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| **Student:** Cole Lamers<br>**Date:** 09/22/19 | **Instructor:** Viktoriya Shcherban<br>**Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban | **Assignment:** 4.3 Monotonic Functions and the First Derivative T |

Answer the following questions about the function whose derivative is $f'(x) = 4x(x-6)$.
**a.** What are the critical points of f?
**b.** On what open intervals is f increasing or decreasing?
**c.** At what points, if any, does f assume local maximum and minimum values?

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**a.** An interior point of the domain of a function f where f′ is zero or undefined is a critical point of f.

Set $f'(x) = 0$ and solve.

$$\begin{aligned} f'(x) &= 0 \\ 4x(x-6) &= 0 \\ x &= 0, 6 \end{aligned}$$

Notice that f′ is a polynomial, which is defined for all real numbers. So there are no values of x where the derivative is undefined.

Thus, the critical points of f are $x = 0$ and $x = 6$.

**b.** To determine on what open intervals f is increasing or decreasing, use the critical points to subdivide the domain into nonoverlapping open intervals in which f′ is either positive or negative.

The open intervals are $(-\infty,0)$, $(0,6)$, and $(6,\infty)$.

Suppose that f is continuous on [a,b] and differentiable on (a,b). If $f'(x) > 0$ at each point $x \in (a,b)$, then f is increasing on [a,b]. If $f'(x) < 0$ at each point $x \in (a,b)$, then f is decreasing on [a,b].

Determine the sign of f′ by evaluating f′ at a convenient point in each interval. For the interval $(-\infty,0)$, evaluate $f'(-1)$.

$$\begin{aligned} f'(-1) &= 4(-1)(-1-6) \\ &= 28 \end{aligned}$$

So, the function is increasing on the open interval $(-\infty,0)$.

Now, for the interval (0,6), evaluate $f'(1)$.

$$\begin{aligned} f'(1) &= 4(1)(1-6) \\ &= -20 \end{aligned}$$

So, the function is decreasing on the open interval (0,6).

Finally, for the interval $(6,\infty)$, evaluate $f'(7)$.

$$\begin{aligned} f'(7) &= 4(7)(7-6) \\ &= 28 \end{aligned}$$

So, the function is increasing on the open interval $(6,\infty)$.

Thus, the function f is increasing on the open intervals $(-\infty,0)$ and $(6,\infty)$, and it is decreasing on the open interval (0,6).

**c.** To determine the location of local maxima and minima, suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right, if f′ changes from negative to positive at c, then f has a local minimum at c. If f′ changes from positive to negative at c, then f has a local maximum at c.

Recall that the sign of f′ changes from positive to negative at $x = 0$. So f has a local maximum at $x = 0$.

Recall that the sign of $f'$ changes from negative to positive at $x = 6$. So f has a local minimum at $x = 6$.

Therefore, the function f has a local maximum at $x = 0$ and a local minimum at $x = 6$.

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Answer the following questions about the function whose derivative is $f'(x) = (x-9)^2(x+10)$.
**a.** What are the critical points of f?
**b.** On what open intervals is f increasing or decreasing?
**c.** At what points, if any, does f assume local maximum and minimum values?

**a.** An interior point of the domain of a function f where f′ is zero or undefined is a critical point of f.

Set $f'(x) = 0$ and solve.

$$\begin{aligned} f'(x) &= 0 \\ (x-9)^2(x+10) &= 0 \\ x &= 9, -10 \end{aligned}$$

Notice that f′ is a polynomial, which is defined for all real numbers. So there are no values of x where the derivative is undefined.

Thus, the critical points of f are $x = 9$ and $x = -10$.

**b.** To determine on what open intervals f is increasing or decreasing, use the critical points to subdivide the domain into nonoverlapping open intervals in which f′ is either positive or negative.

The two critical points split the domain of the function into three nonoverlapping open intervals, $(-\infty,-10)$, $(-10,9)$, and $(9,\infty)$.

Suppose that f is continuous on [a,b] and differentiable on (a,b). If $f'(x) > 0$ at each point $x \in (a,b)$, then f is increasing on [a,b]. If $f'(x) < 0$ at each point $x \in (a,b)$, then f is decreasing on [a,b].

Determine the sign of f′ by evaluating f′ at a convenient point in each interval. For the interval $(-\infty,-10)$, evaluate $f'(-11)$.

$$\begin{aligned} f'(-11) &= (-11-9)^2(-11+10) \\ &= -400 \end{aligned}$$

So, the function is decreasing on the open interval $(-\infty,-10)$.

Now, for the interval $(-10,9)$, evaluate $f'(0)$.

$$\begin{aligned} f'(0) &= (0-9)^2(0+10) \\ &= 810 \end{aligned}$$

So, the function is increasing on the open interval $(-10,9)$.

Finally, for the interval $(9,\infty)$, evaluate $f'(10)$.

$$\begin{aligned} f'(10) &= (10-9)^2(10+10) \\ &= 20 \end{aligned}$$

So, the function is increasing on the open interval $(9,\infty)$.

Note that the sign of f′ changes at the critical point $x = -10$ but does not change at the critical point $x = 9$ that separates the open intervals $(-10,9)$ and $(9,\infty)$. Therefore, there is a local extremum at $x = -10$ but no local extremum at $x = 9$, indicating that the function f is decreasing on the open interval $(-\infty,-10)$ and increasing over the open interval $(-10,\infty)$.

**c.** To determine the location of local maxima and minima, suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right, if f′ changes from negative to positive at c, then f has a local minimum at c. If f′ changes from positive to negative at c, then f has a local maximum at c.

Recall that the sign of $f'$ changes from negative to positive at $x = -10$. so f has a local minimum at $x = -10$.

Recall that the sign of $f'$ does not change sign at $x = 9$. So f does not have an extreme value at $x = 9$.

Therefore, the function f has a local minimum at $x = -10$ and no extreme value at $x = 9$.

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| **Student:** Cole Lamers<br>**Date:** 09/22/19 | **Instructor:** Viktoriya Shcherban<br>**Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban | **Assignment:** 4.3 Monotonic Functions and the First Derivative T |

Answer the questions below about the function whose derivative is $f'(x) = \frac{(x-5)(x+7)}{(x+4)(x-6)}$, $x \neq -4, 6$.

**a.** What are the critical points of f?
**b.** On what open intervals is f increasing or decreasing?
**c.** At what points, if any, does f assume local maximum and minimum values?

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**a.** An interior point of the domain of a function f where f′ is zero or undefined is a critical point of f.

Notice that f′ is a rational function. This means that f′ is equal to zero whenever the numerator is zero and f′ is undefined whenever the denominator is zero.

Find all values of x that make the numerator zero.

$$x = -7, 5$$

The value $x = -7$ is in the domain of f because it does not cause a denominator to equal zero or the radicand of any even roots to be negative.

The value $x = 5$ is in the domain of f because it does not cause a denominator to equal zero or the radicand of any even roots to be negative.

Find all values of x that make the denominator of f′ zero.

$$x = -4, 6$$

The value $x = -4$ is not in the domain of f because it is given in the problem statement that $x \neq -4$.

The value $x = 6$ is not in the domain of f because it is given in the problem statement that $x \neq 6$.

Thus, the critical points of f are at $x = -7$ and $x = 5$.

**b.** Suppose that f is continuous on [a,b] and differentiable on (a,b). If $f'(x) > 0$ at each point $x \in (a,b)$, then f is increasing on [a,b]. If $f'(x) < 0$ at each point $x \in (a,b)$, then f is decreasing on [a,b].

Recall that $f'(x)$ is either zero or undefined at $x = -7, -4, 5$, and 6. This means that the open intervals to examine are $(-\infty, -7)$, $(-7,-4)$, $(-4,5)$, $(5,6)$, and $(6,\infty)$. Select a point in each interval to determine if $f'(x)$ is greater than zero or less than zero.

For $(-\infty, -7)$, let $x = -8$. Substitute $-8$ for x in $f'(x)$ and simplify.

$$\frac{(-8-5)(-8+7)}{(-8+4)(-8-6)} = \frac{13}{56}$$

This value is greater than zero, so f(x) is increasing on $(-\infty, -7)$.

For $(-7,-4)$, let $x = -6$. Substitute $-6$ for x in $f'(x)$ and simplify.

$$\frac{(-6-5)(-6+7)}{(-6+4)(-6-6)} = -\frac{11}{24}$$

This value is less than zero, so f(x) is decreasing on $(-7,-4)$.

For $(-4,5)$, let $x = 0$. Substitute 0 for x in $f'(x)$ and simplify.

$$\frac{(0-5)(0+7)}{(0+4)(0-6)} = \frac{35}{24}$$

This value is greater than zero, so f(x) is increasing on $(-4,5)$.

For (5,6), let x = 5.5. Substitute 5.5 for x in $f'(x)$ and simplify.

$$\frac{(5.5-5)(5.5+7)}{(5.5+4)(5.5-6)} = -\frac{25}{19}$$

This value is less than zero, so f(x) is decreasing on (5,6).

For $(6,\infty)$, let x = 7. Substitute 7 for x in $f'(x)$ and simplify.

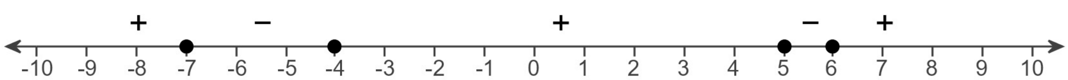
$$\frac{(7-5)(7+7)}{(7+4)(7-6)} = \frac{28}{11}$$

This value is greater than zero, so f(x) is increasing on $(6,\infty)$.

Thus, f(x) is increasing on $(-\infty,-7)$, $(-4,5)$, and $(6,\infty)$ and decreasing on $(-7,-4)$ and (5,6).

**c.** Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right, if f′ changes from positive to negative at c, then f has a local maximum at c. If f′ changes from negative to positive at c, then f has a local minimum at c.

Recall that the critical points of f are at $x = -7$ and x = 5. The number line below shows the values where f′ is zero or undefined, as well as the sign of f′ on the intervals between those values.




Since f′ changes from positive to negative at $x = -7$, f has a local maximum at $x = -7$.

Since f′ changes from positive to negative at x = 5, f has a local maximum at x = 5.

Thus, f has local maxima at $x = -7$ and x = 5. Notice that f′ changes from negative to positive at $x = -4$ and x = 6, yet the values are not minima since they are not in the domain of f. Thus, f has no local minima.

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Answer the following questions about the function whose derivative is given below.
**(a)** What are the critical points of f?
**(b)** On what open intervals is f increasing or decreasing?
**(c)** At what points, if any, does f assume local maximum and minimum values?

$$f'(x) = (\sin x - 1)(2\cos x - 1),\ 0 \le x \le 2\pi$$

**(a)** The critical points of a function f are points in the interior of its domain where $f'$ is zero or undefined.

The derivative $f'(x) = (\sin x - 1)(2\cos x - 1)$ is defined for all real x because both sine and cosine are defined for all real numbers.

Therefore, the critical points must be where $(\sin x - 1)(2\cos x - 1)$ is zero. Set each factor equal to zero and solve for x in the given domain, $0 \le x \le 2\pi$. Start with the first factor.

$$\sin x - 1 = 0$$
$$x = \frac{\pi}{2}$$

Then set $2\cos x - 1$ equal to 0 and solve for x in the given domain.

$$x = \frac{\pi}{3} \text{ and } \frac{5\pi}{3}$$

Therefore, the function has critical points at $\frac{\pi}{3}$, $\frac{\pi}{2}$, and $\frac{5\pi}{3}$.

**(b)** The three critical points divide the domain $0 \le x \le 2\pi$ into four intervals. If a function f is continuous on [a,b] and differentiable on (a,b) and $f'(x) > 0$ for each point $x \in (a,b)$, then f is increasing on [a,b]. If $f'(x) < 0$ for each point $x \in (a,b)$, then f is decreasing on [a,b].

For each interval, evaluate $f'(x)$ in that interval. Start with $x = \frac{\pi}{6}$ in the interval $\left(0, \frac{\pi}{3}\right)$.

$$f'\left(\frac{\pi}{6}\right) = -0.366$$

Do the same for a point in each of the remaining integrals.

| Interval | $f'$ evaluated | Sign of $f'$ | Behavior of $f'$ |
|---|---|---|---|
| $\left(0, \frac{\pi}{3}\right)$ | $f'\left(\frac{\pi}{6}\right) = -0.366$ | | |
| $\left(\frac{\pi}{3}, \frac{\pi}{2}\right)$ | $f'\left(\frac{5\pi}{12}\right) = 0.016$ | | |
| $\left(\frac{\pi}{2}, \frac{5\pi}{3}\right)$ | $f'\left(\frac{13\pi}{12}\right) = 3.691$ | | |
| $\left(\frac{5\pi}{3}, 2\pi\right)$ | $f'\left(\frac{11\pi}{6}\right) = -1.098$ | | |

Then determine if the function is increasing or decreasing on each interval based on the sign of the derivative.

| Interval | f′ evaluated | Sign of f′ | Behavior of f′ |
|---|---|---|---|
| $\left(0,\frac{\pi}{3}\right)$ | $f'\left(\frac{\pi}{6}\right) = -0.366$ | − | decreasing |
| $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$ | $f'\left(\frac{5\pi}{12}\right) = 0.016$ | + | increasing |
| $\left(\frac{\pi}{2},\frac{5\pi}{3}\right)$ | $f'\left(\frac{13\pi}{12}\right) = 3.691$ | + | increasing |
| $\left(\frac{5\pi}{3},2\pi\right)$ | $f'\left(\frac{11\pi}{6}\right) = -1.098$ | − | decreasing |

Therefore, the function is increasing on $\left(\frac{\pi}{3},\frac{\pi}{2}\right)$, $\left(\frac{\pi}{2},\frac{5\pi}{3}\right)$ and decreasing on $\left(0,\frac{\pi}{3}\right)$, $\left(\frac{5\pi}{3},2\pi\right)$.

**(c)** Now use these results to determine the nature of each critical point and the two end points. The function is decreasing on the right of $x = 0$.

The end point at $x = 0$ is a local maximum.

The function is decreasing on the left of $x = \frac{\pi}{3}$ and increasing on the right.

The critical point at $x = \frac{\pi}{3}$ is a local minimum.

The function is increasing on the left of $x = \frac{\pi}{2}$ and increasing on the right.

The critical point at $x = \frac{\pi}{2}$ is not a relative extremum.

The function is increasing on the left of $x = \frac{5\pi}{3}$ and decreasing on the right.

The critical point at $x = \frac{5\pi}{3}$ is a local maximum.

The function is decreasing on the left of $x = 2\pi$.

The end point at $x = 2\pi$ is a local minimum.

Therefore, the function has local maxima at $0, \frac{5\pi}{3}$ and local minima at $\frac{\pi}{3}, 2\pi$.

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**a.** Find the open intervals on which the function is increasing and decreasing.
**b.** Identify the function's local and absolute extreme values, if any, saying where they occur.

$$g(t) = 2t^2 - 3t - 4$$

**a.** To determine on what intervals the function is increasing or decreasing, first find the critical points.

An interior point of the domain of a function f where f′ is zero or undefined is a critical point.

Find $g'(t)$.

$$\begin{aligned} g(t) &= 2t^2 - 3t - 4 \\ g'(t) &= 4t - 3 \end{aligned}$$

Set $g'(t) = 4t - 3$ equal to zero and solve for t.

$$\begin{aligned} 4t - 3 &= 0 \\ t &= \frac{3}{4} \end{aligned}$$

There are no values of t where the derivative $g'(t)$ is undefined, so the only critical point of g is $t = \frac{3}{4}$.

Use the critical point to subdivide the domain into nonoverlapping open intervals in which g′ is either positive or negative.

The intervals are $\left(-\infty, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \infty\right)$.

Determine the sign of g′ by evaluating g at a convenient point in each subinterval. The behavior of g is determined by applying the corollary of the First Derivative Test for Monotonic Functions.

This corollary supposes that f is continuous on [a,b] and differentiable on (a,b).
If $f'(x) > 0$ at each point $x \in (a,b)$, then f is increasing on [a,b]. If $f'(x) < 0$ at each point $x \in (a,b)$, then f is decreasing on [a,b].

For the interval $\left(-\infty, \frac{3}{4}\right)$, evaluate $g'(0)$.

$$g'(0) = 4(0) - 3 = -3$$

Since $g'(t) < 0$, on the subinterval $\left(-\infty, \frac{3}{4}\right)$, the function g is decreasing.

For the interval $\left(\frac{3}{4}, \infty\right)$, evaluate $g'(1)$.

$$g'(1) = 4(1) - 3 = 1$$

Since $g'(t) > 0$, on the subinterval $\left(\frac{3}{4}, \infty\right)$, the function g is increasing.

Note that strict and non-strict inequalities can be used in the corollary above to specify the intervals on which the function is increasing and decreasing.

Thus, the function g is decreasing on the subinterval $\left(-\infty, \frac{3}{4}\right)$ and increasing on the subinterval $\left(\frac{3}{4}, \infty\right)$.

**b.** To identify the function's local extreme values, if any, use the First Derivative Test for Local Extrema.

The First Derivative Test for Local Extrema first supposes that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across c from left to right,
1. if $f'$ changes from negative to positive at c, then f has a local minimum at c;
2. if $f'$ changes from positive to negative at c, then f has a local maximum at c;
3. if $f'$ does not change sign at c (that is, $f'$ is positive on both sides of c or negative on both sides), then f has no local extremum.

Since the sign of $g'$ changes from negative to positive at $t = \frac{3}{4}$, g has a local minimum at $t = \frac{3}{4}$.

There is also an absolute minimum at $t = \frac{3}{4}$ because the function's values fall toward it from the left and rise away from it on the right.

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| **Student:** Cole Lamers<br>**Date:** 09/22/19 | **Instructor:** Viktoriya Shcherban<br>**Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban | **Assignment:** 4.3 Monotonic Functions and the First Derivative T |

**(a)** Find the open intervals on which the function $f(\theta) = 3\theta^2 - 10\theta^3$ is increasing and decreasing.
**(b)** Identify the function's local and absolute extreme values, if any, saying where they occur.

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To find the open intervals on which the function is increasing or decreasing, find the critical points of $f(\theta)$.

Differentiate $f(\theta)$ with respect to $\theta$.

$$f(\theta) = 3\theta^2 - 10\theta^3$$
$$f'(\theta) = 6\theta - 30\theta^2$$

An interior point of the domain of a function f where f′ is zero or undefined is a critical point of f.

Find the points where f′ is zero by equating it to zero.

The expression $6\theta - 30\theta^2$ is zero at $x = 0, \frac{1}{5}$.

Since there is no point at which the function is undefined, 0 and $\frac{1}{5}$ are the only critical points of f. These critical points subdivide the domain of f to create non-overlapping open intervals $(-\infty, 0)$, $\left(0, \frac{1}{5}\right)$, and $\left(\frac{1}{5}, \infty\right)$ on which f′ is either positive or negative.

Suppose that f is continuous on [a, b] and differentiable on (a, b). If $f' < 0$ at each point $x \in (a, b)$, then f is decreasing on [a, b]. If $> 0$ at each point $x \in (a, b)$, then f is increasing on [a, b].

The sign of f′ is negative on the interval $(-\infty, 0)$.

Thus, f is decreasing on the interval $(-\infty, 0)$.

The sign of f′ is positive on the interval $\left(0, \frac{1}{5}\right)$.

So, f is increasing on the interval $\left(0, \frac{1}{5}\right)$.

The sign of f′ is negative on the interval $\left(\frac{1}{5}, \infty\right)$.

So, f is decreasing on the interval $\left(\frac{1}{5}, \infty\right)$.

**(b)** A function f has an absolute maximum value at a point c within its domain D if $f(x) \le f(c)$ for all $x \in D$.

Note that the domain of f is all real numbers, and $f(\theta)$ decreases as $\theta$ increases. So, there is no absolute maximum.

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right, if f′ changes from positive to negative at c, then f has a local maximum at c.

Examine which are the consecutive intervals where f′ changes from positive to negative, and determine the local maximum.

The local maximum occurs at $\left(\frac{1}{5}, \frac{1}{25}\right)$.

A function f has an absolute minimum value at a point c within its domain D if $f(x) \ge f(c)$ for all $x \in D$.

Note that $f(\theta)$ decreases as $\theta$ increases. So, there is no absolute minimum.

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across this interval from left to right, if $f'$ changes from negative to positive at c, then f has a local minimum at c.

Examine which are the consecutive intervals where $f'$ changes from negative to positive, and determine the local minimum.

The local minimum occurs at (0,0).

Therefore, the function f has no absolute extreme values, but the local maximum occurs at $\left(\frac{1}{5},\frac{1}{25}\right)$ and the local minimum occurs at (0,0).

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**a.** Find the open intervals on which the function is increasing and decreasing.
**b.** Identify the function's local and absolute extreme values, if any, saying where they occur.

$$f(x) = x^4 - 72x^2 + 1296$$

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**a.** To determine on what intervals the function is increasing or decreasing, first find the critical points.

An interior point of the domain of a function f where f′ is zero or undefined is a critical point.

Find f′(x).

$$\begin{aligned} f(x) &= x^4 - 72x^2 + 1296 \\ f'(x) &= 4x^3 - 144x \end{aligned}$$

Set $f'(x) = 4x^3 - 144x$ equal to 0 and solve for x.

$$\begin{aligned} 4x^3 - 144x &= 0 \\ 4x\left(x^2 - 36\right) &= 0 \\ x &= -6,0,6 \end{aligned}$$

There are no values of x where the derivative f′(x) is undefined.

The critical points of f are $x = -6$, 0, and 6.

Use the critical points to subdivide the domain into nonoverlapping open intervals in which f′ is either positive or negative.

The intervals are $(-\infty,-6)$, $(-6,0)$, $(0,6)$, and $(-6,\infty)$.

Determine the sign of f′ by evaluating f at a convenient point in each subinterval. The behavior of f is determined by applying the corollary of the First Derivative Test for Monotonic Functions.

This corollary supposes that f is continuous on [a,b] and differentiable on (a,b).
If $f'(x) > 0$ at each point $x \in (a,b)$, then f is increasing on [a,b]. If $f'(x) < 0$ at each point $x \in (a,b)$, then f is decreasing on [a,b].

For the interval $(-\infty,-6)$, evaluate $f'(-7)$.

$$f'(-7) = 4(-7)^3 - 144(-7) = -364$$

Since $f'(x) < 0$, on the subinterval $(-\infty,-6)$, the function f is decreasing.

For the interval $(-6,0)$, evaluate $f'(-1)$.

$$f'(-1) = 4(-1)^3 - 144(-1) = 140$$

Since $f'(x) > 0$, on the subinterval $(-6,0)$, the function f is increasing.

For the interval (0,6), evaluate $f'(1)$.

$$f'(1) = 4(1)^3 - 144(1) = -140$$

Since $f'(x) < 0$, on the subinterval (0,6), the function f is decreasing.

For the interval $(6,\infty)$, evaluate $f'(7)$.

$$f'(7) = 4(7)^3 - 144(7) = 364$$

Since $f'(x) > 0$, on the subinterval $(6,\infty)$, the function f is increasing.

Note that strict and non-strict inequalities can be used in the corollary above to specify the intervals on which the function is increasing and decreasing.

Thus, the function f is decreasing on the subintervals $(-\infty,-6),(0,6)$ and increasing on the subintervals $(-6,0),(6,\infty)$.

**b.** To identify the function's local extreme values, if any, use the First Derivative Test for Local Extrema.

The First Derivative Test for Local Extrema first supposes that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across c from left to right,
1. if $f'$ changes from negative to positive at c, then f has a local minimum at c;
2. if $f'$ changes from positive to negative at c, then f has a local maximum at c;
3. if $f'$ does not change sign at c (that is, f ' is positive on both sides of c or negative on both sides), then f has no local extremum.

Since the sign of $f'$ changes from negative to positive at $x = -6$, f has a local minimum at $x = -6$.

Since the sign of $f'$ changes from positive to negative at $x = 0$, f has a local maximum at $x = 0$.

Since the sign of $f'$ changes from negative to positive at $x = 6$, f has a local minimum at $x = 6$.

Thus, f has a local minimum at $x = -6$ and $x = 6$, and it has a local maximum at $x = 0$.

To determine whether any of the extreme values are absolute, evaluate f at each of the critical points.

$$\begin{aligned} f(-6) &= (-6)^4 - 72(-6)^2 + 1296 = 0 \\ f(0) &= (0)^4 - 72(0)^2 + 1296 = 1296 \\ f(6) &= (6)^4 - 72(6)^2 + 1296 = 0 \end{aligned}$$

For the critical points $x = -6$ and $x = 6$, $f(-6) = f(6)$. Also, for all x in the domain, $f(x) \ge f(-6)$ and $f(x) \ge f(6)$. Thus, the function f has an absolute minimum at $x = -6$ and $x = 6$.

The function does not have an absolute maximum at the critical point $x = 0$. There are values in the domain where $f(x) > f(0)$.

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| **Student:** Cole Lamers<br>**Date:** 09/22/19 | **Instructor:** Viktoriya Shcherban<br>**Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban | **Assignment:** 4.3 Monotonic Functions and the First Derivative T |

**a.** Identify the function's local extreme values in the given domain, and say where they occur.
**b.** Which of the extreme values, if any, are absolute?
**c.** Support your findings with a graphing calculator or computer grapher.

$$f(x) = x^2 - 4x + 10, \quad -1 \le x < \infty$$

Suppose c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. According to the First Derivative Test for Local Extrema, the following rules can be used to identify the behavior at the critical point when moving from left to right across the interval.
1. If $f'$ changes from negative to positive at c, then f has a local minimum at c.
2. If $f'$ changes from positive to negative at c, then f has a local maximum at c.
3. If $f'$ does not change sign at c, then f has no local extremum.

The function $f(x) = x^2 - 4x + 10$ is continuous and differentiable everywhere in the given domain, $-1 \le x < \infty$.

Since the function is continuous and differentiable, the First Derivative Test for Local Extrema applies. The local extrema must occur at critical points.

The critical points are points where $f'$ is zero or undefined. Find $f'(x)$.

$$f'(x) = 2x - 4$$

Then identify all the points in the domain $-1 \le x < \infty$ where $f'(x) = 2x - 4$ is zero or undefined. The derivative is defined for all real numbers. Solving $2x - 4 = 0$ gives $x = 2$, which is in the given domain. Therefore, the only critical point is $x = 2$.

This critical point divides $-1 \le x < \infty$ into the open intervals $(-1,2)$ and $(2,\infty)$.

Choose a convenient point in each interval to evaluate $f'(x) = 2x - 4$ to determine the sign. Start with $x = 1$ in the interval $(-1,2)$.

$$f'(1) = -2$$

Then evaluate $f'(3)$ for the interval $(2,\infty)$.

$$f'(3) = 2$$

Use the corollary of the First Derivative Test for Monotonoic Functions shown below to determine the behavior of the function.

Suppose f is continuous on $[a,b]$ and differentiable on $(a,b)$. If $f'(x) > 0$ at each point $x \in (a,b)$, then f is increasing on $[a,b]$. If $f'(x) < 0$ at each point $x \in (a,b)$, then f is decreasing on $[a,b]$.

Identify the behavior of f in the two intervals.

| Interval | $f'$ evaluated | Sign of $f'$ | Behavior of f |
|---|---|---|---|
| $(-1,2)$ | $f'(1) = -2$ | $-$ | decreasing |
| $(2,\infty)$ | $f'(3) = 2$ | $+$ | increasing |

Then use the First Derivative Test for Local Extrema to determine the nature of the critical point. Since $f'$ changes sign from negative to positive at the point $x = 2$, the point is a local minimum.

Now determine the behavior at the endpoints of the interval. The interval is bounded on the left by $x = -1$.

At this point, the function is decreasing to the right, so the end point must be a local maximum.

The upper limit has no bound. As x increases without bound, the function $f(x) = x^2 - 4x + 10$ increases without bound.

Evaluate the function at the local extrema.

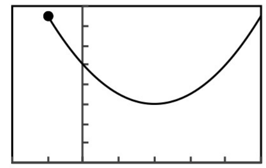
$$f(-1) = 15$$
$$f(2) = 6$$

Therefore, the function $f(x) = x^2 - 4x + 10, \ -1 \le x < \infty$ has a local maximum 15 at $x = -1$ and a local minimum 6 at $x = 2$.

**b.** Use all the information found to determine if the function has an absolute minimum or maximum.

Since the function increases without bound as x increases, there is no absolute maximum. The local minimum at (2,6) is also an absolute minimum because the function increases both to the left and the right of the point.

**c.** Graph the function to verify the answers. Notice that (2,6) is a local and absolute minimum and $(-1,15)$ is a local maximum.



$[-4,8,1]$ by $[0,16,2]$

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For the function $f(t) = 60t - 5t^3$ on the domain $-3 \le t < \infty$, do the following.

**a.** Identify the function's extreme values in the given domain.
**b.** Which of the extreme values, if any, are absolute?
**c.** Support your findings with a graphing calculator or computer grapher.

**a.** In order to determine the extreme values, first find the critical points of f by computing the values of t that make $f' = 0$.

Determine the derivative of f with respect to t.

$$\begin{aligned} f'(t) &= 60 - 15t^2 \\ &= 15(4 - t^2) \end{aligned}$$

Find the values of t that make $f' = 0$.

$$\begin{aligned} 0 &= 15(2 - t)(2 + t) && \text{Factor } (4 - t^2) \\ 0 &= (2 - t)(2 + t) && \text{Divide both sides of the equation by 15 and simplify.} \\ t &= \pm 2 \end{aligned}$$

The two critical points split the domain of the function into three intervals, $(-3,-2)$, $(-2,2)$, and $(2,\infty)$.

Suppose that c is a critical point of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across c from left to right,

1. if $f'$ changes from negative to positive at c, then f has a local minimum at c;
2. if $f'$ changes from positive to negative at c, then f has a local maximum at c;
3. if $f'$ does not change sign at c, then f has no local extremum at c.

Consider the critical point at $t = -2$. At this point, $f'$ changes sign because f is decreasing on the interval $(-3,-2)$ and increasing on the interval $(-2,2)$.

Because $f'$ is changing sign, there is a local extremum at $t = -2$. Moving across from left to right, $f'$ changes sign from negative to positive and therefore, a local minimum is located at this critical point.

Determine the value of $f(-2)$ by substituting $t = -2$ into f.

$$\begin{aligned} f(-2) &= 60(-2) - 5(-2)^3 \\ &= -120 + 40 \\ &= -80 \end{aligned}$$

The local minimum is at $(-2,-80)$.

At the critical point $t = 2$, $f'$ changes sign because f is increasing on the interval $(-2,2)$ and decreasing on the interval $(2,\infty)$.

Moving across from left to right, $f'$ changes sign from positive to negative and therefore, a local maximum is located at this critical point.

Determine the value of $f(2)$ by substituting $t = 2$ into f.

$$\begin{aligned} f(2) &= 60(2) - 5(2)^3 \\ &= 120 - 40 \\ &= 80 \end{aligned}$$

Thus, there is a local maximum at $(2,80)$.

The critical points occur in the interior of the function. Local extreme values may also occur at the endpoints of the domain of the function. Consider the endpoint at $t = -3$.

To the right of $t = -3$, $f'$ is negative because f is decreasing on the interval $(-3, -2)$. Therefore, the endpoint is a local maximum.

Determine the value of $f(-3)$ by substituting $t = -3$ into f.

$$\begin{aligned} f(-3) &= 60(-3) - 5(-3)^3 \\ &= -180 + 135 \\ &= -45 \end{aligned}$$

Thus, there is a second local maximum at $(-3, -45)$.

**b.** To determine which of the extreme values, if any, are absolute, consider the following.

Let f be a function with domain D. Then f has an absolute maximum value on D at a point c if $f(x) \le f(c)$ for all x in D and an absolute minimum value on D at c if $f(x) \ge f(c)$ for all x in D.

The local minimum $(-2, -80)$ is not an absolute minimum because there are infinitely many values of f(t) less than $f(-2) = -80$.

The local maximum (2,80) is an absolute maximum because there is no other value of f(t) greater than $f(2) = 80$.

The local maximum $(-3, -45)$ is not an absolute maximum because there is at least one value of f(t) greater than $f(-3) = -45$.

Thus, the absolute maximum occurs at (2,80) and there is no absolute minimum.

**c.** Support your findings with a graphing calculator or computer grapher.

The graph of $f(t) = 60t - 5t^3$, $-3 \le t < \infty$, is shown on the right.

Notice the local minimum $(-2, -80)$ is not an absolute minimum. The local maximum (2,80) is an absolute maximum whereas the local maximum $(-3, -45)$ is not an absolute maximum.