Unit 5 – Expansion

A brass ring is to be fit over a steel rod. At room temperature (20.0°C), the rod's diameter is 16.234 mm and the inside diameter of the ring is 16.229 mm. To what temperature must the ring be heated to fit?

$$A_{steel\ rod} = \frac{\pi D^2}{4} = \frac{\pi\left(16.234\ \text{mm}\right)^2}{4} = 206.98\ \text{mm}^2$$

$$A_{brass\ ring} = \frac{\pi D^2}{4} = \frac{\pi\left(16.229\ \text{mm}\right)^2}{4} = 206.86\ \text{mm}^2$$

$$\Delta A = 206.98\ \text{mm}^2 - 206.86\ \text{mm}^2 = 0.12\ \text{mm}^2$$

γ = Coeff. of Area Expansion & α = Coeff. of Linear Expansion

$$\gamma_{brass} = 2\alpha_{brass} = 2\left(1.9 \text{ x } 10^{-5}\,{}^\circ\text{C}^{-1}\right) = 3.8 \text{ x } 10^{-5}\,{}^\circ\text{C}^{-1}$$

A_o = Original Area ΔA = Change in Area

$$\Delta A = \gamma A_o \Delta T$$

$$\therefore\ \Delta T = \frac{\Delta A}{\gamma A_o} = \frac{0.12\ \cancel{\text{m}}\text{m}^2}{\left(3.8 \text{ x } 10^{-5}\,{}^\circ\text{C}^{-1}\right)\left(206.86\ \cancel{\text{m}}\text{m}^2\right)} = 15^\circ\text{C}$$

$$\therefore\ T_{final} = 20.0^\circ\text{C} + 15^\circ\text{C} = \boxed{35^\circ\text{C}}$$

To what temperature would the steel rod need to be cooled for it to fit into the ring?

$$\gamma_{steel} = 2\alpha_{steel} = 2\left(1.1 \text{ x } 10^{-5}\,{}^\circ\text{C}^{-1}\right) = 2.2 \text{ x } 10^{-5}\,{}^\circ\text{C}^{-1}$$

$$\Delta T = \frac{\Delta A}{\gamma A_o} = \frac{-0.12 \text{ }\cancel{\text{mm}}^2}{\left(2.2 \text{ x } 10^{-5}\,{}^\circ\text{C}^{-1}\right)\left(206.98 \text{ }\cancel{\text{mm}}^2\right)} = -26^\circ\text{C}$$

$$\therefore\ T_{final} = 20.0^\circ\text{C} + \left(-26^\circ\text{C}\right) = \boxed{-6^\circ\text{C}}$$

A 178 ft length of #14 copper wire is subject to temperatures that range from -18.0°F to 135.0°F. How much expansion would occur over this range?

$$\Delta L = \alpha L_o \Delta T$$

$$\left(153\ ^\circ\cancel{F}\right)\left(\frac{1\ ^\circ C}{1.8\ ^\circ\cancel{F}}\right) = 85.0\ ^\circ C$$

$$\alpha_{copper} = 1.7 \text{ x } 10^{-5}\,^\circ C^{-1}$$

$$\Delta L = \left(1.7 \text{ x } 10^{-5}\,^\circ\cancel{C}^{-1}\right)\left(178 \text{ ft}\right)\left(85.0^\circ\cancel{C}\right) = \boxed{0.26 \text{ ft}}$$

The temperature gradient between the skin and the air is regulated by cutaneous (skin) blood flow. If the cutaneous blood vessels are constricted, the skin temperature and the temperature of the environment will be about the same. When the vessels are dilated, more blood is brought to the surface. Suppose during dilation the skin warms from 72.0 °F to 84.0 °F. Convert **(a)** 72.0 °F and **(b)** 84.0 °F to Celsius and also **(c)** find their difference. Convert **(d)** 72.0 °F and **(e)** 84.0 °F to Kelvin and also **(f)** find their difference.

$$T_C = (0.556)(T_F - 32.0) = (0.556)(72.0 - 32.0) = \boxed{22.2\ °C}$$

$$T_C = (0.556)(T_F - 32.0) = (0.556)(84.0 - 32.0) = \boxed{28.9\ °C}$$

$$\Delta T_C = 28.9\ °C - 22.2\ °C = \boxed{6.7\ °C}$$

$$T_K = T_C + 273.15$$

$$T_K = 22.2\ °C + 273.15 = \boxed{295.4\ K}$$

$$T_K = 28.9\ °C + 273.15 = \boxed{302.0\ K}$$

$$\Delta T_K = 302.0\ K - 295.4\ K = \boxed{6.6\ K}$$

A steel railroad track has a length of 30.000 m when the temperature is 0 °C.

(a) What is its length on a hot day when the temperature is 40.0 °C? Suppose the track is nailed down so that it can't expand.

(b) What stress results in the track due to the temperature change?

$$\Delta L = \alpha L_0 \Delta T = \left[11 \text{ x } 10^{-6} \left(°\text{C}\right)^{-1}\right]\left(30.000 \text{ m}\right)\left(40.0 \text{ °C}\right) = 0.013 \text{ m}$$

$$L = L_0 + \Delta L = \left(30.000 \text{ m}\right) + \left(0.013 \text{ m}\right) = \boxed{30.013 \text{ m}}$$

$$\frac{F}{A} = \frac{Y \Delta L}{L} = \frac{\left(20 \text{ x } 10^{10} \text{ Pa}\right)\left(0.013 \text{ m}\right)}{\left(30.000 \text{ m}\right)} = \boxed{8.7 \text{ x } 10^{7} \text{ Pa}}$$

A circular copper ring at 20.000 °C has a hole with an area of 9.980 cm^2.

(a) What minimum temperature must it have so that it can be slipped onto a steel metal rod having a cross-sectional area of 10.000 cm^2? Suppose the ring and the rod are heated simultaneously.

(b) What minimum change in temperature of both will allow the ring to be slipped onto the end of the rod?

$$\Delta A = \gamma A_0 \Delta T = \left[34 \text{ x } 10^{-6} \left(°\text{C}\right)^{-1}\right]\left(9.980 \text{ cm}^2\right)\Delta T = 0.020 \text{ cm}^2$$

$$\therefore \quad \Delta T = 59\ °\text{C}$$

$$T = T_0 + \Delta T = \left(20.000\ °\text{C}\right) + \left(59\ °\text{C}\right) = \boxed{79\ °\text{C}}$$

$$A_C + \Delta A_C = A_S + \Delta A_S$$

$$A_C + \gamma_C A_C \Delta T = A_S + \gamma_S A_S \Delta T$$

$$\gamma_C A_C \Delta T - \gamma_S A_S \Delta T = A_S - A_C$$

$$\left(\gamma_C A_C - \gamma_S A_S\right)\Delta T = A_S - A_C$$

$$\Delta T = \frac{\left(A_S - A_C\right)}{\left(\gamma_C A_C - \gamma_S A_S\right)}$$

$$= \frac{\left(10.000 \text{ cm}^2 - 9.980 \text{ cm}^2\right)}{\left[\left(34 \text{ x } 10^{-6} \left(°\text{C}\right)^{-1}\right)\left(9.980 \text{ cm}^2\right) - \left(22 \text{ x } 10^{-6} \left(°\text{C}\right)^{-1}\right)\left(10.000 \text{ cm}^2\right)\right]}$$

$$= \boxed{170\ °\text{C}}$$

(a) Estimate the fractional change in the volume of the earth's oceans due to an average temperature change of 1.00 °C.

$\beta_{\text{salt water}} = 2.07 \times 10^{-4} (°C^{-1})$

(b) Use the fact that the average depth of the ocean is 4.00×10^3 m to estimate the approximate increase in depth.

$$\Delta V = \beta V_0 \Delta T$$

$$\therefore \quad \frac{\Delta V}{V_0} = \beta \Delta T = \left[2.07 \text{ x } 10^{-4}\left({}^\circ\mathrm{C}^{-1}\right)\right]\left(1.00\ {}^\circ\mathrm{C}\right) = \boxed{2.07 \text{ x } 10^{-4}}$$

$$\Delta L = \alpha L_0 \Delta T = \left(\frac{\beta}{3}\right) L_0 \Delta T$$

$$\Delta L = \left(\frac{2.07 \text{ x } 10^{-4}\left({}^\circ\mathrm{C}^{-1}\right)}{3}\right)\left(40\bar{0}0 \text{ m}\right)\left(1.00\ {}^\circ\mathrm{C}\right) = \boxed{0.276 \text{ m}}$$