Student: Cole Lamers **Date:** 08/31/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.3 The Precise Definition of a Limit

1. Suppose that the interval (a,b) is on the x-axis with the point c inside the interval. For the given values of a, b, and c, find the value of $\delta > 0$ such that for all x, $0 < |x - c| < \delta \rightarrow a < x < b$.

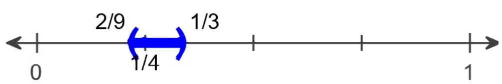
   $a = 7, \quad b = 19, \quad c = 15$

   The value of δ is 4.
   (Simplify your answer.)

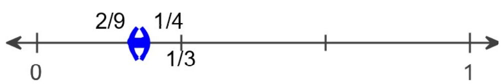
2. Sketch the interval (a,b) on the x-axis with the point c inside. Then find the largest value of $\delta > 0$ such that for all x, $0 < |x - c| < \delta$ implies $a < x < b$.

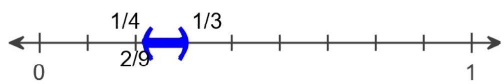
   $$a = \frac{2}{9},\ b = \frac{1}{3},\ c = \frac{1}{4}$$

   Choose the correct sketch below.

   A.
   

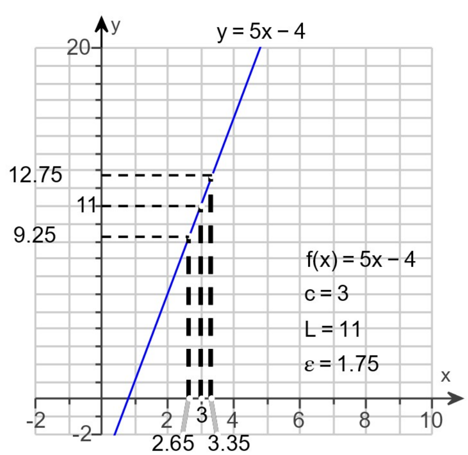
   B.
   

   C.
   

   D.
   

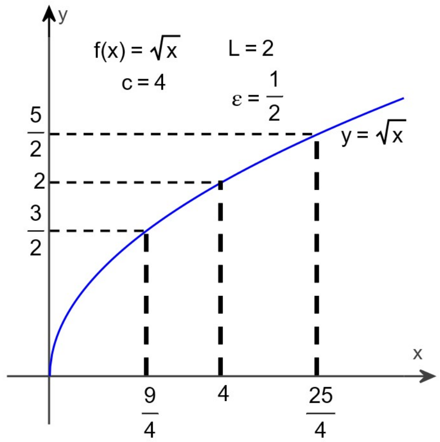
   The largest possible value for δ is $\frac{1}{36}$.
   (Type a simplified fraction.)

3. Use the graph below to find $\delta > 0$ such that for all x, $0 < |x - c| < \delta \rightarrow |f(x) - L| < \varepsilon$.

   The value of δ is .35.
   (Simplify your answer.)

4. Use the graph below to find a $\delta > 0$ such that for all x, $0 < |x - c| < \delta \to |f(x) - L| < \varepsilon$.



The value of δ is $\frac{7}{4}$.

(Type an exact answer, using radicals as needed.)

5. For the given function f(x) and values of L, c, and $\varepsilon > 0$ find the largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then determine the largest value for $\delta > 0$ such that $0 < |x - c| < \delta \to |f(x) - L| < \varepsilon$.

$f(x) = 4x + 2, \quad L = 34, \quad c = 8, \quad \varepsilon = 0.08$

The largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds is (7.98,8.02).
(Use interval notation.)

The largest value of $\delta > 0$ such that $0 < |x - c| < \delta \to |f(x) - L| < \varepsilon$ is .02.
(Simplify your answer.)

6. For the given function f(x) and values of L, c, and $\varepsilon > 0$ find the largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then determine the largest value for $\delta > 0$ such that $0 < |x - c| < \delta \to |f(x) - L| < \varepsilon$.

$f(x) = x^2, \quad L = 49, \quad c = -7, \quad \varepsilon = 0.15$

The largest open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds is $(-7.0107, -6.9893)$.
(Use interval notation. Round to four decimal places.)

The largest value of $\delta > 0$ such that $0 < |x - c| < \delta \to |f(x) - L| < \varepsilon$ is .0107.
(Round to four decimal places.)

7. For the given function f(x) and numbers L, c, and $\varepsilon > 0$, find an open interval about c on which the inequality $|f(x) - L| < \varepsilon$ holds. Then give a value for $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

$f(x) = x^2 - 34, L = 2, c = 6, \varepsilon = 1$

For what open interval does the inequality $|f(x) - L| < \varepsilon$ hold?

$(\sqrt{35}, \sqrt{37})$

(Simplify your answer. Type your answer in interval notation. Type an exact answer, using radicals as needed.)

Find the largest value $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$ the inequality $|f(x) - L| < \varepsilon$ holds.

$\delta = \sqrt{37} - 6$

(Simplify your answer. Type an exact answer, using radicals as needed.)

8. For the given function f(x), the point c, and a positive number ε, find $L = \lim_{x \to c} f(x)$. Then find a number $\delta > 0$ such that for all x, $0 < |x - c| < \delta \to |f(x) - L| < \varepsilon$.

$f(x) = 6 - 5x$, $c = 2$, $\varepsilon = 0.04$

L = −4 (Simplify your answer.)

What is the largest possible value for δ?

δ = .008 (Simplify your answer.)

9. For the given function f(x) and the given values of c and $\varepsilon > 0$, find $L = \lim_{x \to c} f(x)$.

Then determine the largest value for $\delta > 0$ such that $0 < |x - c| < \delta \to |f(x) - L| < \varepsilon$.

$f(x) = \dfrac{x^2 - 64}{x - 8}$, $c = 8$, $\varepsilon = 0.08$

The value of L is 16 .
(Simplify your answer.)

The largest value of $\delta > 0$ such that $0 < |x - c| < \delta \to |f(x) - L| < \varepsilon$ is .08 .
(Round to the nearest hundredth as needed.)

10. Give an ε-δ proof of the limit fact.

$\lim_{x \to 0} (3x + 6) = 6$

Let $\varepsilon > 0$ be given.

- **A.** Choose $\delta = \frac{\varepsilon}{3}$. Then $0 < |x - 0| < \delta \Rightarrow |(3x + 6) - 6| = |3x| = 3|x| < 3\delta = \varepsilon$.
- **B.** Choose $\delta = 3\varepsilon$. Then $0 < |x - 0| < \delta \Rightarrow |(3x + 6) - 6| = |3x| = 3|x| < \frac{\delta}{3} = \varepsilon$.
- **C.** Choose $\delta = \varepsilon$. Then $0 < |x - 0| < \delta \Rightarrow |(3x + 6) - 6| = |3x| < \delta = \varepsilon$.
- **D.** Choose $\delta = \frac{\varepsilon}{6}$. Then $0 < |x - 0| < \delta \Rightarrow |(3x + 6) - 3x| = |6x| = 6|x| < 6\delta = \varepsilon$.
- **E.** None of the above proofs is correct.

11.

Prove that $\lim_{x \to 8} f(x) = 64$ if $f(x) = \begin{cases} x^2 & x \neq 8 \\ 7 & x = 8 \end{cases}$.

For a function f(x) that is defined in an open interval about c, except possibly at c itself, the limit of f(x) as x approaches c is the number L if, for every number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that for all x, $0 < |x - c| < \delta$ implies that $|f(x) - L| < \varepsilon$.

To prove the given limit statement, it is necessary to show that for all x, if $0 < |x - 8| < \delta$, then $|x^2 - 64| < \varepsilon$.

Solve the inequality $|x^2 - 64| < \varepsilon$ to find an open interval about $c = 8$ on which this inequality holds for all $x \neq c$.

$$\begin{aligned} |x^2 - 64| &< \varepsilon \\ -\varepsilon < x^2 - 64 &< \varepsilon \\ 64 - \varepsilon < x^2 &< 64 + \varepsilon \\ \sqrt{64 - \varepsilon} < |x| &< \sqrt{64 + \varepsilon} \quad \text{Assume that } \varepsilon < 64. \\ \sqrt{64 - \varepsilon} < x &< \sqrt{64 + \varepsilon} \end{aligned}$$

The inequality $|x^2 - 64| < \varepsilon$ holds for all $x \neq 8$ in the open interval $\left(\sqrt{64 - \varepsilon}, \sqrt{64 + \varepsilon}\right)$, where $\varepsilon < 64$. Now, find a value of $\delta > 0$ that places the centered interval $(8 - \delta, 8 + \delta)$ inside the interval $\left(\sqrt{64 - \varepsilon}, \sqrt{64 + \varepsilon}\right)$.

Take δ to be the distance from $c = 8$ to the nearer endpoint of $\left(\sqrt{64 - \varepsilon}, \sqrt{64 + \varepsilon}\right)$.

$$\delta = \min\left\{8 - \sqrt{64 - \varepsilon}, \sqrt{64 + \varepsilon} - 8\right\}$$

Thus, for $\varepsilon < 64$, if δ has a value of $\min\left\{8 - \sqrt{64 - \varepsilon}, \sqrt{64 + \varepsilon} - 8\right\}$ or any smaller positive value, the inequality $0 < |x - 8| < \delta$ automatically places x between $\sqrt{64 - \varepsilon}$ and $\sqrt{64 + \varepsilon}$ to make $|x^2 - 64| < \varepsilon$.

If $\varepsilon \geq 64$, take δ to be the distance from $c = 8$ to the nearer endpoint of $\left(0, \sqrt{64 + \varepsilon}\right)$.

$$\delta = \min\left\{8, \sqrt{64 + \varepsilon} - 8\right\}$$

Since there exists a value of δ such that $0 < |x - 8| < \delta$ that makes the inequality $|x^2 - 64| < \varepsilon$ true for all x, the limit as x approaches 8 of the function $\begin{cases} x^2 & x \neq 8 \\ 7 & x = 8 \end{cases}$ is 64.

12. Give an ε-δ proof of $\lim_{x \to 3} \left(\frac{x^2 - 9}{x - 3} \right) = 6$.

Let $\varepsilon > 0$ be given.

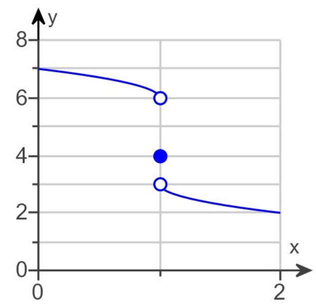
A. Let $\delta = 3\varepsilon$. Then $0 < |x - 3| < \delta \Rightarrow$
$$\left| \left(\frac{x^2 - 9}{x - 3} \right) - 6 \right| = \left| \frac{1}{3}(x + 3 - 6) \right| = \frac{1}{3}|x - 3| < \frac{1}{3}\delta = \varepsilon.$$

B. Let $\delta = \varepsilon$. Then $0 < |x - 3| < \delta \Rightarrow$
$$\left| \left(\frac{x^2 - 9}{x - 3} \right) - 6 \right| = |(x + 3) - 6| = |x - 3| < \delta = \varepsilon.$$

C. Let $\delta = 2\varepsilon$. Then $0 < |x - 3| < \delta \Rightarrow$
$$\left| \left(\frac{x^2 - 9}{x - 3} \right) - 6 \right| = \left| \frac{1}{2}(x + 3 - 6) \right| = \frac{1}{2}|x - 3| < \frac{1}{2}\delta = \varepsilon.$$

D. None of the above proofs is correct.

13. For the function graphed to the right, explain why $\lim_{x \to 1} f(x) \neq 4$.



Choose the correct reason below.

A. The limit of f(x) as x approaches 1 is 3.

B. The limit of f(x) as x approaches 1 is $\frac{9}{2}$.

C. The limit of f(x) as x approaches 1 is 6.

D. The limit of f(x) as x approaches 1 does not exist.