

Student: Cole Lamers
Date: 07/20/19

Instructor: Kelly Galarneau
Course: CA&T Internet (70263)
 Galarneau

Assignment: 6.4 Product-to-Sum and
 Sum-to-Product Formulas

1. Complete the following.

We can rewrite the product of two sines as a difference of two cosines by using the identity **$\sin x \sin y =$** _____.

$$\sin x \sin y = \frac{1}{2} [\cos (x - y) - \cos (x + y)]$$

2. Complete the following.

We can rewrite the product of a sine and a cosine as the sum of two sines by using the identity **$\sin x \cos y =$** _____.

$$\sin x \cos y = \frac{1}{2} [\sin (x - y) + \sin (x + y)]$$

3. Complete the following.

We can rewrite the sum of two cosines as a product of two cosines by using the identity **$\cos x + \cos y =$** _____.

Select the correct choice given below.

- ☒ **A.** $2 \cos \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$
- ☐ **B.** $2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$
- ☐ **C.** $2 \sin \left(\frac{x-y}{2} \right) \cos \left(\frac{x+y}{2} \right)$
- ☐ **D.** $-2 \sin \left(\frac{x+y}{2} \right) \sin \left(\frac{x-y}{2} \right)$

4. Identify whether the following equation is true or false.

$$\sin x + \sin y = \sin (x + y)$$

Choose the correct answer below.

- ☐ **A.** True, because **$\sin x + \sin y = 2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$** which is equal to **$\sin (x + y)$** .
- ☐ **B.** False, because **$\sin x + \sin y = \sin (x + y) \cos (x - y)$** and **$\sin (x + y) = \sin x \cos y + \cos x \sin y$** .
- ☐ **C.** True, because **$\sin x + \sin y = \sin x \cos y + \cos x \sin y$** which is equal to **$\sin (x + y)$** .
- ☒ **D.** False, because **$\sin x + \sin y = 2 \sin \left(\frac{x+y}{2} \right) \cos \left(\frac{x-y}{2} \right)$** and **$\sin (x + y) = \sin x \cos y + \cos x \sin y$** .

5. Express the given product as a sum or difference containing only sines or cosines.

$$\sin (9x) \cos (5x)$$

$$\sin (9x) \cos (5x) = \frac{1}{2} [\sin (14x) + \sin (4x)]$$

(Simplify your answer.)

6. Express the given product as a sum or difference containing only sines or cosines.

$$\cos (4x) \cos (7x)$$

$$\cos (4x) \cos (7x) = \frac{1}{2} [\cos (11x) + \cos (3x)]$$

(Simplify your answer.)

7. Express the given product as a sum or difference containing only sines or cosines.

$$\sin(7x) \sin(4x)$$

$$\sin(7x) \sin(4x) = \frac{1}{2} [\cos(3x) - \cos(11x)]$$

(Simplify your answer.)

8. Use a product-to-sum identity to rewrite the expression.

$$\cos 51^\circ \sin 21^\circ$$

$$\cos 51^\circ \sin 21^\circ = \frac{1}{2} \sin(72) - \frac{1}{4}$$

(Simplify your answer. Do not include the degree symbol in your answer. Use integers or fractions for any numbers in the expression.)

9. Use a product-to-sum identity to rewrite the expression.

$$\cos 50^\circ \sin 20^\circ$$

$$\cos 50^\circ \sin 20^\circ = \frac{1}{2} \sin(70) - \frac{1}{4}$$

(Simplify your answer. Do not include the degree symbol in your answer. Use integers or fractions for any numbers in the expression.)

10. Use the product-to-sum identities to rewrite the expression.

$$\sin\left(\frac{\pi}{5}\right) \cos\left(\frac{\pi}{9}\right)$$

Which of the following is equivalent to $\sin\left(\frac{\pi}{5}\right) \cos\left(\frac{\pi}{9}\right)$?

☐ A. $\frac{1}{2} \left(\cos \frac{14\pi}{45} + \cos \frac{4\pi}{45} \right)$

☐ B. $\frac{1}{2} \left(\sin \frac{14\pi}{45} - \sin \frac{4\pi}{45} \right)$

☐ C. $\sin \frac{14\pi}{45} + \sin \frac{4\pi}{45}$

☒ D. $\frac{1}{2} \left(\sin \frac{14\pi}{45} + \sin \frac{4\pi}{45} \right)$

11. Use the product-to-sum identities to rewrite the expression.

$$\cos\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{7}\right)$$

Which of the following is equivalent to $\cos\left(\frac{\pi}{4}\right) \sin\left(\frac{\pi}{7}\right)$?

☐ A. $\sin \frac{11\pi}{28} - \sin \frac{3\pi}{28}$

☐ B. $\frac{1}{2} \left(\cos \frac{11\pi}{28} - \cos \frac{3\pi}{28} \right)$

☐ C. $\frac{1}{2} \left(\sin \frac{11\pi}{28} + \sin \frac{3\pi}{28} \right)$

☒ D. $\frac{1}{2} \left(\sin \frac{11\pi}{28} - \sin \frac{3\pi}{28} \right)$

12. Express the given product as a sum containing only sines or cosines.

$$\sin(6\theta) \cos(3\theta)$$

$$\sin(6\theta) \cos(3\theta) = \frac{1}{2} [\sin(9\theta) + \sin(3\theta)]$$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

13. Find the exact value of the product $\sin 37.5^\circ \sin 7.5^\circ$.

Which of the following is the exact value of $\sin 37.5^\circ \sin 7.5^\circ$?

- ☐ A. $\frac{\sqrt{3} - \sqrt{2}}{2}$
☐ B. $\frac{\sqrt{2} + 1}{4}$
- ☒ C. $\frac{\sqrt{3} - \sqrt{2}}{4}$
☐ D. 75°

14. Find the exact value of the expression $\sin 165^\circ \cdot \cos 75^\circ$.

Choose the correct answer below.

- ☐ A. 1
 ☐ B. $\frac{\sqrt{2}}{2}$
- ☐ C. $-\frac{\sqrt{3}}{2}$
☒ D. $\frac{1}{2} \left(-\frac{\sqrt{3}}{2} + 1 \right)$

15. Use the sum-to-product identities to rewrite the expression.

$$\sin 14^\circ + \sin 6^\circ$$

Which expression is equal to $\sin 14^\circ + \sin 6^\circ$?

- ☐ A. $2 \cos 10^\circ \sin 4^\circ$
☐ B. $2 \cos 10^\circ \cos 4^\circ$
☒ C. $2 \sin 10^\circ \cos 4^\circ$
☐ D. $-2 \sin 10^\circ \sin 4^\circ$

16. Use a sum-to-product identity to rewrite the expression.

$$\sin 3\alpha + \sin 6\alpha$$

$$\sin 3\alpha + \sin 6\alpha = 2 \sin \left(\frac{9\alpha}{2} \right) \cos \left(\frac{3\alpha}{2} \right)$$

(Use integers or fractions for any numbers in the expression.)

17. Identify the following equation as an identity, conditional, or inconsistent equation.

$$\cos x = 3$$

Choose the correct answer below.

- ☐ an identity
 ☐ a conditional equation
 ☒ an inconsistent equation

18. Identify the following equation as an identity, conditional, or inconsistent equation.

$$\tan^2 x = \sec^2 x - 1$$

Choose the correct answer below.

- ☒ an identity
 ☐ a conditional equation
 ☐ an inconsistent equation

19. Identify the following equation as an identity, conditional, or inconsistent equation. (Assume that $\sec x > 0$.)

$$\sec x = \sqrt{1 + \tan^2 x}$$

Choose the correct answer below.

- ☒ an identity
☐ a conditional equation
☐ an inconsistent equation

20. Watch the video and then solve the problem given below.

[Click here to watch the video.](#)¹

Find the exact value of $\cos\left(\frac{5\pi}{3}\right)\sin\left(\frac{7\pi}{3}\right)$.

$$\cos\left(\frac{5\pi}{3}\right)\sin\left(\frac{7\pi}{3}\right) = \frac{\sqrt{3}}{4}$$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

1: <http://mediaplayer.pearsoncmg.com/assets/v5IM9ASqV75SPyN42Qv1ASX64pguN3vR?clip=2>

21. Watch the video and then solve the problem given below.

[Click here to watch the video.](#)²

Write the expression $\cos 7\theta - \cos 9\theta$ as a product of two trigonometric functions.

$$\cos 7\theta - \cos 9\theta = 2 \sin(8\theta) \sin(2\theta) \quad (\text{Simplify your answer.})$$

2: <http://mediaplayer.pearsoncmg.com/assets/v5IM9ASqV75SPyN42Qv1ASX64pguN3vR?clip=3>