Student: Cole Lamers **Date:** 09/12/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.8 Related Rates

Suppose that a dimension x and the area $A = 5x^2$ of a shape are differentiable functions of t. Write an equation that relates $\dfrac{dA}{dt}$

to $\dfrac{dx}{dt}$.

To find a related rates equation, differentiate the equation relating the variables with respect to t using the chain rule.

In the equation $A = 5x^2$, both A and x are functions of t.

Since A is given as a function of x, differentiate the equation as shown.

$$\frac{dA}{dt} = \frac{dA}{dx}\frac{dx}{dt}$$

Differentiate $A = 5x^2$ with respect to x first.

$$\frac{dA}{dx} = 10x$$

Since there is no expression given for x as a function of t, leave $\dfrac{dx}{dt}$ unchanged. The resulting related rates equation is as shown.

$$\frac{dA}{dt} = 10x\frac{dx}{dt}$$

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Assume that $y = 4x$ and $\dfrac{dx}{dt} = 3$. Find $\dfrac{dy}{dt}$.

The rate of change of one variable x is given and the problem is to find the rate of change of another related variable y.

The relationship between $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ can be found by differentiating the equation $y = 4x$ with respect to t.

Use the Chain Rule to differentiate both sides of the equation with respect to t.

$$\frac{dy}{dt} = 4\frac{dx}{dt}$$

Notice that this is now an equation that relates $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$. Use this equation to calculate $\dfrac{dy}{dt}$ when $\dfrac{dx}{dt}$ is 3.

$$\frac{dy}{dt} = 12$$

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Assume that $x = x(t)$ and $y = y(t)$. Let $y = x^5 + 8$ and $\dfrac{dx}{dt} = 6$ when $x = 1$.

Find $\dfrac{dy}{dt}$ when $x = 1$.

Use implicit differentiation to find $\dfrac{dy}{dt}$.

$$\frac{dy}{dt} = \frac{d}{dt}\left(x^5 + 8\right)$$

$$= 5x^4 \frac{dx}{dt}$$

When $x = 1$, we calculate $\dfrac{dy}{dt}$ as follows.

$$\frac{dy}{dt} = 5x^4 \frac{dx}{dt}$$

$$= 5\left(1^4\right)(6) \qquad \text{Substitute for } \frac{dx}{dt}.$$

$$= 30 \qquad\qquad \text{Simplify.}$$

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Assume that all variables are implicit functions of time t. Find the indicated rate.

$x^2 + 6y^2 + 6y = 36$; $\dfrac{dx}{dt} = 3$ when $x = 6$ and $y = -1$; find $\dfrac{dy}{dt}$

To find $\dfrac{dy}{dt}$, begin by differentiating the given equation, $x^2 + 6y^2 + 6y = 36$, with respect to time t. Remember that since x and y are implicitly functions of time, the chain rule must be applied when differentiating each term.

$$x^2 + 6y^2 + 6y = 36$$

$$2x\frac{dx}{dt} + 12y\frac{dy}{dt} + 6\frac{dy}{dt} = 0 \qquad \text{Take derivatives with respect to time.}$$

Solve the resulting equation, $2x\dfrac{dx}{dt} + 12y\dfrac{dy}{dt} + 6\dfrac{dy}{dt} = 0$, for $\dfrac{dy}{dt}$.

$$2x\frac{dx}{dt} + 12y\frac{dy}{dt} + 6\frac{dy}{dt} = 0$$

$$(12y + 6)\frac{dy}{dt} = -2x\frac{dx}{dt} \qquad \text{Subtract } 2x\frac{dx}{dt} \text{ and combine like terms.}$$

$$\frac{dy}{dt} = -\frac{2x(dx/dt)}{12y + 6} \qquad \text{Divide by } 12y + 6.$$

Evaluate $\dfrac{dy}{dt}$ using the given values of $\dfrac{dx}{dt}$, x, and y.

$$\frac{dy}{dt} = -\frac{2x(dx/dt)}{12y + 6}$$

$$= -\frac{2(6)(3)}{12(-1) + 6} \qquad \text{Substitute.}$$

$$= 6 \qquad \text{Evaluate.}$$

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The dimensions x and y of an object are related to its volume V by the formula $V = 7x^2y$.

a. How is $\dfrac{dV}{dt}$ related to $\dfrac{dy}{dt}$ if x is constant?

b. How is $\dfrac{dV}{dt}$ related to $\dfrac{dx}{dt}$ if y is a constant?

c. How is $\dfrac{dV}{dt}$ related to $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ if neither x nor y is constant?

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a. To find a related rates equation, differentiate the equation using the chain rule. Differentiate both sides of the equation with respect to t. Use the chain rule on the right side.

$$\frac{dV}{dt} = \frac{d}{dy}\left(7x^2y\right)\frac{dy}{dt}$$

Evaluate the derivative with respect to y. Treat x as a constant.

$$\frac{dV}{dt} = \frac{d}{dy}\left(7x^2y\right)\frac{dy}{dt} = \left(7x^2\right)\frac{dy}{dt}$$

Therefore, the equation below relates $\dfrac{dV}{dt}$ and $\dfrac{dy}{dt}$ when x is constant.

$$\frac{dV}{dt} = 7x^2\frac{dy}{dt}$$

b. Now differentiate both sides with respect to t, but treat y as a constant.

$$\frac{dV}{dt} = \frac{d}{dx}\left(7x^2y\right)\frac{dx}{dt}$$

Evaluate the derivative with respect to x. Treat y as a constant.

$$\frac{dV}{dt} = \frac{d}{dx}\left(7x^2y\right)\frac{dx}{dt} = (14xy)\frac{dx}{dt}$$

Therefore, the equation below relates $\dfrac{dV}{dt}$ and $\dfrac{dx}{dt}$ when y is constant.

$$\frac{dV}{dt} = 14xy\frac{dx}{dt}$$

c. Finally, differentiate both sides with respect to t, where neither x nor y is constant. Use the product rule on the right side.

$$\frac{dV}{dt} = (7y)\frac{d}{dt}\left(x^2\right) + \left(7x^2\right)\frac{d}{dt}(y)$$

Then use the chain rule for each term.

$$\frac{dV}{dt} = (7y)\frac{d}{dx}\left(x^2\right)\frac{dx}{dt} + \left(7x^2\right)\frac{d}{dy}(y)\frac{dy}{dt}$$

Evaluate the derivatives with respect to x and y.

$$\frac{dV}{dt} = (7y)(2x)\frac{dx}{dt} + \left(7x^2\right)(1)\frac{dy}{dt}$$

Simplify the results to find the related rates equation.

$$\frac{dV}{dt} = 14xy\frac{dx}{dt} + 7x^2\frac{dy}{dt}$$

Student: Cole Lamers **Date:** 09/12/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.8 Related Rates

When a circular plate of metal is heated in an oven, its radius increases at a rate of 0.04 cm / min. At what rate is the plate's area increasing when the radius is 40 cm?

First write and equation that relates the two variables involved, the area, A, and the radius, r. The plate is a circle, so the area is $A = \pi r^2$.

Using the chain rule, differentiate with respect to t to find the related rates equation.

$$\frac{dA}{dt} = \frac{dA}{dr}\frac{dr}{dt}$$
$$= 2\pi r \frac{dr}{dt}$$

Then use the known values to find $\frac{dA}{dt} = 2\pi r\frac{dr}{dt}$, the rate of the change in area.

$$\frac{dA}{dt} = 2\pi(40 \text{ cm})(0.04 \text{ cm / min}) = 3.2\pi \text{ cm}^2 / \text{min}$$

Therefore, the plate's area is increasing at a rate of 3.2π cm^2 / min.

Student: Cole Lamers **Date:** 09/12/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.8 Related Rates

A 15-ft ladder is leaning against a house when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving away at the rate of 18 ft/sec.
a. At what rate is the top of the ladder sliding down the wall then?
b. At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?
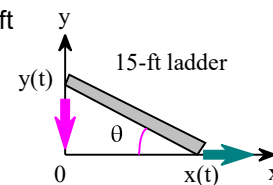
c. At what rate is the angle between the ladder and the ground changing then?

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a. Let L be the length of the ladder. The given values are shown below.

$L = 15$ ft, $x = 12$ ft, $\dfrac{dx}{dt} = 18$ ft/sec

The lengths x and y are related by $x^2 + y^2 = L^2$.

Differentiate $x^2 + y^2 = L^2$ with respect to t using the chain rule. Notice that L is not a function of t.

$2x\dfrac{dx}{dt} + 2y\dfrac{dy}{dt} = 0$

Notice that the rate equation also involves y as an unknown. Use $x^2 + y^2 = L^2$ to solve for y.

$(12)^2 + y^2 = (15)^2$
$\qquad\quad y = 9$ ft

Substitute the known values and solve for the rate of change of the height of the top of the ladder.

$2(12)(18) + 2(9)\dfrac{dy}{dt} = 0$

$\qquad\qquad\quad \dfrac{dy}{dt} = -24$ ft/sec

Therefore, the rate of change of the height of the top of the ladder is -24 ft/sec.

b. The area of the triangle is given by the equation $A = \dfrac{1}{2}xy$.

Use the chain rule and the product rule to differentiate this equation with respect to t. In this case, A, x, and y are all functions of t.

$\dfrac{dA}{dt} = \dfrac{1}{2}x\dfrac{dy}{dt} + \dfrac{1}{2}y\dfrac{dx}{dt}$

Then substitute the known values to find the rate of change of the area of the triangle.

$\dfrac{dA}{dt} = \dfrac{1}{2}(12)(-24) + \dfrac{1}{2}(9)(18)$

$\dfrac{dA}{dt} = -63$ ft^2/sec

c. There are several trigonometric definitions that can be used to relate θ to x, y, and L. Choose $\cos\theta = \dfrac{x}{L}$.

Differentiate this equation with respect to t using the chain rule on the left side. Here, both θ and x are functions of t.

$$-\sin\theta\frac{d\theta}{dt} = \frac{1}{L}\frac{dx}{dt}$$

Use the triangle to find $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$.

$$\sin\theta = \frac{9}{15}$$

Then substitute the known values and solve for the rate of change of the angle.

$$-\frac{9}{15}\frac{d\theta}{dt} = \frac{1}{15}(18)$$

$$\frac{d\theta}{dt} = -2 \text{ rad/sec}$$

| **Student:** Cole Lamers **Date:** 09/12/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 3.8 Related Rates

Water is flowing at the rate of 45 m^3 / min from a shallow concrete conical reservoir (vertex down) of base radius 60 m and height 5 m.
a. How fast (centimeters per minute) is the water level falling when the water is 4 m deep?
b. How fast is the radius of the water's surface changing then? Answer in centimeters per minute.

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a. Begin with a diagram and choose variables to represent the various quantities in the problem. In this case, let V represent the volume of water, let h represent the depth of the water, and let r represent the radius at the water's surface.
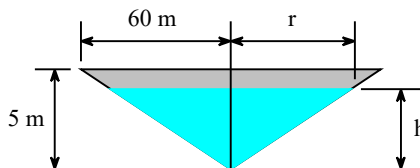
Now write down the given numerical information using these variables.

The given rate is $\dfrac{dV}{dt} = -45$ m^3 / min. The given depth is h = 4 m.

The rate at which the water is falling is $\dfrac{dh}{dt}$.

Therefore, an equation that relates the volume V to the depth h is required. Begin by using the given geometry to write an expression for the volume of water V in terms of the radius r at the surface and the depth h.

$$V = \frac{1}{3}\pi r^2 h$$

Notice that this equation involves V, r, and h. However, both the depth h and the radius of the surface r change as the reservoir drains. The equation must be modified so that it contains only V and h. The value of r and h are related by the geometry of the reservoir.

Use similar triangles where r corresponds to 60 m and h corresponds to 5 m to write an expression for r in terms of h.

$\qquad$ r = 12h
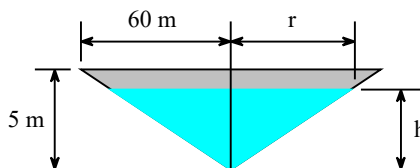
Now use this expression to write the volume $V = \dfrac{1}{3}\pi r^2 h$ in terms of h only.

$$V = 48\pi h^3$$

Differentiate this equation with respect to t.

$$\frac{dV}{dt} = 144\pi h^2 \frac{dh}{dt}$$

This equation gives a relationship between the rate of change in the volume and the rate of change in the depth. Use this equation to find $\dfrac{dh}{dt}$ when $\dfrac{dV}{dt} = -45$ m^3 / min and h = 4 m.

$$\frac{dh}{dt} = -\frac{5}{256\pi} \text{ m / min}$$

The question asks for the rate in centimeters per minute, so use the appropriate conversion.

$$\frac{dh}{dt} = -0.62 \text{ cm / min}$$

The height of the surface is changing at -0.62 cm / min, which means that it is falling at 0.62 cm / min.

b. The rate at which the radius of the water's surface is changing is $\frac{dr}{dt}$. Since $\frac{dh}{dt}$ is now known, the equation r = 12h can be used to find $\frac{dr}{dt}$. Differentiate this equation with respect to t.

$$\frac{dr}{dt} = 12\frac{dh}{dt}$$

Use the rate $\frac{dh}{dt} = -\frac{5}{256\pi}$ m / min to calculate the rate $\frac{dr}{dt}$ in cm / min.

$$\frac{dr}{dt} = -7.46 \text{ cm / min}$$

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Water is flowing at the rate of 40 m^3 / min from a shallow concrete conical reservoir (vertex down) of base radius 50 m and height 8 m.
a. How fast (centimeters per minute) is the water level falling when the water is 6 m deep?
b. How fast is the radius of the water's surface changing then? Answer in centimeters per minute.

a. Begin with a diagram and choose variables to represent the various quantities in the problem. In this case, let V represent the volume of water, let h represent the depth of the water, and let r represent the radius at the water's surface.
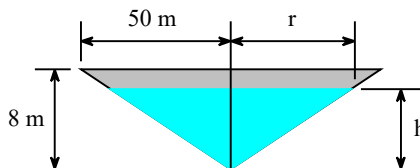
Now write down the given numerical information using these variables.

The given rate is $\dfrac{dV}{dt}$ = − 40 m^3 / min. The given depth is h = 6 m.

The rate at which the water is falling is $\dfrac{dh}{dt}$.

Therefore, an equation that relates the volume V to the depth h is required. Begin by using the given geometry to write an expression for the volume of water V in terms of the radius r at the surface and the depth h.
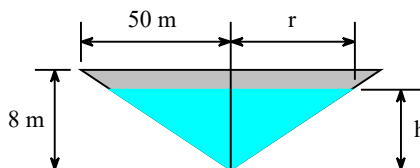
V = $\dfrac{1}{3}\pi r^2 h$ (Type an exact answer, using π as needed.)

Notice that this equation involves V, r, and h. However, both the depth h and the radius of the surface r change as the reservoir drains. The equation must be modified so that it contains only V and h. What other information can be used to eliminate r from the equation?

- ○ **A.** Since derivatives are going to be used, r will be eliminated by differentiating.
- ○ **B.** The depth h and radius r must be the same.
- ✅ **C.** The depth h and radius r are related by the geometry of the reservoir.
- ○ **D.** The maximum radius of the reservoir should be used for r.

Use the given geometry to write an expression for r in terms of h.

r = $\dfrac{25}{4}h$

Now use this expression to write the volume V = $\dfrac{1}{3}\pi r^2 h$ in terms of h only.

V = $\dfrac{625}{48}\pi h^3$ (Type an exact answer, using π as needed.)

Differentiate this equation with respect to t.

$\dfrac{dV}{dt} = \dfrac{625}{16}\pi h^2 \dfrac{dh}{dt}$

(Type an equation. Type an exact answer, using π as needed. Use integers or fractions for any numbers in the equation.)

This equation gives a relationship between the rate of change in the volume and the rate of change in the depth. Use this equation to find $\dfrac{dh}{dt}$ when $\dfrac{dV}{dt} = -40 \text{ m}^3 / \text{min}$ and $h = 6 \text{ m}$.

$$\dfrac{dh}{dt} = \boxed{-\dfrac{32}{1125\pi}} \text{ m / min}$$

(Type an exact answer, using π as needed.)

The question asks for the rate in centimeters per minute, so use the appropriate conversion.

$$\dfrac{dh}{dt} = \boxed{-0.91} \text{ cm / min}$$

(Round to two decimal places as needed.)

The height of the surface is changing at -0.91 cm / min, which means that it is falling at 0.91 cm / min.

b. The rate at which the radius of the water's surface is changing is $\dfrac{dr}{dt}$. Since $\dfrac{dh}{dt}$ is now known, the equation $r = \dfrac{25}{4}h$ can be used to find $\dfrac{dr}{dt}$. Differentiate this equation with respect to t.

$$\dfrac{dr}{dt} = \dfrac{25}{4}\dfrac{dh}{dt}$$

(Use integers or fractions for any numbers in the equation.)

Use the rate $\dfrac{dh}{dt} = -\dfrac{32}{1125\pi} \text{ m / min}$ to calculate the rate $\dfrac{dr}{dt}$ in cm / min.

$$\dfrac{dr}{dt} = \boxed{-5.66} \text{ cm / min}$$

(Round to two decimal places as needed.)

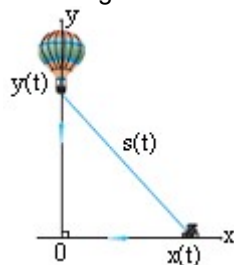YOU ANSWERED: $\dfrac{dr}{dt}$

a

6.25

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$\dfrac{625}{16}\pi h^2$

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A balloon is rising vertically above a level, straight road at a constant rate of 2 ft / sec. Just when the balloon is 30 ft above the ground, a bicycle moving at a constant rate of 9 ft / sec passes under it. How fast is the distance s(t) between the bicycle and balloon increasing 3 seconds later?

Refer to the picture of the problem situation on the left.

x(t) represents the distance that bicycle has traveled t seconds after passing under the balloon.

y(t) represents the height of the balloon above the ground t seconds after the bicycle passes below it.

s(t) represents the distance between the bicycle and the balloon t seconds after the bicycle passes under the balloon.

x(t), y(t), and s(t) are all differentiable functions of time, t. The rates of change of x and y, or $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$, are given in the problem statement.

$$\frac{dx}{dt} = 9 \text{ ft / sec}$$

$$\frac{dy}{dt} = 2 \text{ ft / sec}$$

Determine the rate of increase of s(t), or $\dfrac{ds}{dt}$, at t = 3 seconds.

Start by finding the relationship between the variables x, y, and s. Note that the triangle formed by x, y, and s is a right triangle. Use the Pythagorean theorem to write s^2 in terms of x and y.

$$s^2 = x^2 + y^2$$

Now, use the chain rule to differentiate s with respect to t.

$$\frac{ds}{dt} = \frac{ds}{dx} \bullet \frac{dx}{dt} + \frac{ds}{dy} \bullet \frac{dy}{dt}$$

$$2s\frac{ds}{dt} = 2x\left(\frac{dx}{dt}\right) + 2y\left(\frac{dy}{dt}\right)$$

Solve this equation for $\dfrac{ds}{dt}$ and write s in terms of x and y.

$$\frac{ds}{dt} = \frac{x\left(\dfrac{dx}{dt}\right) + y\left(\dfrac{dy}{dt}\right)}{\sqrt{x^2 + y^2}}$$

Substitute the values of $\dfrac{dx}{dt}$ and $\dfrac{dy}{dt}$ into $\dfrac{ds}{dt}$.

$$\frac{ds}{dt} = \frac{9x + 2y}{\sqrt{x^2 + y^2}}$$

To evaluate $\dfrac{ds}{dt}$ at t = 3 seconds, find x and y, or the positions of the bicycle and the balloon, 3 seconds after the bicycle passes under the balloon.

To evaluate $\frac{ds}{dt}$ at $t = 3$ seconds, find x and y, or the positions of the bicycle and the balloon, 3 seconds after the bicycle passes under the balloon.

Calculate x, or the distance the bicycle has traveled in these 3 seconds.

$x = (9 \text{ ft} / \text{sec})(3 \text{ sec})$

  $= 27$ feet

Now find y, or the height of the balloon above the ground, 3 seconds after the bicycle passes below it.

$y = 30 + (2 \text{ ft} / \text{sec}) \cdot (3 \text{ sec})$

  $= 36$ feet

Substitute these values into the expression for $\frac{ds}{dt}$ and evaluate.

$$\frac{ds}{dt} = \frac{9(27) + 2(36)}{\sqrt{27^2 + 36^2}}$$

$$\frac{ds}{dt} = 7 \text{ ft} / \text{sec}$$

Therefore, 3 seconds after the bicycle passes under the balloon, the distance between the bicycle and the balloon is increasing by 7 ft / sec.

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On a morning of a day when the sun will pass directly overhead, the shadow of a 48-ft building on level ground is 36 ft long. At the moment in question, the angle θ the sun makes with the ground is increasing at the rate of 0.25°/min. At what rate is the shadow decreasing? Remember to use radians in your calculations. Express your answer in inches per minute.

48 '
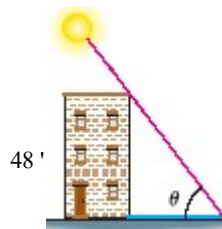
Let x be the length of the shadow, y be the height of the building, and L be the hypotenuse of the triangle. Use tangent to relate x, y, and θ.

$$\tan \theta = \frac{y}{x}$$

Differentiate the equation with respect to t using the chain rule. Notice that x and θ are functions of t, while y is a constant.

$$\left(\sec^2\theta \right) \frac{d\theta}{dt} = -\frac{y}{x^2} \frac{dx}{dt}$$

In $\left(\sec^2\theta \right) \frac{d\theta}{dt} = -\frac{y}{x^2} \frac{dx}{dt}$, $\frac{dx}{dt}$ is the rate at which the shadow's length is changing. The values of x and y are given. Notice that the equation also involves $\sec^2\theta$ and $\frac{d\theta}{dt}$. Use the Pythagorean theorem to find the length of the hypotenuse.

$$L^2 = (36)^2 + (48)^2$$
$$L = 60 \text{ ft}$$

Use the triangle and find the value of $\sec^2\theta$.

$$\sec^2\theta = \left(\frac{60}{36} \right)^2 = \left(\frac{5}{3} \right)^2 = \frac{25}{9}$$

The value for $\frac{d\theta}{dt}$ is given in degrees per minute. This value needs to be changed to radians per minute. Remember, 2π radians equals 360°.

0.25°/min $\approx$ 0.004363 rad/min

Substitute the values into $\left(\sec^2\theta \right) \frac{d\theta}{dt} = -\frac{y}{x^2} \frac{dx}{dt}$ and solve for $\frac{dx}{dt}$.

$$\left(\frac{25}{9} \right)(0.004363) = -\frac{48}{(36)^2} \frac{dx}{dt}$$
$$\frac{dx}{dt} \approx -0.327 \text{ ft/min}$$

The question asks for the rate of change in inches per minute. Multiply by 12 in/ft to convert the units.

-0.327 ft/min ≈ -3.9 in/min

Therefore, the shadow is moving at -3.9 inches per minute, or decreasing at 3.9 inches per minute.