

**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
 (81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value Theorem

Find the value or values of  $c$  that satisfy the equation  $\frac{f(b) - f(a)}{b - a} = f'(c)$  in the conclusion of the Mean Value Theorem for the following function and interval.

$$f(x) = 2x^2 - 5x - 3, \quad [-3, 0]$$

To find the value or values of  $c$  that satisfy the equation, first determine  $a$  and  $b$ .

The value of  $a$  is the left endpoint,  $-3$ . The value of  $b$  is the right endpoint,  $0$ .

Now, evaluate  $f(a)$  and  $f(b)$ .

$$f(a) = 2a^2 - 5a - 3$$

$$f(-3) = 2(-3)^2 - 5(-3) - 3$$

$$f(-3) = 30$$

$$f(b) = 2b^2 - 5b - 3$$

$$f(0) = 2(0)^2 - 5(0) - 3$$

$$f(0) = -3$$

To find  $f'(c)$ , first find  $f'(x)$ .

$$f(x) = 2x^2 - 5x - 3$$

$$f'(x) = 4x - 5$$

Simplify.

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

$$\frac{-3 - 30}{0 - (-3)} = 4c - 5$$

$$-11 = 4c - 5$$

Solve for  $c$ .

$$-11 = 4c - 5$$

$$-6 = 4c$$

$$-\frac{3}{2} = c$$

**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
 (81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value Theorem

Find the value or values of  $c$  that satisfies the equation  $\frac{f(b) - f(a)}{b - a} = f'(c)$  in the conclusion of the mean value theorem for the given function and interval.

$$f(x) = 5x + \frac{5}{x}, \left[ \frac{1}{4}, 4 \right]$$

The mean value theorem supposes that  $y = f(x)$  is continuous on a closed interval  $[a, b]$  and differentiable on the interval's interior  $(a, b)$ . Then there is at least one point  $c$  in  $(a, b)$  at which  $\frac{f(b) - f(a)}{b - a} = f'(c)$ .

To determine the value of  $f'(c)$ , first evaluate  $f(b)$  for  $b = 4$ .

$$f(x) = 5x + \frac{5}{x}$$

$$f(4) = 5(4) + \frac{5}{4} \quad \text{Substitute } x = 4.$$

Simplify.

$$f(4) = \frac{80}{4} + \frac{5}{4} \quad \text{Write each term with a common denominator.}$$

$$= \frac{85}{4} \quad \text{Add the fractions together.}$$

Next, evaluate  $f(a)$  for  $a = \frac{1}{4}$ .

$$f(x) = 5x + \frac{5}{x}$$

$$f\left(\frac{1}{4}\right) = 5\left(\frac{1}{4}\right) + \frac{5}{\frac{1}{4}} \quad \text{Substitute } x = \frac{1}{4}.$$

Simplify.

$$f\left(\frac{1}{4}\right) = \frac{5}{4} + \frac{80}{4} \quad \text{Write each term with a common denominator.}$$

$$= \frac{85}{4} \quad \text{Add the fractions together.}$$

Now, compute the value of  $f'(c)$ .

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$= \frac{\frac{85}{4} - \frac{85}{4}}{4 - \frac{1}{4}} \quad \text{Substitute values.}$$

$$= 0 \quad \text{Simplify.}$$

Determine the value of  $c$  that makes  $f'(c) = 0$  true. Begin by finding  $f'(x)$  given that  $f(x) = 5x + \frac{5}{x}$ .

$$f(x) = 5x + 5x^{-1} \quad \text{Rewrite the equation.}$$

$$f'(x) = 5 + 5(-1 \cdot x^{-2}) \quad \text{Differentiate using the power rule.}$$

$$= 5 - \frac{5}{x^2} \quad \text{Simplify.}$$

Substituting  $x = c$  in  $f'(x)$  results in the following equation.

$$f'(c) = 5 - \frac{5}{c^2}$$

Now find the value(s) of  $c$  that satisfy the equation  $f'(c) = 0$ .

$$f'(c) = 5 - \frac{5}{c^2}$$

$$0 = 5 - \frac{5}{c^2} \quad \text{Substitute } f'(c) = 0.$$

$$c = \pm 1 \quad \text{Simplify.}$$

Because  $-1$  does not lie in the given interval, only  $c = 1$  satisfies the conclusion of the mean value theorem for the function

$$f(x) = 5x + \frac{5}{x} \text{ in the interval } \left[ \frac{1}{4}, 4 \right].$$



## Solution

$$\frac{d}{dx}\left(x + \sin^2\left(\frac{x}{3}\right) - 8\right) = \frac{\sin\left(\frac{2x}{3}\right)}{3} + 1$$

## Steps

$$\frac{d}{dx}\left(x + \sin^2\left(\frac{x}{3}\right) - 8\right)$$

Apply the Sum/Difference Rule:  $(f \pm g)' = f' \pm g'$

$$= \frac{d}{dx}(x) + \frac{d}{dx}\left(\sin^2\left(\frac{x}{3}\right)\right) - \frac{d}{dx}(8)$$

$$\frac{d}{dx}(x) = 1$$

Show Steps

$$\frac{d}{dx}\left(\sin^2\left(\frac{x}{3}\right)\right) = \frac{\sin\left(\frac{2x}{3}\right)}{3}$$

Hide Steps

$$\frac{d}{dx}\left(\sin^2\left(\frac{x}{3}\right)\right)$$

Apply the chain rule:  $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$$f = u^2, \quad u = \sin\left(\frac{x}{3}\right)$$

$$= \frac{d}{du}(u^2) \frac{d}{dx}\left(\sin\left(\frac{x}{3}\right)\right)$$

$$\frac{d}{du}(u^2) = 2u$$

Show Steps

$$\frac{d}{dx}\left(\sin\left(\frac{x}{3}\right)\right) = \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

Hide Steps

$$\frac{d}{dx}\left(\sin\left(\frac{x}{3}\right)\right)$$

Apply the chain rule:  $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$$f = \sin(u), \quad u = \frac{x}{3}$$

$$= \frac{d}{du}(\sin(u)) \frac{d}{dx}\left(\frac{x}{3}\right)$$

$$\frac{d}{du}(\sin(u)) = \cos(u)$$

Show Steps +

$$\frac{d}{dx}\left(\frac{x}{3}\right) = \frac{1}{3}$$

Show Steps +

$$= \cos(u) \frac{1}{3}$$

$$\text{Substitute back } u = \frac{x}{3}$$

$$= \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

$$= 2u \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

$$\text{Substitute back } u = \sin\left(\frac{x}{3}\right)$$

$$= 2\sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

$$\text{Simplify } 2\sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right) \frac{1}{3}: \quad \frac{\sin\left(\frac{2x}{3}\right)}{3}$$

Hide Steps -

$$2\sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

$$\text{Multiply fractions: } a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$$

$$= \frac{1 \cdot 2}{3} \sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right)$$

$$\text{Multiply the numbers: } 1 \cdot 2 = 2$$

$$= \frac{2}{3} \sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right)$$

$$\text{Use the following identity: } \cos(x) \sin(x) = \frac{\sin(2x)}{2}$$

$$= \frac{\sin\left(2 \cdot \frac{x}{3}\right)}{2} \cdot \frac{2}{3}$$

Multiply fractions:  $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

$$= \frac{\sin\left(2 \cdot \frac{x}{3}\right) \cdot 2}{2 \cdot 3}$$

Cancel the common factor: 2

$$= \frac{\sin\left(2 \cdot \frac{x}{3}\right)}{3}$$

Multiply  $2 \cdot \frac{x}{3} : \frac{2x}{3}$

Show Steps 

$$= \frac{\sin\left(\frac{x \cdot 2}{3}\right)}{3}$$

$$= \frac{\sin\left(\frac{2x}{3}\right)}{3}$$

$$\frac{d}{dx}(8) = 0$$

Show Steps 

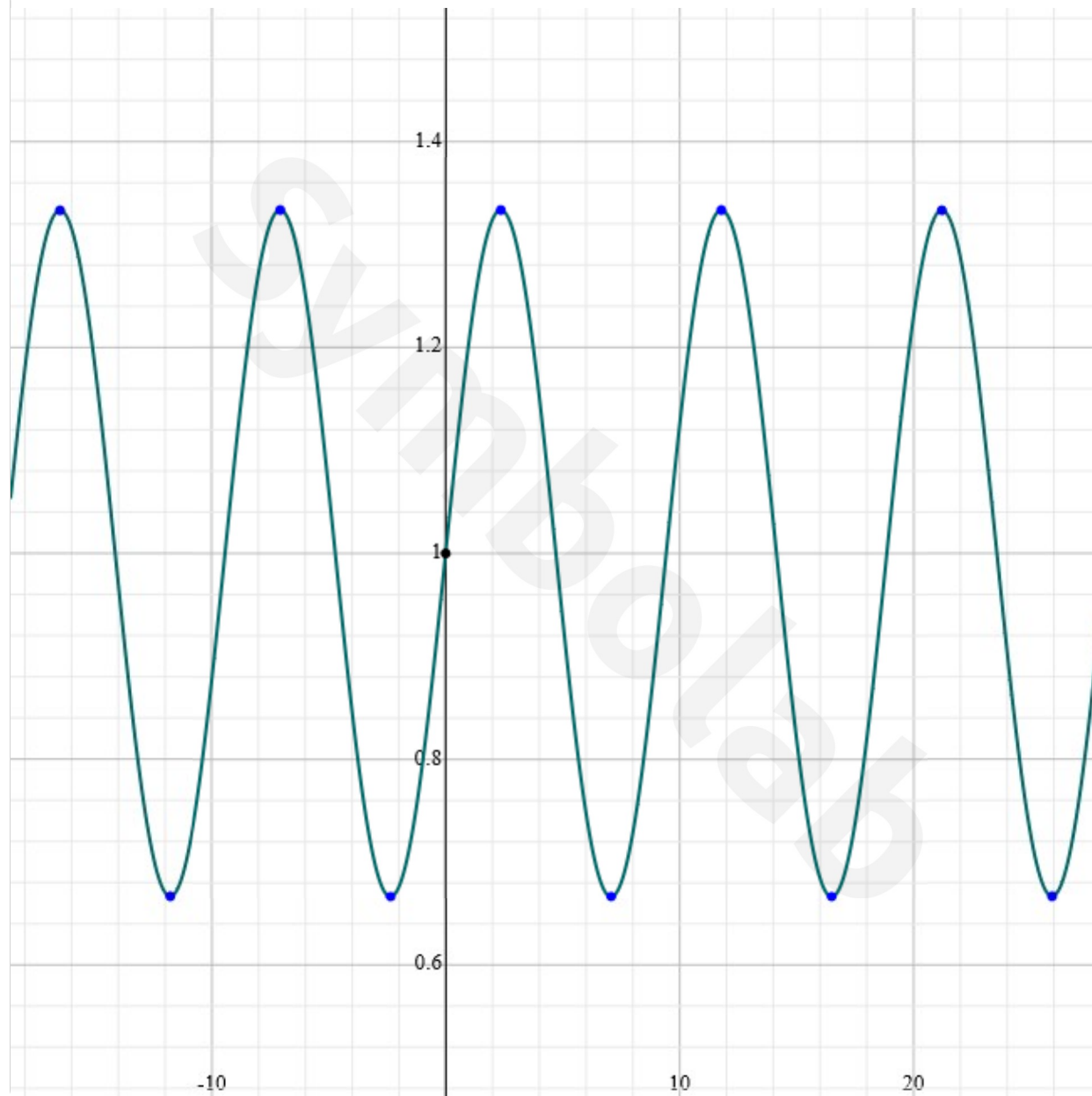
$$= 1 + \frac{\sin\left(\frac{2x}{3}\right)}{3} - 0$$

Simplify

$$= \frac{\sin\left(\frac{2x}{3}\right)}{3} + 1$$

## Graph

Plotting:  $\frac{\sin\left(\frac{2x}{3}\right)}{3} + 1$



**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
 (81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value  
 Theorem

Find all possible functions with the given derivative.

$$f'(t) = \cos 4t + \sin \frac{t}{3}$$

The corollary of Functions with the Same Derivative Differ by a Constant states if

$f'(x) = g'(x)$  at each point in an open interval  $(a,b)$ , then there exists a constant  $C$  such that  $f(x) = g(x) + C$  for all  $x \in (a,b)$ . That is,  $f - g$  is a constant on  $(a,b)$ .

Determine the functions that have the same derivatives as the terms of the sum. Let

$$f(t) = f_1(t) + f_2(t) \text{ and } g(t) = g_1(t) + g_2(t).$$

Find the trigonometric function that has the derivative  $g_1'(t) = \cos 4t$ .

$$g_1(t) = \frac{1}{4} \sin(4t)$$

Find the trigonometric function that has the derivative  $g_2'(t) = \sin \frac{t}{3}$ .

$$g_2(t) = -3 \cos \frac{t}{3}$$

Thus, the function  $f(t) = \frac{1}{4} \sin(4t) - 3 \cos \frac{t}{3} + C$ , for some constant  $C$ .

**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
(81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value  
Theorem

Find all possible functions with the given derivative.

$$f'(x) = x^4$$

The corollary of Functions with the Same Derivative Differ by a Constant states if  $f'(x) = g'(x)$  at each point  $x$  in an open interval  $(a,b)$ , then there exists a constant  $C$  such that  $f(x) = g(x) + C$  for all  $x \in (a,b)$ . That is,  $f - g$  is a constant on  $(a,b)$ .

Find the power function that has the derivative  $f'(x) = x^4$ .

$$g(x) = \frac{x^5}{5}$$

Thus,  $f(x) = \frac{x^5}{5} + C$ , for some constant  $C$ .

**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
 (81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value  
 Theorem

Find the function with the given derivative whose graph passes through the point P.

$$f'(x) = 2x + 3, \quad P(-2, 2)$$

The corollary of Functions with the Same Derivative Differ by a Constant states if  $f'(x) = g'(x)$  at each point in an open interval  $(a, b)$ , then there exists a constant  $C$  such that  $f(x) = g(x) + C$  for all  $x \in (a, b)$ . That is,  $f - g$  is a constant on  $(a, b)$ .

Determine the functions that have the same derivatives as the terms of the sum. Let  $f(x) = f_1(x) + f_2(x)$  and  $g(x) = g_1(x) + g_2(x)$ .

Find the power function that has the same derivative as  $g_1'(x) = 2x$ .

$$g_1(x) = x^2$$

Find the power function that has the same derivative as  $g_2'(x) = 3$ .

$$g_2(x) = 3x$$

Thus, the function  $f(x) = x^2 + 3x + C$ , for some constant  $C$ .

Use the point  $P(-2, 2)$  to find  $C$ .

$$2 = (-2)^2 + 3(-2) + C$$

$$C = 4$$

Therefore, the function with the given derivative whose graph passes through the point P is  $f(x) = x^2 + 3x + 4$ .

**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
(81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value Theorem

Given the velocity  $v = \frac{ds}{dt}$  and the initial position of a body moving along a coordinate line, find the body's position at time  $t$ .

$$v = 9.8t + 10, s(0) = 15$$

The velocity function is the derivative of the position function, that is,  $s(t)$  is some function whose derivative is  $9.8t + 10$ .

If  $f'(t) = g'(t)$  at each point  $t$  in an open interval  $(a, b)$ , then there exists a constant  $C$  such that  $f(t) = g(t) + C$  for all  $t$  in  $(a, b)$ . That is,  $f - g$  is a constant on  $(a, b)$ .

Note that the derivative of the function  $f(t) = 4.9t^2 + 10t$  is  $f'(t) = 9.8t + 10$ .

Since  $f'(t) = 9.8t + 10$  and  $v = 9.8t + 10$ , then  $s(t)$  must vary from  $f(t)$  by only a constant  $C$ , such that  $s(t) = 4.9t^2 + 10t + C$ .

Using  $s(0) = 15$ , determine the value of the constant  $C$ .

$$s(0) = 4.9(0)^2 + 10(0) + C$$

$$15 = 4.9(0)^2 + 10(0) + C$$

$$C = 15$$

Substitute  $C = 15$  in the equation for  $s(t)$  and write the equation for the body's position at time  $t$ .

$$s(t) = 4.9t^2 + 10t + C$$

$$= 4.9t^2 + 10t + 15$$

**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
 (81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value  
 Theorem

Given the velocity  $v = \frac{ds}{dt}$  and initial position of a body moving along a coordinate line, find the body's position at time  $t$ .

$$v = \sin \pi t, \quad s(-8) = 0$$

The corollary of Functions with the Same Derivative Differ by a Constant states if  $f'(x) = g'(x)$  at each point in an open interval  $(a,b)$ , then there exists a constant  $C$  such that  $f(x) = g(x) + C$  for all  $x \in (a,b)$ . That is,  $f - g$  is a constant on  $(a,b)$ .

Find the trigonometric function that has the derivative  $v = \sin \pi t$ .

$$s = -\frac{\cos(\pi t)}{\pi}$$

Thus, the function  $s = -\frac{\cos(\pi t)}{\pi} + C$ , for some constant  $C$ .

Use the initial position to find  $C$ .

$$0 = -\frac{\cos(\pi t)}{\pi} + C$$

$$C = \frac{1}{\pi}$$

The body's position at time  $t$  is  $s = -\frac{\cos(\pi t)}{\pi} + \frac{1}{\pi}$ .

**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
 (81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value Theorem

Consider the following acceleration  $a = \frac{d^2s}{dt^2}$ , initial velocity, and initial position of an object moving on a number line. Find the object's position at time  $t$ .

$$a = 32, v(0) = 10, s(0) = 11$$

The following corollary follows from the Mean Value Theorem.

If  $f'(x) = g'(x)$  at each point  $x$  in an open interval  $(a, b)$ , then there exists a constant  $C$  such that  $f(x) = g(x) + C$  for all  $x \in (a, b)$ .

First, find a function,  $\frac{ds}{dt}$ , with a derivative equal to  $\frac{d^2s}{dt^2}$ . Use the fact that  $\frac{d}{dt}(At) = A$ .

$$\frac{d^2s}{dt^2} = 32$$

$$\frac{ds}{dt} = 32t + C$$

From the above and the mean value theorem,  $\frac{ds}{dt} = 32t + C$ . At time  $t = 0$ ,  $v(0) = 10$  and  $v = \frac{ds}{dt}$ . Substitute  $t = 0$  in the equation and solve for  $C$ .

$$v(0) = 32(0) + C$$

$$10 = 32(0) + C$$

$$10 = C$$

$$\text{So, } \frac{ds}{dt} = 32t + 10.$$

Now, find a function,  $s$ , with a derivative equal to  $\frac{ds}{dt}$ . Use the fact that  $\frac{d}{dt}(At^2) = 2At$  and  $\frac{d}{dt}(At) = A$ .

$$\frac{ds}{dt} = 32t + 10$$

$$s = 16t^2 + 10t + C$$

From above and the mean value theorem,  $s = 16t^2 + 10t + C$ . At time  $t = 0$ ,  $s(0) = 11$ . Substitute  $t = 0$  in the equation and solve for  $C$ .

$$s(0) = 16(0)^2 + 10(0) + C$$

$$11 = 16(0)^2 + 10(0) + C$$

$$11 = C$$

Therefore, the position of the object at time  $t$  is given by  $s = 16t^2 + 10t + 11$ .

**Student:** Cole Lamers  
**Date:** 09/21/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM / Internet  
 (81749&81750) Shcherban

**Assignment:** 4.2 The Mean Value Theorem

Given the acceleration  $a = \frac{d^2s}{dt^2}$ , initial velocity, and initial position of a body moving on a coordinate line, find the body's position at time  $t$ .

$$a = -16 \sin 4t, v(0) = 4, s(0) = -6$$

Acceleration is the derivative of the velocity function, and velocity is the derivative of the position function.

First, determine the velocity function knowing that  $v(t)$  is some function whose derivative is  $-16 \sin 4t$ .

If  $f'(t) = g'(t)$  at each point  $t$  in an open interval  $(a, b)$ , then there exists a constant  $C$  such that  $f(t) = g(t) + C$  for all  $t$  in  $(a, b)$ . That is,  $f - g$  is a constant on  $(a, b)$ .

Note the derivative of the function  $f(t) = 4 \cos 4t$  is  $f'(t) = -16 \sin 4t$ .

Since  $f'(t) = -16 \sin 4t$  and  $a = -16 \sin 4t$ , then  $v(t)$  must vary from  $f(t)$  by only a constant, such that  $v(t) = 4 \cos 4t + C_1$ .

Using  $v(0) = 4$ , determine the value of the constant  $C_1$ .

$$v(0) = 4 \cos 4(0) + C_1$$

$$4 = 4 \cos 0 + C_1$$

$$4 = 4 + C_1$$

$$C_1 = 0$$

Substituting  $C_1 = 0$  into the equation for  $v(t)$  gives the following equation.

$$v(t) = 4 \cos 4t$$

Next, determine the position function knowing that  $s(t)$  is some function whose derivative is  $4 \cos 4t$ .

Note the derivative of the function  $g(t) = \sin 4t$  is  $g'(t) = 4 \cos 4t$ .

Since  $g'(t) = 4 \cos 4t$  and  $v(t) = 4 \cos 4t$ , then  $s(t)$  must vary from  $g(t)$  by only a constant, such that  $s(t) = \sin 4t + C_2$ .

Using  $s(0) = -6$ , determine the value of the constant  $C_2$ .

$$s(0) = \sin 4(0) + C_2$$

$$-6 = \sin 0 + C_2$$

$$-6 = 0 + C_2$$

$$C_2 = -6$$

Substitute  $C_2 = -6$  into the equation for  $s(t)$  and write the equation for the body's position at time  $t$ .

$$s(t) = \sin 4t + C_2$$

$$= \sin(4t) - 6$$