Calculus I Formulas

Derivatives

Definition of Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

Product Rule: $\frac{d}{dx}(uv) = u'v + uv'$

Quotient Rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{u'v-uv'}{v^2}$

Chain Rule: $\frac{d}{dx}(f(g(x)) = f'(g(x)) \cdot g'(x)$

Trigonometric Functions

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

Exponential and Logarithmic Functions

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(a^x) = a^x \ln a$

Inverse Trigonometric Functions

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$

Parametric Equations

If $x = f(t)$ and $y = g(t)$ are differentiable, then:

$y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ and $\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$

Some Common Trigonometric Identities

$\sin^2 \theta + \cos^2 \theta = 1$

$1 + \tan^2 \theta = \sec^2 \theta$

$1 + \cot^2 \theta = \csc^2 \theta$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1$$
$$= 1 - 2\sin^2 \theta$$

$\sin 2\theta = 2 \sin \theta \cos \theta$

$\cos^2 \theta = \frac{1}{2} + \frac{1}{2} cos2\theta$

$\sin^2 \theta = \frac{1}{2} - \frac{1}{2} \cos 2\theta$

Formal Definition of a Limit (Precise Definition)

Given $\lim_{x \to x_0} f(x) = L$, find $\delta > 0$ such that $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \epsilon$.

Mean Value Theorem for Derivatives

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Integrals

$\int x^n dx = \frac{1}{n+1} x^n + C$

$\int \sin kx \, dx = -\frac{1}{k} \cos kx + C$

$\int \cos kx \, dx = \frac{1}{k} \sin kx + C$

$\int \sec^2 kx \, dx = \frac{1}{k} \tan kx + C$

$\int \csc^2 kx \, dx = -\frac{1}{k} \cot kx + C$

$\int \sec kx \tan kx \, dx = \frac{1}{k} \sec kx + C$

$\int \csc kx \cot kx \, dx = -\frac{1}{k} \csc kx + C$

$\int e^{kx} dx = \frac{1}{k} e^{kx} + C$

$\int \frac{1}{x} dx = \ln|x| + C$

$\int \frac{1}{\sqrt{1-k^2x^2}} dx = \frac{1}{k} \sin^{-1} kx + C$

$\int \frac{1}{1+k^2x^2} dx = \frac{1}{k} \tan^{-1} kx + C$

$\int \frac{1}{x\sqrt{k^2x^2-1}} dx = \sec^{-1} kx + C$

$\int a^{kx} dx = \left(\frac{1}{k \ln a}\right) a^{kx} + C$

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Center of Mass

$\bar{x} = \frac{\int \tilde{x} dm}{\int dm}, \bar{y} = \frac{\int \tilde{y} dm}{\int dm}$

Mean Value Theorem for Definite Integrals

$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$

Fundamental Theorem of Calculus

Part 1

$F'(x) = \frac{d}{dx} \int_a^x f(t) dt = f(x)$

Part 2

$\int_a^b f(x) dx = F(b) - F(a)$

Disk Method

$V = \int_a^b \pi [R(x)]^2 dx$

Washer Method

$V = \int_a^b \pi([R(x)]^2 - [r(x)]^2) dx$

Outer radius: $R(x)$

Inner radius: $r(x)$

Shell Method

$V = \int_a^b 2\pi \begin{pmatrix} shell \\ radius \end{pmatrix} \begin{pmatrix} shell \\ height \end{pmatrix} dx$

Length of a Curve (Parametric), (Function)

$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$, $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$

Surface Area of Revolution

Function (about x-axis), (about y-axis)

$S = \int_a^b 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$, $S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \, dy$

Parametric (about x-axis), (about y-axis)

$S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$, $S = \int_c^d 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

Fluid Force Against a Vertical Plate

$F = \int_a^b w \cdot \begin{pmatrix} strip \\ depth \end{pmatrix} \cdot L(y) dy$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

Some Identities

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2 \sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\tanh^2 x = 1 - \operatorname{sech}^2 x$

$\coth^2 x = 1 + \operatorname{csch}^2 x$

Derivatives

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

Integrals

$\int \sinh x \, dx = \cosh x + C$

$\int \cosh x \, dx = \sinh x + C$

$\int \operatorname{sech}^2 x \, dx = \tanh x + C$

$\int \operatorname{csch}^2 x \, dx = -\coth x + C$

$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$

$\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$

Some Derivatives of Inverse Hyperbolics

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}, x > 1$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, |x| < 1$

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}, |x| > 1$

Some Integrals Leading to Inverse Hyperbolics

$\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, a > 0$

$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, x > a > 0$

$$\int \frac{dx}{a^2-x^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C & if\ x^2 < a^2 \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C & if\ x^2 > a^2 \end{cases}$$