Vector Addition

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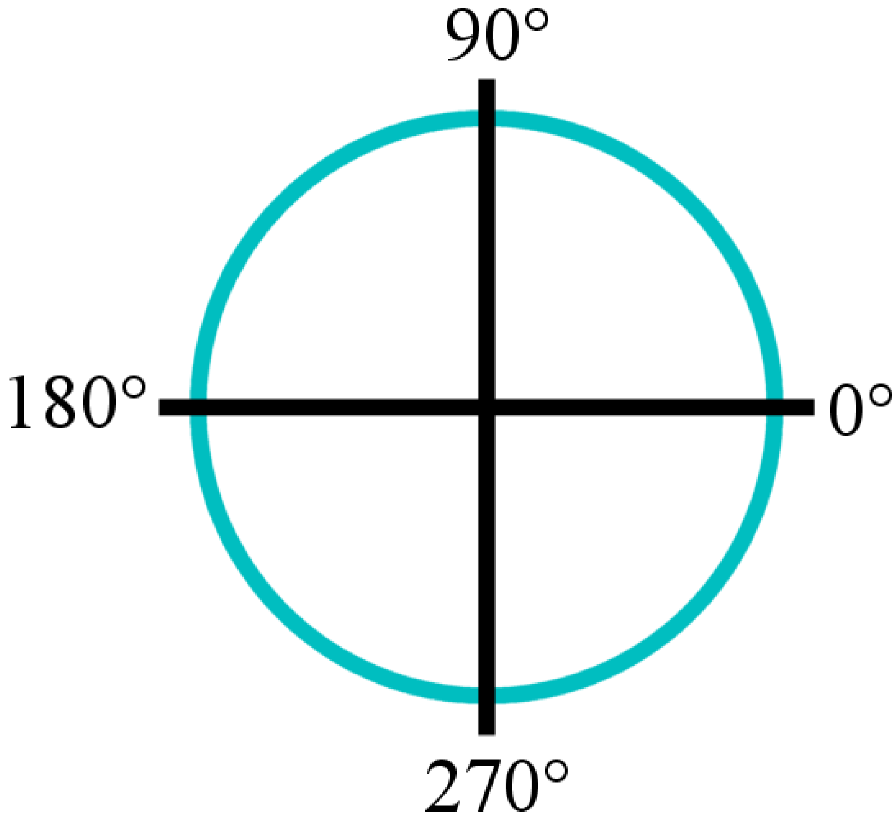
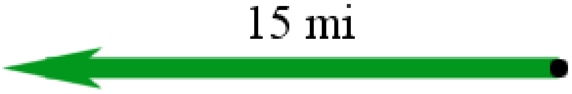
http://phet.colorado.edu/en/simulation/vector-addition

Coordinate System



Starting in Appleton, a car drives 15.0 miles at 180.0 ° (due West), then 10.0 miles at 345°(15.0 ° S of E), and then 20.0 miles at 270.0 ° (due South). Draw a scale diagram to find the resultant displacement.



1.00 cm = 5.00 miles

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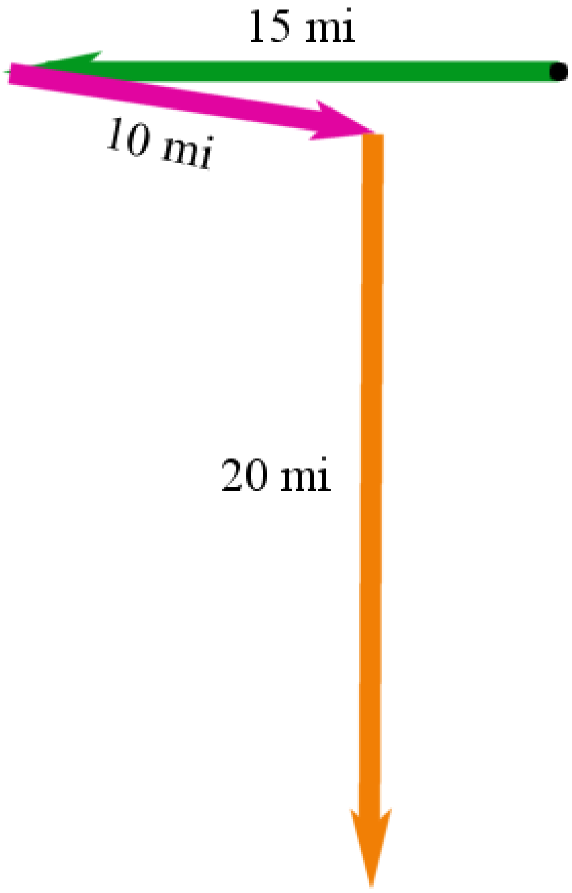
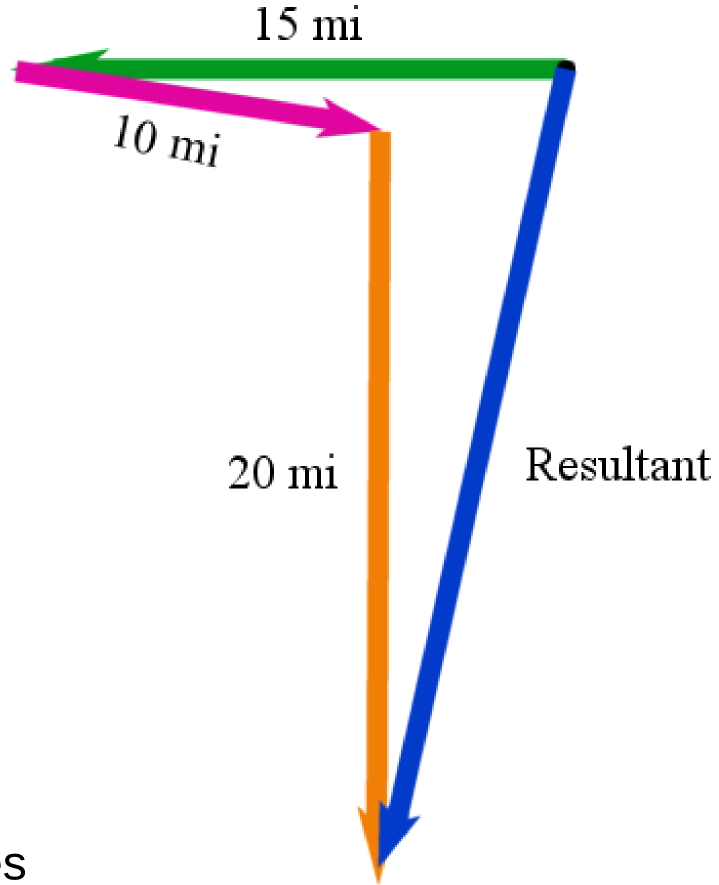
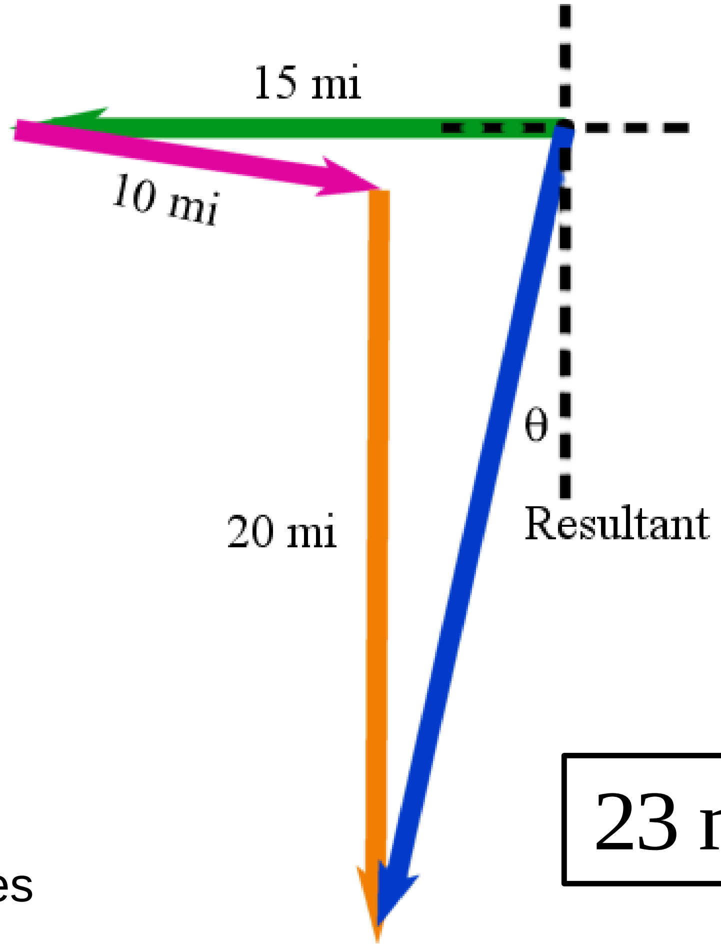


1.00 cm = 5.00 miles



1.00 cm = 5.00 miles

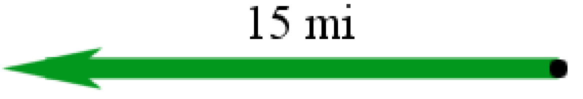


23 miles @ 256°

1.00 cm = 5.00 miles

4.60 cm = 23.0 miles

Using the component method, calculate the resultant displacement for the previous problem.

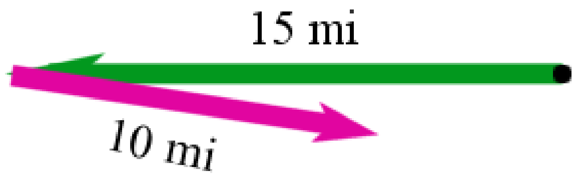


$$x = -15.0 \text{ mi}$$

$$y = 0$$

$$x = \cos\theta(\mathrm{H}) = (\cos 15.0°)(10.0\ \mathrm{mi}) = +9.66\ \mathrm{mi}$$

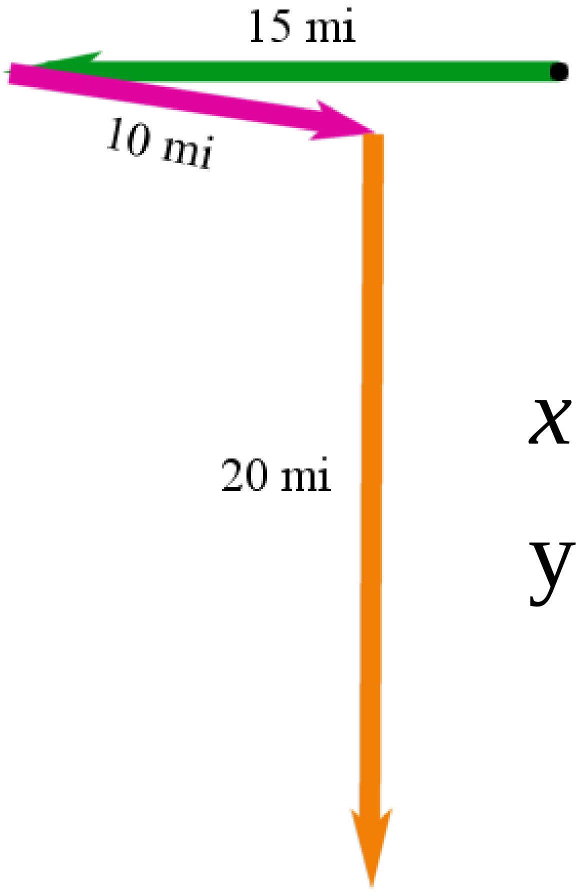
$$y = \sin\theta(\mathrm{H}) = (\sin 15.0°)(10.0\ \mathrm{mi}) = -2.59\ \mathrm{mi}$$



OR

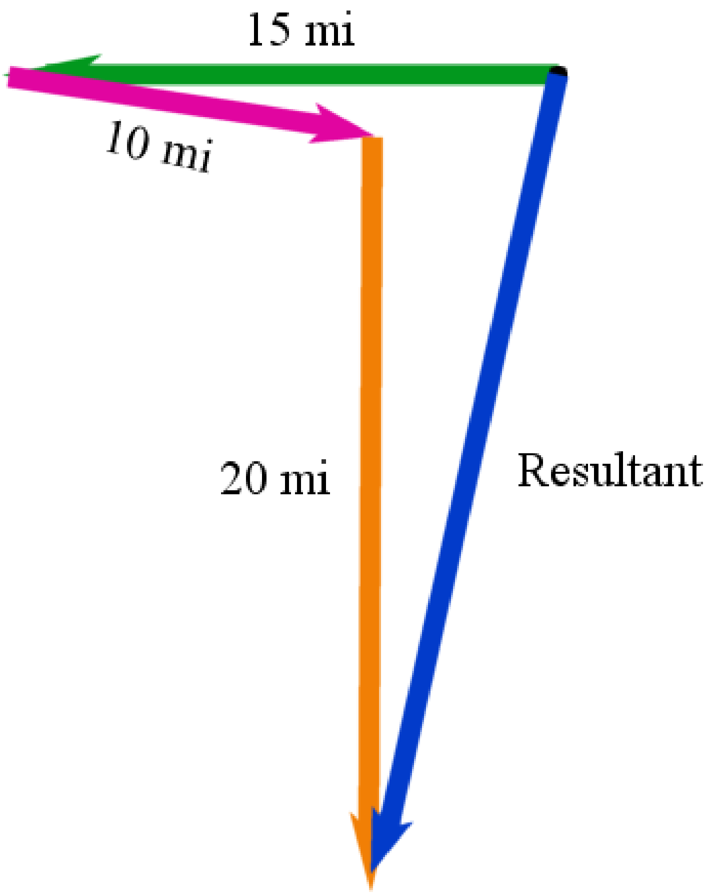
$$x = \cos\theta(\mathrm{H}) = (\cos 345°)(10.0\ \mathrm{mi}) = +9.66\ \mathrm{mi}$$

$$y = \sin\theta(\mathrm{H}) = (\sin 345°)(10.0\ \mathrm{mi}) = -2.59\ \mathrm{mi}$$

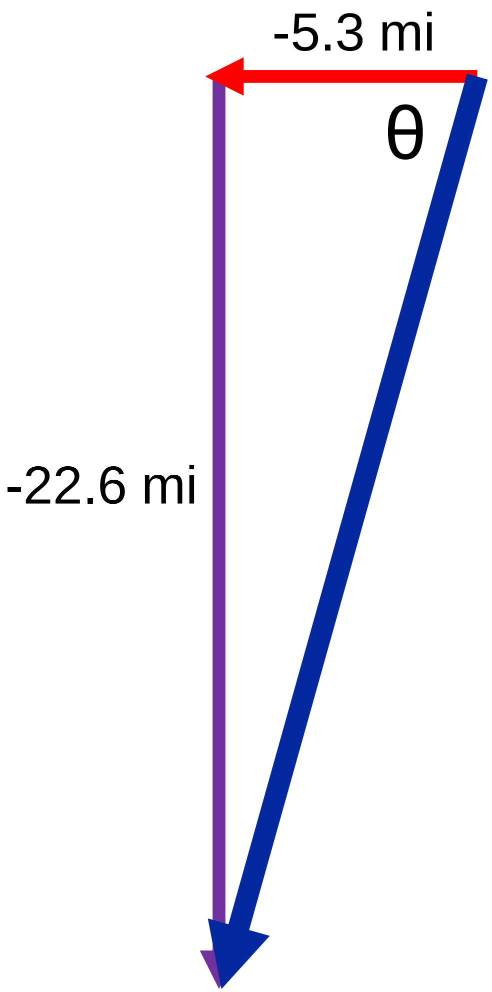


$$x = 0$$

$$y = -20.0 \text{ mi}$$



	x	y
Green	-15.0	0
Pink	9.66	-2.59
Orange	0	-20.0
Totals:	-5.3	-22.6



$$R_x^{\,2} + R_y^{\,2} = R^2$$

$$\therefore \; R = \sqrt{R_x^{\,2} + R_y^{\,2}}$$

$$\therefore \; R = \sqrt{(-5.3\text{ mi})^2 + (-22.6\text{ mi})^2}$$

$$\therefore \; R = \boxed{23\text{ mi}}$$

$$\text{Tan}\,\theta = \frac{\text{O}}{\text{A}}$$

$$\therefore \; \theta = \text{Tan}^{-1}\left(\frac{\text{O}}{\text{A}}\right)$$

$$\therefore \; \theta = \text{Tan}^{-1}\left(\frac{-22.6\,\cancel{\text{mi}}}{-5.3\,\cancel{\text{mi}}}\right)$$

**Remember to add your answer to 180.0° because the vector lies in the third Quadrant.

$$\therefore \; \theta = 77° \quad \therefore \; 77° + 180.0° = \boxed{257°}$$

A long jumper leaves the ground at an angle of 20.0° to the horizontal and at a speed of 11.0 m/s. Determine the horizontal and vertical speed of the athlete.

$$v_x = \cos\theta = \frac{\mathrm{A}}{\mathrm{H}} = (\cos 20.0°)(11.0\ \mathrm{m/s}) = \boxed{10.3\ \mathrm{m/s}}$$

$$v_y = \sin\theta = \frac{\mathrm{O}}{\mathrm{H}} = (\sin 20.0°)(11.0\ \mathrm{m/s}) = \boxed{3.76\ \mathrm{m/s}}$$

Two people pull on a stubborn mule's reins. One person pulls with a force of 120.0 N at 60.0° while the other pulls at a force of 80.0 N at 105.0°. Calculate the resultant of these vectors.

$$x_1 = \cos 60.0°(120.0\text{ N}) = +60.0\text{ N}$$

$$y_1 = \sin 60.0°(120.0\text{ N}) = +104\text{ N}$$

$$x_2 = \sin 15.0°(80.0\text{ N}) = -20.7\text{ N}$$

$$y_2 = \cos 15.0°(80.0\text{ N}) = +77.3\text{ N}$$

Two people pull on a stubborn mule's reins. One person pulls with a force of 120.0 N at 60.0° while the other pulls at a force of 80.0 N at 105.0°. Calculate the resultant of these vectors.

$$x_1 = \cos 60.0°(120.0\ \text{N}) = +60.0\ \text{N}$$

$$y_1 = \sin 60.0°(120.0\ \text{N}) = +104\ \text{N}$$

$$x_2 = \cos 105.0°(80.0\ \text{N}) = -20.7\ \text{N}$$

<u>OR</u>

$$y_2 = \sin 105.0°(80.0\ \text{N}) = +77.3\ \text{N}$$

Two people pull on a stubborn mule's reins. One person pulls with a force of 120.0 N at 60.0° while the other pulls at a force of 80.0 N at 105.0°. Calculate the resultant of these vectors.

	x	y
1st Person	60.0	104
2nd Person	-20.7	77.3
Totals:	39.3	181

Two people pull on a stubborn mule's reins. One person pulls with a force of 120.0 N at 60.0° while the other pulls at a force of 80.0 N at 105.0°. Calculate the resultant of these vectors.

$$R_x^{\ 2} + R_y^{\ 2} = R^2$$

$$\therefore \; R = \sqrt{R_x^{\ 2} + R_y^{\ 2}}$$

$$\therefore \; R = \sqrt{(39.3\text{ N})^2 + (181\text{ N})^2}$$

$$\therefore \; F = \boxed{185\text{ N}}$$

$$\text{Tan}\theta = \frac{\text{O}}{\text{A}}$$

$$\therefore \; \theta = \text{Tan}^{-1}\left(\frac{\text{O}}{\text{A}}\right)$$

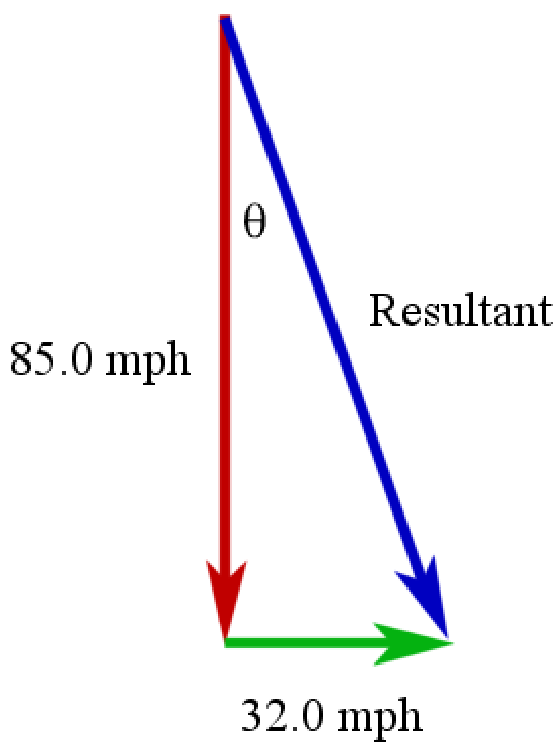
$$\therefore \; \theta = \text{Tan}^{-1}\left(\frac{181\ \cancel{\text{N}}}{39.3\ \cancel{\text{N}}}\right)$$

$$\therefore \; \theta = \boxed{77.7°}$$

Planes & Wind

http://www.physicsclassroom.com/mmedia/vectors/plane.cfm

A plane is directed toward 270.0° (due South) and its indicated airspeed is 85.0 mph. The prevailing wind is traveling at 32.0 mph toward 360.0° (due East). Calculate the resulting speed of the plane.



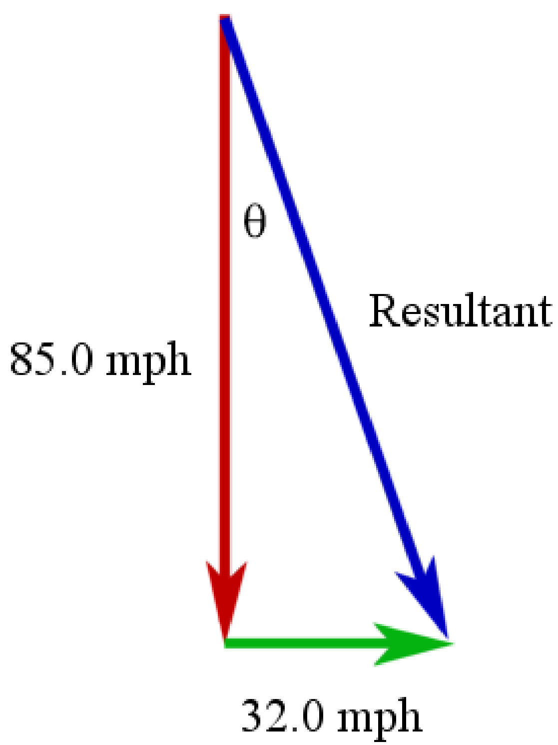
$$R_x^{\,2} + R_y^{\,2} = R^2$$

$$\therefore \; R = \sqrt{R_x^{\,2} + R_y^{\,2}}$$

$$\therefore \; R = \sqrt{(32.0\text{ mph})^2 + (-85.0\text{ mph})^2}$$

$$\therefore \; v = \boxed{90.8\text{ mph}}$$

For the previous problem, calculate the resulting direction for the previous problem.



$$\text{Tan}\,\theta = \frac{\text{O}}{\text{A}}$$

$$\therefore \quad \theta = \text{Tan}^{-1}\left(\frac{\text{O}}{\text{A}}\right)$$

$$\therefore \quad \theta = \text{Tan}^{-1}\left(\frac{32.0\ \text{m}\cancel{\text{p}}\text{h}}{-85.0\ \text{m}\cancel{\text{p}}\text{h}}\right)$$

$$\therefore \quad \theta = -20.6° = \boxed{290.6°}$$

Find the **(a)** horizontal A_x and **(b)** vertical A_y components of the d = 1.00 x 10^2 m displacement of a superhero who flies from the top of a tall building along the path 30.0° below the horizontal (*as shown similarly in lecture Figure 3.11*). Instead of drawing a precise graph, **solve all of these problems mathematically**.

Suppose instead the superhero leaps in the other direction along a displacement vector B to the top of a flagpole where the displacement components are given by B_x = -25.0 m and B_y = 10.0 m. Find the **(c)** magnitude in meters and **(d)** direction in degrees.

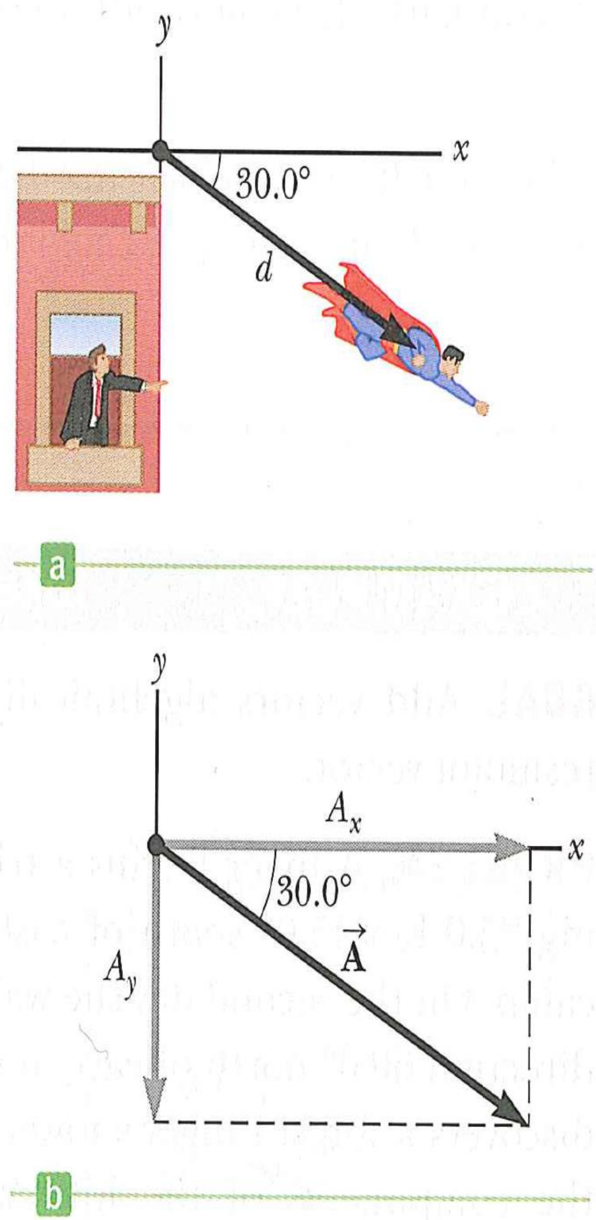
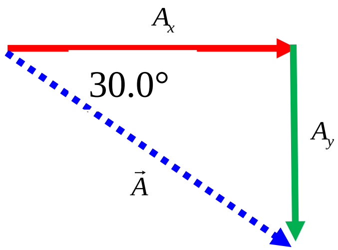


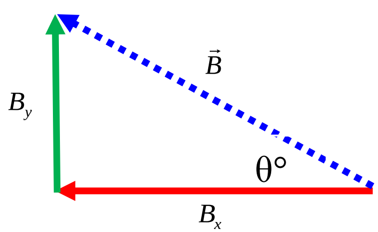
Figure 3.11

Photos/Illustrations courtesy of *College Physics*, 9th edition.



$$A_x = (A)(\cos\ \theta) = \left(10\bar{0}\text{ m}\right)(\cos\ \text{-}30.0°) = \boxed{86.6\text{ m}}$$

$$A_y = (A)(\sin\ \theta) = \left(10\bar{0}\text{ m}\right)(\sin\ \text{-}30.0°) = \boxed{\text{-}50.0\text{ m}}$$



$$\vec{B} = \sqrt{B_x{}^2 + B_y{}^2} = \sqrt{(-25.0\text{ m})^2 + (10.0\text{ m})^2} = \boxed{26.9\text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = \tan^{-1}\left(\frac{10.0\ \cancel{\text{m}}}{-25.0\ \cancel{\text{m}}}\right) = \text{-}21.8°$$

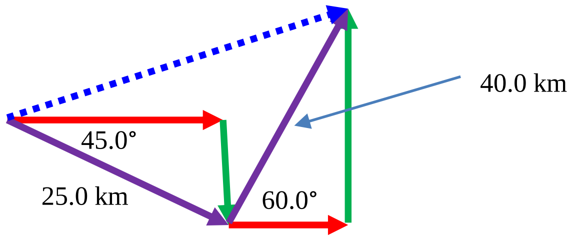
$$\therefore\ \theta = (\text{-}21.8°) + 18\bar{0}.0° = \boxed{158.2°}$$

**Remember to add 180.0° to the answer because the vector lies in the second Quadrant.

A hiker begins a trip by first walking 25.0 km 45.0° south of east (315°) from her base camp. On the second day she walks 40.0 km in a direction 60.0° (60.0°) north of east, at which point she discovers a forest ranger's tower. Determine the components of the hiker's displacements in the **(a)** first and **(b)** second days.

(**Determine all of these values by applying the proper mathematical calculations** – Without drawing a precise graph.) A rough sketch graph is always encouraged to be drawn as a reference; however, the actual answers should be arrived at via a calculation.

(c) Determine the components of the hiker's total displacement for the trip. **(d)** Find the magnitude and direction of the displacement from base camp



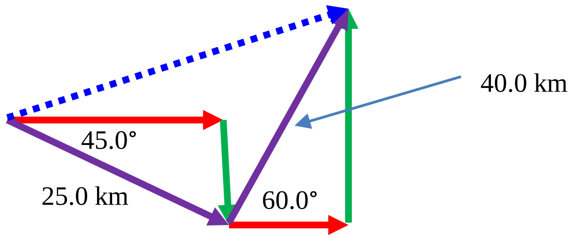
$$A_x = (A)(\cos\ \theta) = (25.0\text{ km})(\cos\ \text{-}45.0°) = \boxed{+17.7\text{ km}}$$

$$A_y = (A)(\sin\ \theta) = (25.0\text{ km})(\sin\ \text{-}45.0°) = \boxed{-17.7\text{ km}}$$

$$B_x = (B)(\cos\ \theta) = (40.0\text{ km})(\cos\ 60.0°) = \boxed{+20.0\text{ km}}$$

$$B_y = (B)(\sin\ \theta) = (40.0\text{ km})(\sin\ 60.0°) = \boxed{+34.6\text{ km}}$$

OR

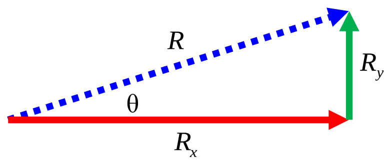


$$A_x = (A)(\cos\ \theta) = (25.0\text{ km})(\cos\ 315°) = \boxed{+17.7\text{ km}}$$

$$A_y = (A)(\sin\ \theta) = (25.0\text{ km})(\sin\ 315°) = \boxed{-17.7\text{ km}}$$

$$B_x = (B)(\cos\ \theta) = (40.0\text{ km})(\cos\ 60.0°) = \boxed{+20.0\text{ km}}$$

$$B_y = (B)(\sin\ \theta) = (40.0\text{ km})(\sin\ 60.0°) = \boxed{+34.6\text{ km}}$$



$$R_x = A_x + B_x = (+17.7 \text{ km}) + (+20.0 \text{ km}) = \boxed{+37.7 \text{ km}}$$

$$R_y = A_y + B_y = (-17.7 \text{ km}) + (+34.6 \text{ km}) = \boxed{+16.9 \text{ km}}$$

$$R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(+37.7 \text{ km})^2 + (+16.9 \text{ km})^2} = \boxed{+41.3 \text{ km}}$$

$$\theta = \tan^{-1}\left(\frac{+16.9 \cancel{\text{km}}}{+37.7 \cancel{\text{km}}}\right) = \boxed{24.1°}$$

(We do not add any additional degree values since our angle lies in the first quadrant.)