Student: Cole Lamers **Date:** 10/02/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.3 The Definite Integral (Set 1)

Express the limit $\lim\limits_{\|P\|\to 0} \sum\limits_{k=1}^{n} c_k^5 \Delta x_k$, P a partition of [3,7], as a definite integral.

The symbolism $\int_a^b f(x)dx$ is related to $\lim\limits_{\|P\|\to 0} \sum\limits_{k=1}^{n} f\left(c_k\right) \Delta x_k$, P a partition of the interval $\left[x_1,x_2\right]$ the following way: $\int_a^b$ replaces $\lim\limits_{\|P\|\to 0} \sum\limits_{k=1}^{n}$, P a partition of $\left[x_1,x_2\right]$ with $a = x_1$, $b = x_2$. $f(c_k)$ becomes f(x) and Δx_k becomes dx.

The integral sign with the limits of integration becomes $\int_3^7$.

In the given summation, $f(c_k) = c_k^5$.

In the integration, f(x) replaces the $f(c_k)$ and dx replaces the Δx_k of the summation.

Thus, $\lim\limits_{\|P\|\to 0} \sum\limits_{k=1}^{n} c_k^5 \Delta x_k$ with P is a partition of [3,7] is, as a definite integral $\int_3^7 x^5 dx$.

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The functions f and g are integrable and $\int_2^4 f(x)dx = -5$, $\int_2^7 f(x)dx = 3$, and $\int_2^7 g(x)dx = 4$. Use these to complete parts **(a)** through **(f)**.

a. $\int_4^4 f(x)dx$

Considering the definite integral as the area under the graph of a function over an interval [a,b], if the interval has zero width, the area is zero and the value of the definite integral is zero. This gives to the Zero Width Interval rule: $\int_a^a f(x)dx = 0$.

So, $\int_4^4 f(x)dx = 0$.

b. $\int_7^2 g(x)dx$

The Order of Integration rule states that $\int_a^b f(x)dx = -\int_b^a f(x)dx$.

So, $\int_7^2 g(x)dx = -4$.

c. $\int_2^7 2g(x)dx$

There is a Constant Multiple rule for definite integrals as there is for limits.

So, $\int_2^7 2g(x)dx = 2\int_2^7 g(x)dx = 2(4) = 8$.

d. $\int_4^7 f(x)dx$

Again considering the definite integral as the area under the graph of a function over an interval [a,b]; for c such that $a < c < b$, the area over [c,b] is equal to the area over [a,b] minus the area over [a,c].

Thus, $\int_4^7 f(x)dx = \int_2^7 f(x)dx - \int_2^4 f(x)dx = 3 - (-5) = 8$.

e. $\int_2^7 [g(x) - f(x)]dx$

There is a Sum and Difference rule for definite integrals as there is for limits.

So, $\int_2^7 [g(x) - f(x)]dx = \int_2^7 g(x)dx - \int_2^7 f(x)dx = 4 - 3 = 1$.

f. $\int_{2}^{7} [4g(x) - f(x)]dx$

Applying both the Sum and Difference and the Constant Multiple rules to $\int_{2}^{7} [4g(x) - f(x)]dx$, the value of the definite integral is 13.

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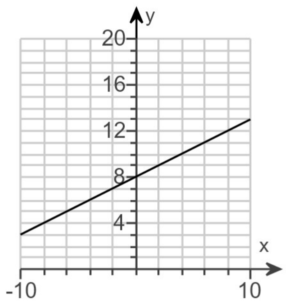
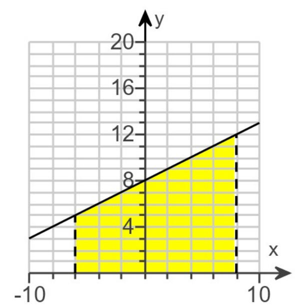
Graph the integrand, and use area to evaluate the definite integral $\int_{-6}^{8} \left(\frac{x}{2} + 8\right) dx$.

The integrand is $f(x) = \frac{x}{2} + 8$.

The graph of the integrand, $f(x) = \frac{x}{2} + 8$, is shown below.



The area equivalent to the value of the definite integral is bounded by the graph, the x-axis, and the lines $x = -6$ and $x = 8$. The boundaries form a trapezoid with area equal to one half the product of the altitude times the sum of the bases.



The area of the trapezoid is one half the product of the altitude times the sum of the bases. Notice that the altitude is 14 and the bases are 5 and 12. Thus, the value of this area is $\frac{14 \cdot (5 + 12)}{2} = 119$.

Thus, $\int_{-6}^{8} \left(\frac{x}{2} + 8\right) dx = 119$.

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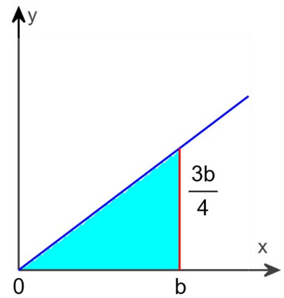
Use area to evaluate the integral $\int_0^b \frac{3x}{4}\,dx$, $b>0$.

The region of interest is a triangle shown to the right.

The area equals the definite integral for a nonnegative function $f(x) = \frac{3x}{4}$. Derive the definite integral by using the formula for the area of a triangle.



The base of the triangle is b.

The height of the triangle is $\frac{3b}{4}$.

Use the formula for the area of a triangle.

$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2}\cdot\text{base}\cdot\text{height}\\ &= \frac{1}{2}\cdot b\cdot\frac{3b}{4}\\ &= \frac{3b^2}{8}\end{aligned}$$

The definition of area $A = \int_a^b f(x)dx$ agrees with the previous notion of area.

Therefore, $\int_0^b \frac{3x}{4}\,dx = \frac{3b^2}{8}$.

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Use areas to evaluate the integral.

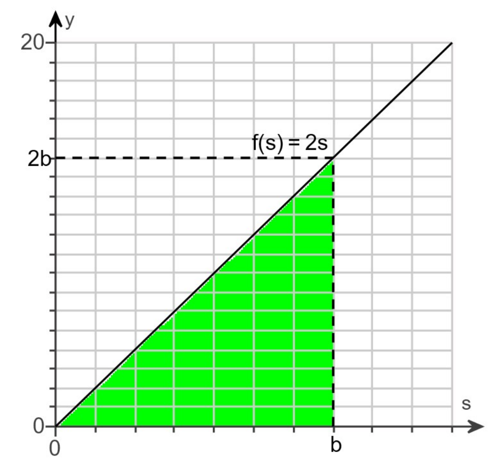
$$\int_{a}^{15b} 2s\,ds,\ 0 < a < b$$

For any integrable function f(x), the value of the integral is related to the area between the curve y = f(x) and the x-axis.

$$\int_{a}^{b} f(x)dx = (\text{area above the x-axis}) - (\text{area below the x-axis})$$

In this problem, the graph of the integrand, f(s) = 2s, is the diagonal line y = 2s.

For any b > 0, the area under y = 2s from 0 to b is that of a triangle in which the base has a measure of b and the height has a measure of 2b. This triangle is above the s-axis for b > 0.

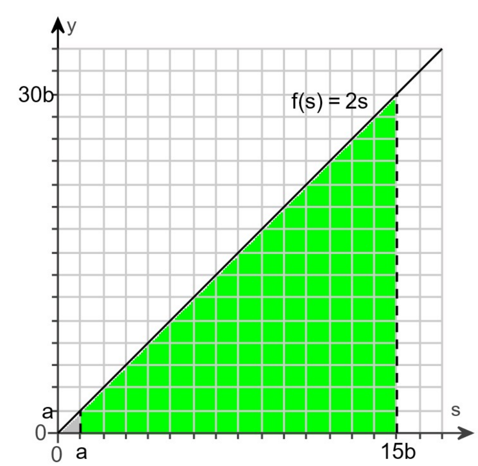


The area of a triangle is half its base length times its height.

$$\text{Area} = \frac{1}{2}bh$$

To find the area between s = a and s = 15b, subtract the area of the small triangle of base length a from the area of the large triangle of base length 15b.

$$\text{Area} = \frac{1}{2}(450)b^2 - \frac{1}{2}(2)a^2$$



Simplify.

$$\text{Area} = 225b^2 - a^2$$

The value of the integral is equal to this area.

$$\int_{a}^{15b} 2s\, ds = 225b^2 - a^2$$

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Evaluate the integral $\int_{6\pi}^{7\pi} \theta \, d\theta$.

For an integrable function $f(x) = x$, the following statement is true.

$$\int_a^b x \, dx = \frac{b^2}{2} - \frac{a^2}{2}, \ a < b$$

Use this rule to evaluate the integral below.

$$\int_{6\pi}^{7\pi} \theta \, d\theta = \frac{49\pi^2}{2} - \frac{36\pi^2}{2}$$

Simplify.

$$\int_{6\pi}^{7\pi} \theta \, d\theta = \frac{49\pi^2}{2} - \frac{36\pi^2}{2}$$

$$= \frac{13\pi^2}{2}$$

Therefore, $\int_{6\pi}^{7\pi} \theta \, d\theta = \frac{13\pi^2}{2}$.