Student: Cole Lamers **Date:** 09/26/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.7 Antiderivatives (Set 1)

Find the antiderivative for each function when C equals 0. Do as many as you can mentally. Check your answers by differentiation.

a. $21x^{20}$ **b.** x^7 **c.** $x^2 + 2x - 48$

A derivative of the form $f(x) = x^n$ has a family of antiderivatives of the form $F(x) = \frac{1}{n+1}x^{n+1} + C$, where $n \neq -1$. Recall also the constant multiple rule, which states that $kf(x)$ has a general antiderivative of the form $kF(x) + C$, where k is a constant and $F(x)$ is the antiderivative of $f(x)$.

a. Let $f(x) = x^{20}$ and $k = 21$. For $f(x) = x^{20}$, identify the value of n to be used in the antiderivative formula.

$n = 20$

Substitute the value of n into the formula $F(x) = \frac{1}{n+1}x^{n+1} + C$ to find the antiderivative of $f(x) = x^{20}$ when $C = 0$.

$f(x) = x^{20}$

$F(x) = \frac{1}{20+1}x^{20+1}$ Substitute and apply the constant multiple rule.

$= \frac{1}{21}x^{21}$ Simplify.

b. For $f(x) = x^7$, identify the value of n to be used in the antiderivative formula.

$n = 7$

Substitute the value of n into the formula to find the antiderivative of $f(x) = x^7$ when $C = 0$.

$f(x) = x^7$

$F(x) = \frac{1}{7+1}x^{7+1}$ Substitute.

$= \frac{1}{8}x^8$ Simplify.

c. Let $f(x) = g(x) + 2h(x) - 48p(x)$, where $g(x) = x^2$, $h(x) = x$, and $p(x) = x^0$. Then $F(x) = G(x) + 2H(x) - 48P(x) + C$ by a combination of the Sum or Difference Rule and the Constant Multiple Rule. Compute the antiderivative of each term individually. First, find $G(x)$ when $C = 0$. Note that $n = 2$ for the purposes of the antiderivative formula.

$g(x) = x^2$

$G(x) = \frac{1}{2+1}x^{2+1}$ Substitute.

$= \frac{1}{3}x^3$ Simplify.

Now, find $H(x)$ when $C = 0$. Note that $n = 1$ for the purposes of the antiderivative formula.

$$\begin{aligned} h(x) &= x \\ H(x) &= \frac{1}{1+1}x^{1+1} && \text{Substitute.} \\ &= \frac{1}{2}x^2 && \text{Simplify.} \end{aligned}$$

Finally, find P(x) when C = 0. Note that n = 0 for the purposes of the antiderivative formula.

$$\begin{aligned} p(x) &= x^0 \\ P(x) &= \frac{1}{0+1}x^{0+1} && \text{Substitute.} \\ &= x && \text{Simplify.} \end{aligned}$$

To find F(x) when C = 0, substitute the expressions found for G(x), H(x) and P(x) into the equation $G(x) + 2H(x) - 48P(x)$ and simplify.

$$\begin{aligned} F(x) &= G(x) + 2H(x) - 48P(x) \\ &= \left(\frac{1}{3}x^3\right) + 2\left(\frac{1}{2}x^2\right) - 48(x) && \text{Substitute and apply the Constant Multiple Rule.} \\ &= \frac{1}{3}x^3 + x^2 - 48x && \text{Simplify.} \end{aligned}$$

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Find the indefinite integral $\int (8x+3)dx$.

The Constant Multiple Rule, an antiderivative linearity rule, states that the general antiderivative for k f(x) is k F(x)+C.

The Sum Rule, another antiderivative linearity rule, states that the general antiderivative for [f(x) + g(x)] is F(x) + G(x) + C.

Alternative representations of these rules are

$$\int k\, f(x)dx = k\int f(x)dx = k\, F(x) + C$$

and $\int (f(x)+g(x))dx = \int f(x)dx + \int g(x)dx = F(x) + G(x) + C$.

Apply these same rules to the expression $\int (8x+3)dx$.

First, apply the Sum Rule.

$$\int (8x+3)dx = \int (8x)dx + \int 3\, dx$$

Then apply the Constant Multiple Rule.

$$\int 8x\, dx = 8\int x\, dx \text{ and } \int 3\, dx = 3\int dx$$

$$\int (8x+3)dx = 8\int x\, dx + 3\int dx = 4x^2 + 3x + C$$

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Find the most general antiderivative or indefinite integral. Check your answer by differentiation.

$$\int \left(3t^2 + \frac{t}{8}\right) dt$$

The sum rule for antiderivatives states that a sum of functions may be antidifferentiated term by term as given by the following formula.

$$\int [f(x) + g(x)]\, dx = \int f(x)\, dx + \int g(x)\, dx$$

Begin by writing the expression according to the sum rule.

$$\int \left(3t^2 + \frac{t}{8}\right) dt = \int 3t^2\, dt + \int \frac{t}{8}\, dt$$

The constant multiple rule states that a constant multiple k may be moved through the integral sign as given by the following formula.

$$\int k\, f(x)\, dx = k \int f(x)\, dx$$

To antidifferentiate $\int 3t^2$ dt, first use the constant multiple rule to rewrite the expression.

$$\int 3t^2\, dt = 3 \int t^2\, dt$$

Now to antidifferentiate $3\int t^2$ dt, use the formula $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$ and multiply by the constant 3.

$$3\int t^2\, dt = 3 \cdot \frac{t^3}{3} + C = t^3 + C$$

Thus, $\int 3t^2\, dt = t^3 + C$.

Next use the constant multiple rule to rewrite $\int \frac{t}{8}$ dt.

$$\int \frac{t}{8}\, dt = \frac{1}{8} \int t\, dt$$

Now to antidifferentiate $\frac{1}{8}\int t$ dt, use the formula $\int x^n\, dx = \frac{x^{n+1}}{n+1} + C$ and multiply by the constant $\frac{1}{8}$.

$$\frac{1}{8}\int t\, dt = \frac{1}{8} \cdot \frac{t^2}{2} + C = \frac{t^2}{16} + C$$

Thus, $\int \frac{t}{8}\, dt = \frac{t^2}{16} + C$.

Now combine the separate terms to find the antiderivative of the original expression. For notational convenience, use only one constant of integration.

$$\int\left(3t^2 + \frac{t}{8}\right) dt = \int 3t^2\, dt + \int \frac{t}{8}\, dt$$

$$= t^3 + \frac{t^2}{16} + C$$

Check your answer by differentiation.

$$\frac{d}{dt}\left(t^3 + \frac{t^2}{16} + C\right) = 3t^2 + \frac{t}{8}$$

Thus, $\int\left(3t^2 + \frac{t}{8}\right) dt = t^3 + \frac{t^2}{16} + C$.

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Find the function y(x) satisfying $\frac{dy}{dx} = 4x - 5$ and $y(3) = 0$.

First, find $y(x) = \int (4x-5)dx$.

$y(x) = \int (4x-5)dx = 4\int x\,dx - 5\int dx$

$4\int x\,dx = 2x^2 + C_1$ and $5\int dx = 5x + C_2$

Letting $C = C_1 + C_2$, $\int (4x-5)dx = 2x^2 - 5x + C$.

Thus, $y(x) = 2x^2 - 5x + C$.

Since $y(3) = 0$, substitute 3 for x and 0 for y(x).

$2(3)^2 - 5(3) + C = 0$

Solving $2(3)^2 - 5(3) + C = 0$ for C, $C = -3$.

Thus, the function y(x) satisfying $\frac{dy}{dx} = 4x - 5$ and $y(3) = 0$ is $y(x) = 2x^2 - 5x - 3$.

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Find the function s(t) satisfying $\frac{ds}{dt} = 5 - 3\cos t$ and $s(0) = 4$.

$\frac{ds}{dt} = 5 - 3\cos t$ is equivalent to $s = \int (5 - 3\cos t)dt$.

Apply the Sum/Difference and Constant Multiple Rules.

$$\begin{aligned} s = \int (5 - 3\cos t)dt &= \int 5\,dt - \int 3\cos t\,dt \\ &= 5\int dt - 3\int \cos t\,dt \\ &= 5t - 3\sin t + C \end{aligned}$$

Since $s(0) = 4$, $5(0) - 3\sin 0 + C = 4$.

Solving $5(0) - 3\sin 0 + C = 4$ for C, $C = 4$.

Thus, the function satisfying $\frac{ds}{dt} = 5 - 3\cos t$ and $s(0) = 4$ is $s(t) = 5t - 3\sin t + 4$.

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Find the function y(x) satisfying $\frac{d^2y}{dx^2} = 4 - 12x$, $y'(0) = 5$, and $y(0) = 7$.

First, find $\frac{dy}{dx}$, the antiderivative of $\frac{d^2y}{dx^2} = 4 - 12x$.

$$\frac{dy}{dx} = \int (4 - 12x)dx = 4x - 6x^2 + C_1$$

The constant C_1 in $\frac{dy}{dx} = y' = 4x - 6x^2 + C_1$ can be evaluated by applying the initial value condition $y'(0) = 5$.

$C_1 = 5$

So $\frac{dy}{dx} = y' = 4x - 6x^2 + 5$.

$$y(x) = \int y'\,dx = \int \left(4x - 6x^2 + 5\right) dx = 2x^2 - 2x^3 + 5x + C_2$$

The constant C_2 in $y = 2x^2 - 2x^3 + 5x + C_2$ can be evaluated by applying the initial value condition $y(0) = 7$.

$C_2 = 7$

So, the function satisfying $\frac{d^2y}{dx^2} = 4 - 12x$, $y'(0) = 5$, and $y(0) = 7$ is $y(x) = 2x^2 - 2x^3 + 5x + 7$.