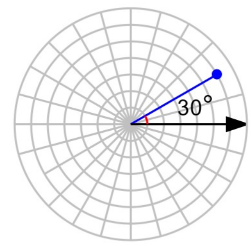
Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Plot the point having the polar coordinates $(6,30^\circ)$. Then find different polar coordinates (r,θ) for the same point for which **(a)** $r<0$ and $0^\circ \le \theta < 360^\circ$, **(b)** $r<0$ and $-360^\circ < \theta < 0^\circ$, **(c)** $r>0$ and $-360^\circ < \theta < 0^\circ$.

Plot the point having the polar coordinates $(6,30^\circ)$.

The point $(6,30^\circ)$ is plotted by first drawing $\theta = 30^\circ$ counterclockwise from the polar axis. Because $r = 6 > 0$, the point is on the terminal side of the angle, 6 units from the pole.

The point is plotted as shown to the right.



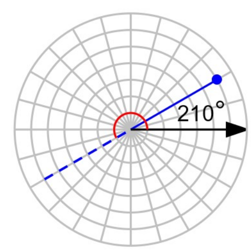
(a) To graph a point (r,θ) for $r<0$, measure $|r|$ units along the terminal sides of $\theta + 180^\circ$.

Since $r<0$, measure 6 units along the terminal sides of $30^\circ + 180^\circ = 210^\circ$.

If an angle is greater than 360°, then subtract 360° from the angle.

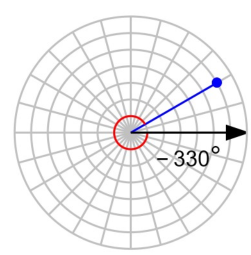
The polar coordinates for the point $(6,30^\circ)$ for which $r<0$ and $0^\circ \le \theta < 360^\circ$ are $(-6,210^\circ)$.

The point $(-6,210^\circ)$ is plotted as shown to the right.



(b) First measure the terminal side for the point $(6,30^\circ)$ from the polar axis in the clockwise direction.

The measure of the angle for $-360^\circ < \theta < 0^\circ$ is -330°.



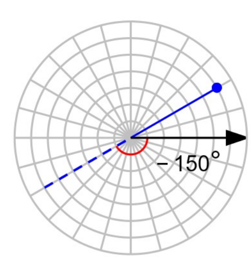
Note that the angle is measured in the clockwise direction, so the extension of the terminal side can be found by subtracting 180° from the angle.

Since $r<0$, measure 6 units along the terminal sides of $-330^\circ - 180^\circ = -510^\circ$.

If an angle is less than -360°, then add 360° to the angle.

The polar coordinates for the point $(6,30^\circ)$ for which $r<0$ and $-360^\circ < \theta < 0^\circ$ are $(-6,-150^\circ)$.

The point $(-6,-150^\circ)$ is plotted as shown to the right.



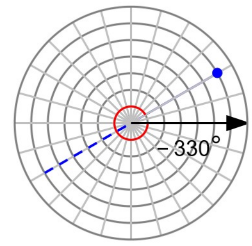
(c) Measure the terminal side for the point $(6,30^\circ)$ from the polar axis in the clockwise direction.

The measure of the angle is -330°.

Since $r>0$, $\theta = -330$.

The polar coordinates for the point $(6,30^\circ)$ for which $r>0$ and $-360^\circ < \theta < 0^\circ$ are $(6,-330^\circ)$.

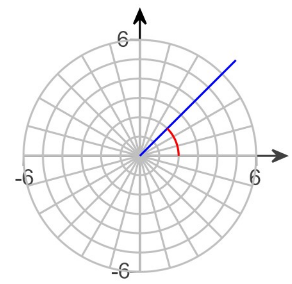
The point $(6, -330^{\circ})$ is plotted as shown to the right.



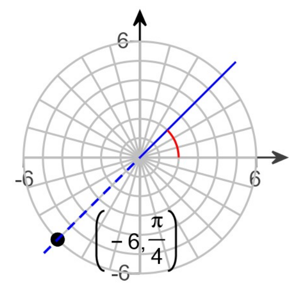
Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Plot the point having the polar coordinates $\left(-6, \frac{\pi}{4}\right)$. Then give two different pairs of polar coordinates of the same point, **(a)** one with the given value of r and **(b)** one with r having the opposite sign of the given value of r.

Begin plotting the point by first drawing an angle of $\frac{\pi}{4}$. That is, place the vertex of the angle at the pole and the initial side of the angle along the polar axis. Since the angle of the point is positive, measure the angle in a counterclockwise direction. Then draw the terminal side. The blue line is the terminal side.



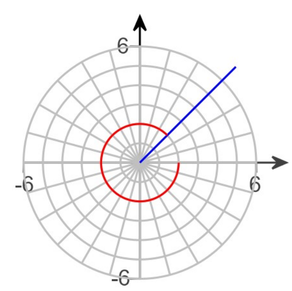
Since the first coordinate of the point is negative, instead of the point being on the terminal side of the angle, it is 6 units on the extension of the terminal side. The extension of the terminal side is the ray from the pole that extends in the direction opposite the terminal side of θ. Move 6 units along the ray from the pole that extends in the direction opposite the terminal side of θ.



a) Since $r < 0$, don't change the first coordinate, and the terminal side of the angle remains the same.

Calculate the new angle measure by subtracting one revolution (2π radians) from the original angle measure.

$$\frac{\pi}{4} - 2\pi = -\frac{7\pi}{4}$$



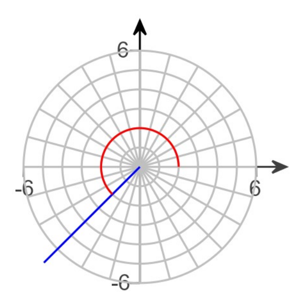
Therefore, the polar coordinates $\left(-6, -\frac{7\pi}{4}\right)$ also represent the point $\left(-6, \frac{\pi}{4}\right)$.

b) Find the pair of polar coordinates of the same point, with r having the opposite sign of the given value of r.

For the given point, $r = -6$. Therefore, the new first coordinate of the point is 6.

Calculate the new angle measure by adding $\frac{1}{2}$ revolution (π) to the original angle measure.

$$\frac{\pi}{4} + \pi = \frac{5\pi}{4}$$

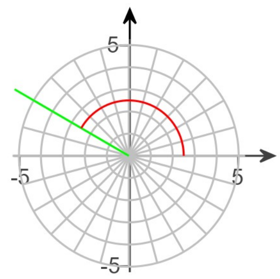


Therefore, the polar coordinates $\left(6, \frac{5\pi}{4}\right)$ also represent the point $\left(-6, \frac{\pi}{4}\right)$.

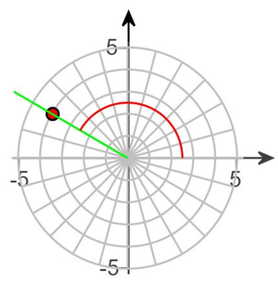
Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Plot the point having the polar coordinates $\left(4, \frac{5\pi}{6}\right)$. Then give two different pairs of polar coordinates of the same point, **(a)** one with the given value of r and **(b)** one with r having the opposite sign of the given value of r.

Begin plotting the point by first drawing an angle of $\frac{5\pi}{6}$. That is, place the vertex of the angle at the pole and the initial side of the angle along the polar axis. Since the angle of the point is positive, measure the angle in a counterclockwise direction. Then draw the terminal side. The green line is the terminal side.



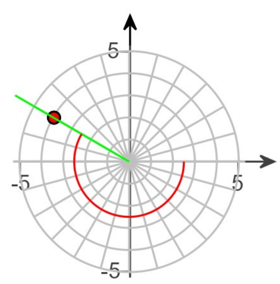
Since the first coordinate of the point is positive, the point is along the terminal side of the angle. Move 4 units along the terminal side and then plot the point.



a) Since $r > 0$, don't change the first coordinate, and the terminal side of the angle remains the same.

Calculate the new angle measure by subtracting one revolution (2π radians) from the original angle measure.

$$\frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$$



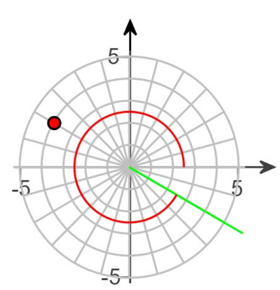
Therefore, the polar coordinates $\left(4, -\frac{7\pi}{6}\right)$ also represent the point $\left(4, \frac{5\pi}{6}\right)$.

b) Find the pair of polar coordinates of the same point, with r having the opposite sign of the given value of r.

For the given point, $r = 4$. Therefore, the new first coordinate of the point is -4.

Calculate the new angle measure by adding $\frac{1}{2}$ revolution (π) to the original angle measure.

$$\frac{5\pi}{6} + \pi = \frac{11\pi}{6}$$



Therefore, the polar coordinates $\left(-4, \frac{11\pi}{6}\right)$ also represent the point $\left(4, \frac{5\pi}{6}\right)$.

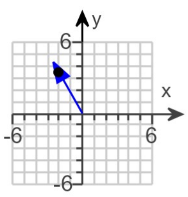
Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Find the rectangular coordinates of the point given in polar coordinates.

$(4, 120^\circ)$

As an aid in converting between polar coordinates and rectangular coordinates, first plot the point in polar coordinates.

To locate the angle, start at the polar axis (positive x-axis) and rotate counter-clockwise for a positive angle. Rotate clockwise for a negative angle. Extend out from the pole (origin) to the value of the radius. The following graph is a plot of the polar coordinates $(4, 120^\circ)$.



Notice that the point lies in quadrant II of the coordinate system, so the x-coordinate should be negative and the y-coordinate should be positive.

The x-coordinate of the point is found using the fact that $\cos 120^\circ = -\frac{1}{2}$.

$$\begin{aligned} x &= r\cos\theta \\ &= 4\cos 120^\circ \\ &= -2 \end{aligned}$$

The y-coordinate of the point is found using the fact that $\sin 120^\circ = \frac{\sqrt{3}}{2}$.

$$\begin{aligned} y &= r\sin\theta \\ &= 4\sin 120^\circ \\ &= 2\sqrt{3} \end{aligned}$$

Thus, the polar coordinates $(4, 120^\circ)$ written in rectangular coordinates are $\left(-2, 2\sqrt{3}\right)$.

Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

The rectangular coordinates of a point are given. Find polar coordinates of the point such that $r > 0$ and $0 \le \theta < 2\pi$.

$(7, -7)$

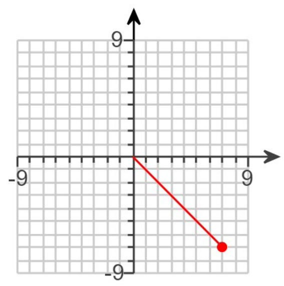
Converting a Point from Rectangular to Polar Coordinates ($r > 0$ and $0 \le \theta < 2\pi$)

1. Plot the point (x,y).

2. Find r by computing the distance from the origin to (x,y):
$r = \sqrt{x^2 + y^2}$.

3. Find θ using $\tan\theta = \frac{y}{x}$ with the terminal side of θ passing through (x,y).

Begin by plotting $(7, -7)$ in a rectangular coordinate system.



If a point has rectangular coordinates (x,y), the distance r from the origin to this point is given by $r = \sqrt{x^2 + y^2}$. Calculate r for the point $(7, -7)$.

$$\begin{aligned} r &= \sqrt{(7)^2 + (-7)^2} \\ &= \sqrt{98} \\ &= 7\sqrt{2} \end{aligned}$$

Now find θ, where θ is in the same quadrant as the point (x,y). First, compute $\tan\theta = \frac{y}{x}$.

$$\tan\theta = \frac{y}{x} = \frac{-7}{7} = -1$$

Since the plotted point $(7, -7)$ lies in quadrant IV, θ also lies in quadrant IV.

Recall that $\tan\frac{\pi}{4} = 1$, where $\frac{\pi}{4}$ is an angle in quadrant I.

Since $\tan\frac{\pi}{4} = 1$ and the point (x,y) lies in quadrant IV, compute θ by subtracting $\frac{\pi}{4}$ from 2π.

$$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$$

Therefore, the point with rectangular coordinates $(7, -7)$ has polar coordinates $\left(7\sqrt{2}, \frac{7\pi}{4}\right)$.

Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Convert to a polar equation.

$x^2 + y^2 = 49$

Use the fact that $r \cos \theta = x$ and $r \sin \theta = y$.

Substitute for x and y.

$(r \cos \theta)^2 + (r \sin \theta)^2 = 49$

$r^2 \cos^2 \theta + r^2 \sin^2 \theta = 49$

$r^2 \left(\cos^2 \theta + \sin^2 \theta \right) = 49$

$r^2 = 49$

$r = 7$

Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Convert the following rectangular equation to polar form.

$$y^2 = 25y - x^2$$

To convert a rectangular equation to a polar equation, replace x with $r\cos\theta$ and y with $r\sin\theta$.

Replace x with $r\cos\theta$ and y with $r\sin\theta$ in the given equation.

$$y^2 = 25y - x^2 \qquad \text{Original equation}$$
$$(r\sin\theta)^2 = 25(r\sin\theta) - (r\cos\theta)^2 \qquad \text{Substitute.}$$

Use the identity $(ab)^2 = a^2b^2$ to simplify the squared brackets.

$$(r\sin\theta)^2 = 25(r\sin\theta) - (r\cos\theta)^2$$
$$r^2\sin^2\theta = 25r\sin\theta - r^2\cos^2\theta$$

Add $r^2\cos^2\theta$ to both sides of the equation.

$$r^2\sin^2\theta = 25r\sin\theta - r^2\cos^2\theta$$
$$r^2\sin^2\theta + r^2\cos^2\theta = 25r\sin\theta - r^2\cos^2\theta + r^2\cos^2\theta$$
$$r^2\sin^2\theta + r^2\cos^2\theta = 25r\sin\theta$$

Now, factor out r^2 from the left side of the equation.

$$r^2\sin^2\theta + r^2\cos^2\theta = 25r\sin\theta$$
$$r^2\left(\sin^2\theta + \cos^2\theta\right) = 25r\sin\theta$$
$$r^2(1) = 25r\sin\theta \qquad \text{Recall that } \sin^2\theta + \cos^2\theta = 1.$$

Finally, simplify.

$$r^2 = 25r\sin\theta$$
$$r = 25\sin\theta \qquad \text{Divide r from both sides.}$$

Therefore, the polar form of the given rectangular equation is $r = 25\sin\theta$.

Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Convert the following polar equation to rectangular form. Identify the curve.

$$r = 26 \cos \theta$$

First multiply both sides of $r = 26 \cos \theta$ by r. Note that multiplying by r results in r^2 on the left side of the equation, which can be easily written in terms of x and y.

$$\begin{aligned} r &= 26 \cos \theta \\ r^2 &= 26r \cos \theta \end{aligned}$$

To convert the polar equation to rectangular form, use the substitutions $r^2 = x^2 + y^2$ and $x = r \cos \theta$.

Replace r^2 with $x^2 + y^2$ and $r \cos \theta$ with x.

$$\begin{aligned} r^2 &= 26r \cos \theta \\ x^2 + y^2 &= 26x \end{aligned}$$

Subtract 26x from both sides and simplify.

$$\begin{aligned} x^2 + y^2 &= 26x \\ x^2 - 26x + y^2 &= 0 \end{aligned}$$

Now to write the equation in standard form, complete the square. Add 169 to $x^2 - 26x$ to create a perfect square trinomial.

$$\begin{aligned} x^2 - 26x + y^2 &= 0 \\ \left(x^2 - 26x + 169\right) + y^2 &= 169 \qquad \text{Add 169 to both sides.} \end{aligned}$$

Factor the perfect-square trinomial.

$$\begin{aligned} \left(x^2 - 26x + 169\right) + y^2 &= 169 \\ (x - 13)^2 + y^2 &= 169 \end{aligned}$$

Therefore, the rectangular form of the polar equation is $(x - 13)^2 + y^2 = 169$.

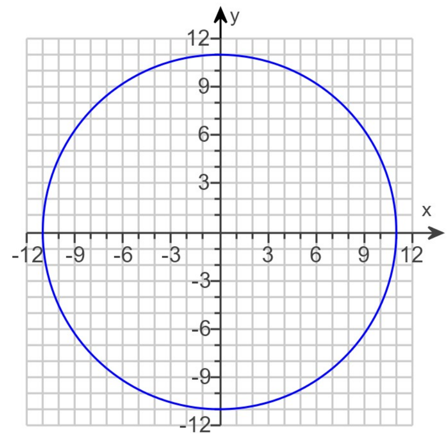
Note that an equation of the form $(x - h)^2 + (y - k)^2 = r^2$ is a circle. For the circle $(x - h)^2 + (y - k)^2 = r^2$, the center is (h,k) and the radius is r.

The graph of $r = 26 \cos \theta$ or $(x - 13)^2 + y^2 = 169$ is a circle centered at (13,0) with a radius of 13 units.

Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Sketch the graph of the following polar equation by transforming it to rectangular coordinates.

$r = 11$



The following formulas can be used to transform equations from one set of coordinates to another.

$x = r\cos\theta \qquad y = r\sin\theta$

$r^2 = x^2 + y^2 \qquad \tan\theta = \dfrac{y}{x}$

Note that the given equation is in terms of r only and not in terms of θ.

To transform $r = 11$ to rectangular coordinates, use the formula $r^2 = x^2 + y^2$.

First, square both sides of the given equation.

$$r = 11$$
$$r^2 = 121$$

Now substitute $x^2 + y^2$ for r^2.

$$r^2 = 121$$
$$x^2 + y^2 = 121$$

Thus, the rectangular form of the polar equation $r = 11$ is $x^2 + y^2 = 121$.

This equation represents a circle.

Note that the standard form for the equation of a circle centered at the origin with a radius r is $x^2 + y^2 = r^2$.

Thus, the graph of $x^2 + y^2 = 121$ is a circle centered at (0,0) with a radius of 11.

A sketch the graph of $x^2 + y^2 = 121$ is shown to the right.

Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Sketch the graph of the following polar equation. Then identify the curve.

$r = 6\,\mathbf{cos}\,\theta$

First test for symmetry about the polar axis, the line $\theta = \frac{\pi}{2}$, and the pole.

To test for symmetry about the polar axis, replace (r,θ) with $(r, -\theta)$. If an equivalent equation results, the graph is symmetric about the polar axis.

Replace (r,θ) with $(r, -\theta)$ in $r = 6\,\mathbf{cos}\,\theta$.

$r = 6\,\mathbf{cos}\,(-\theta)$

$r = 6\,\mathbf{cos}\,\theta$ Use the fact that the cosine function is even, $\mathbf{cos}\,(-\theta) = \mathbf{cos}\,\theta$.

Since the polar equation did not change, it is symmetric about the polar axis.

To test for symmetry about the line $\theta = \frac{\pi}{2}$, replace (r,θ) with $(-r, -\theta)$. If an equivalent equation results, the graph is symmetric about the line $\theta = \frac{\pi}{2}$.

Replace (r, θ) with $(-r, -\theta)$ in $r = 6\,\mathbf{cos}\,\theta$.

$-r = 6\,\mathbf{cos}(-\theta)$

$-r = 6\,\mathbf{cos}\,\theta$ Use the fact that the cosine function is even, $\mathbf{cos}\,(-\theta) = \mathbf{cos}\,\theta$.

$r = -6\,\mathbf{cos}\,\theta$ Multiply both sides by -1.

Since the polar equation changes, it may or may not be symmetric about the line $\theta = \frac{\pi}{2}$.

To test for symmetry about the pole, replace (r,θ) with $(-r,\theta)$. If an equivalent equation results, the graph is symmetric about the pole.

Replace (r,θ) with $(-r,\theta)$ in $r = 6\,\mathbf{cos}\,\theta$.

$-r = 6\,\mathbf{cos}\theta$

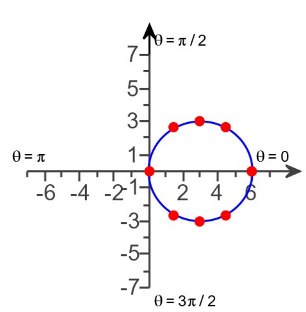
$r = -6\,\mathbf{cos}\,\theta$ Multiply both sides by -1.

Since the polar equation changed, it may or may not be symmetric about the pole.

Construct the table by substituting values for θ and calculating the corresponding values of r.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$r = 6\,\mathbf{cos}\,\theta$	6	5.2	4.24	3	0	-3	-4.24	-5.2	-6

Plot the points (r, θ) in the polar coordinate system. Now draw a smooth curve through them, and complete the graph by symmetry about polar axis. Therefore, the graph of the polar equation $r = 6\,\mathbf{cos}\,\theta$ is as shown to the right.



Therefore, the curve of $r = 6\,\mathbf{cos}$ is a circle.

Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.6 Polar Coordinates

Graph the equation.

$r = 14 \cos 2\theta$

To graph a polar equation, make a table of values, choosing values of θ and calculating corresponding values of r.

Begin with $\theta = 0^\circ$ and calculate its corresponding value of r.

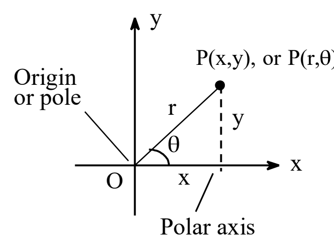
$r = 14 \cos(2 \cdot 0^\circ)$	Substitute 0° for θ.
$= 14 \cos(0^\circ)$	Simplify the argument.
$= 14(1)$	Calculate the value of $\cos 0^\circ$.
$= 14$	Simplify.

A table of points that satisfy the given equation is shown below.

θ	r
0°	14
30°	7
45°	0
60°	−7
90°	−14
120°	−7
135°	0
150°	7

θ	r
180°	14
210°	7
225°	0
240°	−7
270°	−14
300°	−7
315°	0
330°	7

Now plot the points and complete the graph. To plot points on a polar graph, locate the directed angle θ and move a directed distance r from the pole. If $r > 0$, move along ray OP. If $r < 0$, move in the opposite direction of ray OP.



The correct graph of $r = 14 \cos 2\theta$ is shown on the right. Notice that the graph passes through each point from the table of points. The scale on r is from -10 to 10 in increments of 3.