Student: Cole Lamers **Date:** 07/17/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.1 Verifying Identities

1. Complete the following statement.

   An equation that is true for all values of the variable in its domain is called a(n) ______.

   An equation that is true for all values of the variable in its domain is called a(n) identity.

2. Complete the following statements.

   a. $\sin^2 x +$ _____ $= 1$ **b.** $1 +$ _____ $= \sec^2 x$ **c.** $\csc^2 x - \cot^2 x =$ _____

   a. Select the correct response from the choices below.

   - ✔ **A.** $\sin^2 x + \cos^2 x = 1$
   - **B.** $\sin^2 x + \csc^2 x = 1$
   - **C.** $\sin^2 x + \cot^2 x = 1$
   - **D.** $\sin^2 x + \tan^2 x = 1$

   b. Select the correct response from the choices below.

   - **A.** $1 + \cot^2 x = \sec^2 x$
   - **B.** $1 + \csc^2 x = \sec^2 x$
   - **C.** $1 + \cos^2 x = \sec^2 x$
   - ✔ **D.** $1 + \tan^2 x = \sec^2 x$

   c. Select the correct response from the choices below.

   - ✔ **A.** $\csc^2 x - \cot^2 x = 1$
   - **B.** $\csc^2 x - \cot^2 x = 2$
   - **C.** $\csc^2 x - \cot^2 x = 0$
   - **D.** $\csc^2 x - \cot^2 x = -1$

3. State whether the following statement is true or false.

   The value of $x = \frac{3\pi}{2}$ can be used to show that $\sin x = \sqrt{1 - \cos^2 x}$ is not an identity.

   Select the correct choice below.

   - **A.** False, because substituting $x = \frac{3\pi}{2}$ gives $\sin x =$ ______ $= \sqrt{1 - \cos^2 x}$.
   - **B.** False, because squaring the equation results into identity true for $x = \frac{3\pi}{2}$.
   - ✔ **C.** True, because the value of $x = \frac{3\pi}{2}$ results in both sides being defined but not equal.
   - **D.** True, because the value of $x = \frac{3\pi}{2}$ results in both sides being undefined and unequal.

4. Simplify so that no quotients appear in the final expression.

$$\frac{\sec\theta - \csc\theta}{\csc\theta\sec\theta}$$

Choose the correct answer below.

- A. $\cos\theta$
- B. $-\cos\theta\sin\theta$
- C. $\sin\theta\cos\theta$
- D. $\csc\theta - \sec\theta$
- ✔ E. $\sin\theta - \cos\theta$
- F. $\cos\theta - \sin\theta$

5. Simplify the expression $\cos(-x) \cdot \tan(-x)$.

Choose the correct simplified form of $\cos(-x) \cdot \tan(-x)$.

- A. $\sin x$
- B. $\tan x$
- ✔ C. $-\sin x$
- D. $\cos x$

6. Write the expression in terms of sine and cosine, and then simplify so that no quotients appear in the final expression.

$$\sin^2\theta\left(1 + \cot^2\theta\right)$$

Choose the correct answer below.

- A. -1
- B. $-\frac{\cos^2\theta}{\sin^2\theta}$
- C. $\csc^2\theta$
- ✔ D. 1
- E. $\frac{\cos^2\theta}{\sin^2\theta}$
- F. $\cot^2\theta$

7. Use the fundamental identities and appropriate algebraic operations to simplify the following expression.

$$(7 + \cot x)(7 - \cot x) + \csc^2 x$$

$(7 + \cot x)(7 - \cot x) + \csc^2 x =$ 50 (Type an integer or a simplified fraction.)

8. Simplify the expression.

$$(\csc x + \cot x)(\csc x - \cot x)$$

$(\csc x + \cot x)(\csc x - \cot x) =$ 1
(Use integers or decimals for any numbers in the expression.)

9. Simplify the expression.

$$\frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$$

Choose the correct answer for $\frac{\cos^2 x - \sin^2 x}{\cos x - \sin x}$.

- A. $\cos x - \sin x$
- B. $\sin x - \cos x$
- ✔ C. $\cos x + \sin x$
- D. Cannot be simplified

10. Factor and simplify.

$\cos^4 x - \sin^4 x$

Choose the most simplified form of $\cos^4 x - \sin^4 x$ below.

- A. $4\cos x - 4\sin x$
- B. $2\cos^2 x - 1$
- C. $(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$
- D. $\cos^2 x$

11. Multiply and simplify.

$\sin x \cos x(\csc x + \tan x)$

$\sin x \cos x(\csc x + \tan x) = \cos x + \sin^2 x$

12. Verify the identity.

$\csc x - \csc x \cos^2 x = \sin x$

Choose the sequence of steps below that verifies the identity.

- A. $\csc x - \csc x \cos^2 x = \csc x\left(1 - \cos^2 x\right) = \frac{1}{\sin x} \cdot \sin^2 x = \sin x$
- B. $\csc x - \csc x \cos^2 x = \csc x\left(\cos^2 x - 1\right) = \frac{1}{\sin x} \cdot \sin x^2 = \sin x$
- C. $\csc x - \csc x \cos^2 x = \csc x\left(1 - \cos^2 x\right) = \frac{1}{\sin^2 x} \cdot \sin x = \sin x$
- D. $\csc x - \csc x \cos^2 x = 1 - \cos^2 x = \sin x$

13. Establish the identity.

$(\sec\theta + 1)(\sec\theta - 1) = \tan^2\theta$

Which of the following four statements establishes the identity?

- A. $(\sec\theta + 1)(\sec\theta - 1) = \tan\theta\tan\theta = \tan^2\theta$
- B. $(\sec\theta + 1)(\sec\theta - 1) = \sec^2\theta + 1 = \tan^2\theta$
- C. $(\sec\theta + 1)(\sec\theta - 1) = \sec^2\theta - 1 = \tan^2\theta$
- D. $(\sec\theta + 1)(\sec\theta - 1) = 1 - \sec^2\theta = \tan^2\theta$

14. Verify the identity.

$\sin^2 x \cot^2 x + \sin^2 x = 1$

To verify that an equation is an identity, start with the more complicated side and transform the side of the equation into the other side by a sequence of steps, each of which produces an identity.

$\sin^2 x \cot^2 x + \sin^2 x$	Begin with left side
$= \sin^2 x\left(\frac{\cos^2 x}{\sin^2 x}\right) + \sin^2 x$ (Do not simplify.)	Write in terms of sine and cosine.
$= \cos^2 x + \sin^2 x$	Divide out the common factor.
$= 1$	Use a Pythagorean identity.

15. Establish the identity.

$$\frac{1}{\csc\theta-1}-\frac{1}{\csc\theta+1}=2\tan^2\theta$$

Which of the following statements establishes the identity?

A. $\frac{1}{\csc\theta-1}-\frac{1}{\csc\theta+1}=\frac{\csc\theta+1-(\csc\theta-1)}{(\csc\theta-1)(\csc\theta+1)}=\frac{2}{\csc^2\theta-1}=\frac{2}{\cot^2\theta}=2\tan^2\theta$

B. $\frac{1}{\csc\theta-1}-\frac{1}{\csc\theta+1}=\frac{2}{\sec^2\theta-1}=\frac{2}{\csc^2\theta+1}=\frac{2}{\sin^2\theta}=2\tan^2\theta$

C. $\frac{1}{\csc\theta-1}-\frac{1}{\csc\theta+1}=\frac{\csc\theta+1-(\csc\theta-1)}{(\csc\theta-1)(\csc\theta+1)}=\frac{2}{\csc^2\theta-1}=\frac{2}{\sin^2\theta}=2\tan^2\theta$

D. $\frac{1}{\csc\theta-1}-\frac{1}{\csc\theta+1}=\frac{2}{\sec^2\theta-1}=\frac{2}{\csc^2\theta}=2\tan^2\theta$

16. Verify the identity.

$$\sec^2 x\csc^2 x=\sec^2 x+\csc^2 x$$

Which of the following four statements establishes the identity?

A. $\sec^2 x\csc^2 x=\sin^2 x\cos^2 x=\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}=\sec^2 x+\csc^2 x$

B. $\sec^2 x\csc^2 x=\frac{1}{\sin^2 x\cos^2 x}=\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}=\sec^2 x+\csc^2 x$

C. $\sec^2 x\csc^2 x=\frac{1}{\sin^2 x\cos^2 x}=\cos^2 x+\frac{1}{\sin^2 x}=\sec^2 x+\csc^2 x$

D. $\sec^2 x\csc^2 x=\sin^2 x\cos^2 x=\cos^2 x+\frac{1}{\sin^2 x}=\sec^2 x+\csc^2 x$

17. Verify the identity.

$$(\cos x-\sin x)^2=1-2\cos x\sin x$$

Choose the sequence of steps below that verifies the identity.

A. $(\cos x-\sin x)^2=\cos^2 x+2\cos x\sin x-\sin^2 x=1-2\cos x\sin x$

B. $(\cos x-\sin x)^2=\cos^2 x-\sin^2 x=1-2\cos x\sin x$

C. $(\cos x-\sin x)^2=\cos^2 x-2\cos x\sin x+\sin^2 x=1-2\cos x\sin x$

D. $(\cos x-\sin x)^2=\cos^2 x+\sin^2 x=1-2\cos x\sin x$

18. Verify the identity.

$$(1+\tan x)^2=\sec^2 x+2\tan x$$

Choose the sequence of steps below that verifies the identity.

A. $(1+\tan x)^2=1-2\tan x+\tan^2 x=\sec^2 x+2\tan x$

B. $(1+\tan x)^2=1-\tan^2 x=\sec^2 x+2\tan x$

C. $(1+\tan x)^2=1+\tan^2 x=\sec^2 x+2\tan x$

D. $(1+\tan x)^2=1+2\tan x+\tan^2 x=\sec^2 x+2\tan x$

19. Verify the identity.

$$\frac{\cos x + \cot x}{\sin x + 1} = \cot x$$

To verify that an equation is an identity, start with more complicated side and transform the side of the equation into the other side by a sequence of steps, each of which produces an identity.

$\dfrac{\cos x + \cot x}{\sin x + 1}$	Begin with left side
$= \dfrac{\cos x + \dfrac{\cos x}{\sin x}}{\sin x + 1}$ (Do not simplify.)	Write in terms of sine and cosine.
$= \dfrac{\dfrac{\sin x \cos x + \cos x}{\sin x}}{\sin x + 1}$	Rewrite the numerator using the LCD.
$= \dfrac{\cos x}{\sin x}$	Multiply and divide by the conjugate of the denominator and simplify.
$= \cot x$	Use the quotient identities.

20. Make the indicated trigonometric substitution in the given algebraic expression and simplify. Assume $0 < \theta < \frac{\pi}{2}$.

$\sqrt{x^2 + 25}$, $x = 5\tan\theta$

$\sqrt{x^2 + 25} = 5\sec\theta$

(Simplify your answer.)