Student: Cole Lamers **Date:** 10/03/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.4 The Fundamental Theorem of Calculus

Evaluate the integral.

$$\int_0^{\pi/3} 4\,\mathbf{sec}^2 x\,dx$$

According to the second part of the fundamental theorem of calculus, if f is continuous at every point of [a,b] and F is any antiderivative of f on [a,b], then the definite integral can be evaluated using the following formula.

$$\int_a^b f(x)dx = F(b) - F(a)$$

Let $f(x) = 4\,\mathbf{sec}^2 x$. To evaluate the integral, first find F(x). Remember that the general antiderivative of $g(x) = \mathbf{sec}^2 x$ is $G(x) = \mathbf{tan}\ x + C$.

$F(x) = 4\mathbf{tan}\ x + C$

Because of the subtraction, a constant in F(x) will not affect the value of F(b) − F(a). Thus, there is no need to include the constant C.

$$\begin{aligned}\int_0^{\pi/3} 4\,\mathbf{sec}^2 x\,dx &= F\left(\frac{\pi}{3}\right) - F(0)\\ &= 4\,\mathbf{tan}\ x\Big]_0^{\pi/3}\end{aligned}$$

Next, calculate $F\left(\frac{\pi}{3}\right)$.

$$\begin{aligned}F\left(\frac{\pi}{3}\right) &= 4\,\mathbf{tan}\left(\frac{\pi}{3}\right)\\ &= 4\cdot\sqrt{3}\\ &= 4\sqrt{3}\end{aligned}$$

Calculate F(0).

$$\begin{aligned}F(0) &= 4\,\mathbf{tan}\,(0)\\ &= 4\cdot 0\\ &= 0\end{aligned}$$

Finally, evaluate the integral.

$$\begin{aligned}\int_0^{\pi/3} 4\,\mathbf{sec}^2 x\,dx &= F\left(\frac{\pi}{3}\right) - F(0)\\ &= 4\sqrt{3} - 0\\ &= 4\sqrt{3}\end{aligned}$$

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Evaluate the integral.

$$\int_{5\pi/4}^{7\pi/4} 2\csc\theta\cot\theta\, d\theta$$

According to the second part of the fundamental theorem of calculus, if f is continuous at every point of [a,b] and F is any antiderivative of f on [a,b], then the definite integral can be evaluated using the following formula.

$$\int_a^b f(x)dx = F(b) - F(a)$$

Let $f(\theta) = 2\csc\theta\cot\theta$. To evaluate the integral, first find $F(\theta)$. Remember that the general antiderivative of $g(x) = \csc x \cot x$ is $G(x) = -\csc x + C$.

$F(\theta) = -2\csc\theta + C$

Because of the subtraction, a constant in F(x) will not affect the value of $F(b) - F(a)$. Thus, there is no need to include the constant C.

$$\begin{aligned}\int_{5\pi/4}^{7\pi/4} 2\csc\theta\cot\theta\, d\theta &= F\left(\frac{7\pi}{4}\right) - F\left(\frac{5\pi}{4}\right)\\ &= -2\csc\theta\Big]_{5\pi/4}^{7\pi/4}\end{aligned}$$

Next, calculate $F\left(\frac{7\pi}{4}\right)$.

$$\begin{aligned}F\left(\frac{7\pi}{4}\right) &= -2\csc\left(\frac{7\pi}{4}\right)\\ &= 2\sqrt{2}\end{aligned}$$

Calculate $F\left(\frac{5\pi}{4}\right)$.

$$\begin{aligned}F\left(\frac{5\pi}{4}\right) &= -2\csc\left(\frac{5\pi}{4}\right)\\ &= 2\sqrt{2}\end{aligned}$$

Finally, evaluate the integral.

$$\begin{aligned}\int_{5\pi/4}^{7\pi/4} 2\csc\theta\cot\theta\, d\theta &= F\left(\frac{7\pi}{4}\right) - F\left(\frac{5\pi}{4}\right)\\ &= 2\sqrt{2} - \left(2\sqrt{2}\right)\\ &= 0\end{aligned}$$

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Find the derivative

a. by evaluating the integral and differentiating the result.
b. by differentiating the integral directly.

$$\frac{d}{dx}\int_{2\pi}^{\sqrt[3]{x}} \mathbf{cos}\, t\, dt$$

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a. According to the second part of the fundamental theorem of calculus, if f is continuous at every point of [a,b] and F is any antiderivative of f on [a,b], then the definite integral can be evaluated using the following formula.

$$\int_a^b f(x)dx = F(b) - F(a)$$

Let $f(t) = \mathbf{cos}\, t$. To evaluate the integral, first find F(t). Recall that the general antiderivative of $f(x) = \mathbf{cos}\, x$ is $F(x) = \mathbf{sin}\, x + C$.

$F(t) = \mathbf{sin}\, t + C$

Because of the subtraction, a constant in F(t) will not affect the value of $F(b) - F(a)$. Thus, there is no need to include the constant C.

$$\int_{2\pi}^{\sqrt[3]{x}} \mathbf{cos}\, t\, dt = F\left(\sqrt[3]{x}\right) - F(2\pi)$$

$$= [\mathbf{sin}\, t]_{2\pi}^{\sqrt[3]{x}}$$

Evaluate $F\left(\sqrt[3]{x}\right)$.

$F\left(\sqrt[3]{x}\right) = \mathbf{sin}\, \sqrt[3]{x}$

Evaluate $F(2\pi)$.

$$\begin{aligned} F(2\pi) &= \mathbf{sin}\, 2\pi \\ &= 0 \end{aligned}$$

Apply the second part of the fundamental theorem of calculus to evaluate the integral.

$$\int_{2\pi}^{\sqrt[3]{x}} \mathbf{cos}\, t\, dt = F\left(\sqrt[3]{x}\right) - F(2\pi)$$

$$= \mathbf{sin}\, \sqrt[3]{x}$$

Now, differentiate the value of the integral. Remember to use the chain rule for derivatives, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.

$$\frac{d}{dx}\int_{2\pi}^{\sqrt[3]{x}} \mathbf{cos}\, t\, dt = \frac{d}{dx}\left(\mathbf{sin}\ \sqrt[3]{x}\right)$$

$$= \mathbf{cos}\ \sqrt[3]{x}\left(\frac{d}{dx}\sqrt[3]{x}\right)$$

$$\frac{d}{dx}\int_{2\pi}^{{}^{3}x} \mathbf{cos}\, t\, dt = \mathbf{cos}\ \sqrt[3]{x}\left(\frac{1}{3}x^{-\frac{2}{3}}\right)$$

Therefore, $\frac{d}{dx}\int_{2\pi}^{\sqrt[3]{x}} \mathbf{cos}\, t\, dt = \frac{1}{3}x^{-\frac{2}{3}}\ \mathbf{cos}\ \sqrt[3]{x}$.

b. According to the first part of the fundamental theorem of calculus, if f is continuous on [a,b] then $F(x) = \int_a^x f(t)dt$ is continuous on [a,b] and differentiable on (a,b) and the following equation is true.

$$F\,'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

In the given derivative, $\frac{d}{dx}\int_{2\pi}^{\sqrt[3]{x}} \mathbf{cos}\, t\, dt$, the upper bound is $\sqrt[3]{x}$. Because $x \neq \sqrt[3]{x}$, the derivative is a composite of two functions, which means the chain rule is used to differentiate.

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dx}$$

Apply the first part of the fundamental theorem of calculus, $F\,'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$, to find the derivative. To use the chain rule, let $u = \sqrt[3]{x}$.

$$\frac{d}{dx}\int_{2\pi}^{\sqrt[3]{x}} \mathbf{cos}\, t\, dt = \mathbf{cos}\ \sqrt[3]{x}\left(\frac{d}{dx}\sqrt[3]{x}\right)$$

$$= \frac{1}{3}x^{-\frac{2}{3}}\ \mathbf{cos}\ \sqrt[3]{x}$$

Notice that both methods have the same result.

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Find the derivative

a. by evaluating the integral and differentiating the result.
b. by differentiating the integral directly.

$$\frac{d}{dt}\int_0^{t^{10}} \sqrt[5]{u}\,du$$

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a. To find $\frac{d}{dt}\int_0^{t^{10}} \sqrt[5]{u}\,du$ by evaluating the integral and differentiating the result, first find $\int_0^{t^{10}} \sqrt[5]{u}\,du$.

According to the second part of the fundamental theorem of calculus, if f is continuous at every point of [a,b] and F is any antiderivative of f on [a,b], then the definite integral can be evaluated using the following formula.

$$\int_a^b f(x)dx = F(b) - F(a)$$

Let $f(t) = \sqrt[5]{u}$. To evaluate the integral, first find F(t).

$$\begin{aligned} F(t) &= \int \sqrt[5]{u}\,du \\ &= \int u^{1/5}du \\ &= \frac{5}{6}u^{6/5} + C \end{aligned}$$

Because of the subtraction, a constant in F(t) will not affect the value of F(b) − F(a). Thus, there is no need to include the constant C.

$$\begin{aligned} \int_0^{t^{10}} \sqrt[5]{u}\,du &= F\left(t^{10}\right) - F(0) \\ &= \frac{5}{6}u^{6/5}\Big]_0^{t^{10}} \end{aligned}$$

Evaluate $F\left(t^{10}\right)$.

$$F\left(t^{10}\right) = \frac{5}{6}t^{12}$$

Evaluate F(0).

$$F(0) = 0$$

Apply the fundamental theorem to evaluate the integral.

$$\int_0^{t^{10}} \sqrt[5]{u}\,du = F\left(t^{10}\right) - F(0)$$

$$= \frac{5}{6}t^{12}$$

Differentiating $\frac{5}{6}t^{12}$, you get $10t^{11}$.

b. According to the first part of the fundamental theorem of calculus, if f is continuous on [a,b] then $F(x) = \int_a^x f(t)dt$ is continuous on [a,b] and differentiable on (a,b) and the following equation is true.

$$F'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

To differentiate $\int_0^{t^{10}} \sqrt[5]{u}\,du$ directly, the upper limit is not t but t^{10}.

So $y = \int_0^{t^{10}} \sqrt[5]{u}\,du$ is a composite of $y(v) = \int_0^{v} \sqrt[5]{u}\,du$ and $v = t^{10}$. Use the chain rule to find $\frac{dy}{dt}$.

Rewrite the chain rule so that it applies to the functions y(v) and v(t).

$$\frac{dy}{dt} = \frac{dy}{dv}\frac{dv}{dt}$$

Thus, $\frac{dy}{dt} = \frac{dy}{dv} \cdot \frac{dv}{dt} = \left(\frac{d}{dv}\int_0^{v} \sqrt[5]{u}\,du\right) \cdot \frac{dv}{dt}$.

By the first part of the fundamental theorem of calculus, $\frac{dy}{dv} = \frac{d}{dv}\int_0^{v} u^{1/5} = v^{1/5}$. Since $v = t^{10}$, $\frac{dv}{dt} = 10t^9$.

Substituting $v = t^{10}$ into $\frac{dy}{dv} = v^{1/5}$, $\frac{dy}{dv} = t^{10/5}$.

Finally, $\frac{dy}{dt} = \frac{dy}{dv} \cdot \frac{dv}{dt} = 10t^{11}$.

Notice that both methods have the same result.

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Find the total area of the region between the x-axis and the graph.

$y = 2x^3 - 6x^2 + 4x,\ 0 \le x \le 2$

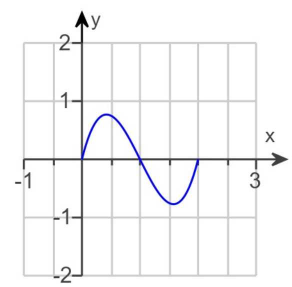
To determine the total area, find the subintervals where y is positive and where y is negative, and integrate over these subintervals. The total area is the sum of the absolute values of each integral.

First calculate the zeros of y. Solve the equation $2x^3 - 6x^2 + 4x = 0$ for x.

$$\begin{aligned} 2x^3 - 6x^2 + 4x &= 0 \\ 2x(x-1)(x-2) &= 0 \\ x &= 0,1,2 \end{aligned}$$

These zeros subdivide the interval $0 \le x \le 2$ into two subintervals, [0,1] and [1,2].

On the interval [0,1], y is greater than or equal to zero and on the interval [1,2], y is less than or equal to zero.



According to the second part of the fundamental theorem of calculus, if f is continuous at every point of [a,b] and F is any antiderivative of f on [a,b], then the definite integral can be evaluated using the following formula.

$$\int_a^b f(x)dx = F(b) - F(a)$$

An antiderivative of $f(x) = 2x^3 - 6x^2 + 4x$ is $F(x) = \frac{1}{2}x^4 - 2x^3 + 2x^2 + C$.

Now evaluate the integral over each subinterval. Start with the subinterval [0,1].

$$\begin{aligned} \int_0^1 \left(2x^3 - 6x^2 + 4x\right) dx &= \left[\frac{1}{2}x^4 - 2x^3 + 2x^2\right]_0^1 \\ &= \frac{1}{2}(1)^4 - 2(1)^3 + 2(1)^2 - 0 \\ &= \frac{1}{2} \end{aligned}$$

Next evaluate the integral over the subinterval [1,2].

$$\int_1^2 \left(2x^3 - 6x^2 + 4x\right) dx = \left[\frac{1}{2}x^4 - 2x^3 + 2x^2\right]_1^2$$
$$= \frac{1}{2}(2)^4 - 2(2)^3 + 2(2)^2 - \left(\frac{1}{2}(1)^4 - 2(1)^3 + 2(1)^2\right)$$
$$= -\frac{1}{2}$$

The total area is the sum of the absolute value of each integral. Calculate the total area.

$$\text{Total area} = \left|\frac{1}{2}\right| + \left|-\frac{1}{2}\right|$$
$$= 1$$

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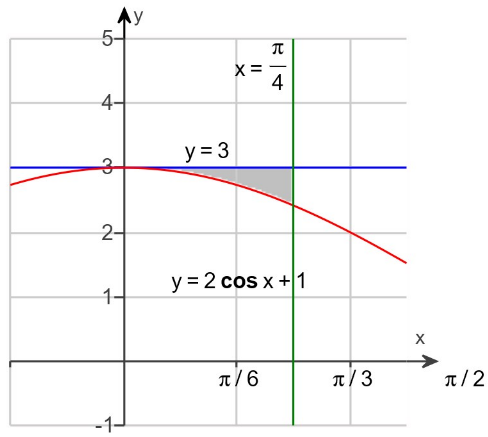
Find the shaded region in the graph.



The shaded area is the difference between the area of the rectangle bounded by the x- and y-axes, y = 3 and $x = \frac{\pi}{4}$ and the area bounded by the x-axis, the y-axis, y = 2 **cos** x + 1, and $x = \frac{\pi}{4}$.

The area of the rectangle is $\frac{3\pi}{4}$.

The area under the curve is $\int_0^{\pi/4} (2\cos x + 1)dx$.

$$\int_0^{\pi/4} (2\cos x + 1)dx = [2\sin x + x]_0^{\pi/4}$$

$$= \sqrt{2} + \frac{\pi}{4}$$

Thus, the shaded area is $\frac{\pi}{2} - \sqrt{2}$.