Student: Cole Lamers **Date:** 07/08/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 5.2 Right Triangle Trigonometry

Use the definition or identities to find the exact value of each of the remaining five trigonometric functions of the acute angle θ.

$$\cos\theta = \frac{1}{4}$$

We use the following Pythagorean Identity to find the exact value of **sin** θ.

$$\sin^2\theta + \cos^2\theta = 1$$

First, solve for **sin** θ.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta \qquad \text{Subtract } \cos^2\theta \text{ from each side.}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} \qquad \text{Take the square root of each side.}$$

The positive square root was chosen because θ is an acute angle.

Evaluate the equation for **sin** θ using the given exact value for cosine.

$$\sin\theta = \sqrt{1 - \cos^2\theta}$$

$$\sin\theta = \sqrt{1 - \left(\frac{1}{4}\right)^2} \qquad \text{Substitute } \frac{1}{4} \text{ for } \cos\theta.$$

$$\sin\theta = \frac{\sqrt{15}}{4} \qquad \text{Simplify.}$$

Now, use the quotient identity for **tan** θ to find its exact value.

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{15}}{4}}{\frac{1}{4}} = \sqrt{15}$$

Use the reciprocal identity for **cot** θ to find its exact value.

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{\sqrt{15}} = \frac{\sqrt{15}}{15}$$

Use the reciprocal identity for **csc** θ to find its exact value. Recall that $\sin\theta = \frac{\sqrt{15}}{4}$.

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{\sqrt{15}}{4}} = \frac{4}{\sqrt{15}} = \frac{4\sqrt{15}}{15}$$

Use the reciprocal identity for **sec** θ to find its exact value.

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{4}} = 4$$

We have found the exact value of each of the five remaining trigonometric functions.

$$\sin\theta = \frac{\sqrt{15}}{4} \quad \tan\theta = \sqrt{15} \quad \cot\theta = \frac{\sqrt{15}}{15} \quad \csc\theta = \frac{4\sqrt{15}}{15} \quad \sec\theta = 4$$

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Use table for trigonometric function values of some common angles and simplify the resulting expression.

$\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

Note that the table for trigonometric function values of some common angles is given below.

θ (degrees)	θ (radians)	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	$\sqrt{2}$	$\sqrt{2}$	1
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2	$\frac{\sqrt{3}}{3}$

To simplify the trigonometric expression $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$, first find the values of $\sin 30^\circ$, $\cos 45^\circ$, $\cos 30^\circ$, and $\sin 45^\circ$ by using the table for trigonometric function values of some common angles.

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

Substitute $\frac{1}{2}$ for $\sin 30^\circ$, $\frac{\sqrt{2}}{2}$ for $\cos 45^\circ$, $\frac{\sqrt{3}}{2}$ for $\cos 30^\circ$, and $\frac{\sqrt{2}}{2}$ for $\sin 45^\circ$ in the trigonometric expression $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$.

$$\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

Simplify.

$$\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

Thus, $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ = \frac{\sqrt{2} + \sqrt{6}}{4}$.

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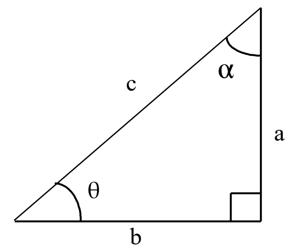
Use the figure shown to the right and the given values to find **sin** θ, b, and c.

$\theta = 30^\circ$, $a = 11$



Find the value of **sin** θ by substituting 30° for θ in **sin** θ.

$$\begin{aligned} \sin\theta &= \sin 30^\circ && \text{Substitute.} \\ &= 0.5 && \text{Simplify.} \end{aligned}$$

To find b, use the fact that **tan** $30^\circ = \frac{11}{b}$.

Solve for b.

$$\begin{aligned} \tan 30^\circ &= \frac{11}{b} \\ b \cdot \tan 30^\circ &= 11 && \text{Multiply both sides by b.} \\ b &= \frac{11}{\tan 30^\circ} && \text{Divide both sides by } \tan 30^\circ. \\ b &\approx 19.053 \end{aligned}$$

Thus, the length of side b is approximately 19.053.

To find c, use the fact that **sin** $30^\circ = \frac{11}{c}$.

Solve for c.

$$\begin{aligned} \sin 30^\circ &= \frac{11}{c} \\ c \cdot \sin 30^\circ &= 11 && \text{Multiply both sides by c.} \\ c &= \frac{11}{\sin 30^\circ} && \text{Divide both sides by } \sin 30^\circ. \\ c &= 22 \end{aligned}$$

Thus, the length of the side c is 22.

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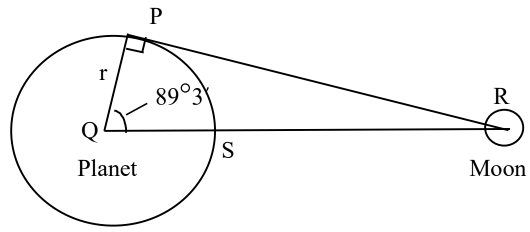
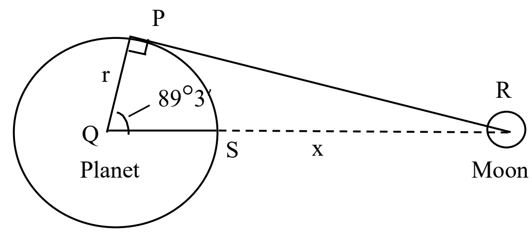
If the radius of a planet is $r = 3985$ miles and the angle at the planet's center is $A = 89^\circ 3'$, find the shortest distance between its moon and the surface of the planet.



To begin, it is important to understand what part of the triangle will be used to find the distance. A portion of the length of side QR is the distance.

In the figure to the right, both points P and S are on the surface, while Q is in the center of the planet. The length of side PR and the length of side SR give distances from the surface to the moon.



The length of SR is considered the distance since it is the shortest way to get from the surface to the moon. Notice that angle P is 90º.

The distance between the surface of the planet and the moon is x. Because there is a right triangle, trigonometric properties will be used to find x. However, the entire length of QR must be used with the trigonometric values.

Since $r = 3985$ miles (the distance between S and Q), the distance between Q and R is $3985 + x$.

Note that if A is any acute angle in a right triangle, $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$.

As $\angle P$ is a right triangle, $\cos 89.05 = \dfrac{3985}{3985 + x}$.

Solve for x. Multiply both sides by $3985 + x$.

$$\begin{aligned} \cos 89.05 &= \frac{3985}{3985 + x} \\ (3985 + x)\cos 89.05 &= 3985 \end{aligned}$$

Use the distributive property.

$$\begin{aligned} (3985 + x)\cos 89.05 &= 3985 \\ 3985\cos 89.05 + x\cos 89.05 &= 3985 \end{aligned}$$

Subtract $3985\cos 89.05$ from both sides in order to isolate the x-term.

$$\begin{aligned} 3985 \cos 89.05 + x \cos 89.05 &= 3985 \\ x\cos 89.05 &= 3985 - 3985 \cos 89.05 \end{aligned}$$

Divide both sides by $\cos 89.05$.

$$\begin{aligned} x\cos 89.05 &= 3985 - 3985 \cos 89.05 \\ x &= \frac{3985 - 3985 \cos 89.05}{\cos 89.05} \\ &\approx 236{,}367 \qquad \text{Simplify by using a calculator.} \end{aligned}$$

The distance between its moon and the surface of the planet is approximately 236,367 miles.

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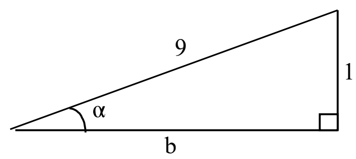
If α is an acute angle and **sin** $\alpha = \frac{1}{9}$, find the value of **cos** α **csc** α + **tan** α **sec** α.

To find the value of the expression **cos** α **csc** α + **tan** α **sec** α first, find the values for each trigonometric function of α. Then, use those values and evaluate **cos** α **csc** α + **tan** α **sec** α by substituting the values of the trigonometric functions, **cos** α, **csc** α,**tan** α, **sec** α.

It is given that **sin** $\alpha = \frac{1}{9}$. Here 1 denotes the length of the side opposite α, and 9 denotes the length of the hypotenuse.

Consider a right triangle with hypotenuse of length 9 and the side opposite α of length 1.



Find the value of b by using the Pythagorean theorem.

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean theorem} \\ b^2 &= c^2 - a^2 \\ b^2 &= (9)^2 - (1)^2 && \text{Replace a with 1 and c with 9.} \end{aligned}$$

Take the squares of 9 and 1.

$$\begin{aligned} b^2 &= (9)^2 - (1)^2 && \text{Replace a with 1 and c with 9.} \\ b^2 &= 81 - 1 \\ b^2 &= 81 - 1 \\ b^2 &= 80 && \text{Simplify.} \end{aligned}$$

Since b is a length, it must be a positive number. Take the square root of both sides of the equation to find b.

$$b = 4\sqrt{5}$$

Recall that if a = length of the side opposite α, b = length of the side adjacent to α, and c = length of the hypotenuse then,
cos $\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$, **csc** $\alpha = \frac{\text{hypotenuse}}{\text{opposite}}$, **tan** $\alpha = \frac{\text{opposite}}{\text{adjacent}}$, and **sec** $\alpha = \frac{\text{hypotenuse}}{\text{adjacent}}$.

With the length of the adjacent side, $b = 4\sqrt{5}$, the length of the opposite side, a = 1 and the length of the hypotenuse, c = 9, find the values of the trigonometric functions.

$$\cos\alpha = \frac{4\sqrt{5}}{9},\ \csc\alpha = 9,\ \tan\alpha = \frac{1}{4\sqrt{5}},\ \text{and } \sec\alpha = \frac{9}{4\sqrt{5}}$$

Substitute the values of **cos** α, **csc** α, **tan** α, and **sec** α in the expression **cos** α **csc** α + **tan** α **sec** α.

$$\begin{aligned} \cos\alpha\csc\alpha + \tan\alpha\sec\alpha &= \frac{4\sqrt{5}}{9} \cdot 9 + \frac{1}{4\sqrt{5}} \cdot \frac{9}{4\sqrt{5}} \\ &= \frac{4\sqrt{5}}{\cancel{9}} \cdot \cancel{9} + \frac{1}{4\sqrt{5}} \cdot \frac{9}{4\sqrt{5}} && \text{Cancel out the common term.} \\ &= 4\sqrt{5} + \frac{9}{80} && \text{Multiply.} \end{aligned}$$

Simplify.

$$4\sqrt{5} + \frac{9}{80} = \frac{320\sqrt{5} + 9}{80}$$

Thus, if α is an acute angle with **sin** $\alpha = \frac{1}{9}$, then the value of **cos** α **csc** α + **tan** α **sec** $\alpha = \frac{320\sqrt{5} + 9}{80}$.

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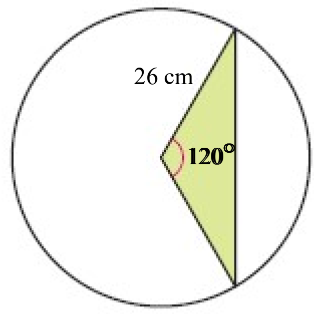
Find the area of the triangle in the given figure.



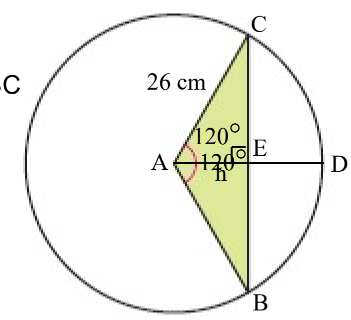
Note that the area of a triangle is $\frac{1}{2} \cdot \text{base} \cdot \text{height}$.

To find the area of the triangle, first determine the height.

Since AC = 26 cm is the radius of the circle and one side of the triangle ABC, 26 is not the height of the triangle.

Let h be the height of the triangle. Name the vertices of the triangle as A, B, and C respectively.

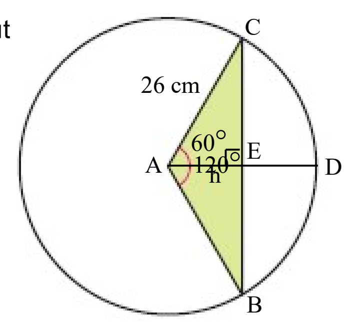
Let D be the point on the circle such that radius AD bisects ∠BAC and E is the midpoint of side BC such that CE = EB.



Since the three sides of one triangle are equal to the corresponding sides of a second triangle, the two triangles AEC and AEB are congruent.

Since the radius AD bisects ∠BAC, the measure of ∠EAC is 60°.

Now use a trigonometric function to determine the length of the side h. Given the information about triangle AEC, sine function should be used because it involves the adjecent side h, and the hypotenuse AC.

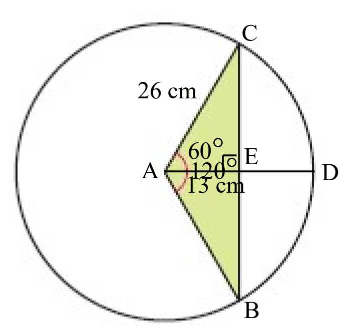


Since AEC is a right triangle, $\textbf{cos}\ 60 = \frac{h}{26}$.

Multiply both sides by 26.

$$26\ \textbf{cos}\ 60 = h$$
$$13 = h \qquad \text{Simplify.}$$

Thus, the height of the triangle is 13 cm.



Now find the base of the triangle. The base is the length of side BC.

Apply the Pythagorean theorem to the triangle ABC, to find the base BC. Let y be the length of segment EC, which is one-half the length of the base.

$$(26)^2 = (13)^2 + y^2$$
$$676 = 169 + y^2 \qquad \text{Simplify.}$$
$$507 = y^2 \qquad \text{Subtract 169 from both sides.}$$

Take the square root of both sides.

$$\begin{aligned} y^2 &= 507 \\ y &= \pm\sqrt{507} \\ &= \pm 13\sqrt{3} \qquad \text{Simplify.} \end{aligned}$$

Since distance is always positive, $y = 13\sqrt{3}$.

Determine the length of side BC.

$$\begin{aligned} BC &= 2 \cdot y \\ &= 2 \cdot 13\sqrt{3} \\ &= 26\sqrt{3} \qquad \text{Multiply.} \end{aligned}$$

Thus, the base is $26\sqrt{3}$ cm.

Find the area of the triangle.

$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot \text{base} \cdot \text{height} \\ &= \frac{1}{2}\left(26\sqrt{3}\right)(13) \\ &= 169\sqrt{3} \qquad \text{Simplify.} \end{aligned}$$

Therefore, the area of the triangle is $169\sqrt{3}$ cm^2.