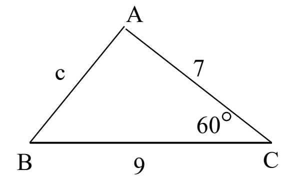
Student: Cole Lamers **Date:** 07/22/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.2 The Law of Cosines

If $a = 9$, $b = 7$, and $C = 60^\circ$ find c in the triangle ABC.

Draw and label the triangle as shown at the right.

Note that the measures of two sides and the included angle are known.



Note that the square of any side of a triangle is equal to the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other sides and the cosine of their included angle.

Thus, use the Law of Cosines shown below to find side c.

$$c^2 = a^2 + b^2 - 2ab\,\mathbf{cos}\,C$$

Substitute the values $a = 9$, $b = 7$, and $C = 60^\circ$ into $c^2 = a^2 + b^2 - 2ab\,\mathbf{cos}\,C$ and simplify.

$$\begin{aligned} c^2 &= a^2 + b^2 - 2ab\,\mathbf{cos}\,C \\ &= 9^2 + 7^2 - 2(9)(7)\,\mathbf{cos}\,60^\circ \\ &= 81 + 49 - 2(9)(7)\,\mathbf{cos}\,60^\circ && \text{Evaluate the exponential expressions.} \end{aligned}$$

Evaluate $\mathbf{cos}\,60^\circ$.

$$\begin{aligned} c^2 &= 81 + 4981 + 49 - 2(9)(7)\,\mathbf{cos}\,60^\circ \\ &= 81 + 49 - 2(9)(7)\left(\frac{1}{2}\right) \\ &= 67 && \text{Simplify.} \end{aligned}$$

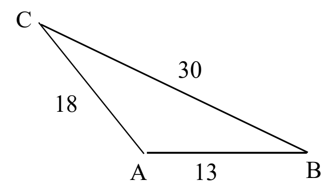
Take the square root of both sides of the equation.

$$\begin{aligned} c^2 &= 67 \\ c &= \sqrt{67} \end{aligned}$$

Therefore, $c = \sqrt{67}$.

Student: Cole Lamers **Date:** 07/22/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.2 The Law of Cosines

Solve the triangle given to the right.



The SSS case is the situation where the lengths of all three sides of a triangle are given. Thus, the missing values are the measures of all three angles.

Since a is the longest side, first find angle A using the Law of Cosines.

The Law of Cosines is shown below for triangle ABC with sides of lengths a, b, and c.

$$a^2 = b^2 + c^2 - 2bc\,\mathbf{cos}\,A$$
$$b^2 = a^2 + c^2 - 2ac\,\mathbf{cos}\,B$$
$$c^2 = a^2 + b^2 - 2ab\,\mathbf{cos}\,C$$

Solve $a^2 = b^2 + c^2 - 2bc\,\mathbf{cos}\,A$ for **cos** A.

$$a^2 = b^2 + c^2 - 2bc\,\mathbf{cos}\,A$$
$$2bc\,\mathbf{cos}\,A = b^2 + c^2 - a^2 \qquad \text{Add } 2bc\,\mathbf{cos}\,A - a^2 \text{ to both sides.}$$

Divide both sides of the equation by 2bc.

$$2bc\,\mathbf{cos}\,A = b^2 + c^2 - a^2$$
$$\mathbf{cos}\,A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \text{Simplify.}$$

Substitute the values a = 30, b = 18, and c = 13 into the formula. Use a calculator to simplify.

$$\mathbf{cos}\,A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$= \frac{(18)^2 + (13)^2 - (30)^2}{2(18)(13)} \qquad \text{Substitute.}$$
$$\approx -0.8697 \qquad \text{Use a calculator. Round to four decimal places.}$$

Take the inverse cosine of both sides.

$$\mathbf{cos}\,A = -0.8697$$
$$A = \mathbf{cos}^{-1}(-0.8697)$$
$$\approx 150.4^\circ \qquad \text{Use a calculator. Round to one decimal place.}$$

Next, determine the measure of angle B. Use either the Law of Sines or the Law of Cosines. In this example, use the Law of Sines.

The Law of Sines is given by $\frac{\mathbf{sin}\,A}{a} = \frac{\mathbf{sin}\,B}{b} = \frac{\mathbf{sin}\,C}{c}$.

Solve $\frac{\mathbf{sin}\,A}{a} = \frac{\mathbf{sin}\,B}{b}$ for B.

$$\frac{\mathbf{sin}\,A}{a} = \frac{\mathbf{sin}\,B}{b}$$
$$\frac{b\,\mathbf{sin}\,A}{a} = \mathbf{sin}\,B \qquad \text{Multiply both sides by b.}$$
$$\mathbf{sin}^{-1}\left(\frac{b\,\mathbf{sin}\,A}{a}\right) = B \qquad \text{Take the inverse sine of both sides.}$$

Substitute the values a = 30, b = 18, and A = 150.4° into the formula. Use a calculator to evaluate.

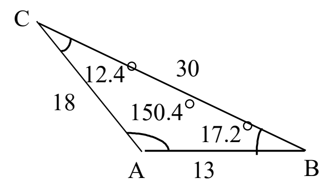
$$B = \mathbf{sin}^{-1}\left(\frac{b\,\mathbf{sin}\,A}{a}\right)$$
$$= \mathbf{sin}^{-1}\left(\frac{18\,\mathbf{sin}\,150.4^\circ}{30}\right) \qquad \text{Substitute.}$$
$$\approx 17.2^\circ \qquad \text{Use a calculator. Round to one decimal place.}$$

To find the measure of the remaining angle, C, recall that the sum of the measures of the three angles of a triangle is 180°. This means that A + B + C = 180°.

Subtract the measures of angles A and B from 180°.

$C \approx 180° - 150.4° - 17.2°$
$C = 12.4°$

Therefore, the solved triangle is shown at the right.



Student: Cole Lamers **Date:** 07/22/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.2 The Law of Cosines

Solve the triangle.

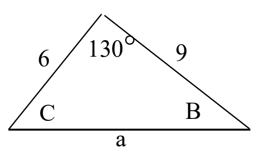
$b = 6,\ \ c = 9,\ \ A = 130^\circ$

Law of Cosines

For a triangle with sides a,b,c and opposite angles A,B,C respectively,

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

A figure can be helpful to use in visualizing the relationships between sides and angles.



We are given the measures of two sides and the included angle (SAS). We begin by finding the side opposite the included angle.

We use the law of cosines to find the length of side a.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\ \mathbf{cos}\ A \\ &= 36 + 81 - 2 \cdot 6 \cdot 9 \cdot \mathbf{cos}\ 130^\circ \\ &\approx 186.4210618 \end{aligned}$$

Take the square root of both sides.

$$\begin{aligned} a &\approx \sqrt{186.4210618} \\ &\approx 13.65 \end{aligned}$$

Next we use the law of sines to find the angle opposite the shorter side, angle B.

$$\frac{\mathbf{sin}\,B}{b} = \frac{\mathbf{sin}\,A}{a}$$

$a\ \mathbf{sin}\ B = b\,\mathbf{sin}\,A$ Cross multiply.

$\mathbf{sin}\ B = \dfrac{b\,\mathbf{sin}\,A}{a}$ Solve for **sin** B.

Now substitute in the values of a, b and A. Do not use rounded numbers.

$$\begin{aligned} \mathbf{sin}\ B &\approx \frac{6\,\mathbf{sin}\,130^\circ}{13.65360985} \\ &\approx 0.3366338 \end{aligned}$$

$$\begin{aligned} B &\approx \mathbf{sin}^{-1}0.3366338 \\ &\approx 19.7^\circ \end{aligned}$$ Take the inverse sine of both sides.

We could use the law of sines again to find angle C. However, it is much easier to use the following.

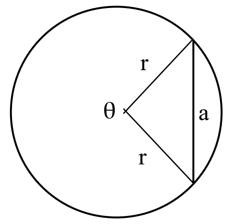
$$A + B + C = 180^\circ$$
$$C = 180^\circ - A - B$$
$$C \approx 180^\circ - 130^\circ - 19.7^\circ$$
$$C \approx 30.3^\circ$$

Therefore, the solution is $a \approx 13.65$, $B \approx 19.7^\circ$, and $C \approx 30.3^\circ$.

Student: Cole Lamers **Date:** 07/22/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.2 The Law of Cosines

What is the length of a chord intercepted by a central angle of 17° in a circle of radius 35 ft?

If a central angle θ in a circle of radius r intercepts a chord of length a, then a formula for a can be written in terms of θ and r.



Apply the Law of Cosines to the triangle formed by the chord and the radii at the endpoints of the chord.

$$a^2 = r^2 + r^2 - 2r^2 \cos\theta$$

Solve for a.

$$a^2 = r^2 + r^2 - 2r^2 \cos\theta$$
$$a^2 = 2r^2 - 2r^2 \cos\theta$$
$$a^2 = r^2(2 - 2\cos\theta)$$
$$a = r\sqrt{2 - 2\cos\theta}$$

Substitute in the values of θ and r.

$$a = 35\sqrt{2 - 2\cos 17^\circ}$$
$$\approx 10.3$$

Therefore, the length of the chord is 10.3 ft.