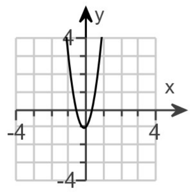
Student: Cole Lamers **Date:** 09/25/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.6 Newton's Method

Use Newton's method to estimate the solutions of the equation $5x^2 + x - 1 = 0$. Start with $x_0 = -1$ for the left solution and $x_0 = 1$ for the right solution. Find x_2 in each case.

Newton's method uses a 'guess' at a solution, perhaps from a graph, to get the first approximation termed x_0. Successive approximations, $x_1, x_2, \ldots, x_n, x_{n+1}$ are calculated by $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$.

The function $f(x) = 5x^2 + x - 1$ is graphed below.



Because Newton's method involves a lot of computation, the 'guess' values, x_0, are chosen to simplify the computation x_1.

Appropriate choices for x_0 for this exercise are -1 for the left zero of f(x) and 1 for the right zero of f(x).

f(x) is $5x^2 + x - 1$

$f'(x) = 10x + 1$

x_1 for the left 0 of f(x) is $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = -1 - \dfrac{5(-1)^2 + 1(-1) - 1}{10(-1) + 1} = -\dfrac{2}{3}$.

Substituting $x_1 = -\dfrac{2}{3}$ into $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} \approx -0.5686$.

x_1 for the right 0 of f(x) is $x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = \dfrac{6}{11}$.

The third approximation, rounded to four decimal places, x_2, such that $f(x_2) = 0$, is

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{6}{11} - \frac{(5)\left(\frac{6}{11}\right)^2 + (1)\left(\frac{6}{11}\right) - 1}{(10)\left(\frac{6}{11}\right) + (1)} \approx 0.3854.$$

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Use Newton's method to estimate the one real solution of $x^3 + 7x + 3 = 0$. Start with $x_0 = 0$ and then find x_2.

To use Newton's method, start with an approximation to a solution, x_0. Then, use the first approximation to get a second, the second to get a third, and so on, using the formula below.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \text{ if } f'(x_n) \neq 0$$

First determine $f'(x)$.

$$\begin{aligned} f(x) &= x^3 + 7x + 3 \\ f'(x) &= 3x^2 + 7 \end{aligned}$$

Now evaluate $f(x_0)$ and $f'(x_0)$ where $x_0 = 0$.

$$\begin{aligned} f(x) &= x^3 + 7x + 3 \\ f(0) &= (0)^3 + 7(0) + 3 \\ f(0) &= 3 \end{aligned}$$

and

$$\begin{aligned} f'(x) &= 3x^2 + 7 \\ f'(0) &= 3(0)^2 + 7 \\ f'(0) &= 7 \end{aligned}$$

Use the formula to determine x_1.

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 0 - \frac{(3)}{(7)} \\ &= -\frac{3}{7} \end{aligned}$$

Now use the value of x_1 to calculate x_2. Substitute and simplify.

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= -\frac{3}{7} - \frac{(-3/7)^3 + 7(-3/7) + 3}{3(-3/7)^2 + 7} \\ &= -\frac{1083}{2590} \\ &\approx -0.4181 \end{aligned}$$

The one real solution of $x^3 + 7x + 3 = 0$ is approximately -0.4181.

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Use Newton's method to estimate the two zeros of the function $f(x) = x^4 + 2x - 7$. Start with $x_0 = -1$ for the left-hand zero and with $x_0 = 1$ for the zero on the right. Then, in each case, find x_2.

To use Newton's method, start with an approximation to a solution, x_0. Then, use the first approximation to get a second, the second to get a third, and so on, using the formula below.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \text{ if } f'(x_n) \neq 0$$

First determine $f'(x)$.

$$\begin{aligned} f(x) &= x^4 + 2x - 7 \\ f'(x) &= 4x^3 + 2 \end{aligned}$$

To find the zero on the left, evaluate $f(x_0)$ and $f'(x_0)$ where $x_0 = -1$.

$$\begin{aligned} f(x) &= x^4 + 2x - 7 \\ f(-1) &= (-1)^4 + 2(-1) - 7 \\ f(-1) &= -8 \end{aligned} \qquad \text{and} \qquad \begin{aligned} f'(x) &= 4x^3 + 2 \\ f'(-1) &= 4(-1)^3 + 2 \\ f'(-1) &= -2 \end{aligned}$$

Use the formula to determine x_1.

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= -1 - \frac{(-8)}{(-2)} \\ &= -5 \end{aligned}$$

Now use the value of x_1 to calculate x_2. Substitute and simplify. Round to four decimal places.

$$\begin{aligned} x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &= -5 - \frac{(-5)^4 + 2(-5) - 7}{4(-5)^3 + 2} \\ &\approx -3.7791 \end{aligned}$$

The zero on the left for $x^4 + 2x - 7 = 0$ is approximately $x = -3.7791$.

To find the zero on the right, evaluate $f(x_0)$ and $f'(x_0)$ where $x_0 = 1$.

$$\begin{aligned} f(x) &= x^4 + 2x - 7 \\ f(1) &= (1)^4 + 2(1) - 7 \\ f(1) &= -4 \end{aligned} \qquad \text{and} \qquad \begin{aligned} f'(x) &= 4x^3 + 2 \\ f'(1) &= 4(1)^3 + 2 \\ f'(1) &= 6 \end{aligned}$$

Use the formula to determine x_1. Round to four decimal places.

$$
\begin{aligned}
x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\
&= 1 - \frac{(-4)}{(6)} \\
&\approx 1.6667
\end{aligned}
$$

Now use the value of x_1 to calculate x_2. Substitute and simplify. Round to four decimal places.

$$
\begin{aligned}
x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\
&= 1.6667 - \frac{(1.6667)^4 + 2(1.6667) - 7}{4(1.6667)^3 + 2} \\
&\approx 1.4693
\end{aligned}
$$

The zero on the right for $x^4 + 2x - 7 = 0$ is approximately $x = 1.4693$.

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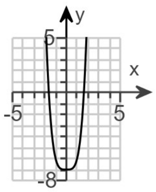
Use Newton's method to obtain the third approximation, x_2, of the positive fourth root of 7 by calculating the third approximation of the right 0 of $f(x) = x^4 - 7$. Start with $x_0 = 1$.

Newton's method uses a 'guess' at a solution, perhaps from a graph, to get the first approximation termed x_0. Successive approximations, $x_1, x_2, \ldots, x_n, x_{n+1}$ are calculated by $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$.

The function $f(x) = x^4 - (7)$ is graphed below.



Because Newton's method involves a lot of computation, the 'guess' value, x_0 is chosen to simplify the computation of x_1.

An appropriate choice for x_0 is 1.

$f(x)$ is $x^4 - 7$

$f'(x) = 4x^3$

x_1 is $x_0 - \dfrac{f(x_0)}{f'(x_0)} = 1 - \dfrac{(1)^4 - 7}{4(1)^3} = 2.5$.

Substituting $x_1 = 2.5$ into $x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 1.987$.