Student: Cole Lamers **Date:** 09/24/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.5 Applied Optimization

Find the smallest perimeter and the dimensions for a rectangle with an area of 121 in^2.

With ℓ representing the length and w representing the width of a rectangle, the perimeter, P, and area, A, are $P = 2\ell + 2w$ and $A = \ell w$.

Substitute 121 for A, and solve for the variable ℓ.

$$\ell = \frac{A}{w} = \frac{121}{w}$$

Substitute the expression for ℓ into the formula for perimeter.

$$P = 2\frac{121}{w} + 2w$$

Find the derivative with respect to w.

$$\frac{dP}{dw} = -\frac{242}{w^2} + 2$$

Set the derivative equal to zero, and solve.

$$-\frac{242}{w^2} + 2 = 0$$

$$w = \pm 11$$

Since a rectangle dimension cannot be negative, discard -11 and test 11.

The second derivative of the perimeter function $P = \frac{242}{w} + 2w$ is $\frac{d^2P}{dw^2} = \frac{484}{w^3}$.

Since $\frac{484}{w} > 0$ when w is positive, the perimeter $\frac{242}{w} + 2w$ for $w = 11$ is the smallest perimeter.

Substituting $w = 11$ into the area formula, $121 = \ell w$, the length is 11 in.

The smallest perimeter is $2(11) + 2(11) = 44$ in.

The dimensions of the rectangle of smallest perimeter with area 121 in^2 are 11 in by 11 in.

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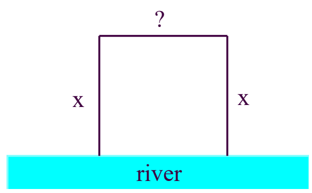
A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 2800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

Write expressions in one variable for the lengths of the three sides of the rectangular plot such that the sum of the three lengths is 2800 m.

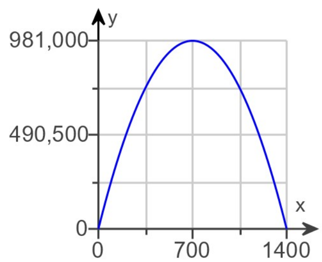
A picture is often helpful. If the lengths of the sides that are perpendicular to the river are x meters, the length of the side parallel to the river in terms of x is $2800 - 2x$ m.



The dimensions of the rectangular plot are $2800 - 2x$ by x. Use the fact that the area of a rectangle is length times width to create a function for the area.

$$\begin{aligned} A(x) &= x(2800 - 2x) \\ &= 2800x - 2x^2 \end{aligned}$$

The graph of the area function on the right suggests a maximum area of about 981,000 m^2 when x is about 700 m.



To determine the exact value of x that maximizes the area, first find the derivative of the area function.

$$\begin{aligned} A(x) &= 2800x - 2x^2 \\ A'(x) &= 2800 - 4x \end{aligned}$$

Now determine the critical points of A(x).

$$\begin{aligned} 2800 - 4x &= 0 \\ x &= 700 \text{ m} \end{aligned}$$

This value agrees with the approximation from the graph. Substitute 700 for x in the area function and determine the maximum area.

$$\begin{aligned} A(x) &= 2800x - 2x^2 \\ A(700) &= 2800(700) - 2(700)^2 \\ &= 980{,}000 \text{ m}^2 \end{aligned}$$

The maximum area is 980,000 m^2.

Recall that x is the length of each side perpendicular to the river. Calculate the length of the side parallel to the river.

$$\begin{aligned} 2800 - 2x &= 2800 - 2(700) \\ &= 1400 \text{ m} \end{aligned}$$

The rectangular plot that maximizes the area is 1400 m by 700 m.

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A rectangular tank that is 1372 ft^3 with a square base and open top is to be constructed of sheet steel of a given thickness. Find the dimensions of the tank with minimum weight.

The weight can be minimized by minimizing the amount of steel used. This means that the area of the base and sides should be minimized for the given volume.

With h representing the height of the tank and s representing the edges of the base, the expression for the area of the tank is $s^2 + 4sh$.

To express the area function in terms of one variable, note that the volume, V, is related to h and s: $1372 = s^2h$. So $h = \frac{1372}{s^2}$.

Thus, $A = s^2 + \frac{5488}{s}$.

To find the dimensions for which the area is a minimum, take the derivative of the area function, set it equal to 0, and solve for s.

$$\frac{dA}{ds} = \frac{d}{ds}\left(s^2 + \frac{5488}{s}\right) = 2s - \frac{5488}{s^2}$$

Set the derivative equal to 0 and solve.

$$2s - \frac{5488}{s^2} = 0$$

$$s = \sqrt[3]{2744}$$

$$s = 14$$

The second derivative of the area function is $2 + \frac{10976}{s^3}$, which has a positive sign for $s = 14$.

Since the second derivative is positive at $s = 14$, the area function has a minimum at that value of s.

Solving $V = s^2h$ for $s = 14$ and $V = 1372$, $h = 7$. The dimensions of the tank are 14 ft by 14 ft by 7 ft.

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You are designing a 864 cm^3 right circular cylindrical can whose manufacture will take waste into account. There is no waste in cutting the aluminum for the side, but the top and bottom of radius r will be cut from squares that measure 4r units on a side. The total amount of aluminum used up by the can will therefore be $A = 32r^2 + 2\pi rh$, where h is the height of the right circular cylindrical can. What is the ratio of h to r for the most economical can?

Interpret the phrase "most economical."

Find the ratio of h to r that makes the total surface area as small as possible while satisfying the given constraint of the volume, ignoring the thickness of the material wasted in manufacturing.

In order to make the total surface area as small as possible while satisfying the given constraint of the volume, find the formula for the volume of the can. If r is the radius of the bottom, and h is the height of the can, the volume of the can is $\pi r^2 h = 864$.

To express the surface area as a function of one variable, solve for one of the variables in $\pi r^2 h = 864$ and substitute that expression into the given surface area formula, $A = 32r^2 + 2\pi rh$. In this case, solving for h is easier.

$$h = \frac{864}{\pi r^2}$$

Substitute $\frac{864}{\pi r^2}$ for h into $A = 32r^2 + 2\pi rh$ and simplify.

$$\begin{aligned} A &= 32r^2 + 2\pi rh \\ &= 32r^2 + 2\pi r\left(\frac{864}{\pi r^2}\right) && \text{Substitute.} \\ &= 32r^2 + \frac{1728}{r} && \text{Simplify.} \end{aligned}$$

The goal is to find the ratio of h to r that minimizes the value of A. Since A is differentiable on $r > 0$, an interval with no endpoints, it can have a minimum value only where its first derivative is zero. Find the first derivative of A with respect to r.

$$\frac{dA}{dr} = 64r - \frac{1728}{r^2}$$

Let $\frac{dA}{dr}$ be equal to 0 and solve for r.

$$\begin{aligned} \frac{dA}{dr} &= 64r - \frac{1728}{r^2} \\ 0 &= 64r - \frac{1728}{r^2} && \text{Set } \frac{dA}{dr} = 0. \\ 64r^3 &= 1728 && \text{Multiply by } r^2. \\ r &= 3 && \text{Solve for r.} \end{aligned}$$

Find the second derivative of A with respect to r.

$$\frac{d^2A}{dr^2} = 64 + \frac{3456}{r^3}$$

Since the domain of A is $r > 0$, the second derivative is always positive. Thus, the value of A at $r = 3$ is an absolute minimum.

Recall that $h = \frac{864}{\pi r^2}$, find the indicated ratio of h to r.

$$\frac{h}{r} = \frac{32}{\pi}$$

Therefore, the ratio of h to r for the most economical can is $\frac{32}{\pi}$.

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The height (feet) of an object moving vertically is given by $s = -16t^2 + 112t + 84$, where t is in seconds. Find the object's velocity at $t = 5$, its maximum height and when it occurs, and its velocity when $s = 0$.

Velocity, v(t), is s'(t). $s(t) = -16t^2 + 112t + 84$. So, $v(t) = -32t + 112$.

Since $v(t) = -32t + 112$, to find the velocity at $t = 5$ sec, find v(5).

$v(5) = -48$ ft/sec

The maximum height occurs when the object stops rising. This is when $v(t) = 0$.
Set $v(t) = 0$ and solve.

$v(t) = -32t + 112 = 0$
$32t = 112$
$t = 3.5$ sec

The height of the object at 3.5 sec is $s(3.5) = 280$ ft.

To find velocity when $s = 0$, first solve $s(t) = 0$.

$s(t) = -16t^2 + 112t + 84 = 0$

Using the quadratic formula, the solutions are

$$\frac{-112 + \sqrt{(112)^2 + 64(84)}}{-32} \text{ and } \frac{-112 - \sqrt{(112)^2 + 64(84)}}{-32},$$

or -0.68 or 7.68. (rounded to the nearest hundredth)

Since the time cannot negative, the solution is 7.68 sec.

Substituting $t = 7.68$ sec into the expression for v(t), the velocity when $s = 0$ is -133.76 ft/sec, rounded to the nearest hundredth.

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It costs 8 dollars to manufacture and distribute a backpack. If the backpacks sell at x dollars each, the number sold, n, is given by $n = \frac{3}{x-8} + 4(100 - x)$. Find the selling price that will maximize profit.

The cost of producing n backpacks is \$8n.

The revenue from selling n backpacks is \$nx.

Thus, the profit from selling n backpacks is P = \$nx − 8n.

To optimize profit, first simplify P(x).

$$\begin{aligned} P(x) &= nx - 8n \\ &= n(x-8) \\ &= \left[\frac{3}{x-8} + 4(100-x)\right](x-8) \\ &= 3 + 4(100-x)(x-8) \\ &= 3 + 400x - 3200 - 4x^2 + 32x \\ &= -4x^2 + 432x - 3197 \end{aligned}$$

Now, take the derivative of P(x).

$$P'(x) = -8x + 432$$

Set the derivative equal to 0 and solve.

$$\begin{aligned} -8x + 432 &= 0 \\ x &= 54 \end{aligned}$$

The second derivative of P(x), $P''(x) = -8$ is always negative. The price of \$54 maximizes profit.

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Suppose that $c(x) = 8x^3 - 64x^2 + 12{,}000x$ is the cost of manufacturing x items. Find a production level that will minimize the average cost of making x items.

You are given that $c(x) = 8x^3 - 64x^2 + 12{,}000x$ is the cost of manufacturing x items. The *average* cost $\bar{c}(x)$ equals this cost function divided by x. Therefore, the average cost function is $\bar{c}(x) = 8x^2 - 64x + 12{,}000$.

Now minimize the the average cost. Notice that for smaller x-values, or as $x \to -\infty$, the average cost becomes large and for larger x-values, or as $x \to \infty$, the average cost becomes large. This suggests that a minimum value exists.

To find the minimum average cost, find the derivative of the average cost function, set it equal to 0, and solve for x. First find the derivative of the average cost function.

$\bar{c}'(x) = 16x - 64$

Set this expression equal to 0 and solve for x.

$$\begin{aligned} 16x - 64 &= 0 && \text{Set the derivative equal to 0.} \\ 16x &= 64 && \text{Add 64 to both sides.} \\ x &= 4 && \text{Divide both sides by 16.} \end{aligned}$$

Apply the second derivative test. Recall that $\bar{c}'(x) = 16x - 64$. Find $\bar{c}''(x)$.

$\bar{c}''(x) = 16$

Thus, $\bar{c}'(4) = 0$ and $\bar{c}''(4) > 0$. By the second derivative test, $\bar{c}(x)$ has a local minimum at $x = 4$. Therefore, the production level that will minimize the average cost is reached when $x = 4$.

What is this minimum average cost? To find it, substitute $x = 4$ into the average cost function, $\bar{c}(x) = 8x^2 - 64x + 12{,}000$, and simplify.

$$\begin{aligned} \bar{c}(4) &= 8(4)^2 - 64(4) + 12{,}000 && \text{Substitute } x = 4. \\ &= 128 - 256 + 12{,}000 && \text{Simplify.} \\ &= 11{,}872 \end{aligned}$$

At the production level $x = 4$, the minimum average cost is \$11,872.00 per item.