

# Unit 4 – Impulse & Momentum

## Post-Lecture

In a simulated head-on collision, a car traveling 24.6 m/s hits a solid wall and comes abruptly to rest. A crash test dummy (mass = 95.0 kg), not wearing a seatbelt, is brought to rest by an airbag. If the airbag delivers a force of 12,500 N, how long is the dummy in contact with the airbag?

$$Ft = m\Delta v \quad \therefore t = \frac{m\Delta v}{F}$$

$$\therefore t = \frac{(95.0 \text{ kg})(24.6 \text{ m/s})}{12,500 \text{ N}} = \boxed{0.187 \text{ s}}$$

# Conservation of Momentum

- The principle of conservation of momentum states that, when no external forces act on a system consisting of two objects. The total initial momentum of both objects will equal the total final momentum of both objects.

A 44.0 g bullet is fired with a muzzle velocity of 1004 m/s from a 6.23 kg rifle. What is the recoil velocity of the rifle?

$$\sum p_i = \sum p_f \quad \therefore m_1 v_{o_1} + m_2 v_{o_2} = m_1 v_1 + m_2 v_2$$

Due to the system initially being at rest, there is no initial momentum  $\therefore m_1 v_1 = -m_2 v_2$

$$\therefore -v_2 = \frac{m_1 v_1}{m_2} = \frac{(0.0440 \text{ kg})(1004 \text{ m/s})}{6.23 \text{ kg}} = \boxed{-7.09 \text{ m/s}}$$

A 44.0 g bullet is fired with a muzzle velocity of 1004 m/s from a 6.23 kg rifle. What is the kinetic energy of the rifle?

$$KE_r = \frac{1}{2}mv_r^2 = \frac{1}{2}(6.23\text{ kg})(7.09\text{ m/s})^2 = \boxed{156\text{ J}}$$

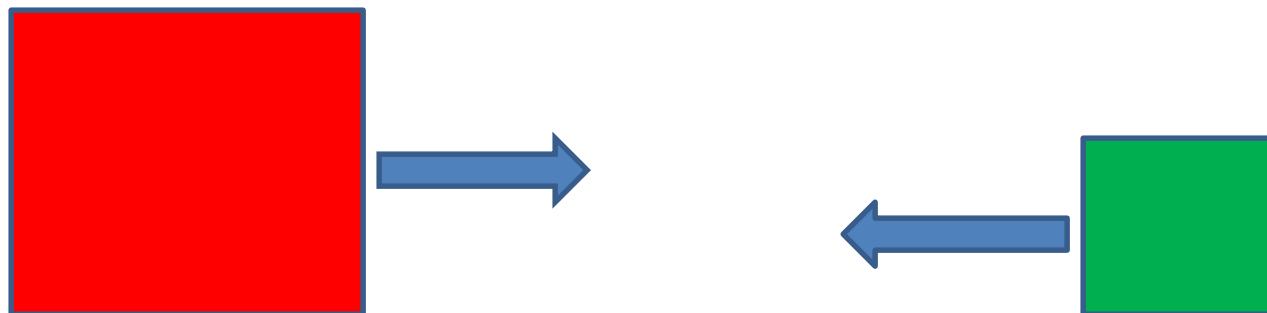
A 44.0 g bullet is fired with a muzzle velocity of 1004 m/s from a 6.23 kg rifle. What is the kinetic energy of the bullet?

$$KE_b = \frac{1}{2}mv_b^2 = \frac{1}{2}(0.0440\text{ kg})(1004\text{ m/s})^2 = \boxed{22,200\text{ J}}$$

# Elastic & Inelastic Collisions

- An elastic collision is one in which both momentum and kinetic energy are conserved.
- An inelastic collision is one in which momentum is conserved but kinetic energy is not.
- A perfectly inelastic collision is an inelastic collision in which the two objects stick together after the collision, so that their final velocities are the same and the momentum of the system is conserved.

Object “A” ( $m = 52.00 \text{ kg}$ ) and object “B” ( $m = 18.0 \text{ kg}$ ) approach each other moving at the same speed of  $27.00 \text{ m/s}$ . Determine the velocity of each after impact if the collision is completely inelastic.



$$m_1 v_{o_1} + m_2 v_{o_2} = (m_1 + m_2) v$$

$$\therefore v = \frac{(m_1 v_{o_1} + m_2 v_{o_2})}{(m_1 + m_2)}$$

$$\therefore v = \frac{[(52.00 \text{ kg})(+27.00 \text{ m/s}) + (18.0 \text{ kg})(-27.00 \text{ m/s})]}{(52.00 \text{ kg} + 18.0 \text{ kg})}$$

$$\therefore v = \frac{\left[ \left( 1404 \frac{\text{kg} \cdot \text{m}}{\text{s}} \right) + \left( -486 \frac{\text{kg} \cdot \text{m}}{\text{s}} \right) \right]}{(70.0 \text{ kg})}$$

$$\therefore v = \boxed{13.1 \text{ m/s}}$$

Determine the velocities of objects “A” and “B” from the previous problem if the collision was perfectly elastic.

For perfectly elastic collisions:

$$\frac{v_2 - v_1}{v_{o_1} - v_{o_2}} = 1 \quad \therefore v_{o_1} - v_{o_2} = v_2 - v_1$$

$$\therefore 27.00 \text{ m/s} - (-27.00 \text{ m/s}) = v_2 - v_1$$

$$\therefore 54.00 \text{ m/s} + v_1 = v_2$$

$$\Sigma p_i = \Sigma p_i \quad \therefore m_1 v_{o_1} + m_2 v_{o_2} = m_1 v_1 + m_2 v_2$$

$$(52.00 \text{ kg})(27.00 \text{ m/s}) + (18.0 \text{ kg})(-27.00 \text{ m/s}) = (52.00 \text{ kg})v_1 + (18.0 \text{ kg})(54.00 \text{ m/s} + v_1)$$

$$\left(1404 \frac{\text{kg}\cdot\text{m}}{\text{s}}\right) + \left(-486 \frac{\text{kg}\cdot\text{m}}{\text{s}}\right) = (52.00 \text{ kg})v_1 + \left(972 \frac{\text{kg}\cdot\text{m}}{\text{s}}\right) + (18.0 \text{ kg})v_1$$

$$\therefore 918 \frac{\text{kg}\cdot\text{m}}{\text{s}} = (70.0 \text{ kg})v_1 + \left(972 \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)$$

$$\therefore -54 \frac{\text{kg}\cdot\text{m}}{\text{s}} = (70.0 \text{ kg})v_1$$

$$\therefore v_1 = \boxed{-0.77 \text{ m/s}} \quad \therefore 54.0 \text{ m/s} + (-0.77 \text{ m/s}) = v_2 = \boxed{53.2 \text{ m/s}}$$

A 1350 kg car traveling east at 25.0 m/s collides with a 1490 kg car traveling west at 21.0 m/s. The impact causes their bumpers to lock together. What is the velocity of the combination after the collision?

$$m_1 v_{o_1} + m_2 v_{o_2} = (m_1 + m_2) v$$

$$(1350 \text{ kg})(25.0 \text{ m/s}) + (1490 \text{ kg})(-21.0 \text{ m/s}) = (1350 \text{ kg} + 1490 \text{ kg})v$$

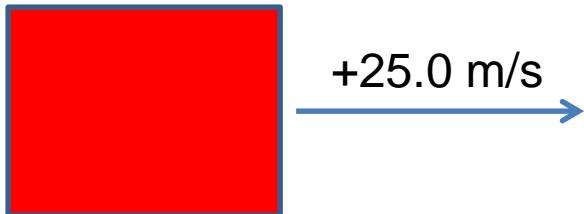
$$33,800 \frac{\text{kg}\cdot\text{m}}{\text{s}} + \left(-31,300 \frac{\text{kg}\cdot\text{m}}{\text{s}}\right) = (2840 \text{ kg})v$$

$$2500 \frac{\text{kg}\cdot\text{m}}{\text{s}} = (2840 \text{ kg})v$$

$\therefore v =$  0.880 m/s toward the east.

At an intersection, a 1500.0 kg car traveling east at 25.0 m/s collides with a 2500.0 kg van traveling north at 20.0 m/s. Find the magnitude and direction of the velocity of the wreckage immediately after the collision, assuming that the vehicles undergo a perfectly inelastic collision.

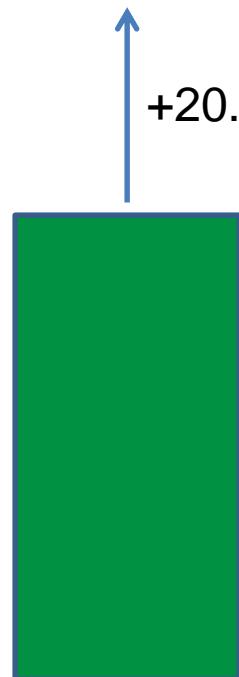
1500.0 kg



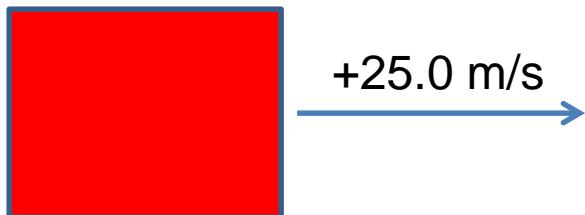
+25.0 m/s

+20.0 m/s

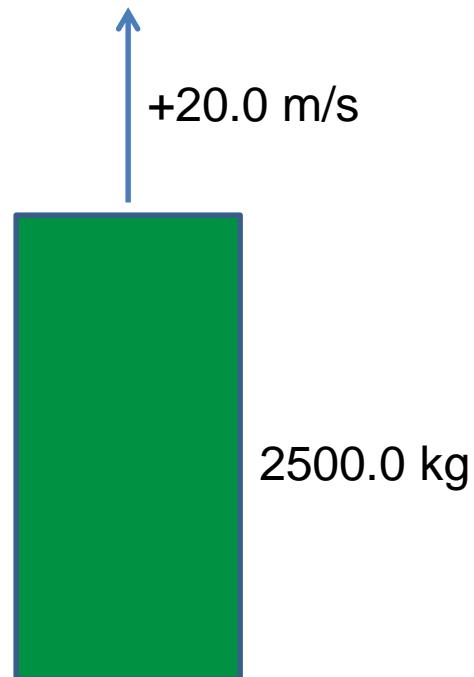
2500.0 kg



1500.0 kg



+25.0 m/s

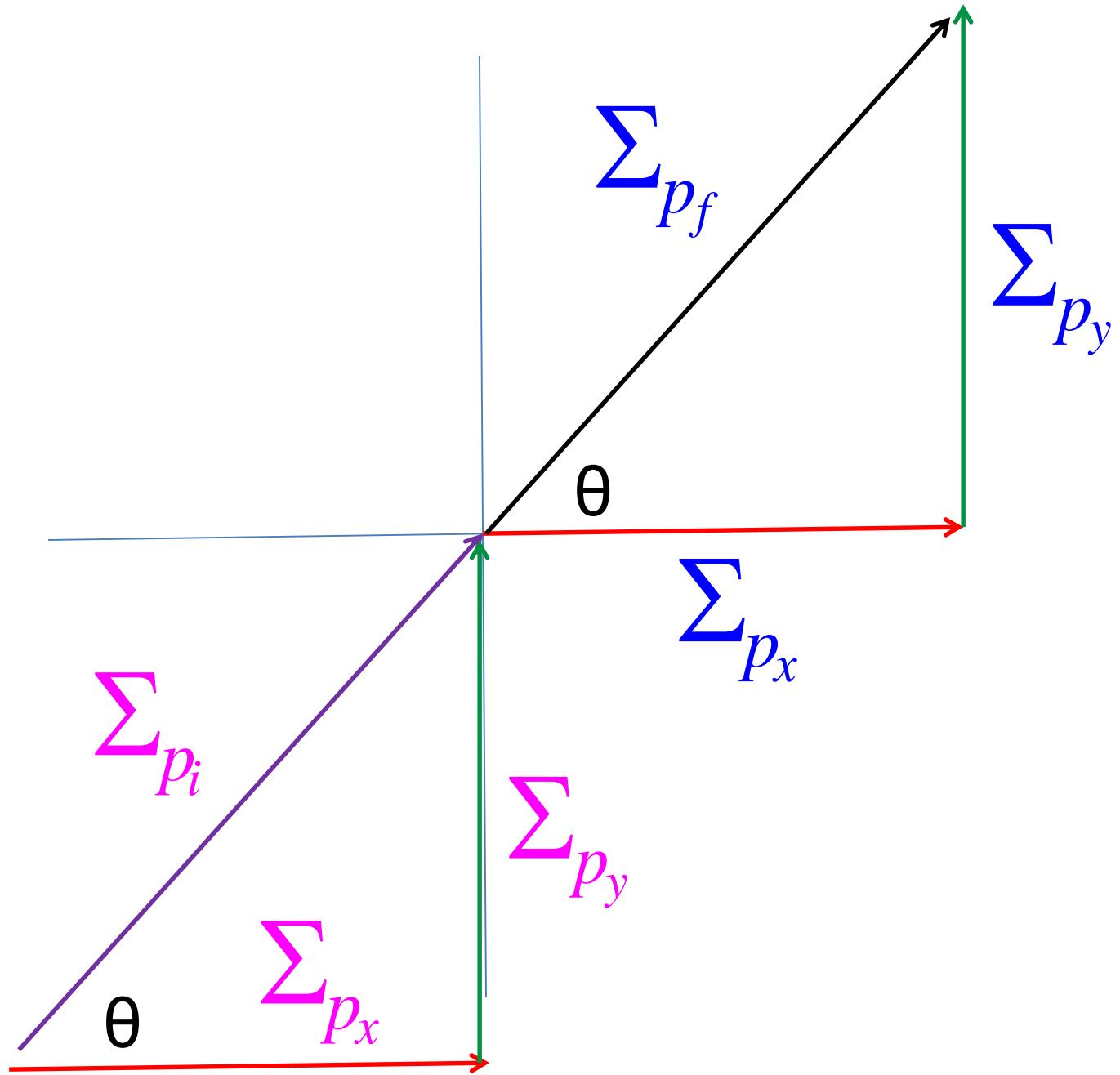


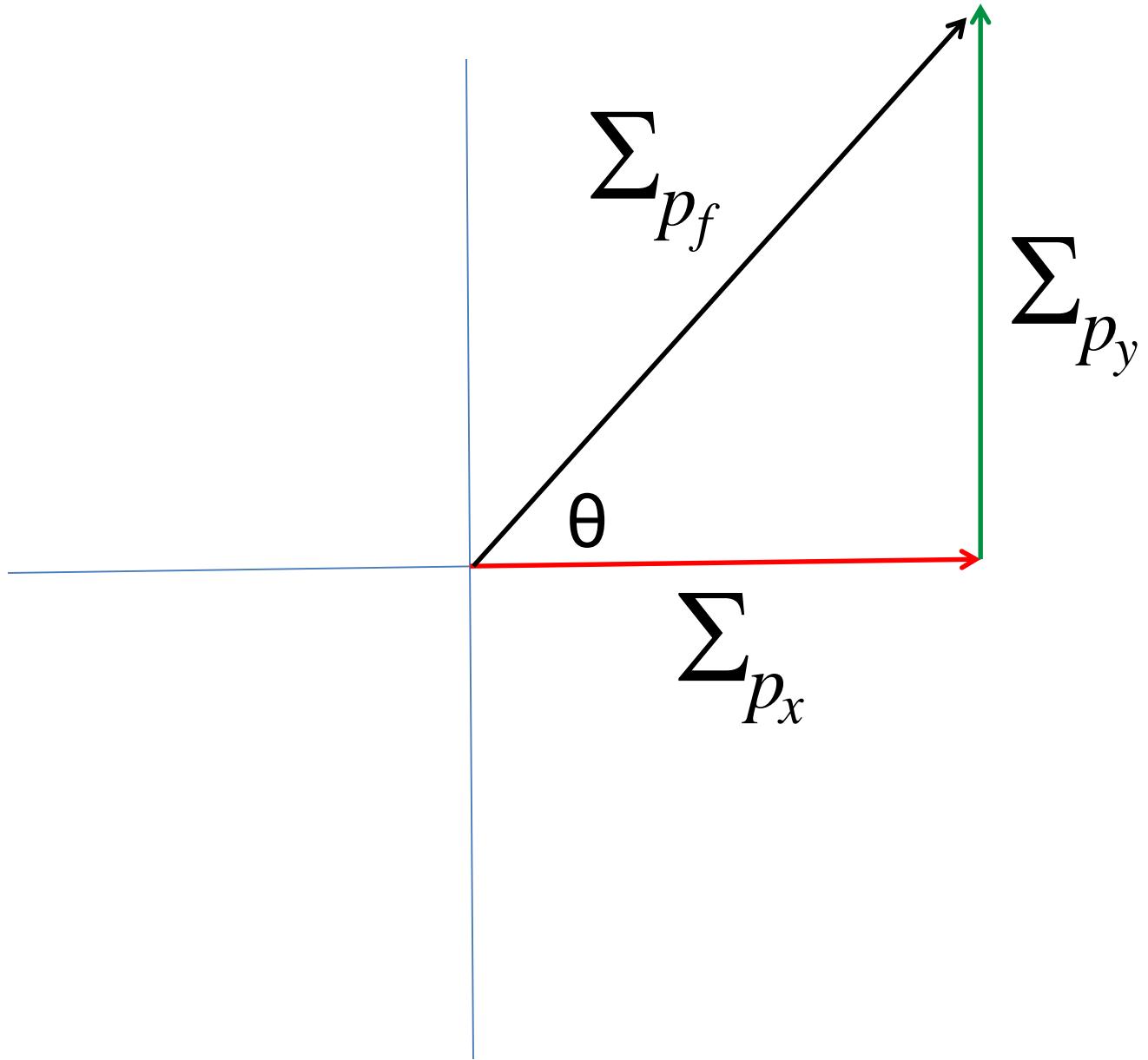
2500.0 kg

$$\sum p_i = \sum p_f$$

$$\sum p_{x_i} = \sum p_{x_f}$$

$$\sum p_{y_i} = \sum p_{y_f}$$



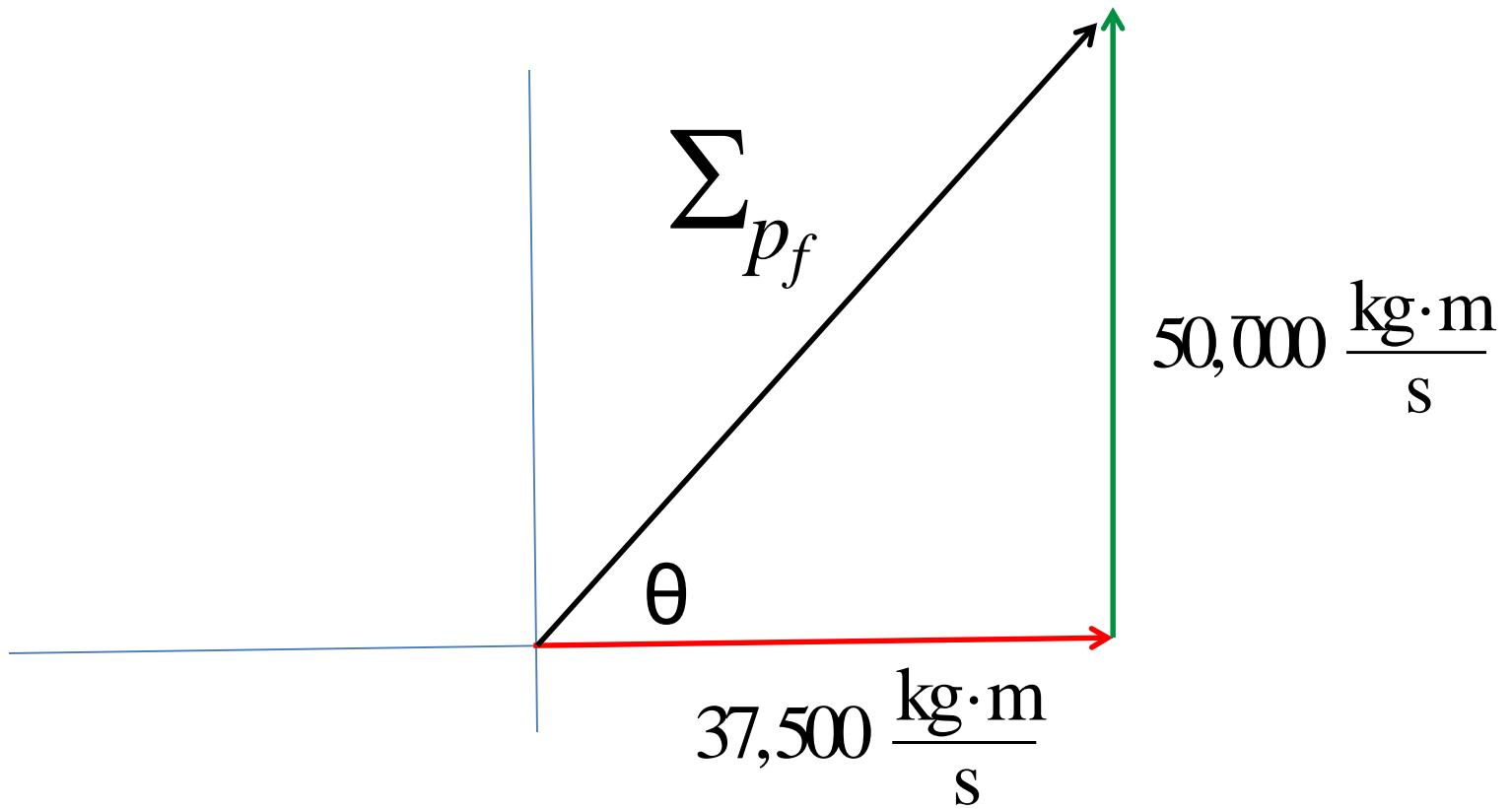


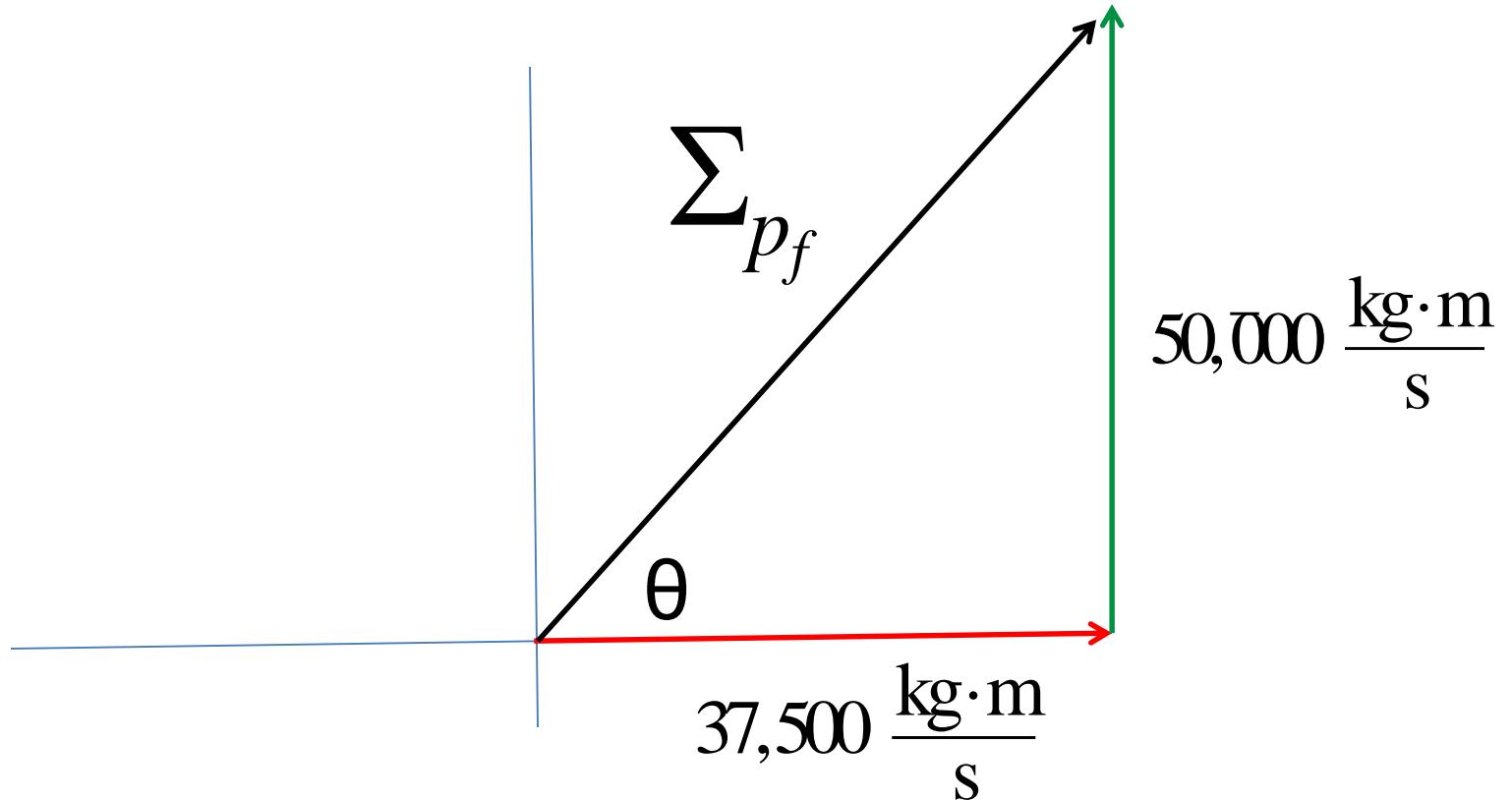
$$\Sigma p_{x_i} = m_1 v_{x_{o1}} + m_2 v_{x_{o2}} = \Sigma p_{x_f}$$

$$\Sigma p_{x_i} = (1500.0 \text{ kg})(+25.0 \text{ m/s}) + 0 \frac{\text{kg}\cdot\text{m}}{\text{s}} = 37,500 \frac{\text{kg}\cdot\text{m}}{\text{s}} = \Sigma p_{x_f}$$

$$\Sigma p_{y_i} = m_1 v_{y_{o1}} + m_2 v_{y_{o2}} = \Sigma p_{y_f}$$

$$\Sigma p_{y_i} = 0 \frac{\text{kg}\cdot\text{m}}{\text{s}} + (2500.0 \text{ kg})(+20.0 \text{ m/s}) = 50,000 \frac{\text{kg}\cdot\text{m}}{\text{s}} = \Sigma p_{y_f}$$





$$\Sigma p_f = \sqrt{\left(37,500 \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2 + \left(50,000 \frac{\text{kg}\cdot\text{m}}{\text{s}}\right)^2}$$

$$\Sigma p_f = 62,500 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$m_1 v_{o_1} + m_2 v_{o_2} = (m_1 + m_2) v = 62,500 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

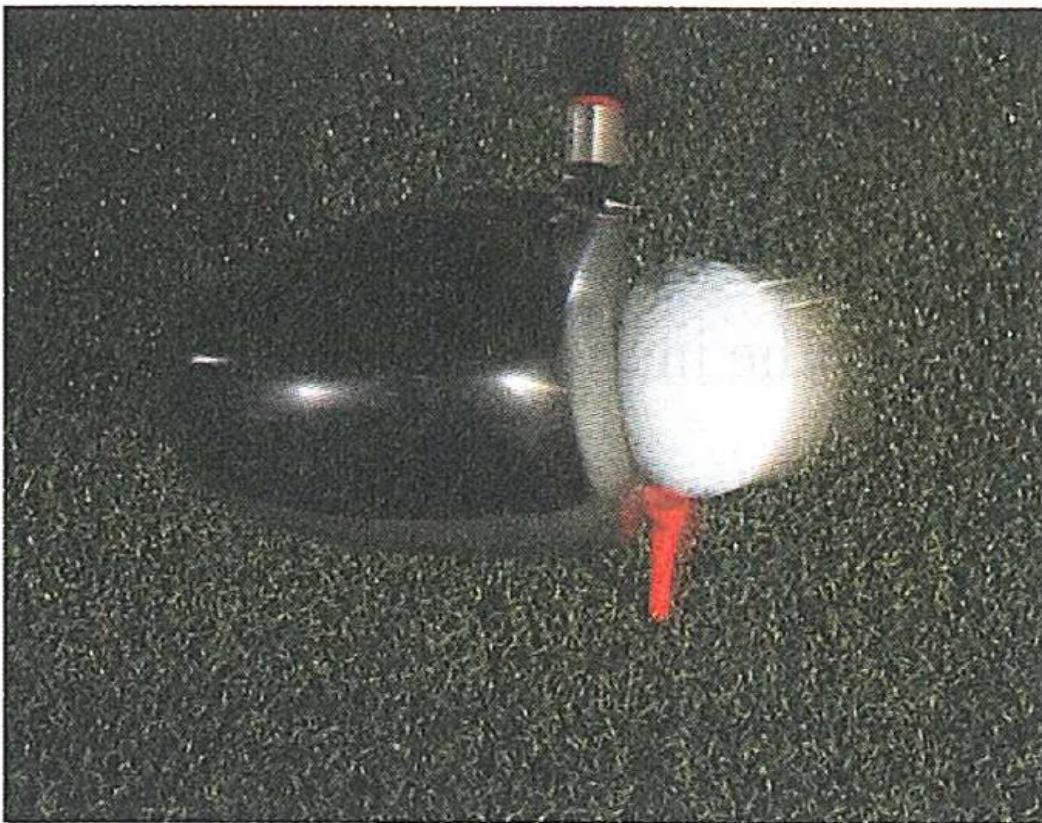
$$\therefore (1500 \text{ kg} + 2500 \text{ kg}) v = 62,500 \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$\therefore v = \boxed{15.6 \text{ m/s}}$$

$$\theta = \tan^{-1} \left( \frac{50,000 \frac{\text{kg}\cdot\text{m}}{\text{s}}}{37,500 \frac{\text{kg}\cdot\text{m}}{\text{s}}} \right) = \boxed{53.1^\circ}$$

A golf ball with mass 0.050 kg is struck by a club (*as in lecture Figure 6.3*). The force on the ball varies from zero when contact is made up to some maximum value (when the ball is maximally deformed) and then back to zero when the ball leaves the club, (*as in the graph in Figure 6.1*). Assume that the ball leaves the club face with a velocity of +44 m/s.

- (a)** Find the magnitude of the impulse due to the collision.
- (b)** Estimate the duration of the collision (assuming that the ball travels a distance of 2.0 cm on the face of the club) and
- (c)** the average force acting on the ball.



**Figure 6.3**

Photos/Illustrations courtesy of *College Physics*, 9<sup>th</sup> edition.

$$\begin{aligned}
 I &= \Delta p = p_f - p_i \\
 &= (0.050 \text{ kg})(+44.0 \text{ m/s}) - 0 \\
 &= \boxed{+2.2 \text{ kg} \cdot \text{m/s}}
 \end{aligned}$$

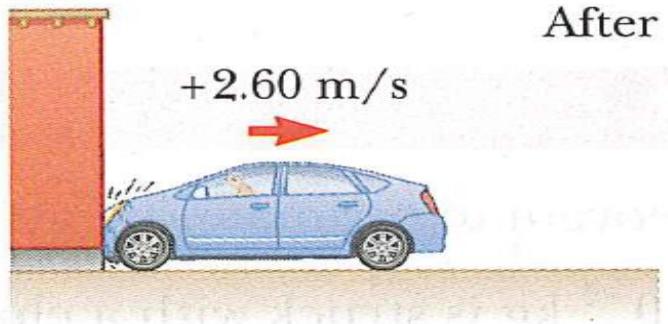
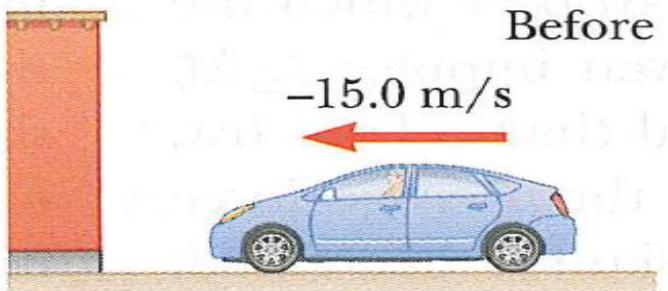
$$\Delta t = \frac{\Delta x}{v_{avg}} = \frac{(0.020 \text{ m})}{(+22 \text{ m/s})} = \boxed{9.1 \times 10^{-4} \text{ s}}$$

$$F_{avg} = \frac{\Delta p}{\Delta t} = \frac{(+2.2 \text{ kg} \cdot \text{m/s})}{(9.1 \times 10^{-4} \text{ s})} = \boxed{2400 \text{ N}}$$

In a crash test, a car of mass  $1.50 \times 10^3$  kg collides with a wall and rebounds (*as shown similarly in lecture Figure 6.4*). The initial and final velocities of the car are  $v_i = -15.0$  m/s and  $v_f = 2.60$  m/s, respectively. If the collision lasts for 0.150 s,

find **(a)** the impulse delivered to the car due to the collision

and **(b)** the size / direction of the average force exerted on the car.



a

Hyundai Motors/HO/Landov



b

**Figure 6.4**

Photos/Illustrations courtesy of *College Physics*, 9<sup>th</sup> edition.

$$p_i = m v_i = (1500 \text{ kg})(-15.0 \text{ m/s}) \\ = -2.25 \times 10^4 \text{ kg} \cdot \text{m/s}$$

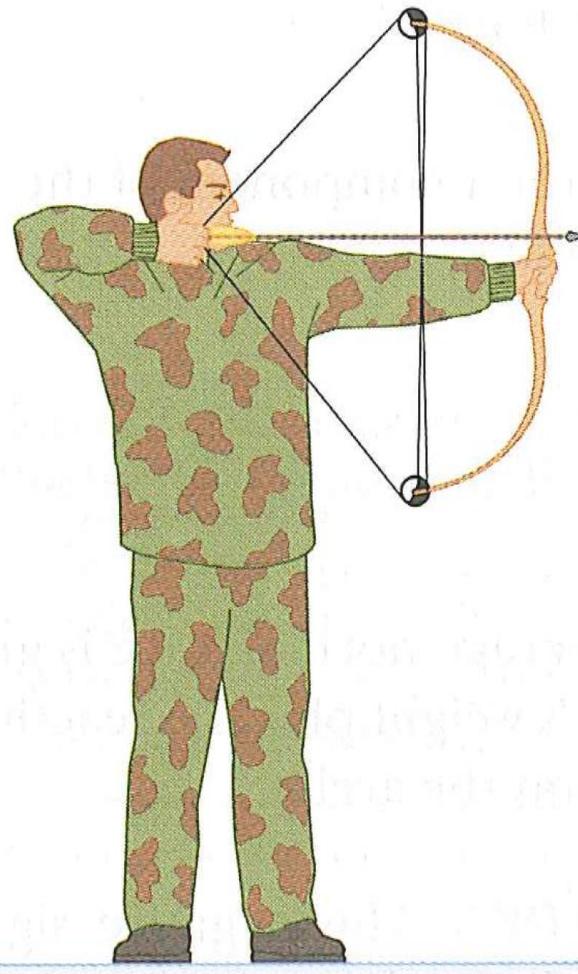
$$p_f = m v_f = (1500 \text{ kg})(+2.60 \text{ m/s}) \\ = +3900 \text{ kg} \cdot \text{m/s}$$

$$I = p_f - p_i = (+3900 \text{ kg} \cdot \text{m/s}) - (-2.25 \times 10^4 \text{ kg} \cdot \text{m/s}) \\ = +2.64 \times 10^4 \text{ kg} \cdot \text{m/s}$$

$$F_{avg} = \frac{\Delta p}{\Delta t} = \frac{(2.64 \times 10^4 \text{ kg} \cdot \text{m/s})}{(0.150 \text{ s})} = +1.76 \times 10^5 \text{ N}$$

An archer stands at rest on frictionless ice; his total mass including his bow and quiver of arrows is 60.00 kg (as shown similarly in lecture Figure 6.9). If the archer fires a 0.0300 kg arrow horizontally at 50.0 m/s in the positive x-direction,

- (a)** what is his subsequent velocity across the ice? He then fires a second identical arrow at the same speed relative to the ground but at an angle of  $30.0^\circ$  above the horizontal.
- (b)** Find his new speed.
- (c)** Estimate the average normal force acting on the archer as the second arrow is accelerated by the bowstring.  
Assume a draw length of 0.800 m.



**Figure 6.9**

Photos/Illustrations courtesy of *College Physics*, 9<sup>th</sup> edition.

$$p_{i_x} = p_{f_x} \quad 0 = m_1 v_{1_f} + m_2 v_{2_f}$$

$$\therefore v_{1_f} = -\left(\frac{m_2 v_{2_f}}{m_1}\right) = -\left(\frac{(0.0300 \text{ kg})(50.0 \text{ m/s})}{(59.97 \text{ kg})}\right)$$

$$\therefore v_{1_f} = \boxed{-0.0250 \text{ m/s}}$$

$$m_1 v_{1_i} = (m_1 - m_2) v_{1_f} + m_2 v_{2_f} (\cos \theta)$$

$$\therefore v_{1_f} = \frac{m_1 v_{1_i}}{(m_1 - m_2)} - \frac{m_2 v_{2_f} (\cos \theta)}{(m_1 - m_2)}$$

$$\therefore v_{1_f} = \frac{(59.97 \text{ kg})(-0.0250 \text{ m/s})}{(59.94 \text{ kg})} - \frac{(0.0300 \text{ kg})(50.0 \text{ m/s})(\cos 30.0^\circ)}{(59.94 \text{ kg})}$$

$$\therefore v_{1_f} = \boxed{-0.0467 \text{ m/s}}$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$\therefore a = \frac{v_{2f}^2 - v_o^2}{2\Delta x} = \frac{(50.0 \text{ m/s})^2 - 0}{2(0.800 \text{ m})} = 1560 \text{ m/s}^2$$

$$t = \frac{v_{2f} - v_o}{a} = \frac{(50.0 \text{ m/s}) - 0}{(1560 \text{ m/s}^2)} = 0.0320 \text{ s}$$

$$\left( F_{y_{avg}} \right) (\Delta t) = \Delta p_y$$

$$\therefore F_{y_{avg}} = \frac{\Delta p_y}{\Delta t} = \frac{m_2 v_{2f} (\sin \theta)}{\Delta t}$$

$$\therefore F_{y_{avg}} = \frac{(0.0300 \text{ kg})(50.0 \text{ m/s})(\sin 30.0^\circ)}{0.0320 \text{ s}} = 23.4 \text{ N}$$

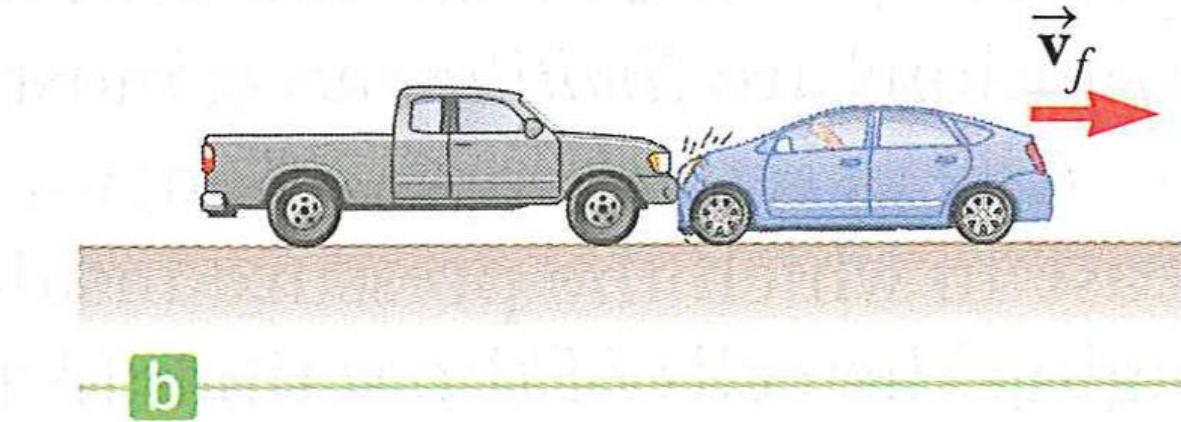
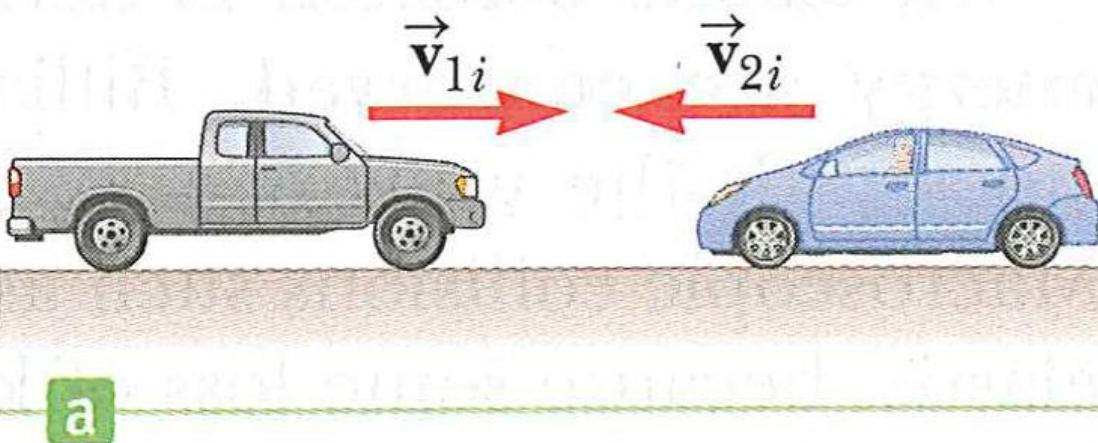
$$\Sigma F_y = n - mg - R = 0$$

$$\therefore n = mg + R = (59.94 \text{ kg})(9.80 \text{ m/s}^2) + (23.4 \text{ N})$$

$$\therefore n = \boxed{611 \text{ N}}$$

A pickup truck with mass  $1.80 \times 10^3$  kg is traveling eastbound at +15.0 m/s, while a compact car with mass  $9.00 \times 10^2$  kg is traveling westbound at -15.0 m/s (*as shown in lecture Figure 6.11*). The vehicles become entangled.

- (a) Find the speed of the entangled vehicles after the collision.
- (b) & (c) Find the change in the velocity of each vehicle.
- (d) Find the change in the kinetic energy of the system consisting of both vehicles.



**Figure 6.11**

Photos/Illustrations courtesy of *College Physics*, 9<sup>th</sup> edition.

$$p_i = p_f$$

$$m_1 v_{1_i} + m_2 v_{2_i} = (m_1 + m_2) v_f$$

$$(1800 \text{ kg})(+15.0 \text{ m/s}) + (900 \text{ kg})(-15.0 \text{ m/s}) = (2700 \text{ kg})v_f$$

$$\therefore v_f = \boxed{+5.00 \text{ m/s}}$$

$$\Delta v_1 = v_f - v_{1_i} = (+5.00 \text{ m/s}) - (+15.0 \text{ m/s}) = \boxed{-10.0 \text{ m/s}}$$

$$\Delta v_2 = v_f - v_{2_i} = (+5.00 \text{ m/s}) - (-15.0 \text{ m/s}) = \boxed{+20.0 \text{ m/s}}$$

$$KE_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2$$

$$= \frac{1}{2}(1800 \text{ kg})(+15.0 \text{ m/s})^2 + \frac{1}{2}(900 \text{ kg})(-15.0 \text{ m/s})^2$$

$$= 3.04 \times 10^5 \text{ J}$$

$$KE_f = \frac{1}{2}(m_1 + m_2)v_f^2$$

$$= \frac{1}{2}(2700 \text{ kg})(+5.00 \text{ m/s})^2$$

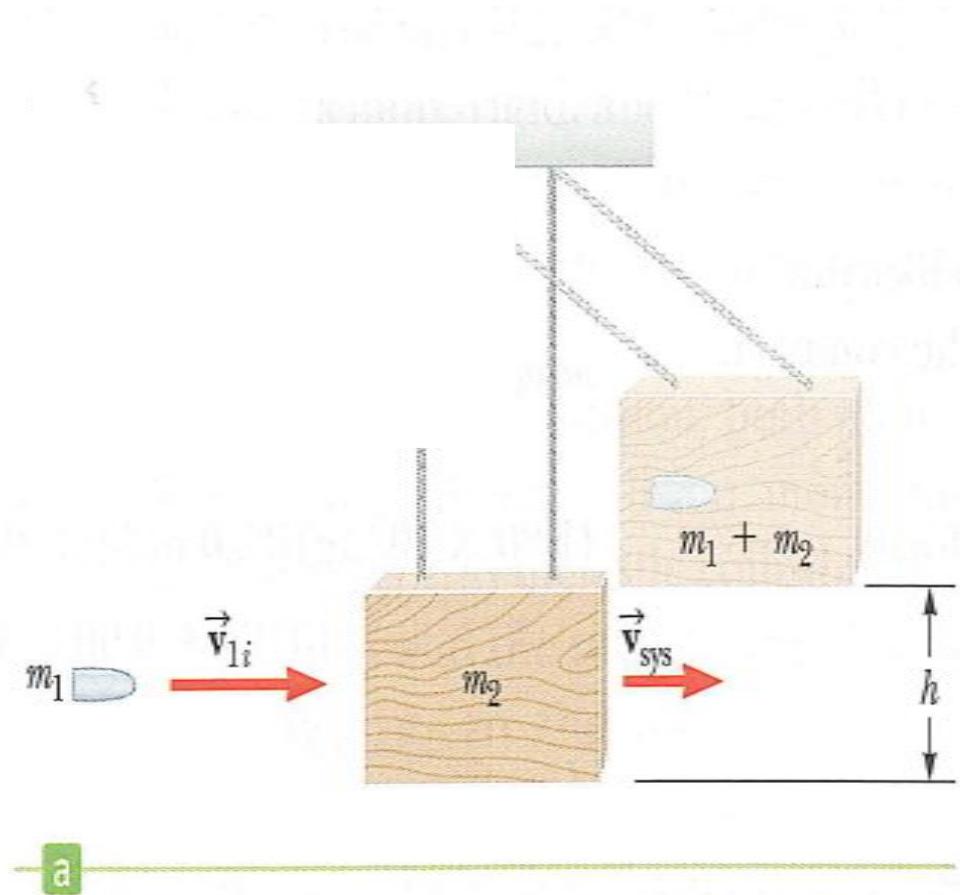
$$= 3.38 \times 10^4 \text{ J}$$

$$\Delta KE = KE_f - KE_i = (3.38 \times 10^4 \text{ J}) - (3.04 \times 10^5 \text{ J})$$

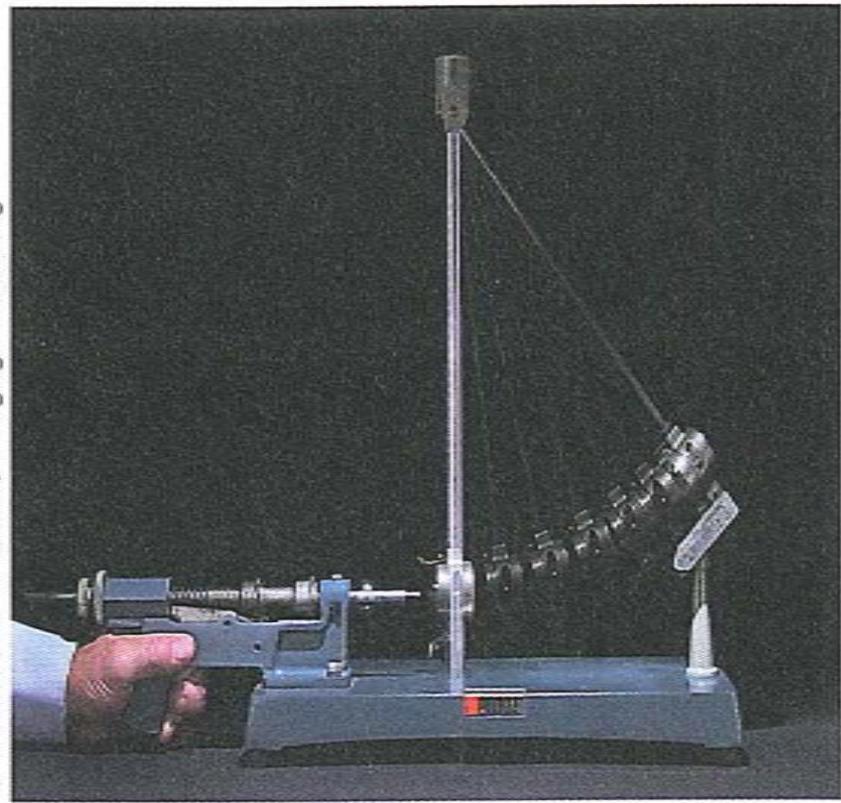
$$\Delta KE = \boxed{-2.70 \times 10^5 \text{ J}}$$

The ballistic pendulum (*as shown in lecture Figure 6.12*) is a device used to measure the speed of a fast-moving projectile such as a bullet. The bullet is fired into a large block of wood suspended from some light wires. The bullet is stopped by the block, and the entire system swings up to a height  $h$ . It is possible to obtain the initial speed of the bullet by measuring  $h$  and the two masses. As an example of the technique, assume that the mass of the bullet,  $m_1$ , is 5.00 g, the mass of the pendulum,  $m_2$ , is 1.000 kg, and  $h$  is 5.00 cm.

- (a) Find the velocity of the system after the bullet embeds in the block.
- (b) Calculate the initial speed of the bullet.



a



b

Photos/Illustrations courtesy of *College Physics*, 9<sup>th</sup> edition.

$$(KE + PE)_{\text{after collision}} = (KE + PE)_{\text{top}}$$

$$\frac{1}{2}(m_1 + m_2)v_{\text{sys}}^2 + 0 = (m_1 + m_2)gh$$

$$\therefore v_{\text{sys}}^2 = 2gh \quad \therefore v_{\text{sys}} = \sqrt{2(9.80 \text{ m/s}^2)(0.0500 \text{ m})}$$

$$\therefore v_{\text{sys}} = \boxed{0.990 \text{ m/s}}$$

$$p_i = p_f$$

$$m_1 v_{1_i} + m_2 v_{2_i} = (m_1 + m_2) v_{sys}$$

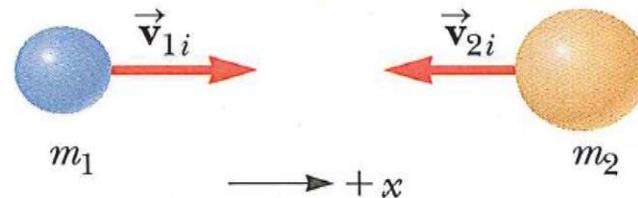
$$\therefore v_{1_i} = \frac{(m_1 + m_2) v_{sys}}{m_1} = \frac{(1.000 \text{ kg} + 0.00500 \text{ kg})(0.990 \text{ m/s})}{(0.00500 \text{ kg})}$$

$$\therefore v_{1_i} = \boxed{199 \text{ m/s}}$$

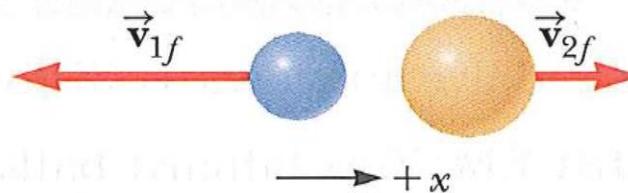
Two billiard balls of identical mass move toward each other (*as in lecture Figure 6.13*), with the positive  $x$ -axis to the right. Assume that the collision between them is perfectly elastic. If the initial velocities of the balls are +30.0 cm/s and -20.0 cm/s,

**(a) & (b)** what are the velocities of the balls after the collision? Assume friction and rotation are unimportant.

Before an elastic collision  
the two objects move  
independently.



After the collision the object  
velocities change, but *both* the  
energy and momentum of the  
system are conserved.



**Figure 6.13**

Photos/Illustrations courtesy of *College Physics*, 9<sup>th</sup> edition.

$$m_1 v_{1_i} + m_2 v_{2_i} = m_1 v_{1_f} + m_2 v_{2_f} \quad \text{***} \\ m_1 = m_2$$

$$( +30.0 \text{ cm/s}) + (-20.0 \text{ cm/s}) = v_{1_f} + v_{2_f}$$

$$\therefore ( +10.0 \text{ cm/s}) = v_{1_f} + v_{2_f}$$

$$v_{1_i} - v_{2_i} = -(v_{1_f} - v_{2_f})$$

$$( +30.0 \text{ cm/s}) - (-20.0 \text{ cm/s}) = v_{2_f} - v_{1_f}$$

$$( +50.0 \text{ cm/s}) = v_{2_f} - v_{1_f}$$

$$(+10.0 \text{ cm/s}) + (+50.0 \text{ cm/s}) = (v_{1_f} + v_{2_f}) + (v_{2_f} - v_{1_f})$$

$$(+60.0 \text{ cm/s}) = 2v_{2_f} \quad \therefore v_{2_f} = \boxed{+30.0 \text{ cm/s}}$$

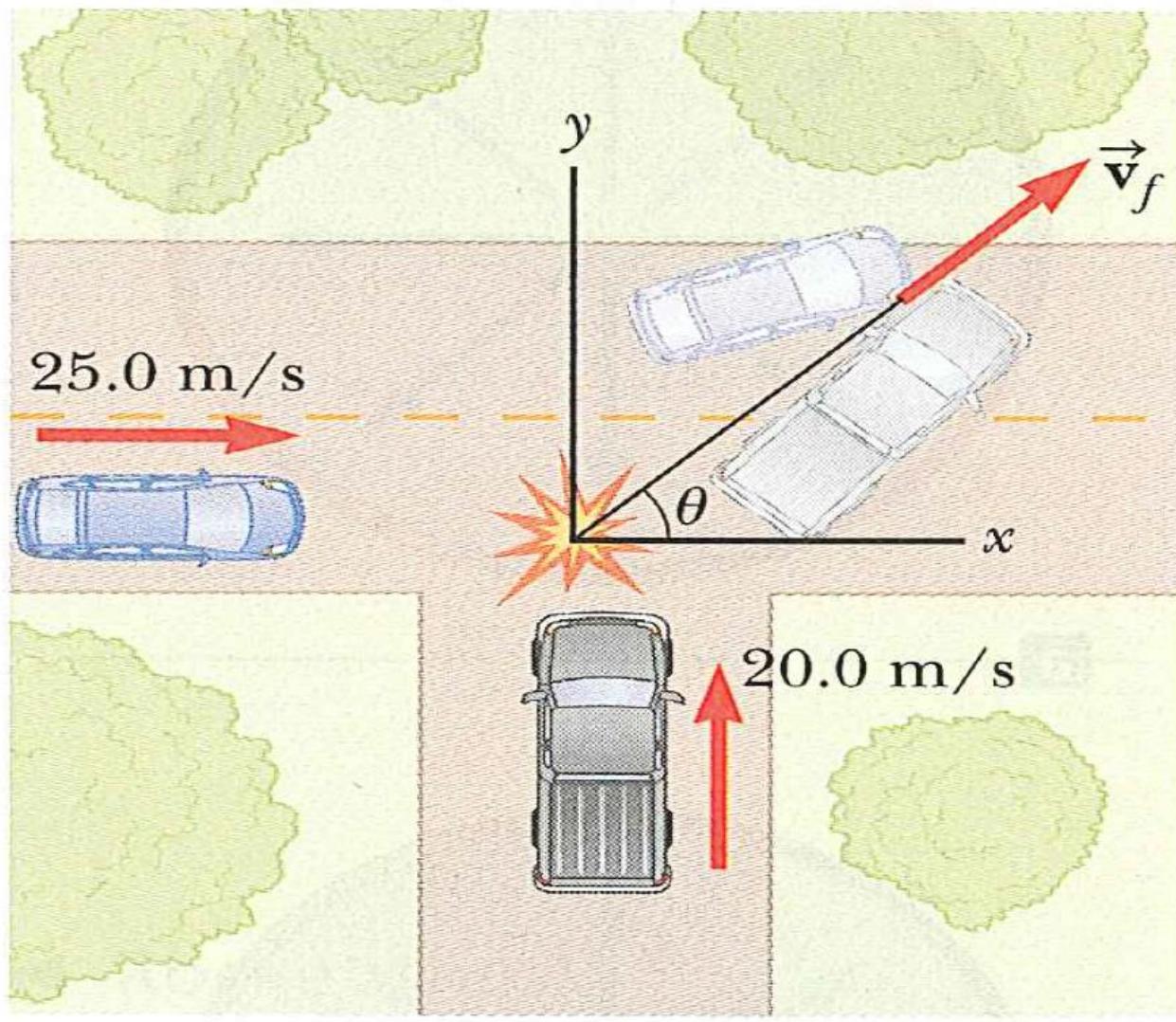
$$(+10.0 \text{ cm/s}) = v_{1_f} + (+30.0 \text{ cm/s})$$

$$\therefore v_{1_f} = \boxed{-20.0 \text{ cm/s}}$$

A car with mass  $1.50 \times 10^3$  kg traveling east at a speed of 25.0 m/s collides at an intersection with a  $2.50 \times 10^3$  kg van traveling north at a speed of 20.0 m/s, (*as shown in lecture Figure 6.16* ).

(a) Find the magnitude

and (b) direction of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision and assuming that friction between the vehicles and the road can be neglected.



**Figure 6.16**

Photos/Illustrations courtesy of *College Physics*, 9<sup>th</sup> edition.

$$\Sigma p_{x_i} = m_{car} v_{car} = (1500 \text{ kg})(25.0 \text{ m/s}) = 37,500 \text{ kg}\cdot\text{m/s}$$

$$\Sigma p_{x_f} = (m_{car} + m_{van}) v_f (\cos \theta)$$

$$\therefore \Sigma p_{x_f} = (1500 \text{ kg} + 2500 \text{ kg}) v_f (\cos \theta)$$

$$\therefore \Sigma p_{x_f} = (4000 \text{ kg}) v_f (\cos \theta)$$

$$\Sigma p_{x_i} = \Sigma p_{x_f}$$

$$\therefore 37,500 \text{ kg}\cdot\text{m/s} = (4000 \text{ kg}) v_f (\cos \theta) \quad [1]$$

$$\Sigma p_{y_i} = m_{van} v_{van} = (2500 \text{ kg})(20.0 \text{ m/s}) = 50,000 \text{ kg}\cdot\text{m/s}$$

$$\Sigma p_{y_f} = (m_{car} + m_{van}) v_f (\sin \theta)$$

$$\therefore \Sigma p_{x_f} = (1500 \text{ kg} + 2500 \text{ kg}) v_f (\sin \theta)$$

$$\therefore \Sigma p_{x_f} = (4000 \text{ kg}) v_f (\sin \theta)$$

$$\Sigma p_{y_i} = \Sigma p_{y_f}$$

$$\therefore 50,000 \text{ kg}\cdot\text{m/s} = (4000 \text{ kg}) v_f (\sin \theta) \quad [2]$$

$$\frac{[2]}{[1]} = \frac{(4000 \text{ kg})v_f(\sin \theta)}{(4000 \text{ kg})v_f(\cos \theta)} = \frac{50,000 \text{ kg} \cdot \text{m/s}}{37,500 \text{ kg} \cdot \text{m/s}}$$

$$\tan \theta = 1.33 \quad \therefore \theta = \boxed{53.1^\circ}$$

$$50,000 \text{ kg} \cdot \text{m/s} = (4000 \text{ kg})v_f(\sin \theta)$$

$$50,000 \text{ kg} \cdot \text{m/s} = (4000 \text{ kg})v_f(\sin 53.1^\circ)$$

$$\therefore v_f = \frac{(50,000 \text{ kg} \cdot \text{m/s})}{(4000 \text{ kg})(\sin 53.1^\circ)} = \boxed{15.6 \text{ m/s}}$$