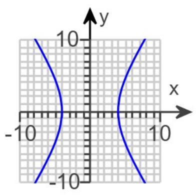
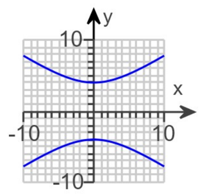
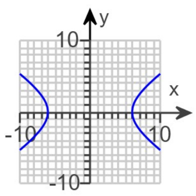
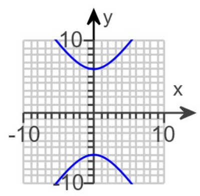
Student: Cole Lamers **Date:** 07/03/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.4 The Hyperbola

Match the equation $\frac{x^2}{36} - \frac{y^2}{16} = 1$ with one of the following graphs.









The first step in determining the correct graph is to write the equation in standard form. The standard equation of a hyperbola centered at the origin is given by the following.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

The given equation has the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Therefore, it is in standard form.

Next determine if the hyperbola has a horizontal or a vertical transverse axis.

A hyperbola of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has a horizontal transverse axis.

A hyperbola of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ has a vertical transverse axis.

Since the given equation has the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, it has a horizontal transverse axis.

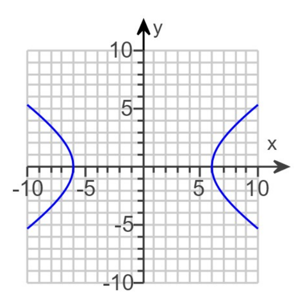
Next find the vertices of the given hyperbola. A hyperbola of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has vertices at $(-a, 0)$ and $(a, 0)$.

In order to find the vertices, first determine the value of a^2. Recall that a^2 is the denominator of the first term.

$a^2 = 36$

Since a^2 is 36, a is 6. Therefore, the hyperbola has vertices at $(-6, 0)$ and $(6, 0)$.

The correct graph is shown below. Notice that it has a horizontal transverse axis and the vertices are at $(-6, 0)$ and $(6, 0)$.



Student: Cole Lamers
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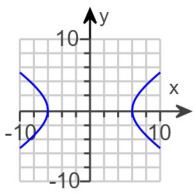
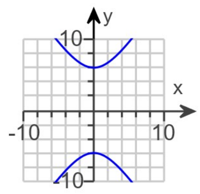
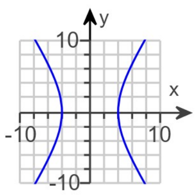
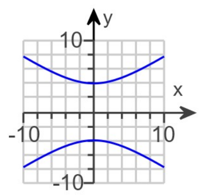
Assignment: 10.4 The Hyperbola

Match the equation $16x^2 - 36y^2 = 576$ with one of the following graphs.









The first step in determining the correct graph is to write the equation in standard form. The standard equation of a hyperbola centered at the origin is given by the following.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

Since the right side of the equation is not equal to 1, the equation for the hyperbola is not written in standard form.

To write $16x^2 - 36y^2 = 576$ in standard form, divide both sides of the equation by 576 and simplify.

$$\frac{16x^2}{576} - \frac{36y^2}{576} = 1$$

$$\frac{x^2}{36} - \frac{y^2}{16} = 1$$

Next determine if the hyperbola has a horizontal or a vertical transverse axis.

A hyperbola of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has a horizontal transverse axis.

A hyperbola of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ has a vertical transverse axis.

Since the hyperbola when written in standard form is similar to $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, it has a horizontal transverse axis.

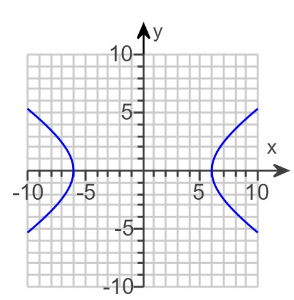
Next find the vertices of the given hyperbola. A hyperbola of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has vertices at $(-a, 0)$ and $(a, 0)$.

In order to determine the vertices, first determine the value of a^2. Recall that the equation for the hyperbola is $\frac{x^2}{36} - \frac{y^2}{16} = 1$ and a^2 is the denominator of the first term.

$a^2 = 36$

Since a^2 is 36, a is 6. Therefore, the hyperbola has vertices at $(-6, 0)$ and $(6, 0)$.

The correct graph is shown below. Notice that it has a horizontal transverse axis and the vertices are at $(-6, 0)$ and $(6, 0)$.



Student: Cole Lamers **Date:** 07/03/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.4 The Hyperbola

The equation of a hyperbola is given. Complete parts **(a)** through **(e)**.

$$x^2 - \frac{y^2}{169} = 1$$

(a) Find the vertices, the foci, and the transverse axis of the hyperbola.

The equation of the hyperbola is in standard form. Rewrite the equation as $\frac{x^2}{1} - \frac{y^2}{169} = 1$. The hyperbola $\frac{x^2}{1} - \frac{y^2}{169} = 1$ is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with center (0,0). The vertices of the hyperbola are at $(\pm a,0)$.

The value of a is 1.

Thus, the vertices of the hyperbola are $(-1,0)$ and (1,0).

The foci of the hyperbola of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with center (0,0) are at $(\pm c,0)$, where $c^2 = a^2 + b^2$.

To find the value of c, first find the value of b. The value of b is 13.

Find c by solving the equation $c^2 = a^2 + b^2$. Use $a^2 = 1^2 = 1$ and $b^2 = 13^2 = 169$.

$$\begin{aligned} c^2 &= 1 + 169 \\ c^2 &= 170 && \text{Add.} \\ c &= \sqrt{170} \end{aligned}$$

Thus, the foci of the hyperbola are $\left(-\sqrt{170},0\right)$ and $\left(\sqrt{170},0\right)$.

The line segment joining the two vertices is called the transverse axis. The transverse axis is on the x-axis when the minus sign precedes the y^2-term and is on the y-axis when the minus sign precedes the x^2-term.

In the equation $x^2 - \frac{y^2}{169} = 1$, the minus sign precedes the y^2-term; so the transverse axis is on the x-axis.

(b) State how the hyperbola opens.

The orientation (left-right branches or up-down branches) of a hyperbola is determined by noting where the minus sign occurs in the standard equation. A hyperbola opens left and right, if the transverse axis is on the x-axis and opens up and down, if the transverse axis is on the x-axis.

Since the transverse axis is on the x-axis, the hyperbola opens left and right.

(c) Find the vertices of the fundamental rectangle.

The vertices of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $(-a,0)$ and $(a,0)$. The endpoints of the conjugate axis are $(0,-b)$ and $(0,b)$. The rectangle with vertices (a,b), $(-a,b)$, $(-a,-b)$, and $(a,-b)$ is called the fundamental rectangle of the hyperbola.

Recall that a = 1 and b = 13.

The vertices of the fundamental rectangle are (1,13), $(-1,13)$, $(-1,-13)$ and $(1,-13)$.

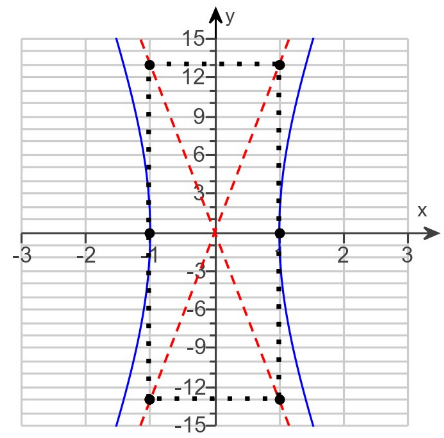
(d) Write the equations of the asymptotes.

The graph of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has a transverse axis along the x-axis and has two asymptotes, $y = \pm \frac{b}{a}x$.

Recall that a = 1 and b = 13.

Thus, the equations of the asymptotes are $y = \pm \frac{13}{1}x = \pm 13x$.

(e) Graph the hyperbola by using the vertices and the asymptotes. Use all the information found in the previous steps to graph the hyperbola. The correct graph is shown on the right.



Student: Cole Lamers **Date:** 07/03/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.4 The Hyperbola

Find an equation for the hyperbola satisfying the following conditions. Graph the hyperbola.

Center (0,0), vertex (0,6), focus (0,7)

The standard form for the equation of a hyperbola with vertices $(0, \pm a)$, foci $(0, \pm c)$, and center (0,0) is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$. The standard form for the equation of a hyperbola with vertices $(\pm a, 0)$, foci $(\pm c, 0)$, and center (0,0) is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. For both equations, $c^2 = a^2 + b^2$.

The hyperbola has its center at (0,0). One vertex of the hyperbola is (0,6), so it follows the formula that has vertices at $(0, \pm a)$. That means the other vertex is $(0, -6)$. Likewise, the foci are at $(\pm c, 0)$, and one focus is at (0,7). So there is a second focus at $(0, -7)$.

Use the formula for the vertices given in the first paragraph to find the value of a.

The value of a is 6.

Use the formula for the foci given in the first paragraph to find the value of c.

The value of c is 7.

To find the equation of the hyperbola in standard form, the values of a^2 and b^2 must be calculated. The value for a^2 can be obtained from a above, but the value for b^2 must be evaluated from the equation $c^2 = a^2 + b^2$. First find a^2 and c^2.

$$\begin{aligned} a^2 &= 6 \cdot 6 \\ &= 36 \end{aligned} \qquad\qquad \begin{aligned} c^2 &= 7 \cdot 7 \\ &= 49 \end{aligned}$$

Now use the equation $c^2 = a^2 + b^2$ to find b^2.

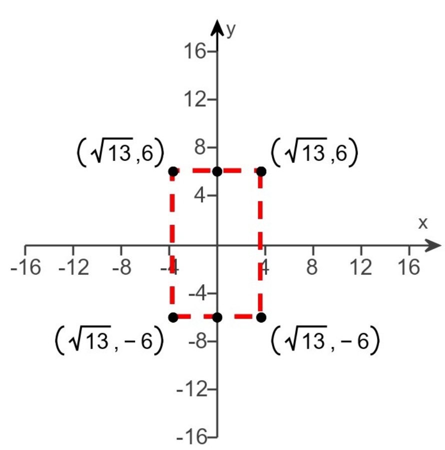
$$\begin{aligned} c^2 &= a^2 + b^2 \\ 49 &= 36 + b^2 \\ 49 - 36 &= b^2 \\ 13 &= b^2 \end{aligned}$$

From the hyperbola facts given in the first paragraph, the equation of a hyperbola with vertices $(0, \pm a)$, foci $(0, \pm c)$, and center (0,0) is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

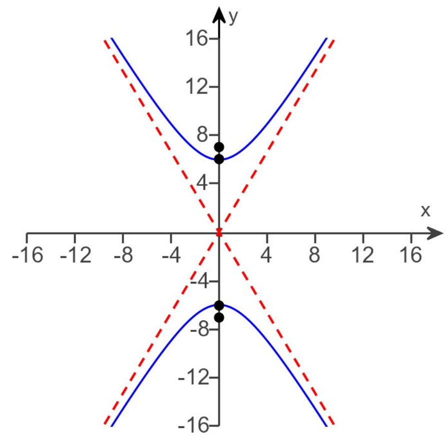
$$\frac{y^2}{36} - \frac{x^2}{13} = 1 \qquad \text{Substitute 36 for } a^2 \text{ and 13 for } b^2.$$

To graph the hyperbola, first draw dashed lines $y = 6$, $y = -6$, $x = \sqrt{13}$, $x = -\sqrt{13}$ to form the fundamental rectangle with vertices $(\sqrt{13}, 6)$, $(-\sqrt{13}, 6)$, $(-\sqrt{13}, -6)$, and $(\sqrt{13}, -6)$.



The asymptotes of the hyperbola are extended diagonals of the fundamental rectangle. Now graph both branches of the hyperbola through the vertices, approaching asymptotes.

A more detailed graph of the given hyperbola is shown to the right.



Student: Cole Lamers **Date:** 07/03/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.4 The Hyperbola

Find the equation of a hyperbola with foci $(\pm\sqrt{50},0)$ and asymptotes $y = \pm 7x$. Graph the hyperbola.

First find the center of the hyperbola. The center of the hyperbola is midway between the foci. Since the foci are $(\sqrt{50},0)$ and $(-\sqrt{50},0)$, the hyperbola is centered at (0,0).

There are two types of hyperbolas centered at (0,0), one with the transverse axis lying on the x-axis and the one with the transverse axis lying on the y-axis.

Note that the foci lie on the x-axis. Thus, the transverse axis is along the x-axis.

The equation of the hyperbola that has foci $(\pm c,0)$ and its transverse axis along the x-axis is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > 0$, $b > 0$, and $c^2 = a^2 + b^2$.

The value of c is the distance from the center (0,0) to either focus. Given that the hyperbola has foci $(\pm\sqrt{50},0)$, the value of c is $\sqrt{50}$.

The equation for the asymptotes of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with center (0,0) and the transverse axis along the x-axis is $y = \pm\frac{b}{a}x$.

Given that the hyperbola has asymptotes $y = \pm 7x$. Compare this with $y = \pm\frac{b}{a}x$.

$\frac{b}{a} = 7$

$b = 7a$ Write b in terms of a.

Use the equation $c^2 = a^2 + b^2$ to find the values of a^2 and b^2. Substitute the values of c and b in $c^2 = a^2 + b^2$.

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
\left(\sqrt{50}\right)^2 &= a^2 + (7a)^2 && \text{Substitute } c = \sqrt{50} \text{ and } b = 7a. \\
50 &= a^2 + 49a^2 && \text{Simplify.} \\
1 &= a^2 && \text{Solve for } a^2.
\end{aligned}
$$

Now to find the value of b^2, subsitute the value of c^2 and a^2 in the equation $c^2 = a^2 + b^2$.

$$
\begin{aligned}
c^2 &= a^2 + b^2 \\
50 &= 1 + b^2 && \text{Substitute } c^2 = 50 \text{ and } a^2 = 1. \\
b^2 &= 49 && \text{Simplify.}
\end{aligned}
$$

Recall that the equation of the hyperbola that has the transverse axis along the x-axis is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The values of a^2 and b^2 are already found. So the equation is $\frac{x^2}{1} - \frac{y^2}{49} = 1$.

Thus, the equation of the hyperbola with foci $(\pm\sqrt{50},0)$ and asymptotes $y = \pm 7x$ is $x^2 - \frac{y^2}{49} = 1$.

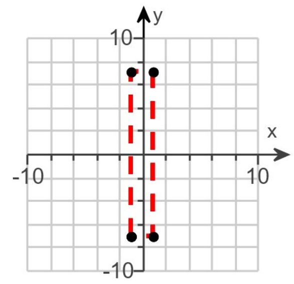
In order to graph the hyperbola, first determine the orientation of the hyperbola. Since a minus sign precedes the y^2-term and the transverse axis is along the x-axis, the hyperbola $x^2 - \frac{y^2}{49} = 1$ opens left and right.

To graph the hyperbola, sketch the fundamental rectangle by drawing dashed lines parallel to the coordinate axes through vertices and the endpoints of the conjugate axis.

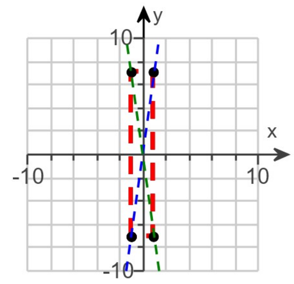
The vertices of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are (a,0) and $(-a,0)$. Thus, the vertices of the hyperbola $x^2 - \frac{y^2}{49} = 1$ are (1,0) and $(-1,0)$.

The endpoints of the conjugate axis of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ are $(0,-b)$ and (0,b). Thus, the endpoints of the conjugate axis of the hyperbola $x^2 - \frac{y^2}{49} = 1$ are (0,7) and $(0,-7)$.

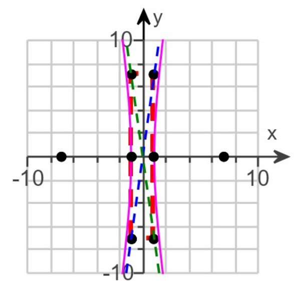
Draw dashed lines x = 1, x = − 1, y = 7, and y = − 7 to form the fundamental rectangle with vertices (1,7), (− 1,7), (− 1, − 7), and (1, − 7). The graph of the fundamental rectangle is shown to the right.



Note that the graph of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ has a transverse axis along the x-axis and has two asymptotes $y = \frac{b}{a}x$ and $y = -\frac{b}{a}x$. Sketch the asymptotes of the hyperbola $x^2 - \frac{y^2}{49} = 1$. The asymptotes are extended diagonals of the fundamental rectangle.



Draw both branches of the hyperbola through the vertices, approaching the asymptotes. The graph of the hyperbola $x^2 - \frac{y^2}{49} = 1$ is shown to the right.



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The eccentricity of a hyperbola, denoted by e, is defined as follows.

$$e = \frac{\text{Distance between the foci}}{\text{Distance between the vertices}} = \frac{2c}{2a} = \frac{c}{a}.$$

Find the eccentricity and the length of the latus rectum of the following hyperbola.

$$4x^2 - 25y^2 = 100$$

The standard form of a hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. First put the given equation in the standard form by dividing it by the constant term on the right side of the equation.

Divide the equation by the constant numeric term, 100.

$$\begin{aligned} 4x^2 - 25y^2 &= 100 \\ \frac{1}{100} \cdot \left(4x^2 - 25y^2\right) &= \frac{1}{100} \cdot 100 \\ \frac{x^2}{25} - \frac{y^2}{4} &= 1 \end{aligned}$$

The standard form of the given equation is $\frac{x^2}{25} - \frac{y^2}{4} = 1$.

Compare the above equation with the standard equation of a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The value of $a^2 = 25$ and the value of $b^2 = 4$.

Now find the values of a and b.

$$\begin{aligned} a^2 &= 25 \\ a &= 5 \end{aligned} \qquad \text{and} \qquad \begin{aligned} b^2 &= 4 \\ b &= 2 \end{aligned}$$

To find the value of c, use the formula $c = \sqrt{a^2 + b^2}$. Substitute the values of a^2 and b^2 in the formula.

$$\begin{aligned} c &= \sqrt{a^2 + b^2} \\ &= \sqrt{25 + 4} \\ &= \sqrt{29} \end{aligned}$$

The eccentricity is given by the formula $e = \frac{c}{a}$. Substitute the values of c and a, and find the value of e.

$$\begin{aligned} e &= \frac{c}{a} \\ &= \frac{\sqrt{29}}{5} \end{aligned}$$

The latus rectum of a hyperbola is a line segment that passes through a focus, is perpendicular to the transverse axis, and has endpoints on the hyperbola. The length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $L = \frac{2b^2}{a}$.

Substitute the values of a and b^2 in the formula $L = \frac{2b^2}{a}$.

$$\begin{aligned} L &= \frac{2b^2}{a} \\ &= \frac{2 \cdot 4}{5} \\ &= \frac{8}{5} \end{aligned}$$

Therefore, the eccentricity e for the given equation of the hyperbola is $\frac{\sqrt{29}}{5}$ and the length of the latus rectum is $\frac{8}{5}$.