

**Student:** Cole Lamers**Date:** 10/02/19**Instructor:** Viktoriya Shcherban**Course:** Calc 1 11:30 AM / Internet  
(81749&81750) Shcherban**Assignment:** 5.3 The Definite Integral  
(Set 2)

Evaluate the integral  $\int_a^{14a} x \, dx$ .

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The definite integral  $\int_a^b x \, dx = \frac{b^2}{2} - \frac{a^2}{2}$ .

$$\text{So, } \int_a^{14a} x \, dx = \frac{(14a)^2}{2} - \frac{a^2}{2}.$$

$$\text{Thus, } \int_a^{14a} x \, dx = \frac{195}{2} a^2.$$

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Evaluate the integral  $\int_8^4 3 \, dx$ .

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For any constant  $c$ ,  $\int_a^b c \, dx = c(b - a)$ .

So,  $\int_8^4 3 \, dx = 3(4 - 8)$ .

Thus,  $\int_8^4 3 \, dx = -12$ .

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Evaluate the integral  $\int_1^4 (2t - 3) dt$ .

First apply the Difference rule for definite integrals.

$$\int_1^4 (2t - 3) dt = \int_1^4 2t \, dt - \int_1^4 3 \, dt$$

Then, apply the Constant Multiple rule.

$$\int_1^4 2t \, dt - \int_1^4 3 \, dt = 2 \int_1^4 t \, dt - \int_1^4 3 \, dt$$

$$2 \int_1^4 t \, dt = 2 \left( \frac{4^2}{2} - \frac{1^2}{2} \right) = 15$$

$$\int_1^4 3 \, dt = 3(4 - 1) = 9$$

$$\text{So, } \int_1^4 (2t - 3) dt = 15 - 9 = 6.$$

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Use a definite integral to find the area of the region between the given curve and the x-axis on the interval  $[0, b]$ .

$$y = 6x^2$$

If  $y = f(x)$  is nonnegative and integrable over a closed interval  $[a, b]$ , then the area under the curve  $y = f(x)$  over  $[a, b]$  is the integral of  $f$  from  $a$  to  $b$ .

$$A = \int_a^b f(x) \, dx$$

First, write the definite integral that corresponds to the area.

$$A = \int_0^b 6x^2 \, dx$$

Use the rules satisfied by definite integrals to rewrite the given integral in terms of simpler integrals. Try to write the integrand  $x^2$  so the rule below can be used.

$$\int_a^b x^2 \, dx = \frac{b^3}{3} - \frac{a^3}{3}$$

Notice that the given integration rule is for  $f(x) = x^2$ . Therefore, use the constant multiple rule shown below to rewrite the integral so the integrand is  $x^2$ .

$$\int_a^b kf(x) \, dx = k \int_a^b f(x) \, dx$$

For the integral  $\int_0^b 6x^2 \, dx$ , the value of the constant,  $k$ , is 6.

Therefore, the integral can be rewritten as shown.

$$\int_0^b 6x^2 \, dx = 6 \int_0^b x^2 \, dx$$

Now use the integration rule for  $x^2$  to evaluate the integral. Substitute  $a = 0$  and  $b = b$  into the formula for the integral and simplify.

$$\begin{aligned} 6 \int_0^b x^2 \, dx &= 6 \left( \frac{b^3}{3} - \frac{a^3}{3} \right) \\ &= 6 \left( \frac{b^3}{3} - \frac{0^3}{3} \right) \\ &= 2b^3 \end{aligned}$$

Therefore, the area between the curve and the x-axis over the interval  $[0, b]$  is  $2b^3$ .

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Use a definite integral to find the area of the region between the curve  $y = 9x$  and the x-axis on the interval  $[0, b]$ .

The area between the curve  $y = f(x)$  and the x-axis over  $[a, b]$  is defined to be the definite integral of  $f$  from  $a$  to  $b$ . In symbolic

terms,  $A = \int_a^b f(x) dx$ .

In this situation,  $f(x) = 9x$  and  $[a, b]$  is  $[0, b]$ .

So the area,  $A$ , is  $A = \int_0^b 9x dx$ .

Apply the constant multiple rule.

$$\int_0^b 9x dx = 9 \int_0^b x dx$$

Finally, evaluate  $9 \int_0^b x dx$ .

$$9 \int_0^b x dx = 9 \left( \frac{b^2}{2} - \frac{0^2}{2} \right) = \frac{9}{2} b^2$$

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**Assignment:** 5.3 The Definite Integral  
 (Set 2)

Find the average value of the function  $f(x) = x^2 - 5$  on  $[0, \sqrt{10}]$ .

The average value of  $f(x)$  on  $[a, b]$  is defined as  $\frac{1}{b-a} \int_a^b f(x) dx$ .

So, the average value of  $f(x) = x^2 - 5$  on  $[0, \sqrt{10}]$  is  $\frac{1}{\sqrt{10} - 0} \int_0^{\sqrt{10}} (x^2 - 5) dx$ .

Applying the Difference and Constant Multiple rules,  $\frac{1}{\sqrt{10} - 0} \int_a^b [x^2 - 5] dx = \frac{1}{\sqrt{10}} \left( \int_0^{\sqrt{10}} x^2 dx - 5 \int_0^{\sqrt{10}} dx \right)$ .

$$\int_0^{\sqrt{10}} x^2 dx = \frac{10^{3/2}}{3}$$

Since  $\int_0^{\sqrt{10}} dx = \sqrt{10}$ , the following is true.

$$\begin{aligned} \frac{1}{\sqrt{10}} \left( \int_0^{\sqrt{10}} x^2 dx - 5 \int_0^{\sqrt{10}} dx \right) &= \frac{1}{\sqrt{10}} \left( \frac{10^{3/2}}{3} - 5\sqrt{10} \right) \\ &= \frac{10}{3} - 5 \\ &= -\frac{5}{3} \end{aligned}$$

So, the average value of  $f(x) = x^2 - 5$  on  $[0, \sqrt{10}]$  is  $-\frac{5}{3}$ .

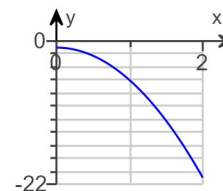
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**Assignment:** 5.3 The Definite Integral  
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Graph the function  $f(x) = -5x^2 - 1$  on  $[0, 2]$  and find its average value over the interval.

The graph of the function on the interval  $[0, 2]$  is shown to the right.



If  $f$  is integrable on  $[a, b]$ , then its average or mean value on  $[a, b]$  is given by the definite integral below.

$$\text{av}(f) = \frac{1}{b-a} \int_a^b f(x) \, dx$$

The lower limit of integration,  $a$ , is 0 and the upper limit of integration,  $b$ , is 2.

Therefore, the average is  $\frac{1}{2} \int_0^2 (-5x^2 - 1) \, dx$ . Use the sum and difference rule as well as the constant multiple rule to rewrite the expression.

$$\frac{1}{2} \int_0^2 (-5x^2 - 1) \, dx = -\frac{5}{2} \int_0^2 x^2 \, dx - \frac{1}{2} \int_0^2 1 \, dx$$

The remaining integrals can be evaluated using the integration rules below.

$$\int_a^b c \, dx = c(b-a) \qquad \int_a^b x^2 \, dx = \frac{b^3}{3} - \frac{a^3}{3}$$

Substitute  $a = 0$  and  $b = 2$  into the formulas for the integrals and simplify.

$$\begin{aligned} -\frac{5}{2} \int_0^2 x^2 \, dx - \frac{1}{2} \int_0^2 1 \, dx &= -\frac{5}{2} \left( \frac{2^3}{3} - \frac{0^3}{3} \right) - \frac{1}{2} (2 - 0) \\ &= -\frac{20}{3} - 1 \\ &= -\frac{23}{3} \end{aligned}$$

The average value of the function  $f(x) = -5x^2 - 1$  on the interval  $[0, 2]$  is  $-\frac{23}{3}$ .