Student: Cole Lamers **Date:** 09/21/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 4.2 The Mean Value Theorem

1. Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for the following function and interval.

$f(x) = 4x^2 - 4x - 3, \qquad [-1,2]$

The value(s) of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ is/are $\frac{1}{2}$.

(Type a simplified fraction. Use a comma to separate answers as needed.)

2. Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the mean value theorem for the given function and interval.

$f(x) = x + \frac{1}{x}, \left[\frac{1}{13}, 13\right]$

c = 1 (Use a comma to separate answers as needed.)

3. Show that the function $f(x) = x^4 + 5x + 3$ has exactly one zero in the interval $[-1, 0]$.

Which theorem can be used to determine whether a function f(x) has any zeros in a given interval?

- **A.** Extreme value theorem
- **B.** Rolle's Theorem
- **C.** Intermediate value theorem
- **D.** Mean value theorem

To apply this theorem, evaluate the function $f(x) = x^4 + 5x + 3$ at each endpoint of the interval $[-1, 0]$.

$f(-1) =$ -1 (Simplify your answer.)

$f(0) =$ 3 (Simplify your answer.)

According to the intermediate value theorem, $f(x) = x^4 + 5x + 3$ has at least one zero in the given interval.

Now, determine whether there can be more than one zero in the given interval.

Rolle's Theorem states that for a function f(x) that is continuous at every point over the closed interval [a,b] and differentiable at every point of its interior (a,b), if f(a) = f(b), then there is at least one number c in (a,b) at which $f'(c) = 0$.

Find the derivative of $f(x) = x^4 + 5x + 3$.

$f'(x) = 4x^3 + 5$

Can the derivative of f(x) be zero in the interval $[-1, 0]$?

- Yes
- No

The function $f(x) = x^4 + 5x + 3$ has at least one zero at some point x = a in the interval $[-1, 0]$. According to Rolle's Theorem, can there be another point x = b in this interval where f(a) = f(b) = 0?

- Yes
- No

Thus, since the intermediate value theorem shows that $f(x) = x^4 + 5x + 3$ has at least one zero in the interval $[-1, 0]$ and Rolle's Theorem shows that there cannot be two points x = a and x = b for which f(a) = f(b) in this interval, the function f(x) has exactly one zero in the interval $[-1, 0]$.

4. Show that the function $r(\theta) = \theta + \sin^2\left(\frac{\theta}{3}\right) - 8$ has exactly one zero in the interval $(-\infty,\infty)$.

Rolle's Theorem states that for a function f(x) that is continuous at every point over the closed interval [a,b] and differentiable at every point of its interior (a,b), if f(a) = f(b), then there is at least one number c in (a,b) at which $f'(c) = 0$.

Find the derivative of $r(\theta) = \theta + \sin^2\left(\frac{\theta}{3}\right) - 8$.

$r'(\theta) = 1 + \frac{1}{3}\sin\left(\frac{2\theta}{3}\right)$

Can the derivative of $r(\theta)$ be zero in the interval $(-\infty,\infty)$?

According to Rolle's Theorem, since the derivative of $r(\theta)$ is never zero in the interval $(-\infty,\infty)$, there cannot be two points $\theta = a$ and $\theta = b$ for which $r(a) = r(b)$ in this interval. In other words, the function $r(\theta)$ has at most one zero in the interval $(-\infty,\infty)$.

Which theorem can be used to determine whether a function has any zeros in a given closed interval?

- **A.** Intermediate value theorem
- **B.** Extreme value theorem
- **C.** Mean value theorem

To apply this theorem, evaluate the function $r(\theta) = \theta + \sin^2\left(\frac{\theta}{3}\right) - 8$ at each endpoint of the interval $[-3\pi,3\pi]$.

$r(-3\pi) =$ $-3\pi - 8$ (Type an exact answer, using π as needed.)
$r(3\pi) =$ $3\pi - 8$ (Type an exact answer, using π as needed.)

According to the intermediate value theorem, $r(\theta) = \theta + \sin^2\left(\frac{\theta}{3}\right) - 8$ has at least one zero in the interval $[-3\pi,3\pi]$.

If $r(\theta)$ has at least one zero in the interval $[-3\pi,3\pi]$, and this interval is fully contained within the interval $(-\infty,\infty)$, then $r(\theta)$ has at least one zero in the interval $(-\infty,\infty)$.

Thus, since the intermediate value theorem shows that $r(\theta) = \theta + \sin^2\left(\frac{\theta}{3}\right) - 8$ has at least one zero in the interval $(-\infty,\infty)$ and Rolle's Theorem shows that $r(\theta)$ has at most one zero in this interval, the function $r(\theta)$ has exactly one zero in the interval $(-\infty,\infty)$.

5. Suppose that $f(-1) = 4$ and that $f'(x) = 0$ for all x. Must $f(x) = 4$ for all x? Give reasons for your answer.

- **A.** No. Since $f(-1) = 4$, f is a constant function with slope 4. The value of f is different for all values of x.
- **B.** Yes. Since $f'(x) = 0$ for all x, and 0 is a constant, the value of f equals $f(-1)$ for all values of x.
- **C.** No. Since $f(-1) = 4$ is greater than -1, f(x) is greater than x for all values of x.
- **D.** Yes. Since $f'(x) = 0$ for all x, f is a constant function. The value of f is the same for all values of x.

6. Find all possible functions with the given derivative.

$f'(x) = x^6$

$f(x) = \frac{x^7}{7} + c$

(Use C as the arbitrary constant.)

7. Find all possible functions with the given derivative.

$f'(t) = \sin 6t + \cos \frac{t}{7}$

$f(t) = -\frac{1}{6}\cos 6t + 7\sin\left(\frac{t}{7}\right) + C$

(Use C as the arbitrary constant.)

8. Find the function with the given derivative whose graph passes through the point P.

$f'(x) = 2x - 3, \quad P(5,3)$

The function with the given derivative whose graph passes through the point P is $f(x) = x^2 - 3x - 7$.

9. Given the velocity $v = \frac{ds}{dt}$ and the initial position of a body moving along a coordinate line, find the body's position at time t.

$v = 9.8t + 15,\ s(0) = 20$

$s(t) = 4.9t^2 + 15t + 20$

10. Given the velocity $v = \frac{ds}{dt}$ and initial position of a body moving along a coordinate line, find the body's position at time t.

$v = \sin(\pi t), \quad s(5) = 0$

The body's position at time t is $s = \frac{-\cos(\pi t)}{\pi} - \frac{1}{\pi}$.

(Type an exact answer.)

11. Consider the following acceleration $a = \frac{d^2s}{dt^2}$, initial velocity, and initial position of an object moving on a number line. Find the object's position at time t.

$a = 32,\ v(0) = 15,\ s(0) = 3$

The position of the object at time t is given by $s = 16t^2 + 15t + 3$.

12. Given the acceleration $a = \frac{d^2s}{dt^2}$, initial velocity, and initial position of a body moving on a coordinate line, find the body's position at time t.

$a = -9 \sin 3t$, $v(0) = 3$, $s(0) = -10$

$s(t) =$ $\sin(3t) - 10$