Student: Cole Lamers **Date:** 08/27/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.1 Rates of Change and Tangents to Curves

Find the average rate of change of the function over the given intervals.

$f(x) = 3x^3 + 3$; **a**) [2,4], **b**) [−2,2]

a) The average rate of change of a function f(x) over the interval $\left[x_1, x_2\right]$ is as follows.

$$\frac{\Delta y}{\Delta x} = \frac{f\left(x_2\right) - f\left(x_1\right)}{x_2 - x_1}$$

For the function $f(x) = 3x^3 + 3$ and interval [2,4], the average rate of change is as follows.

$$\frac{\Delta y}{\Delta x} = \frac{\left[(3)(4)^3 + 3\right] - \left[(3)(2)^3 + 3\right]}{4 - 2}$$

$$\frac{\Delta y}{\Delta x} = 84$$

Thus, the average rate of change of the function $3x^3 + 3$ over the interval [2,4] is 84.

b) For the function $3x^3 + 3$ and interval [−2,2],

$$\frac{\Delta y}{\Delta x} = \frac{\left[(3)(2)^3 + 3\right] - \left[(3)(-2)^3 + 3\right]}{2 - (-2)}.$$

$$\frac{\Delta y}{\Delta x} = 12$$

Thus, the average rate of change of the function $3x^3 + 3$ over the interval [−2,2] is 12.

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Find the average rate of change of the function over the given interval.

$f(t) = 3 + \mathbf{cos}\, t$

a. $\left[\frac{\pi}{2}, \frac{4\pi}{3}\right]$

b. $\left[-\frac{3\pi}{2}, \frac{\pi}{2}\right]$

The average rate of change of a function f(x) over the interval $\left[x_1, x_2\right]$ is given by the formula below.

$$\frac{\Delta y}{\Delta x} = \frac{f\left(x_2\right) - f\left(x_1\right)}{x_2 - x_1}$$

For the function h(t) = f(t) and interval $\left[\frac{\pi}{2}, \frac{4\pi}{3}\right]$, $\frac{\Delta y}{\Delta x} = \frac{\left(3 + \mathbf{cos}\left(\frac{4\pi}{3}\right)\right) - \left(3 + \mathbf{cos}\left(\frac{\pi}{2}\right)\right)}{\frac{4\pi}{3} - \frac{\pi}{2}}$.

Simplify the numerator.

$$\frac{\Delta y}{\Delta x} = \frac{-\frac{1}{2}}{\frac{4\pi}{3} - \frac{\pi}{2}}$$

Simplify the denominator.

$$\frac{\Delta y}{\Delta x} = \frac{-\frac{1}{2}}{\frac{5\pi}{6}}$$

Now simplify the complex fraction. The average rate of change is $-\frac{3}{5\pi}$.

b. Now find the average rate of change over $\left[-\frac{3\pi}{2}, \frac{\pi}{2}\right]$.

For the function $f(x) = 3 + \mathbf{cos}\, t$ and interval $\left[-\frac{3\pi}{2}, \frac{\pi}{2}\right]$, $\frac{\Delta y}{\Delta x} = \frac{\left(3 + \mathbf{cos}\left(\frac{\pi}{2}\right)\right) - \left(3 + \mathbf{cos}\left(-\frac{3\pi}{2}\right)\right)}{\frac{\pi}{2} - \left(-\frac{3\pi}{2}\right)}$

Simplify the numerator.

$$\frac{\Delta y}{\Delta x} = \frac{0}{\frac{\pi}{2} - \left(-\frac{3\pi}{2}\right)}$$

The average rate of change is 0.

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Find **(a)** the slope of the curve at the given point P, and **(b)** an equation of the tangent line at P.

$y = 7 - 4x^2; \quad P(-2,-9).$

a. Start with a secant line through $P(-2,-9)$ and $Q(-2+h, 7-4(-2+h)^2)$ nearby. Write an expression for the slope of the secant PQ, and determine what happens to the slope as Q approaches P; that is, as h approaches 0.

$$\text{Secant slope} = \frac{\Delta y}{\Delta x} = \frac{\left(7-4(-2+h)^2\right) - \left(7-4(-2)^2\right)}{h}$$

Simplify the numerator.

$$\text{Secant slope} = \frac{\Delta y}{\Delta x} = \frac{16h - 4h^2}{h}$$

Now simplify the entire fraction to get $16 - 4h$. Notice that this expression approaches 16 as h approaches 0.

Thus, the slope of the curve at $P(-2,-9)$ is 16.

b. Use this slope and the given point $(-2,-9)$ to write the equation for the tangent line.

$$\begin{aligned} y-(-9) &= 16(x-(-2)) && \text{This is the point-slope equation.}\\ y &= 16x+32-9 && \text{Simplify.}\\ y &= 16x+23 \end{aligned}$$

The equation of the line tangent to $y = 7-4x^2$ at $P(-2,-9)$ is $y = 16x + 23$.

Example

Find the equation of the tangent line to the graph of $f(x) = 1 - x^3$ at $x = 2$.

- Since the y-coordinate at $x = 2$ is $f(2) = -7$, we can use point-slope form to write the equation for the tangent line as: $y + 7 = m_{\tan}(x - 2)$
- The slope $m_{\tan}$ is the instantaneous rate of change of $f(x)$ at $x = 2$. As before, we will calculate the average rate of change over an x-interval of length h.
- We want the slope of the secant line connecting the points $(2, f(2))$ and $(2 + h, f(2 + h))$:

$$\begin{aligned} m_{\sec} &= \frac{f(2+h) - f(2)}{(2+h) - 2} = \frac{(1 - (2+h)^3) - (-7)}{h} \\ &= \frac{1 - 8 - 12h - 6h^2 - h^3 + 7}{h} = \frac{-12h - 6h^2 - h^3}{h} = -12 - 6h - h^2 \\ m_{\tan} &= -12 - 6(0) - (0)^2 = -12 \end{aligned}$$

- The equation of the tangent line is $y + 7 = -12(x - 2)$.