Unit 5 – Heat

$$\text{Heat} = Q = mc\Delta T$$

m = mass

c = specific heat

ΔT = change in temperature

Specific Heat

- We can think of specific heat as thermal inertia. Recall that inertia signifies the resistance to change in motion.
- Therefore, specific heat is like thermal inertia since it signifies the resistance of a substance to change its temperature.

How much heat (cal) must be supplied to raise the temperature of a 64.0 g copper fitting from 25.0°C to 425.0°C? [The melting point of copper is 1083°C.]

$$Q = mc\Delta T$$

$$Q = \left(64.0\ \cancel{\text{g}}\right)\left(\frac{0.093\ \text{cal}}{\cancel{\text{g}} \cdot {}^\circ\cancel{\text{C}}}\right)\left(400.0^\circ\cancel{\text{C}}\right)$$

$$Q = \boxed{2400\ \text{cal}}$$

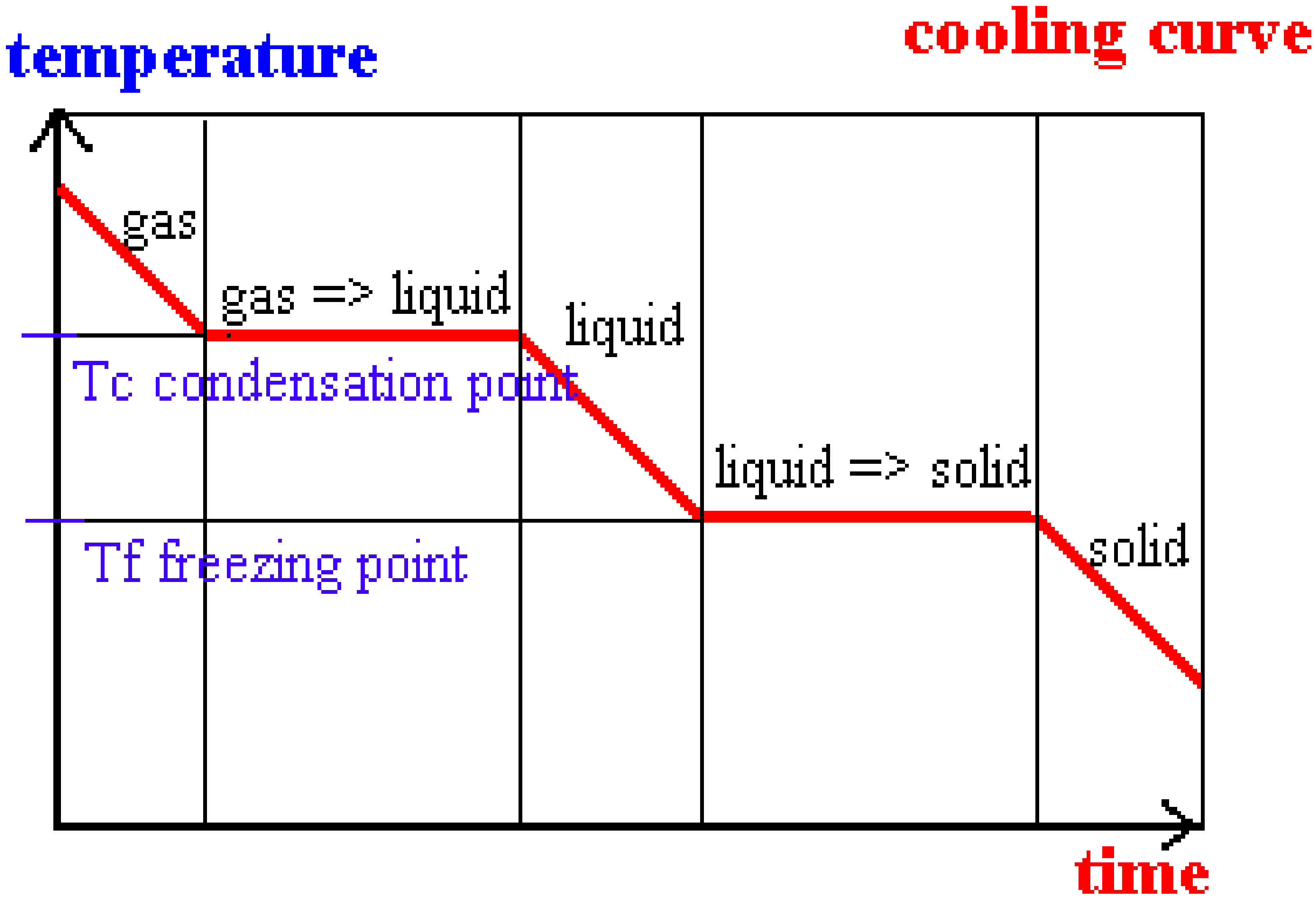
Definitions

Heat of Fusion L_f

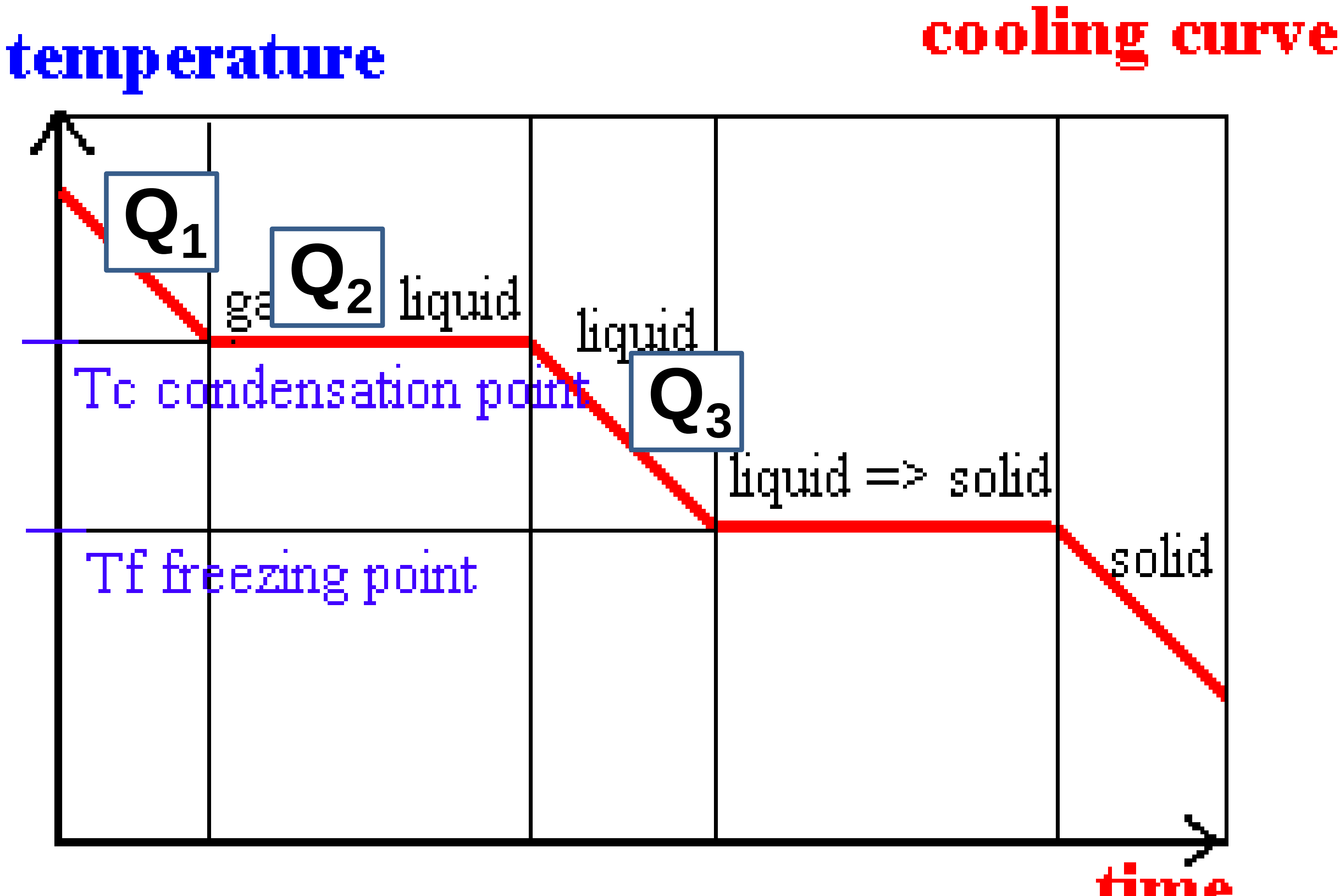
- The quantity of heat absorbed when a specific quantity of a solid is converted to a liquid at its melting point (also known as freezing point).

Heat of Vaporization L_v

- The quantity of heat absorbed when a specific quantity of a liquid is converted to a gas at the boiling point (also known as condensation point).


cooling curve
temperature
gas
gas => liquid
liquid
Tc condensation point
liquid => solid
solid
Tf freezing point
time

How much heat (Btu) is removed when 3.50 lbs of steam at 235.0°F is converted to liquid water at 35.0°F?



cooling curve
temperature
Q_1
Q_2
Q_3
gas
liquid
liquid
Tc condensation point
liquid => solid
solid
Tf freezing point
time

$$L_v = 97\bar{0}\ \frac{\text{Btu}}{\text{lb}} \qquad c_{H_2O\,(l)} = 1.00\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{F}} \qquad c_{H_2O\,(g)} = 0.480\ \frac{\text{Btu}}{\text{lb}\cdot{}^\circ\text{F}}$$

$$Q_1 = mc\Delta T = (3.50\ \cancel{\text{lbs}})\left(0.480\ \frac{\text{Btu}}{\cancel{\text{lb}}\cdot{}^\circ\cancel{\text{F}}}\right)(23.0^\circ\cancel{\text{F}}) = 38.6\ \text{Btu}$$

$$Q_2 = mL_v = (3.50\ \cancel{\text{lbs}})\left(97\bar{0}\ \frac{\text{Btu}}{\cancel{\text{lb}}}\right) = 34\bar{0}0\ \text{Btu}$$

$$Q_3 = mc\Delta T = (3.50\ \cancel{\text{lbs}})\left(1.00\ \frac{\text{Btu}}{\cancel{\text{lb}}\cdot{}^\circ\cancel{\text{F}}}\right)(177^\circ\cancel{\text{F}}) = 62\bar{0}\ \text{Btu}$$

$$\Sigma Q = Q_1 + Q_2 + Q_3 = \boxed{4060\ \text{Btu}}$$

Calculate the heat (Btu) needed to vaporize 15.3 lbs of water if it is already at the boiling point.

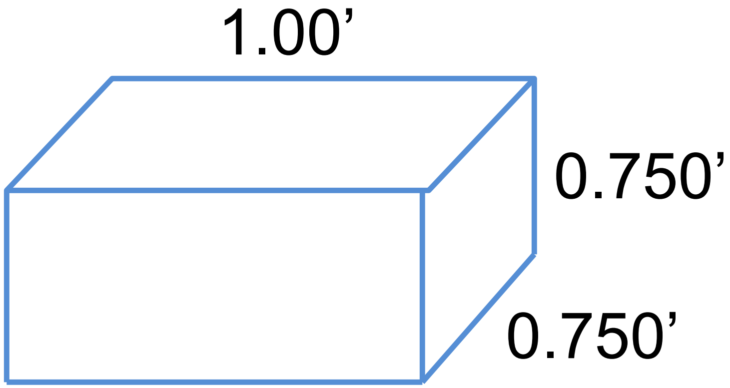
$$Q = mL_v = \left(15.3\ \cancel{\text{lb}}\text{s}\right)\left(97\overline{0}\ \frac{\text{Btu}}{\cancel{\text{lb}}}\right) = \boxed{14{,}800\ \text{Btu}}$$

An ice chest (1.00’ x 0.750’ x 0.750’) is filled with 2.50 lbs of ice. The chest is made of foam plastic 1.00” thick. If the inside of the ice chest is at 32.0°F and the outside temperature is 69.0°F, how much ice will melt in 2.85 hours?

($k_{foam\ plastic}$ = 0.300 Btu·in/ft^2·hr·°F)

(L_f = 144 Btu/lb)

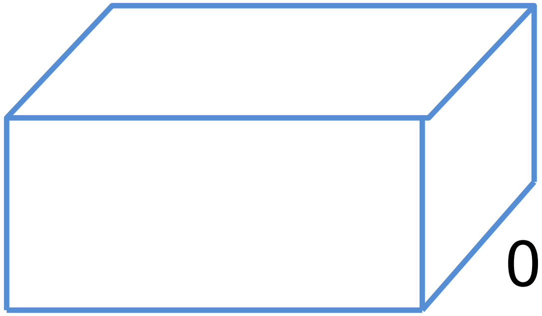
$$Q = \frac{k(A)(\Delta T)(\tau)}{L}$$



k = Thermal Conductivity A = Area of Slab

ΔT = Temperature Difference Between Slabs

τ = Time in Hours L = Thickness of Slab



$$4 \text{ area sides} = 4\left(1.00'\right)\left(0.750'\right) = 3.00 \text{ ft}^2$$

$$2 \text{ area sides} = 2\left(0.750'\right)\left(0.750'\right) = 1.12 \text{ ft}^2$$

$$\therefore \; \Sigma A = 3.00 \text{ ft}^2 + 1.12 \text{ ft}^2 = 4.12 \text{ ft}^2$$

$$Q = \frac{k(A)(\Delta T)(\tau)}{L}$$

$$Q = \frac{\left(\dfrac{0.300\ \text{Btu}\cdot\cancel{\text{in}}}{\cancel{\text{ft}}^2\cdot\cancel{\text{hr}}\cdot{}^\circ\cancel{\text{F}}}\right)\left(4.12\ \cancel{\text{ft}}^2\right)\left(37.0^\circ\cancel{\text{F}}\right)\left(2.85\ \cancel{\text{h}}\text{rs}\right)}{\left(1.00\ \cancel{\text{in}}\right)}$$

$$Q = 13\bar{0}\ \text{Btu} = Q_{fusion} = mL_f = m\left(\frac{144\ \text{Btu}}{\text{lb}}\right)$$

$$\therefore\ m = \boxed{0.903\ \text{lbs}}$$

A mixing valve allows 3.26 gallons of hot water at 125.0°F to mix with 1.37 gallons of cold water at 35.4°F. What is the final temperature of the mixture in °C ? Assume no heat loss to the surroundings.

(Density of water = 8.34 lb/gal)

$$(3.26\ \cancel{\text{gal}})\left(\frac{8.34\ \cancel{\text{lbs}}}{\cancel{\text{gal}}}\right)\left(\frac{454\ \text{g}}{1\ \cancel{\text{lb}}}\right) = 12{,}300\ \text{g}$$

$$(1.37\ \cancel{\text{gal}})\left(\frac{8.34\ \cancel{\text{lbs}}}{\cancel{\text{gal}}}\right)\left(\frac{454\ \text{g}}{1\ \cancel{\text{lb}}}\right) = 5190\ \text{g}$$

$$\left(\frac{125\ ^\circ\cancel{\text{F}} - 32\ ^\circ\cancel{\text{F}}}{1.8\ ^\circ\cancel{\text{F}}\ /\ ^\circ\text{C}}\right) = 51.7\ ^\circ\text{C}$$

$$\left(\frac{35.4\ ^\circ\cancel{\text{F}} - 32\ ^\circ\cancel{\text{F}}}{1.8\ ^\circ\cancel{\text{F}}\ /\ ^\circ\text{C}}\right) = 1.89\ ^\circ\text{C}$$

$$Q_{HW} + Q_{CW} = 0$$

$$\mathrm{m_{CW}}\cancel{\mathrm{c}}_{\mathrm{W}}\Delta\mathrm{T_{CW}} + \mathrm{m_{HW}}\cancel{\mathrm{c}}_{\mathrm{W}}\Delta\mathrm{T_{HW}} = 0$$

$$\left(5190\text{ g}\right)\left(T - 1.89\ °\text{C}\right) + \left(12{,}300\text{ g}\right)\left(T - 51.7\ °\text{C}\right) = 0$$

$$\left(5190\text{ g}\right)T - 9810\text{ g}\cdot°\text{C} + \left(12{,}300\text{ g}\right)T - 636{,}000\text{ g}\cdot°\text{C} = 0$$

$$\left(17{,}500\text{ g}\right)T = 646{,}000\text{ g}\cdot°\text{C}$$

$$\therefore\ T = \boxed{36.9\ °\text{C}}$$

A steel strut near a ship's furnace is 2.00 m long, with a mass of 1.57 kg and cross-sectional area of 0.000100 m^2. During the operation of the furnace, the strut absorbs a net thermal energy of 2.50×10^5 J.

(a) Find the change in temperature (°C) of the strut.

(b) Find the increase in length (m) of the strut.

If the strut is not allowed to expand because it's bolted at each end, **(c)** find the compressional stress (Pa) developed in the strut.

$$Q = m_s c_s \Delta T \qquad \therefore \quad \Delta T = \frac{Q}{m_s c_s}$$

$$\Delta T = \frac{\left(2.50 \text{ x } 10^5 \text{ J}\right)}{\left(1.57 \text{ kg}\right)\left(448 \ \dfrac{\text{J}}{\text{kg} \cdot {}^\circ\text{C}}\right)} = \boxed{355\ {}^\circ\text{C}}$$

$$\Delta L = \alpha L_0 \Delta T = \left(11 \text{ x } 10^{-6}\ {}^\circ\text{C}^{-1}\right)\left(2.00 \text{ m}\right)\left(355\ {}^\circ\text{C}\right) = \boxed{0.0078 \text{ m}}$$

$$\frac{F}{A} = \frac{Y \Delta L}{L_o} = \frac{\left(2.00 \text{ x } 10^{11} \text{ Pa}\right)\left(0.0078 \text{ m}\right)}{\left(2.00 \text{ m}\right)} = \boxed{7.8 \text{ x } 10^8 \text{ Pa}}$$

A 125-g block of an unknown substance with a temperature of 90.0 °C is placed in a Styrofoam cup containing 0.326 kg of water at 20.0 °C. The system reaches an equilibrium temperature of 22.4 °C.

(a) What is the specific heat (J/kg·°C), c_x , of the unknown substance if the heat capacity of the cup is neglected?

$$Q_{cold} = -Q_{hot}$$

$$m_w c_w \left(T - T_w\right) = -m_x c_x \left(T - T_x\right)$$

$$c_x = \frac{m_w c_w \left(T - T_w\right)}{m_x \left(T_x - T\right)}$$

$$c_x = \frac{\left(0.326 \text{ kg}\right)\left(4190 \text{ J/kg}\cdot{}^\circ\text{C}\right)\left(22.4\ {}^\circ\text{C} - 20.0\ {}^\circ\text{C}\right)}{\left(0.125 \text{ kg}\right)\left(90.0\ {}^\circ\text{C} - 22.4\ {}^\circ\text{C}\right)}$$

$$c_x = \boxed{390 \text{ J/kg}\cdot{}^\circ\text{C}}$$

Suppose 0.400 kg of water initially at 40.00 °C is poured into a 0.300-kg glass beaker having a temperature of 25.00 °C. A 0.500-kg block of aluminum at 37.00 °C is placed in the water and the system insulated.

(a) Calculate the final equilibrium temperature (°C) of the system.

Q (J)	m (kg)	c (J/kg·°C)	T_f (°C)	T_i (°C)
Q_w	0.400	4190	T	40.00
Q_{Al}	0.500	900.	T	37.00
Q_g	0.300	837	T	25.00

$$Q_w + Q_{Al} + Q_g = 0$$

$$m_w c_w \left(T - T_w\right) + m_{Al} c_{Al} \left(T - T_{Al}\right) + m_g c_g \left(T - T_g\right) = 0$$

$$\begin{aligned} & \left(0.400\ \text{kg}\right)\left(4190\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(T - 40.00\ {}^\circ\text{C}\right) \\ + & \left(0.500\ \text{kg}\right)\left(900.\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(T - 37.00\ {}^\circ\text{C}\right) \\ + & \left(0.300\ \text{kg}\right)\left(837\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(T - 25.00\ {}^\circ\text{C}\right) = 0 \end{aligned}$$

$$\left(1680\ \text{J/°C}\right)T + \left(-67{,}\overline{0}00\ \text{J}\right)$$

$$+\ \left(45\overline{0}\ \text{J/°C}\right)T + \left(-16{,}700\ \text{J}\right)$$

$$+\ \left(251\ \text{J/°C}\right)T + \left(-6280\ \text{J}\right) \quad = \ 0$$

$$\left(2380\ \text{J/°C}\right)T \ = \ \left(9.00 \times 10^{4}\ \text{J}\right)$$

$$\therefore\ T \ = \ \boxed{37.8\ \text{°C}}$$

At a party, 6.00 kg of ice at -5.00 °C is added to a cooler holding 30.0 liters of water at 20.00 °C.

(a) What is the temperature of the water when it comes to equilibrium?

Q	m (kg)	c (J/kg·°C)	L (J/kg)	T_f (°C)	T_i (°C)	Expression
Q_{ice}	6.00	2090		0	-5.00	$m_{ice}\, c_{ice}\, (T_f - T_i)$
Q_{melt}	6.00		3.33×10^5	0	0	$m_{ice}\, L_f$
$Q_{ice\text{-}water}$	6.00	4190		T	0	$m_{ice}\, c_{water}\, (T_f - T_i)$
Q_{water}	30.0	4190		T	20.00	$m_{water}\, c_{water}\, (T_f - T_i)$

$$m_{\text{water}} = \rho_{\text{water}} V = \left(10\bar{0}0 \text{ kg/m}^3\right)\left(30.0 \text{ L}\right)\left(\frac{1.00 \text{ m}^3}{10\bar{0}0 \text{ L}}\right) = 30.0 \text{ kg}$$

$$Q_{\text{ice}} + Q_{\text{melt}} + Q_{\text{ice-water}} + Q_{\text{water}} = 0$$

$$\begin{aligned}
& \left(6.00 \text{ kg}\right)\left(2090 \text{ J/kg}\cdot{}^\circ\text{C}\right)\left(5.00\ {}^\circ\text{C}\right) \\
+ \ & \left(6.00 \text{ kg}\right)\left(3.33 \text{ x } 10^5 \text{ J/kg}\right) \\
+ \ & \left(6.00 \text{ kg}\right)\left(4190 \text{ J/kg}\cdot{}^\circ\text{C}\right)\left(T - 0\ {}^\circ\text{C}\right) \\
+ \ & \left(30.00 \text{ kg}\right)\left(4190 \text{ J/kg}\cdot{}^\circ\text{C}\right)\left(T - 20.00\ {}^\circ\text{C}\right) = 0
\end{aligned}$$

$$
\begin{aligned}
&\left(62{,}700\ \mathrm{J}\right) \;+\; \left(2.00 \times 10^{6}\ \mathrm{J}\right) \\
&+\;\left(25{,}100\ \mathrm{J/^\circ C}\right)\left(T - 0\ {}^\circ\mathrm{C}\right) \\
&+\;\left(126{,}000\ \mathrm{J/^\circ C}\right)\left(T - 20.00\ {}^\circ\mathrm{C}\right) \;=\; 0
\end{aligned}
$$

$$\left(62{,}700\ \text{J}\right) + \left(2.00 \times 10^6\ \text{J}\right)$$

$$+ \left(25{,}100\ \text{J/°C}\right)\left(\text{T}\right)$$

$$+ \left(126{,}000\ \text{J/°C}\right)\left(\text{T}\right) + \left(-\ 2.52 \times 10^6\ \text{J}\right) = 0$$

$$\left(151{,}000\ \text{J/°C}\right)\left(\text{T}\right) + \left(-\ 460{,}000\ \text{J}\right) = 0$$

$$\therefore\ T = \boxed{3.05\ °\text{C}}$$

(a) Find the energy transferred in 1.00 hour by conduction through a concrete wall 2.00 m high, 3.65 m long, and 0.200 m thick if one side of the wall is held to 5.00 °C and the other side is at 20.00 °C. Assume the concrete has a thermal conductivity of 0.800 $J/s\cdot m^2\cdot °C$. The owner of the home decides to increase the insulation, so he installs 0.500 in. of thick sheathing, 3.50 in. of fiberglass batting, and a drywall 0.500 in. thick. **(b)** Calculate the R-factor. **(c)** Calculate the energy transferred in 1.00 hour by conduction. **(d)** What is the temperature between the concrete wall and the sheathing? Assume there is an air layer on the exterior of the concrete wall but not between the concrete and the sheathing.

Table 11.4 conversion factor to: $\dfrac{m^2 \cdot s \cdot °C}{J}$

$$\frac{1\ m^2}{10.76\ ft^2} \cdot \frac{1\ °C}{1.80\ °F} \cdot \frac{3600\ s}{1\ hr} \cdot \frac{1\ Btu}{252\ cal} \cdot \frac{1\ cal}{4.186\ J} = 0.1761$$

$$\Sigma R = \overset{\downarrow\, R_{\text{concrete}}}{\frac{L}{k}} + 2R_{\text{air layer}} = \frac{0.200\text{ m}}{0.800\text{ J/s}\cdot\text{m}^2\cdot{}^\circ\text{C}} + 2\overset{\text{stagnant air}\ \downarrow\ (0.17)(0.1761)}{\left(0.030\ \frac{\text{m}^2}{\text{J/s}\cdot{}^\circ\text{C}}\right)} = 0.31\ \frac{\text{m}^2}{\text{J/s}\cdot{}^\circ\text{C}}$$

$$P = \frac{A\left(T_h - T_c\right)}{\Sigma R} = \frac{\left(7.30\text{ m}^2\right)\left(20.00\ {}^\circ\text{C} - 5.00\ {}^\circ\text{C}\right)}{\left(0.31\ \frac{\text{m}^2}{\text{J/s}\cdot{}^\circ\text{C}}\right)} = 350\text{ W}$$

$$Q = P\Delta T = \left(350\text{ W}\right)\left(1.00\text{ hr}\right)\left(\frac{3600\text{ s}}{1\text{ hr}}\right) = \boxed{1.3 \times 10^6\ \text{J}}$$

$$\Sigma R = R_{\text{total}} = \overset{\left(\frac{0.200}{0.800}\right)\downarrow}{R_{\text{outside air layer}}} + \overset{(1.32)(0.1761)\downarrow}{R_{\text{concrete}}} + R_{\text{sheath}} + \overset{\downarrow(10.90)(0.1761)}{R_{\text{fiberglass}}} + \overset{\downarrow(0.45)(0.1761)}{R_{\text{drywall}}} + R_{\text{inside air layer}}$$

$$= (0.030 + 0.250 + 0.232 + 1.920 + 0.079 + 0.030)$$

$$= \boxed{2.541 \ \frac{\text{m}^2}{\text{J/s}\cdot{}^{\circ}\text{C}}}$$

$$P = \frac{A(T_h - T_c)}{\Sigma R} = \frac{(7.30 \text{ m}^2)(20.00\ {}^{o}\text{C} - 5.00\ {}^{o}\text{C})}{\left(2.541\ \dfrac{\text{m}^2}{\text{J/s}\cdot{}^{o}\text{C}}\right)} = 43.1 \text{ W}$$

$$Q = P\Delta T = (43.1 \text{ W})(1.00 \text{ hr})\left(\frac{3600 \text{ s}}{1 \text{ hr}}\right) = \boxed{1.55 \text{ x } 10^5 \text{ J}}$$

Temp of Hotter $\downarrow$ Surface $\downarrow$ Temp of Cooler Surface

$$P = \frac{A\left(T_h - T_c\right)}{\Sigma R} \qquad \therefore \quad P\Sigma R = A\left(T_h - T_c\right) \qquad \therefore \quad \left(T_h - T_c\right) = \frac{P\Sigma R}{A}$$

$$\therefore \quad T_h = \frac{P\Sigma R}{A} + T_c$$

$$\therefore \quad T_h = \frac{\left(43.1\ \text{W}\right)\left(0.31\ \dfrac{\text{m}^2}{\text{J/s}\cdot{}^{\text{o}}\text{C}} - 0.030\ \dfrac{\text{m}^2}{\text{J/s}\cdot{}^{\text{o}}\text{C}}\right)}{\left(7.3\ \text{m}^2\right)} + \left(5.00\ {}^{\text{o}}\text{C}\right)$$

$$\therefore \quad T_h = \boxed{6.7\ {}^{\text{o}}\text{C}}$$