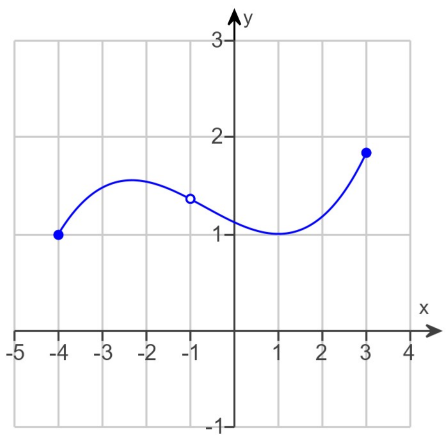
Student: Cole Lamers **Date:** 09/01/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.5 Continuity

Say whether the function graph below is continuous on $[-4,3]$. If not, where does it fail to be continuous?



Recall that a function is continuous over a closed interval [a,b] if it is right-continuous at a, left-continuous at b, and continuous at all interior points of the interval.

In this case, the function is defined on $[-4,3]$. Notice that $\lim_{x \to -4^+} f(x) = 1$, so the function is right-continuous at $x = -4$.

Since $\lim_{x \to 3^-} f(x) = 1.84$, the function is left-continuous at $x = 3$.

An open circle occurs on the graph, so the function is not continuous at all interior points of $[-4,3]$.

The open circle occurs at an x-value of $x = -1$.

Therefore, the function fails to be continuous at $x = -1$.

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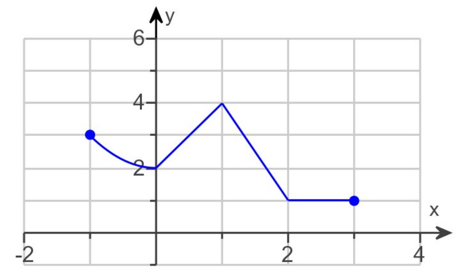
State whether the function graphed is continuous on $[-1,3]$. If not, where does it fail to be continuous and why?



A function f(x) is continuous at $x = c$ if and only if it meets the following three conditions.

1. f(c) exists
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

An interior point is discontinuous if $\lim_{x \to c} f(x) \neq f(c)$. There are four different types of discontinuities. A removable discontinuity occurs when the limit exists as $x \to c$ and the discontinuity can be removed by setting f(c) equal to this limit. A jump discontinuity occurs when the one-sided limits exist but have different values.

An infinity discontinuity occurs when the limit as $x \to c$ goes to $\pm\infty$. An oscillating discontinuity occurs when the function oscillates too much around $x = c$ for a limit to exist.

A function is continuous on an interval if and only if it is continuous at every point in the interval. The function is continuous at all points on the interval $[-1,3]$.

The function shown does not contain any discontinuities between $x = -1$ and $x = 3$ inclusive, and is therefore continuous on the interval $[-1,3]$.

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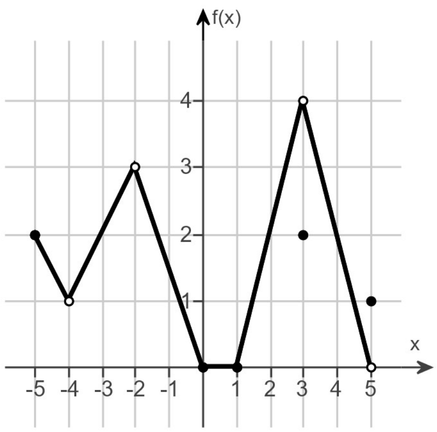
Use the graph to answer the questions about existence, limits, and continuity.



The existence of $f(-1)$ can be determined either by examining the algebraic definition of the function to determine if -1 is in the domain or by examining the graph to determine if the point $(-1, f(-1))$ is on the graph.

Either (or both) of these approaches shows that $f(-1)$ does exist.

$$f(-1) = \frac{3}{2}$$

Examine the behavior of the function near the interior point -1 on the graph. As x gets closer and closer to -1 from either side, f(x) gets closer and closer to $\frac{3}{2}$.

$$\lim_{x \to -1} f(x) = \frac{3}{2}$$

A function $y = f(x)$ is continuous at an interior point c of its domain if $\lim\limits_{x \to c} f(x) = f(c)$.

A function $y = f(x)$ is continuous at a left endpoint a or a right endpoint b of its domain if $\lim\limits_{x \to a^+} f(x) = f(a)$ or $\lim\limits_{x \to b^-} f(x) = f(b)$.

Compare $f(-1)$ to the limit as x approaches -1 from either side. Notice that they are equal.

$$\lim_{x \to -1} f(x) = f(-1) = \frac{3}{2}$$

Thus, the function is continuous at $x = -1$.

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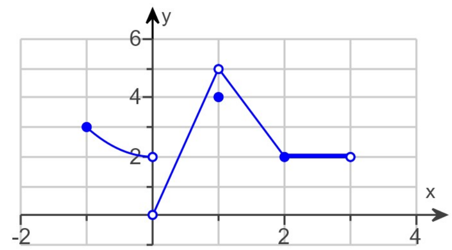
Use the function and the accompanying figure to answer the following questions.

$$f(x) = \begin{cases} x^2 - 2, & -1 \le x < 0 \\ 5x, & 0 < x < 1 \\ 4, & x = 1 \\ -3x + 8, & 1 < x < 2 \\ 2, & 2 \le x < 3 \end{cases}$$



a. Is f defined at x = 2?

Look at the piecewise definition of f(x). At x = 2 the function does have a value.

According to the last part of the piecewise definition of the function, f(2) is 2.

b. Is f continuous at x = 2?

A function is continuous at an interior point c of its domain if the following statement is true.

$\lim_{x \to c} f(x) = f(c)$

From the graph, it can be determined that the function approaches 2 from both sides as x approaches 2. Therefore, $\lim_{x \to 2} f(x) = f(2)$ is a true statement.

Therefore, f is continuous at x = 2.

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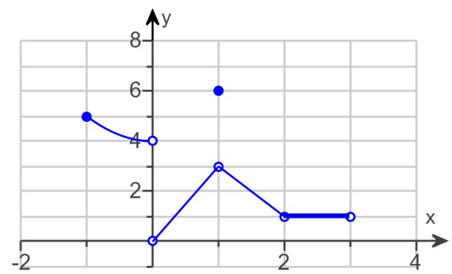
Assignment: 2.5 Continuity

What value should be assigned to f(2) to make the extended function continuous at $x = 2$?

$$f(x) = \begin{cases} x^2 - 4, & -1 \le x < 0 \\ 3x, & 0 < x < 1 \\ 6, & x = 1 \\ -2x + 5, & 1 < x < 2 \\ 1, & 2 < x < 3 \end{cases}$$



The continuous extension of f(x) at $x = c$ makes the function continuous at that point. The continuous extension of f to $x = c$ is defined by the new function F(x) below.

$$F(x) = \begin{cases} f(x), & \text{if } x \text{ is in the domain of } f \\ L, & x = c \end{cases}$$

where f(c) is not defined, but $\lim_{x \to c} f(x) = L$ exists.

Let $c = 2$. Start by finding $\lim_{x \to 2} f(x)$. The limit is the value that f(x) approaches as x approaches 2.

$$\begin{aligned} \lim_{x \to 2} f(x) &= L \\ &= 1 \end{aligned}$$

The value of L that should be assigned to f(2) to make the extended function continuous at $x = 2$ is shown below.

$f(2) = 1$

The continuous extension is shown below.

$$F(x) = \begin{cases} x^2 - 4, & -1 \le x < 0 \\ 3x, & 0 < x < 1 \\ 6, & x = 1 \\ -2x + 5, & 1 < x < 2 \\ 1, & 2 < x < 3 \\ 1, & x = 2 \end{cases}$$

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Determine the point(s) at which the given function f(x) is continuous.

$$f(x) = \frac{4}{x-5} - 6x$$

The given function is the difference of two functions, $g(x) = \frac{4}{x-5}$ and $h(x) = 6x$.

The function h(x) is the product of the constant function and the identity function, both of which are continuous over $(-\infty,\infty)$.

The $\lim_{x \to c} \frac{4}{x-5}$ exists and is equal to and $f(c) = \frac{4}{c-5}$ on $(-\infty,\infty)$ except at the point at which f(c) is undefined. The x-coordinate of this point is 5.

A function y = f(x) is continuous at an interior point c of its domain if $\lim_{x \to c} f(x) = f(c)$ and is continuous at a left endpoint a or a right endpoint b of its domain if $\lim_{x \to a^+} f(x) = f(a)$ or $\lim_{x \to b^-} f(x) = f(b)$.

So, $g(x) = \frac{4}{x-5}$ is continuous on $(-\infty,\infty)$ except at x = 5.

Thus, f(x) is continuous on $(-\infty,\infty)$ except at x = 5. In interval notation, this is $(-\infty,5) \cup (5,\infty)$.

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At what points is the function $y = \frac{\sin x}{28x}$ continuous?

Express $\frac{\sin x}{28x}$ as a quotient of continuous functions and use the properties of continuous functions to find the values of x for which the function is continuous. Let $f(x) = \sin x$ and $g(x) = 28x$.

Trigonometric functions are continuous wherever they are defined. Because **sin** x is defined for all x, it is continuous for all values of x. Polynomial functions are continuous for all x. Thus, g(x) is continuous over all x.

If the functions f and g are continuous at $x = c$, then the quotient $\frac{f(x)}{g(x)}$ is also continuous, provided that $g(x) \neq 0$.

Find all the values of x such that $28x = 0$.

$x = 0$

Thus, the function $\frac{\sin x}{28x}$ is undefined when $x = 0$, and the function is continuous for all x in its domain except for $x = 0$. In interval notation, the function is continuous at the set of x-values $(-\infty,0) \cup (0,\infty)$.

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Determine the point(s) at which the given function f(x) is continuous.

$f(x) = 10 \textbf{ csc } (14x)$

Use the following definition to rewrite the given function as a quotient of continuous functions.

$$\textbf{csc}\,(x) = \frac{1}{\textbf{sin}\,(x)}$$

Use this definition to find an equivalent expression for 10 **csc** (14x) using a sine function.

$$10\,\textbf{csc}\,(14x) = \frac{10}{\textbf{sin}\,(14x)}$$

Now use the properties of continuous functions to find the values of x for which the function is continuous. Let f(x) = 10 and g(x) = **sin** (14x).

Constant functions are continuous everywhere. Thus, f(x) is continuous over all x. Trigonometric functions are continuous wherever they are defined. Because **sin** (14x) is defined for all x, it is continuous for all values of x.

If the functions f and g are continuous at x = c, then the quotient $\frac{f(x)}{g(x)}$ is also continuous, provided that $g(x) \neq 0$.

Find all the values of x such that **sin** (14x) = 0.

To do this, first find all values for which **sin** (x) = 0.

$x = n\pi$

As a result, **sin** (14x) = 0 only when $14x = n\pi$. To find all the values of x such that **sin** (14x) = 0, solve this equation for x.

$$x = \frac{n\pi}{14}$$

Thus, the function 10 **csc** (14x) is undefined when $x = \frac{n\pi}{14}$.

The function f(x) = 10 **csc** (14x) is continuous on $(-\infty,\infty)$ except for $\frac{n\pi}{14}$, where n is an integer.

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Determine the point(s) at which the given function f(x) is continuous.

$f(x) = \sqrt{4x + 12}$

A function $y = f(x)$ is continuous at an interior point c of its domain if $\lim\limits_{x \to c} f(x) = f(c)$. The function is continuous at a left endpoint a or a right endpoint b of its domain if $\lim\limits_{x \to a^+} f(x) = f(a)$ or $\lim\limits_{x \to b^-} f(x) = f(b)$, respectively.

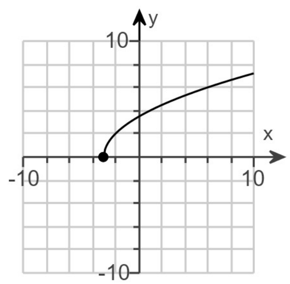
First find the domain of $\sqrt{4x + 12}$.

The square root function is defined for arguments ≥ 0.

For $\sqrt{4x + 12}$ to be defined, then, $4x + 12 \geq 0$ or $x \geq -3$.

$\sqrt{4x + 12}$ is defined on $[-3, \infty)$.

A graph of the function $f(x) = \sqrt{4x + 12}$ shows that $\lim\limits_{x \to c} f(x)$ exists and is equal to f(c) and $-3 \leq x < \infty$.



Thus, in interval notation, f(x) is continuous at the set of x-values $[-3, \infty)$.

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Determine the limit as x approaches the given x-coordinate and the continuity of the function at that x-coordinate.

$$\lim_{x \to 7\pi/6} \cos(12x - \cos(3x))$$

Express $\cos(12x - \cos(3x))$ as the composite of two functions f(x) and g(x).

Let $f(x) = 12x - \cos(3x)$ and $g(x) = \cos(x)$.

If f is continuous at c and g is continuous at f(c), then the composite $g \circ f$ is continuous at c.

Notice that f(x) is the difference between two terms, 12x and $\cos(3x)$. Therefore, their difference, f(x), is continuous at $x = \frac{7\pi}{6}$ by the difference property of continuous functions.

Trigonometric functions are continuous wherever they are defined. Because the function $g(x) = \cos(x)$ is defined at $x = f\left(\frac{7\pi}{6}\right)$, it is continuous at $x = f\left(\frac{7\pi}{6}\right)$.

Since f(x) is continuous at $x = \frac{7\pi}{6}$ and g(x) is continuous at $x = f\left(\frac{7\pi}{6}\right)$, their composite $g(f(x)) = \cos(12x - \cos(3x))$, is continuous at $x = \frac{7\pi}{6}$.

Use this fact to determine the limit as x approaches $\frac{7\pi}{6}$.

A function f(x) is continuous at x = c if and only if it meets the following three conditions.

1. f(c) exists
2. $\lim_{x \to c} f(x)$ exists
3. $\lim_{x \to c} f(x) = f(c)$

Since the given function is continuous, use the third condition to rewrite its limit.

$$\lim_{x \to 7\pi/6} \cos(12x - \cos(3x)) = f\left(\frac{7\pi}{6}\right)$$

Calculate this value. Begin by simplifying the terms in parentheses.

$$\begin{aligned} f\left(\frac{7\pi}{6}\right) &= \cos\left(12\left(\frac{7\pi}{6}\right) - \cos\left(3\left(\frac{7\pi}{6}\right)\right)\right) \\ &= \cos(14\pi - 0) \end{aligned}$$

Then simplify the result.

$$\cos(14\pi - 0) = 1$$

Therefore, $\lim_{x \to 7\pi/6} \cos(12x - \cos(3x)) = 1$ and $\cos(12x - \cos(3x))$ is continuous at $x = \frac{7\pi}{6}$.

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Find the following limit. Is the function continuous at the point being approached?

$$\lim_{t\to 0} \sin\left(\frac{\pi}{\sqrt{15-6\cos 4t}}\right)$$

To find the limit of a given function, use the limits of continuous functions theorem which states that if g is continuous at the point b and $\lim_{x\to c} f(x) = b$, then $\lim_{x\to c} g(f(x)) = g(b) = g\left(\lim_{x\to c} f(x)\right)$.

Apply the limit to the given function.

$$\lim_{t\to 0} \sin\left(\frac{\pi}{\sqrt{15-6\cos 4t}}\right) = \sin \lim_{t\to 0}\left(\frac{\pi}{\sqrt{15-6\cos 4t}}\right)$$
$$= \sin\left(\frac{\pi}{3}\right)$$

Simplify $\sin\left(\frac{\pi}{3}\right)$.

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Therefore, $\lim_{t\to 0} \sin\left(\frac{\pi}{\sqrt{15-6\cos 4t}}\right) = \frac{\sqrt{3}}{2}$.

Note that a function f(x) is continuous at an interior point x = c of its domain if and only if it meets the following three conditions.

1. The function value f(c) exists, when c lies in the domain of f.
2. The $\lim_{x\to c} f(x)$ exists, that is, f has a limit as $x\to c$.
3. The limit equals the function value, that is, $\lim_{x\to c} f(x) = f(c)$.

Now substitute 0 for t in the given function.

$$\sin\left(\frac{\pi}{\sqrt{15-6\cos 4t}}\right) = \sin\left(\frac{\pi}{\sqrt{15-6\cos 4(0)}}\right)$$
$$= \sin\left(\frac{\pi}{3}\right)$$

Simplify $\sin\left(\frac{\pi}{3}\right)$.

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Since the function is defined at t = 0, the limit exists at that point, and it matches with the value of the function at that point, the function $\lim_{t\to c} f(t)$ is continuous at t = 0.

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For what value of a is the following function continuous at every x?

$$f(x) = \begin{cases} x^2 - 72, & x < 18 \\ 2ax, & x \geq 18 \end{cases}$$

In order for the function to be continuous for all values of x, the function must be continuous at the boundary between the two pieces of the function. For the function to be continuous at $x = 18$, the limit of f as x approaches 18 must equal f(18).

What is the limit as x approaches 18 from the left of $x^2 - 72$?

$$\lim_{x \to 18^-} \left(x^2 - 72\right) = 252$$ (Simplify your answer.)

Set this limit equal to f(18) and solve for a.

$$f(18) = 252$$
$$2a(18) = 252$$
$$a = 7$$

(Simplify your answer.)

Therefore, the function below is continuous at all values of x.

$$f(x) = \begin{cases} x^2 - 72, & x < 18 \\ 14x, & x \geq 18 \end{cases}$$