Student: Cole Lamers **Date:** 09/02/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.6 Limits Involving Infinity; Asymptotes of Graph

Find the limit of $f(x) = \frac{2}{x} - 3$ as x approaches ∞ and as x approaches $-\infty$.

Apply various limit laws.

$$\lim_{x\to\infty}\left(\frac{2}{x} - 3\right) = 2\lim_{x\to\infty}\frac{1}{x} - \lim_{x\to\infty} 3 = 2(0) - 3$$

Thus, $\lim_{x\to\infty}\left(\frac{2}{x} - 3\right) = -3$.

Similarly, $\lim_{x\to-\infty}\left(\frac{2}{x} - 3\right) = 2\lim_{x\to-\infty}\frac{1}{x} - \lim_{x\to-\infty} 3 = 2(0) - 3$.

Thus, $\lim_{x\to-\infty}\left(\frac{2}{x} - 3\right) = -3$.

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Find the limit.

$$\lim_{x\to\infty} \frac{\sin 2x}{4x}$$

The sandwich theorem states that if $g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself, and if $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$, then, $\lim_{x\to c} f(x) = L$.

Start by examining the numerator of the given function, $\sin 2x$. The **sin** function has a minimum absolute value of 0 and a maximum absolute value of 1.

Thus, the range of the absolute value of $\sin 2x$ is the following.

$$0 \le |\sin 2x| \le 1$$

Divide each part of the inequality by $|4x|$.

$$0 \le \left|\frac{\sin 2x}{4x}\right| \le \left|\frac{1}{4x}\right|$$

The $\lim_{x\to\infty} 0$ is 0.

The $\lim_{x\to\infty} \left|\frac{1}{4x}\right|$ is 0.

Therefore, by the sandwich theorem, $\lim_{x\to\infty} \left|\frac{\sin 2x}{4x}\right| = 0$.

For any function f(x), if $\lim_{x\to c} |f(x)| = 0$, then $\lim_{x\to c} f(x)$ is also 0. This can also be proved using the sandwich theorem. Given $-|f(x)| \le f(x) \le |f(x)|$, then, $\lim_{x\to c} -|f(x)|$ is 0 if $\lim_{x\to c} |f(x)|$ is 0, which makes $\lim_{x\to c} f(x) = 0$ as well.

Therefore, $\lim_{x\to\infty} \frac{\sin 2x}{4x} = 0$.

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Find the limit of $f(x) = \frac{2x+10}{4x+11}$ as x approaches ∞ and as x approaches $-\infty$.

Divide both numerator and denominator by x.

$$f(x) = \frac{2x+10}{4x+11} = f(x) = \frac{\frac{2x+10}{x}}{\frac{4x+11}{x}} = \frac{2+\frac{10}{x}}{4+\frac{11}{x}}$$

$$\lim_{x \to \pm\infty} \frac{10}{x} = 0$$

$$\lim_{x \to \pm\infty} \frac{11}{x} = 0$$

Thus, $\lim\limits_{x \to \pm\infty} \frac{2x+10}{4x+11} = \frac{1}{2}$.

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Find the limit of the rational function **a.** as $x \to \infty$ and **b.** as $x \to -\infty$.

$$h(x) = \frac{10x^3}{2x^3 + 3x^2 + 6x}$$

a. Find the limit of h(x) as $x \to \infty$. The function h(x) is a rational function. To determine the limit, first divide the numerator and denominator by the highest power of x in the denominator.

The highest power of x in the denominator is 3 so divide the numerator and denominator by x^3.

$$\lim_{x\to\infty} \frac{10x^3}{2x^3 + 3x^2 + 6x} = \frac{\left(10x^3 / x^3\right)}{\left(2x^3 / x^3\right) + \left(3x^2 / x^3\right) + \left(6x / x^3\right)}$$

$$= \frac{(10)}{(2) + (3/x) + \left(6/x^2\right)}$$

Find the limit as x approaches ∞ for the term in the numerator.

$$\lim_{x\to\infty} 10 = 10$$

Find the limit as x approaches ∞ for each term in the denominator.

$$\lim_{x\to\infty} 2 = 2$$

$$\lim_{x\to\infty} \frac{3}{x} = \lim_{x\to\infty} 3 \cdot \lim_{x\to\infty} \frac{1}{x} = 3 \cdot 0 = 0$$

$$\lim_{x\to\infty} \frac{6}{x^2} = \lim_{x\to\infty} 6 \cdot \lim_{x\to\infty} \frac{1}{x} \cdot \lim_{x\to\infty} \frac{1}{x} = 6 \cdot 0 \cdot 0 = 0$$

Substitute these values into the given limit and simplify.

$$\lim_{x\to\infty} \frac{10x^3}{2x^3 + 3x^2 + 6x} = \lim_{x\to\infty} \frac{(10)}{(2) + (3/x) + \left(6/x^2\right)}$$

$$= \frac{10}{2 + 0 + 0}$$

$$= 5$$

b. Find the limit of h(x) as $x \to -\infty$. Recall that dividing the numerator and the denominator by the highest power of x in the denominator gives the following limit.

$$\lim_{x\to-\infty} \frac{10x^3}{2x^3 + 3x^2 + 6x} = \lim_{x\to-\infty} \frac{(10)}{(2) + (3/x) + \left(6/x^2\right)}$$

Find the limit as x approaches $-\infty$ for the term in the numerator.

$$\lim_{x\to-\infty} 10 = 10$$

Find the limit as x approaches $-\infty$ for each term in the denominator.

$$\lim_{x \to -\infty} 2 = 2$$

$$\lim_{x \to -\infty} \frac{3}{x} = \lim_{x \to -\infty} 3 \bullet \lim_{x \to -\infty} \frac{1}{x} = 0$$

$$\lim_{x \to -\infty} \frac{6}{x^2} = \lim_{x \to -\infty} 6 \bullet \lim_{x \to -\infty} \frac{1}{x} \bullet \lim_{x \to -\infty} \frac{1}{x} = 0$$

Substitute these values into the given limit and simplify.

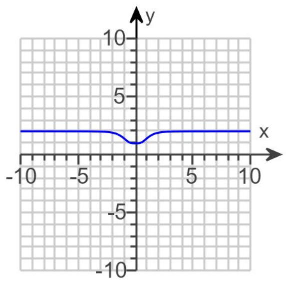
$$\begin{aligned}\lim_{x \to -\infty} \frac{10x^3}{2x^3 + 3x^2 + 6x} &= \lim_{x \to -\infty} \frac{(10)}{(2) + (3/x) + \left(6/x^2\right)} \\ &= \frac{10}{2 + 0 + 0} \\ &= 5\end{aligned}$$

Hence, $\lim_{x \to \infty} \frac{10x^3}{2x^3 + 3x^2 + 6x} = \lim_{x \to -\infty} \frac{10x^3}{2x^3 + 3x^2 + 6x} = 5$.

Student: Cole Lamers **Date:** 09/02/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.6 Limits Involving Infinity; Asymptotes of Graph

Find $\lim\limits_{x\to\infty} \sqrt[5]{\dfrac{5+32x^4}{x^4+6}}$.

The graph of $f(x) = \sqrt[5]{\dfrac{5+32x^4}{x^4+6}}$ is shown below.



To find $\lim\limits_{x\to\infty} \sqrt[5]{\dfrac{5+32x^4}{x^4+6}}$, we first simplify the limit using limit laws.

The limit law for nth roots states that the limit of an nth root is the same as the nth root of the limit. However, this law cannot be applied when the radicand is negative and the index of the radical is even, because the calculation would not result in a real answer.

Since $x \to \infty$, the numerator and denominator of the radicand will both be positive values, so we can apply the limit law. Also, because the index is 5, we could apply the law even with a negative quotient in the radicand.

We apply the limit law.

$$\lim_{x\to\infty} \sqrt[5]{\frac{5+32x^4}{x^4+6}} = \sqrt[5]{\lim_{x\to\infty} \frac{5+32x^4}{x^4+6}}$$

Now we divide the numerator and denominator by the largest power of x in the denominator, x^4.

$$\sqrt[5]{\lim_{x\to\infty} \frac{5+32x^4}{x^4+6}} = \sqrt[5]{\lim_{x\to\infty} \frac{\dfrac{5+32x^4}{x^4}}{\dfrac{x^4+6}{x^4}}}$$

We break up the numerator and denominator into simpler parts.

$$\sqrt[5]{\lim_{x\to\infty} \frac{\dfrac{5+32x^4}{x^4}}{\dfrac{x^4+6}{x^4}}} = \sqrt[5]{\lim_{x\to\infty} \frac{\dfrac{5}{x^4}+32}{1+\dfrac{6}{x^4}}}$$

Next find the limit of the numerator.

$$\lim_{x\to\infty} \frac{5}{x^4} + 32 = 32$$

Then find the limit of the denominator.

$$\lim_{x\to\infty} 1 + \frac{6}{x^4} = 1$$

We divide these two values to find the limit of the radicand.

$$\sqrt[5]{\lim_{x\to\infty} \frac{5 + 32x^4}{x^4 + 6}} = \sqrt[5]{\frac{32}{1}} = \sqrt[5]{32}$$

Finally, we take the fifth root to find the final answer.

$$\lim_{x\to\infty} \sqrt[5]{\frac{5 + 32x^4}{x^4 + 6}} = \sqrt[5]{32} = 2$$

Student: Cole Lamers **Date:** 09/02/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.6 Limits Involving Infinity; Asymptotes of Graph

Find the limit.

$$\lim_{x \to 0^+} \frac{1}{2x}$$

To determine $\lim_{x \to 0^+} \frac{1}{2x}$, begin by examining the behavior of $\frac{1}{2x}$ as values of x get closer and closer to zero from the right.

For $x = 0.0001$, $\frac{1}{2x} = 5000$.

For $x = 0.000001$, $\frac{1}{2x} = 500000$.

By choosing a value for x close enough to 0, make the value of $\frac{1}{2x}$ larger than any positive number, B, that can be chosen.

Describe this situation by saying that the limit is infinite and by writing $\lim_{x \to 0^+} \frac{1}{2x} = \infty$.

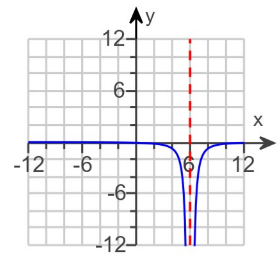
Student: Cole Lamers **Date:** 09/02/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.6 Limits Involving Infinity; Asymptotes of Graph

Find the limit.

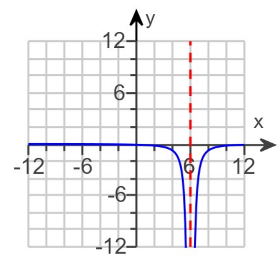
$$\lim_{x \to 6} \frac{-3}{(x-6)^2}$$

To find the limit, graph the function $y = \frac{-3}{(x-6)^2}$ and determine the value of y as x approaches 6 from the left and the right.

The graph of $y = \frac{-3}{(x-6)^2}$ is on the right. Notice that, as x approaches 6 from the left, the values of y become arbitrarily large and negative.



Also notice that, as x approaches 6 from the right, the values of y become arbitrarily large and negative.



Since the values of y become arbitrarily large and negative as x approaches 6 from both sides, $\lim_{x \to 6} \frac{-3}{(x-6)^2} = -\infty$.

Student: Cole Lamers **Date:** 09/02/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.6 Limits Involving Infinity; Asymptotes of Graph

Find $\lim \dfrac{x^2-10x+21}{x^3-7x^2}$ as

a. $x\to 0^+$ **b.** $x\to 7^+$ **c.** $x\to 7^-$ **d.** $x\to 7$

e. Determine what, if anything, can be said about the limit as $x\to 0$.

a. To find the limit, first factor the numerator and the denominator of the expression and eliminate any common factors.

Factor the numerator and the denominator of $\dfrac{x^2-10x+21}{x^3-7x^2}$.

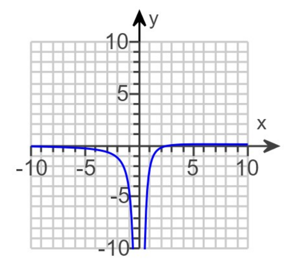
$$\frac{x^2-10x+21}{x^3-7x^2}=\frac{(x-7)(x-3)}{x^2(x-7)}$$

Now cancel out common factors in the numerator and the denominator.

$$\frac{(x-7)(x-3)}{x^2(x-7)}=\frac{x-3}{x^2}$$

Since substituting $x=0$ in the expression still results in a denominator of zero, graph the function $y=\dfrac{x-3}{x^2}$.

The graph of $y=\dfrac{x-3}{x^2}$ is on the right. Notice that, as x approaches 0 from the right, the values of y become arbitrarily large and negative.



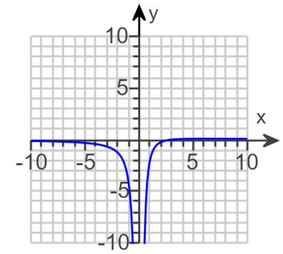
Since the values of y become arbitrarily large and negative as x approaches 0 from the right,

$$\lim_{x\to 0^+}\frac{x^2-10x+21}{x^3-7x^2}=\lim_{x\to 0^+}\frac{x-3}{x^2}=-\infty.$$

b. Since substituting $x=7$ in the simplified expression does not result in a denominator of zero, calculate the limit by evaluating the simplified expression at $x=7$.

$$\begin{aligned}\lim_{x\to 7^+}\frac{x^2-10x+21}{x^3-7x^2}&=\lim_{x\to 7^+}\frac{x-3}{x^2}\\&=\frac{(7)-3}{(7)^2}\\&=\frac{4}{49}\end{aligned}$$

c. The graph of $\frac{x^2-10x+21}{x^3-7x^2}$ is continuous at x = 7, so the limit as x approaches 7 from the left is the same as the limit as x approaches 7 from the right. Therefore, the $\lim_{x \to 7^-} \frac{x^2-10x+21}{x^3-7x^2}$ is also equal to $\frac{4}{49}$.

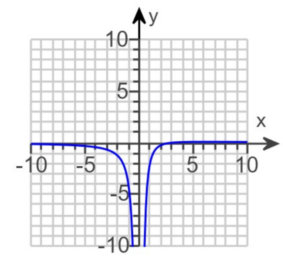


d. Since the values for the limit of the expression as $x \to 7^+$ and as $x \to 7^-$ are the same, this value is the limit of the expression as $x \to 7$.

$$\lim_{x \to 7} \frac{x^2-10x+21}{x^3-7x^2} = \frac{4}{49}$$

e. Evaluate the limit as $x \to 0^-$ and compare it to the limit as $x \to 0^+$ to determine what, if anything, can be said about the limit as $x \to 0$.

The graph of $y = \frac{x-3}{x^2}$ is on the right. Notice that, as x approaches 0 from the left, the values of y become arbitrarily large and negative.



Since the values of y become arbitrarily large and negative as x approaches 0 from the left,

$$\lim_{x \to 0^-} \frac{x^2-10x+21}{x^3-7x^2} = \lim_{x \to 0^-} \frac{x-3}{x^2} = -\infty.$$

Since the values for limit of the expression as $x \to 0^+$ and as $x \to 0^-$ are the same, this value is the limit of the expression as $x \to 0$.

$$\lim_{x \to 0} \frac{x^2-10x+21}{x^3-7x^2} = -\infty$$

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Find the horizontal and vertical asymptotes of f(x). Then graph f(x).

$$f(x) = \frac{x+4}{x+2}$$

Begin by transforming $f(x) = \frac{x+4}{x+2}$ by algebraically dividing. The long division shown below shows that $\frac{x+4}{x+2} = 1 + \frac{2}{x+2}$.

$$\begin{array}{r} 1 \\ x+2\overline{)x+4} \\ \underline{x+2} \\ 2 \end{array}$$

Recall that the line y = b is a horizontal asymptote of the graph of y = f(x) if either $\lim_{x\to\infty} f(x) = b$ or $\lim_{x\to-\infty} f(x) = b$.

$$\lim_{x\to\infty}\left(1 + \frac{2}{x+2}\right) = 1$$

$$\lim_{x\to-\infty}\left(1 + \frac{2}{x+2}\right) = 1$$

So, the horizontal asymptote is y = 1.

Similarly, the line x = a is a vertical asymptote of the graph of y = f(x) if either $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$.

By examining the behavior of $f(x) = 1 + \frac{2}{x+2}$ as $x\to -2$ from either side, $\lim_{x\to-2^+}\left(1 + \frac{2}{x+2}\right) = \infty$ and $\lim_{x\to-2^-}\left(1 + \frac{2}{x+2}\right) = -\infty$.

Thus x = −2 is the vertical asymptote.

To graph f(x), identify the vertical asymptote x = −2 and the horizontal asymptote y = 1. Note that for x > −2 the function is greater than 1. The graph of f(x) is shown to the right.