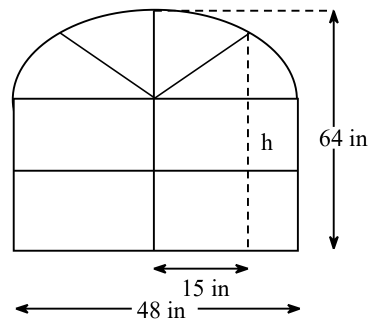
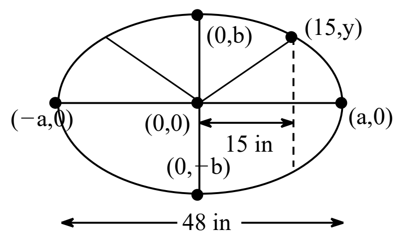
Student: Cole Lamers **Date:** 07/03/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.3 The Ellipse

A window is constructed with the top half of an ellipse on top of a square. The square portion of the window has a 48 inch base. If the window is 64 inches tall at its highest point, find the height, h, of the window 15 inches from the center of the base.



Find the equation of the ellipse that describes the top portion of the window. Then, use the equation to find the height.

Set up a rectangular coordinate system so that the x-axis coincides with the base of the upper half of the ellipse and the center of the ellipse is the origin.



The equation of the ellipse with center at (0,0), vertices at $(-a,0)$ and $(a,0)$, and endpoints of the vertical axis at (0,b) and $(0,-b)$ is shown below. The horizontal axis is the x-axis.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a > b > 0.$$

The major axis lies on the x-axis and coincides with the top of the square portion of the window.

The length of the major axis is 48 inches, the length of the base of the window.

Use the fact that the length of the major axis is the distance between the vertices $(-a,0)$ and (a,0) (which is 2a) to find a.

$$\begin{aligned} 2a &= 48 \\ a &= 24 \end{aligned}$$

To determine b, notice that the distance from the origin (0,0) to (0,b) is the difference between the height of the window at its highest point and the height of the square portion of the window.

The height of the window at its highest point is 64 inches.

The height of the square portion of the window is 48 inches.

Find b.

$$\begin{aligned} b &= 64 - 48 \\ &= 16 \end{aligned}$$

Find the equation of the ellipse descibing the upper portion of the window.

$$\begin{aligned} \frac{x^2}{a^2} + \frac{y^2}{b^2} &= 1 \\ \frac{x^2}{24^2} + \frac{y^2}{16^2} &= 1 \\ \frac{x^2}{576} + \frac{y^2}{256} &= 1 \qquad \text{Evaluate the squares.} \end{aligned}$$

To find the height of the window 15 inches from the center of the base, notice that the height is the sum of the height of the square and the value of y when $x = 15$.

Use the equation of the ellipse to find y when $x = 15$.

$$\begin{aligned} \frac{15^2}{576} + \frac{y^2}{256} &= 1 \\ \frac{225}{576} + \frac{y^2}{256} &= 1 \qquad \text{Evaluate the square.} \end{aligned}$$

Subtract $\frac{225}{576}$ from both sides.

$$\frac{y^2}{256} = 1 - \frac{225}{576}$$
$$= \frac{39}{64}$$

Multiply both sides by 256.

$$y^2 = 256\left(\frac{39}{64}\right)$$
$$= 156$$

Find the square root. Remember that y represents a length and so should be a nonnegative number.

$$y = \sqrt{156}$$
$$= 12.49$$

Find the height, h, of the window 15 inches from the center.

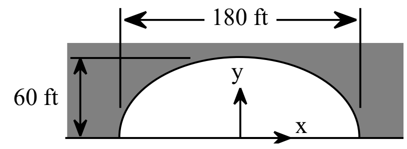
$$h = 48 + 12.49$$
$$= 60.49$$

Thus, the height of the window 15 inches from the center of the base is 60.49 inches.

Student: Cole Lamers **Date:** 07/03/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.3 The Ellipse

A bridge is built in the shape of a semielliptical arch. The bridge has a span of 180 feet and a maximum height of 60 feet. Choose a suitable rectangular coordinate system and find the height of the arch at a distance of 50 feet from the center.

Set up a rectangular coordinate system so that the x-axis coincides with the ground under the bridge, and the origin is in the middle of the span.



The equation of the ellipse with center at (0,0), foci at $(-c,0)$ and $(c,0)$, and vertices at $(-a,0)$ and $(a,0)$ is shown below. The major axis is the x-axis.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2$$

The major axis is along the x-axis. For the bridge, this is equal to the span, or 180 feet.

The vertices, $(-a,0)$ and $(a,0)$, are the points where the ellipse meets the ground. The distance between the vertices and the center of the ellipse is a. Therefore, a is half the major axis of the ellipse.

$a = 90$ feet

The intercept, (0,b), is the highest point of the arch, and is b feet away from the origin. From the problem statement, the maximum height is 60 feet.

$b = 60$ feet

Use the values $a = 90$ and $b = 60$ to write the equation of the ellipse without the units.

$$\frac{x^2}{8100} + \frac{y^2}{3600} = 1$$

The x-coordinate of a point 50 feet from the center could be either -50 or 50. Since the height is the same for either point, choose $x = 50$.

Solve the equation for the ellipse for y. Begin by isolating y^2.

$$\frac{x^2}{8100} + \frac{y^2}{3600} = 1$$

$$y^2 = 3600\left(1 - \frac{x^2}{8100}\right)$$

Then solve for y.

$$y^2 = 3600\left(1 - \frac{x^2}{8100}\right)$$

$$y = 60\sqrt{1 - \frac{x^2}{8100}}$$

Substitute the value of x and evaluate the expression for y.

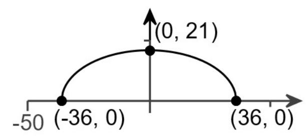
$$y = 60\sqrt{1 - \frac{(50)^2}{8100}}$$

$$y \approx 49.89$$

Therefore, the height of the arch 50 feet from the center is about 49.89 feet.

Student: Cole Lamers **Date:** 07/03/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.3 The Ellipse

The elliptical ceiling of a building is 72 ft long and 21 ft tall. Use the rectangular coordinate system in the figure shown to write the standard form of the equation for the elliptical ceiling. A man discovered that he could hear conversations of colleagues in the entire room if he stood at the focus, $(-c, 0)$, where $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. How far along the major axis did the man stand to hear the conversations?



Standard Forms of the Equations of an Ellipse

The standard form of the equation of an ellipse with center at the origin, and major axis of length 2a and minor axis of length 2b (where a and b are positive, and $a^2 > b^2$) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The vertices are on the major axis, a units from the center. The foci are on the major axis, c units from the center. For both equations, $b^2 = a^2 - c^2$. Equivalently, $c^2 = a^2 - b^2$. For the first form, the major axis is horizontal with length 2a and the foci are $(c, 0)$ and $(-c, 0)$. For the second form, the major axis is vertical with length 2a and the foci are $(0, c)$ and $(0, -c)$.

The distance from the center to the wall of the room is $a = 36$.

The distance from the center to the ceiling of the room is $b = 21$.

Since the ellipse has a horizontal major axis and the center is located at (0,0), the equation takes the form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The standard form of the equation using the values of a and b from above is $\frac{x^2}{1296} + \frac{y^2}{441} = 1$.

Now find the foci. Since the major axis is horizontal, the foci are located at $(c,0)$ and $(-c,0)$. Use the formula $c^2 = a^2 - b^2$ to determine c^2. (Recall from above, $a = 36$ and $b = 21$.)

$c^2 = 855$

If $c^2 = 855$, the value of c, the distance between the center and where the man stands is $c \approx 29$.

Student: Cole Lamers **Date:** 07/03/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.3 The Ellipse

The vertical cross section of a dome is semi-elliptical in shape. The cross section is 310 feet long with a maximum height of 120 feet. Where should two people stand in order to maximize the whispering effect?

The reflecting property for an ellipse says that a ray of light originating at one focus will be reflected to the other focus. Sound waves also follow such paths and this property is used in the construction of whispering galleries. To maximize the whispering effects, the two people should stand at the foci of the ellipse on opposite sides of the room from each other.

The major axis of the ellipse is the longest axis of symmetry of the ellipse and the minor axis is the smallest axis of symmetry of the ellipse. Because the dome is semi-elliptical in shape, its cross-sectional length is the major axis and its maximum height is the semi-minor axis.

To locate the foci of the ellipse, determine the values of a and b, that is, the lengths of the semi-major and semi-minor axes, respectively.

$$a = \frac{310}{2} = 155 \text{ feet}$$
$$b = 120 \text{ feet}$$

Use the equation $c^2 = a^2 - b^2$ to locate the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Solve the equation $c^2 = a^2 - b^2$ for c. Substitute 155 feet and 120 feet for a and b, respectively.

$$\begin{aligned} c^2 &= a^2 - b^2 \\ c &= \pm\sqrt{a^2 - b^2} && \text{Take the square root of both sides.} \\ c &= \sqrt{a^2 - b^2} && \text{Since c is a distance, it must be positive.} \\ c &\approx 98.11 \text{ feet} \end{aligned}$$

The foci are located at approximately $(\pm 98.11, 0)$.

The distance each person should stand from the wall is the difference of half the major axis and the distance to the foci.

$$\begin{aligned} a - c &= 155 \text{ feet} - 98.11 \text{ feet} \\ &\approx 57 \text{ feet} \end{aligned}$$

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The standard equation of an ellipse with center (0,0), vertices $(0, \pm a)$, and foci $(0, \pm c)$ is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, where $b^2 = a^2 - c^2$. The eccentricity of an ellipse, denoted by e, is defined as given below.

$$e = \frac{\text{Distance between the foci}}{\text{Distance between the vertices}} = \frac{2c}{2a} = \frac{c}{a}$$

Find the eccentricity of the ellipse $\frac{x^2}{24} + \frac{y^2}{49} = 1$.

First compare the given equation of the ellipse to the standard equations of the ellipse to find the value of a.

The standard equations of the ellipse are as follows.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1;\ a > b > 0 \text{ and } \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1;\ a > b > 0$$

Note that the given equation of the ellipse, $\frac{x^2}{24} + \frac{y^2}{49} = 1$, is of the form $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ since $49 > 24$.

Compare the two equations to find the value of a.

$$a = 7$$

Given that the relationship between a, b, and c is $b^2 = a^2 - c^2$.

Find the value of c.

$$\begin{aligned} b^2 &= a^2 - c^2 \\ 24 &= 49 - c^2 && \text{Substitute 24 for } b^2 \text{ and 49 for } c^2. \\ -25 &= -c^2 && \text{Subtract 49 from both sides.} \end{aligned}$$

Solve the equation for c.

$$\begin{aligned} -25 &= -c^2 \\ 25 &= c^2 && \text{Divide both sides by } -1. \\ 5 &= c && \text{Take the square root on both sides.} \end{aligned}$$

Thus, $a = 7$ and $c = 5$.

Use the definition of the eccentricity of an ellipse to find the eccentricity.

$$\begin{aligned} e &= \frac{\text{Distance between the foci}}{\text{Distance between the vertices}} \\ &= \frac{c}{a} \\ &= \frac{5}{7} \end{aligned}$$

Therefore, the eccentricity of the ellipse, $\frac{x^2}{24} + \frac{y^2}{49} = 1$, is $\frac{5}{7}$.

Find the center, vertices, and foci of the ellipse with equation

$$3x^2+4y^2+12x-8y-32=0\ .$$

$3x^2+12x+\quad +4y^2-8y+\quad =32$

$3(x^2+4x+4)+4(y^2-2y+1)=32+12+4$

$\dfrac{\cancel{3}(x+2)^2}{\cancel{48}\,16}+\dfrac{\cancel{4}(y-1)^2}{\cancel{48}\,12}=\dfrac{48}{48}=1$

$\dfrac{(x+2)^2}{16}+\dfrac{(y-1)^2}{12}=1$ $\qquad b=\sqrt{12}$

$b=2\sqrt{3}$

$a=4 \quad b^2=12 \nearrow \quad c^2=a^2-b^2=4$

$\dfrac{(x+2)^2}{16}+\dfrac{(y-1)^2}{12}=1$

$a=4 \quad b=2\sqrt{3} \quad c=2$

Center: $(h,k)=(-2,1)$

Foci: $(h\pm c,k)=(-2\pm 2,1)$

$(0,1)$ & $(-4,1)$

Vertices: $(h\pm a,k)=(-2\pm 4,1)$

$(2,1)$ & $(-6,1)$

endpoints: $(h,k\pm b)=(-2,1\pm 2\sqrt{3})$

$(-2,1+2\sqrt{3})$ &

$(-2,1-2\sqrt{3})$

lengths

major $=2a$
$=8$

minor $=2b$
$=4\sqrt{3}$

Sketch a graph of the ellipse whose equation is $9x^2 + 4y^2 = 36$. Find the foci of the ellipse.

$$\frac{\cancel{9}x^2}{\cancel{36}\,4} + \frac{\cancel{4}y^2}{\cancel{36}\,9} = \frac{36}{36}$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

$a = 3 \quad b = 2$

$c^2 = a^2 - b^2 = 9 - 4 = 5$

$c = \pm\sqrt{5}$

Vertex: $(0, \pm 3)$

Foci: $(0, \pm\sqrt{5})$

length major

$2 \cdot 3 = 6$

length minor

$2 \cdot 2 = 4$

Student: Cole Lamers **Date:** 07/02/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.3 The Ellipse

Find the vertices and foci of the ellipse. Graph the equation.

$$x^2 + y^2 = 100$$

The equation of an ellipse can be written in one of the two forms below, where $a > b > 0$.

The major axis is the x-axis.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The major axis is the y-axis.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

First write the given equation in the standard form.

$$\frac{x^2 + y^2}{100} = \frac{100}{100}$$

$$\frac{x^2}{100} + \frac{y^2}{100} = 1$$

Notice that the two denominators are equal. Since the axes are equal, the equation describes a circle.

The equation for a circle with center at (0,0), focus at (0,0), and vertices at $(-r,0)$, $(r,0)$, $(0,-r)$, and $(0,r)$ is shown below.

$$x^2 + y^2 = r^2$$

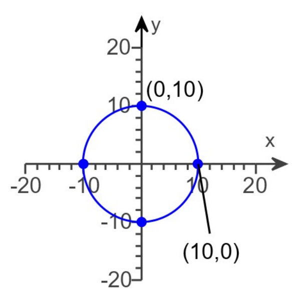
Identify the radius from the given equation.

$r = \sqrt{100} = 10$

The vertices are $(-10,0)$, $(10,0)$, $(0,-10)$, and $(0,10)$.

The coordinates of the focus are (0,0).

The graph of the circle is shown to the right.



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Find the standard form of the equation for the ellipse with foci $(0, \pm 5)$, and vertex $(0,8)$. Graph the equation.

Note that the foci of the ellipse are $(0,5)$ and $(0, -5)$, through which the major axis passes.

Thus, the major axis of the given ellipse is the y-axis because the foci are points on the y-axis.

So, the given ellipse is a vertical ellipse. The standard equation of the vertical ellipse is shown below.

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \ a > b > 0$$

In a vertical ellipse with center $(0,0)$, the foci have coordinates $(0, \pm c)$ and the vertices are $(0, \pm a)$. Also, $b^2 = a^2 - c^2$. To find the equation of the ellipse determine the values of a^2 and b^2. Use foci $(0,5)$ and $(0, -5)$, and vertex $(0,8)$ to find a and c.

The value for a is taken from the given vertex, and the value for c is taken from the given foci.

$a = 8 \qquad\qquad c = 5$

Substitute the values of a and c into $b^2 = a^2 - c^2$ and find b^2.

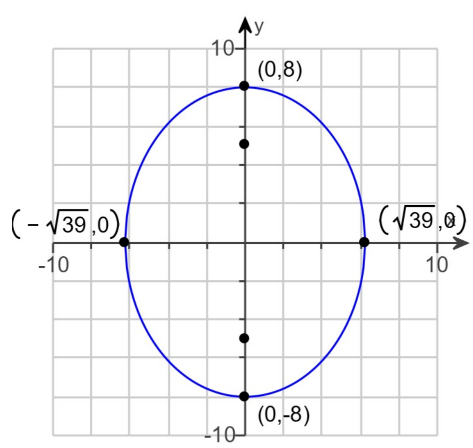
$$\begin{aligned} b^2 &= a^2 - c^2 \\ &= 8^2 - 5^2 \\ &= 64 - 25 \\ &= 39 \end{aligned}$$

Substitute the values of a^2 and b^2 in the formula $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$, in order to find the equation for the given ellipse. The value for $a^2 = 64$ and $b^2 = 39$.

$$\frac{x^2}{39} + \frac{y^2}{64} = 1$$

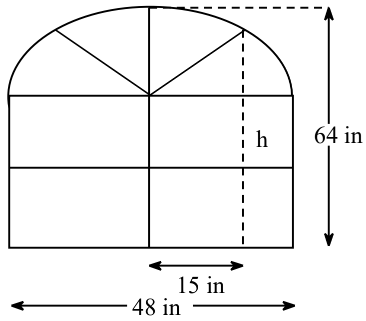
So, the given ellipse is a vertical ellipse with foci $(0,5)$ and $(0, -5)$, and vertices $(0,8)$ and $(0, -8)$. The length of its major axis is 2a and the length of its minor axis is 2b, where $a = 8$ and $b = \sqrt{39}$. These points and lengths help in graphing the equation of an ellipse.

Use foci $(0,5)$ and $(0, -5)$, and vertices $(0,8)$ and $(0, -8)$. The length of its major axis is 16 and the length of its minor axis is $2\sqrt{39}$. Note that the center of the ellipse is at $(0,0)$. A more detailed graph of the given ellipse is shown to the right.



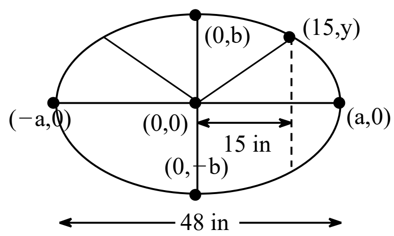
Student: Cole Lamers **Date:** 07/02/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.3 The Ellipse

A window is constructed with the top half of an ellipse on top of a square. The square portion of the window has a 48 inch base. If the window is 64 inches tall at its highest point, find the height, h, of the window 15 inches from the center of the base.



Find the equation of the ellipse that describes the top portion of the window. Then, use the equation to find the height.

Set up a rectangular coordinate system so that the x-axis coincides with the base of the upper half of the ellipse and the center of the ellipse is the origin.



The equation of the ellipse with center at (0,0), vertices at $(-a,0)$ and $(a,0)$, and endpoints of the vertical axis at (0,b) and $(0,-b)$ is shown below. The horizontal axis is the x-axis.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ where } a > b > 0.$$

The major axis lies on the x-axis and coincides with the top of the square portion of the window.

The length of the major axis is 48 inches, the length of the base of the window.

Use the fact that the length of the major axis is the distance between the vertices $(-a,0)$ and $(a,0)$ (which is 2a) to find a.

$$\begin{aligned} 2a &= 48 \\ a &= 24 \end{aligned}$$

To determine b, notice that the distance from the origin (0,0) to (0,b) is the difference between the height of the window at its highest point and the height of the square portion of the window.

The height of the window at its highest point is 64 inches.

The height of the square portion of the window is 48 inches.

Find b.

$$\begin{aligned} b &= 64 - 48 \\ &= 16 \end{aligned}$$

Find the equation of the ellipse descibing the upper portion of the window.

$$\begin{aligned} \frac{x^2}{a^2} + \frac{y^2}{b^2} &= 1 \\ \frac{x^2}{24^2} + \frac{y^2}{16^2} &= 1 \\ \frac{x^2}{576} + \frac{y^2}{256} &= 1 \qquad \text{Evaluate the squares.} \end{aligned}$$

To find the height of the window 15 inches from the center of the base, notice that the height is the sum of the height of the square and the value of y when $x = 15$.

Use the equation of the ellipse to find y when $x = 15$.

$$\begin{aligned} \frac{15^2}{576} + \frac{y^2}{256} &= 1 \\ \frac{225}{576} + \frac{y^2}{256} &= 1 \qquad \text{Evaluate the square.} \end{aligned}$$

Subtract $\frac{225}{576}$ from both sides.

$$\begin{aligned}\frac{y^2}{256} &= 1 - \frac{225}{576}\\ &= \frac{39}{64}\end{aligned}$$

Multiply both sides by 256.

$$\begin{aligned}y^2 &= 256\left(\frac{39}{64}\right)\\ &= 156\end{aligned}$$

Find the square root. Remember that y represents a length and so should be a nonnegative number.

$$\begin{aligned}y &= \sqrt{156}\\ &= 12.49\end{aligned}$$

Find the height, h, of the window 15 inches from the center.

$$\begin{aligned}h &= 48 + 12.49\\ &= 60.49\end{aligned}$$

Thus, the height of the window 15 inches from the center of the base is 60.49 inches.

Student: Cole Lamers **Date:** 07/02/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 10.3 The Ellipse

Find the vertices and foci for the ellipse. Graph the equation.

$$\frac{x^2}{49} + \frac{y^2}{9} = 1$$

Use the following theorem.

Equation of an Ellipse; Center at (0,0); Foci at (± c,0); Major Axis along the x-Axis

An equation of the ellipse with center at (0,0) and foci at (c,0) and (− c,0) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \text{where } a > b > 0 \text{ and } b^2 = a^2 - c^2$$

The major axis is the x-axis. The vertices are at (a,0) and (− a,0). The y-intercepts are (0,b) and (0, − b).

The equation given in the problem statement is in the form of the equation in the theorem with $a^2 = 49$ and $b^2 = 9$. We find the values of a and b by taking the square root, so $a = 7$and $b = 3$.

The value of c is found as follows.

$b^2 = a^2 - c^2$	Equation given in the theorem.
$c^2 = a^2 - b^2$	Solve for c^2.
$c = \sqrt{a^2 - b^2}$	Take the square root of both sides.
$c = \sqrt{49 - 9}$	$a^2 = 49$, $b^2 = 9$.
$c = 2\sqrt{10}$	Simplify.

The vertices are at (a,0) and (− a,0), so the vertices are (7,0) and (− 7,0).

The foci are at (c,0) and (− c,0), so the foci are $\left(2\sqrt{10},0\right)$ and $\left(-2\sqrt{10},0\right)$.

The equation is that of an ellipse with center (0,0) and major axis along the x-axis. The vertices are at (7,0) and (− 7,0) and the y-intercepts are (0,3) and (0, − 3). The foci are at $\left(2\sqrt{10},0\right)$ and $\left(-2\sqrt{10},0\right)$, or about (6.3,0) and (− 6.3,0). Therefore, the following is the correct graph of the equation $\frac{x^2}{49} + \frac{y^2}{9} = 1$.