Student: Cole Lamers **Date:** 07/04/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 2.2, 10.2 Circle and The Parabola

Find the focus and directrix of the parabola with the equation $12x^2 = -14y$. Then graph the parabola.

The standard form of the equations of a parabola with vertex at the origin are $y^2 = 4ax$, $y^2 = -4ax$, $x^2 = 4ay$, or $x^2 = -4ay$.

For the equation $y^2 = 4ax$ or $y^2 = -4ax$, the focus is on the x-axis, which is the axis of symmetry. For the equation $x^2 = 4ay$ or $x^2 = -4ay$, the focus is on the y-axis, which is the axis of symmetry.

Write the equation $12x^2 = -14y$ in the standard form $x^2 = -4ay$.

$$12x^2 = -14y$$

$$x^2 = -\frac{7}{6}y \qquad \text{Divide both sides by 12.}$$

Equate the coefficients of y by comparing with $x^2 = -4ay$. Determine the value of a.

$$-4a = -\frac{7}{6}$$

$$a = \frac{7}{24} \qquad \text{Divide both sides by } -4.$$

The focus of a parabola with an equation of the form $x^2 = -4ay$ is at the point $(0, -a)$.

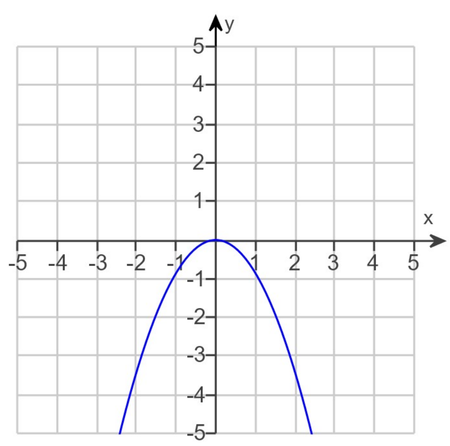
The focus of the parabola $12x^2 = -14y$ is $\left(0, -\frac{7}{24}\right)$.

The directrix of a parabola with an equation of the form $x^2 = -4ay$ has the form $y = a$.

The directrix of the parabola $12x^2 = -14y$ is $y = \frac{7}{24}$.

Because the equation of the parabola is of the form $x^2 = -4ay$ with $a > 0$, the parabola, with its y-axis symmetry, opens downward.

The graph of the parabola with the equation $12x^2 = -14y$ is shown to the right.



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a. Find the center and radius of the given circle.
b. Find the x- and y-intercepts of the graph of the given circle.

$$x^2 + y^2 + 8x - 18y + 16 = 0$$

To find the center and radius of the given circle, convert the general form of the equation of a circle to standard form.

Note that the standard form of an equation of a circle with center (h,k) and radius r is $(x-h)^2 + (y-k)^2 = r^2$.

Complete the square on both x-terms and y-terms to get the standard form.

To complete the square, group the x-terms and y-terms and put the constant on the right side of the equation.

$$x^2 + y^2 + 8x - 18y + 16 = 0$$
$$\left(x^2 + 8x\right) + \left(y^2 - 18y\right) = -16$$

Next, complete the square of each expression in parentheses on the left side.

$$\left(x^2 + 8x\right) + \left(y^2 - 18y\right) = -16$$
$$\left(x^2 + 8x + 16\right) + \left(y^2 - 18y + 81\right) = -16$$

Add the numbers 16 and 81 to the right side as those added on the left. The resulting number on the right side is 81.

The constant after completing the square is 81.

Now factor the left side.

$$\left(x^2 + 8x + 16\right) + \left(y^2 - 18y + 81\right) = 81$$
$$(x+4)^2 + (y-9)^2 = 81$$

Now compare the obtained equation, $(x+4)^2 + (y-9)^2 = 81$ to the standard form of the equation $(x-h)^2 + (y-k)^2 = r^2$ of a circle.

Now determine the center (h,k) and radius r of the circle.

$$(h,k) = (-4,9) \qquad r = 9$$

To find the x-intercepts, set y = 0 in the standard form of the equation and solve for x.

$$(x+4)^2 + (y-9)^2 = 81$$
$$(x+4)^2 + (0-9)^2 = 81 \qquad \text{Set } y = 0.$$
$$(x+4)^2 + 81 = 81 \qquad \text{Simplify.}$$
$$(x+4)^2 = 0 \qquad \text{Simplify.}$$

Apply the square root property and solve for x.

$$(x+4)^2 = 0$$
$$x + 4 = \pm 0 \qquad \text{Take square root.}$$
$$x = -4$$

Therefore, the x-intercept is -4.

To find the y-intercepts, set x = 0 in the standard form of the equation and solve for y.

$$(x+4)^2 + (y-9)^2 = 81$$
$$(0+4)^2 + (y-9)^2 = 81 \qquad \text{Set } x = 0.$$
$$16 + (y-9)^2 = 81 \qquad \text{Simplify.}$$
$$(y-9)^2 = 65 \qquad \text{Simplify.}$$

Apply the square root property and solve for y.

$$(y-9)^2 = 65$$
$$y - 9 = \pm\sqrt{65}$$
$$y = 9 + \sqrt{65} \text{ and } 9 - \sqrt{65}$$

Therefore, the y-intercepts are $9 + \sqrt{65}$ and $9 - \sqrt{65}$.

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Find the standard equation of the parabola that satisfies the given conditions. Also, find the length of the latus rectum of each parabola.

focus: (1,0); directrix: $x = 10$

Notice that the directrix is a vertical line, so the parabola opens to the left or right. Determine which way the parabola opens by examining on which side of the directrix the focus lies.

The x-value for the focus, 1, is smaller than $x = 10$ so the focus lies to the left of the directrix.

Since a parabola always opens away from its directrix, the parabola with focus (1,0) and directrix $x = 10$ opens left.

The standard equation for the given parabola has vertex (h,k), $a > 0$, and the standard equation $(y-k)^2 = -4a(x-h)$. Use the facts about the parabola of this form, shown below, to find the values of h, k, and a.

Standard Equation	$(y-k)^2 = -4a(x-h)$
Axis of Symmetry	$y = k$
Description	Opens left
Vertex	(h,k)
Focus	$(h-a,k)$
Directrix	$x = h + a$

The focus (1,0) has general form $(h-a,k)$. The value of $h-a$ is 1 and the value of k is 0.

The directrix $x = 10$ has general form $x = h + a$. The value of $h + a$ is 10.

The equations $h - a = 1$ and $h + a = 10$ are two equations with two unknowns. Notice that the coefficients of a are opposite values. Solve the system by elimination.

Add the equations.

$$\begin{array}{l} h - a = 1 \\ \underline{h + a = 10} \\ \quad 2h = 11 \end{array}$$

Solve for h.

$$h = \frac{11}{2}$$

Substitute the value of h back into either equation and solve for a. For this problem, use $h + a = 10$.

Solve for a.

$$\frac{11}{2} + a = 10 \qquad \text{Substitute } h = \frac{11}{2}.$$

$$a = \frac{9}{2} \qquad \text{Subtract } \frac{11}{2} \text{ from both sides and simplify.}$$

Substitute the values of h, k, and a into the standard equation for a parabola.

$$(y-k)^2 = -4a(x-h)$$

$$(y-0)^2 = -4\left(\frac{9}{2}\right)\left(x - \frac{11}{2}\right) \qquad \text{Substitute the values of h, k, and a.}$$

$$y^2 = -18\left(x - \frac{11}{2}\right) \qquad \text{Simplify.}$$

Thus, the parabola with focus (1,0) and directrix $x = 10$ has the standard equation shown below.

$$y^2 = -18\left(x - \frac{11}{2}\right)$$

Now find the length of the latus rectum of the parabola. The line segment that passes through the focus of a parabola, is perpendicular to the axis of the parabola, and has endpoints on the parabola is called the latus rectum of the parabola. The length of the latus rectum for the graph of a parabola with $a > 0$ is 4a.

Use the fact that $a = \frac{9}{2}$ to determine the length of the latus rectum.

$$4a = 4 \cdot \frac{9}{2} = 18$$

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Find the standard equation of the parabola that satisfies the given conditions. Also find the length of the latus rectum of the parabola.

Focus: (0,11); directrix: $y = 9$

The axis of symmetry must be perpendicular to the directrix and must pass through the focus. Because the directrix is the vertical line $y = 9$ and the focus is at (0,11) the parabola has the y-axis ($x = 0$) symmetry.

The vertex of the parabola lies on the axis of symmetry $x = 0$, midway between the focus, (0,11), and the directrix, $y = 9$.

The vertex of the parabola is $\left(0, \frac{11+9}{2}\right) = (0,10)$.

Observe that the vertex of the parabola is (0,10), the axis is the y-axis, and the focus is (0,11).

The focus is $11 - 10 = 1$ unit above the vertex (0,10).

Because the focus is 1 unit above the vertex (0,10), the parabola opens up.

The main facts about a parabola with vertex (h,k) and $p > 0$ are given below.

Standard Equation	$(y-k)^2 = 4p(x-h)$	$(y-k)^2 = -4p(x-h)$	$(x-h)^2 = 4p(y-k)$	$(x-h)^2 = -4p(y-k)$
Equation of axis	$y = k$	$y = k$	$x = h$	$x = h$
Description	Opens right	Opens left	Opens up	Opens down
Vertex	(h,k)	(h,k)	(h,k)	(h,k)
Focus	$(h+p,k)$	$(h-p,k)$	$(h,k+p)$	$(h,k-p)$
Directrix	$x = h - p$	$x = h + p$	$y = k - p$	$y = k + p$

The parabola opens up and the equation of the parabola is of the form $(x-h)^2 = 4p(y-k)$.

The vertex of the parabola is $(h,k) = (0,10)$. Thus, $h = 0$ and $k = 10$.

Find the value of p which is the directed distance from the vertex to the focus.

$p = 1$

Substitute $h = 0$, $k = 10$, and $p = 1$ in the standard form of the equation $(x-h)^2 = 4p(y-k)$.

$$\begin{aligned} (x-h)^2 &= 4p(y-k) && \\ (x-0)^2 &= 4(1)(y-10) && \text{Substitute 0 for h, 10 for k, and 1 for p.} \\ x^2 &= 4(y-10) && \text{Simplify.} \end{aligned}$$

Thus, the standard equation of the parabola that satisfies the given conditions is $x^2 = 4(y-10)$.

The line segment that passes through the focus of a parabola, is perpendicular to the axis, and has endpoints on the parabola is called the latus rectum of the parabola. The length of the latus rectum is 4p.

Recall that $p = 1$.

Thus, the length of the latus rectum is $4(1) = 4$.