

Student: Cole Lamers
Date: 09/21/19

Instructor: Viktoriya Shcherban
Course: Calc 1 11:30 AM / Internet
(81749&81750) Shcherban

Assignment: 4.2 The Mean Value Theorem

Find the value or values of c that satisfy the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the Mean Value Theorem for the following function and interval.

$$f(x) = 2x^2 - 5x - 3, \quad [-3, 0]$$

To find the value or values of c that satisfy the equation, first determine a and b .

The value of a is the left endpoint, -3 . The value of b is the right endpoint, 0 .

Now, evaluate $f(a)$ and $f(b)$.

$$\begin{aligned} f(a) &= 2a^2 - 5a - 3 \\ f(-3) &= 2(-3)^2 - 5(-3) - 3 \\ f(-3) &= 30 \end{aligned}$$

$$\begin{aligned} f(b) &= 2b^2 - 5b - 3 \\ f(0) &= 2(0)^2 - 5(0) - 3 \\ f(0) &= -3 \end{aligned}$$

To find $f'(c)$, first find $f'(x)$.

$$\begin{aligned} f(x) &= 2x^2 - 5x - 3 \\ f'(x) &= 4x - 5 \end{aligned}$$

Simplify.

$$\begin{aligned} \frac{f(b) - f(a)}{b - a} &= f'(c) \\ \frac{-3 - 30}{0 - (-3)} &= 4c - 5 \\ -11 &= 4c - 5 \end{aligned}$$

Solve for c .

$$\begin{aligned} -11 &= 4c - 5 \\ -6 &= 4c \\ -\frac{3}{2} &= c \end{aligned}$$

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Find the value or values of c that satisfies the equation $\frac{f(b) - f(a)}{b - a} = f'(c)$ in the conclusion of the mean value theorem for the given function and interval.

$$f(x) = 5x + \frac{5}{x}, \left[\frac{1}{4}, 4 \right]$$

The mean value theorem supposes that $y = f(x)$ is continuous on a closed interval $[a, b]$ and differentiable on the interval's interior (a, b) . Then there is at least one point c in (a, b) at which $\frac{f(b) - f(a)}{b - a} = f'(c)$.

To determine the value of $f'(c)$, first evaluate $f(b)$ for $b = 4$.

$$f(x) = 5x + \frac{5}{x}$$

$$f(4) = 5(4) + \frac{5}{4} \quad \text{Substitute } x = 4.$$

Simplify.

$$\begin{aligned} f(4) &= \frac{80}{4} + \frac{5}{4} && \text{Write each term with a common denominator.} \\ &= \frac{85}{4} && \text{Add the fractions together.} \end{aligned}$$

Next, evaluate $f(a)$ for $a = \frac{1}{4}$.

$$\begin{aligned} f(x) &= 5x + \frac{5}{x} \\ f\left(\frac{1}{4}\right) &= 5\left(\frac{1}{4}\right) + \frac{5}{\frac{1}{4}} && \text{Substitute } x = \frac{1}{4}. \end{aligned}$$

Simplify.

$$\begin{aligned} f\left(\frac{1}{4}\right) &= \frac{5}{4} + \frac{80}{4} && \text{Write each term with a common denominator.} \\ &= \frac{85}{4} && \text{Add the fractions together.} \end{aligned}$$

Now, compute the value of $f'(c)$.

$$\begin{aligned} f'(c) &= \frac{f(b) - f(a)}{b - a} \\ &= \frac{\frac{85}{4} - \frac{85}{4}}{4 - \frac{1}{4}} && \text{Substitute values.} \\ &= 0 && \text{Simplify.} \end{aligned}$$

Determine the value of c that makes $f'(c) = 0$ true. Begin by finding $f'(x)$ given that $f(x) = 5x + \frac{5}{x}$.

$$f(x) = 5x + 5x^{-1} \quad \text{Rewrite the equation.}$$

$$f'(x) = 5 + 5(-1 \cdot x^{-2}) \quad \text{Differentiate using the power rule.}$$

$$= 5 - \frac{5}{x^2} \quad \text{Simplify.}$$

Substituting $x = c$ in $f'(x)$ results in the following equation.

$$f'(c) = 5 - \frac{5}{c^2}$$

Now find the value(s) of c that satisfy the equation $f'(c) = 0$.

$$f'(c) = 5 - \frac{5}{c^2}$$

$$0 = 5 - \frac{5}{c^2} \quad \text{Substitute } f'(c) = 0.$$

$$c = \pm 1 \quad \text{Simplify.}$$

Because -1 does not lie in the given interval, only $c = 1$ satisfies the conclusion of the mean value theorem for the function

$f(x) = 5x + \frac{5}{x}$ in the interval $\left[\frac{1}{4}, 4\right]$.



Solution

$$\frac{d}{dx} \left(x + \sin^2 \left(\frac{x}{3} \right) - 8 \right) = \frac{\sin \left(\frac{2x}{3} \right)}{3} + 1$$

Steps

$$\frac{d}{dx} \left(x + \sin^2 \left(\frac{x}{3} \right) - 8 \right)$$

Apply the Sum/Difference Rule: $(f \pm g)' = f' \pm g'$

$$= \frac{d}{dx}(x) + \frac{d}{dx} \left(\sin^2 \left(\frac{x}{3} \right) \right) - \frac{d}{dx}(8)$$

Show Steps

$$\frac{d}{dx}(x) = 1$$

Hide Steps

$$\frac{d}{dx} \left(\sin^2 \left(\frac{x}{3} \right) \right) = \frac{\sin \left(\frac{2x}{3} \right)}{3}$$

$$\frac{d}{dx} \left(\sin^2 \left(\frac{x}{3} \right) \right)$$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$$f = u^2, \quad u = \sin \left(\frac{x}{3} \right)$$

$$= \frac{d}{du}(u^2) \frac{d}{dx} \left(\sin \left(\frac{x}{3} \right) \right)$$

$$\frac{d}{du}(u^2) = 2u$$

Show Steps

$$\frac{d}{dx} \left(\sin \left(\frac{x}{3} \right) \right) = \cos \left(\frac{x}{3} \right) \frac{1}{3}$$

Hide Steps

$$\frac{d}{dx} \left(\sin \left(\frac{x}{3} \right) \right)$$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

$$f = \sin(u), \quad u = \frac{x}{3}$$

$$= \frac{d}{du}(\sin(u)) \frac{d}{dx}\left(\frac{x}{3}\right)$$

$$\frac{d}{du}(\sin(u)) = \cos(u)$$

[Show Steps +](#)

$$\frac{d}{dx}\left(\frac{x}{3}\right) = \frac{1}{3}$$

[Show Steps +](#)

$$= \cos(u) \frac{1}{3}$$

$$\text{Substitute back } u = \frac{x}{3}$$

$$= \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

$$= 2u \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

$$\text{Substitute back } u = \sin\left(\frac{x}{3}\right)$$

$$= 2\sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

$$\text{Simplify } 2\sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right) \frac{1}{3}: \quad \frac{\sin\left(\frac{2x}{3}\right)}{3}$$

[Hide Steps -](#)

$$2\sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right) \frac{1}{3}$$

$$\text{Multiply fractions: } a \cdot \frac{b}{c} = \frac{a \cdot b}{c}$$

$$= \frac{1 \cdot 2}{3} \sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right)$$

$$\text{Multiply the numbers: } 1 \cdot 2 = 2$$

$$= \frac{2}{3} \sin\left(\frac{x}{3}\right) \cos\left(\frac{x}{3}\right)$$

$$\text{Use the following identity: } \cos(x) \sin(x) = \frac{\sin(2x)}{2}$$

$$= \frac{\sin\left(2 \cdot \frac{x}{3}\right)}{2} \cdot \frac{2}{3}$$

Multiply fractions: $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$

$$= \frac{\sin\left(2 \cdot \frac{x}{3}\right) \cdot 2}{2 \cdot 3}$$

Cancel the common factor: 2

$$= \frac{\sin\left(2 \cdot \frac{x}{3}\right)}{3}$$

Multiply $2 \cdot \frac{x}{3}$: $\frac{2x}{3}$

Show Steps +

$$= \frac{\sin\left(\frac{x \cdot 2}{3}\right)}{3}$$

$$= \frac{\sin\left(\frac{2x}{3}\right)}{3}$$

$$\frac{d}{dx}(8) = 0$$

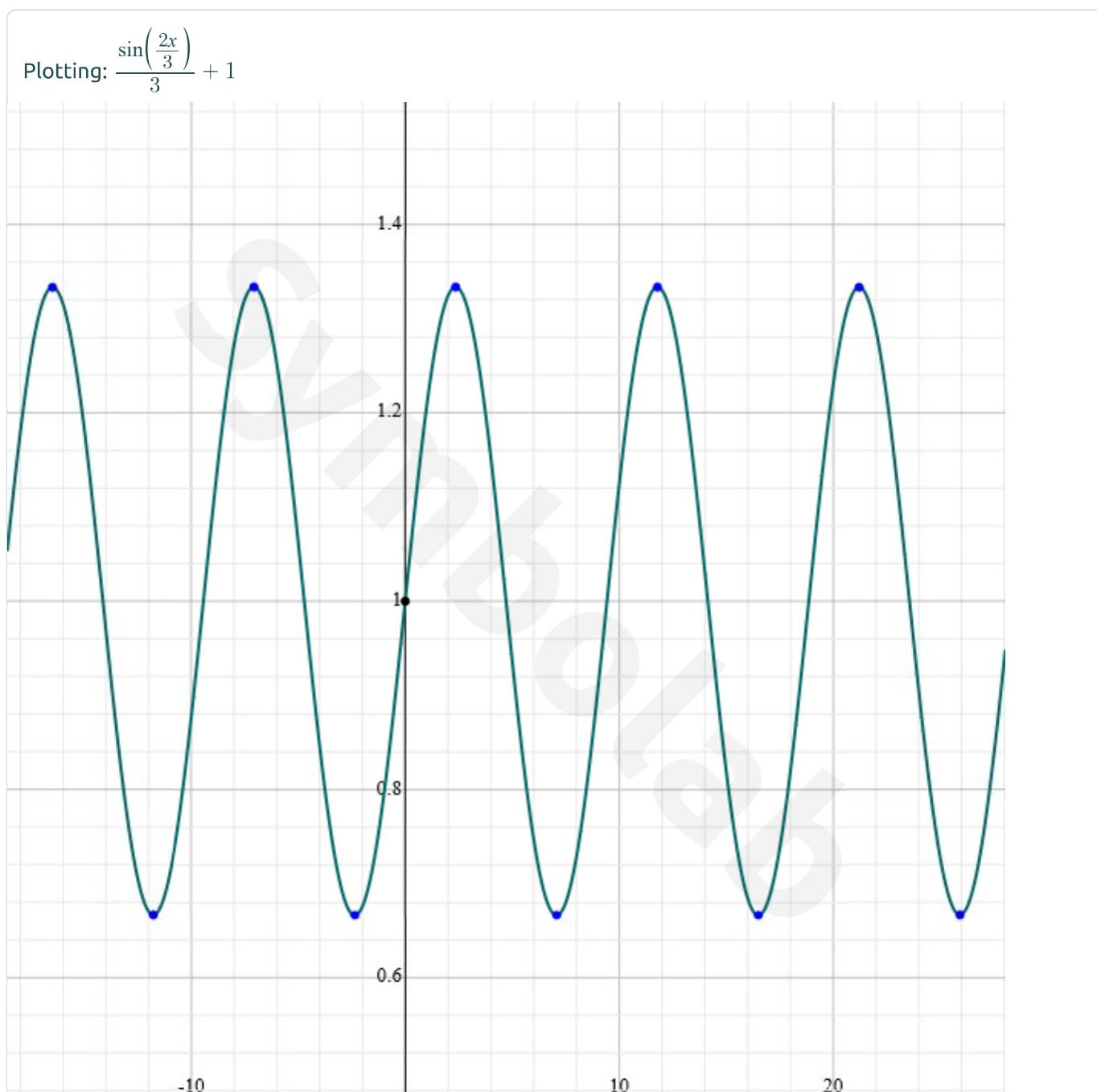
Show Steps +

$$= 1 + \frac{\sin\left(\frac{2x}{3}\right)}{3} - 0$$

Simplify

$$= \frac{\sin\left(\frac{2x}{3}\right)}{3} + 1$$

Graph



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Find all possible functions with the given derivative.

$$f'(t) = \cos 4t + \sin \frac{t}{3}$$

The corollary of Functions with the Same Derivative Differ by a Constant states if $f'(x) = g'(x)$ at each point in an open interval (a,b) , then there exists a constant C such that $f(x) = g(x) + C$ for all $x \in (a,b)$. That is, $f - g$ is a constant on (a,b) .

Determine the functions that have the same derivatives as the terms of the sum. Let $f(t) = f_1(t) + f_2(t)$ and $g(t) = g_1(t) + g_2(t)$.

Find the trigonometric function that has the derivative $g_1'(t) = \cos 4t$.

$$g_1(t) = \frac{1}{4} \sin(4t)$$

Find the trigonometric function that has the derivative $g_2'(t) = \sin \frac{t}{3}$.

$$g_2(t) = -3 \cos \frac{t}{3}$$

Thus, the function $f(t) = \frac{1}{4} \sin(4t) - 3 \cos \frac{t}{3} + C$, for some constant C .

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Assignment: 4.2 The Mean Value Theorem

Find all possible functions with the given derivative.

$$f'(x) = x^4$$

The corollary of Functions with the Same Derivative Differ by a Constant states if $f'(x) = g'(x)$ at each point x in an open interval (a,b) , then there exists a constant C such that $f(x) = g(x) + C$ for all $x \in (a,b)$. That is, $f - g$ is a constant on (a,b) .

Find the power function that has the derivative $f'(x) = x^4$.

$$g(x) = \frac{x^5}{5}$$

Thus, $f(x) = \frac{x^5}{5} + C$, for some constant C .

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Find the function with the given derivative whose graph passes through the point P.

$$f'(x) = 2x + 3, \quad P(-2, 2)$$

The corollary of Functions with the Same Derivative Differ by a Constant states if $f'(x) = g'(x)$ at each point in an open interval (a, b) , then there exists a constant C such that $f(x) = g(x) + C$ for all $x \in (a, b)$. That is, $f - g$ is a constant on (a, b) .

Determine the functions that have the same derivatives as the terms of the sum. Let $f(x) = f_1(x) + f_2(x)$ and $g(x) = g_1(x) + g_2(x)$.

Find the power function that has the same derivative as $g_1'(x) = 2x$.

$$g_1(x) = x^2$$

Find the power function that has the same derivative as $g_2'(x) = 3$.

$$g_2(x) = 3x$$

Thus, the function $f(x) = x^2 + 3x + C$, for some constant C.

Use the point $P(-2, 2)$ to find C.

$$\begin{aligned} 2 &= (-2)^2 + 3(-2) + C \\ C &= 4 \end{aligned}$$

Therefore, the function with the given derivative whose graph passes through the point P is $f(x) = x^2 + 3x + 4$.

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Assignment: 4.2 The Mean Value Theorem

Given the velocity $v = \frac{ds}{dt}$ and the initial position of a body moving along a coordinate line, find the body's position at time t.

$$v = 9.8t + 10, s(0) = 15$$

The velocity function is the derivative of the position function, that is, $s(t)$ is some function whose derivative is $9.8t + 10$.

If $f'(t) = g'(t)$ at each point t in an open interval (a,b) , then there exists a constant C such that $f(t) = g(t) + C$ for all t in (a,b) . That is, $f - g$ is a constant on (a,b) .

Note that the derivative of the function $f(t) = 4.9t^2 + 10t$ is $f'(t) = 9.8t + 10$.

Since $f'(t) = 9.8t + 10$ and $v = 9.8t + 10$, then $s(t)$ must vary from $f(t)$ by only a constant C, such that $s(t) = 4.9t^2 + 10t + C$.

Using $s(0) = 15$, determine the value of the constant C.

$$s(0) = 4.9(0)^2 + 10(0) + C$$

$$15 = 4.9(0)^2 + 10(0) + C$$

$$C = 15$$

Substitute C = 15 in the equation for $s(t)$ and write the equation for the body's position at time t.

$$\begin{aligned}s(t) &= 4.9t^2 + 10t + C \\ &= 4.9t^2 + 10t + 15\end{aligned}$$

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Assignment: 4.2 The Mean Value Theorem

Given the velocity $v = \frac{ds}{dt}$ and initial position of a body moving along a coordinate line, find the body's position at time t.

$$v = \sin \pi t, \quad s(-8) = 0$$

The corollary of Functions with the Same Derivative Differ by a Constant states if $f'(x) = g'(x)$ at each point in an open interval (a,b) , then there exists a constant C such that $f(x) = g(x) + C$ for all $x \in (a,b)$. That is, $f - g$ is a constant on (a,b) .

Find the trigonometric function that has the derivative $v = \sin \pi t$.

$$s = -\frac{\cos(\pi t)}{\pi}$$

Thus, the function $s = -\frac{\cos(\pi t)}{\pi} + C$, for some constant C.

Use the initial position to find C.

$$0 = -\frac{\cos(\pi t)}{\pi} + C$$

$$C = \frac{1}{\pi}$$

The body's position at time t is $s = -\frac{\cos(\pi t)}{\pi} + \frac{1}{\pi}$.

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Assignment: 4.2 The Mean Value Theorem

Consider the following acceleration $a = \frac{d^2s}{dt^2}$, initial velocity, and initial position of an object moving on a number line. Find the object's position at time t .

$$a = 32, v(0) = 10, s(0) = 11$$

The following corollary follows from the Mean Value Theorem.

If $f'(x) = g'(x)$ at each point x in an open interval (a,b) , then there exists a constant C such that $f(x) = g(x) + C$ for all $x \in (a,b)$.

First, find a function, $\frac{ds}{dt}$, with a derivative equal to $\frac{d^2s}{dt^2}$. Use the fact that $\frac{d}{dt}(At) = A$.

$$\frac{d^2s}{dt^2} = 32$$

$$\frac{ds}{dt} = 32t + C$$

From the above and the mean value theorem, $\frac{ds}{dt} = 32t + C$. At time $t = 0, v(0) = 10$ and $v = \frac{ds}{dt}$. Substitute $t = 0$ in the equation and solve for C .

$$v(0) = 32(0) + C$$

$$10 = 32(0) + C$$

$$10 = C$$

$$\text{So, } \frac{ds}{dt} = 32t + 10.$$

Now, find a function, s , with a derivative equal to $\frac{ds}{dt}$. Use the fact that $\frac{d}{dt}(At^2) = 2At$ and $\frac{d}{dt}(At) = A$.

$$\frac{ds}{dt} = 32t + 10$$

$$s = 16t^2 + 10t + C$$

From above and the mean value theorem, $s = 16t^2 + 10t + C$. At time $t = 0, s(0) = 11$. Substitute $t = 0$ in the equation and solve for C .

$$s(0) = 16(0)^2 + 10(0) + C$$

$$11 = 16(0)^2 + 10(0) + C$$

$$11 = C$$

Therefore, the position of the object at time t is given by $s = 16t^2 + 10t + 11$.

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Assignment: 4.2 The Mean Value Theorem

Given the acceleration $a = \frac{d^2s}{dt^2}$, initial velocity, and initial position of a body moving on a coordinate line, find the body's position at time t.

$$a = -16 \sin 4t, v(0) = 4, s(0) = -6$$

Acceleration is the derivative of the velocity function, and velocity is the derivative of the position function.

First, determine the velocity function knowing that $v(t)$ is some function whose derivative is $-16 \sin 4t$.

If $f'(t) = g'(t)$ at each point t in an open interval (a,b) , then there exists a constant C such that $f(t) = g(t) + C$ for all t in (a,b) . That is, $f - g$ is a constant on (a,b) .

Note the derivative of the function $f(t) = 4 \cos 4t$ is $f'(t) = -16 \sin 4t$.

Since $f'(t) = -16 \sin 4t$ and $a = -16 \sin 4t$, then $v(t)$ must vary from $f(t)$ by only a constant, such that $v(t) = 4 \cos 4t + C_1$.

Using $v(0) = 4$, determine the value of the constant C_1 .

$$\begin{aligned} v(0) &= 4 \cos 4(0) + C_1 \\ 4 &= 4 \cos 0 + C_1 \\ 4 &= 4 + C_1 \\ C_1 &= 0 \end{aligned}$$

Substituting $C_1 = 0$ into the equation for $v(t)$ gives the following equation.

$$v(t) = 4 \cos 4t$$

Next, determine the position function knowing that $s(t)$ is some function whose derivative is $4 \cos 4t$.

Note the derivative of the function $g(t) = \sin 4t$ is $g'(t) = 4 \cos 4t$.

Since $g'(t) = 4 \cos 4t$ and $v(t) = 4 \cos 4t$, then $s(t)$ must vary from $g(t)$ by only a constant, such that $s(t) = \sin 4t + C_2$.

Using $s(0) = -6$, determine the value of the constant C_2 .

$$\begin{aligned} s(0) &= \sin 4(0) + C_2 \\ -6 &= \sin 0 + C_2 \\ -6 &= 0 + C_2 \\ C_2 &= -6 \end{aligned}$$

Substitute $C_2 = -6$ into the equation for $s(t)$ and write the equation for the body's position at time t.

$$\begin{aligned} s(t) &= \sin 4t + C_2 \\ &= \sin(4t) - 6 \end{aligned}$$