Student: Cole Lamers **Date:** 09/13/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.9 Linearization and Differentials

Find the linearization L(x) at x = a.

$f(x) = 2x^3 - 3x - 4 \qquad a = -1$

The definition of linearization states if f is differentiable at x = a, then the approximating function $L(x) = f(a) + f'(a)(x - a)$ is the linearization of f at a.

Use the definition of linearization.

$L(x) = f(a) + f'(a)(x - a)$
$L(x) = f(-1) + f'(-1)(x - (-1))$

Evaluate $f(-1)$.

$f(x) = 2x^3 - 3x - 4$
$f(-1) = 2(-1)^3 - 3(-1) - 4$
$f(-1) = -3$

To evaluate $f'(-1)$, first find $f'(x)$.

$f(x) = 2x^3 - 3x - 4$
$f'(x) = 6x^2 - 3$

Now evaluate $f'(-1)$.

$f'(x) = 6x^2 - 3$
$f'(-1) = 6(-1)^2 - 3$
$f'(-1) = 3$

Simplify.

$L(x) = f(-1) + f'(-1)(x - (-1))$
$L(x) = -3 + 3(x + 1)$
$L(x) = 3x$

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Find the linearization L(x) at x = a.

$f(x) = 4x^3 + 5x + 2 \qquad a = 1$

The definition of linearization states if f is differentiable at x = a, then the approximating function L(x) = f(a) + f '(a)(x − a) is the linearization of f at a.

Use the definition of linearization.

$L(x) = f(a) + f'(a)(x - a)$
$L(x) = f(1) + f'(1)(x - (1))$

Evaluate f(1).

$f(x) = 4x^3 + 5x + 2$
$f(1) = 4(1)^3 + 5(1) + 2$
$f(1) =$ 11
(Simplify your answer.)

To evaluate f '(1), first find f '(x).

$f(x) = 4x^3 + 5x + 2$
$f'(x) =$ $12x^2 + 5$
(Simplify your answer. Do not factor.)

Now evaluate f '(1).

$f'(x) = 12x^2 + 5$
$f'(1) = 12(1)^2 + 5$
$f'(1) =$ 17
(Simplify your answer.)

Simplify.

$L(x) = f(1) + f'(1)(x - (1))$
$L(x) = 11 + 17(x - 1)$
$L(x) =$ $17x - 6$
(Simplify your answer. Do not factor.)

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Find the linearization L(x) at x = a.

$$f(x) = -x - \frac{1}{x}, \quad a = 1$$

The definition of linearization states if f is differentiable at x = a, then the approximating function $L(x) = f(a) + f'(a)(x - a)$ is the linearization of f at a.

Use the definition of linearization.

$$L(x) = f(a) + f'(a)(x - a)$$
$$L(x) = f(1) + f'(1)(x - (1))$$

Evaluate f(1).

$$f(x) = -x - \frac{1}{x}$$
$$f(1) = -(1) - \frac{1}{1}$$
$$f(1) = -2$$

To evaluate f '(1), first find f '(x).

$$f(x) = -x - \frac{1}{x}$$
$$f'(x) = -1 + \frac{1}{x^2}$$

Now evaluate f '(1).

$$f'(x) = -1 + \frac{1}{x^2}$$
$$f'(1) = -1 + \frac{1}{1^2}$$
$$f'(1) = 0$$

Simplify.

$$L(x) = f(1) + f'(1)(x - (1))$$
$$L(x) = -2 + 0(x - 1)$$
$$L(x) = -2$$

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Find the linearization L(x) of $f(x) = \mathbf{cot}\, x$ at $x = \frac{\pi}{2}$.

The linearization L(x) of f(x) at x = a is $L(x) = f(a) + f'(a)(x - a)$.

At $a = \frac{\pi}{2}$, $f(a) = \mathbf{cot}\left(\frac{\pi}{2}\right) = 0$.

If $f(x) = \mathbf{cot}\, x$, then $f'(x) = -\mathbf{csc}^2 x$. Then, $f'(a) = -\mathbf{csc}^2\left(\frac{\pi}{2}\right) = -1$.

Substitute these values in the equation for L(x).

$$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) \\ &= 0 + (-1)\left(x - \frac{\pi}{2}\right) \\ &= \frac{\pi}{2} - x \end{aligned}$$

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Find a linearization that will replace the function over an interval that includes the given point x_0. Center each linearization not at x_0 but at a nearby integer, $x = a$, at which the given function and its derivative are easy to evaluate.

$f(x) = x^4 + 13x$, $x_0 = .01$

A linear approximation normally loses accuracy away from its center. The closer the center is to the desired point of linearization the better the linearization approximation will be. If $x_0 = .01$, the linearization should be centered at the nearest integer.

Set the center of the linearization as $x = 0$.

If f is differentiable at $x = a$, then the approximating function $L(x) = f(a) + f'(a)(x - a)$ is the linearization of f at a. Start by finding f(a) for $a = 0$.

$f(0) = 0$

Now find the derivative of the function $f(x) = x^4 + 13x$.

$f'(x) = 4x^3 + 13$

Evaluate $f'(x)$ at $a = 0$.

$f'(0) = 13$

Substitute and simplify.

$$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) \\ &= 0 + 13(x - 0) \\ &= 13x \end{aligned}$$

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Find a linearization at a suitably chosen integer near a at which the given function and its derivative are easy to evaluate.

$f(x) = 2x^2 + 12x - 4$, $a = -0.9$

A linear approximation normally loses accuracy away from its center. The closer the center is to the desired point of linearization the better the linearization approximation will be. If $a = -0.9$, the linearization should be centered at the nearest integer.

Set the center of the linearization as $x = -1$.

If f is differentiable at $x = a$, then the approximating function $L(x) = f(a) + f'(a)(x - a)$ is the linearization of f at a, where, $x = a$ is the center of linearization; in this case $a = -1$. Start by finding f(a) for $a = -1$.

$$f(-1) = 2(-1)^2 + 12(-1) - 4$$
$$f(-1) = -14$$

Now find the derivative of the function $f(x) = 2x^2 + 12x - 4$.

$$f'(x) = 4x + 12$$

Evaluate $f'(x)$ at $a = -1$.

$$f'(-1) = 4(-1) + 12$$
$$f'(-1) = 8$$

Substitute and simplify.

$$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) \\ &= -14 + 8(x + 1) \\ &= 8x - 6 \end{aligned}$$

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Find a linearization that will replace the function over an interval that includes the given point x_0. Center each linearization not at x_0 but at a nearby integer, $x = a$, at which the given function and its derivative are easy to evaluate.

$$f(x) = \sqrt[5]{x},\ x_0 = 31.7$$

A linear approximation normally loses accuracy away from its center. The closer the center is to the desired point of linearization the better the linearization approximation will be. If $x_0 = 31.7$, the linearization should be centered at the nearest integer.

Set the center of the linearization as $x = 32$.

If f is differentiable at $x = a$, then the approximating function $L(x) = f(a) + f'(a)(x - a)$ is the linearization of f at a. Start by finding $f(a) = \sqrt[5]{a}$ for $a = 32$.

$f(32) = 2$

Now find the derivative of the function $f(x) = \sqrt[5]{x}$.

$$f'(x) = \frac{1}{5}x^{-\frac{4}{5}}$$

Evaluate $f'(x)$ at $a = 32$.

$$f'(32) = \frac{1}{80}$$

Substitute and simplify.

$$\begin{aligned} L(x) &= f(a) + f'(a)(x - a) \\ &= 2 + \frac{1}{80}(x - 32) \\ &= \frac{1}{80}x + \frac{8}{5} \end{aligned}$$

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Find dy for $y = 2x^4 - 3\sqrt{5x}$.

Let y = f(x) be a differentiable function. The differential dx is an independent variable. The differential dy is dy = f '(x) dx.

First, find f '(x).

$$f(x) = 2x^4 - 3\sqrt{5x}$$

$$f'(x) = 8x^3 - 3 \cdot \frac{1}{2}(5x)^{-1/2} \cdot 5$$

$$f'(x) = 8x^3 - \frac{15}{2\sqrt{5x}}$$

Use the definition of differential to find dy.

$$dy = \left(8x^3 - \frac{15}{2\sqrt{5x}}\right)dx$$

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Find dy.

$$y = \cos\left(27\sqrt{x}\right)$$

The differential dy is $f'(x)dx$. To find the differential dy it is necessary to use the chain rule shown below.

if $y = f(u)$ and $u = g(x)$ then

$$\frac{dy}{dx} = \frac{dy}{du} \bullet \frac{du}{dx}$$

The differential dy is defined by $f'(x)dx$. To find the differential dy, use the chain rule shown below.

If $y = f(u)$ and $u = g(x)$ then $f'(x) = \dfrac{dy}{dx} = \dfrac{dy}{du} \bullet \dfrac{du}{dx}$.

Start by identifying f(u) and g(x). Let $f(u) = \cos u$. Find g(x).

$$g(x) = 27\sqrt{x}$$

Use $f(u) = \cos u$, $g(x) = 27\sqrt{x}$ to find $\dfrac{dy}{du}$ and $\dfrac{du}{dx}$.

$$\frac{dy}{du} = -\sin u$$

$$\frac{du}{dx} = \frac{27}{2\sqrt{x}}$$

Now substitute back into the chain rule formula, $\dfrac{dy}{dx} = \dfrac{dy}{du} \bullet \dfrac{du}{dx}$. Remember that $u = 27\sqrt{x}$.

$$dy = -\frac{27 \sin 27\sqrt{x}}{2\sqrt{x}}\,dx$$

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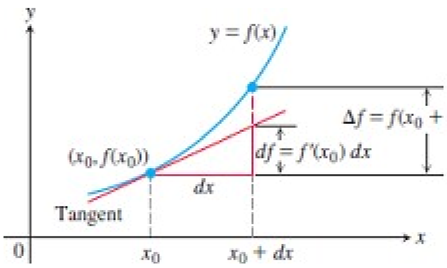
Each function f(x) changes value when x changes from x_0 to $x_0 + dx$. Find the change $\Delta f = f(x_0 + dx) - f(x_0)$, the value of the estimate $df = f'(x_0)\,dx$, and the approximate error $|\Delta f - df|$.

$f(x) = 8x^2 - 9x,\ \ x_0 = -1,\ \ dx = 0.1$



To find the change Δf, substitute the values of x_0 and dx into the equation for Δf and evaluate.

$$\begin{aligned}\Delta f &= f(x_0 + dx) - f(x_0)\\ &= f(-1 + 0.1) - f(-1)\\ &= 8(-1 + 0.1)^2 - 9(-1 + 0.1) - \\ &\quad (8(-1)^2 - 9(-1))\\ &= -2.42\end{aligned}$$

To find the value of the estimate df, first find $f'(x)$.

$$f(x) = 8x^2 - 9x$$
$$f'(x) = 16x - 9$$

Now substitute the values of x_0 and dx into the equation and evaluate to find df.

$$\begin{aligned}df &= f'(x_0)\,dx\\ &= (16x_0 - 9)\,dx\\ &= (16(-1) - 9)(0.1)\\ &= -2.5\end{aligned}$$

To find the approximate error, substitute the values of Δf and df into the expression for the approximate error and evaluate.

$$\begin{aligned}|\Delta f - df| &= |-2.42 - (-2.5)|\\ &= 0.08\end{aligned}$$

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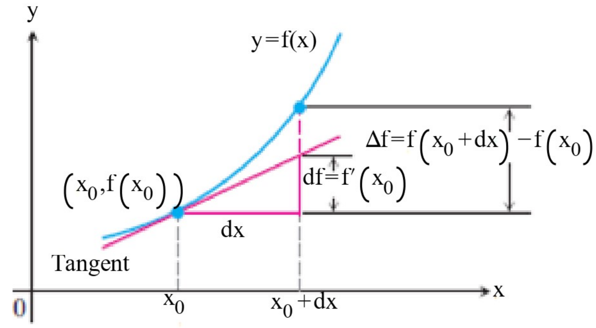
The function f(x) changes value when x changes from x_0 to $x_0 + dx$.

$$f(x) = 10x^2 + 7x - 3,\ x_0 = 2,\ dx = 0.1$$

a. Find the change $\Delta f = f(x_0 + dx) - f(x_0)$.

b. Find the value of the estimate $df = f'(x_0)\ dx$.

c. Find the approximate error $|\Delta f - df|$.



a. To find the change Δf, substitute the values of x_0 and dx into the equation for Δf and evaluate.

$$\Delta f = f(x_0 + dx) - f(x_0)$$
$$\Delta f = f(2 + 0.1) - f(2)$$
$$\Delta f = \left[10(2 + 0.1)^2 + 7(2 + 0.1) - 3\right] - \left[\left(10(2)^2 + 7(2)\right) - 3\right]$$
$$\Delta f = 4.8$$

b. First determine f′(x) of $f(x) = 10x^2 + 7x - 3$.

$$f'(x) = 20x + 7$$

Next substitute the values of x_0 and dx into the equation and evaluate.

$$df = f'(x_0)\ dx$$
$$df = (20x_0 + 7)\ dx$$
$$df = (20(2) + 7)(0.1)$$
$$df = 4.7$$

c. To find the approximate error, substitute the values of Δf and df into the expression for the approximate error and evaluate.

$$|\Delta f - df| = |4.8 - (4.7)|$$
$$|\Delta f - df| = 0.1$$

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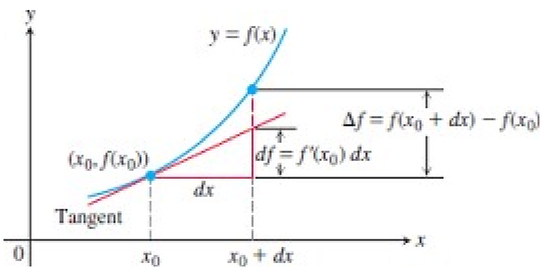
Each function f(x) changes value when x changes from x_0 to $x_0 + dx$. Find the change $\Delta f = f(x_0 + dx) - f(x_0)$, the value of the estimate $df = f'\left(x_0\right)\, dx$, and the approximate error $\left|\Delta f - df\right|$.

$f(x) = 5x^{-2}, \; x_0 = -1.1, \; dx = 0.1$



To find the change Δf, substitute the values of x_0 and dx into the equation for Δf and evaluate.

$$\begin{aligned}\Delta f &= f(x_0 + dx) - f(x_0)\\ &= f(-1.1 + 0.1) - f(-1.1)\\ &= 5(-1.1 + 0.1)^{-2} - 5(-1.1)^{-2}\\ &= 0.868\end{aligned}$$

(rounded to the nearest thousandth)

To find the value of the estimate df, first find f′(x).

$$f(x) = 5x^{-2}$$
$$f'(x) = -10x^{-3}$$

Now substitute the values of x_0 and dx into the equation and evaluate to find df.

$$\begin{aligned}df &= f'(x_0)\, dx\\ &= \left(-10x_0^{\ -3}\right) dx\\ &= \left(-10(-1.1)^{-3}\right)(0.1)\\ &= 0.751\end{aligned}$$

(rounded to the nearest thousandth)

To find the approximate error, substitute the values of Δf and df into the expression for the approximate error and evaluate.

$$\begin{aligned}\left|\Delta f - df\right| &= \left|0.868 - (0.751)\right|\\ &= 0.117\end{aligned}$$

(rounded to the nearest thousandth)

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Estimate the volume of material in a cylindrical shell with height 36 in., radius 3 in., and shell thickness 0.7 in. Assume the height is fixed.

The volume of a cylinder is $V(r) = \pi r^2 h$, where r is the radius and h is the height. The estimated volume of material in a cylindrical shell is $dV = V'(r)\ dr$.

To find the estimated volume of material, first find V '(r) and dr.

$V(r) = \pi r^2 h$
$V'(r) = 2\pi rh$

The value of dr is the shell thickness. Thus, dr = 0.7 in.

Now, substitutite the values of r, h, and dr into the equation and evaluate to find dV.

$$\begin{aligned} dV &= V'(r)\ dr \\ &= (2\pi rh)dr \\ &= 2\pi(3)(36)(0.7) \\ &= 475 \end{aligned}$$
(rounded to the nearest tenth)

Thus, the estimated volume of material is 475 $in.^3$.

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The volume of a solid can be expressed as $V = 8x^3$. The volume is to be calculated with an error of no more than 6% of the true value. Find approximately the greatest error that can be tolerated in the measurement of x, expressed as a percentage of x.

Use the fact that the change in volume is approximately equal to the differential of the volume, $\Delta V \approx dV$, and the change in volume should be less than or equal to 6% of the true value. This second condition corresponds to the inequality, $|\Delta V| \le (6\%)V$.

Next find the differential term dV of the volume $V = 8x^3$.

$dV = 24x^2 dx$

Because $\Delta V \approx dV$, substitute dV and V into the inequality.

$$\begin{aligned} |\Delta V| &\le (6\%)V \\ |dV| &\le (6\%)V \\ 24x^2 dx &\le \left(\frac{3}{50}\right) 8x^3 \end{aligned}$$

Solve for dx.

$$\begin{aligned} 24x^2 dx &\le \left(\frac{3}{50}\right) 8x^3 \\ dx &\le \frac{1}{50} x \end{aligned}$$

Now multiply dx by 100 to get the percent tolerence in terms of x.

The greatest tolerated error in the measurement of x is 2%.