Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the first and second derivatives.

$y = -5x^4 + 7$

To find $\frac{dy}{dx}$, first use the Derivative Sum Rule, which states if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

$$\frac{dy}{dx} = \frac{d}{dx}(-5x^4 + 7) = \frac{d}{dx}(-5x^4) + \frac{d}{dx}(7)$$

To find $\frac{d}{dx}(-5x^4)$, use the Constant Multiple Rule and the Power Rule for Positive Integers.

The Constant Multiple Rule states that if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

The Power Rule for Positive Integers states that if n is a positive integer, then
$\frac{d}{dx}(x^n) = nx^{n-1}$.

$$\begin{aligned}\frac{d}{dx}(-5x^4) &= -5\frac{d}{dx}(x^4)\\ &= -5 \cdot 4x^3\\ &= -20x^3\end{aligned}$$

To find $\frac{d}{dx}(7)$, use the Derivative of a Constant Function Rule, which states if f has the constant value f(x) = c, then
$\frac{df}{dx} = \frac{d}{dx}(c) = 0$.

$$\frac{d}{dx}(7) = 0$$

Thus, $\frac{dy}{dx} = \frac{d}{dx}(-5x^4 + 7) = -20x^3$.

Now find $\frac{d^2y}{dx^2}$. Note that $f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy'}{dx} = y''$.

To find $\frac{d}{dx}(-20x^3)$, again use the Constant Multiple Rule and the Power Rule for Positive Integers.

$$\frac{d}{dx}(-20x^3) = -60x^2$$

Thus, $\frac{dy}{dx} = -20x^3$ and $\frac{d^2y}{dx^2} = -60x^2$.

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the first and second derivatives.

$s = 4t^3 - 3t^4$

The derivative sum rule states that if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

Begin by finding the first derivative. By the above rule it is possible to differentiate the polynomial $4t^3 - 3t^4$ term by term. Write $\frac{d}{dt}\left(4t^3 - 3t^4\right)$ as a sum of differentiable functions.

$$\frac{d}{dt}\left(4t^3 - 3t^4\right) = \frac{d}{dt}\left(4t^3\right) + \frac{d}{dt}\left(-3t^4\right)$$

The constant multiple rule states that if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

Differentiate the first term. Begin by rewriting the expression using the constant multiple rule.

$$\frac{d}{dt}(4t^3) = 4\frac{d}{dt}t^3$$

The power rule for positive integers states that if n is a positive integer, then $\frac{d}{dx}x^n = nx^{n-1}$.

Differentiate the function using the power rule for positive integers.

$$4 \cdot \frac{d}{dt}t^3 = 4 \cdot 3t^2$$

Now simplify.

$$4 \cdot 3t^2 = 12t^2$$

Thus, $\frac{d}{dt}(4t^3) = 12t^2$.

Now use the same method to find the derivative of the second term.

$$\frac{d}{dt}\left(-3t^4\right) = -3\frac{d}{dt}t^4 = -12t^3$$

Thus, the derivatives of each term individually are $12t^2$ and $-12t^3$.

Combine all derivatives to find the first derivative of the original expression.

$$\begin{aligned}\frac{ds}{dt} &= \frac{d}{dt}\left(4t^3 - 3t^4\right)\\ &= 12t^2 - 12t^3\end{aligned}$$

Now find the second derivative of $s = 4t^3 - 3t^4$ by differentiating $\frac{ds}{dt}$.

$$\frac{d^2}{dt^2}\left(4t^3 - 3t^4\right) = \frac{d}{dt}\left(12t^2 - 12t^3\right)$$

Use the sum rule and write $\frac{d}{dt}(12t^2 - 12t^3)$ as a sum of differentiable functions.

$$\frac{d}{dt}(12t^2 - 12t^3) = \frac{d}{dt}\left(12t^2\right) + \frac{d}{dt}\left(-12t^3\right)$$

Differentiate the first term.

$$\frac{d}{dt}\left(12t^2\right) = 24t$$

Now find the derivative of the last term.

$$\frac{d}{dt}\left(-12t^3\right) = -36t^2$$

Thus, the derivatives of each term individually are 24t and $-36t^2$.

Now combine all derivatives to find the second derivative of the original expression.

$$\begin{aligned}\frac{d^2s}{dt^2} &= \frac{d}{dt}(12t^2 - 12t^3)\\ &= 24t - 36t^2\end{aligned}$$

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the first and second derivatives.

$y = 4x^{-2} - \frac{3}{x}$

To find $\frac{dy}{dx}$, first use the Derivative Sum Rule, which states if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^{-2} - \frac{3}{x}\right) = \frac{d}{dx}(4x^{-2}) - \frac{d}{dx}\left(\frac{3}{x}\right)$$

To find $\frac{d}{dx}(4x^{-2})$, use the Constant Multiple Rule and the Power Rule for Negative Integers.

The Constant Multiple Rule states that if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

The Power Rule for Negative Integers states that if n is a negative integer and $x \neq 0$, then $\frac{d}{dx}(x^n) = nx^{n-1}$.

$$\begin{aligned}\frac{d}{dx}(4x^{-2}) &= 4\frac{d}{dx}(x^{-2})\\ &= 4(-2x^{-3})\\ &= -8x^{-3}\end{aligned}$$

Before evaluating $\frac{d}{dx}\left(\frac{3}{x}\right)$, write $\frac{3}{x}$ using exponents.

$$\frac{3}{x} = 3x^{-1}$$

To find $\frac{d}{dx}(3x^{-1})$, use the Constant Multiple Rule and the Power Rule for Negative Integers.

$$\frac{d}{dx}(3x^{-1}) = -3x^{-2}$$

Thus, $$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(4x^{-2} - \frac{3}{x}\right)\\ &= -8x^{-3} - (-3x^{-2})\\ &= -8x^{-3} + 3x^{-2}\\ &= -\frac{8}{x^3} + \frac{3}{x^2}.\end{aligned}$$

Now find $\frac{d^2y}{dx^2}$. Note that $f''(x) = \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy'}{dx} = y''$.

To find $\frac{d}{dx}(-8x^{-3} + 3x^{-2})$, again first use the Derivative Sum Rule, and then use the Constant Multiple Rule and the Power

Rule for Negative Integers.

$$\frac{d}{dx}(-8x^{-3}+3x^{-2}) = 24x^{-4}-6x^{-3}$$
$$= \frac{24}{x^4}-\frac{6}{x^3}$$

Thus, $\frac{dy}{dx} = -\frac{8}{x^3}+\frac{3}{x^2}$ and $\frac{d^2y}{dx^2} = \frac{24}{x^4}-\frac{6}{x^3}$.

Solution

Simplify $3x^{-1}$: $\frac{3}{x}$

Steps

$3x^{-1}$

Apply exponent rule: $a^{-1}=\frac{1}{a}$

$=3\cdot\frac{1}{x}$

Multiply fractions: $a\cdot\frac{b}{c}=\frac{a\,\cdot\,b}{c}$

$=\frac{1\cdot 3}{x}$

Multiply the numbers: $1\cdot\ 3=3$

$=\frac{3}{x}$

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the first and second derivatives.

$$y = 4x^2 - 15x - 5x^{-2}$$

The derivative sum rule states that if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

Begin by finding the first derivative. By the above rule it is possible to differentiate$4x^2 - 15x - 5x^{-2}$ term by term. Write $\frac{d}{dx}\left(4x^2 - 15x - 5x^{-2}\right)$ as a sum of differentiable functions.

$$\frac{d}{dx}\left(4x^2 - 15x - 5x^{-2}\right) = \frac{d}{dx}\left(4x^2\right) + \frac{d}{dx}(-15x) + \frac{d}{dx}\left(-5x^{-2}\right)$$

The constant multiple rule states that if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

Differentiate the first term. Begin by rewriting the expression using the constant multiple rule.

$$\frac{d}{dx}\left(4x^2\right) = 4 \bullet \frac{d}{dx}x^2$$

The power rule for positive integers states that if n is a positive integer, then $\frac{d}{dx}x^n = nx^{n-1}$.

Differentiate the function using the power rule for positive integers.

$$4 \bullet \frac{d}{dx}x^2 = 4 \bullet 2x$$

Now simplify.

$$4 \bullet 2x = 8x$$

Thus, $\frac{d}{dx}\left(4x^2\right) = 8x$.

Now use the same method to find the derivative of the second term.

$$\frac{d}{dx}(-15x) = -15\frac{d}{dx}(x) = -15$$

The power rule for negative integers states that if n is a negative integer and $x \neq 0$, then $\frac{d}{dx}x^n = nx^{n-1}$.

Find the derivative of the last term.

$$\frac{d}{dx}\left(-5x^{-2}\right) = 10x^{-3}$$

Thus, the derivatives of each term individually are $8x$, -15, and $10x^{-3}$.

Combine all derivatives to find the first derivative of the original expression.

$$\frac{dy}{dx} = \frac{d}{dx}\left(4x^2 - 15x - 5x^{-2}\right)$$
$$= 8x - 15 + 10x^{-3}$$
$$= 8x - 15 + \frac{10}{x^3}$$

Now find the second derivative of $y = 4x^2 - 15x - 5x^{-2}$ by differentiating $\frac{dy}{dx}$.

$$\frac{d^2}{dx^2}\left(4x^2 - 15x - 5x^{-2}\right) = \frac{d}{dx}\left(8x - 15 + 10x^{-3}\right)$$

Use the sum rule and write $\frac{d}{dx}\left(8x - 15 + 10x^{-3}\right)$ as a sum of differentiable functions.

$$\frac{d}{dx}\left(8x - 15 + 10x^{-3}\right) = \frac{d}{dx}(8x) + \frac{d}{dx}(-15) + \frac{d}{dx}\left(10x^{-3}\right)$$

Differentiate the first term.

$$\frac{d}{dx}(8x) = 8$$

The rule of differentiation of a constant states that if f has the constant value $f(x) = c$, then $\frac{df}{dx} = \frac{d}{dx}(c) = 0$.

Now find the derivative of the second term.

$$\frac{d}{dx}(-15) = 0$$

Differentiate the last term.

$$\frac{d}{dx}\left(10x^{-3}\right) = -30x^{-4}$$

Thus, the derivatives of each term individually are 8, 0, and $-30x^{-4}$.

Now combine all derivatives to find the second derivative of the original expression.

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(8x - 15 + 10x^{-3}\right)$$
$$= 8 - 30x^{-4}$$
$$= 8 - \frac{30}{x^4}$$

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the first and second derivatives.

$$r = \frac{1}{5s^4} - \frac{5}{2s}$$

The derivative sum rule states that if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

Begin by finding the first derivative. By the above rule it is possible to differentiate $\frac{1}{5s^4} - \frac{5}{2s}$ term by term. Write $\frac{d}{ds}\left(\frac{1}{5s^4} - \frac{5}{2s}\right)$ as a sum of differentiable functions.

$$\frac{d}{ds}\left(\frac{1}{5s^4} - \frac{5}{2s}\right) = \frac{d}{ds}\left(\frac{1}{5s^4}\right) + \frac{d}{ds}\left(-\frac{5}{2s}\right)$$

The constant multiple rule states that if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

Differentiate the first term. Begin by rewriting the expression using the constant multiple rule.

$$\frac{d}{ds}\left(\frac{1}{5s^4}\right) = \frac{d}{ds}\left(\frac{1}{5}s^{-4}\right) = \frac{1}{5} \bullet \frac{d}{ds}s^{-4}$$

The power rule for negative integers states that if n is a negative integer and $x \neq 0$, then $\frac{d}{dx}x^n = nx^{n-1}$.

Differentiate the function using the power rule for negative integers.

$$\frac{1}{5} \bullet \frac{d}{ds}s^{-4} = \frac{1}{5} \bullet \left(-4s^{-5}\right)$$

Now simplify.

$$\frac{1}{5} \bullet \left(-4s^{-5}\right) = -\frac{4}{5}s^{-5}$$

Thus, $\frac{d}{ds}\left(\frac{1}{5s^4}\right) = -\frac{4}{5}s^{-5}$.

Now use the same method to find the derivative of the second term.

$$\frac{d}{ds}\left(-\frac{5}{2s}\right) = \frac{d}{ds}\left(-\frac{5}{2}s^{-1}\right) = \frac{5}{2}s^{-2}$$

Thus, the derivatives of each term individually are $-\frac{4}{5}s^{-5}$ and $\frac{5}{2}s^{-2}$.

Combine all derivatives to find the first derivative of the original expression.

$$\begin{aligned}\frac{dr}{ds} &= \frac{d}{ds}\left(\frac{1}{5s^4} - \frac{5}{2s}\right)\\ &= -\frac{4}{5}s^{-5} + \frac{5}{2}s^{-2}\\ &= -\frac{4}{5s^5} + \frac{5}{2s^2}\end{aligned}$$

Now find the second derivative of $r = \frac{1}{5s^4} - \frac{5}{2s}$ by differentiating $\frac{dr}{ds}$.

$$\frac{d^2}{ds^2}\left(\frac{1}{5s^4} - \frac{5}{2s}\right) = \frac{d}{ds}\left(-\frac{4}{5s^5} + \frac{5}{2s^2}\right)$$

Use the sum rule and write $\frac{d}{ds}\left(-\frac{4}{5s^5} + \frac{5}{2s^2}\right)$ as a sum of differentiable functions.

$$\frac{d}{ds}\left(-\frac{4}{5s^5} + \frac{5}{2s^2}\right) = \frac{d}{ds}\left(-\frac{4}{5s^5}\right) + \frac{d}{ds}\left(\frac{5}{2s^2}\right)$$

Differentiate the first term.

$$\frac{d}{ds}\left(-\frac{4}{5s^5}\right) = \frac{d}{ds}\left(-\frac{4}{5}s^{-5}\right) = 4s^{-6}$$

Differentiate the last term.

$$\frac{d}{ds}\left(\frac{5}{2s^2}\right) = \frac{d}{ds}\left(\frac{5}{2}s^{-2}\right) = -5s^{-3}$$

Thus, the derivatives of each term individually are $4s^{-6}$ and $-5s^{-3}$.

Now combine all derivatives to find the second derivative of the original expression.

$$\begin{aligned}\frac{d^2r}{ds^2} &= \frac{d}{ds}\left(-\frac{4}{5s^5} + \frac{5}{2s^2}\right)\\ &= 4s^{-6} - 5s^{-3}\\ &= \frac{4}{s^6} - \frac{5}{s^3}\end{aligned}$$

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find y′ by **(a)** applying the product rule and **(b)** multiplying the factors to produce a sum of simpler terms to differentiate.

$$y = \left(7 - x^2\right)\left(x^3 - x + 5\right)$$

(a) First find y′ by applying the product rule.

The derivative product rule states that if u and v are differentiable functions of x, then so is their product uv. The derivative of uv can be calculated as shown below.

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Begin by rewriting the expression using the product rule.

$$\frac{d}{dx}\left(\left(7 - x^2\right)\left(x^3 - x + 5\right)\right) = \left(7 - x^2\right)\frac{d}{dx}\left(x^3 - x + 5\right) + \left(x^3 - x + 5\right)\frac{d}{dx}\left(7 - x^2\right)$$

The derivative sum rule states that if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$.

By this rule it is possible to differentiate the polynomial $x^3 - x + 5$ term by term. Then add the derivatives together.

$$\frac{d}{dx}\left(x^3 - x + 5\right) = \left(3x^2 - 1\right)$$

Use the same method to differentiate $7 - x^2$.

$$\frac{d}{dx}\left(7 - x^2\right) = -2x$$

Now substitute the derivatives found above into the product rule.

$$\begin{aligned}\frac{d}{dx}\left(\left(7 - x^2\right)\left(x^3 - x + 5\right)\right) &= \left(7 - x^2\right)\frac{d}{dx}\left(x^3 - x + 5\right) + \left(x^3 - x + 5\right)\frac{d}{dx}\left(7 - x^2\right)\\ &= \left(7 - x^2\right)\left(3x^2 - 1\right) + \left(x^3 - x + 5\right)(-2x)\end{aligned}$$

Simplify to find y′.

$$\begin{aligned}y' &= \left(7 - x^2\right)\left(\left(3x^2 - 1\right)\right) + \left(x^3 - x + 5\right)(-2x)\\ &= \left(21x^2 - 7 - 3x^4 + x^2\right) + \left(-2x^4 + 2x^2 - 10x\right)\\ &= 21x^2 - 7 - 3x^4 + x^2 - 2x^4 + 2x^2 - 10x\\ &= -5x^4 + 24x^2 - 10x - 7\end{aligned}$$

(b) Next find y′ by multiplying the factors to produce a sum of simpler terms to differentiate.

Multiply.

$$\begin{aligned}y &= \left(\left(7 - x^2\right)\left(x^3 - x + 5\right)\right)\\ &= 7x^3 - 7x + 35 - x^5 + x^3 - 5x^2\\ &= -x^5 + 8x^3 - 5x^2 - 7x + 35\end{aligned}$$

Now differentiate $-x^5 + 8x^3 - 5x^2 - 7x + 35$.

$y' = -5x^4 + 24x^2 - 10x - 7$

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find y ′ by applying the Product Rule, and then find y ′ by multiplying the factors to produce a sum of simpler terms to differentiate.

$$y = (4x^2 + 9)\left(2x - 3 + \frac{7}{x}\right)$$

First, find y ′ by applying the Derivative Product Rule, which states if u and v are differentiable at x, then so is their product uv, and $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$.

$$y' = \frac{d}{dx}\left(\left(4x^2 + 9\right)\left(2x - 3 + \frac{7}{x}\right)\right)$$
$$= (4x^2 + 9)\frac{d}{dx}\left(2x - 3 + \frac{7}{x}\right) + \left(2x - 3 + \frac{7}{x}\right)\frac{d}{dx}(4x^2 + 9)$$

To find $\frac{d}{dx}\left(2x - 3 + \frac{7}{x}\right)$, use the Derivative Sum Rule, which states if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$.

$$\frac{d}{dx}\left(2x - 3 + \frac{7}{x}\right) = \frac{d}{dx}(2x) + \frac{d}{dx}(-3) + \frac{d}{dx}\left(\frac{7}{x}\right)$$

To find $\frac{d}{dx}(2x)$, use the Constant Multiple Rule, which states if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

$$\frac{d}{dx}(2x) = 2\frac{d}{dx}(x) = 2$$

To find $\frac{d}{dx}(-3)$, use the Derivative of a Constant Function Rule, which states if f has the constant value f(x) = c, then $\frac{df}{dx} = \frac{d}{dx}(c) = 0$.

$$\frac{d}{dx}(-3) = 0$$

Before evaluating $\frac{d}{dx}\left(\frac{7}{x}\right)$, write $\frac{7}{x}$ using exponents.

$$\frac{7}{x} = 7x^{-1}$$

To find $\frac{d}{dx}(7x^{-1})$, use the Constant Multiple Rule and the Power Rule for Negative Integers. The Power Rule for Negative Integers states that if n is a negative integer and $x \neq 0$, then $\frac{d}{dx}(x^n) = nx^{n-1}$.

$$\frac{d}{dx}(7x^{-1}) = -\frac{7}{x^2}$$

Thus, $\frac{d}{dx}\left(2x - 3 + \frac{7}{x}\right) = 2 - \frac{7}{x^2}$.

To find $\frac{d}{dx}(4x^2 + 9)$, use the Derivative Sum Rule.

$$\frac{d}{dx}(4x^2 + 9) = \frac{d}{dx}(4x^2) + \frac{d}{dx}(9)$$

To find $\frac{d}{dx}(4x^2)$, use the Constant Multiple Rule and the Power Rule for Positive Integers. The Power Rule for Positive Integers states that if n is a positive integer, then $\frac{d}{dx}(x^n) = nx^{n-1}$.

$$\frac{d}{dx}(4x^2) = 8x$$

To find $\frac{d}{dx}(9)$, use the Derivative of a Constant Function Rule.

$$\frac{d}{dx}(9) = 0$$

Thus, $\frac{d}{dx}(4x^2 + 9) = 8x$.

Now simplify.

$$(4x^2 + 9)\frac{d}{dx}\left(2x - 3 + \frac{7}{x}\right) + \left(2x - 3 + \frac{7}{x}\right)\frac{d}{dx}(4x^2 + 9)$$
$$= (4x^2 + 9)\left(2 - \frac{7}{x^2}\right) + \left(2x - 3 + \frac{7}{x}\right)(8x)$$
$$= 24x^2 - 24x + 46 - \frac{63}{x^2}$$

Thus, by applying the Derivative Product Rule, $y' = 24x^2 - 24x + 46 - \frac{63}{x^2}$.

Now find y' by multiplying the factors to produce a sum of simpler terms to differentiate.

Multiply.

$$(4x^2 + 9)\left(2x - 3 + \frac{7}{x}\right) = 8x^3 - 12x^2 + 46x - 27 + \frac{63}{x}$$

To find $\frac{d}{dx}\left(8x^3 - 12x^2 + 46x - 27 + \frac{63}{x}\right)$, use the Derivative Sum Rule.

$$\frac{d}{dx}\left(8x^3 - 12x^2 + 46x - 27 + \frac{63}{x}\right)$$
$$= \frac{d}{dx}(8x^3) + \frac{d}{dx}(-12x^2) + \frac{d}{dx}(46x) + \frac{d}{dx}(-27) + \frac{d}{dx}\left(\frac{63}{x}\right)$$

Find the derivatives of each of the differentiable functions.

$\frac{d}{dx}(8x^3) = 24x^2$ Constant Multiple Rule, Power Rule for Positive Integers

$\frac{d}{dx}(-12x^2) = 24x$ Constant Multiple Rule, Power Rule for Positive Integers

$\frac{d}{dx}(46x) = 46$ Constant Multiple Rule, Power Rule for Positive Integers

$\frac{d}{dx}(-27) = 0$ Derivative of a Constant Function Rule

$\frac{d}{dx}\left(\frac{63}{x}\right) = -\frac{63}{x^2}$ Constant Multiple Rule, Power Rule for Negative Integers

Thus, by multiplying the factors to produce a sum of simpler terms to differentiate, $y' = 24x^2 - 24x + 46 - \frac{63}{x^2}$.

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the derivative of the function.

$$y = \frac{5x-2}{4x+3}$$

To find y', use the Derivative Quotient Rule, which states if u and v are differentiable at x and if $v(x) \neq 0$, then the quotient $\frac{u}{v}$ is differentiable at x, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}.$$

$$y' = \frac{d}{dx}\left(\frac{5x-2}{4x+3}\right) = \frac{(4x+3)\frac{d}{dx}(5x-2) - (5x-2)\frac{d}{dx}(4x+3)}{(4x+3)^2}$$

To find $\frac{d}{dx}(5x-2)$, use the Derivative Sum Rule, which states if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

$$\frac{d}{dx}(5x-2) = \frac{d}{dx}(5x) - \frac{d}{dx}(2)$$

To find $\frac{d}{dx}(5x)$, use the Constant Multiple Rule and the Power Rule for Positive Integers.

The Constant Multiple Rule states if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

The Power Rule for Positive Integers states if n is a positive integer, then
$\frac{d}{dx}(x^n) = nx^{n-1}$.

$$\frac{d}{dx}(5x) = 5$$

To find $\frac{d}{dx}(2)$, use the Derivative of a Constant Function Rule, which states if f has the constant value f(x) = c, then
$\frac{df}{dx} = \frac{d}{dx}(c) = 0$.

$$\frac{d}{dx}(2) = 0$$

Thus, $\frac{d}{dx}(5x-2) = \frac{d}{dx}(5x) - \frac{d}{dx}(2)$
$= 5 - 0$
$= 5.$

To find $\frac{d}{dx}(4x+3)$, use the Derivative Sum Rule.

$$\frac{d}{dx}(4x+3) = \frac{d}{dx}(4x) + \frac{d}{dx}(3)$$

To find $\frac{d}{dx}(4x)$, use the Constant Multiple Rule and the Power Rule for Positive Integers.

$$\frac{d}{dx}(4x) = 4$$

To find $\frac{d}{dx}(3)$, use the Derivative of a Constant Function Rule.

$$\frac{d}{dx}(3) = 0$$

Thus, $\frac{d}{dx}(4x+3) = 4$.

Now simplify.

$$\begin{aligned}\frac{(4x+3)\frac{d}{dx}(5x-2) - (5x-2)\frac{d}{dx}(4x+3)}{(4x+3)^2} &= \frac{(4x+3)(5) - (5x-2)(4)}{(4x+3)^2} \\ &= \frac{23}{(4x+3)^2}\end{aligned}$$

Thus, $y' = \frac{d}{dx}\left(\frac{5x-2}{4x+3}\right) = \frac{23}{(4x+3)^2}$.

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the derivative of the function.

$$y = \frac{5x-7}{x^2+8x}$$

According to the Quotient Rule if u and v are differentiable functions of x and if $v(x) \neq 0$, then the quotient is differentiable at x.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Notice that $y = \frac{5x-7}{x^2+8x}$ is the quotient of two functions. Let $v(x) = x^2 + 8x$ and $u(x) = 5x - 7$.

Set up the derivative.

$$y = \frac{5x-7}{x^2+8x}$$

$$y' = \frac{\left(x^2+8x\right) \cdot \frac{d}{dx}(5x-7) - (5x-7) \cdot \frac{d}{dx}\left(x^2+8x\right)}{\left(x^2+8x\right)^2}$$

Now differentiate. Evaluate the expression $\frac{d}{dx}(5x-7)$ first. Use the Constant Multiple Rule and the Constant Rule.

$$\begin{aligned} y' &= \frac{\left(x^2+8x\right) \cdot \frac{d}{dx}(5x-7) - (5x-7) \cdot \frac{d}{dx}\left(x^2+8x\right)}{\left(x^2+8x\right)^2} \\ &= \frac{\left(x^2+8x\right)(5) - (5x-7) \cdot \frac{d}{dx}\left(x^2+8x\right)}{\left(x^2+8x\right)^2} \end{aligned}$$

Evaluate the expression $\frac{d}{dx}\left(x^2+8x\right)$. Use the Power Rule and the Constant Multiple Rule.

$$\begin{aligned} y' &= \frac{\left(x^2+8x\right)(5) - (5x-7) \cdot \frac{d}{dx}\left(x^2+8x\right)}{\left(x^2+8x\right)^2} \\ &= \frac{\left(x^2+8x\right)(5) - (5x-7)(2x+8)}{\left(x^2+8x\right)^2} \end{aligned}$$

Now simplify the numerator.

$$y' = \frac{5x^2 + 40x - \left(10x^2 + 40x - 14x - 56\right)}{\left(x^2+8x\right)^2}$$ Use the distributive property and the FOIL method.

$$= \frac{5x^2 + 40x - 10x^2 - 40x + 14x + 56}{\left(x^2 + 8x\right)^2}$$

Multiply the negative sign through the parentheses.

$$= \frac{-5x^2 + 14x + 56}{\left(x^2 + 8x\right)^2}$$

Combine like terms in the numerator.

Therefore, if $y = \frac{5x - 7}{x^2 + 8x}$, then $y' = \frac{-5x^2 + 14x + 56}{\left(x^2 + 8x\right)^2}$.

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the derivative of the function.

$$y = (7t - 1)(8t - 3)^{-1}$$

Begin by rewriting the function with all positive exponents.

$$(7t - 1)(8t - 3)^{-1} = \frac{7t - 1}{8t - 3}$$

Next apply the Quotient Rule, which states that if u and v are differentiable at x and if $v(x) \neq 0$, then the quotient $\frac{u}{v}$ is differentiable at t, with derivative as shown below.

$$\frac{d}{dt}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dt} - u\frac{dv}{dt}}{v^2}$$

Define u and v.

$$u = 7t - 1$$
$$v = 8t - 3$$

Notice that for the quotient rule, u' and v' are needed. Find the derivative of $7t - 1$.

$$\begin{aligned} u &= 7t - 1 \\ u' &= 7 \end{aligned}$$

Find the derivative of $8t - 3$.

$$\begin{aligned} v &= 8t - 3 \\ v' &= 8 \end{aligned}$$

Apply the quotient rule. Remember that $u = 7t - 1$, $u' = 7$, $v = 8t - 3$, and $v' = 8$.

$$\begin{aligned} \frac{d}{dt}\left(\frac{u}{v}\right) &= \frac{v\frac{du}{dt} - u\frac{dv}{dt}}{v^2} \\ \frac{dy}{dt}\left(\frac{7t - 1}{8t - 3}\right) &= \frac{(8t - 3)(7) - (7t - 1)(8)}{[8t - 3]^2} && \text{Substitute.} \\ \frac{dy}{dt}\ \frac{7t - 1}{8t - 3} &= -\frac{13}{(8t - 3)^2} && \text{Simplify.} \end{aligned}$$

Therefore, the derivative of the given function is shown below.

$$\frac{dy}{dt}(7t - 1)(8t - 3)^{-1} = -\frac{13}{(8t - 3)^2}$$

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the derivative of the function.

$$f(s) = \frac{\sqrt{s} - 9}{\sqrt{s} + 9}$$

The derivative quotient rule states that if u and v are differentiable at x and $v(x) \neq 0$, then the quotient $\frac{u}{v}$ is differentiable at x.

The derivative of a quotient is calculated as follows.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

In the expression $\frac{\sqrt{s} - 9}{\sqrt{s} + 9}$, $\sqrt{s} - 9$ corresponds to u and $\sqrt{s} + 9$ corresponds to v.

Rewrite $\frac{\sqrt{s} - 9}{\sqrt{s} + 9}$ according to the derivative quotient rule.

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{d}{ds}\left(\frac{\sqrt{s} - 9}{\sqrt{s} + 9}\right) = \frac{\left(\sqrt{s} + 9\right) \cdot \frac{d}{ds}\left(\sqrt{s} - 9\right) - \left(\sqrt{s} - 9\right) \cdot \frac{d}{ds}\left(\sqrt{s} + 9\right)}{\left(\sqrt{s} + 9\right)^2}$$

Now evaluate $\frac{d}{ds}\left(\sqrt{s} - 9\right)$.

$$\frac{d}{ds}\left(\sqrt{s} - 9\right) = \frac{1}{2\sqrt{s}}$$

Next evaluate $\frac{d}{ds}\left(\sqrt{s} + 9\right)$.

$$\frac{d}{ds}\left(\sqrt{s} + 9\right) = \frac{1}{2\sqrt{s}}$$

Substitute $\frac{d}{ds}\left(\sqrt{s} - 9\right)$ and $\frac{d}{ds}\left(\sqrt{s} + 9\right)$ into the expression for $\frac{d}{ds}\left(\frac{\sqrt{s} - 9}{\sqrt{s} + 9}\right)$.

$$\frac{d}{ds}\left(\frac{\sqrt{s}-9}{\sqrt{s}+9}\right) = \frac{\left(\sqrt{s}+9\right)\cdot\frac{d}{ds}\left(\sqrt{s}-9\right)-\left(\sqrt{s}-9\right)\cdot\frac{d}{ds}\left(\sqrt{s}+9\right)}{\left(\sqrt{s}+9\right)^2}$$

$$= \frac{\left(\sqrt{s}+9\right)\cdot\left(\frac{1}{2\sqrt{s}}\right)-\left(\sqrt{s}-9\right)\cdot\left(\frac{1}{2\sqrt{s}}\right)}{\left(\sqrt{s}+9\right)^2}$$

Now simplify the numerator.

$$\frac{d}{ds}\left(\frac{\sqrt{s}-9}{\sqrt{s}+9}\right) = \frac{\left(\sqrt{s}+9\right)\cdot\left(\frac{1}{2\sqrt{s}}\right)-\left(\sqrt{s}-9\right)\cdot\left(\frac{1}{2\sqrt{s}}\right)}{\left(\sqrt{s}+9\right)^2}$$

$$= \frac{\left(\sqrt{s}+9\right)-\left(\sqrt{s}-9\right)}{2\sqrt{s}\left(\sqrt{s}+9\right)^2}$$

$$\frac{d}{ds}\ \frac{s-9}{s+9} = \frac{18}{2\sqrt{s}\left(\sqrt{s}+9\right)^2}$$

$$= \frac{9}{\sqrt{s}\left(\sqrt{s}+9\right)^2}$$

Thus, $f'(s) = \frac{9}{\sqrt{s}\left(\sqrt{s}+9\right)^2}$.

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the derivative of all orders of the function.

$$y = \frac{x^4}{4} + \frac{5}{6}x^3 - x^2 + 7x - 8$$

To find the derivatives of all orders of a function means to find the derivative of the function and the derivatives of derivatives until a derivative is zero.

To find y', use the Derivative Sum Rule, which states if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

$$\begin{aligned} y' = \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{x^4}{4} + \frac{5}{6}x^3 - x^2 + 7x - 8\right) \\ &= \frac{d}{dx}\left(\frac{x^4}{4}\right) + \frac{d}{dx}\left(\frac{5}{6}x^3\right) - \frac{d}{dx}(x^2) + \frac{d}{dx}(7x) - \frac{d}{dx}(8) \end{aligned}$$

To find $\frac{d}{dx}\left(\frac{x^4}{4}\right)$, use the Constant Multiple Rule and the Power Rule for Positive Integers.

The Constant Multiple Rule states that if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

The Power Rule for Positive Integers states that if n is a positive integer, then $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$.

$$\frac{d}{dx}\left(\frac{x^4}{4}\right) = \frac{4x^{4-1}}{4} = x^3$$

Similarly, $\frac{d}{dx}\left(\frac{5}{6}x^3\right) = \frac{5}{2}x^2$.

Using the Power Rule for Positive Integers, $\frac{d}{dx}(x^2) = 2x$.

Using the Constant Multiple Rule, $\frac{d}{dx}(7x) = 7$.

To find $\frac{d}{dx}(8)$, use the Derivative of a Constant Function Rule, which states if f has the constant value f(x) = c, then $\frac{df}{dx} = \frac{d}{dx}(c) = 0$.

$$\frac{d}{dx}(8) = 0$$

Thus, $$\begin{aligned} y' = \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{x^4}{4} + \frac{5}{6}x^3 - x^2 + 7x - 8\right) \\ &= \frac{d}{dx}\left(\frac{x^4}{4}\right) + \frac{d}{dx}\left(\frac{5}{6}x^3\right) - \frac{d}{dx}(x^2) + \frac{d}{dx}(7x) - \frac{d}{dx}(8) \end{aligned}$$

$$= x^3 + \frac{5}{2}x^2 - 2x + 7.$$

To find y'', use the rules of differentiation.

$$y'' = \frac{dy'}{dx} = \frac{d}{dx}\left(x^3 + \frac{5}{2}x^2 - 2x + 7\right) = 3x^2 + 5x - 2$$

To find y''', again use the rules of differentiation.

$$y''' = \frac{dy''}{dx} = \frac{d}{dx}\left(3x^2 + 5x - 2\right) = 6x + 5$$

To find $y^{(4)}$, again use the rules of differentiation.

$$y^{(4)} = \frac{dy'''}{dx} = \frac{d}{dx}(6x + 5) = 6$$

To find $y^{(5)}$, use the Derivative of a Constant Function Rule.

$$y^{(5)} = \frac{dy^{(4)}}{dx} = \frac{d}{dx}(6) = 0$$

Since the fifth derivative is zero, the sixth and all later derivatives are 0.

Thus, here are the derivatives of all the orders of the function.

$y' = x^3 + \frac{5}{2}x^2 - 2x + 7$
$y'' = 3x^2 + 5x - 2$
$y''' = 6x + 5$
$y^{(4)} = 6$
$y^{(n)} = 0$ for $n \ge 5$

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the derivatives of all orders of the following function.

$$y = (x - 7)(x + 5)(x + 6)$$

Since the given function is written as a product of simpler functions, the Derivative Product Rule could be a good first step for finding the derivative of y. Alternatively, the expression can be multiplied out to form a polynomial and then the polynomial can be differentiated using a combination of rules. Either method could be used. Here we will use the method of expanding the expression first.

Multiply the three terms together and expand to form a polynomial in x.

$$(x - 7)(x + 5)(x + 6) = x^3 + 4x^2 - 47x - 210$$

Use the Derivative Sum Rule to write the derivative of this polynomial as a sum of derivatives.

$$y' = \frac{d}{dx}\left(x^3\right) + \frac{d}{dx}\left(4x^2\right) + \frac{d}{dx}(-47x) + \frac{d}{dx}(-210)$$

Use the Derivative Power Rule to evaluate the first term.

$$\frac{d}{dx}\left(x^3\right) = 3x^2$$

Proceed similarly, differentiating each term and then adding the results together.

$$y' = \frac{d}{dx}\left(x^3 + 4x^2 - 47x + 210\right) = 3x^2 + 8x - 47$$

Differentiate y' to find y''.

$$y'' = \frac{d}{dx}\left(3x^2 + 8x - 47\right) = 6x + 8$$

Use the same procedure as in the previous steps to find y'''.

$$y''' = \frac{d}{dx}(6x + 8) = 6$$

Continuing in the same manner, find $y^{(4)}$.

$$y^{(4)} = \frac{d}{dx}(6) = 0$$

Find the derivative of $y^{(4)}$.

$$y^{(5)} = \frac{d}{dx}\left(y^{(4)}\right) = \frac{d}{dx}(0) = 0$$

Thus, all higher derivatives of order greater than 4 are 0.

Student: Cole Lamers **Date:** 09/06/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.3 Differentiation Rules

Find the first and second derivatives.

$$y = \frac{2x^5 + 8}{x^3}$$

The choice of which rules to use in solving a differentiation problem can make a difference in how much work you have to do. In this case rather than using the quotient rule to find the derivative of $\frac{2x^5+8}{x^3}$, divide each term in the numerator by x^3.

$$\frac{2x^5+8}{x^3} = 2x^2 + 8x^{-3}$$

The derivative sum rule states that if u and v are differentiable functions of x, then their sum u + v is differentiable at every point where u and v are both differentiable. At such points, $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$.

Begin by finding the first derivative. By the sum rule it is possible to differentiate the $2x^2 + 8x^{-3}$ term by term. Write $\frac{d}{dx}\left(2x^2 + 8x^{-3}\right)$ as a sum of differentiable functions.

$$\frac{d}{dx}\left(2x^2 + 8x^{-3}\right) = \frac{d}{dx}\left(2x^2\right) + \frac{d}{dx}\left(8x^{-3}\right)$$

The constant multiple rule states that if u is a differentiable function of x, and c is a constant, then $\frac{d}{dx}(cu) = c\frac{du}{dx}$.

Differentiate the first term. Begin by rewriting the expression using the constant multiple rule.

$$\frac{d}{dx}\left(2x^2\right) = 2 \cdot \frac{d}{dx}x^2$$

The power rule for positive integers states that if n is a positive integer, then $\frac{d}{dx}x^n = nx^{n-1}$.

Differentiate the function using the power rule for positive integers.

$$2 \cdot \frac{d}{dx}x^2 = 2 \cdot 2x$$

Now simplify.

$$2 \cdot 2x = 4x$$

Thus, $\frac{d}{dx}(2x^2) = 4x$.

Now use the same method to find the derivative of the second term.

$$\frac{d}{dx}\left(8x^{-3}\right) = -24x^{-4}$$

Thus, the derivatives of each term individually are $4x$ and $-24x^{-4}$.

Combine all derivatives to find the first derivative of the original expression.

$$\begin{aligned} y' &= \frac{d}{dx}\left(2x^2 + 8x^{-3}\right) \\ &= 4x - 24x^{-4} \\ &= 4x - \frac{24}{x^4} \end{aligned}$$

Therefore, $y' = 4x - \frac{24}{x^4}$. Note that this first derivative could also have been found using the quotient rule.

Now find the second derivative of $\frac{2x^5 + 8}{x^3}$ by differentiating $\frac{dy}{dx}$.

$$\frac{d^2}{dx^2}\left(\frac{2x^5 + 8}{x^3}\right) = \frac{d}{dx}\left(4x - \frac{24}{x^4}\right)$$

Use the sum rule and write $\frac{d}{dx}\left(4x - \frac{24}{x^4}\right)$ as a sum of differentiable functions.

$$\frac{d}{dx}\left(4x - \frac{24}{x^4}\right) = \frac{d}{dx}(4x) + \frac{d}{dx}\left(-\frac{24}{x^4}\right)$$

Differentiate the first term.

$$\frac{d}{dx}(4x) = 4$$

Now find the derivative of the last term.

$$\frac{d}{dx}\left(-\frac{24}{x^4}\right) = \frac{d}{dx}\left(-24x^{-4}\right) = 96x^{-5}$$

Thus, the derivatives of each term individually are 4 and $96x^{-5}$.

Now combine all derivatives to find the second derivative of the original expression.

$$\begin{aligned} y'' &= \frac{d}{dx}\left(4x - \frac{24}{x^4}\right) \\ &= 4 + 96x^{-5} \\ &= 4 + \frac{96}{x^5} \end{aligned}$$