Unit 4 – Work, Energy, & Power
Post-Lecture

A student pushes horizontally on a box of books (mass = 89.0 kg) that rests on a tile floor. Assume the μ_s = 0.632 and the μ_k = 0.338. He pushes just hard enough to get the box moving and then continues to push with the same amount of force for a distance of 9.54 m. How much work does he accomplish on the box of books?

$$f_s = \mu_s \left(mg \right) = \left(0.632 \right)\left(89.0\,\text{kg} \right)g = 551\,\text{N} = F_x$$

$$f_k = \mu_k \left(mg \right) = \left(0.338 \right)\left(89.0\,\text{kg} \right)g = 295\,\text{N}$$

$$F_{net} = F_x - f_k = ma = 551\,\text{N} - 295\,\text{N} = 256\,\text{N}$$

$$W = Fd = \left(256\,\text{N} \right)\left(9.54\,\text{m} \right) = 2440\,\text{N} \cdot \text{m} = \boxed{2440\,\text{J}}$$

A 115 kg "Superman" is traveling at 135 m/s hits a tree and is slowed quickly and uniformly to a stop. If he leaves an 11.0 cm impression in the tree, what force was exerted on this super-hero causing him to stop?

$$a = \frac{v^2 - v_o{}^2}{2x} = \frac{(0\,\mathrm{m/s})^2 - (135\,\mathrm{m/s})^2}{2(0.110\,\mathrm{m})} = -82{,}800\,\mathrm{m/s^2}$$

$$F = ma = (115\,\mathrm{kg})(-82{,}800\,\mathrm{m/s^2}) = \boxed{-9.52 \times 10^6\ \mathrm{N}}$$

Alternate Energy Referenced Method

$$\Delta KE = KE_f - KE_i = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2 = W$$

$$\therefore \Delta KE = -KE_i = -\frac{1}{2}mv_o^2 = -\frac{1}{2}(115\,\text{kg})(135\,\text{m/s})^2$$

$$\therefore \Delta KE = W = -1.05 \times 10^6 \text{ J}$$

$$W = Fd \quad \therefore F = \frac{W}{d} = \frac{-1.05 \times 10^6 \text{ J}}{0.110\,\text{m}} = \boxed{-9.54 \times 10^6 \text{ N}}$$

A child sleds down a hill that is 15.0 m vertically tall. If her initial speed at the top is 2.50 m/s, what is her speed at the bottom?

Conservative Work Method for this Problem:

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_o{}^2 + mgy_o = \frac{1}{2}mv^2 + mgy$$

$$\therefore \frac{1}{2}mv_o{}^2 + mgy_o = \frac{1}{2}mv^2 \qquad \therefore \frac{1}{2}v_o{}^2 + gy_o = \frac{1}{2}v^2$$

$$\therefore \frac{1}{2}(2.50\,\text{m/s})^2 + g(15.0\,\text{m}) = \frac{1}{2}v^2$$

$$\therefore 150\,\text{m}^2/\text{s}^2 = \frac{1}{2}v^2 \qquad \therefore v = \boxed{17.3\,\text{m/s}}$$

A 1500.0 kg car can go from 0 to 135 km/hr in 6.20 s. What amount of power (watts & hp) is required to accomplish this?

$$\frac{135\,\cancel{km}}{\cancel{hr}} \cdot \frac{1\,\cancel{hr}}{3600\,s} \cdot \frac{1000\,m}{1\,\cancel{km}} = 37.5\,m/s$$

$$W = \Delta KE = KE_f - KE_i = \frac{1}{2}mv^2 - \frac{1}{2}mv_o^2$$

$$W = \frac{1}{2}(1500.0\,kg)(37.5\,m/s)^2 - 0\,J = 1.05 \times 10^6\,J$$

$$P = \frac{W}{t} = \frac{1.05 \times 10^6\,J}{6.20\,s} = \boxed{169{,}000\,W}$$

$$\therefore 169{,}000\,\cancel{W} \cdot \frac{1\,hp}{746\,\cancel{W}} = \boxed{226\,hp}$$

Coal is lifted out of a mine that is 637 m deep using a motor rated at 3.50 hp. Assuming the motor to be 75.0% efficient, how many kilograms of coal can be lifted in 10.0 minutes.

$$10.0 \, \text{min} \cdot \frac{60 \, \text{s}}{1 \, \text{min}} = 600 \, \text{s}$$

$$P = 3.50 \, \text{hp} \cdot \frac{746 \, \text{W}}{1 \, \text{hp}} = 2610 \, \text{W} = 2610 \, \text{J/s}$$

$$(0.750)(2610 \, \text{J/s}) = 1960 \, \text{J/s}$$

$$P = \frac{W}{t} \quad \therefore W = Pt = (1960 \, \text{J/s})(600 \, \text{s}) = 1.18 \times 10^6 \, \text{J}$$

$$W = \Delta PE = mg(\Delta y) = 1.18 \times 10^6 \, \text{J}$$

$$\therefore m = \frac{W}{g(\Delta y)} = \frac{1.18 \times 10^6 \, \text{J}}{g(637 \, \text{m})} = \boxed{189 \, \text{kg}}$$

A person returning from a successful fishing trip pulls a sled loaded with salmon. The total mass of the sled and salmon is 50.0 kg, and the person exerts a force of magnitude 1.20×10^2 N on the sled by pulling on the rope.

(a) How much work does he do on the sled if the rope is horizontal to the ground ($\theta = 0°$ *as in lecture Figure 5.6*) and he pulls the sled 5.00 m? (b) How much work does he do on the sled if $\theta = 30.0°$ and he pulls it the same distance? At a coordinate position of 12.4 m, the person lets up on the applied force. A friction force of 45.0 N between the ice and the sled brings the sled to rest at a coordinate position of 18.2 m. (c) How much work does friction do on the sled?
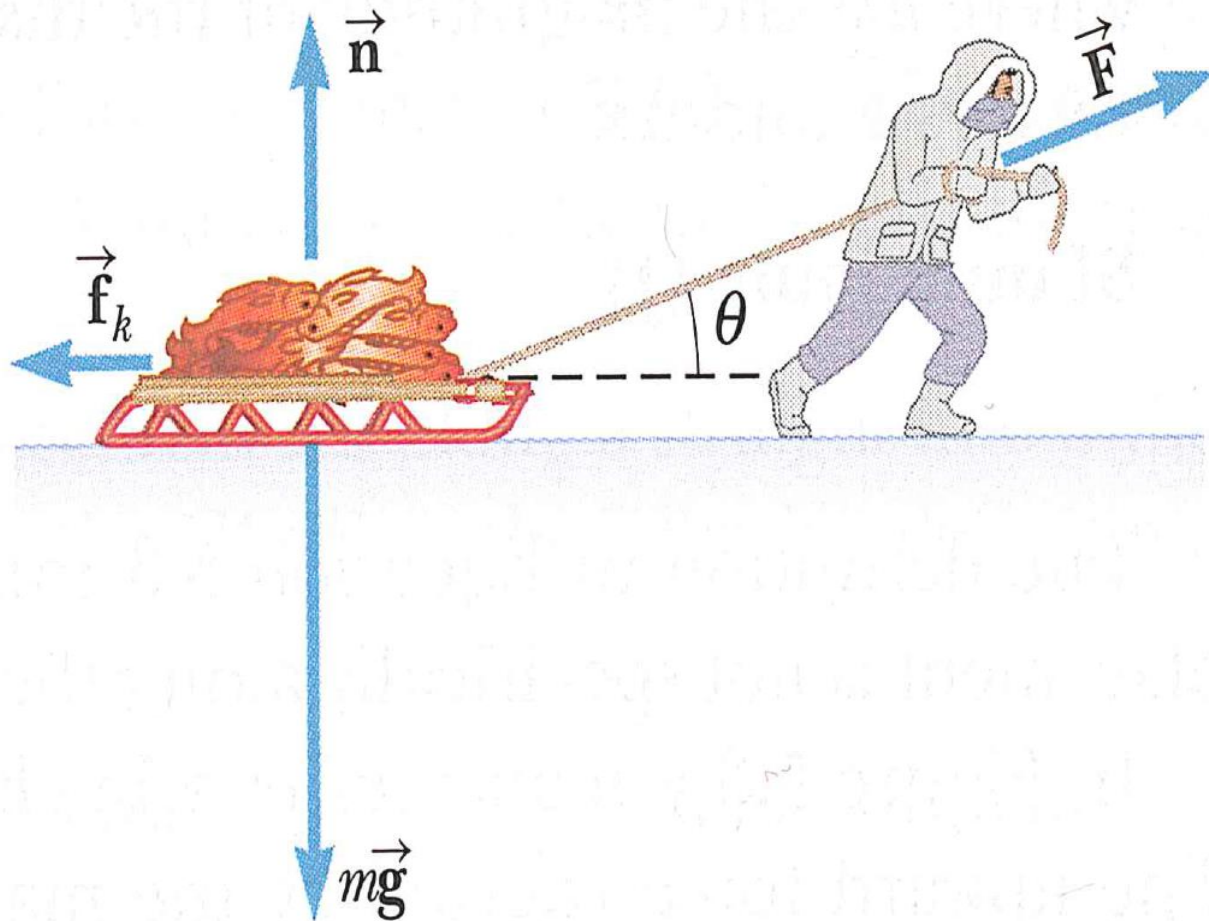
Figure 5.6

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$W = F_x \Delta_x = \left(120\,\text{N}\right)\left(5.00\,\text{m}\right) = \ \textcolor{blue}{600\,\text{J}}$$

$$W = \left(F\cos\theta\right)d = \left(120\,\text{N}\right)\left(\cos 30.0°\right)\left(5.00\,\text{m}\right) = \ \textcolor{blue}{520\,\text{J}}$$

$$W_{fric} = F_x \Delta_x = f_k\left(x_f - x_i\right)$$

$$\therefore \ W_{fric} = \left(-45.0\,\text{N}\right)\left(18.2\,\text{m} - 12.4\,\text{m}\right) = \boxed{\textcolor{blue}{-260\,\text{J}}}$$

Suppose that in *Problem #1* the coefficient of kinetic friction between the loaded 50.0 kg sled and snow is 0.200. The person again pulls the sled 5.00 m, exerting a force of 1.20×10^2 N at an angle of 30.0°

(a) Find the work done on the sled by friction,

and **(b)** the net work.

$$\Sigma F_y = n + F_y - mg = 0$$

$$\therefore \ n = mg - F_y$$

$$\therefore \ n = (50.0 \, \text{kg})(9.80 \, \text{m/s}^2) - (\sin 30.0°)(120 \, \text{N})$$

$$\therefore \ n = 430 \, \text{N}$$

$$f_k = -\mu_k n = -(0.200)(430 \, \text{N}) = -86.0 \, \text{N}$$

$$W_{fric} = f_k d = (-86.0 \, \text{N})(5.00 \, \text{m}) = \boxed{-430 \, \text{J}}$$

$$W_{net} = W_{applied} + W_{fric} + W_n + W_g$$

$$W_{applied} = (\cos 30.0°)(120\,\text{N})(5.00\,\text{m}) = 520\,\text{J}$$

$$W_{net} = (520\,\text{J}) + (-430\,\text{J}) + 0 + 0$$

$$W_{net} = \boxed{90\,\text{J}}$$

The driver of a 1.00×10^3 kg car traveling on the interstate at 35.0 m/s slams on his brakes to avoid hitting a second vehicle in front of him, which had come to rest because of congestion ahead (*as shown similarly in lecture Figure 5.9*). After the brakes are applied, a constant kinetic friction force of magnitude 8.00×10^3 N acts on the car. Ignore air resistance.

(a) At what minimum distance should the brakes be applied to avoid a collision with the other vehicle? If the distance between the vehicles is initially only 30.0 m, **(b)** at what speed would the collision occur?
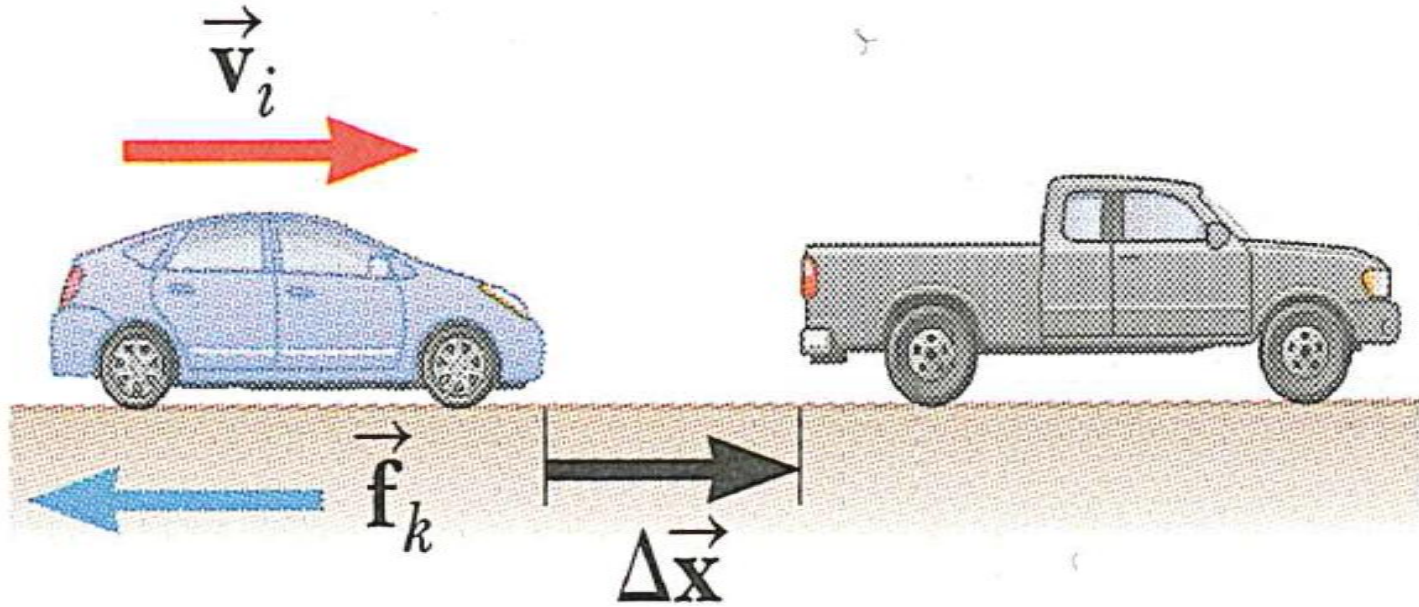
Figure 5.9

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$W_{net} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$-f_k\Delta x = 0 - \tfrac{1}{2}mv_i^2$$

$$-(8000\,\text{N})\Delta x = -\tfrac{1}{2}(1000\,\text{kg})(35.0\,\text{m/s})^2$$

$$\therefore \Delta x = \boxed{76.6\,\text{m}}$$

$$W_{net} = W_{fric} = -f_k \Delta x = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$v_f^2 = v_i^2 - \frac{2f_k \Delta x}{m}$$

$$v_f = \sqrt{(35.0\,\text{m/s})^2 - \frac{2(8000\,\text{N})(30.0\,\text{m})}{(1000\,\text{kg})}} = \boxed{27.3\,\text{m/s}}$$

A police investigator measures straight skid marks 27.0 m long in an accident investigation. Assuming a friction force and car mass the same as in the previous problem,

(a) what was the minimum speed of the car when the brakes locked?

$$W_{net} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$-f_k\Delta x = 0 - \tfrac{1}{2}mv_i^2$$

$$f_k\Delta x = \tfrac{1}{2}mv_i^2$$

$$v_i = \sqrt{\frac{2f_k\Delta x}{m}}$$

$$v_i = \sqrt{\frac{2(8000\,\text{N})(27.0\,\text{m})}{(1000\,\text{kg})}} = \boxed{20.8\,\text{m/s}}$$

Waterslides are nearly frictionless, hence they can provide bored students with high-speed thrills. One such slide is 21.9 m tall.

(a) Determine the speed of a 60.0 kg person at the bottom of such a slide, assuming no friction present. If the person is clocked at 18.0 m/s at the bottom of the slide,
(b) find the work done on the person by friction.

$$W_c = \left(KE_i + PE_i \right) = \left(KE_f + PE_f \right)$$

$$\tfrac{1}{2}mv_i^2 + mgy_i = \tfrac{1}{2}mv_f^2 + mgy_f$$

$$0 + mgy_i = \tfrac{1}{2}mv_f^2 + 0$$

$$v_f = \sqrt{2gy_i} = \sqrt{2\left(9.80\,\text{m/s}^2\right)\left(21.9\,\text{m}\right)} = \boxed{20.7\,\text{m/s}}$$

$$W_{nc} = \left(KE_f - KE_i \right) + \left(PE_f - PE_i \right)$$

$$= \left(\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \right) + \left(mgy_f - mgy_i \right)$$

$$= \left[\tfrac{1}{2}(60.0\,\text{kg})(18.0\,\text{m/s})^2 - (0\,\text{J}) \right] + \left[(0\,\text{J}) - (60.0\,\text{kg})(9.80\,\text{m/s}^2)(21.9\,\text{m}) \right]$$

$$= \boxed{-3160\,\text{J}}$$

A skier starts from rest at the top of a frictionless incline of height 20.0 m (*as shown similarly in lecture Figure 5.19*). At the bottom of the incline, the skier encounters a horizontal surface where the coefficient of kinetic friction between his skis and snow is 0.210

(a) Find the skier's speed at the bottom.
(b) How far does the skier travel on the horizontal surface before coming to rest? Neglect air resistance.

$h = 20.0$ m

y

x

θ

Ⓐ

Ⓑ

Ⓒ

d
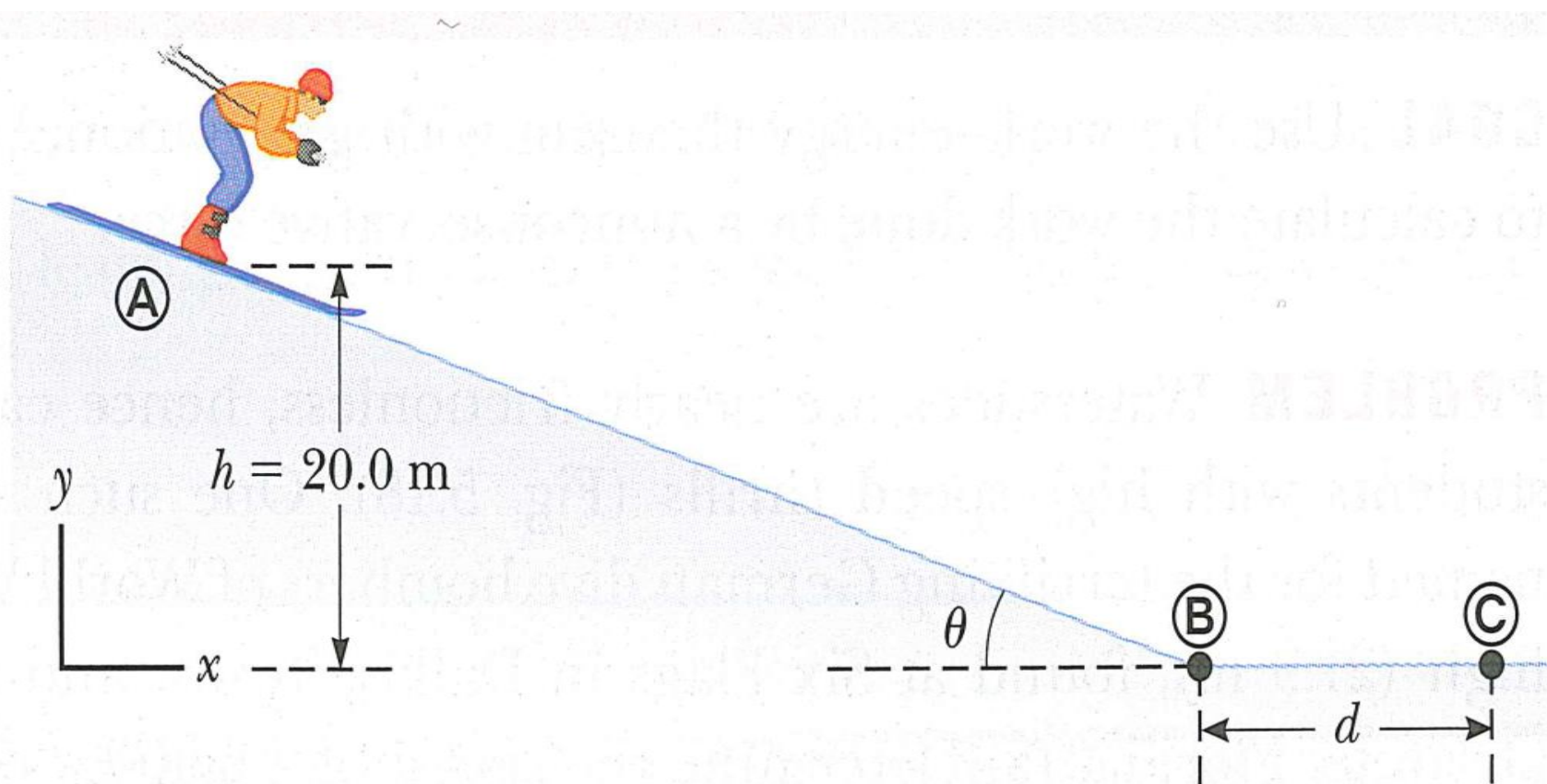
Figure 5.19

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$\frac{1}{2}mv_A^2 + mgy_i = \frac{1}{2}mv_B^2 + mgy_B$$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0$$

$$v_B = \sqrt{2gh} = \sqrt{2(9.80\,\text{m/s}^2)(20.0\,\text{m})} = \boxed{19.8\,\text{m/s}}$$

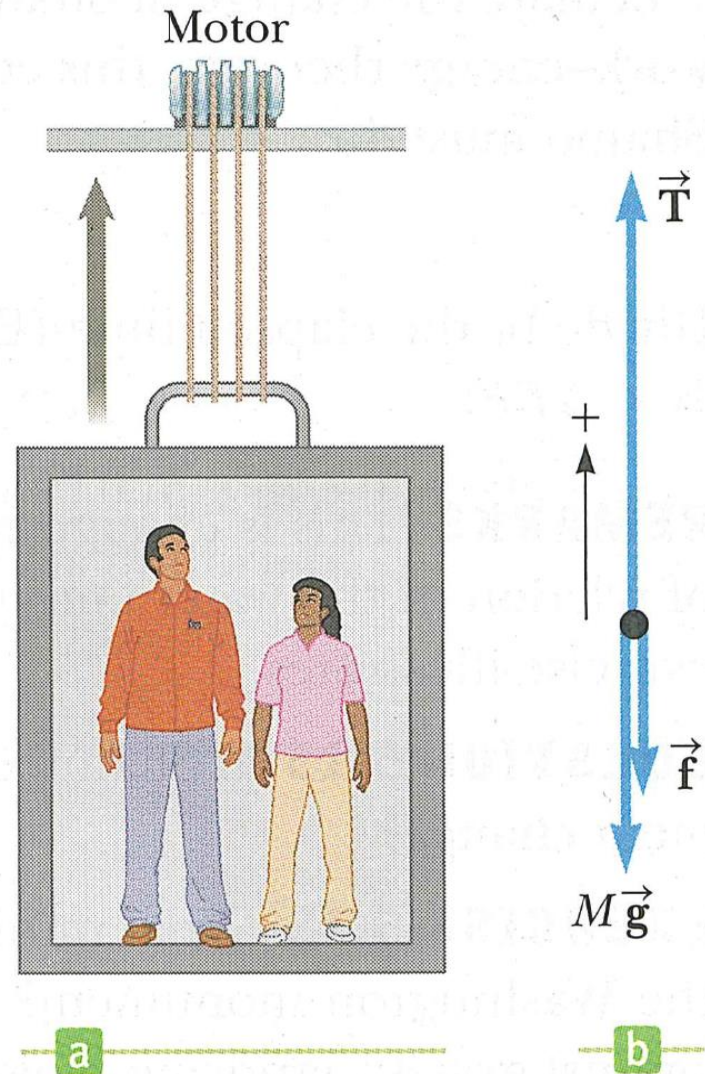$$W_{net} = -f_k d = \Delta KE = \tfrac{1}{2}mv_C^2 - \tfrac{1}{2}mv_B^2$$

$$= -\mu_k mgd = -\tfrac{1}{2}mv_B^2$$

$$\therefore \; d = \frac{v_B^2}{2\mu_k g} = \frac{(19.8\,\text{m/s})^2}{2(0.210)(9.80\,\text{m/s}^2)} = \boxed{95.2\,\text{m}}$$

A 1.00 x 10^3 kg elevator car carries a maximum load of 8.00 x 10^2 kg. A constant frictional force of 4.00 x 10^3 N impedes its motion upward (*as shown similarly in lecture Figure 5.26*).

What minimum power, **(a)** kilowatts

and **(b)** in horsepower, must the motor deliver to lift the fully loaded elevator car at a constant speed of 3.00 m/s?

Motor

$\vec{T}$

$+$

$\vec{f}$

$M\vec{g}$

a   b

Figure 5.26

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$\Sigma F = T - f - Mg = 0$$

$$T = f + Mg$$

$$= (4000\,\text{N}) + (1800\,\text{kg})(9.80\,\text{m/s}^2)$$

$$= 21{,}600\,\text{N}$$

$$P = Fv = (21{,}600\,\text{N})(3.00\,\text{m/s})$$

$$= 64{,}800\,\text{W} = \boxed{64.8\,\text{kW}} = \boxed{86.9\,\text{hp}}$$