Projectile Motion Simplified Post-Lecture

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Common Equations

$$v = v_0 + at \qquad \Delta x = v_0 t + \tfrac{1}{2} a t^2 \qquad v^2 = v_0^2 + 2a\Delta x$$

$$v_x = v_{0x} + a_x t \qquad \Delta x = v_{0x} t + \tfrac{1}{2} a_x t^2 \qquad v_x^2 = v_{0x}^2 + 2a_x \Delta x$$

$$v_y = v_{0y} + a_y t \qquad \Delta y = v_{0y} t + \tfrac{1}{2} a_y t^2 \qquad v_y^2 = v_{0y}^2 + 2a_y \Delta y$$

$$v_y = v_{0y} + gt \qquad \Delta y = v_{0y} t + \tfrac{1}{2} g t^2 \qquad v_y^2 = v_{0y}^2 + 2g\Delta y$$

$$ax^2 + bx + c = 0 \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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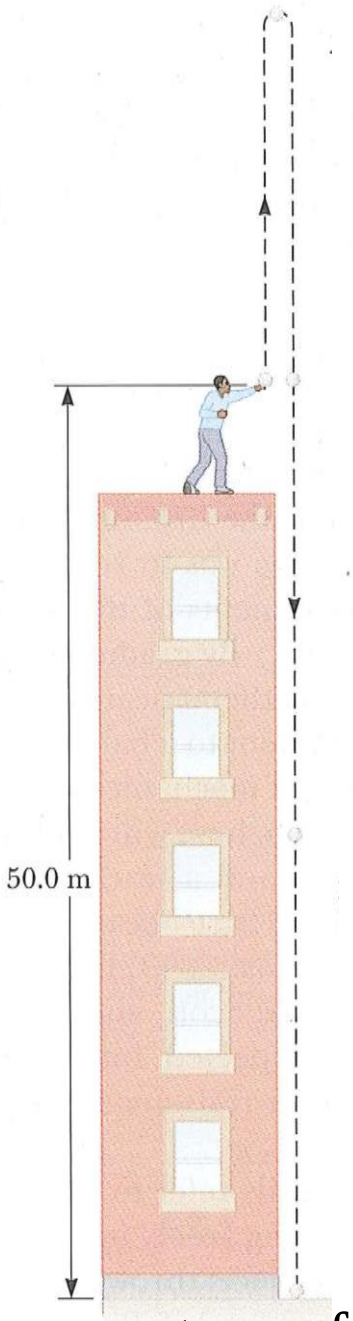
Figure 2.20



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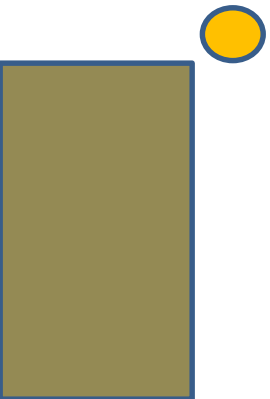
- Vertical Velocity Only (VVO)

"Ball Toss"



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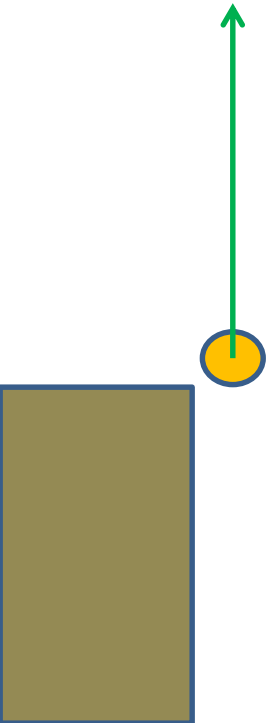
v_{oy} = +19.6 m/s Δy = 0 m v_y = +19.6 m/s Time = 0 s



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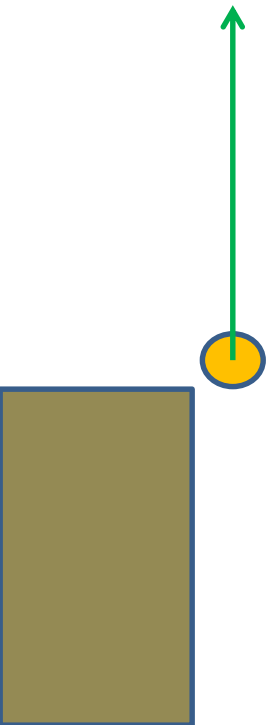
v_{oy} = +19.6 m/s Δy = ? v_y = ? Time = 0.500 s



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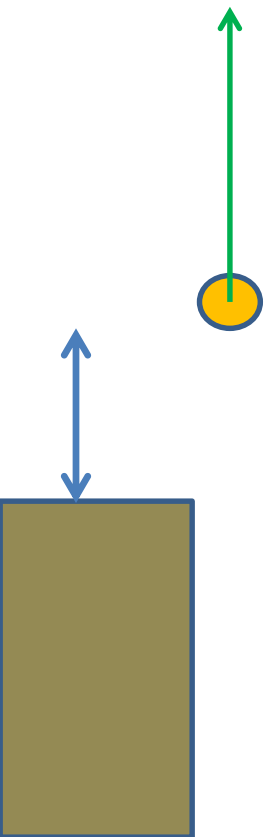
v_{oy} = +19.6 m/s Δy = +8.58 m v_y = +14.7 m/s Time = 0.500 s

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+19.6\ \text{m/s})(0.500\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(0.500\ \text{s})^2 = +8.58\ \text{m}$$

$$v_y = v_{0y} + gt = (+19.6\ \text{m/s}) + (-9.80\ \text{m/s}^2)(0.500\ \text{s}) = +14.7\ \text{m/s}$$



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v_{oy} = +19.6 m/s Δy = ? v_y = ? Time = 1.00 s

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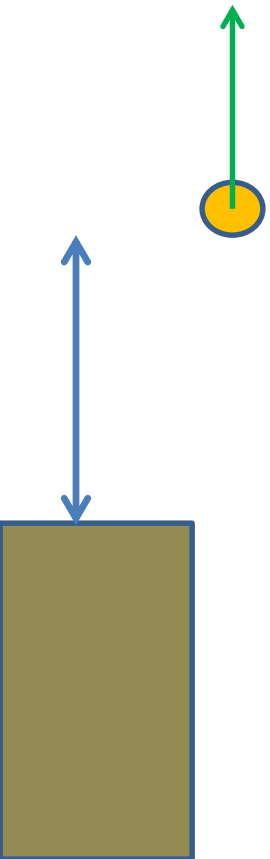
v_{oy} = +19.6 m/s Δy = +14.7 m v_y = +9.80 m/s Time = 1.00 s

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+19.6\ \text{m/s})(1.00\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(1.00\ \text{s})^2 = +14.7\ \text{m}$$

$$v_y = v_{0y} + gt = (+19.6\ \text{m/s}) + (-9.80\ \text{m/s}^2)(1.00\ \text{s}) = +9.80\ \text{m/s}$$



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v_{oy} = +19.6 m/s Δy = ? v_y = ? Time = 1.50 s

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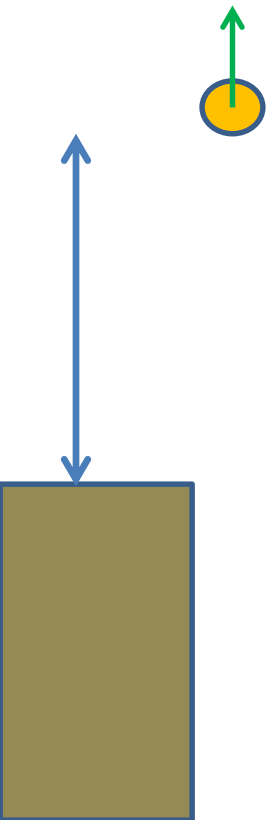
v_{oy} = +19.6 m/s Δy = +18.4 v_y = +4.90 m/s Time = 1.50 s

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+19.6\ \text{m/s})(1.50\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(1.50\ \text{s})^2 = +18.4\ \text{m}$$

$$v_y = v_{0y} + gt = (+19.6\ \text{m/s}) + (-9.80\ \text{m/s}^2)(1.50\ \text{s}) = +4.90\ \text{m/s}$$



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v_{oy} = +19.6 m/s Δy = ? v_y = ? Time = 2.00 s

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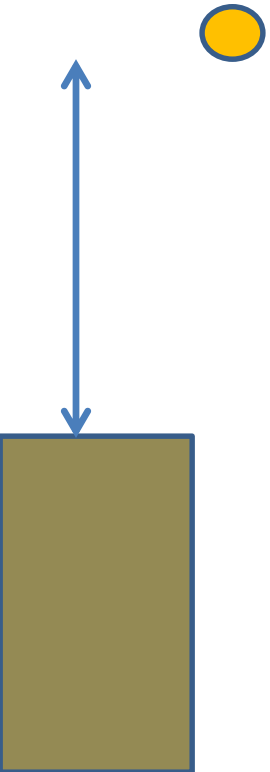
v_{oy} = +19.6 m/s Δy = +19.6 m v_y = 0 m/s Time = 2.00 s

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+19.6\ \text{m/s})(2.00\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(2.00\ \text{s})^2 = +19.6\ \text{m}$$

$$v_y = v_{0y} + gt = (+19.6\ \text{m/s}) + (-9.80\ \text{m/s}^2)(2.00\ \text{s}) = 0\ \text{m/s}$$



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v_{oy} = +19.6 m/s Δy = ? v_y = ? Time = 3.00 s

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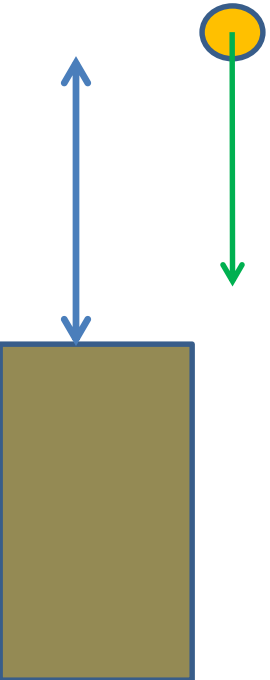
v_{oy} = +19.6 m/s Δy = +14.7 m v_y = -9.80 m/s Time = 3.00 s

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+19.6\ \text{m/s})(3.00\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(3.00\ \text{s})^2 = +14.7\ \text{m}$$

$$v_y = v_{0y} + gt = (+19.6\ \text{m/s}) + (-9.80\ \text{m/s}^2)(3.00\ \text{s}) = -9.80\ \text{m/s}$$



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v_{oy} = +19.6 m/s Δy = ? v_y = ? Time = 4.00 s

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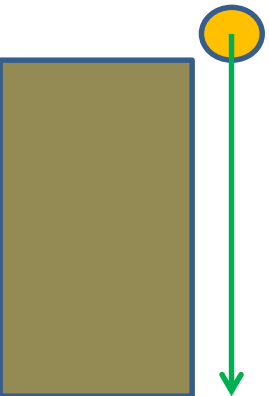
v_{oy} = +19.6 m/s Δy = 0 m v_y = -19.6 m/s Time = 4.00 s

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+19.6\ \text{m/s})(4.00\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(4.00\ \text{s})^2 = 0\ \text{m}$$

$$v_y = v_{0y} + gt = (+19.6\ \text{m/s}) + (-9.80\ \text{m/s}^2)(4.00\ \text{s}) = -19.6\ \text{m/s}$$



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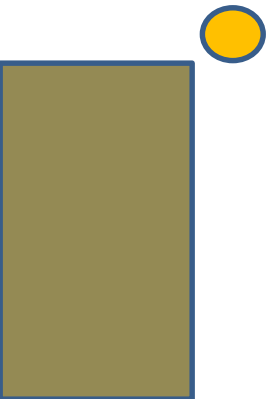
v_{oy} = +19.6 m/s Δy = ? v_y = ? Time = 4.50 s



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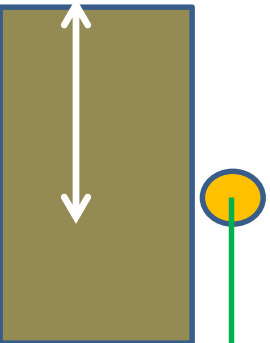
v_{oy} = +19.6 m/s Δy = -11.0 m v_y = -24.5 m/s Time = 4.50 s

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+19.6\ \text{m/s})(4.50\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(4.50\ \text{s})^2 = -11.0\ \text{m}$$

$$v_y = v_{0y} + gt = (+19.6\ \text{m/s}) + (-9.80\ \text{m/s}^2)(4.50\ \text{s}) = -24.5\ \text{m/s}$$



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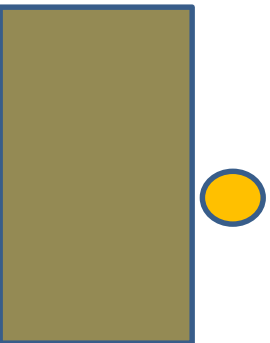
v_{oy} = +19.6 m/s Δy = ? v_y = ? Time = 4.84 s



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v_{oy} = +19.6 m/s Δy = -19.9 m v_y = -27.8 m/s Time = 4.84 s

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+19.6\ \text{m/s})(4.84\ \text{s}) + \tfrac{1}{2}(-9.80\ \text{m/s}^2)(4.84\ \text{s})^2 = -19.9\ \text{m}$$

$$v_y = v_{0y} + gt = (+19.6\ \text{m/s}) + (-9.80\ \text{m/s}^2)(4.84\ \text{s}) = -27.8\ \text{m/s}$$

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How much time does it take the object to reach $\Delta y = -50.0$ m?

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How much time does it take the object to reach $\Delta y = -50.0$ m?

$$\Delta y = y - y_0 = v_{0y}t + \tfrac{1}{2}gt^2$$

$$= (-50.0\text{ m}) - (0\text{ m}) = (19.6\text{ m/s})t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$\therefore\ (4.90\text{ m/s}^2)t^2 + (-19.6\text{ m/s})t + (-50.0\text{ m}) = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-19.6) \pm \sqrt{(-19.6)^2 - 4(4.90)(-50.0)}}{2(4.90)}$$

$$= \frac{19.6 \pm 36.9}{9.80} = \boxed{5.77\text{ s}}$$

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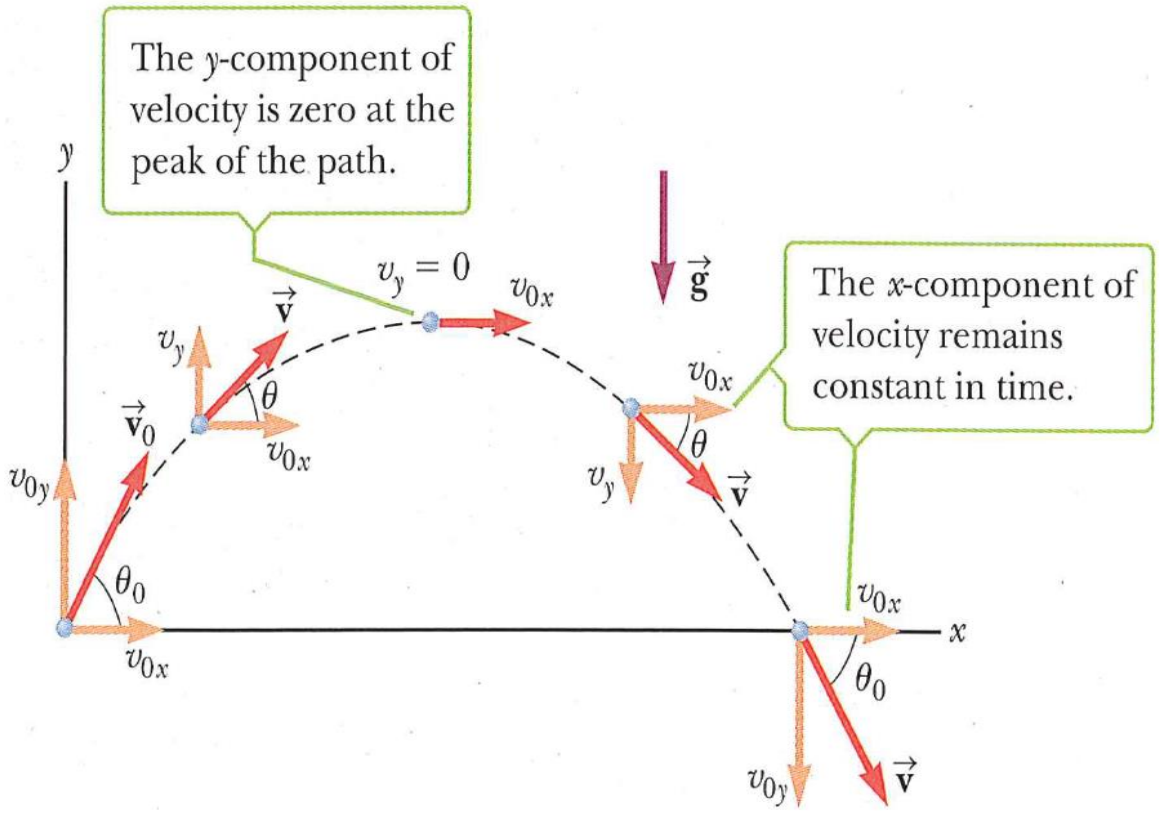

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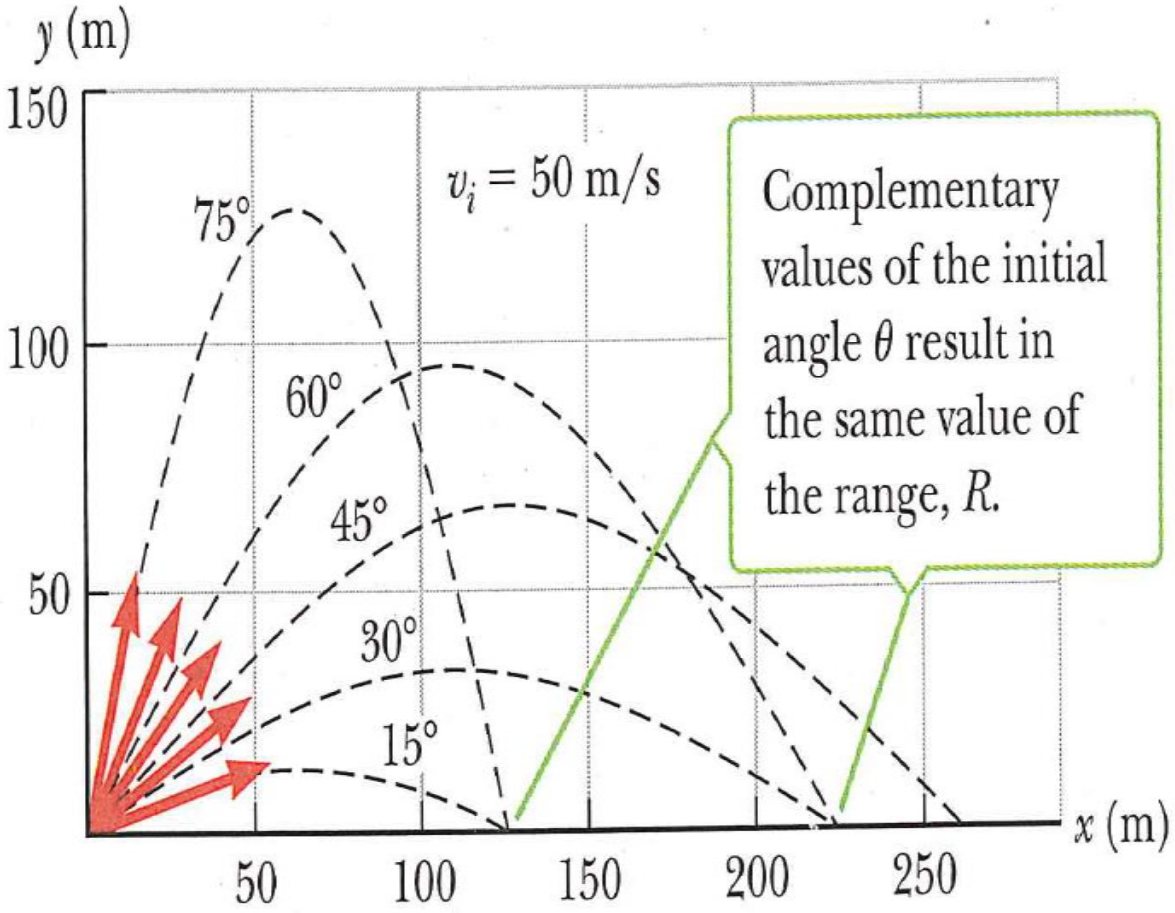

Photo/Illustration courtesy of *College Physics*, 9th Ed.

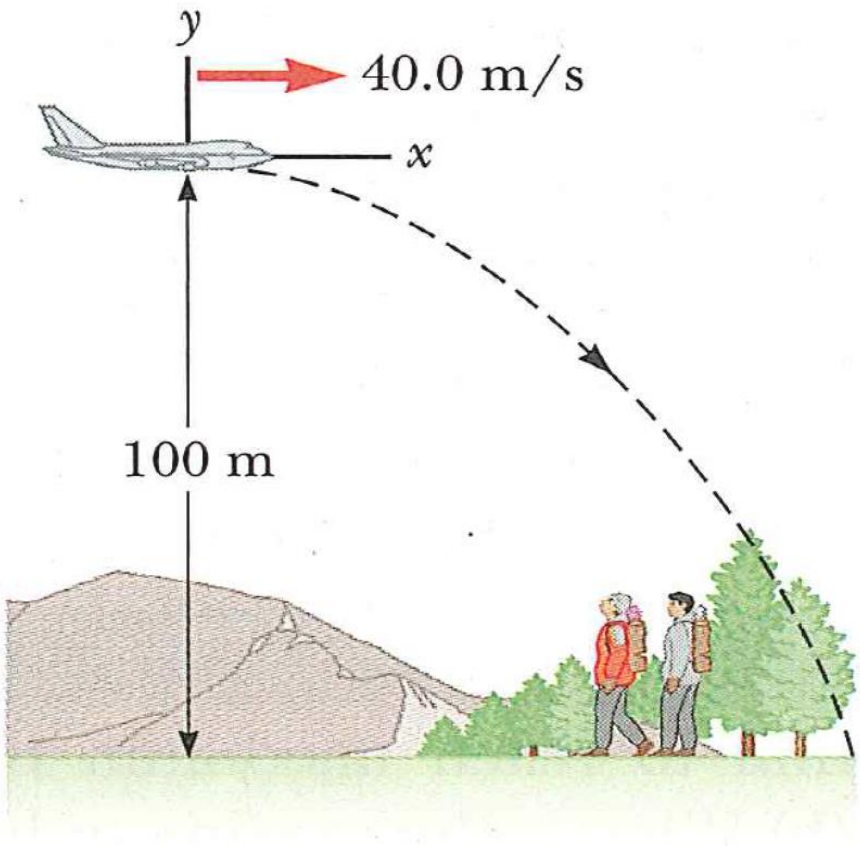

Figure 3.19

Photo/Illustration courtesy of *College Physics*, 9th Ed.

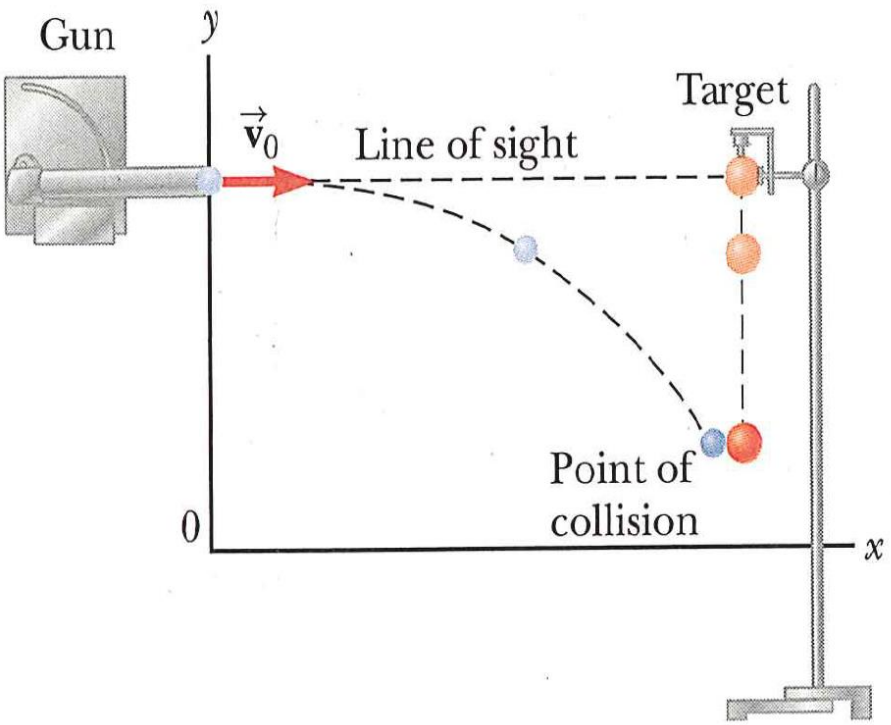

Figure 3.16

Photo/Illustration courtesy of *College Physics*, 9th Ed.

- Horizontal Velocity Only (HVO)

"Mug Slide"

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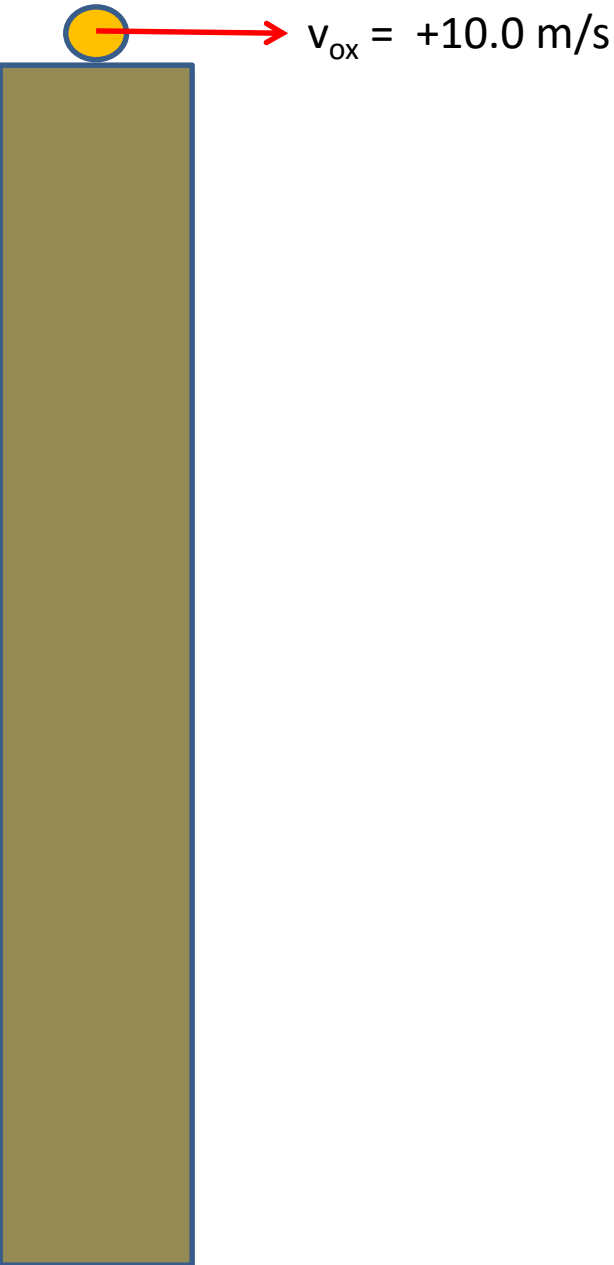
$v_{oy} = 0$ m/s



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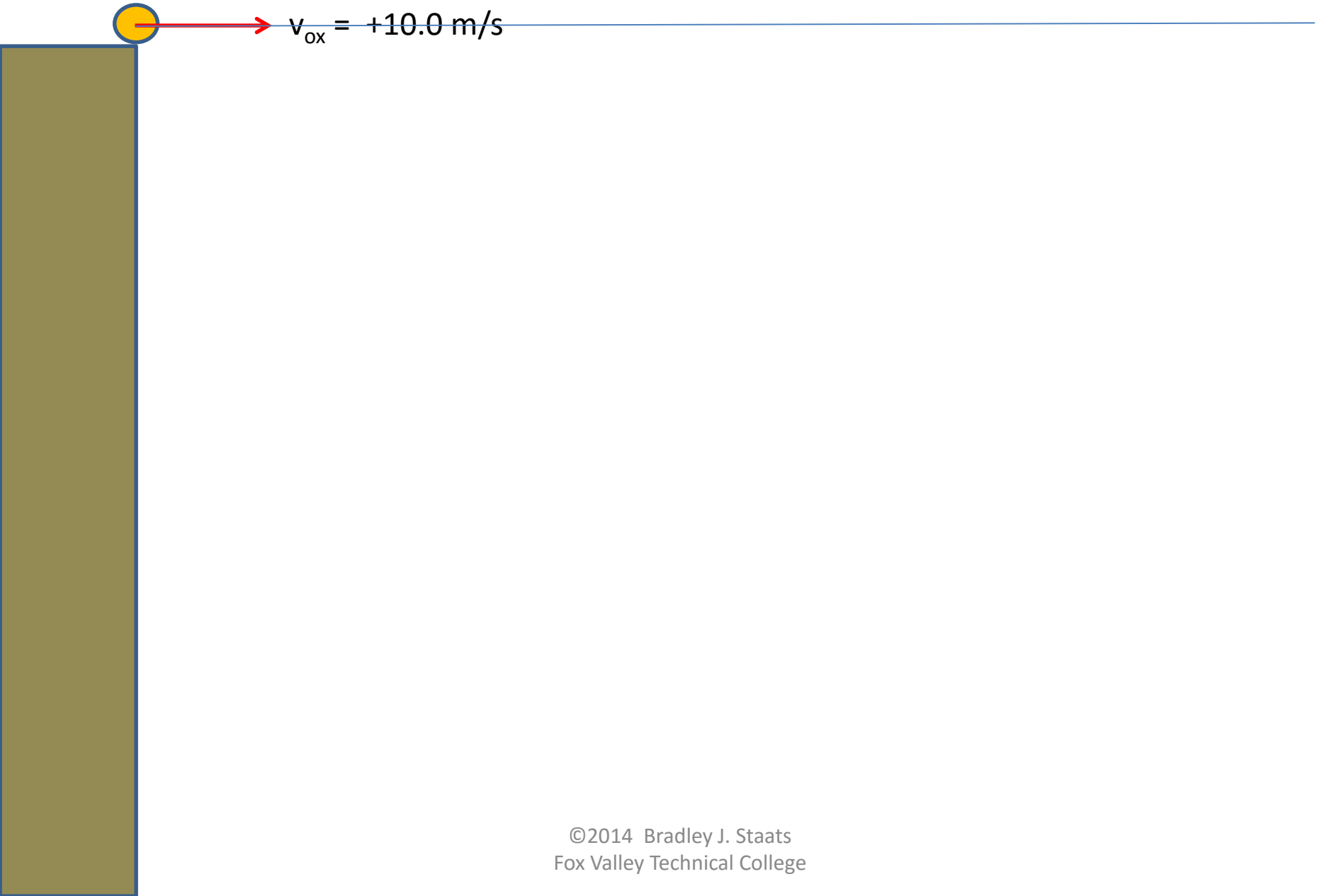
v_{oy} = 0 m/s

Time = 0 s



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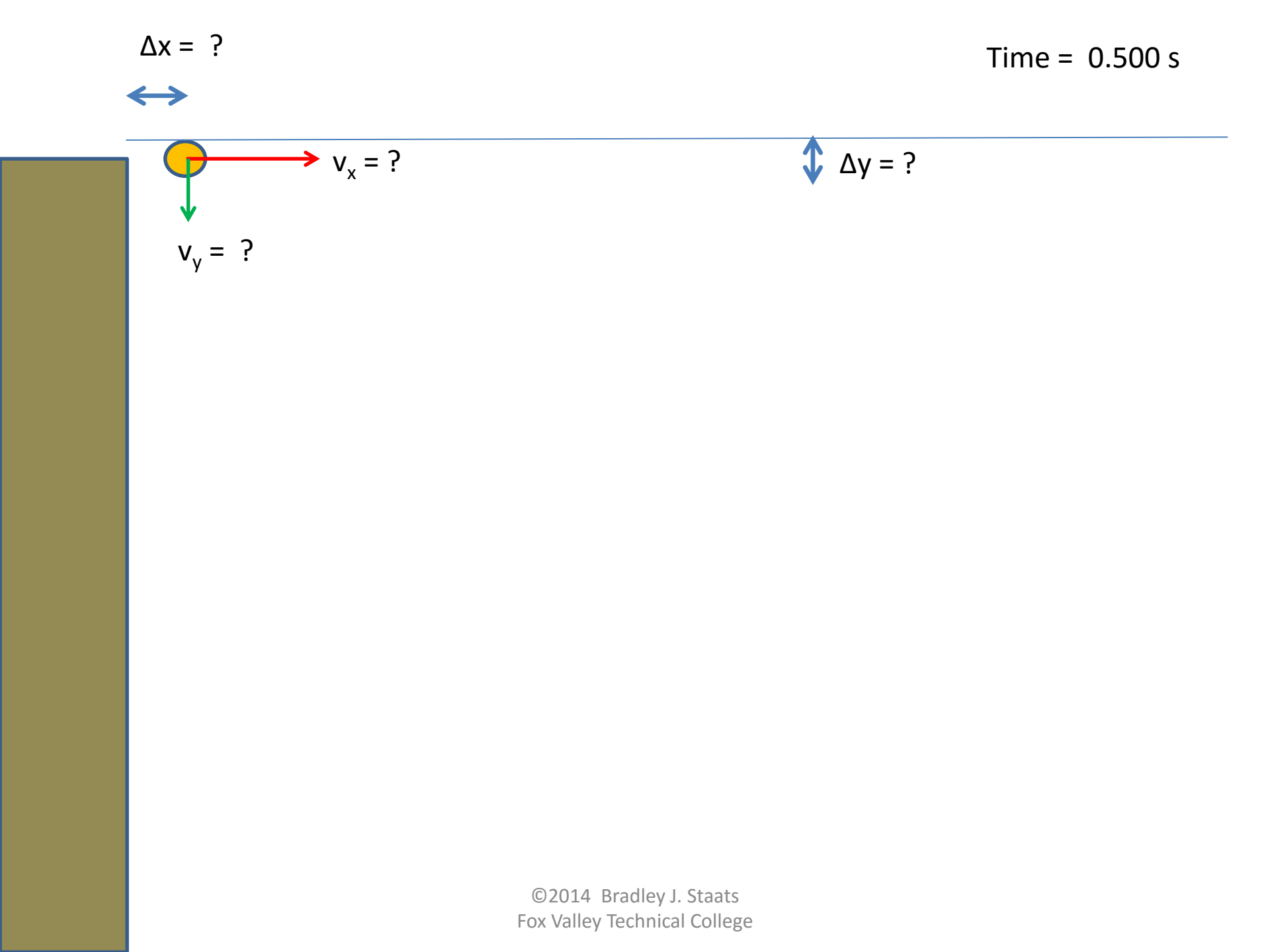

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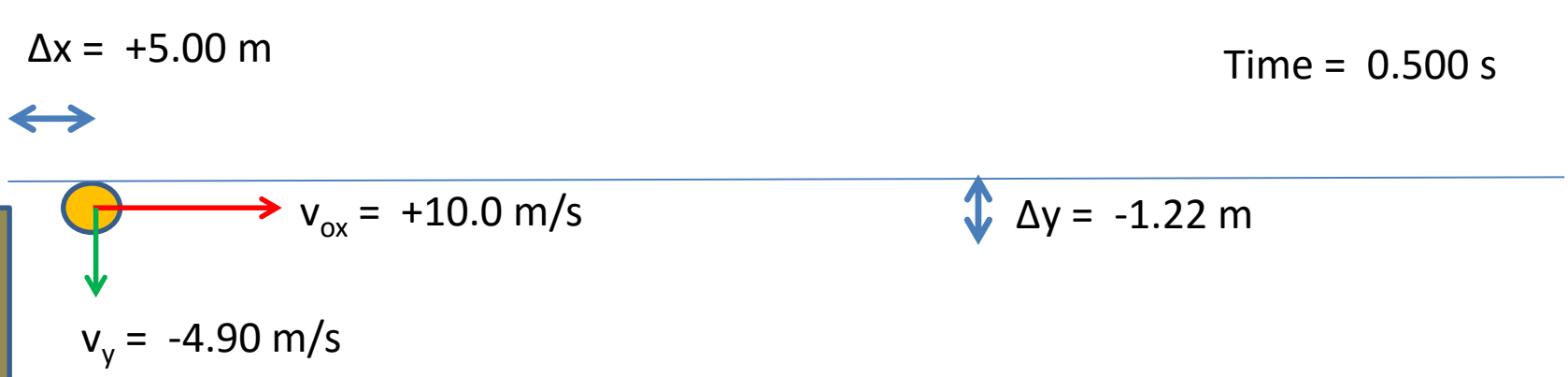

$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(0.500 \text{ s}) = +5.00 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (0 \text{ m/s})(0.500 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(0.500 \text{ s})^2 = -1.22 \text{ m}$$

$$v_y = v_{0y} + gt = (0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(0.500 \text{ s}) = -4.90 \text{ m/s}$$

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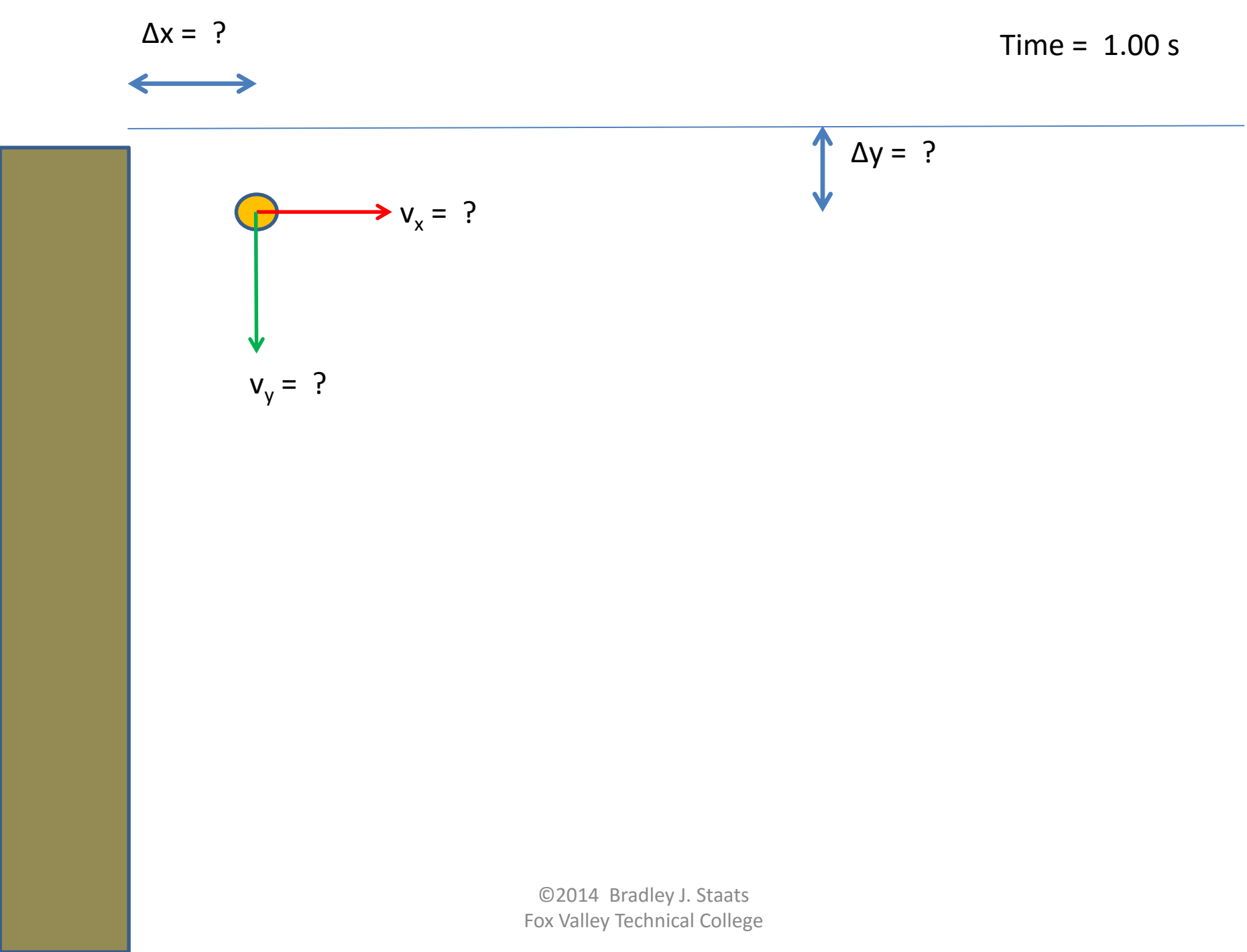

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(1.00 \text{ s}) = +10.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (0 \text{ m/s})(1.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = -4.90 \text{ m}$$

$$v_y = v_{0y} + gt = (0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(1.00 \text{ s}) = -9.80 \text{ m/s}$$

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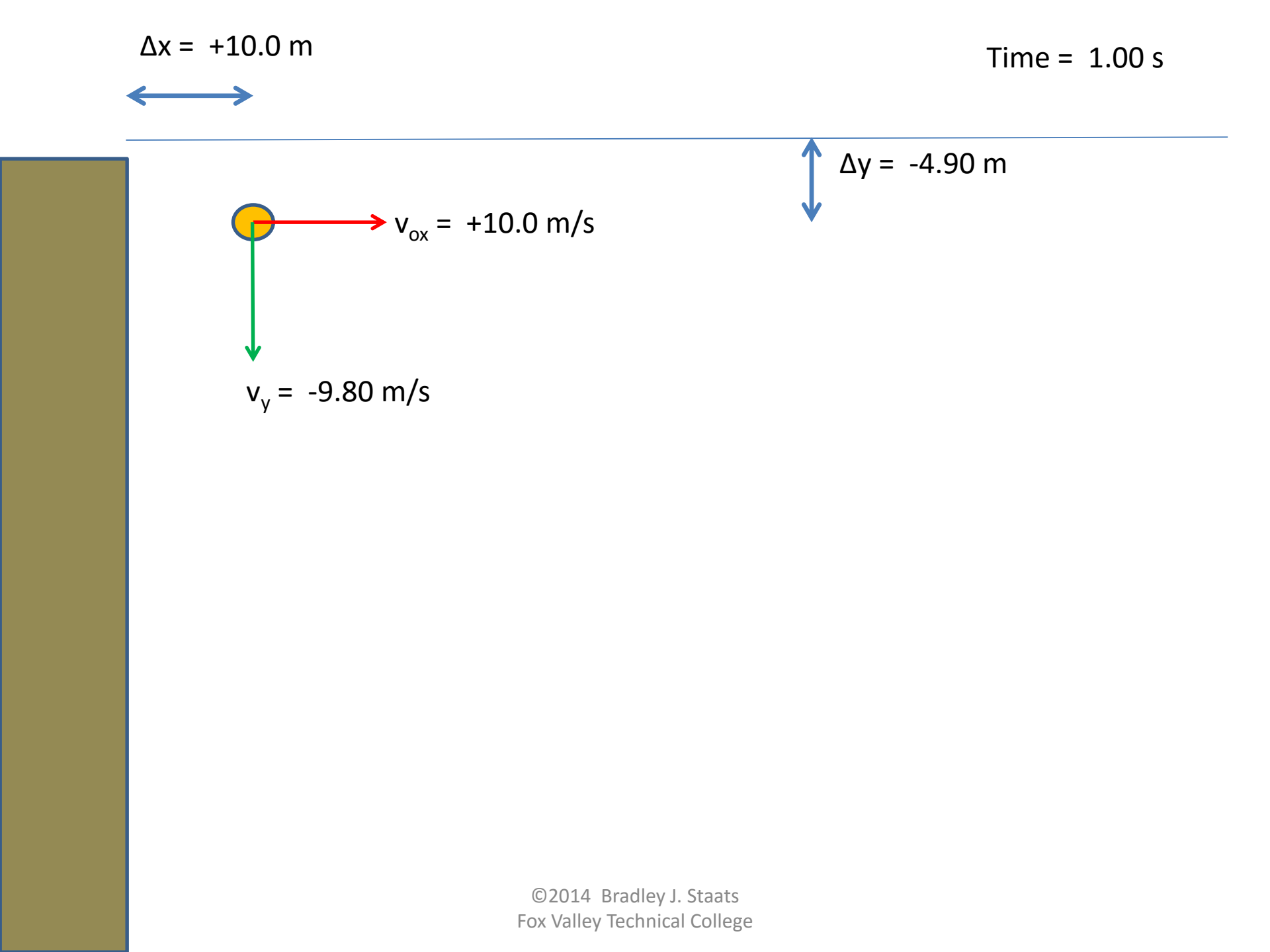

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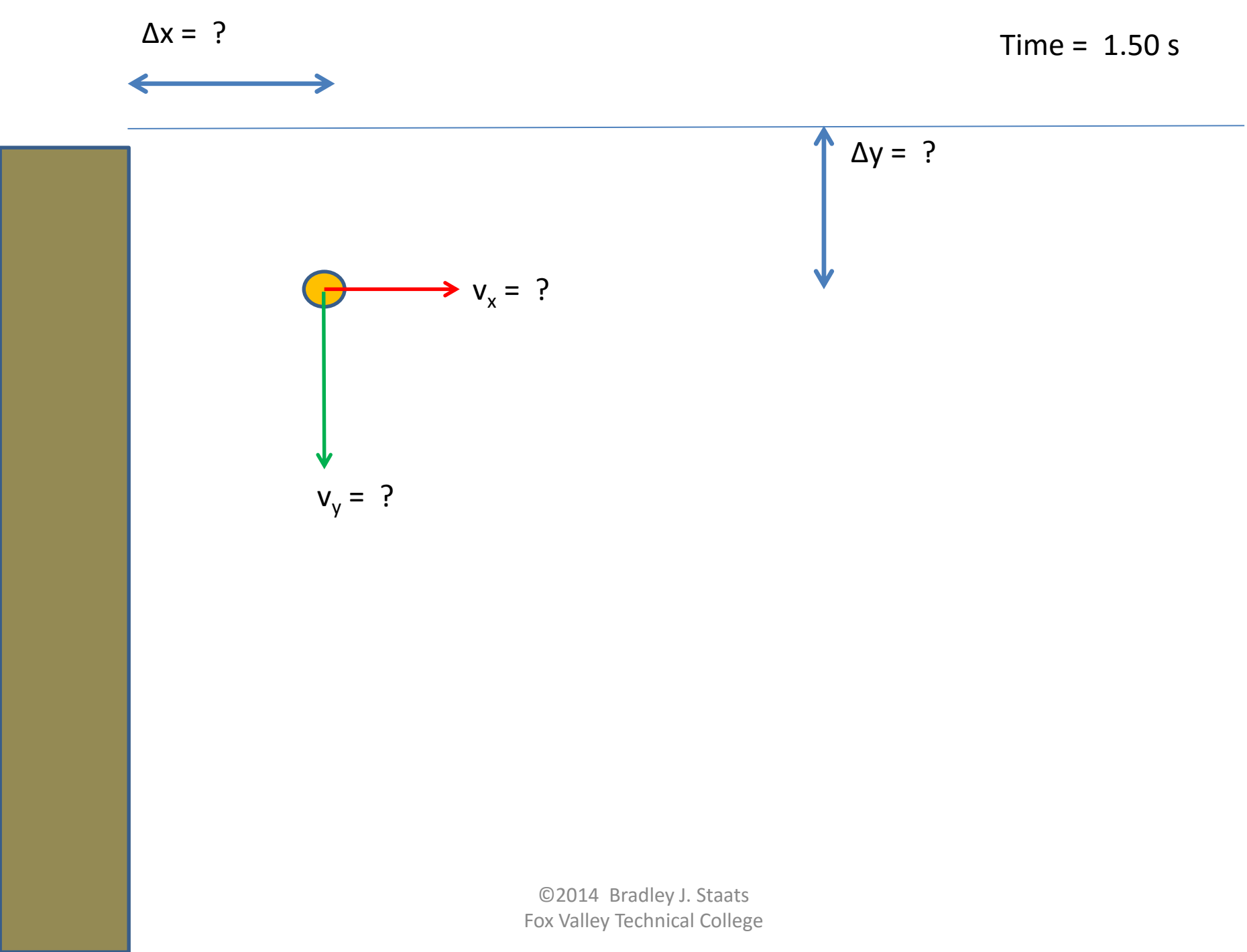

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(1.50 \text{ s}) = +15.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (0 \text{ m/s})(1.50 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(1.50 \text{ s})^2 = -11.0 \text{ m}$$

$$v_y = v_{0y} + gt = (0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(1.50 \text{ s}) = -14.7 \text{ m/s}$$

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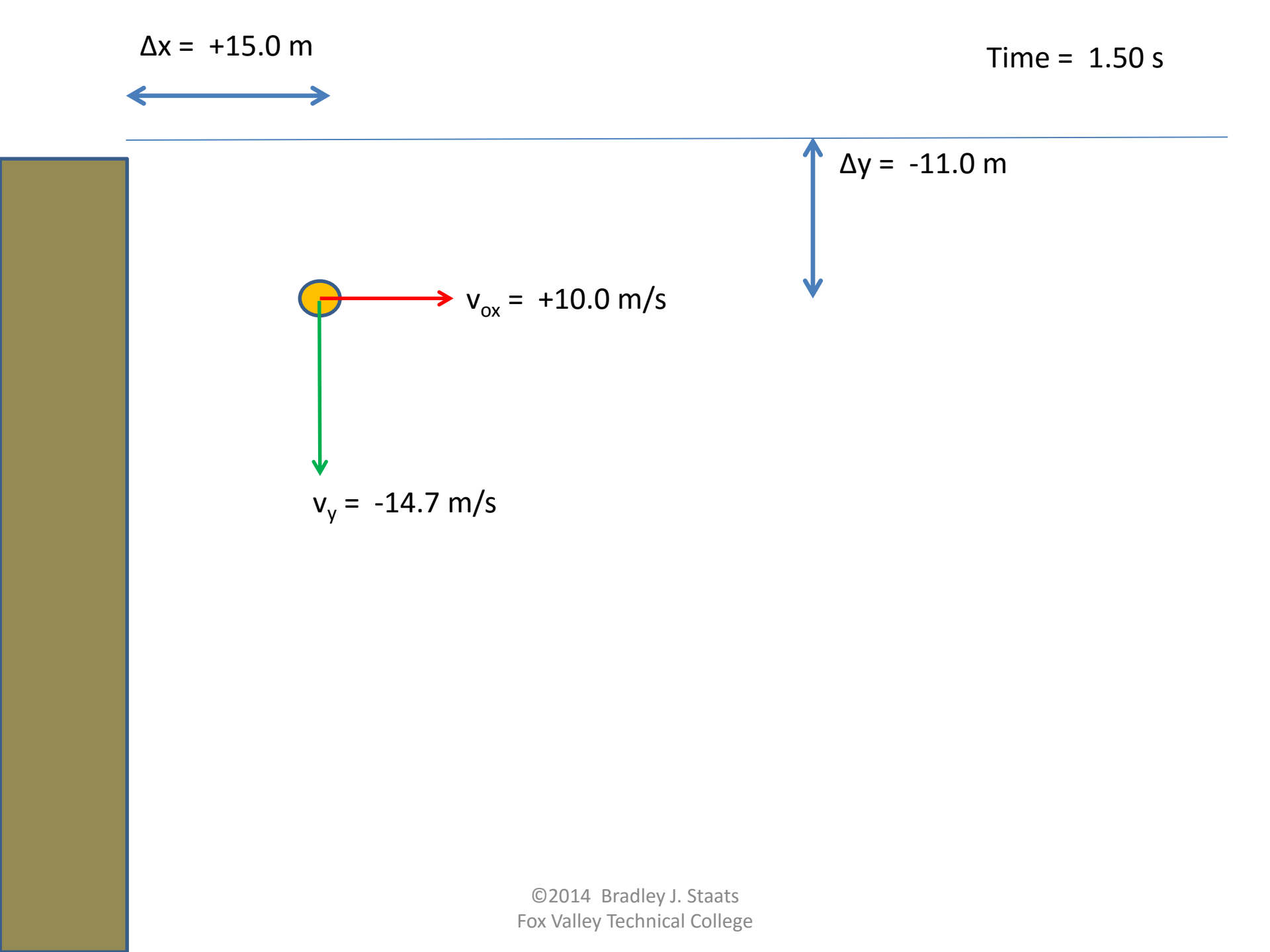

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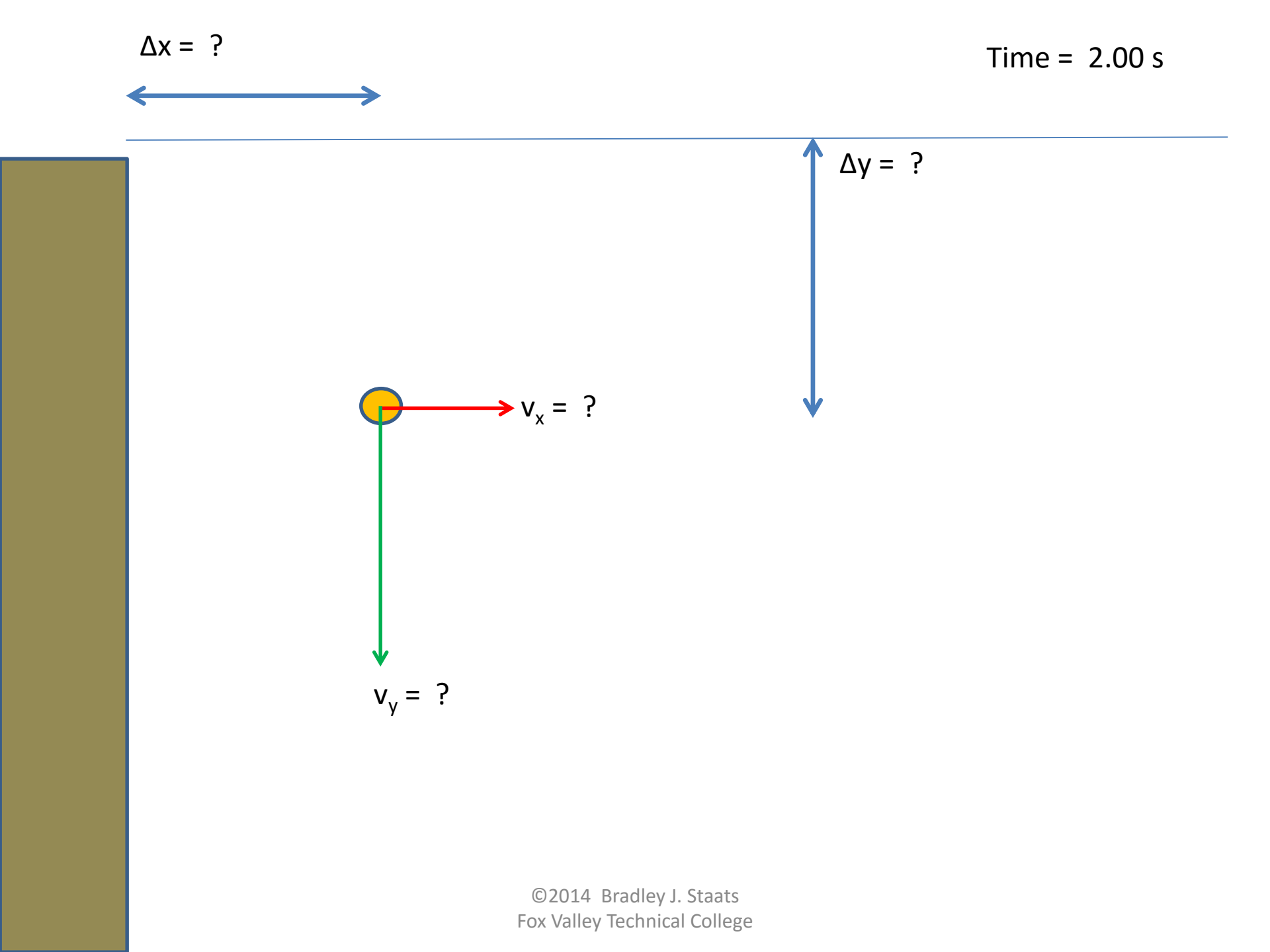

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(2.00 \text{ s}) = +20.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (0 \text{ m/s})(2.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.00 \text{ s})^2 = -19.6 \text{ m}$$

$$v_y = v_{0y} + gt = (0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(2.00 \text{ s}) = -19.6 \text{ m/s}$$

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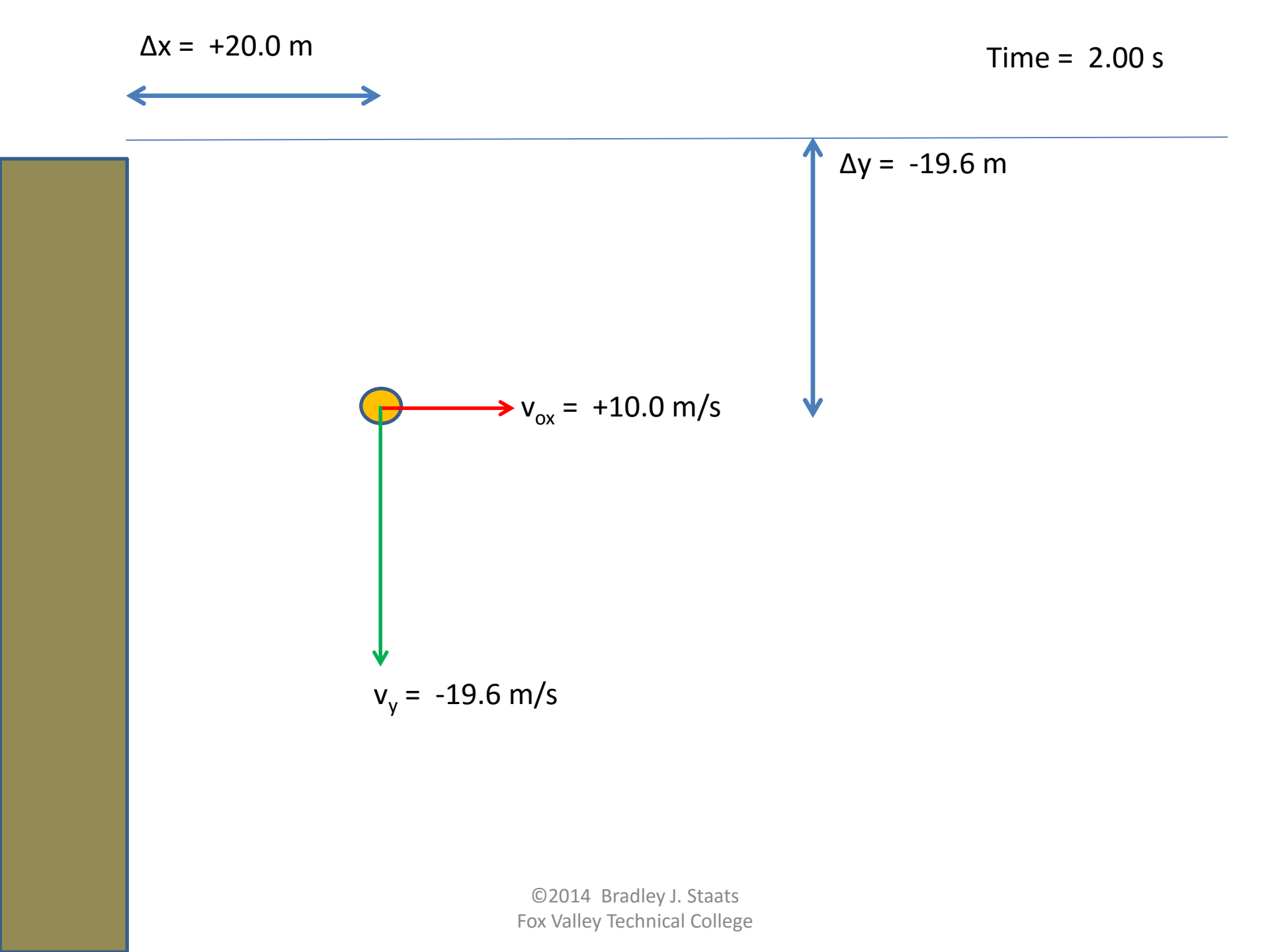

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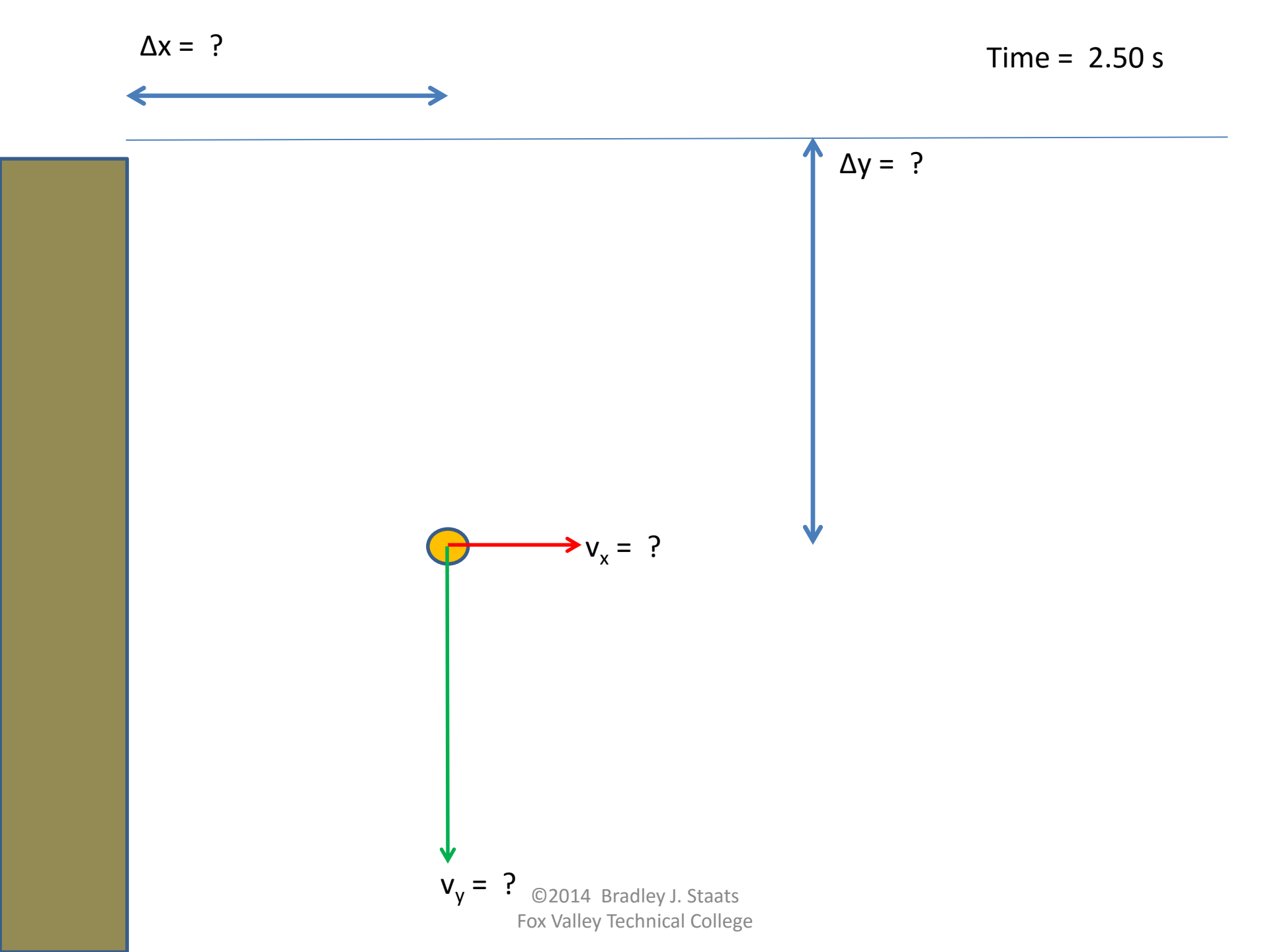

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(2.50 \text{ s}) = +25.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (0 \text{ m/s})(2.50 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.50 \text{ s})^2 = -30.6 \text{ m}$$

$$v_y = v_{0y} + gt = (0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(2.50 \text{ s}) = -24.5 \text{ m/s}$$

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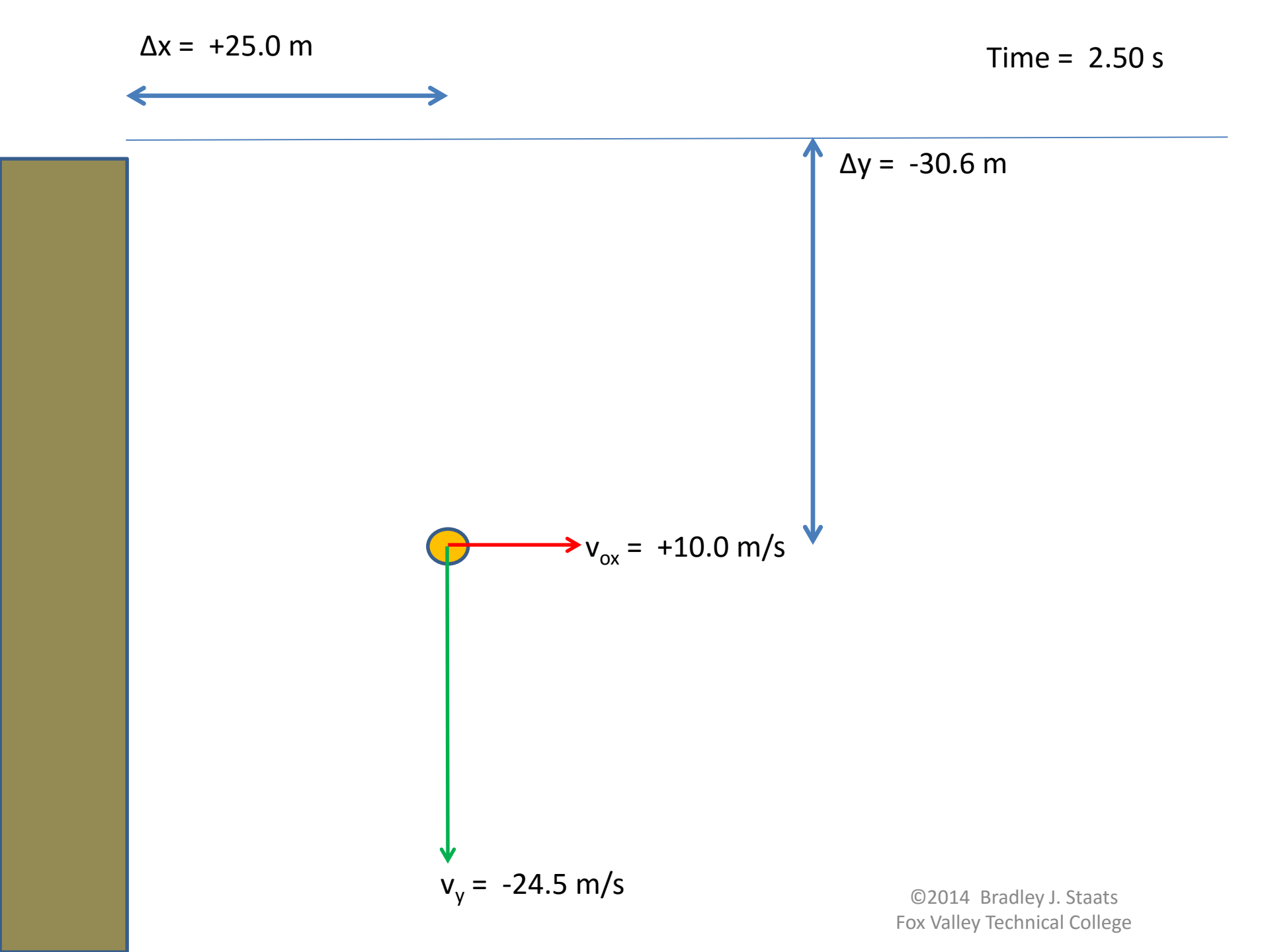

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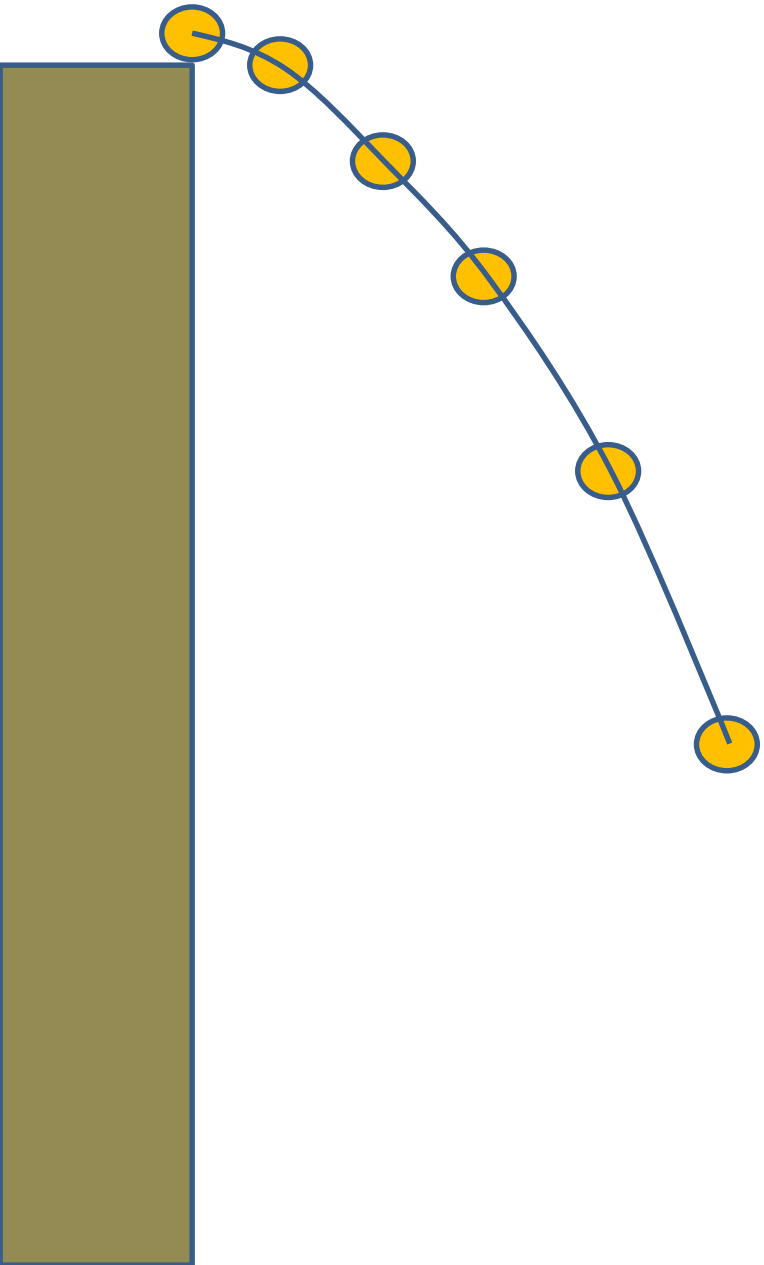
Overall Motion



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How much time does it take the object to reach $\Delta y = -40.0$ m?

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How much time does it take the object to reach $\Delta y = -40.0$ m?

$$\Delta y = y - y_0 = v_{0y}t + \tfrac{1}{2}gt^2$$

$$= (-40.0\text{ m}) - (0\text{ m}) = (0\text{ m/s})t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$\therefore \quad (4.90\text{ m/s}^2)t^2 = 40.0\text{ m}$$

$$\therefore \quad t = \sqrt{\frac{40.0\text{ m}}{4.90\text{ m/s}^2}}$$

$$\therefore \quad t = \boxed{2.86\text{ s}}$$

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If the surface is elevated 60.0 m above the ground and the object's horizontal displacement is 42.0 m, what was it's initial horizontal velocity?

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If the surface is elevated 60.0 m above the ground and the object's horizontal displacement is 42.0 m, what was it's initial horizontal velocity?

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = \left(-60.0\text{ m}\right) = \left(0\text{ m/s}\right)\left(t\right) + \tfrac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(t\right)^2$$

$$\therefore \quad t = \sqrt{\frac{\left(-60.0\text{ m}\right)}{\left(-4.90\text{ m/s}^2\right)}} = 3.50\text{ s}$$

$$\Delta x = v_{0x}t = \left(+42.0\text{ m}\right) = v_{0x}\left(3.50\text{ s}\right)$$

$$\therefore \quad v_{0x} = \frac{\left(+42.0\text{ m}\right)}{\left(3.50\text{ s}\right)} = \boxed{+12.0\text{ m/s}}$$

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If the object's horizontal displacement is 32.0 m and it's initial horizontal velocity is 15.0 m/s, how high is the elevated surface?

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If the object's horizontal displacement is 32.0 m and it's initial horizontal velocity is 15.0 m/s, how high is the elevated surface?

$$\Delta x = v_{0x}t = (+32.0\text{ m}) = (15.0\text{ m/s})(t)$$

$$\therefore \quad t = \frac{(+32.0\text{ m})}{(15.0\text{ m/s})} = 2.13\text{ s}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (0\text{ m/s})(2.13\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(2.13\text{ s})^2$$

$$\therefore \quad \Delta y = \boxed{-22.2\text{ m}}$$

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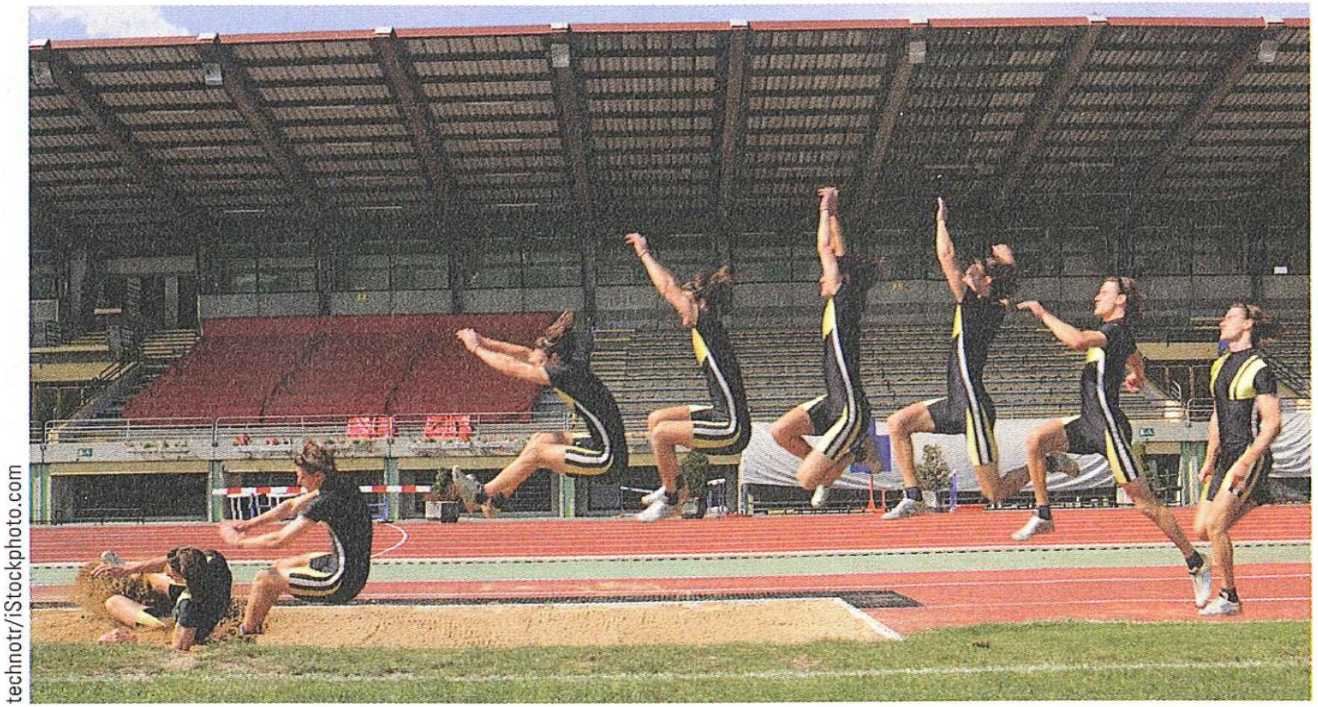

technotr/iStockphoto.com

Figure 3.20

Photo/Illustration courtesy of *College Physics*, 9th Ed.

- Dual Velocity Flat Surface (DVFS)

  “Long Jumper”

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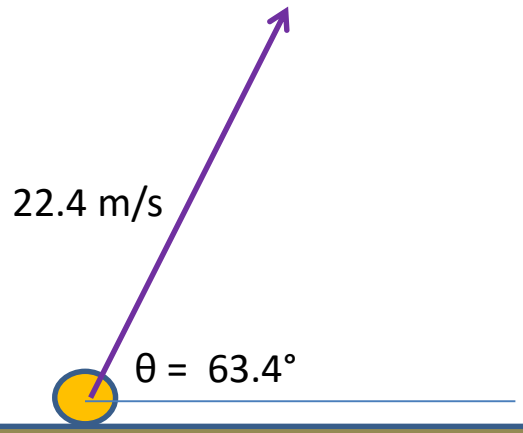
v_{ox} = ?

v_{oy} = ?



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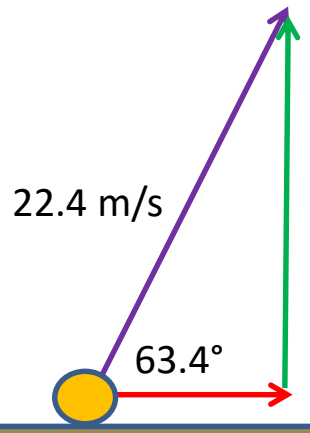
v_{ox} = +10.0 m/s

v_{oy} = +20.0 m/s

$$v_{0x} = (\cos 63.4°)(22.4 \text{ m/s}) = +10.0 \text{ m/s}$$

$$v_{0y} = (\sin 63.4°)(22.4 \text{ m/s}) = +20.0 \text{ m/s}$$



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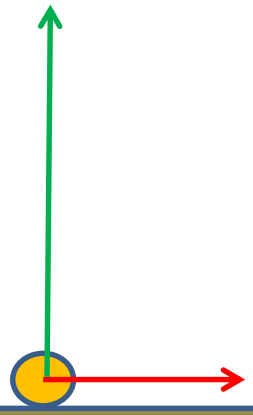
v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s



Time = 0 s

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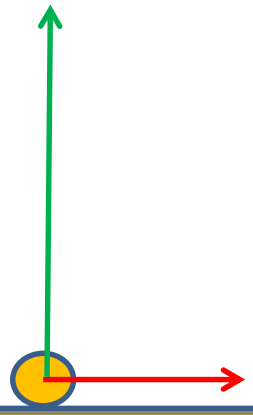
v_{ox} = +10.0 m/s Δx = 0 m v_x = +10.0 m/s Δy = 0 m v_y = +20.0 m/s

v_{oy} = +20.0 m/s



Time = 0 s

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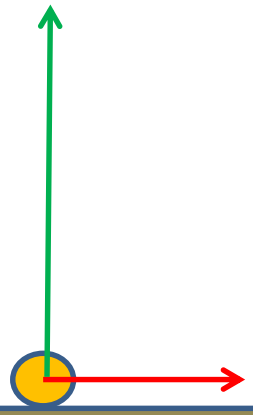
v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s



Time = 0.500 s

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(0.500 \text{ s}) = +5.00 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+20.0 \text{ m/s})(0.500 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(0.500 \text{ s})^2 = +8.78 \text{ m}$$

$$v_y = v_{0y} + gt = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(0.500 \text{ s}) = +15.1 \text{ m/s}$$

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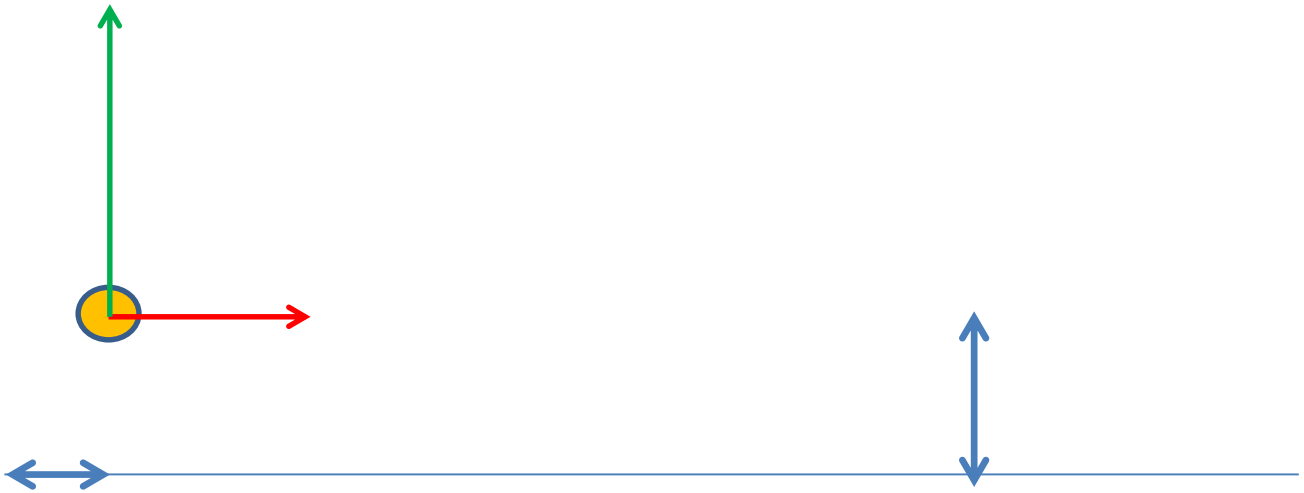
v_{ox} = +10.0 m/s Δx = +5.00 m v_x = +10.0 m/s Δy = +8.78 m v_y = +15.1 m/s

v_{oy} = +20.0 m/s



Time = 0.500 s

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v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s

Time = 1.00 s

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(1.00 \text{ s}) = +10.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+20.0 \text{ m/s})(1.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = +15.1 \text{ m}$$

$$v_y = v_{0y} + gt = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(1.00 \text{ s}) = +10.2 \text{ m/s}$$

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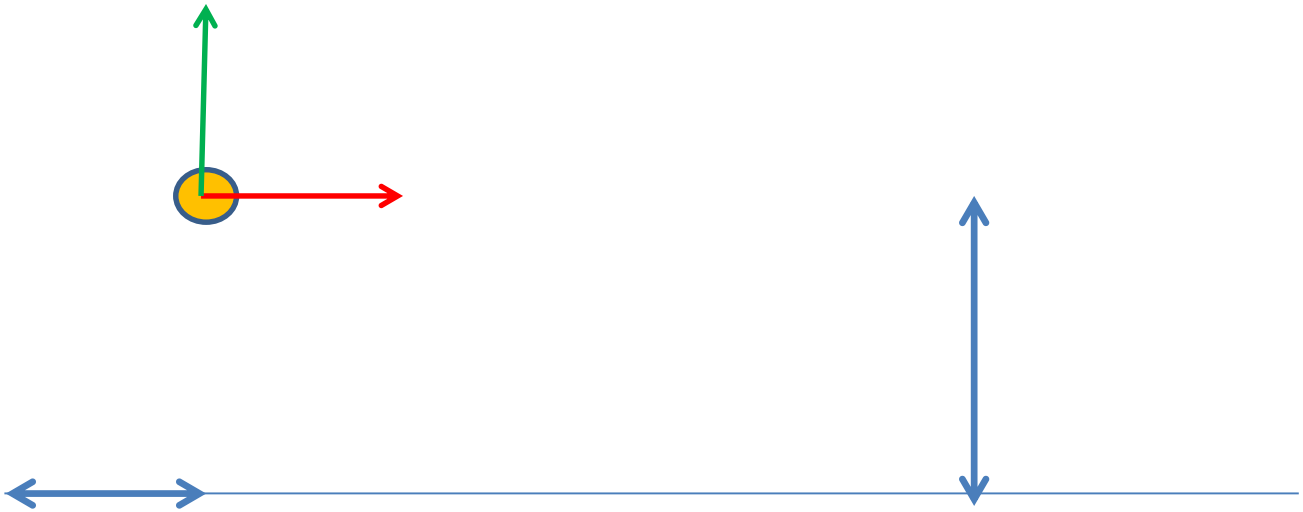
v_{ox} = +10.0 m/s Δx = +10.0 m v_x = +10.0 m/s Δy = +15.1 m v_y = +10.2 m/s

v_{oy} = +20.0 m/s



Time = 1.00 s

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v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s

Time = 1.50 s

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(1.50 \text{ s}) = +15.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+20.0 \text{ m/s})(1.50 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(1.50 \text{ s})^2 = +19.0 \text{ m}$$

$$v_y = v_{0y} + gt = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(1.50 \text{ s}) = +5.30 \text{ m/s}$$

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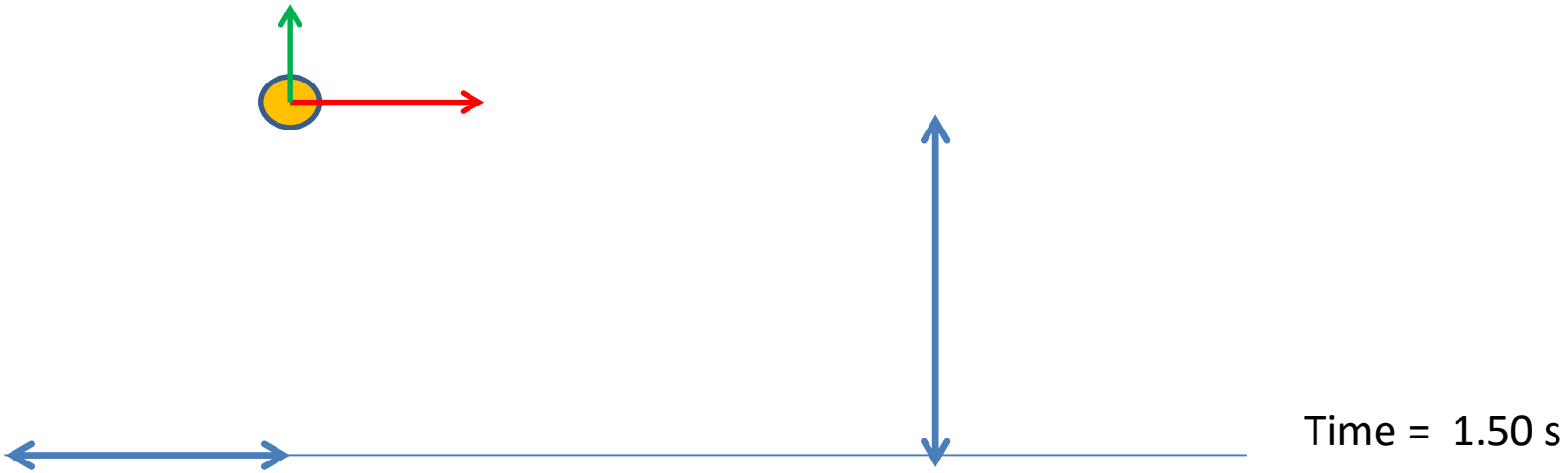
v_{ox} = +10.0 m/s Δx = +15.0 m v_x = +10.0 m/s Δy = +19.0 m v_y = +5.30 m/s

v_{oy} = +20.0 m/s



Time = 1.50 s

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v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s

Time = 2.04 s

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(2.04 \text{ s}) = +20.4 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2 = +20.4 \text{ m}$$

$$v_y = v_{0y} + gt = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(2.04 \text{ s}) = +0.00800 \text{ m/s}$$

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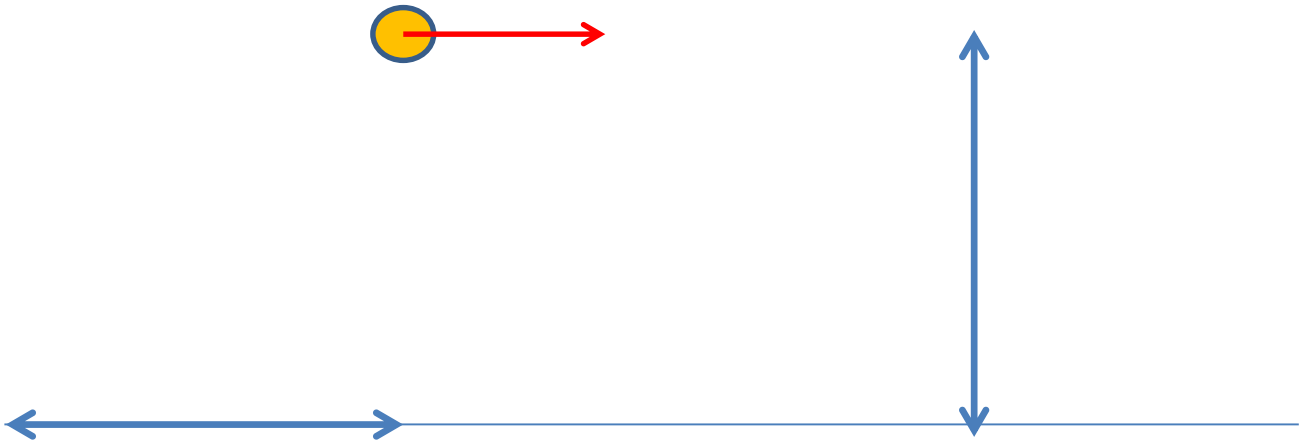
v_{ox} = +10.0 m/s Δx = +20.4 m v_x = +10.0 m/s Δy = +20.4 m v_y = +0.008 m/s

v_{oy} = +20.0 m/s



Time = 2.04 s

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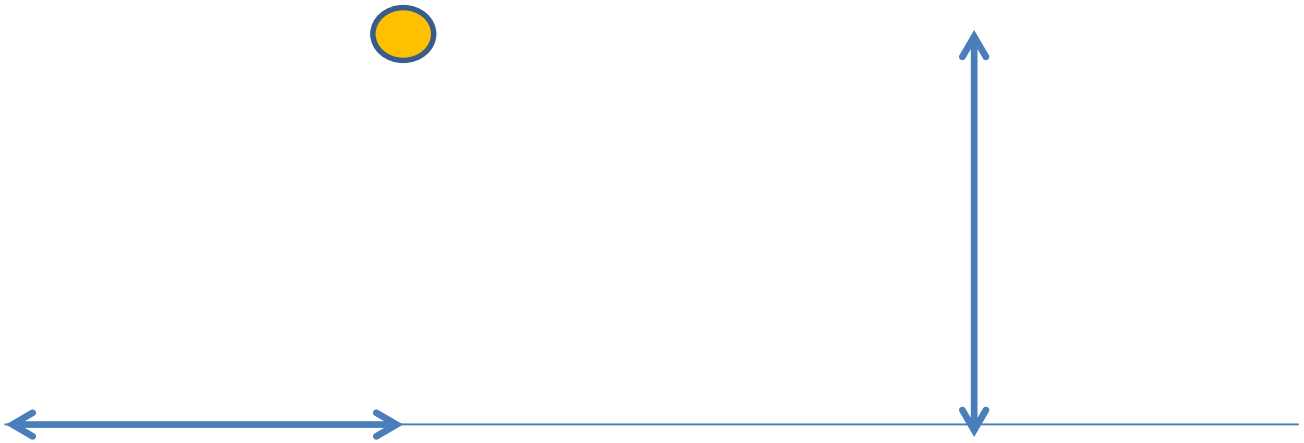
v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s



Time = 2.50 s

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(2.50 \text{ s}) = +25.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+20.0 \text{ m/s})(2.50 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.50 \text{ s})^2 = +19.4 \text{ m}$$

$$v_y = v_{0y} + gt = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(2.50 \text{ s}) = -4.50 \text{ m/s}$$

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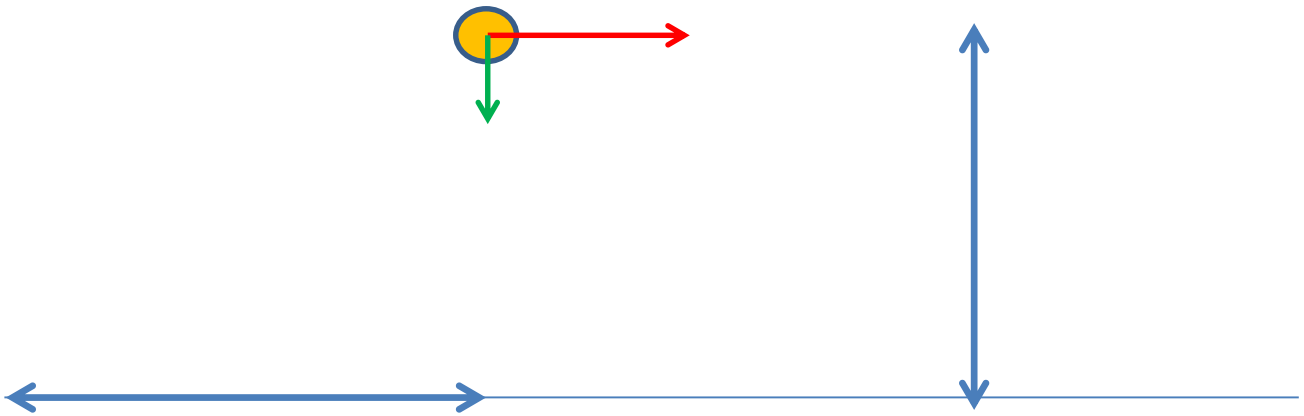
v_{ox} = +10.0 m/s Δx = +25.0 m v_x = +10.0 m/s Δy = +19.4 m v_y = -4.50 m/s

v_{oy} = +20.0 m/s



Time = 2.50 s

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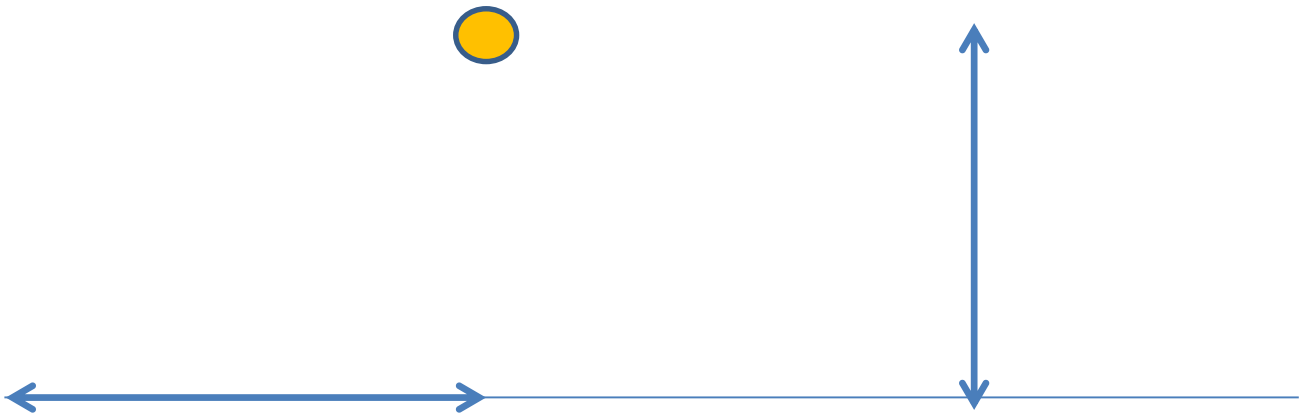
v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s



Time = 3.00 s

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(3.00 \text{ s}) = +30.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+20.0 \text{ m/s})(3.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(3.00 \text{ s})^2 = +15.9 \text{ m}$$

$$v_y = v_{0y} + gt = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(3.00 \text{ s}) = -9.40 \text{ m/s}$$

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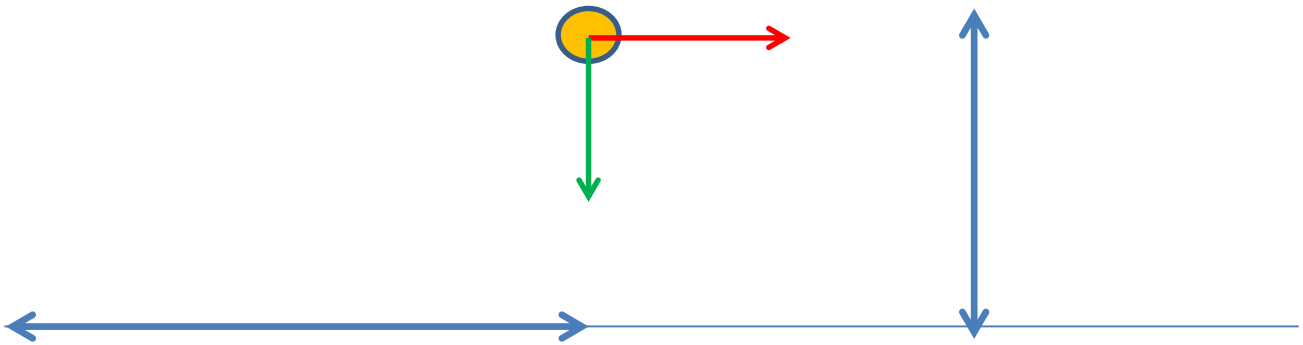
v_{ox} = +10.0 m/s Δx = +30.0 m v_x = +10.0 m/s Δy = +15.9 m v_y = -9.40 m/s

v_{oy} = +20.0 m/s



Time = 3.00 s

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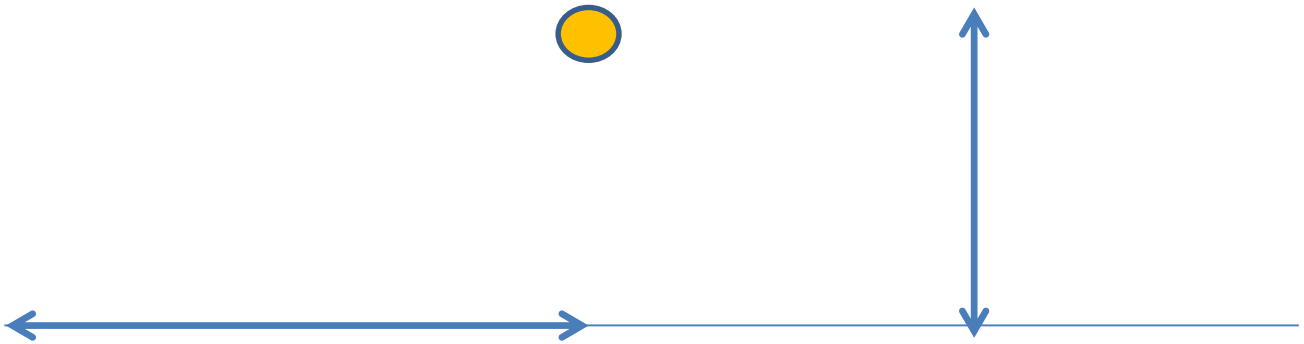
v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s



Time = 3.50 s

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(3.50 \text{ s}) = +35.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+20.0 \text{ m/s})(3.50 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(3.50 \text{ s})^2 = +9.98 \text{ m}$$

$$v_y = v_{0y} + gt = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(3.50 \text{ s}) = -14.3 \text{ m/s}$$

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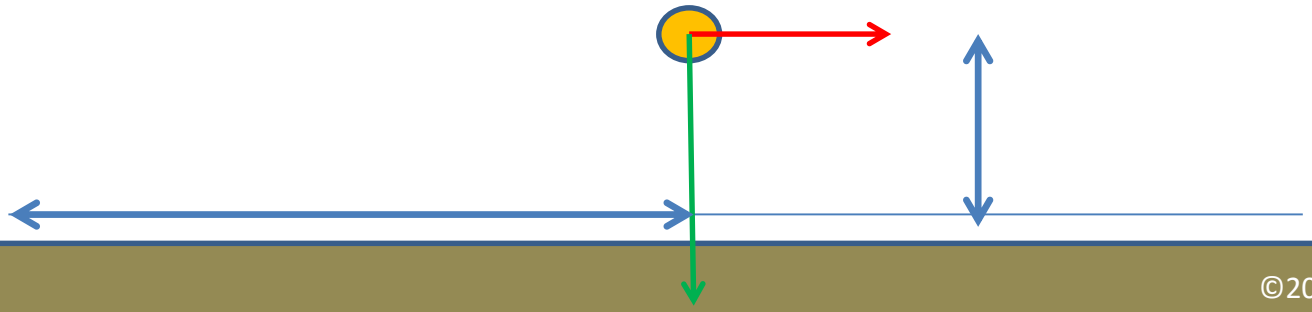
v_{ox} = +10.0 m/s Δx = +35.0 m v_x = +10.0 m/s Δy = +9.98 m v_y = -14.3 m/s

v_{oy} = +20.0 m/s



Time = 3.50 s

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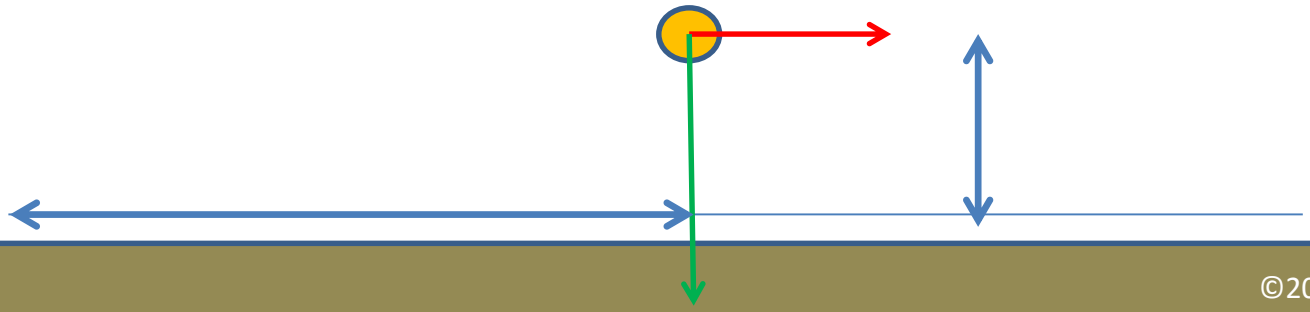
v_{ox} = +10.0 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +20.0 m/s



Time = 4.00 s

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$$v_{0x} = v_x = \text{constant} = +10.0 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+10.0 \text{ m/s})(4.00 \text{ s}) = +40.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+20.0 \text{ m/s})(4.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(4.00 \text{ s})^2 = +1.60 \text{ m}$$

$$v_y = v_{0y} + gt = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(4.00 \text{ s}) = -19.2 \text{ m/s}$$

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v_{ox} = +10.0 m/s Δx = +40.0 m v_x = +10.0 m/s Δy = +1.60 m v_y = -19.2 m/s

v_{oy} = +20.0 m/s

Time = 4.00 s

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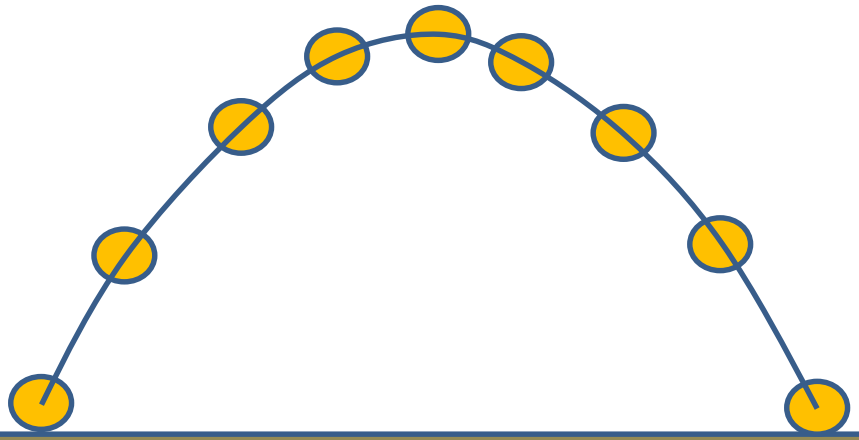
Overall Motion



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How much time does it take the object to reach $\Delta y = +13.0$ m?

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How much time does it take the object to reach Δy = +13.0 m?

$$\Delta y = y - y_0 = v_{0y}t + \tfrac{1}{2}gt^2$$

$$= (13.0\text{ m}) - (0\text{ m}) = (20.0\text{ m/s})t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$\therefore\ (4.90\text{ m/s}^2)t^2 + (-20.0\text{ m/s})t + (13.0\text{ m}) = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-20.0) \pm \sqrt{(-20.0)^2 - 4(4.90)(13.0)}}{2(4.90)}$$

$$= \frac{20.0 \pm 12.0}{9.80} = \boxed{0.816\text{ s} \quad \text{or} \quad 3.27\text{ s}}$$

** The two times represent the time to rise to +13.0 m and the time to fall back down to +13.0 m respectively.

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What is the maximum height that it achieves?

$$v_y^{\,2} = v_{0y}^{\,2} + 2g\Delta y$$

$$\therefore \quad \Delta y = \frac{v_y^{\,2} - v_{0y}^{\,2}}{2g} = \frac{(0\ \text{m/s})^2 - (20.0\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)}$$

$$\therefore \quad \Delta y = \boxed{20.4\ \text{m}}$$

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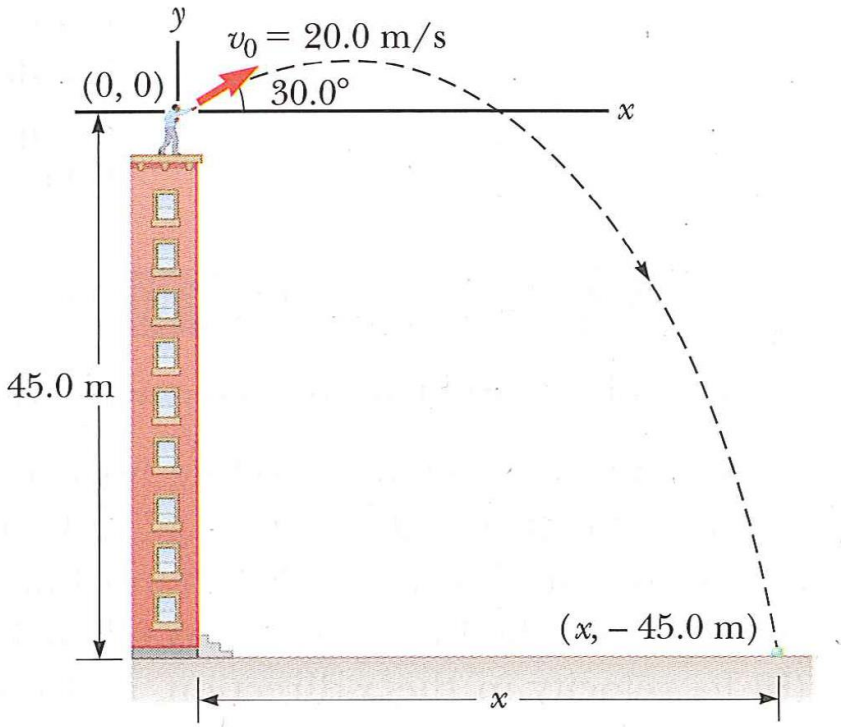

Figure 3.21

Photo/Illustration courtesy of *College Physics*, 9th Ed.

- Dual Velocity Elevated Launch (DVEL)

"Stone's Throw"

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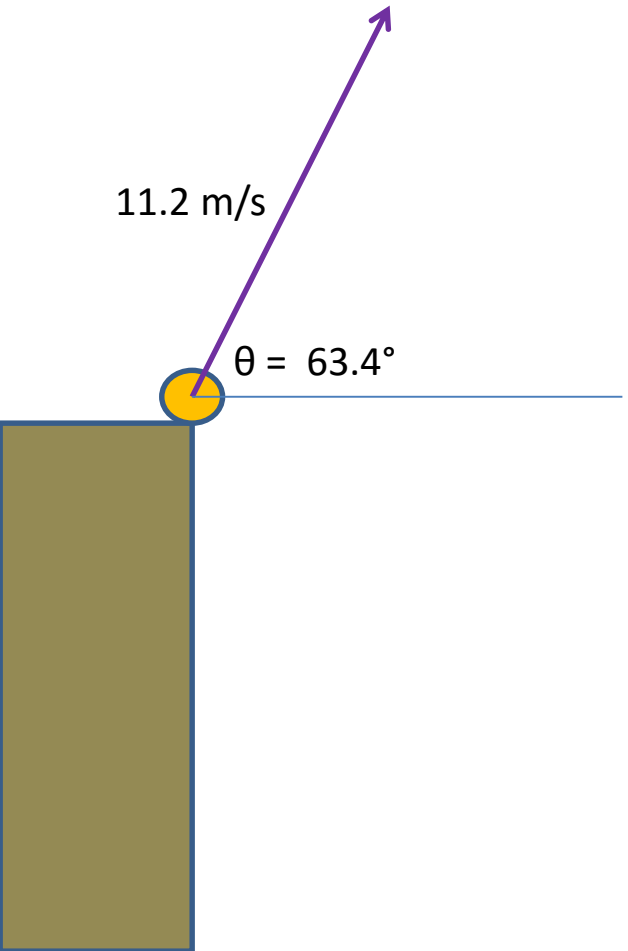
v_{ox} = ?

v_{oy} = ?



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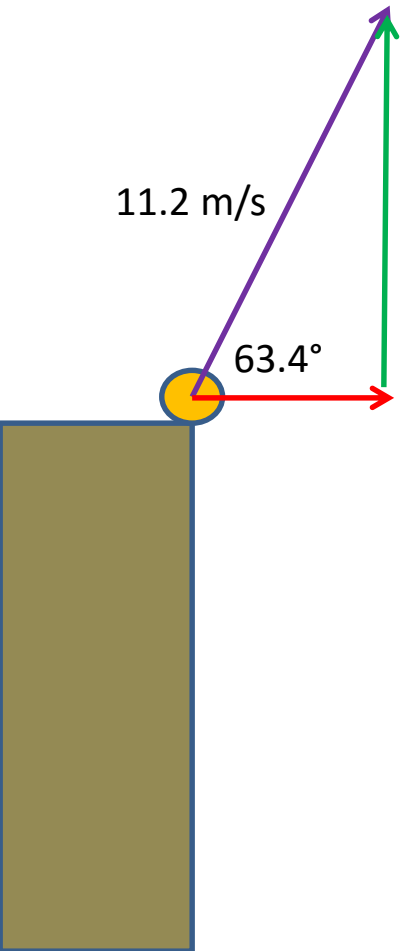
v_{ox} = +5.01 m/s

v_{oy} = +10.0 m/s



$$v_{0x} = (\cos 63.4°)(11.2 \text{ m/s}) = +5.01 \text{ m/s}$$

$$v_{0y} = (\sin 63.4°)(11.2 \text{ m/s}) = +10.0 \text{ m/s}$$

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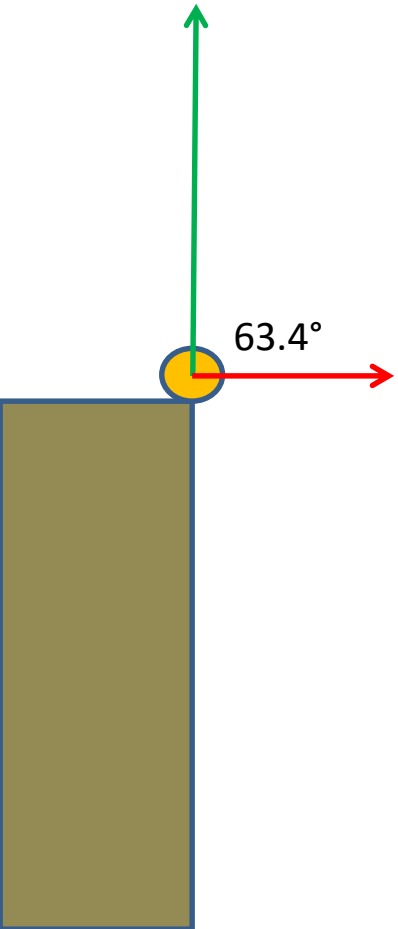
v_{ox} = +5.01 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +10.0 m/s



Time = 0 s

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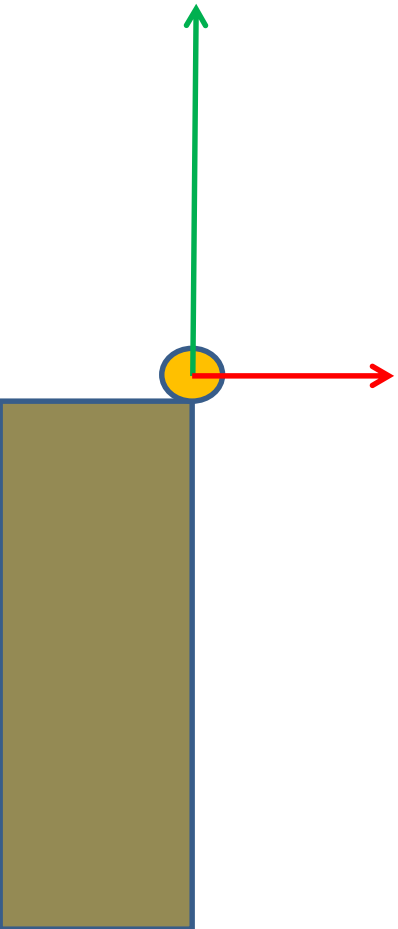
v_{ox} = +5.01 m/s Δx = 0 m v_x = +5.01 m/s Δy = 0 m v_y = +10.0 m/s

v_{oy} = +10.0 m/s



Time = 0 s

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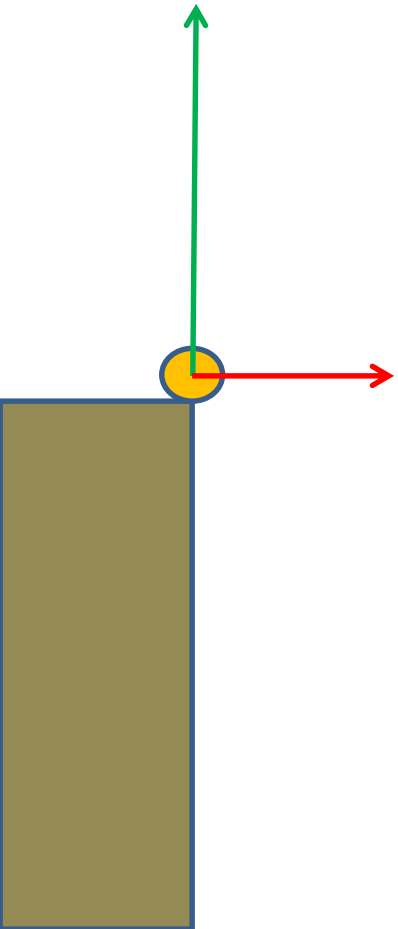
v_{ox} = +5.01 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +10.0 m/s



Time = 0.500 s

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$$v_{0x} = v_x = \text{constant} = +5.01 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+5.01 \text{ m/s})(0.500 \text{ s}) = +2.50 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+10.0 \text{ m/s})(0.500 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(0.500 \text{ s})^2 = +3.78 \text{ m}$$

$$v_y = v_{0y} + gt = (+10.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(0.500 \text{ s}) = +5.10 \text{ m/s}$$

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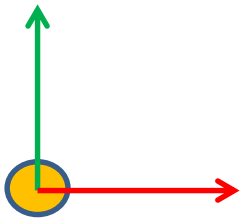
v_{ox} = +5.01 m/s Δx = +2.50 m v_x = +5.01 m/s Δy = +3.78 m v_y = +5.10 m/s

v_{oy} = +10.0 m/s



Time = 0.500 s

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v_{ox} = +5.01 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +10.0 m/s

Time = 1.00 s

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$$v_{0x} = v_x = \text{constant} = +5.01 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+5.01 \text{ m/s})(1.00 \text{ s}) = +5.01 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+10.0 \text{ m/s})(1.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = +5.10 \text{ m}$$

$$v_y = v_{0y} + gt = (+10.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(1.00 \text{ s}) = +0.200 \text{ m/s}$$

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v_{ox} = +5.01 m/s Δx = +5.01 m v_x = +5.01 m/s Δy = +5.10 m v_y = +0.200 m/s

v_{oy} = +10.0 m/s

Time = 1.00 s

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v_{ox} = +5.01 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +10.0 m/s

Time = 1.50 s

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$$v_{0x} = v_x = \text{constant} = +5.01 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+5.01 \text{ m/s})(1.50 \text{ s}) = +7.52 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+10.0 \text{ m/s})(1.50 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(1.50 \text{ s})^2 = +3.98 \text{ m}$$

$$v_y = v_{0y} + gt = (+10.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(1.50 \text{ s}) = -4.70 \text{ m/s}$$

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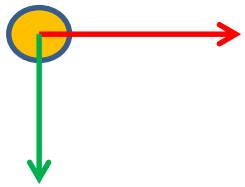
v_{ox} = +5.01 m/s Δx = +7.52 m v_x = +5.01 m/s Δy = +3.98 m v_y = -4.70 m/s

v_{oy} = +10.0 m/s



Time = 1.50 s

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v_{ox} = +5.01 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +10.0 m/s

Time = 2.00 s

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$$v_{0x} = v_x = \text{constant} = +5.01 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+5.01 \text{ m/s})(2.00 \text{ s}) = +10.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+10.0 \text{ m/s})(2.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.00 \text{ s})^2 = +0.400 \text{ m}$$

$$v_y = v_{0y} + gt = (+10.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(2.00 \text{ s}) = -9.60 \text{ m/s}$$

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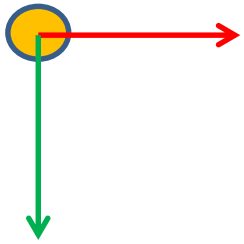
v_{ox} = +5.01 m/s Δx = +10.0 m v_x = +5.01 m/s Δy = +0.400 m v_y = -9.60 m/s

v_{oy} = +10.0 m/s



Time = 2.00 s

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v_{ox} = +5.01 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +10.0 m/s

Time = 2.50 s

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$$v_{0x} = v_x = \text{constant} = +5.01 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+5.01 \text{ m/s})(2.50 \text{ s}) = +12.5 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+10.0 \text{ m/s})(2.50 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.50 \text{ s})^2 = -5.62 \text{ m}$$

$$v_y = v_{0y} + gt = (+10.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(2.50 \text{ s}) = -14.5 \text{ m/s}$$

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v_{ox} = +5.01 m/s Δx = +12.5 m v_x = +5.01 m/s Δy = -5.62 m v_y = -14.5 m/s

v_{oy} = +10.0 m/s

Time = 2.50 s

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v_{ox} = +5.01 m/s Δx = ? v_x = ? Δy = ? v_y = ?

v_{oy} = +10.0 m/s

Time = 3.00 s

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$$v_{0x} = v_x = \text{constant} = +5.01 \text{ m/s}$$

$$\Delta x = v_{0x}t = (+5.01 \text{ m/s})(3.00 \text{ s}) = +15.0 \text{ m}$$

$$\Delta y = v_{0y}t + \tfrac{1}{2}gt^2 = (+10.0 \text{ m/s})(3.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(3.00 \text{ s})^2 = -14.1 \text{ m}$$

$$v_y = v_{0y} + gt = (+10.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(3.00 \text{ s}) = -19.4 \text{ m/s}$$

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v_{ox} = +5.01 m/s Δx = +15.0 m v_x = +5.01 m/s Δy = -14.1 m v_y = -19.4 m/s

v_{oy} = +10.0 m/s

Time = 3.00 s

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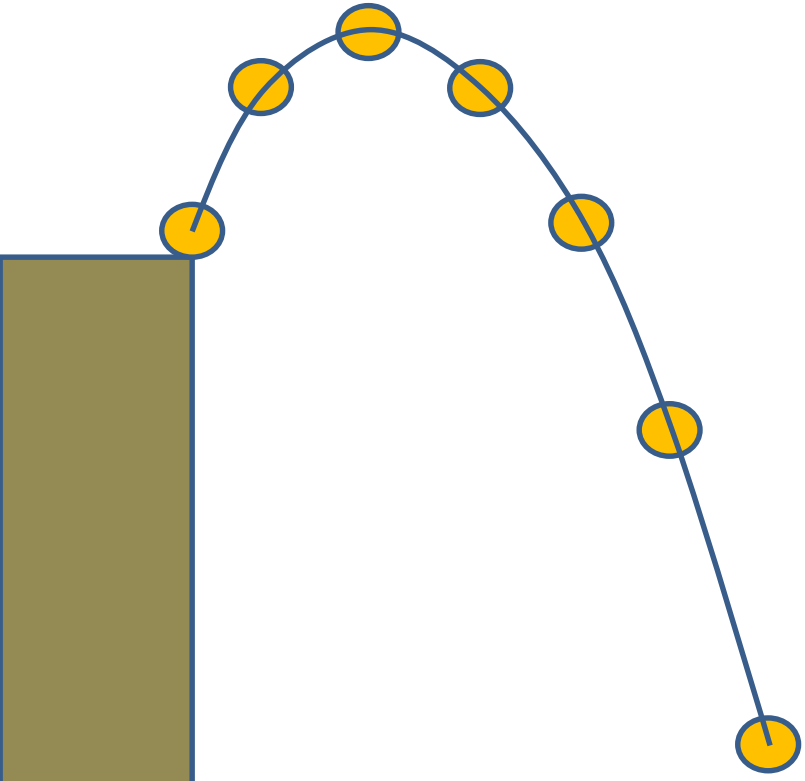
Overall Motion



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How much time does it take the object to reach $\Delta y = -20.0$ m?

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How much time does it take the object to reach Δy = -20.0 m?

$$\Delta y = y - y_0 = v_{0y}t + \tfrac{1}{2}gt^2$$

$$= (-20.0\text{ m}) - (0\text{ m}) = (10.0\text{ m/s})t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$\therefore\ (4.90\text{ m/s}^2)t^2 + (-10.0\text{ m/s})t + (-20.0\text{ m}) = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-10.0) \pm \sqrt{(-10.0)^2 - 4(4.90)(-20.0)}}{2(4.90)}$$

$$= \frac{10.0 \pm 22.2}{9.80} = \boxed{3.29\text{ s}}$$

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