Student: Cole Lamers **Date:** 09/21/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.1 Extreme Values of Functions (set 2)

Find the absolute maximum and minimum values of the function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur.

$$f(x) = -\frac{5}{x^2}, \quad 0.5 \le x \le 8$$

For a continuous function f on a closed interval, the absolute extrema must occur at critical points or endpoints.

The only value of x where the given function is not continuous or defined is $x = 0$. Since this point is not in the domain, the function meets the criteria stated above.

A critical point is an interior point of the domain of f where f' is zero or undefined.

Differentiate the function to find f' using the power rule.

$$f'(x) = -5(-2)x^{-3} = \frac{10}{x^3}$$

There are no points on the interior of the domain $0.5 \le x \le 8$ where $\frac{10}{x^3}$ is undefined or zero so there are no critical points.

Therefore, the absolute extrema must occur at the endpoints. Evaluate the function at the endpoints of the domain.

$$f(0.5) = -20$$

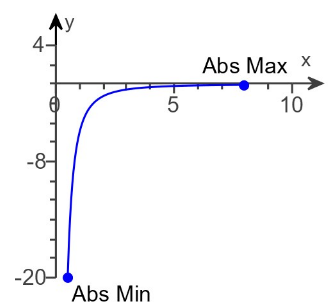
$$f(8) = -\frac{5}{64}$$

The absolute maximum is the largest of these values. The absolute minimum is the smallest of these values.

The absolute maximum is $-\frac{5}{64}$ at $x = 8$.

The absolute minimum is -20 at $x = 0.5$.

The graph of the function is as shown.



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Find the absolute maximum and minimum values of the function on the given interval.

$f(x) = \sqrt{-x^2+9}$, $\quad -3 \le x \le 2$

Let f be a function with domain D. Then f has an absolute maximum value on D at a point c if $f(x) \le f(c)$ for all x in D. The function f has an absolute minimum value on D at c if $f(x) \ge f(c)$ for all x in D.

To find the absolute extrema of a continuous function f on a finite closed interval, evaluate f at all critical points and endpoints, and take the largest and smallest of the values.

A critical point of f is an interior point of the domain of a function f where f ' is zero or undefined.

To find all critical points, first find f '(x).

$$f(x) = \sqrt{-x^2+9}$$

$$f'(x) = -\frac{x}{\sqrt{-x^2+9}}$$

Now, set $f'(x) = 0$.

$$-\frac{x}{\sqrt{-x^2+9}} = 0$$

Find the value of x where the numerator is equal to zero and the denominator does not equal zero.

The derivative $f'(x) = 0$ when $x = 0$.

Find the value of x on the given interval where the denominator is equal to zero.

The derivative $f'(x)$ is undefined when $x = -3$.

There are two endpoints, $x = -3$ and $x = 2$. Since $x = -3$ is an endpoint, it is not a critical point. Thus, there is only one critical point, $x = 0$.

Evaluate f(x) at the endpoints and critical points.

$$f(x) = \sqrt{-x^2+9} \qquad f(x) = \sqrt{-x^2+9} \qquad f(x) = \sqrt{-x^2+9}$$
$$f(-3) = \sqrt{-(-3)^2+9} \qquad f(0) = \sqrt{-(0)^2+9} \qquad f(2) = \sqrt{-(2)^2+9}$$
$$f(-3) = 0 \qquad f(0) = 3 \qquad f(2) = \sqrt{5}$$

The absolute maximum of the function $f(x) = \sqrt{-x^2+9}$ on the interval $-3 \le x \le 2$ has a value of 3.

The absolute minimum of the function $f(x) = \sqrt{-x^2+9}$ on the interval $-3 \le x \le 2$ has a value of 0.

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Find the absolute maximum and minimum values of the function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur.

$$f(\theta) = \cos\theta, \quad -\frac{5\pi}{4} \le \theta \le 0$$

For a continuous function f on a closed interval, the absolute extrema must occur at critical points or endpoints.

The given function is defined for all real θ, and the domain is a closed interval.

A critical point is an interior point of the domain of f where f' is zero or undefined.

Differentiate the function to find f'.

$f'(\theta) = -\sin\theta$

Since $-\sin\theta$ is defined for all real θ, there are no points in the interior of the domain where the derivative is undefined.

However, there is at least one point on the interior of the domain where $-\sin\theta$ is zero.

Find all the solutions to $-\sin\theta = 0$ in the interior of the domain $-\frac{5\pi}{4} \le \theta \le 0$. Note, 0 is not a critical point because it is not in the interior of the domain.

$\theta = -\pi$

Therefore, the absolute extrema can only occur at points $-\frac{5\pi}{4}$, $-\pi$, or 0. Evaluate the function at these three points.

$f\left(-\frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$f(-\pi) = -1$

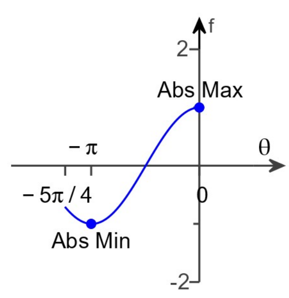
$f(0) = 1$

The absolute maximum is the largest of these values. The absolute minimum is the smallest of these values.

The absolute maximum is 1 at $\theta = 0$.

The absolute minimum is -1 at $\theta = -\pi$.

The graph of the function is as shown.



Student: Cole Lamers **Date:** 09/21/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.1 Extreme Values of Functions (set 2)

Find the absolute maximum and minimum values of the function on the given interval.

$f(x) = -6 \csc x, \; -\frac{3\pi}{4} \le x \le -\frac{\pi}{6}$

Let f be a function with domain D. Then f has an absolute maximum value on D at a point c if $f(x) \le f(c)$ for all x in D. The function f has an absolute minimum value on D at c if $f(x) \ge f(c)$ for all x in D.

To find the absolute extrema of a continuous function f on a finite closed interval, evaluate f at all critical points and endpoints, and take the largest and smallest of the values.

A critical point of f is an interior point of the domain of a function f where f ' is zero or undefined.

To find all critical points, first find f '(x).

$f(x) = -6 \csc x$
$f'(x) = 6 \csc x \cot x$

Now, set $f'(x) = 0$.

$6 \csc x \cot x = 0$

Find the value of x on the interval where $\csc x = 0$ and/or $\cot x = 0$.

On the interval, the derivative $f'(x) = 0$ when $x = -\frac{\pi}{2}$.

There are no values of x on the interval where the derivative f '(x) is undefined.

Evaluate f(x) at the critical point, $x = -\frac{\pi}{2}$.

$f(x) = -6 \csc x$
$f\left(-\frac{\pi}{2}\right) = -6 \csc \left(-\frac{\pi}{2}\right)$
$f\left(-\frac{\pi}{2}\right) = 6$

Evaluate f(x) at the endpoints, $x = -\frac{3\pi}{4}$ and $x = -\frac{\pi}{6}$.

$f(x) = -6 \csc x$
$f\left(-\frac{3\pi}{4}\right) = -6 \csc \left(-\frac{3\pi}{4}\right)$
$f\left(-\frac{3\pi}{4}\right) = 6\sqrt{2}$

$f(x) = -6 \csc x$
$f\left(-\frac{\pi}{6}\right) = -6 \csc \left(-\frac{\pi}{6}\right)$
$f\left(-\frac{\pi}{6}\right) = 12$

The absolute maximum of the function $f(x) = -6 \csc x$ on the interval $-\frac{3\pi}{4} \le x \le -\frac{\pi}{6}$ has a value of 12.

The absolute minimum of the function $f(x) = -6 \csc x$ on the interval $-\frac{3\pi}{4} \le x \le -\frac{\pi}{6}$ has a value of 6.

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Determine all critical points for the following function.

$$f(x) = x^2 - 16x + 10$$

Start by determining the domain of the function. The domain of a function is the set of all values of x for which there is a y value.

The domain of f is $(-\infty, \infty)$.

The critical value of a function f is an interior point of the domain of a function f where f' is zero or undefined.

Find the derivative of the function f.

$$f'(x) = 2x - 16$$

Since the derivative exists for all real x, the critical values will occur only at the x-values for which $f'(x) = 0$. To find the critical values of the given function, solve $2x - 16 = 0$.

Now using the the addition property and the multiplication property of equations, isolate the variable on the left side of the equation.

$$\begin{aligned} 2x - 16 &= 0 \\ x &= 8 \end{aligned}$$

Therefore, $x = 8$ is the only critical value of the function.

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Determine all critical points for the function.

$$f(x) = 15x^2 - 2x^3$$

An interior point of the domain of a function f where f′ is zero or undefined is a critical point of f.

To find the critical points, begin by finding $f'(x)$.

$$\begin{aligned} f(x) &= 15x^2 - 2x^3 \\ f'(x) &= 30x - 6x^2 \end{aligned}$$

Since it is possible to replace x in $f'(x) = 30x - 6x^2$ with any real number, $f'(x)$ exists for all real numbers.

The only possibilities for critical values are where $f'(x) = 0$. Solve $f'(x) = 0$ for x.

$$\begin{aligned} f'(x) &= 0 \\ 30x - 6x^2 &= 0 \\ 6x(5 - x) &= 0 \qquad \text{Factor the left side of the equation.} \end{aligned}$$

$6x(5 - x)$

Use the principle of zero products and solve for x.

$$\begin{aligned} 6x &= 0 \quad \text{or} \quad 5 - x = 0 \\ x &= 0 \qquad\qquad\quad x = 5 \end{aligned}$$

Therefore, the critical points for the function are $x = 0$ and $x = 5$.

Student: Cole Lamers **Date:** 09/21/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.1 Extreme Values of Functions (set 2)

Find the extreme values of the following function and where they occur.

$$y = x^3 - 4x^2 - 3x + 3$$

To find the extreme values, evaluate f at all critical points and endpoints, and take the largest and smallest of the values.

A critical point of f is an interior point of the domain of the function f where $f'(c)$ is zero or does not exist.

To find all critical values, first find y'.

$$\begin{aligned} y &= x^3 - 4x^2 - 3x + 3 \\ y' &= 3x^2 - 8x - 3 \end{aligned}$$

Now set $y' = 0$ and solve for x.

$$\begin{aligned} 3x^2 - 8x - 3 &= 0 \\ x = 3 \text{ and } x &= -\frac{1}{3} \end{aligned}$$

Critical points also occur when the derivative fails to exist. Notice that $y' = 3x^2 - 8x - 3$ exists for all values of x in the domain.

There are two critical values. Evaluate the function at each value of x.

Evaluate y at $x = 3$.

$$\begin{aligned} y &= x^3 - 4x^2 - 3x + 3 \\ y(3) &= -15 \end{aligned}$$

Evaluate y at $x = -\frac{1}{3}$.

$$\begin{aligned} y &= x^3 - 4x^2 - 3x + 3 \\ y\left(-\frac{1}{3}\right) &= \frac{95}{27} \end{aligned}$$

Recall that $y(3) = -15$ and $y\left(-\frac{1}{3}\right) = \frac{95}{27}$, identify the extrema.

The local maximum is $\frac{95}{27}$ at $x = -\frac{1}{3}$ and the local minimum is -15 at $x = 3$.

Student: Cole Lamers **Date:** 09/21/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.1 Extreme Values of Functions (set 2)

Find the extreme values of the following function and where they occur.

$$y = \sqrt{x^2 - 121}$$

To find the extreme values of the given function, find the critical points of the function. A critical point of f is an interior point of the domain of the function f where f′ is zero or does not exist.

Determine the domain of $y = \sqrt{x^2 - 121}$. Remember, the argument of a square root must be greater than or equal to zero.

The domain is $(-\infty, -11] \cup [11, \infty)$.

To find all critical values, first find y′.

$$\begin{aligned} y &= \sqrt{x^2 - 121} \\ y' &= \frac{x}{\sqrt{x^2 - 121}} \end{aligned}$$

Find for which values of x f′ is zero. Set $y' = 0$ and solve for x.

$$\begin{aligned} \frac{x}{\sqrt{x^2 - 121}} &= 0 \\ x &= 0 \end{aligned}$$

Notice that when $x = 0$ the function $y = \sqrt{x^2 - 121}$ is undefined. This means that $x = 0$ is not in the domain of y and is not a critical value.

Critical points also occur when the derivative fails to exist. Recall that when the denominator is zero, then the derivative is undefined.

Now set the denominator equal to zero and solve for x.

$$\begin{aligned} \sqrt{x^2 - 121} &= 0 \\ x &= -11 \quad \text{and} \quad x = 11 \end{aligned}$$

Thus, there are two critical values. Evaluate the function at each critical value.

Evaluate the function at $x = -11$. Substitute -11 for x into $y = \sqrt{x^2 - 121}$ and simplify.

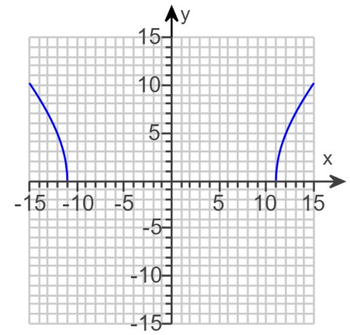
$$\begin{aligned} y &= \sqrt{x^2 - 121} \\ y(-11) &= \sqrt{(-11)^2 - 121} \\ y(-11) &= 0 \end{aligned}$$

Evaluate the function at $x = 11$. Substitute 11 for x into $y = \sqrt{x^2 - 121}$ and simplify.

$$\begin{aligned} y &= \sqrt{x^2 - 121} \\ y(11) &= \sqrt{(11)^2 - 121} \\ y(11) &= 0 \end{aligned}$$

Recall that $y(-11) = 0$ and $y(11) = 0$, identify the extrema.

Therefore, the absolute minimum value is 0 at $x = -11$ and $x = 11$.



Student: Cole Lamers **Date:** 09/21/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.1 Extreme Values of Functions (set 2)

Find the extreme values of the following function and where they occur.

$$y = \frac{x+5}{x^2+8x+40}$$

To find the extreme values, evaluate f at all critical points and endpoints, and take the largest and smallest of the values.

A critical point of f is an interior point of the domain of the function f where y′ is zero or does not exist.

Determine the domain of $y = \frac{x+5}{x^2+8x+40}$.

The domain is $(-\infty,\infty)$.

Since the domain of the function is defined for all real numbers, there are no domain endpoints where there could be extreme values.

To find the critical points, differentiate the function with respect to x.

$$y' = \frac{-x^2-10x}{\left(x^2+8x+40\right)^2}$$

To find the critical values, set the numerator of y′ equal to zero and solve for x.

$$\begin{aligned} -x^2-10x &= 0 \\ x &= 0 \text{ and } x = -10 \end{aligned}$$

Notice that there are no values of x for which the derivative is undefined.

So the x-values for the critical points are $x=0$ and $x=-10$. Evaluate the function at both points. First evaluate at $x=0$.

$$f(0) = \frac{1}{8}$$

Now evaluate at $x=-10$.

$$f(-10) = -\frac{1}{12}$$

Therefore, the function has an absolute maximum of $\frac{1}{8}$ at $x=0$.

The function has an absolute minimum of $-\frac{1}{12}$ at $x=-10$.

Student: Cole Lamers **Date:** 09/21/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.1 Extreme Values of Functions (set 2)

Find the critical points, domain endpoints, and local extreme values for the function.

$$y = x^{4/7}(x+2)$$

A critical point of f is an interior point of the domain of a function f where f′ is zero or undefined.

First identify the domain of the function.

The domain is $(-\infty,\infty)$.

Since the domain of the function is defined for all real numbers, there are no domain endpoints where there could be an extreme value.

To find the critical points, differentiate the function with respect to x.

First use the distributive property to multiply $x^{4/7}$ and simplify.

$$\begin{aligned} y &= x^{4/7}(x+2) \\ &= x^{4/7}x^{7/7} + 2x^{4/7} \\ &= x^{11/7} + 2x^{4/7} \end{aligned}$$

Find y′.

$$\begin{aligned} y &= x^{11/7} + 2x^{4/7} \\ y' &= \frac{11}{7}x^{4/7} + \frac{8}{7}x^{-3/7} \end{aligned}$$

To find the critical points, first write y′ as one rational expression to make computations easier. Factor the left side of the equation.

$$\begin{aligned} \frac{11}{7}x^{4/7} + \frac{8}{7}x^{-3/7} &= 0 \\ \frac{1}{7}x^{-3/7}(11x+8) &= 0 \end{aligned}$$

Write the left side as a fraction by writing the negative exponent in the denominator.

$$\begin{aligned} \frac{1}{7}x^{-3/7}(11x+8) &= 0 \\ \frac{11x+8}{7x^{3/7}} &= 0 \end{aligned}$$

To find the critical points where y′ is zero, set the numerator of y′ equal to zero and solve for x.

$$\begin{aligned} 11x + 8 &= 0 \\ x &= -\frac{8}{11} \end{aligned}$$

Therefore, the critical point where y′ is zero is at $x = -\frac{8}{11}$.

A point where y′ is undefined could also be a critical point. To find the critical points where y′ is undefined, set the denominator of y′ equal to zero and solve for x.

$$7x^{3/7} = 0$$

The denominator is equal to zero at $x = 0$.

Therefore, the critical point where y′ is undefined is at $x = 0$.

Evaluate y at the critical points, $-\frac{8}{11}$ and 0.

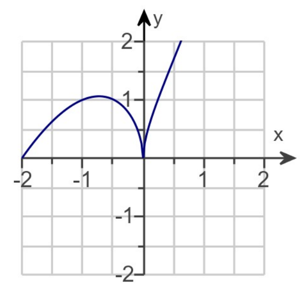
$$y = x^{4/7}(x+2) \qquad\qquad y = x^{4/7}(x+2)$$

$$y = \left(-\frac{8}{11}\right)^{4/7}\left(-\frac{8}{11}+2\right) \qquad\qquad y = (0)^{4/7}(0+2)$$

$$y \approx 1.061 \qquad\qquad y = 0$$

A function f has a local maximum at $x = c$ if $f(x) \le f(c)$ for all x in some open interval containing c, and f has a local minimum at $x = c$ if $f(x) \ge f(c)$ for all x in some open interval containing c.

Determine whether each of the critical points found above corresponds to a local maximum or minimum by evaluating or graphing y for x-values near $x = -\frac{8}{11} \approx -0.727$ and $x = 0$.



From the graph, a local maximum of approximately 1.061 occurs at $x \approx -0.727$ and a local minimum of 0 occurs at $x = 0$.

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Find the critical points, domain endpoints, and local extreme values (absolute and local) for the function.

$$y = \begin{cases} 7 - 5x, & x \le 2 \\ x - 5, & x > 2 \end{cases}$$

A critical point of f is an interior point of the domain of a function f where f′ is zero or undefined.

First identify the domain of the function.

The domain of the first piece of the function is $(-\infty, 2]$.

The domain of the second piece of the function is $(2, \infty)$.

Thus, the domain of the piecewise function is $(-\infty, \infty)$.

Since the domain of the function is all real numbers, there are no domain endpoints where there could be an extreme value.

To find the critical points, first find y′ for each piece of the function.

$$\begin{aligned} y &= 7 - 5x \\ y' &= -5 \end{aligned} \qquad\qquad \begin{aligned} y &= x - 5 \\ y' &= 1 \end{aligned}$$

The definition of y′ for $x \ne 2$ is shown below.

$$y' = \begin{cases} -5, & x < 2 \\ 1, & x > 2 \end{cases}$$

Since there are no points where either derivative is equal to 0, there are no critical points obtained from the first derivatives.

Since a common endpoint of each piece of the domain of y occurs at $x = 2$, the derivative of y at $x = 2$ may or may not exist. If it does exist, it is given by the limit as x approaches 2 of y′. This limit exists if the left and right limits both exist and are equal.

The value of y′ is undefined when $x = 2$.

Since the derivative is not defined at $x = 2$, this point is considered a critical point. Evaluate the given function at $x = 2$.

When $x = 2$, $y = -3$.

A function f has a local maximum at $x = c$ if $f(x) \le f(c)$ for all x in some open interval containing c, and f has a local minimum at $x = c$ if $f(x) \ge f(c)$ for all x in some open interval containing c.

Determine whether the critical point found above corresponds to a local maximum or minimum by evaluating or graphing y for $x = 2$.

The point $(2, -3)$ corresponds to the local minimum.

Note that the point $(2, -3)$ is also corresponds to the absolute minimum because there are no smaller values of y anywhere.