Student: Cole Lamers **Date:** 09/27/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.7 Antiderivatives (Set 2)

Find the most general antiderivative or indefinite integral. Check your answer by differentiation.

$$\int \frac{1+\cos 3t}{4}\,dt$$

To find the antiderivative, begin by dividing each term in the numerator by 4.

$$\int \frac{1+\cos 3t}{4}\,dt = \int \left(\frac{1}{4}+\frac{\cos 3t}{4}\right) dt$$

The sum rule for antiderivatives states that a sum of functions may be antidifferentiated term by term as shown by the following formula.

$$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

Begin by writing the expression according to the sum rule.

$$\int \left(\frac{1}{4}+\frac{\cos 3t}{4}\right) dt = \int \frac{1}{4}\,dt + \int \frac{\cos 3t}{4}\,dt$$

The constant multiple rule states that a constant multiple k may be moved through the integral sign and as by the following formula.

$$\int k\,f(x)\,dx = k\int f(x)\,dx$$

To find $\int \frac{1}{4}\,dt$, first use the constant multiple rule to rewrite the expression.

$$\int \frac{1}{4}\,dt = \frac{1}{4}\int dt$$

To find $\frac{1}{4}\int dt$, use the formula $\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$ and multiply by the constant $\frac{1}{4}$. Notice that $t^0 = 1$.

$$\frac{1}{4}\int dt = \frac{t}{4} + C$$

Thus, $\int \frac{1}{4}\,dt = \frac{t}{4} + C$.

Next use the constant multiple rule to rewrite $\int \frac{\cos 3t}{4}\,dt$.

$$\int \frac{\cos 3t}{4}\,dt = \frac{1}{4}\int \cos 3t\,dt$$

Now to find $\frac{1}{4}\int \cos 3t\,dt$, use the formula $\int \cos kx\,dx = \frac{\sin kx}{k} + C$ and multiply by the constant $\frac{1}{4}$.

$$\frac{1}{4}\int \cos 3t\,dt = \frac{1}{4}\cdot\frac{\sin 3t}{3} + C = \frac{\sin 3t}{12} + C$$

Thus, $\int \frac{\cos 3t}{4}\,dt = \frac{\sin 3t}{12} + C$.

Now combine the separate terms to find the antiderivative of the original expression. For notational convenience, use only one constant of integration.

$$\int \left(\frac{1}{4} + \frac{\cos 3t}{4} \right) dt = \int \frac{1}{4}\, dt + \int \frac{\cos 3t}{4}\, dt$$

$$= \frac{t}{4} + \frac{\sin 3t}{12} + C$$

Check your answer by differentiation.

$$\frac{d}{dt}\left(\frac{t}{4} + \frac{\sin 3t}{12} + C \right) = \frac{1 + \cos 3t}{4}$$

Thus, $\displaystyle\int \frac{1 + \cos 3t}{4}\, dt = \frac{t}{4} + \frac{\sin 3t}{12} + C.$

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Find the function y(x) satisfying $\frac{d^2y}{dx^2} = 4 - 12x$, $y'(0) = 5$, and $y(0) = 7$.

First, find $\frac{dy}{dx}$, the antiderivative of $\frac{d^2y}{dx^2} = 4 - 12x$.

$$\frac{dy}{dx} = \int (4 - 12x)dx = 4x - 6x^2 + C_1$$

The constant C_1 in $\frac{dy}{dx} = y' = 4x - 6x^2 + C_1$ can be evaluated by applying the initial value condition $y'(0) = 5$.

$C_1 = 5$

So $\frac{dy}{dx} = y' = 4x - 6x^2 + 5$.

$$y(x) = \int y' \, dx = \int \left(4x - 6x^2 + 5\right) dx = 2x^2 - 2x^3 + 5x + C_2$$

The constant C_2 in $y = 2x^2 - 2x^3 + 5x + C_2$ can be evaluated by applying the initial value condition $y(0) = 7$.

$C_2 = 7$

So, the function satisfying $\frac{d^2y}{dx^2} = 4 - 12x$, $y'(0) = 5$, and $y(0) = 7$ is $y(x) = 2x^2 - 2x^3 + 5x + 7$.

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Solve the initial value problem.

$$\frac{d^2r}{dt^2} = \frac{2}{t^3}; \quad \left.\frac{dr}{dt}\right|_{t=1} = 4, \; r(2) = 2$$

To find r(t), first find all antiderivatives of the function $\frac{d^2r}{dt^2} = \frac{2}{t^3}$. That is, $\frac{dr}{dt} = \int \frac{2}{t^3}\,dt$.

$$\int \frac{2}{t^3}\,dt = \int 2t^{-3}\,dt = -t^{-2} + C$$

Thus, $\frac{dr}{dt} = -t^{-2} + C$.

To find $\frac{dr}{dt}$ such that $\left.\frac{dr}{dt}\right|_{t=1} = 4$,set $\frac{dr}{dt} = 4$ and $t = 1$ and solve for C.

$$\begin{aligned} \frac{dr}{dt} &= -t^{-2} + C \\ 4 &= -(1)^{-2} + C \\ 5 &= C \end{aligned}$$

Now substitute the value of C into $\frac{dr}{dt} = -t^{-2} + C$.

$$\frac{dr}{dt} = -t^{-2} + 5$$

To find r(t), find all antiderivatives of the function $\frac{dr}{dt} = -t^{-2} + 5$. That is, $r(t) = \int \left(-t^{-2} + 5\right)\,dt$.

$$\int \left(-t^{-2} + 5\right)\,dt = t^{-1} + 5t + C$$

Thus, $r(t) = t^{-1} + 5t + C = \frac{1}{t} + 5t + C$.

To find r(t) such that $r(2) = 2$, set $r(2) = 2$ and solve for C.

$$\begin{aligned} r(t) &= \frac{1}{t} + 5t + C \\ 2 &= \frac{1}{2} + 5(2) + C \\ -\frac{17}{2} &= C \end{aligned}$$

Now substitute the value of C into $r(t) = \frac{1}{t} + 5t + C$ to obtain the final answer.

$$r(t) = \frac{1}{t} + 5t - \frac{17}{2}$$