Unit 3.4 Cheat Sheet

Speed, velocity, Acceleration

- Start with Polynomial$_2$ − Polynomial$_1$ = Δs
- $\Delta x = x_1 \le x_0 \le x_2 \quad x_2 - x_1$
- Then $\frac{\Delta s}{\Delta x}$
- Then take Polynomial$_1^{-1}$ & polynomial$_2^{-1}$ — velocity

left & right (x_1, x_2) — these are positive

- Then $(\text{polynomial}_1^{-1})^{-1}$ $(\text{polynomial}_2^{-1})^{-1}$ acceleration
- Lastly $\left(\text{Polynomial}_{1 \text{ or } 2}^{-1}\right)^{-1} = ax \pm b = y$ solve for x

if $y = x_1 \le x_0 \le$ then it is a solution in the time interval

if it is not within the domain, it is not a solution.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

A body moves on a coordinate line such that it has a position $s = f(t) = t^2 - 4t + 3$ on the interval $0 \le t \le 5$, with s in meters and t in seconds.
a. Find the body's displacement and average velocity for the given time interval.
b. Find the body's speed and acceleration at the endpoints of the interval.
c. When, if ever, during the interval does the body change direction?

The displacement of the object over the time interval from t to $t + \Delta t$ is $\Delta s = f(t + \Delta t) - f(t)$.

Since the interval is $0 \le t \le 5$, the value of t is 0, and the value of $t + \Delta t$ is 5.

Substitute t and $t + \Delta t$ in the formula for displacement.

$$\Delta s = f(t + \Delta t) - f(t)$$
$$\Delta s = f(5) - f(0)$$
$$\Delta s = (5)^2 - 4(5) + 3 - ((0)^2 - 4(0) + 3)$$
$$\Delta s = 5$$

The body's displacement for the given time interval is 5 m.

The average velocity of the object over the time interval from t to $t + \Delta t$ is $v_{av} = \frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$.

The displacement has already been calculated. Now find Δt.

Since the value of t is 0, and the value of $t + \Delta t$ is 5, solve for Δt.

$$t + \Delta t = 5$$
$$0 + \Delta t = 5$$
$$\Delta t = 5$$

Substitute Δs and Δt in the formula for average velocity.

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{5}{5} = 1 \ \frac{\text{m}}{\text{sec}}$$

The body's average velocity for the given time interval is $1 \ \frac{\text{m}}{\text{sec}}$.

Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

Velocity is the derivative of the position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is $v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$.

Use the rules of differentiation to find v(t).

$$v(t) = \frac{ds}{dt} = \frac{d}{dt}(t^2 - 4t + 3) = 2t - 4$$

Substitute the endpoints in v(t) to find the velocity at time t.

$$v(0) = -4, \ \ v(5) = 6$$

Since speed is the absolute value of velocity, the body's speeds at the left and right endpoints of the interval are 4 $\frac{\text{m}}{\text{sec}}$ and 6 $\frac{\text{m}}{\text{sec}}$, repectively.

Acceleration is the derivative of velocity with respect to time. If a body's position at time t is s = f(t), then the body's acceleration at time t is $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

Use the rules of differentiation to find a(t).

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(2t - 4) = 2$$

Since the acceleration is a constant, the acceleration is the same for all t over the time interval.

Thus, the acceleration at the endpoints of the interval is 2 $\frac{\text{m}}{\text{sec}^2}$.

To determine whether the body will change direction during the interval, first set v(t) = 0 and solve for t.

$$\begin{aligned} 2t - 4 &= 0 \\ t &= 2 \end{aligned}$$

Now determine the sign of v(t) between the left endpoint, 0, and 2 and the sign of v(t) between 2 and the right endpoint, 5. If the signs change, the body will change direction where v(t) = 0.

Choose a value between 0 and 2, and evaluate v(t) at that value. The sign of v(t) at that value is negative.

Choose a value between 2 and 5, and evaluate v(t) at that value. The sign of v(t) at that value is positive.

Since the signs change, the body will change direction at t = 2 sec.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

The function $s = -t^3 + 18t^2 - 108t$, $0 \le t \le 6$, gives the position of a body moving on a coordinate line, with s in meters and t in seconds.
a. Find the body's displacement and average velocity for the given time interval.
b. Find the body's speed and acceleration at the endpoints of the interval.
c. When, if ever, during the interval does the body change direction?

a. If a body's position at time t is $s = f(t)$, then the displacement of the body over the time interval from t to $t + \Delta t$ is given below.

$$\Delta s = f(t + \Delta t) - f(t)$$

For the given interval, $t = 0$ and $\Delta t = 6$. Find s(6).

$s(6) = -216$

Next find s(0).

$s(0) = 0$

Now determine Δs, the body's displacement for the given time interval.

$$\begin{aligned} \Delta s &= s(6) - s(0) \\ &= -216 - 0 \\ &= -216 \text{ m} \end{aligned}$$

The average velocity of the body over the time interval is given below.

$$v_{av} = \frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

The value Δs was determined to be -216. The value of Δt is 6.

Calculate the average velocity for the given time interval.

$$v_{av} = \frac{-216}{6} = -36 \text{ m/s}$$

b. Velocity (instantaneous velocity) is the derivative of position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is given below.

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

First derive v(t) by finding the derivative of s with respect to t.

$$v(t) = \frac{d}{dt}(-t^3 + 18t^2 - 108t) = -3t^2 + 36t - 108$$

Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

Use the function $v(t) = -3t^2 + 36t - 108$ to find the body's speed at $t = 0$.

$|v(0)| = \left|-3(0)^2 + 36(0) - 108\right| = 108$ m/s

Now find the body's speed at $t = 6$.

$|v(6)| = \left| -3(6)^2 + 36(6) - 108 \right| = 0$ m/s

Acceleration is the derivative of velocity with respect to time. If a body's position at time t is s = f(t), then the body's acceleration at time t is given below.

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Now derive a(t) by finding the derivative of $v(t) = -3t^2 + 36t - 108$ with respect to t.

$$a(t) = \frac{d}{dt}(-3t^2 + 36t - 108) = -6t + 36$$

Find the body's acceleration at t = 0 by evaluating a(0).

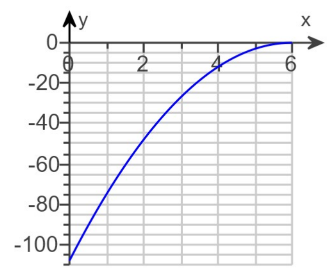
$a(0) = -6(0) + 36 = 36 \text{ m/s}^2$

Next find the body's acceleration at t = 6.

$a(6) = -6(6) + 36 = 0 \text{ m/s}^2$

c. When the object is moving forward (s is increasing), the velocity is positive. When the body is moving backward (s is decreasing), the velocity is negative.

The graph of the velocity, v(t) is shown on the right.



Notice that v(t) can be written as $v(t) = -3(t-6)^2$. Since $(t-6)^2$ is never negative, $-3(t-6)^2$ is never positive, which means that v(t) does not cross the t-axis.

There are no points on the graph of $v(t) = -3t^2 + 36t - 108$ where the graph crosses the t-axis in the interval $0 \le t \le 6$. Therefore, the body does not change direction in the interval.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

A body moves on a coordinate line such that it has a position $s = f(t) = \frac{9}{t^2} - \frac{3}{t}$ on the interval $1 \le t \le 3$, with s in meters and t in seconds.

a. Find the body's displacement and average velocity for the given time interval.
b. Find the body's speed and acceleration at the endpoints of the interval.
c. When, if ever, during the interval does the body change direction?

The displacement of the object over the time interval from t to $t + \Delta t$ is $\Delta s = f(t + \Delta t) - f(t)$.

Since the interval is $1 \le t \le 3$, the value of t is 1, and the value of $t + \Delta t$ is 3.

Substitute t and $t + \Delta t$ in the formula for displacement.

$$\Delta s = f(t + \Delta t) - f(t)$$
$$\Delta s = f(3) - f(1)$$
$$\Delta s = \frac{9}{(3)^2} - \frac{3}{3} - \left(\frac{9}{(1)^2} - \frac{3}{1}\right)$$
$$\Delta s = -6$$

The body's displacement for the given time interval is -6 m.

The average velocity of the object over the time interval from t to $t + \Delta t$ is $v_{av} = \frac{\text{displacement}}{\text{travel time}} = \frac{\Delta s}{\Delta t} = \frac{f(t + \Delta t) - f(t)}{\Delta t}$.

The displacement has already been calculated. Now find Δt.

Since the value of t is 1, and the value of $t + \Delta t$ is 3, solve for Δt.

$$t + \Delta t = 3$$
$$1 + \Delta t = 3$$
$$\Delta t = 2$$

Substitute Δs and Δt in the formula for average velocity.

$$v_{av} = \frac{\Delta s}{\Delta t} = \frac{-6}{2} = -3 \frac{\text{m}}{\text{sec}}$$

The body's average velocity for the given time interval is $-3 \frac{\text{m}}{\text{sec}}$.

Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

Velocity is the derivative of the position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is $v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$.

Use the rules of differentiation to find v(t).

$$v(t) = \frac{ds}{dt} = \frac{d}{dt}\left(\frac{9}{t^2} - \frac{3}{t}\right) = -\frac{18}{t^3} + \frac{3}{t^2}$$

Substitute the endpoints in v(t) to find the velocity at time t.

$v(1) = -15, \ v(3) = -\frac{1}{3}$

Since speed is the absolute value of velocity, the body's speeds at the endpoints of the interval are $15 \frac{m}{sec}$ and $\frac{1}{3} \frac{m}{sec}$, repectively.

Acceleration is the derivative of velocity with respect to time. If a body's position at time t is s = f(t), then the body's acceleration at time t is $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

Use the rules of differentiation to find a(t).

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(-\frac{18}{t^3} + \frac{3}{t^2}\right) = \frac{54}{t^4} - \frac{6}{t^3}$$

Substitute the endpoints in a(t) to find the acceleration at time t.

$a(1) = 48, \ a(3) = \frac{4}{9}$

The accelerations at the endpoints of the interval are $48 \frac{m}{sec^2}$ and $\frac{4}{9} \frac{m}{sec^2}$, respectively.

To determine whether the body will change direction during the interval, first set v(t) = 0 and solve for t. Note that $t \neq 0$.

$$-\frac{18}{t^3} + \frac{3}{t^2} = 0$$
$$t^3\left(-\frac{18}{t^3} + \frac{3}{t^2}\right) = t^3(0)$$
$$t = 6$$

Since there are no values within the interval for which v(t) = 0, the body will not change direction during the interval.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

At time t, the position of a body moving along the s-axis is $s = t^3 - 18t^2 + 105t$ m.
a. Find the body's acceleration each time the velocity is zero.
b. Find the body's speed each time the acceleration is zero.
c. Find the total distance traveled by the body from $t = 0$ to $t = 6$.

---

a. Velocity (instantaneous velocity) is the derivative of position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is given below.

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

First derive v(t) by finding the derivative of s with respect to t.

$$v(t) = \frac{d}{dt}(t^3 - 18t^2 + 105t) = 3t^2 - 36t + 105$$

Acceleration is the derivative of velocity with respect to time. If a body's position at time t is $s = f(t)$, then the body's acceleration at time t is given below.

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Now derive a(t) by finding the derivative of $v(t) = 3t^2 - 36t + 105$ with respect to t.

$$a(t) = \frac{d}{dt}(3t^2 - 36t + 105) = 6t - 36$$

Next find the times when the velocity is zero by solving $v(t) = 3t^2 - 36t + 105 = 0$.

$$\begin{aligned} 3t^2 - 36t + 105 &= 0 \\ 3(t-5)(t-7) &= 0 \\ t &= 5 \text{ sec or } 7 \text{ sec} \end{aligned}$$

To find the body's acceleration each time the velocity is zero, substitute $t = 5$ and $t = 7$ into the equation for acceleration, $a(t) = 6t - 36$.

$a(5) = 6(5) - 36 = -6 \text{ m/s}^2$
$a(7) = 6(7) - 36 = 6 \text{ m/s}^2$

b. Find when the acceleration is zero by solving $a(t) = 6t - 36 = 0$.

$$\begin{aligned} 6t - 36 &= 0 \\ 6t &= 36 \\ t &= 6 \text{ sec} \end{aligned}$$

Speed is the absolute value of velocity.

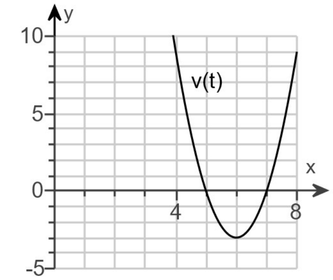
$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

To find the body's speed when the acceleration is zero, substitute $t = 6$ into the equation for velocity $v(t) = 3t^2 - 36t + 105$, and then take the absolute value.

$|v(6)| = \left|3(6)^2 - 36(6) + 105\right| = 3$ m/s

c. When the body is moving forward (s is increasing), the velocity is positive. When the body is moving backward (s is decreasing), the velocity is negative. Find the displacement, when s is increasing and when s is decreasing, for t = 0 to t = 6.

The graph of v(t) is shown to the right.



Given that the zeros of the function $v(t) = 3t^2 - 36t + 105$ are at t = 5 and t = 7, the interval for which the function s is increasing between t = 0 and t = 6 is $0 \le t < 5$.

The interval for which the function s is decreasing between t = 0 and t = 6 is $5 < t \le 6$.

If a body's position at time t is s = f(t), then the displacement of the object over the time interval from t to t + Δt is given below.

$$\Delta s = f(t + \Delta t) - f(t)$$

The total distance traveled by the body from time t = 0 to t = 6 is the sum of the absolute value of the displacements. Start by finding the displacement on the interval $0 \le t < 5$.

$$\begin{aligned} \Delta s &= s(5) - s(0) \\ &= (5)^3 - 18(5)^2 + 105(5) - \left[(0)^3 - 18(0)^2 + 105(0)\right] \\ &= 200 \text{ m} \end{aligned}$$

Next find the displacement on the interval $5 < t \le 6$.

$$\begin{aligned} \Delta s &= s(6) - s(5) \\ &= (6)^3 - 18(6)^2 + 105(6) - \left[(5)^3 - 18(5)^2 + 105(5)\right] \\ &= -2 \text{ m} \end{aligned}$$

The total distance traveled by the body from t = 0 to t = 6 is given below.

$|200| + |-2| = 202$ m

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

The equation for free fall at the surface of a celestial body in outer space (s in meters, t in seconds) is $s = 1.98t^2$. How long does it take a rock falling from rest to reach a velocity of 26.7 $\frac{m}{sec}$ on this celestial body in outer space?

Velocity is the derivative of position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is $v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$.

Use the rules of differentiation to find v(t).

$$v(t) = \frac{ds}{dt} = \frac{d}{dt}(1.98t^2) = 3.96t$$

Now, set v(t) equal to 26.7 $\frac{m}{sec}$ and solve for t.

$3.96t = 26.7$
$t = 6.7$

It takes 6.7 sec for a rock falling from rest to reach a velocity of 26.7 $\frac{m}{sec}$ on this celestial body in outer space.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

Explorers on a small airless planet used a spring gun to launch a ball bearing vertically upward from the surface at a launch velocity of 31.5 m/sec. Because the acceleration of gravity at the planet's surface was g_s m/sec^2, the explorers expected the ball bearing to reach a height of $s = 31.5t - (1/2)g_s t^2$ m, t sec later. The ball bearing reached its maximum height 30 sec after being launched. What was the value of g_s?

Note that when the ball bearing reaches its maximum height, its instantaneous velocity is zero.

Velocity is the derivative of position with respect to time. If a body's position at time t is s = f(t), then the body's velocity at time t is given by the formula below.

$$v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

Take the derivative of the position function to find the velocity function.

$$\begin{aligned} s &= 31.5t - (1/2)g_s t^2 \\ \frac{ds}{dt} &= 31.5 - g_s t \end{aligned}$$

It is given that the ball bearing reached its maximum height 30 sec after being launched. Also, at its maximum height, its velocity is zero (that is, v(30) = 0). Use this information to solve for g_s.

$$\begin{aligned} v(t) &= 31.5 - g_s t \\ v(30) &= 31.5 - g_s(30) \\ 0 &= 31.5 - g_s(30) \\ 1.05 &= g_s \end{aligned}$$

Thus, the gravitational acceleration on the small airless planet is $g_s = 1.05$ m/sec^2.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

An object is dropped from a tower, 192 ft above the ground. The object's height above ground t sec into the fall is $s = 192 - 16t^2$.
a. What is the object's velocity, speed, and acceleration at time t?
b. About how long does it take the object to hit the ground?
c. What is the object's velocity at the moment of impact?

Velocity is the derivative of position with respect to time. If a body's position at time t is $s = f(t)$, then the body's velocity at time t is $v(t) = \frac{ds}{dt} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$.

Use the rules of differentiation to find v(t).

$$v(t) = \frac{ds}{dt} = \frac{d}{dt}(192 - 16t^2) = -32t$$

The object's velocity at time t is $-32t \ \frac{ft}{sec}$.

Speed is the absolute value of velocity.

$$\text{Speed} = |v(t)| = \left|\frac{ds}{dt}\right|$$

Since $t \geq 0$, $|t| = t$. Also, it is known that $|-32| = 32$. Therefore, the object's speed at time t is $|-32t| = 32t \ \frac{ft}{sec}$.

Acceleration is the derivative of velocity with respect to time. If a body's position at time t is $s = f(t)$, then the body's acceleration at time t is $a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$.

Use the rules of differentiation to find a(t).

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}(-32t) = -32$$

The object's acceleration at time t is $-32 \ \frac{ft}{sec^2}$.

To determine how long it takes the object to hit the ground, set the equation for the object's height equal to 0 and solve for t.

$$\begin{aligned} s &= 192 - 16t^2 \\ 0 &= 192 - 16t^2 \\ 16t^2 &= 192 \\ t^2 &= \frac{192}{16} \\ t &= 3.5 \qquad \text{(rounded to the nearest tenth)} \end{aligned}$$

It takes 3.5 sec for the object to hit the ground.

To find the object's velocity at the moment of impact, substitute the length of time it takes for the object to hit the ground in v(t).

$v(t) = -32t$
$v(3.5) = -112$

The object's velocity at the moment of impact is $-112\ \frac{\text{ft}}{\text{sec}}$.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

Suppose that the dollar cost of producing x appliances is $c(x) = 1100 + 140x - 0.3x^2$.
a. Find the average cost per appliance of producing the first 150 appliances.
b. Find the marginal cost when 150 appliances are produced.
c. Show that the marginal cost when 150 appliances are produced is approximately the cost of producing one more appliance after the first 150 have been made, by calculating the latter cost directly.

The average cost per appliance of producing the first 150 appliances is given by the formula $AC = \frac{c(x)}{x}$, where x is the number of appliances.

Substitute the initial value of x into the formula for the average cost.

$$\begin{aligned} AC &= \frac{c(x)}{x} \\ &= \frac{c(150)}{150} \\ &= \frac{1100 + 140(150) - 0.3(150)^2}{150} \\ &= 102.33 \end{aligned}$$

(rounded to the nearest hundredth)

The average cost per appliance of producing the first 150 appliances is \$102.33 / appliance.

The marginal cost of production is given by the formula $MC = c'(x) = \frac{dc}{dx} = \lim_{h\to 0} \frac{c(x+h) - c(x)}{h}$.

Use the rules of differentiation to find the marginal cost.

$$MC = c'(x) = \frac{dc}{dx} = \frac{d}{dx}(1100 + 140x - 0.3x^2) = 140 - 0.6x$$

Substitute 150 for x in the equation for marginal cost.

$$\begin{aligned} c'(x) &= 140 - 0.6x \\ c'(150) &= 140 - 0.6(150) \\ c'(150) &= 50 \end{aligned}$$

The marginal cost when 150 appliances are produced is \$50.

The marginal cost of production is loosely defined to be the extra cost of producing one unit beyond the number of units already produced.

To find the exact cost of producing one additional unit beyond the x units already produced, compute $c(x + 1) - c(x)$.

$$\begin{aligned} c(x+1) - c(x) &= c(151) - c(150) \\ &= 1100 + 140(151) - 0.3(151)^2 - (1100 + 140(150) - 0.3(150)^2 \\ &= 49.70 \end{aligned}$$

The cost of producing one more appliance beyond 150 appliances is \$49.70.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

When a bactericide is added to a nutrient broth in which bacteria are growing, the bacterium population continues to grow for a while, but then stops growing and begins to decline. The size of the population at time t (hours) is $b = 8^7 + 8^5 t - 8^4 t^2$. Find the growth rates at $t = 0$ hours, $t = 4$ hours, and $t = 8$ hours.

To find the equation of the growth rate, find the derivative of the size of the population at time t.

$b(t) = 8^7 + 8^5 t - 8^4 t^2$
$b'(t) = 8^5 - 2 \cdot 8^4 t$

To find the growth rate at $t = 0$ hours, substitute 0 for t and solve.

$b'(t) = 8^5 - 2 \cdot 8^4 t$
$b'(0) = 8^5 - 2 \cdot 8^4 (0)$
$b'(0) = 8^5$

The growth rate at $t = 0$ hours is 8^5 bacteria per hour.

To find the growth rate at $t = 4$ hours, substitute 4 for t and solve.

$b'(t) = 8^5 - 2 \cdot 8^4 t$
$b'(4) = 8^5 - 2 \cdot 8^4 (4)$
$b'(4) = 8^5 - 8 \cdot 8^4$
$b'(4) = 0$

The growth rate at $t = 4$ hours is 0 bacteria per hour.

To find the growth rate at $t = 8$ hours, substitute 8 for t and solve.

$b'(t) = 8^5 - 2 \cdot 8^4 t$
$b'(8) = 8^5 - 2 \cdot 8^4 (8)$
$b'(8) = -8^5$

The growth rate at $t = 8$ hours is -8^5 bacteria per hour.

Student: Cole Lamers **Date:** 09/07/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.4 The Derivative as a Rate of Change

The volume $V = \frac{4}{3}\pi r^3$ of a spherical balloon changes with the radius.

a. At what rate (ft^3/ft) does the volume change with respect to the radius when r = 4 ft?
b. Using the rate from part **a**, by approximately how much does the volume increase when the radius changes from 4 to 4.6 ft?

---

a. The instantaneous rate of change of f with respect to x at x_0 is the derivative of f at x_0, provided the limit exists.

$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

First differentiate the volume with respect to r.

$$\frac{dV}{dr} = \frac{d}{dr}\frac{4}{3}\pi r^3 = 4\pi r^2 \text{ ft}^3/\text{ft}$$

Now evaluate the rate when the radius is 4 feet.

$$4\pi(4)^2 = 64\pi \text{ ft}^3/\text{ft}$$

b. The average rate of change in volume with respect to radius, or $\frac{\Delta V}{\Delta r}$, can be approximated by the instantaneous rate of change, $\frac{dV}{dr}$, at r when the slope of the graph of V does not change quickly near r.

Find the change in the volume as the radius changes from 4 to 4.6 ft. When r = 4, $\frac{dV}{dr} = 64\pi$ so that when r changes by 1 unit, we expect V to change by approximately 64π.

As the radius changes from 4 to 4.6 ft, the change in r is 0.6 ft.

Therefore, when r changes by 0.6 units, V changes by approximately $(64\pi)(0.6) = 38.4\pi \approx 120.64 \text{ ft}^3$.