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**Course:** Calc 1 11:30 AM / Internet  
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**Assignment:** 6.1 Volumes Using Cross-Sections

1. A solid lies between planes perpendicular to the  $x$ -axis at  $x = 0$  and  $x = 9$ . The cross-sections perpendicular to the axis on the interval  $0 \leq x \leq 9$  are squares with diagonals that run from the parabola  $y = -2\sqrt{x}$  to the parabola  $y = 2\sqrt{x}$ . Find the volume of the solid.

The volume of the solid is  cubic units.  
 (Type an exact answer, using  $\pi$  as needed.)

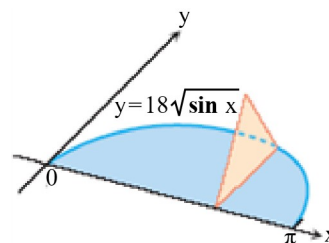
2. A solid lies between planes perpendicular to the  $x$ -axis at  $x = -4$  and  $x = 4$ . The cross-sections perpendicular to the  $x$ -axis between these planes are squares whose bases run from the semicircle  $y = -\sqrt{16 - x^2}$  to the semicircle  $y = \sqrt{16 - x^2}$ . Find the volume of the solid.

The volume of the solid is  cubic units.  
 (Simplify your answer.)

3. Find the volume of the following solids.

The base of a solid is the region between the curve  $y = 18\sqrt{\sin x}$  and the interval  $[0, \pi]$  on the  $x$ -axis. The cross-sections perpendicular to the  $x$ -axis are

- a. equilateral triangles with bases running from the  $x$ -axis to the curve as shown in the figure.  
 b. squares with bases running from the  $x$ -axis to the curve.



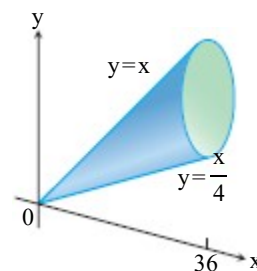
a.  $V =$   (Type an exact answer, using radicals as needed.)

b.  $V =$   (Type an exact answer, using radicals as needed.)

4. A solid lies between planes perpendicular to the  $y$ -axis at  $y = 0$  and  $y = 2$ . The cross-sections perpendicular to the  $y$ -axis are circular disks with diameters running from the  $y$ -axis to the parabola  $x = \sqrt{14}y^2$ . Find the volume of the solid.

The volume of the solid is  cubic units.  
 (Type an exact answer, using  $\pi$  as needed.)

5. A solid lies between two planes perpendicular to the  $x$ -axis at  $x = 0$  and  $x = 36$ . The cross-sections by planes perpendicular to the  $x$ -axis are circular disks whose diameters run from the line  $y = \frac{x}{4}$  to the line  $y = x$  as shown in the accompanying figure. Explain why the solid has the same volume as a right circular cone with base radius 13.5 and height 36.



The radius of a circular cross section of the solid at any value of  $x$  is  $\frac{3}{8}x$ . The height of the solid is 36.

Locate the right circular cone with base radius 13.5 and height 36 so that its vertex is at the origin and its height is along the  $x$ -axis. This cone is the surface of revolution of the function  $y = \frac{3}{8}x$  about the  $x$ -axis.

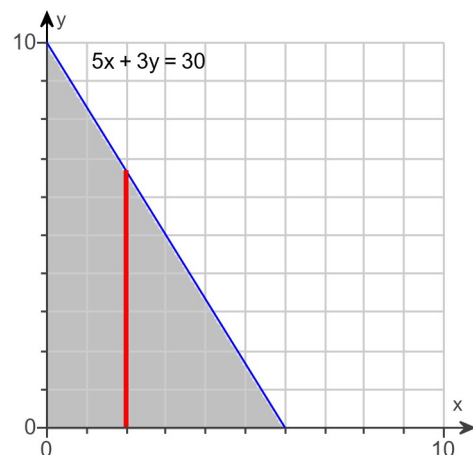
(Type an equation.)

The radius of the cross-section of this right circular cone at any value of  $x$  is  $\frac{3}{8}x$ . The height of it is 36.

Apply Cavalieri's principle to state the conclusion. Choose the correct answer below.

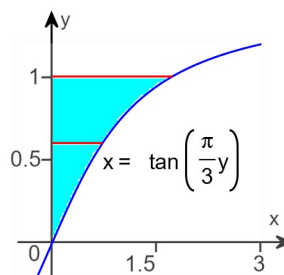
- ☒ A. Since the radii of the cross-sections of the solids are equal, the cross-sectional areas are equal. Since the solids have equal cross-sectional areas and equal heights, the solids have the same volume by Cavalieri's principle.
- ☐ B. Since the base radii of the solids are equal, the base areas are equal. Since the base areas and the heights of the solids are equal, the solids have the same volume by Cavalieri's principle.
- ☐ C. Since the base radii of the solids are equal, the areas of the bases are equal. The solids therefore have the same volume by Cavalieri's principle.
- ☐ D. Since the radii of the cross-sections of the solids are equal, the cross-sectional areas are equal. Since the solids have equal cross-sectional areas, the solids have the same volume by Cavalieri's principle.

6. Find the volume of the solid generated by revolving the shaded region about the  $x$ -axis.



The volume of the solid is  $200\pi$  cubic units.  
(Type an exact answer, using  $\pi$  as needed.)

7. Find the volume of the solid generated by revolving the shaded region about the y-axis.



The volume of the solid generated by revolving the shaded region about the y-axis is  $3\left(\frac{3}{2}\right) - \pi$ .  
(Type an exact answer, using  $\pi$  as needed.)

8. Find the volume of the solid generated by revolving the region bounded by  $y = 4x^2$ ,  $y = 0$ , and  $x = 3$  about the x-axis.

The volume of the solid generated by revolving the region bounded by  $y = 4x^2$ ,  $y = 0$ , and  $x = 3$  about the x-axis is

$$\frac{3888\pi}{5} \text{ cubic units.}$$

(Type an exact answer, using  $\pi$  as needed.)

9. Find the volume of the solid generated by revolving the region bounded by the given line and curve about the x-axis.

$$y = \sqrt{81 - x^2}, y = 0$$

The volume of the solid is  $972\pi$ . (Type an exact answer, using  $\pi$  as needed.)

10. Find the volume of the solid generated by revolving the region bounded by  $y = 4\sqrt{\sin x}$ ,  $y = 0$ , and  $x_1 = \frac{\pi}{4}$  and  $x_2 = \frac{5\pi}{6}$  about the x-axis.

The volume of the solid generated by revolving the region bounded by  $y = 4\sqrt{\sin x}$ ,  $y = 0$ , and  $x_1 = \frac{\pi}{4}$  and  $x_2 = \frac{5\pi}{6}$  about the x-axis is  $79.07$  cubic units.  
(Round to the nearest hundredth.)

11. Find the volume of the solid generated by revolving the region about the given line.

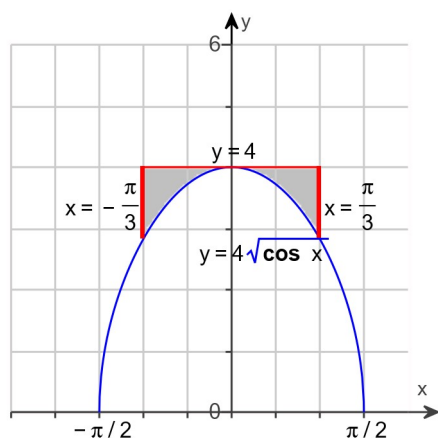
The region in the first quadrant bounded above by the line  $y = \sqrt{2}$ , below by the curve  $y = \sec x \tan x$ , and on the left by the y-axis, about the line  $y = \sqrt{2}$ .

The volume of the solid generated is  $\pi\left(\frac{\pi}{2} + 2\sqrt{2} - \frac{11}{3}\right)$  cubic units.

(Simplify your answer. Type an exact answer, using  $\pi$  and radicals as needed.)

12.

Use the washer method to find the volume of the solid generated by revolving the shaded region about the x-axis.



The volume of the solid generated by revolving the shaded region about the x-axis is 18.21 cubic units.  
(Round to the nearest hundredth as needed.)

13. Find the volume of the solid generated by revolving the region bounded by the given curve and lines about the x-axis.

$$y = 5x, \quad y = 5, \quad x = 0$$

$$V = \frac{50\pi}{3}$$

(Type an exact answer, using  $\pi$  as needed.)

14. Find the volume of the solid generated by revolving the region bounded by the graphs of  $y = 2x^2 + 1$  and  $y = 2x + 10$  about the x-axis.

The volume of the solid generated by revolving the region bounded by the graphs of  $y = 2x^2 + 1$  and  $y = 2x + 10$  about the x-axis is 1248.88 cubic units.  
(Round to the nearest hundredth.)

15.

Find the volume of the solid generated by revolving the region bounded by the graphs of  $y = \csc x$  and  $y = \frac{2\sqrt{3}}{3}$  about the x-axis where  $0 \leq x \leq \pi$ .

The volume of the solid generated by revolving the region bounded by the graphs of  $y = \csc x$  and  $y = \frac{2\sqrt{3}}{3}$  about the x-axis is .76.  
(Round to the nearest hundredth.)

16. Find the volume of the solid generated by revolving the region enclosed by the triangle with vertices (2,3), (2,5), and (6,5) about the y-axis.

The volume of the solid generated by revolving the region enclosed by the triangle with vertices (2,3), (2,5) and (6,5) about the y-axis is  $\frac{80\pi}{3}$  cubic units.  
(Type an exact answer, using  $\pi$  as needed.)

17. Find the volume of the solid generated by revolving the region bounded by  $y = 4\sqrt{x}$  and the lines  $y = 4\sqrt{13}$  and  $x = 0$  about
- a. the x-axis.    b. the y-axis.    c. the line  $y = 4\sqrt{13}$ .    d. the line  $x = 13$ .

a. The volume of the solid generated by revolving the region bounded by  $y = 4\sqrt{x}$  and the lines  $y = 4\sqrt{13}$  and  $x = 0$  about the x-axis is  cubic units.

(Type an exact answer, using  $\pi$  and radicals as needed, or type the answer as a decimal rounded to the nearest tenth.)

b. The volume of the solid generated by revolving the region bounded by  $y = 4\sqrt{x}$  and the lines  $y = 4\sqrt{13}$  and  $x = 0$  about the y-axis is  cubic units.

(Type an exact answer, using  $\pi$  and radicals as needed, or type the answer as a decimal rounded to the nearest tenth.)

c. The volume of the solid generated by revolving the region bounded by  $y = 4\sqrt{x}$  and the lines  $y = 4\sqrt{13}$  and  $x = 0$  about the line  $y = 4\sqrt{13}$  is  cubic units.

(Type an exact answer, using  $\pi$  and radicals as needed, or type the answer as a decimal rounded to the nearest tenth.)

d. The volume of the solid generated by revolving the region bounded by  $y = 4\sqrt{x}$  and the lines  $y = 4\sqrt{13}$  and  $x = 0$  about the line  $x = 13$  is  cubic units.

(Type an exact answer, using  $\pi$  and radicals as needed, or type the answer as a decimal rounded to the nearest tenth.)

18. Find the volume of the solid generated by revolving the region bounded by the parabola  $y = -\frac{x^2}{16}$  and the line  $y = -1$  about the following lines.

a. The line  $y = -1$

b. The line  $y = -2$

c. The line  $y = 1$

a. The volume of the given solid is .

(Type an exact answer, using  $\pi$  as needed.)

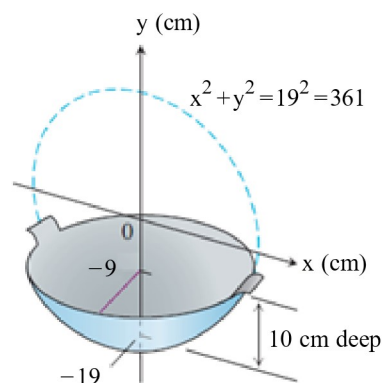
b. The volume of the given solid is .

(Type an exact answer, using  $\pi$  as needed.)

c. The volume of the given solid is .

(Type an exact answer, using  $\pi$  as needed.)

19. A wok frying pan is shaped like a spherical bowl with handles. A bit of experimentation at home persuades a cook that a wok holds about 3 L if it is 10 cm deep and the sphere has a radius of 19 cm. To be sure, picture the wok as a solid of revolution, as shown in the figure, and calculate its volume with an integral. What is the real volume of the wok? (1L = 1000cm<sup>3</sup>)



The volume of the wok is  cm<sup>3</sup>. (Round to the nearest cubic centimeter as needed.)