Student: Cole Lamers **Date:** 07/17/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.3 Double-Angle and Half-Angle Formulas

Use a double-angle formula to find the exact value of the given expression.

$1 - 2\sin^2 112.5^\circ$

Recall the double-angle formulas.

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

The given expression, $1 - 2\sin^2 112.5^\circ$, is equivalent to the right side of one of the formulas for $\cos 2x$ when x is equal to 112.5°.

$$\cos 2x = 1 - 2\sin^2 x$$

Substitute 112.5° for x in the double-angle formula for the cosine function.

$$\begin{aligned} 1 - 2\sin^2 x &= \cos 2x \\ 1 - 2\sin^2 112.5^\circ &= \cos (2 \cdot 112.5)^\circ \\ 1 - 2\sin^2 112.5^\circ &= \cos 225^\circ \end{aligned}$$

Use the unit circle to find the exact value.

$$1 - 2\sin^2 112.5^\circ = \cos 225^\circ = -\frac{\sqrt{2}}{2}$$

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Use a double-angle formula to find the exact value of the given expression.

$$2\cos^2 112.5^\circ - 1$$

Recall the double-angle formulas shown below.

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

The given expression, $2\cos^2 112.5^\circ - 1$, is equivalent to the right side of one of the formulas for $\cos 2x$ when x is equal to 112.5°.

$$\cos 2x = 2\cos^2 x - 1$$

Substitute 112.5° for x in the double-angle formula for the cosine function.

$$\begin{aligned} 2\cos^2 x - 1 &= \cos 2x \\ 2\cos^2 112.5^\circ - 1 &= \cos(2 \cdot 112.5)^\circ \\ 2\cos^2 112.5^\circ - 1 &= \cos 225^\circ \end{aligned}$$

Use the unit circle to find the exact value. The value of $\cos 225^\circ$ is the x-coordinate of the point that corresponds to 225° on the unit circle.

$$2\cos^2 112.5^\circ - 1 = \cos 225^\circ = -\frac{\sqrt{2}}{2}$$

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Use the power-reducing identities to rewrite the expression to one that contains a single trigonometric function of power 1.

$4 \sin 11x \cos 11x\left(1-2\sin^2 11x\right)$

To simplify the expression, first factor out 2.

$$4 \sin 11x \cos 11x\left(1-2\sin^2 11x\right) = 2(2\sin 11x \cos 11x)\left(1-2\sin^2 11x\right)$$

Use the double angle identity to simplify $2\sin 11x \cos 11x$.

$$2(2\sin 11x \cos 11x)\left(1-2\sin^2 11x\right) = 2\sin 22x\left(1-2\sin^2 11x\right)$$

Next, use the power reducing identity shown below to simplify $1-2\sin^2 11x$.

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

Isolate $\cos 2x$ on one side of the equation $\sin^2 x = \dfrac{1-\cos 2x}{2}$.

$$\cos 2x = 1-2\sin^2 x$$

Now, use $\cos 2x = 1-2\sin^2 x$ to simplify $1-2\sin^2 11x$.

$$2\sin 22x\left(1-2\sin^2 11x\right) = 2\sin 22x \cos 22x$$

Finally, use the double angle identity of sine to simplify.

$$2\sin 22x \cos 22x = \sin 44x$$

Therefore, $4 \sin 11x \cos 11x\left(1-2\sin^2 11x\right) = \sin 44x$.

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Use the half-angle formulas to find the exact value of the trigonometric function **tan** $\left(-\frac{\pi}{8}\right)$.

Since **tan** $\left(-\frac{\pi}{8}\right) = \mathbf{tan}\,\frac{-\frac{\pi}{4}}{2}$, apply the half-angle formula for **tan** $\frac{\theta}{2}$ with $\theta = -\frac{\pi}{4}$.

The half-angle formula **tan** $\frac{\theta}{2} = \pm\sqrt{\frac{1-\mathbf{cos}\,\theta}{1+\mathbf{cos}\,\theta}}$ can be used to find **tan** $\left(-\frac{\pi}{8}\right)$.

First determine which sign to use.

Since $\frac{\theta}{2}$ is in quadrant IV, the sign $-$ will be used in the half-angle formula.

Find **tan** $\left(-\frac{\pi}{8}\right)$.

$$\mathbf{tan}\left(-\frac{\pi}{8}\right) = \mathbf{tan}\left(\frac{-\frac{\pi}{4}}{2}\right)$$

$$= -\sqrt{\frac{1-\mathbf{cos}\left(-\frac{\pi}{4}\right)}{1+\mathbf{cos}\left(-\frac{\pi}{4}\right)}}$$ Use the half-angle formula for **tan** $\frac{\theta}{2}$.

$$= -\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}}$$ Find **cos** $\left(-\frac{\pi}{4}\right)$.

Next, clear the fractions from the expression. Then write the result as a quotient of two radicals. Begin to rationalize the denominator by multiplying the numerator and denominator by the expression in the denominator

$$-\sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}} = -\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$$

$$= -\frac{\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\cdot\frac{\sqrt{2+\sqrt{2}}}{\sqrt{2+\sqrt{2}}}$$

$$= -\frac{\sqrt{2}}{2+\sqrt{2}}$$

Finish rationalizing the denominator by multiplying the numerator and denominator by $2-\sqrt{2}$.

$$-\frac{\sqrt{2}}{2+\sqrt{2}}\cdot\frac{2-\sqrt{2}}{2-\sqrt{2}} = -\frac{2\sqrt{2}-2}{4-2}$$

$$= -\frac{2\sqrt{2}-2}{2}$$

$$= -\sqrt{2}+1$$

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Given that **cos** $\alpha = \frac{1}{4}$, and $0^\circ < \alpha < 90^\circ$, find **sin** $\frac{\alpha}{2}$, **cos** $\frac{\alpha}{2}$, and **tan** $\frac{\alpha}{2}$.

Since you know the value of **cos** α, and you're looking for the value of **sin** $\frac{\alpha}{2}$, you should use the half-angle identity for sine.

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

In this expression, **cos** $\alpha = \frac{1}{4}$.

Substitute.

$$\cos\alpha = = \pm\sqrt{\frac{1-\frac{1}{4}}{2}}$$

$$= \pm\sqrt{\frac{\frac{3}{4}}{2}}$$

$$= \pm\sqrt{\frac{3}{8}}$$

Rationalize and simplify.

$$\sin\frac{\alpha}{2} = \pm\frac{\sqrt{6}}{4}$$

Notice that $0^\circ < \alpha < 90^\circ$, so $0^\circ < \frac{\alpha}{2} < 45^\circ$. This means that $\frac{\alpha}{2}$ is in the first quadrant. Remember that sine is positive in the first quadrant.

So, **sin** $\frac{\alpha}{2} = \frac{\sqrt{6}}{4}$.

Similarly, to find **cos** $\frac{\alpha}{2}$, use the half-angle identity for cosine:

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$= \pm\sqrt{\frac{1+\frac{1}{4}}{2}}$$

$$= \pm\sqrt{\frac{5}{8}}$$

Rationalizing and simplifying, you get:

$$\cos\frac{\alpha}{2} = \pm\frac{\sqrt{10}}{4}$$

Since $\frac{\alpha}{2}$ is in the first quadrant, and cosine is positive in the first quadrant, you get:

$$\cos\frac{\alpha}{2} = \frac{\sqrt{10}}{4}$$

To find the value of the tangent, use the values of sine and cosine.

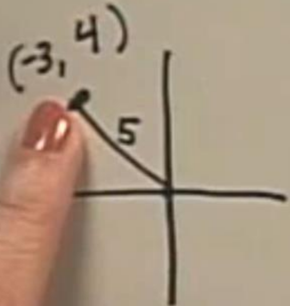
$$\tan\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\frac{\sqrt{6}}{4}}{\frac{\sqrt{10}}{4}} = \frac{\sqrt{15}}{5}$$

If $\cos\theta = -\frac{3}{5}$ and θ is in quadrant II, find the exact value of the expression.

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{5}$$

$$\sin\theta = \frac{y}{r}$$



$$r = \sqrt{x^2+y^2}$$

$$5 = \sqrt{(-3)^2+y^2}$$

$$25 = \left(\sqrt{9+y^2}\right)^2$$

$$25 = 9+y^2$$

$$16 = y^2$$

$$4 = y$$

If $\cos\theta = -\frac{3}{5}$ and θ is in quadrant II, find the exact value of the expression.

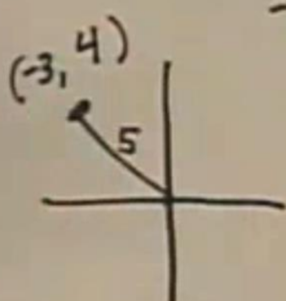
$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= \frac{2}{1}\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) =$$

$$= \frac{-24}{25}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{5}$$

$$\sin\theta = \frac{y}{r} = \frac{4}{5}$$



$$r = \sqrt{x^2+y^2}$$

$$5 = \sqrt{(-3)^2+y^2}$$

$$25 = \left(\sqrt{9+y^2}\right)^2$$

If $\cos\theta = -\frac{3}{5}$ and θ is in quadrant II, find the exact value of the expression.

$\cos\theta = -\frac{3}{5}$

$\sin\theta = \frac{4}{5}$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(-\frac{3}{5}\right)^2 - \left(\frac{4}{5}\right)^2$$

$$= \frac{9}{25} - \frac{16}{25} = \frac{-7}{25}$$

If $\cos\theta = -\frac{3}{5}$ and θ is in quadrant II, find the exact value of the expression.

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{4}{5}}{-\frac{3}{5}}$$

$\tan\theta = -\frac{4}{3}$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$= \frac{2\left(-\frac{4}{3}\right)}{1 - \left(-\frac{4}{3}\right)^2}$$

$$= \frac{-\frac{8}{3}}{\frac{9}{9} - \frac{16}{9}} = \frac{-\frac{8}{\cancel{3}}}{-\frac{7}{\cancel{9}}} \cdot \frac{\overset{3}{\cancel{9}}}{9} = \frac{-24}{-7} = \frac{24}{7}$$