Student: Cole Lamers **Date:** 10/02/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.1-5.2 Area, Sigma Notation and Limits of Finite

1. Use finite approximation to estimate the area under the graph of $f(x) = 6x^2$ and above the graph of $f(x) = 0$ from $x_0 = 0$ to $x_n = 10$ using

i) a lower sum with two rectangles of equal width.
ii) a lower sum with four rectangles of equal width.
iii) an upper sum with two rectangles of equal width.
iv) an upper sum with four rectangles of equal width.

The estimated area using a lower sum with two rectangles of equal width is 750 square units.
(Simplify your answer. Type an integer or a decimal.)

The estimated area using a lower sum with four rectangles of equal width is 1312.5 square units.
(Simplify your answer. Type an integer or a decimal.)

The estimated area using an upper sum with two rectangles of equal width is 3750 square units.
(Simplify your answer. Type an integer or a decimal.)

The estimated area using an upper sum with four rectangles of equal width is 2812.5 square units.
(Simplify your answer. Type an integer or a decimal.)

2. Using rectangles whose height is given by the value of the function at the midpoint of the rectangle's base, estimate the area under the graph using first two and then four rectangles.

$f(x) = x^2$ between $x = 2$ and $x = 4$

Using two rectangles to estimate, the area under f(x) is approximately $\frac{37}{2}$.
(Type an integer or a simplified fraction.)

Using four rectangles to estimate, the area under f(x) is approximately _____.
(Type an integer or a simplified fraction.)

YOU ANSWERED: $\frac{29}{16}$

3. Write the sum without sigma notation. Then evaluate the sum.

$$\sum_{k=1}^{2} \frac{24k}{k+2}$$

Write the sum without sigma notation. Choose the correct answer below.

- A. $\frac{24k}{2+2}$
- B. $\left(\frac{24 \cdot 1}{1+2}\right) + \left(\frac{24 \cdot 2}{2+2}\right) + \left(\frac{24 \cdot 3}{3+2}\right)$
- C. $\left(\frac{24 \cdot 1}{1+2}\right) + \left(\frac{24 \cdot 2}{2+2}\right)$
- D. $\frac{24 \cdot 2}{}$

The value of the sum is 20.
(Simplify your answer.)

4. Write the sum without sigma notation. Then evaluate.

$$\sum_{k=4}^{8} \sin k\pi$$

Write out the sum.

$$\sum_{k=4}^{8} \sin k\pi = \sin(4\pi) + \sin(5\pi) + \sin(6\pi) + \sin(7\pi) + \sin(8\pi)$$

Evaluate the sum.

$$\sum_{k=4}^{8} \sin k\pi = 0$$ (Simplify your answer.)

5. Write the sum without sigma notation. Then evaluate.

$$\sum_{k=1}^{3} (-1)^{k+4} \cos\frac{\pi}{k}$$

Write the sum without sigma notation. Choose the correct answer below.

- **A.** $(-1)^{3+4} \cos\frac{\pi}{3}$
- **B.** $(-1)^{1+4}\cos\frac{\pi}{1} + (-1)^{3+4}\cos\frac{\pi}{3}$
- **C.** $(-1)^{k+4} \cos\frac{\pi}{k}$
- **D.** $(-1)^{1+4}\cos\frac{\pi}{1} + (-1)^{2+4}\cos\frac{\pi}{2} + (-1)^{3+4}\cos\frac{\pi}{3}$

Evaluate the sum.

$$\sum_{k=1}^{3} (-1)^{k+4} \cos\frac{\pi}{k} = \frac{1}{2}$$

(Simplify your answer. Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression.)

6. Which of the following expresses 1 + 4 + 16 + 64 + 256 + 1024 in sigma notation?

(a) $\sum_{k=1}^{6} 4^{k-1}$ **(b)** $\sum_{k=0}^{5} 4^{k}$ **(c)** $\sum_{k=-1}^{4} 4^{k+1}$

Choose the correct answer below.

- **A.** $\sum_{k=1}^{6} 4^{k-1}$
- **B.** $\sum_{k=0}^{5} 4^{k}$
- **C.** $\sum_{k=-1}^{4} 4^{k+1}$
- **D.** All of them

7. Which formula is not equivalent to the other two?

(a) $\sum_{k=10}^{12} \frac{(-1)^{k-9}}{k-9}$ **(b)** $\sum_{k=-2}^{0} \frac{(-1)^{k}}{k+3}$ **(c)** $\sum_{k=-9}^{-7} \frac{(-1)^{k}}{k+10}$

Choose the correct answer below.

- $\sum^{-7} \frac{(-1)^{k}}{k+10}$

8. Express the following sum in sigma notation. Use 1 as the lower limit of summation and k for the index of summation.

$1+2+3+\cdots+37$

Choose the correct answer below.

- **A.** $\sum_{k=1}^{37} k$
- **B.** $\sum_{k=1}^{37} (k+1)$
- **C.** $\sum_{k=1}^{36} k$
- **D.** $\sum_{k=1}^{36} (k+1)$

9. Rewrite the following sum using sigma notation.

$1+8+27+64$

Choose the correct answer.

- **A.** $\sum_{k=1}^{4} -k^3$
- **B.** $\sum_{k=1}^{4} k^3$
- **C.** $\sum_{k=0}^{4} k^3$
- **D.** $\sum_{k=1}^{4} k^4$

10. Express the sum in sigma notation.

$$\frac{7}{4}+\frac{7}{16}+\frac{7}{64}+\frac{7}{256}+\frac{7}{1024}+\frac{7}{4096}$$

$$\frac{7}{4}+\frac{7}{16}+\frac{7}{64}+\frac{7}{256}+\frac{7}{1024}+\frac{7}{4096}=\sum_{k=1}^{6}\frac{7}{4^k}$$

(Type an expression using k as the variable.)

11. If $\sum_{k=1}^{n} a_k = 3$ and $\sum_{k=1}^{n} b_k = 18$, find the following values.

$$\sum_{k=1}^{n} 9a_k,\quad \sum_{k=1}^{n}\frac{b_k}{18},\quad \sum_{k=1}^{n}\left(a_k+b_k\right),\quad \sum_{k=1}^{n}\left(a_k-b_k\right),\quad \sum_{k=1}^{n}\left(b_k-3a_k\right)$$

$\sum_{k=1}^{n} 9a_k = 27$

(Simplify your answer.)

$\sum_{k=1}^{n}\frac{b_k}{18} = 1$

(Simplify your answer.)

$\sum_{k=1}^{n}\left(a_k+b_k\right) = 21$

(Simplify your answer.)

$\sum_{k=1}^{n}\left(a_k-b_k\right) = -15$

(Simplify your answer.)

$\sum_{k=1}^{n}\left(b_k-3a_k\right) = 9$

(Simplify your answer.)

12. Evaluate the following sums.

a. $\sum_{k=1}^{17} k$ **b.** $\sum_{k=1}^{17} k^2$ **c.** $\sum_{k=1}^{17} k^3$

a. $\sum_{k=1}^{17} k =$ 153

(Type an integer or a simplified fraction.)

b. $\sum_{k=1}^{17} k^2 =$ 1785

(Type an integer or a simplified fraction.)

c. $\sum_{k=1}^{17} k^3 =$ 23409

(Type an integer or a simplified fraction.)

13. Evaluate the sum.

$$\sum_{n=1}^{7} \left(1 + n^2\right)$$

$\sum_{n=1}^{7} \left(1 + n^2\right) =$ 147 (Type an integer or a simplified fraction.)

14. Evaluate the following sums.

(a) $\sum_{k=1}^{8} 4$ **(b)** $\sum_{k=1}^{500} 8$ **(c)** $\sum_{k=2}^{261} 20$

(a) $\sum_{k=1}^{8} 4 =$ 32 (Type an integer or a decimal.)

(b) $\sum_{k=1}^{500} 8 =$ 4000 (Type an integer or a decimal.)

(c) $\sum_{k=2}^{261} 20 =$ 5200 (Type an integer or a decimal.)

15. Graph the function $f(x) = x^2 - 1$ over the interval [0, 2]. Partition the interval into four subintervals of equal length. Then add to your sketch the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$, given that c_k is the **(a)** left-hand endpoint, **(b)** right-hand endpoint, **(c)** midpoint of the kth subinterval.

(a) Choose the correct graph below.

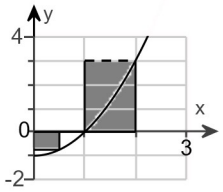
A. 

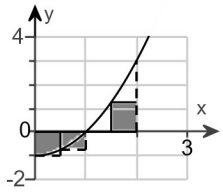
B. 

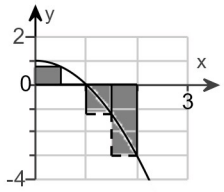
C. ✔ 

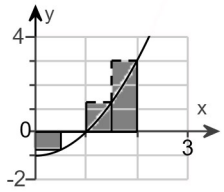
D. 

(b) Choose the correct graph below.

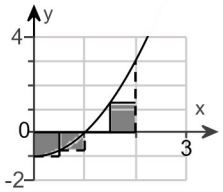
A. 

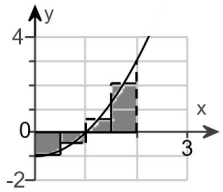
B. 

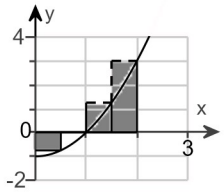
C. 

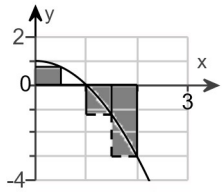
D. ✔ 

(c) Choose the correct graph below.

A. 

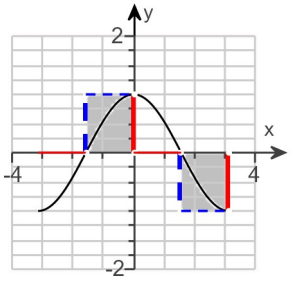
B. ✔ 

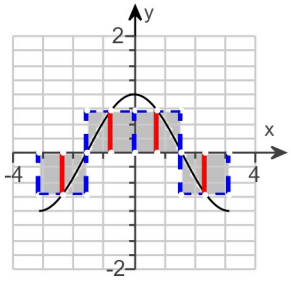
C. 

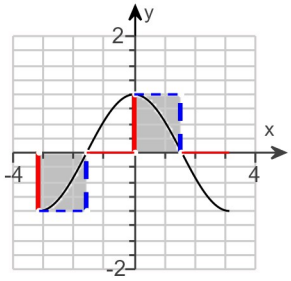
D. 

16. Graph the function $f(x) = \cos x$ on the interval $[-\pi, \pi]$, showing the addition of the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$ given that c_k is the left endpoint of the kth subinterval.

Choose the correct graph.

A. 

B. 

C. ✔ 

17. For the function given below, find a formula for the Riemann sum obtained by dividing the interval [0,1] into n equal subintervals and using the right-hand endpoint for each c_k. Then take a limit of this sum as $n \to \infty$ to calculate the area under the curve over [0,1].

$f(x) = x + x^2$

Which of the following expressions gives the formula for the Riemann sum for the given function over the interval [0,1]?

- **A.** $\sum_{k=1}^{n} \left(\frac{k}{n}\right)\left(\frac{1}{n}\right)$
- **B.** $\sum_{k=1}^{n} \left(\frac{k}{n} - \frac{k^2}{n}\right)\left(\frac{1}{n}\right)$
- **C.** $\sum_{k=1}^{n} \left(\frac{k^2}{n}\right)\left(\frac{1}{n}\right)$
- **D.** $\sum_{k=1}^{n} \left(\frac{k}{n} + \frac{k^2}{n^2}\right)\left(\frac{1}{n}\right)$

Simplify the sigma notation to find the sum in terms of n only. Choose the correct answer below.

- **A.** $S_n = \frac{6n+1}{6n^2}$
- **B.** $S_n = \frac{5}{6} - \frac{6n+1}{6n^2}$
- **C.** $S_n = \frac{5}{6} + \frac{6n+1}{6n^2}$
- **D.** $S_n = \frac{5}{6} + \frac{6n-1}{6n^2}$

The area under the curve over [0,1] is $\frac{5}{6}$ square units.

(Simplify your answer. Type an integer or a simplified fraction.)