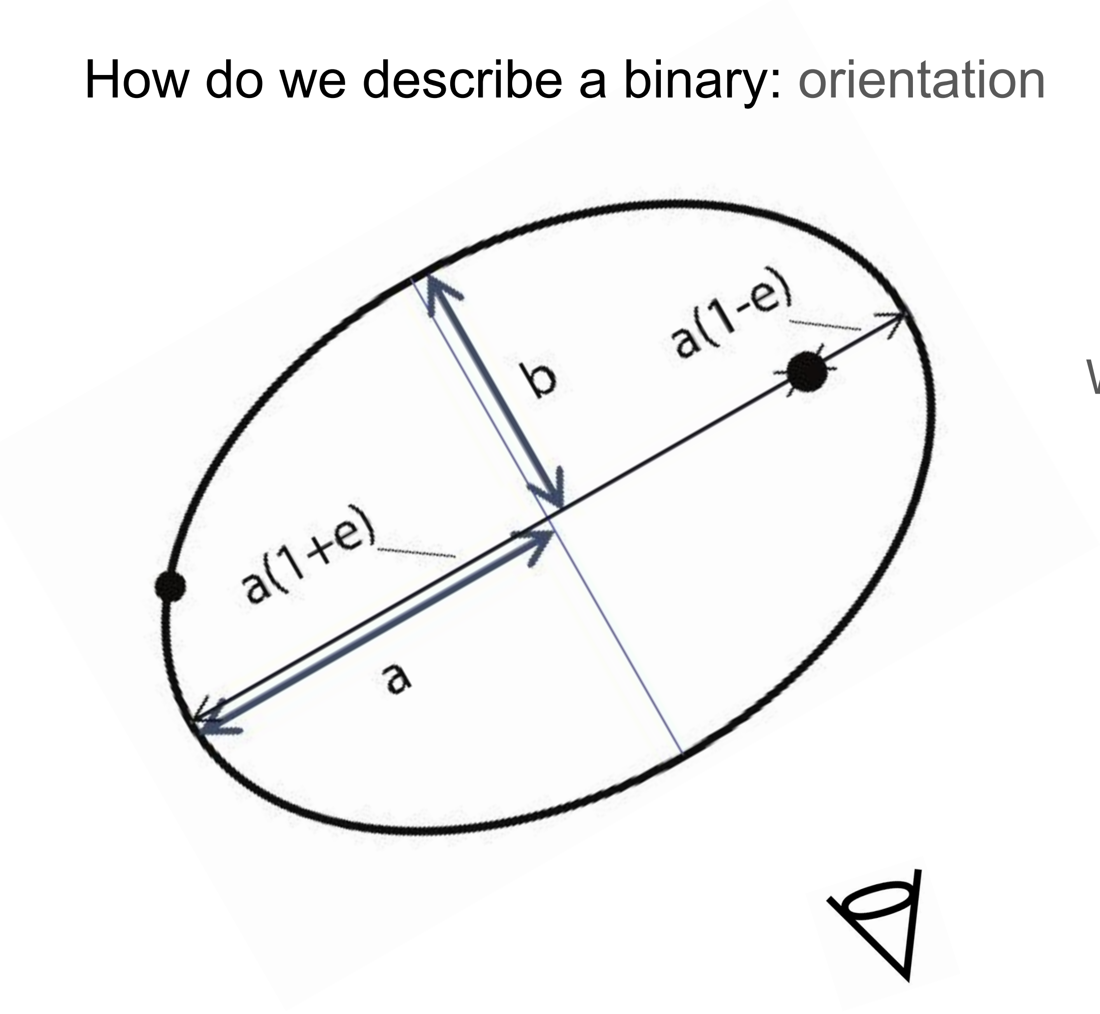
- Size & shape
- Orientation
- Timings
- Where the stars are

How do we describe a binary: Size & shape



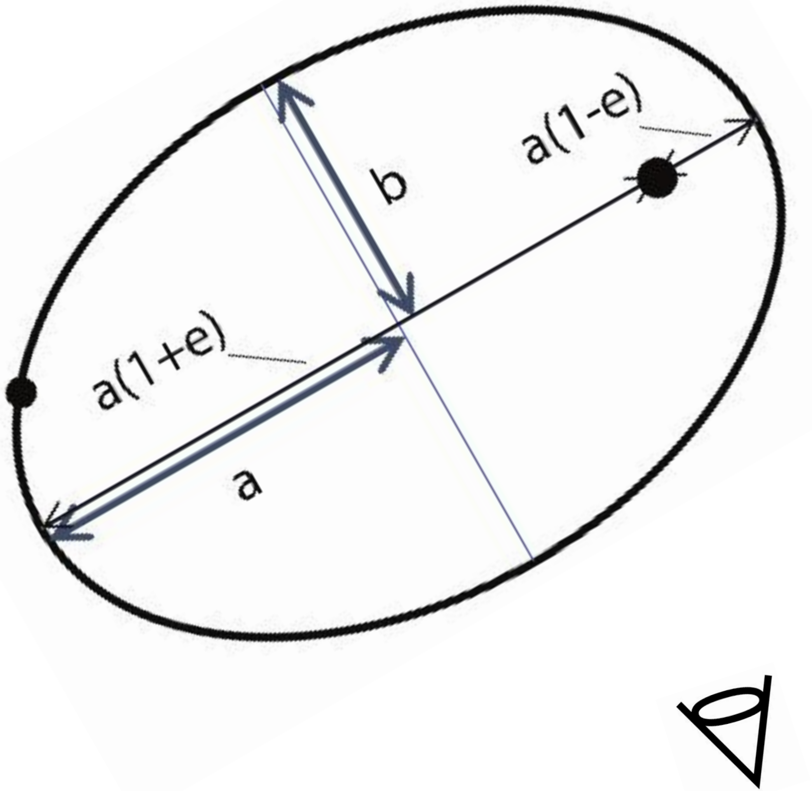
Semi-major axis: a
Eccentricity: e

How do we describe a binary: orientation



What angles do we need?

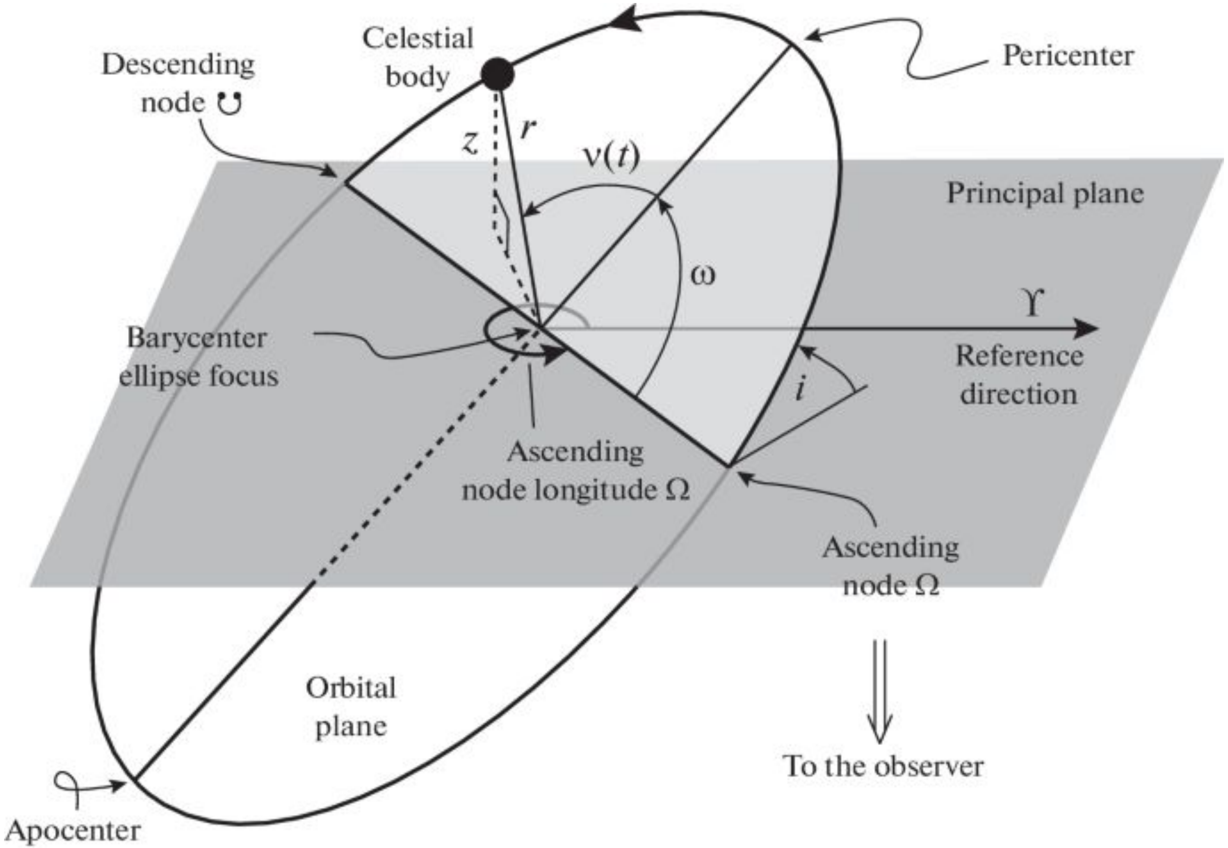
How do we describe a binary: orientation



What angles do we need?

- inclination: i
- Rotation: argument of periastron
- Ascending node

How do we describe a binary: orientation

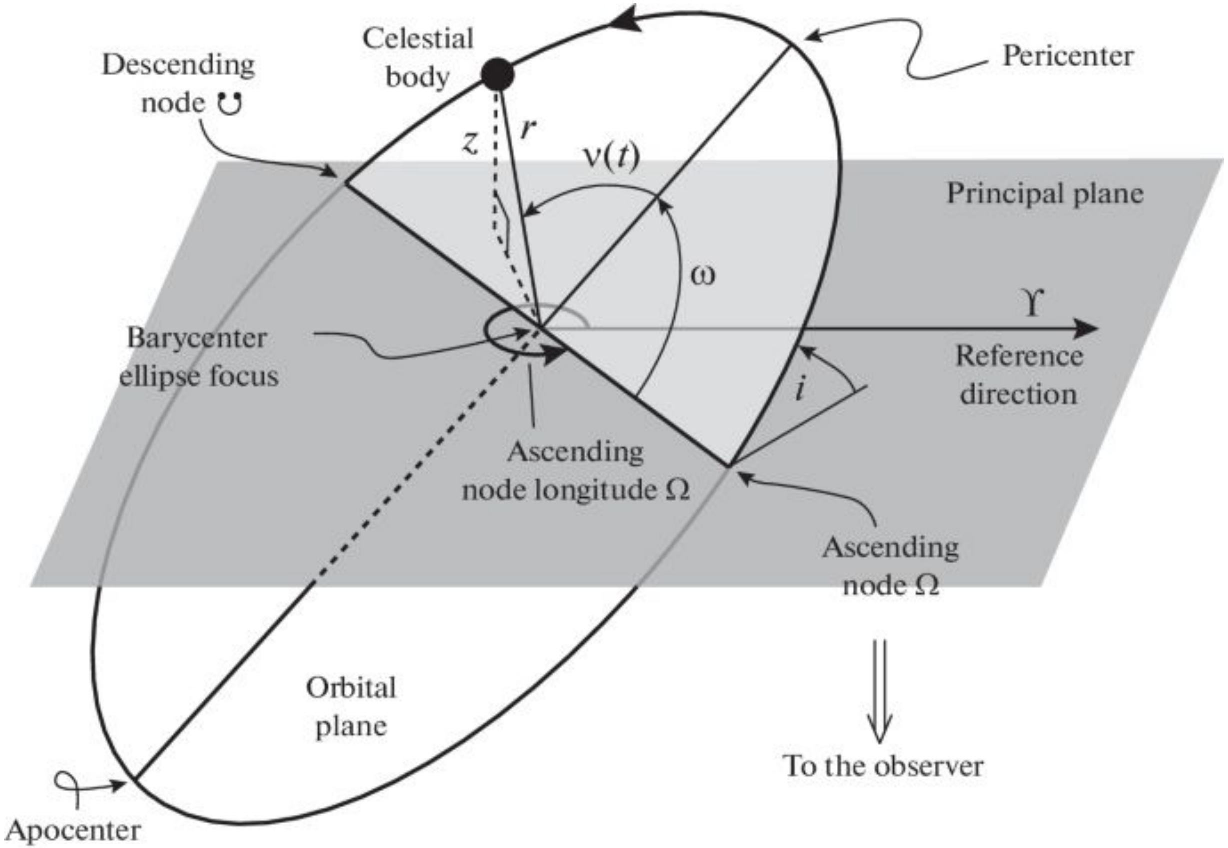


<u>Reference plane</u>: plane of the sky

<u>Inclination</u>: (i) the tilt of the orbit with respect to the reference plane

0° = face on ; in the plane
90° = edge on ; perpendicular

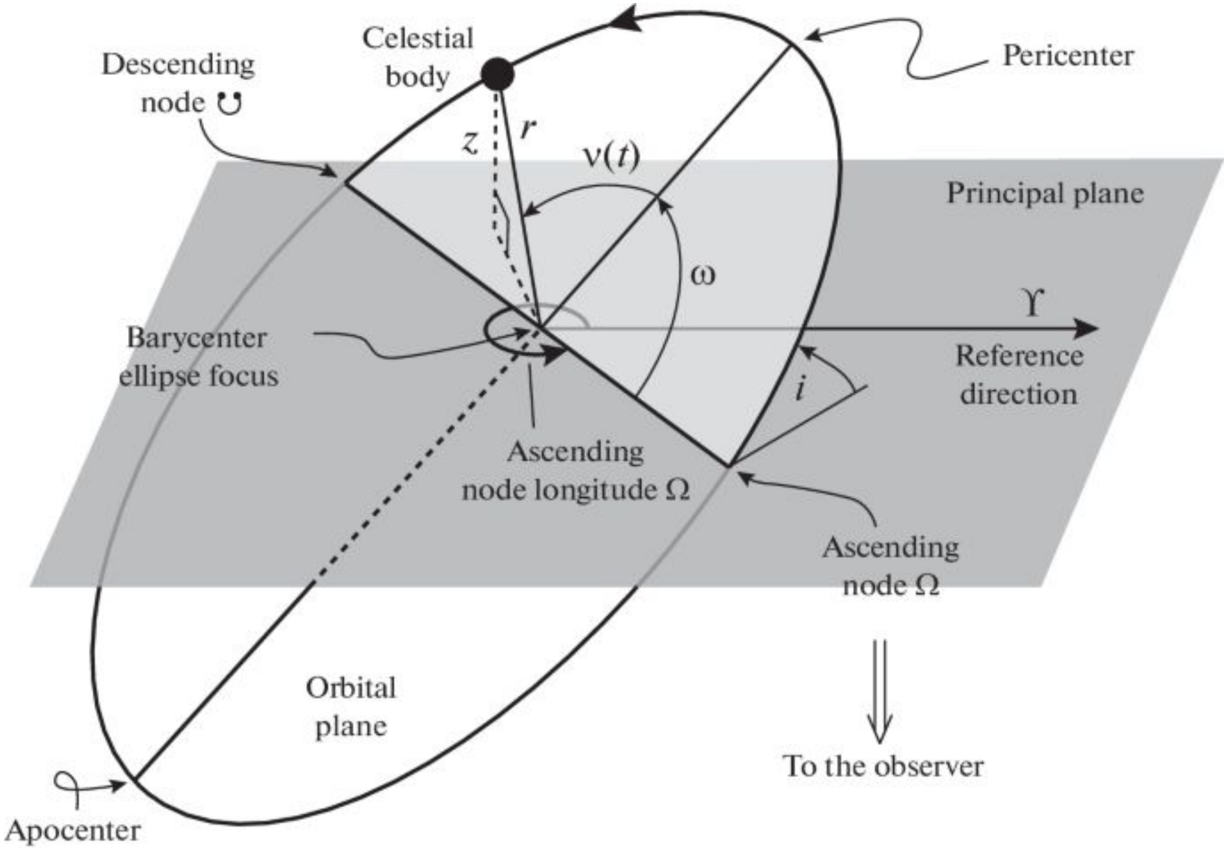
How do we describe a binary: orientation



<u>Ascending node</u>: point where the orbiting star passes through the reference plane ; the point of maximum receding velocity

<u>Longitude of the ascending node</u>: (**Ω**) angle between the reference direction and the ascending node (ccw)

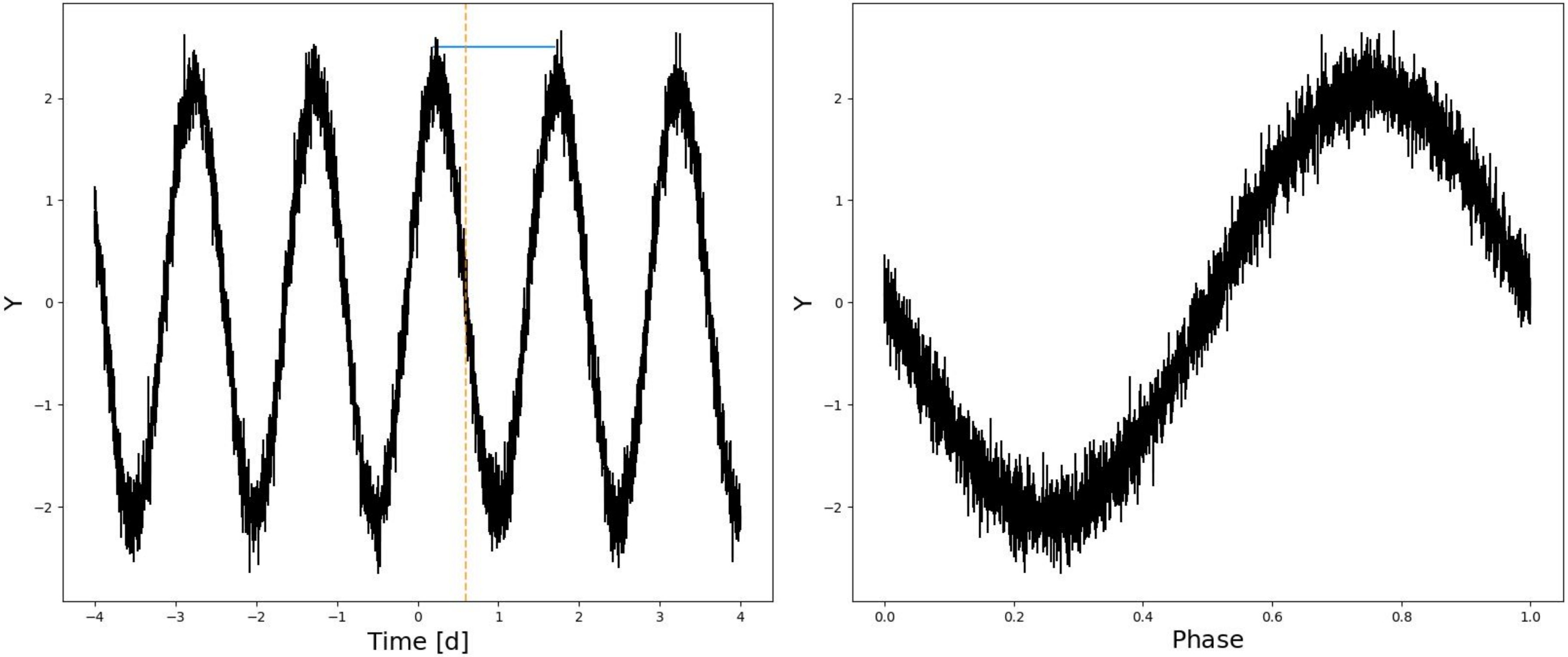
How do we describe a binary: orientation



Argument of periastron: (ω) the angle between the ascending node and periastron (ccw)

How do we describe a binary: timing

$$\Phi = \frac{(t - t_0)}{P} \bmod 1$$



How do we describe a binary: where are the stars?

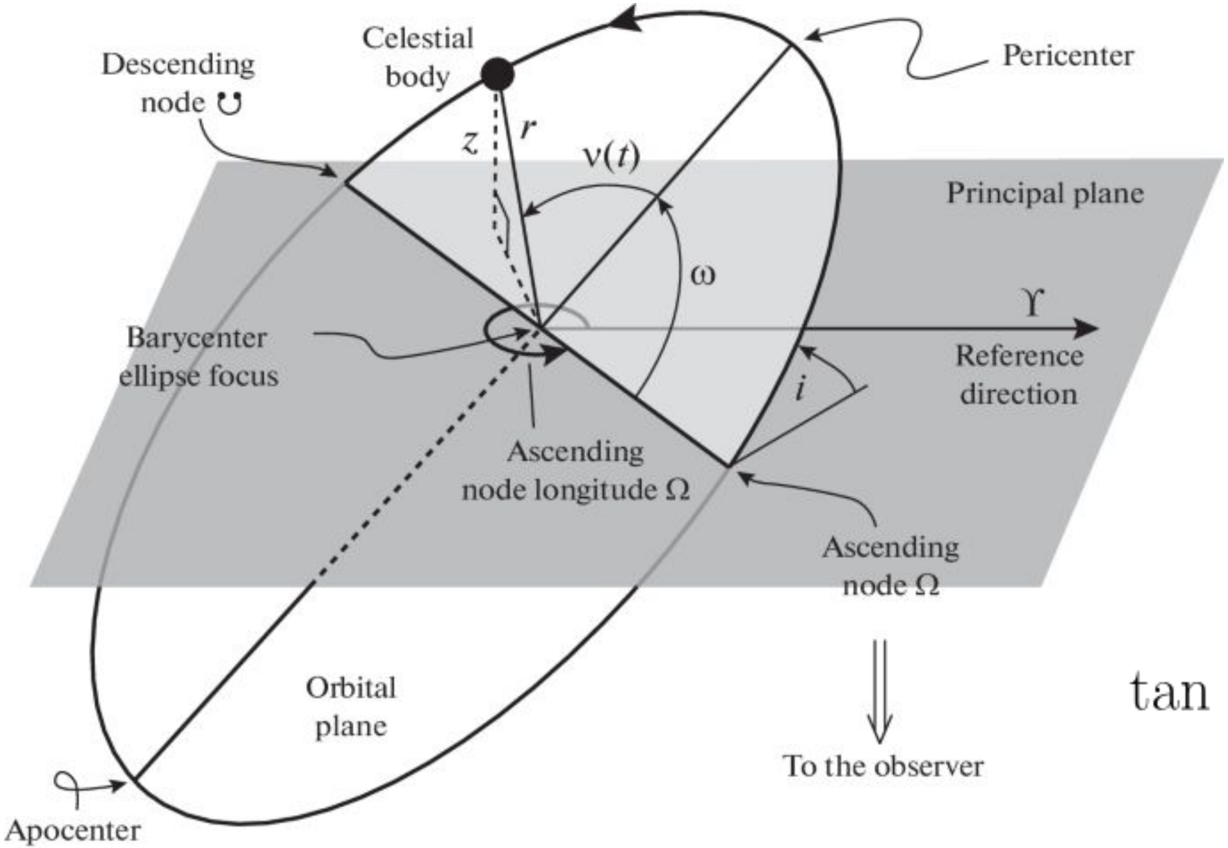
Mean anomaly: (M) how far along its orbit an object would be if it moved at a constant angular speed in a circular orbit.

$$M = \frac{2\pi}{P}(t - t_0)$$

Eccentric anomaly: (E) an angular parameter that maps the position of an object in an elliptical orbit to a corresponding position in a circular orbit of the same size

$$M = \frac{2\pi}{P}(t - t_0) = E - e\sin(E)$$

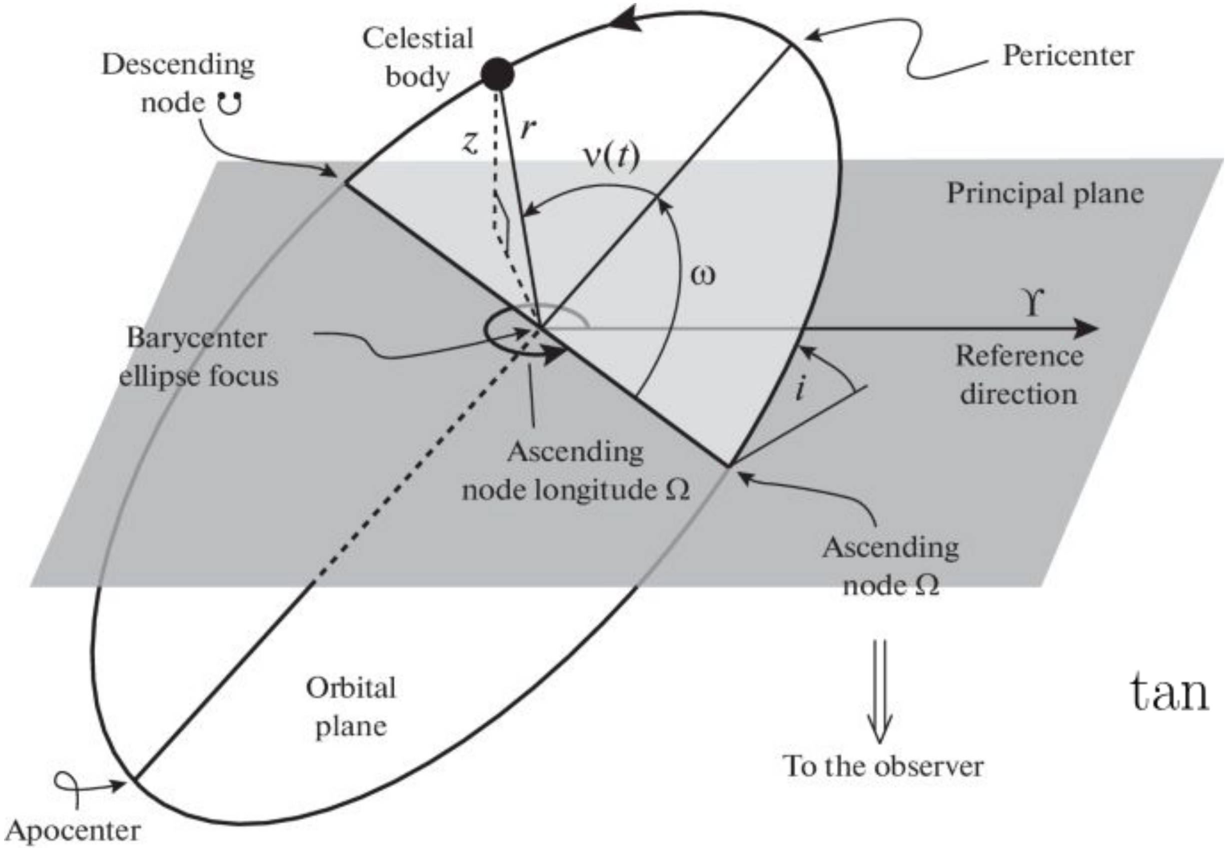
How do we describe a binary: where are the stars?



True anomaly: (ν) the angle between the direction of periastron and the current position of the object.

$$\tan\left(\frac{\nu}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

How do we describe a binary: where are the stars?



<u>True anomaly</u>: ($\boldsymbol{\nu}$) the angle between the direction of periastron and the current position of the object.

$$\tan\left(\frac{\nu}{2}\right) = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right)$$

How do we observe a binary?

How do we observe a binary?

Time series

- Photometry
- Radial velocities
- Imaging
- Astrometry
- Interferometry
- Polarimetry
- Pulsations

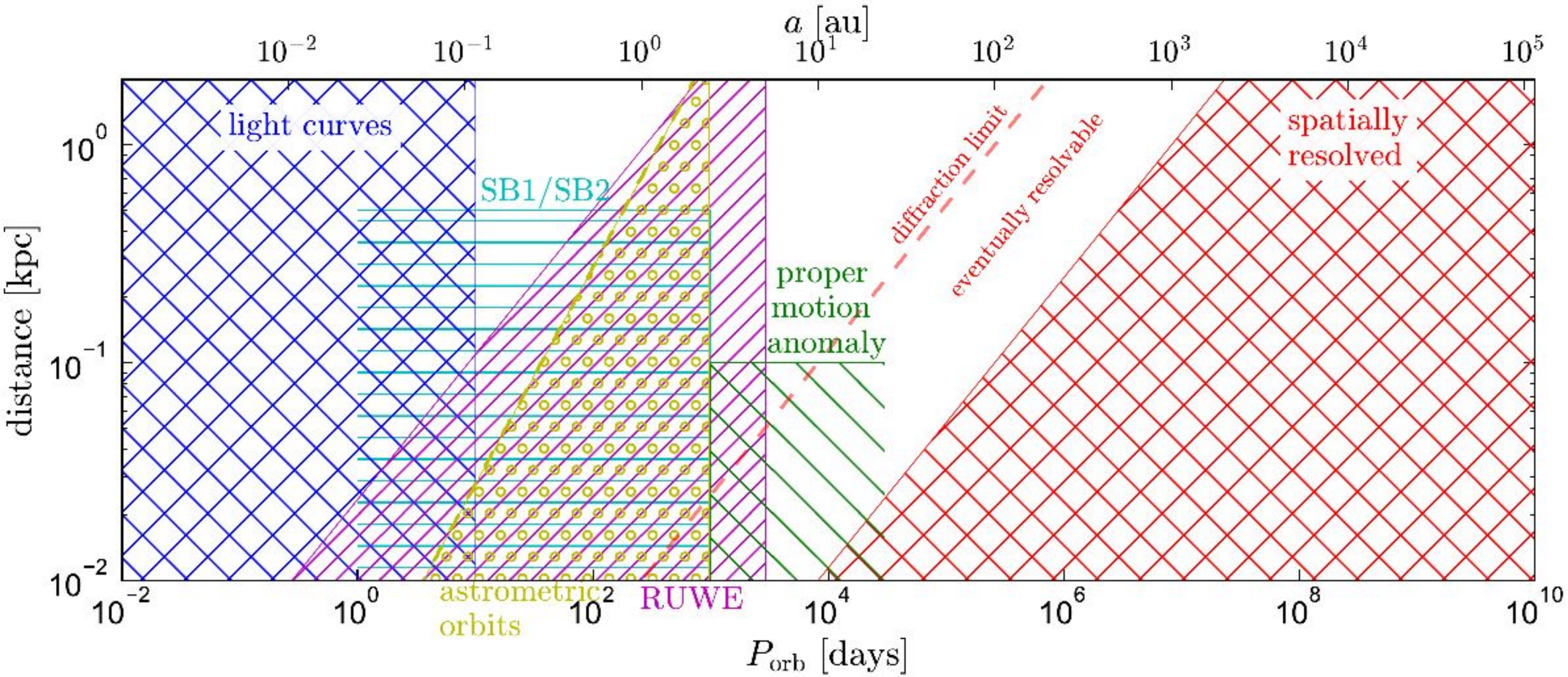
Snapshot

- Spectra
- SEDs
- Direct imaging
- Interferometry

Inference

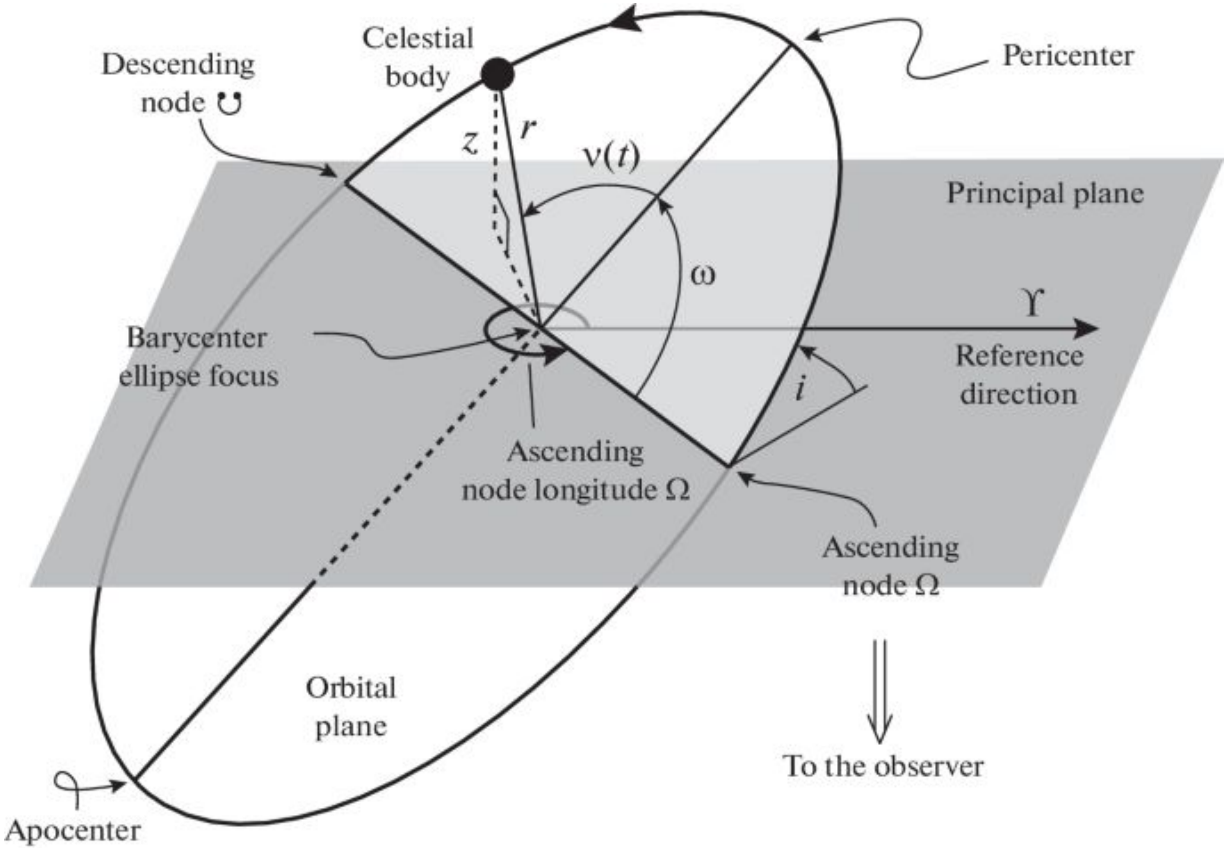
- Populations
- Colour
- REWE
- Abundances

How do we observe a binary?



El Badry, 2024

How do we observe a binary: radial velocities



$$z = r\sin(i)\sin(\nu + \omega)$$

sin(*i*) → projects r along the plane of the sky

sin(ν+ω) → projects r along the line of sight

How do we observe a binary: radial velocities

$$z = r \sin(i) \sin(\nu + \omega)$$

$$\frac{dz}{dt} = \left(r \cos(\nu + \omega) \frac{d\nu}{dt} + \frac{dr}{dt} \sin(\nu + \omega) \right) \sin(i)$$

$$v_r = \frac{dz}{dt} = \frac{2\pi a \sin i}{P\sqrt{1 - e^2}} \left(\cos(\nu + \omega) + e \cos(\omega)\right)$$

How do we observe a binary: radial velocities

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Derive the bottom equation from this

You need:

- Kepler's 2nd law
- Motion on an ellipse (r)

How do we observe a binary: radial velocities

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How do we observe a binary: radial velocities

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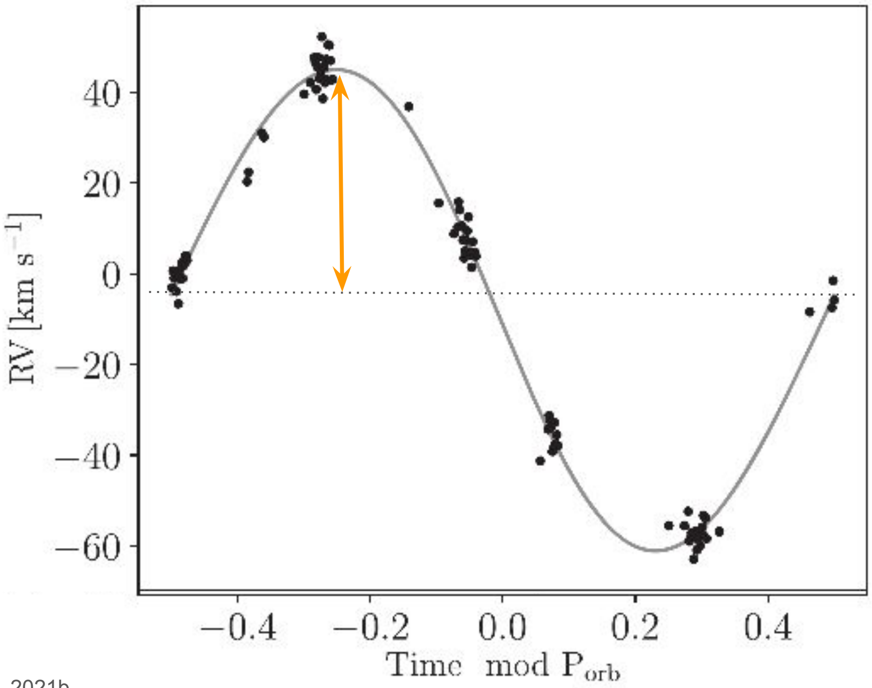


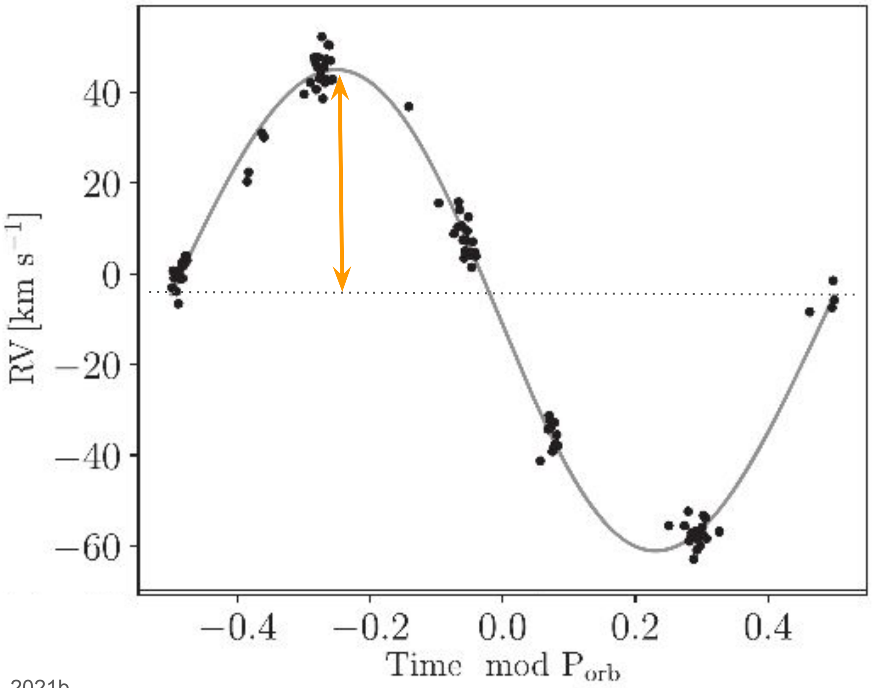
What do we actually observe?

$$K_{1,2} = \frac{2\pi a \sin i}{P\sqrt{1-e^2}}$$

Johnston et al., 2021b

How do we observe a binary: radial velocities

$$v_r = \frac{dz}{dt} = \frac{2\pi a \sin i}{P\sqrt{1-e^2}}\left(\cos\left(\nu+\omega\right) + e\cos\left(\omega\right)\right)+\gamma$$



$$K_{1,2} = \frac{2\pi a \sin i}{P\sqrt{1-e^2}}$$

$$a_{1,2}\sin i = \frac{\sqrt{1-e^2}}{2\pi}PK_{1,2}$$

Johnston et al., 2021b

How do we observe a binary: radial velocities

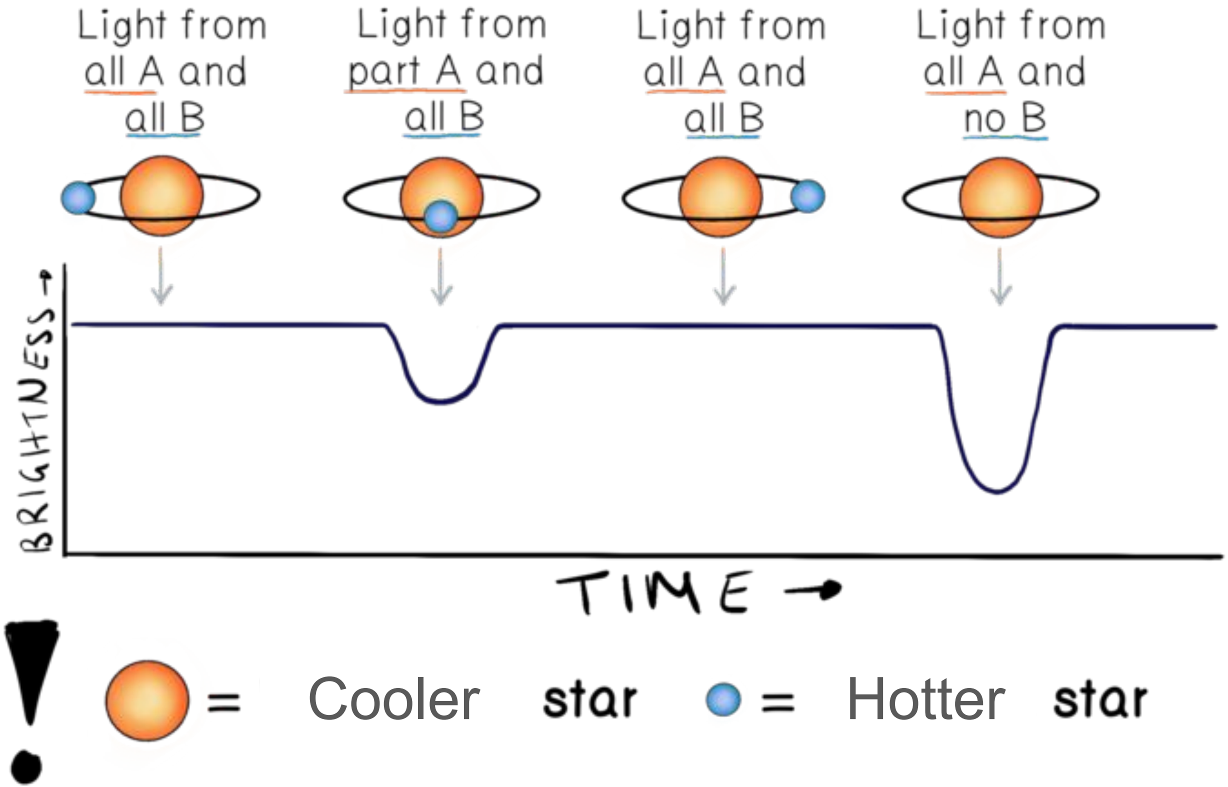
$$m_{1,2} \sin^3 i = \frac{P(1-e^2)^{3/2}}{2\pi G}(K_1 + K_2)K_{2,1}$$

Minimum masses (only
accurate when i = 90°

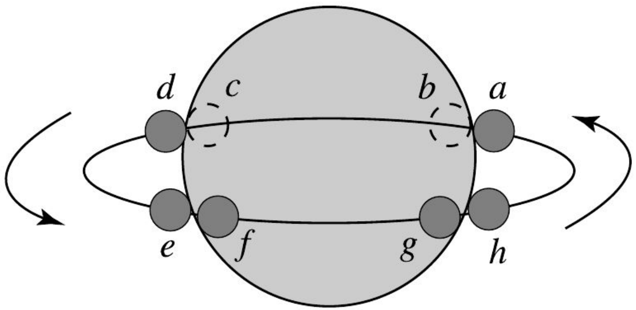
$$f(m) = \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2}$$

Single-lined binary
Can make assumptions on i and m_1

How do we observe a binary: eclipses

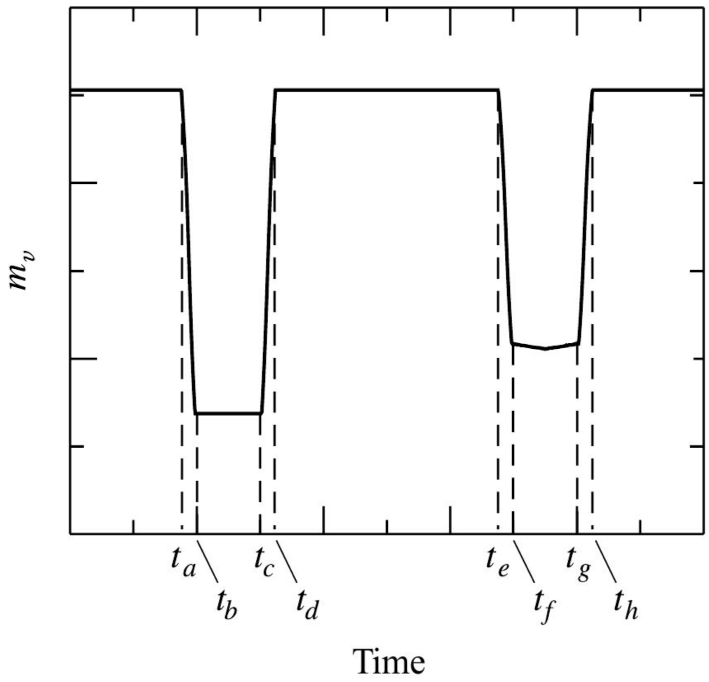


How do we observe a binary: eclipses



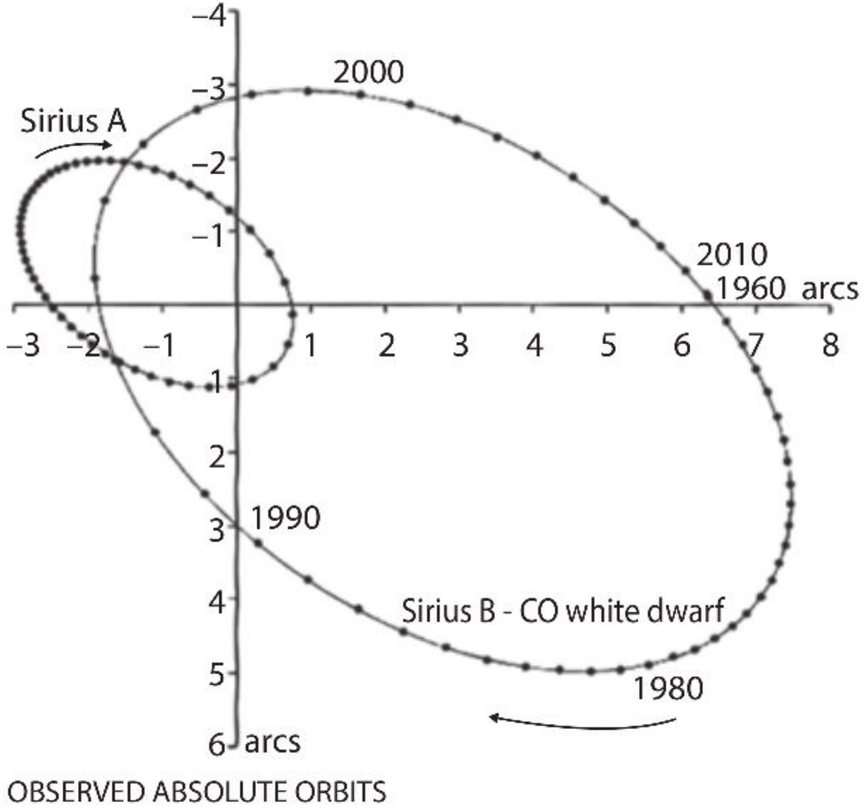
$$\Delta^2 = v^2 + w^2 = a^2\left(\sin^2\Phi + \cos^2\Phi\cos^2 i\right)$$

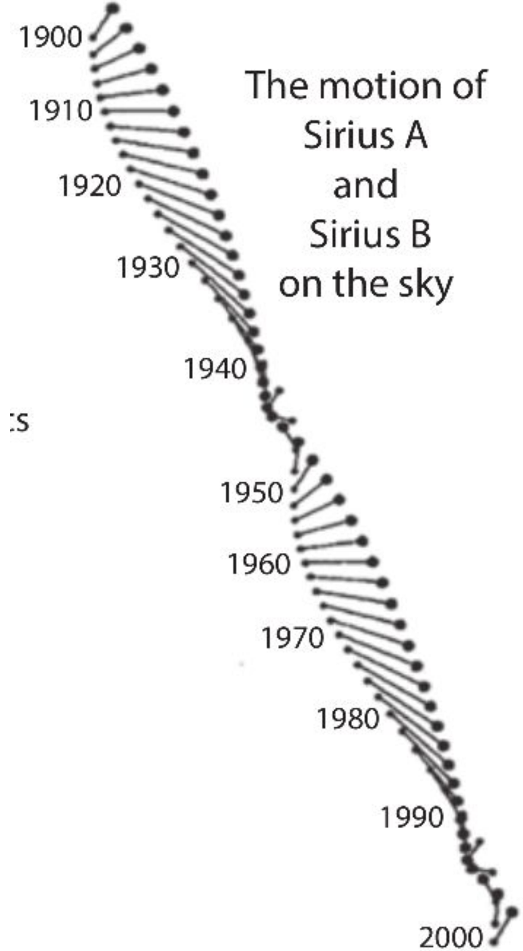
$$\begin{array}{rcccl} R_1 + R_2 & < & \Delta & & : \text{ no eclipses} \\ R_1 - R_2 & < & \Delta & < R_1 + R_2 & : \text{ partial eclipse} \\ & & \Delta & < R_1 - R_2 & : \text{ total eclipse} \end{array}$$



Carrol & Ostie 2007

How do we observe a binary: astrometry



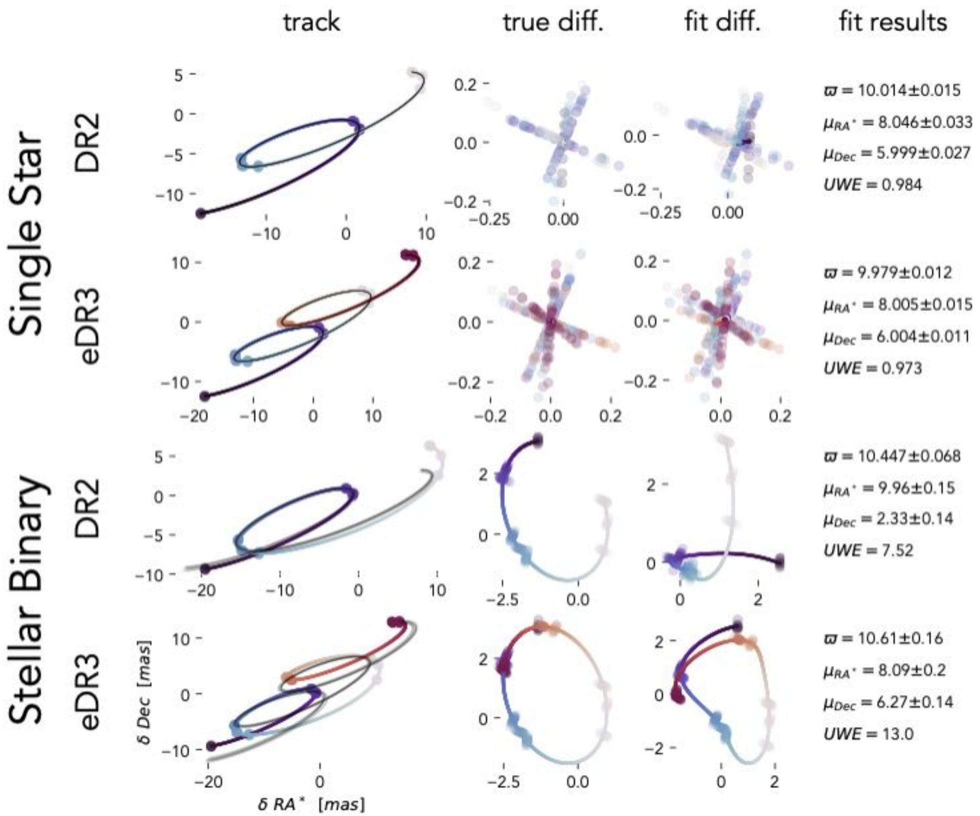


Taurus & van den Heuvel, 2022

How do we observe a binary: astrometry

4.1. RUWE

Gaia's standard 5-parameter astrometric model assumes that the motion of a source can be explained as a consequence of parallax and proper motion alone. This assumption in general does not hold for binaries. In some cases, binary orbits can be constrained by fitting more complicated astrometric models (Section 5). But binary models were only published in DR3, and only for a modest number of targets. Many binaries can nevertheless be detected – and to some extent, characterized – on the basis of a poor single-star astrometric model fit. This is illustrated in the left pane of Figure 5, which shows that the expected residuals of a single-star fit for nearby binaries can be significantly larger than the uncertainty of the astrometric measurements.

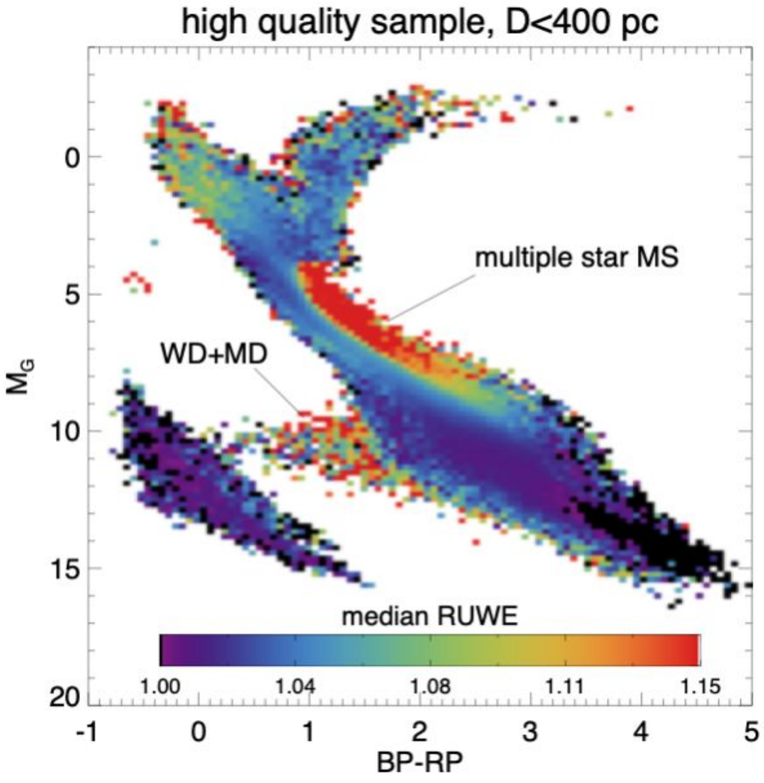


Penoyre+2022

El Badry, 2024

How do we observe a binary: astrometry

(2022). For binaries with orbital periods shorter than the *Gaia* observational baseline, the expected amplitude of the `ruwe` signal due to an unresolved binary scales approximately linearly with the angular size of the photocenter orbit (e.g. Stassun and Torres, 2021). As of *Gaia* DR3, `ruwe` is thus most sensitive to binaries that are nearby, have orbital periods of order 1000 days, and have massive but faint companions.



Belokurov+2020

How do we observe a binary: cheat sheet

Name	Symbol	Astrometric binary			Spectroscopic binary		Eclipsing binary		
		alone	with distance	with RVs	SB1	SB2	alone	with RVs (SB1)	with RVs (SB2)
Orbital parameters									
Orbital period	P	*	*	*	*	*	*	*	*
Orbital eccentricity	e	*	*	*	*	*	*	*	*
Argument of periastron	ω	*	*	*	*	*	*	*	*
Longitude of ascending node	Ω	*	*	*					
Projected semimajor axis	$a \sin i$		*	*		*			*
True semimajor axis	a (au)		*	*					*
Orbital inclination	i	*	*	*			*	*	*
Distance	d			*					*
Spectroscopic parameters									
Velocity amplitude of star 1	K_1			*	*	*		*	*
Velocity amplitude of star 2	K_2			*		*			*
Systemic velocity	V_γ			*	*	*		*	*
Mass function	$f(M)$				*	*		*	*
Mass ratio	$q = M_2/M_1$			*		*			*
Mass sum	$M_1 + M_2$		*	*		*			*
Minimum masses	$M_{1,2} \sin^3 i$			*		*			*
Mass of primary star	M_1			*					*
Mass of secondary star	M_2			*					*

Southworth, 2020

How do we observe a binary: cheat sheet

		Astrometric binary			Spectroscopic binary		Eclipsing binary		
Name	Symbol	alone	with distance	with RVs	SB1	SB2	alone	with RVs (SB1)	with RVs (SB2)
Size parameters									
Fractional radii	r_1 and r_2						*	*	*
Radius of primary star	R_1								*
Radius of secondary star	R_2								*
Surface gravity of primary	$\log g_1$								*
Surface gravity of secondary	$\log g_2$							*	*
Density of primary star	ρ_1								*
Density of secondary star	ρ_2								*
Radiative parameters									
Temperature of primary star	$T_{\mathrm{eff},1}$	*	*	*	*	*		*	*
Temperature of secondary star	$T_{\mathrm{eff},2}$	*	*	*		*			*
Luminosity of primary star	L_1			*					*
Luminosity of secondary star	L_2			*					*

Southworth, 2020