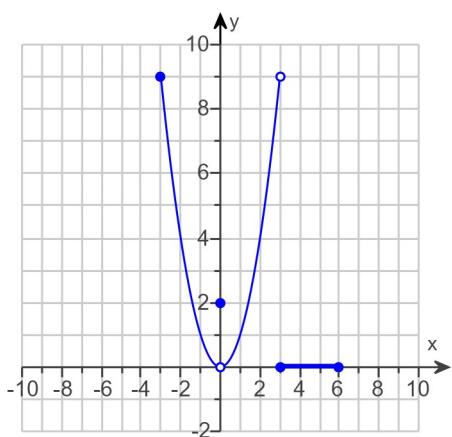


Use the graph below to determine whether the statements about the function  $y = f(x)$  are true or false.



Determine whether the statement  $\lim_{x \rightarrow -3^+} f(x) = 9$  is true or false.

The symbol  $x \rightarrow -3^+$  means that values of  $x$  are getting closer and closer to the value  $-3$  from the side that has numbers larger than  $-3$ .

As  $x$  gets closer and closer to  $-3$  from the side with the values greater than  $-3$ , the values of  $f(x)$  are getting larger and larger; closer and closer to 9. So, as  $x \rightarrow -3^+$ ,  $f(x) \rightarrow 9$ . This is essentially the definition of  $\lim_{x \rightarrow -3^+} f(x) = 9$ . So, the statement is true.

Determine whether the statement  $\lim_{x \rightarrow 0^-} f(x) = 2$  is true or false.

As  $x$  approaches 0 from values less than 0,  $f(x)$  assumes values smaller and smaller; closer and closer to 0. Although the value of  $f(x)$  at 0 is 2, as  $x$  moves towards 0 from negative values,  $f(x)$  moves toward 0, not 2, as a value. Thus, the statement is false.

Determine whether the statement  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$  is true or false.

As  $x$  approaches 0 from either side, the value of  $f(x)$  gets closer and closer to 0. Thus, regardless of the value of  $f(x)$  at 0, the statement  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$  is true.

Determine whether the statement ' $\lim_{x \rightarrow 0} f(x)$  exists' is true or false.

Since  $\lim_{x \rightarrow a^-} f(x) = \lim_{x \rightarrow a^+} f(x) \rightarrow \lim_{x \rightarrow a} f(x)$  exists, the statement ' $\lim_{x \rightarrow 0} f(x)$  exists' is true.

Determine whether the statement  $\lim_{x \rightarrow 3} f(x) = 2$  is true or false.

As  $x$  approaches 3 from values less than 3,  $f(x)$  gets closer and closer to the value 9. As  $x$  approaches 3 from values larger than 3, the value of  $f(x)$  stays at 0. Thus, the statement  $\lim_{x \rightarrow 3} f(x) = 2$  is false.

Determine whether the statement  $\lim_{x \rightarrow 6^-} f(x) = 6$  is true or false.

As  $x$  approaches 6 from values smaller than 6,  $f(x)$  approaches the value 0, not 6.

So,  $\lim_{x \rightarrow 6^-} f(x) = 6$  is false.

2.4.1

Use the graph below to determine whether the statements about the function  $y = f(x)$  are true or false.

True or false:  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$ .

False  
 True

True or false:  $\lim_{x \rightarrow 0} f(x)$  exists.

Sorry, that's not correct.

Compare the values approached by  $f(x)$  as  $x$  moves towards 2 from the values both larger than and smaller than 2. Since these are different, the limit does not exist.

True or false:  $\lim_{x \rightarrow 2} f(x) = 4$ .

False  
 True

Click to select your answer and then click Check Answer.

**Score:** 0.71 of 1 pt

◀ 1 of 11 (1 complete) ▶

**HW Score:** 6.49%, 0.71 of 11 pt

2.4.1

Use the graph below to determine whether the statements about the function  $y = f(x)$  are true or false.

True or false:  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$ .

False  
 True

True or false:  $\lim_{x \rightarrow 0} f(x)$  exists.

Sorry, that's not correct.

The limit is the value approached by the function, not the value approached by  $x$ .

True or false:  $\lim_{x \rightarrow 4^-} f(x) = 4$ .

False  
 True

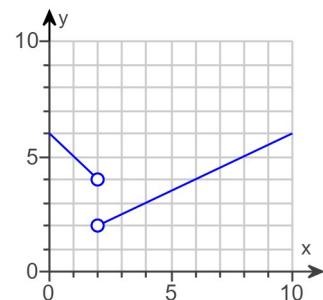
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**Date:** 09/01/19

**Instructor:** Viktoriya Shcherban  
**Course:** Calc 1 11:30 AM Internet  
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**Assignment:** 2.4 One Sided Lits

Use the following function and its graph to answer (a) through (d) below.

$$\text{Let } f(x) = \begin{cases} 6 - x, & x < 2 \\ \frac{x}{2} + 1, & x > 2. \end{cases}$$



- a. Find  $\lim_{x \rightarrow 2^+} f(x)$  and  $\lim_{x \rightarrow 2^-} f(x)$ .

Begin by determining the right-hand limit  $\lim_{x \rightarrow 2^+} f(x)$

If  $f(x)$  is defined on the interval  $(c, b)$ , where  $c < b$ , and approaches arbitrarily close to  $L$  as  $x$  approaches  $c$  from within the interval, then  $f$  has right-hand limit  $L$  at  $c$ , denoted  $\lim_{x \rightarrow c^+} f(x) = L$

The expression  $\frac{x}{2} + 1$  is used to calculate  $\lim_{x \rightarrow 2^+} f(x)$  because, as  $x$  approaches 2 from the right,  $x > 2$

Determine the value of  $\lim_{x \rightarrow 2^+} \left( \frac{x}{2} + 1 \right)$  by evaluating  $\frac{x}{2} + 1$  for  $x = 2$

$$\lim_{x \rightarrow 2^+} \left( \frac{x}{2} + 1 \right) = \frac{(2)}{2} + 1 = 2$$

Therefore,  $\lim_{x \rightarrow 2^+} f(x) = 2$ . Next, find the value of  $\lim_{x \rightarrow 2^-} f(x)$ .

If  $f(x)$  is defined on the interval  $(a, c)$ , where  $a < c$ , and approaches arbitrarily close to  $M$  as  $x$  approaches  $c$  from within that interval, then  $f$  has left-hand limit  $M$  at  $c$ , denoted  $\lim_{x \rightarrow c^-} f(x) = M$

The expression  $6 - x$  is used to calculate  $\lim_{x \rightarrow 2^-} f(x)$  because, as  $x$  approaches 2 from the right,  $x < 2$

Determine the value of  $\lim_{x \rightarrow 2^-} (6 - x)$  by evaluating  $6 - x$  for  $x = 2$

$$\lim_{x \rightarrow 2^-} (6 - x) = 6 - (2) = 4$$

Therefore,  $\lim_{x \rightarrow 2^-} f(x) = 4$ .

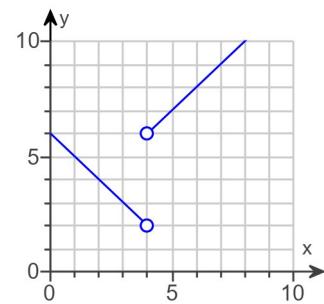
- b. Does  $\lim_{x \rightarrow 2} f(x)$  exist? If so, what is it? If not, why not?

A function  $f(x)$  has a limit as  $x$  approaches  $c$  if and only if it has left-hand and right-hand limits there and these one-sided limits are equal.

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Use the following function and its graph to answer (a) through (d) below.

Let  $f(x) = \begin{cases} 6 - x, & x < 4 \\ x + 2, & x > 4. \end{cases}$



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**Assignment:** 2.4 One Sided Limits

Find the limit

$$\lim_{h \rightarrow 0^+} \frac{\sqrt{h^2 + 4h + 21} - \sqrt{21}}{h}$$

Because there is an  $h$  in the denominator of the function,  $h = 0$  cannot be substituted in right away to calculate the value of the limit. Determine if there are any common factors.

The function in the limit does not have any common factors between the numerator and the denominator.

Hence, a common factor needs to be created.

To create a common factor, multiply the numerator and the denominator of the function by the conjugate of the numerator.

$$\sqrt{h^2 + 4h + 21} + \sqrt{21}$$

Multiply the numerator and denominator by the expression and simplify.

$$\frac{\sqrt{h^2 + 4h + 21} - \sqrt{21}}{h} \cdot \frac{\sqrt{h^2 + 4h + 21} + \sqrt{21}}{\sqrt{h^2 + 4h + 21} + \sqrt{21}} = \frac{h^2 + 4h + 21 - 21}{h(\sqrt{h^2 + 4h + 21} + \sqrt{21})}$$

Simplifying the numerator, the product becomes the following.

$$\begin{aligned} \frac{h^2 + 4h + 21 - 21}{h(\sqrt{h^2 + 4h + 21} + \sqrt{21})} &= \frac{h^2 + 4h}{h(\sqrt{h^2 + 4h + 21} + \sqrt{21})} \\ &= \frac{h(h + 4)}{h(\sqrt{h^2 + 4h + 21} + \sqrt{21})} \end{aligned}$$

Cancel the common factor that occurs in the numerator and the denominator.

$$\frac{h(h + 4)}{h(\sqrt{h^2 + 4h + 21} + \sqrt{21})} = \frac{h + 4}{\sqrt{h^2 + 4h + 21} + \sqrt{21}}$$

Determine the value of  $\lim_{h \rightarrow 0^+} \frac{h + 4}{(\sqrt{h^2 + 4h + 21} + \sqrt{21})}$  by substituting 0 for  $h$ .

$$\begin{aligned} \lim_{h \rightarrow 0^+} \frac{h + 4}{(\sqrt{h^2 + 4h + 21} + \sqrt{21})} &= \frac{(0) + 4}{(\sqrt{(0)^2 + 4(0) + 21} + \sqrt{21})} \\ &= \frac{4}{2\sqrt{21}} \\ &= \frac{2}{\sqrt{21}} \end{aligned}$$

$$\text{Therefore, } \lim_{h \rightarrow 0^+} \frac{\sqrt{h^2 + 4h + 21} - \sqrt{21}}{h} = \frac{2}{\sqrt{21}}.$$

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**Assignment:** 2.4 One Sided Li~~its~~

Use the relation  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$  to determine the limit of the given function.

$$f(\theta) = \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta}$$

As  $\theta \rightarrow 0$ ,  $\sqrt{2}\theta \rightarrow 0$ .

$$\text{So, } \lim_{\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta} = \lim_{\sqrt{2}\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta}.$$

Let  $x = \sqrt{2}\theta$ .

Use substitution to simplify.

$$\lim_{\sqrt{2}\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta} = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\text{Thus, } \lim_{\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta} = \lim_{\sqrt{2}\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta} = \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$$

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**Assignment:** 2.4 One Sided Liits

Find the liit

$$\lim_{x \rightarrow 0} \frac{x \csc 2x}{\cos 7x}$$

Use the theorem  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \neq$  to find the liit

The given liit is not of the form  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$  because there is no **sin** in the function.

To make the given liit of the form  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$ , first apply the quotient rule as follows.

$$\lim_{x \rightarrow 0} \frac{x \csc 2x}{\cos 7x} = \frac{\lim_{x \rightarrow 0} x \csc 2x}{\lim_{x \rightarrow 0} \cos 7x}$$

Evaluate the liit in the denominator.

$$\frac{\lim_{x \rightarrow 0} x \csc 2x}{\lim_{x \rightarrow 0} \cos 7x} = \frac{\lim_{x \rightarrow 0} x \csc 2x}{1}$$

$$\text{Thus, } \lim_{x \rightarrow 0} \frac{x \csc 2x}{\cos 7x} = \lim_{x \rightarrow 0} x \csc 2x.$$

Write **csc** 2x in terms of **sin**.

$$\csc 2x = \frac{1}{\sin 2x}$$

Substituting this into the given liit gives the following liit

$$\lim_{x \rightarrow 0} \frac{x \csc 2x}{\cos 7x} = \lim_{x \rightarrow 0} \frac{x}{\sin 2x}$$

The numerator and the denominator must be multiplied by so that the numerator is equal to the argument of **sin**.

$$\lim_{x \rightarrow 0} \frac{x \csc 2x}{\cos 7x} = \lim_{x \rightarrow 0} \frac{2x}{2 \sin 2x}$$

Notice that the liit is almost of the form  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$ , except the **sin** is in the denominator and the x-term is in the numerator.

Apply the constant multiple rule.

$$\lim_{x \rightarrow 0} \frac{2x}{2 \sin 2x} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{2x}{\sin 2x}$$

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**Assignment:** 2.4 One Sided Lits

Find the liith

$$\lim_{x \rightarrow 0} \frac{x \csc 11x}{\cos 8x}$$

Use the theorem  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$  ≠ to find the liith

Is the given liith of the form  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$

Yes

No

To make the given liith of the form  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$ , first apply the quotient rule as follows.

$$\lim_{x \rightarrow 0} \frac{x \csc 11x}{\cos 8x} = \frac{\lim_{x \rightarrow 0} x \csc 11x}{\lim_{x \rightarrow 0} \cos 8x}$$

Evaluate the liith in the denominator.

$$\frac{\lim_{x \rightarrow 0} x \csc 11x}{\lim_{x \rightarrow 0} \cos 8x} = \frac{\lim_{x \rightarrow 0} x \csc 11x}{1} \quad (\text{Type an integer or a simplified fraction.})$$

Thus,  $\lim_{x \rightarrow 0} \frac{x \csc 11x}{\cos 8x} = \lim_{x \rightarrow 0} x \csc 11x$ .

Write  $\csc 11x$  in terms of  $\sin$ .

$$\csc 11x = \frac{1}{\sin 11x}$$

Substituting this into the given liith gives the following liith

$$\lim_{x \rightarrow 0} \frac{x \csc 11x}{\cos 8x} = \lim_{x \rightarrow 0} \frac{x}{\sin 11x}$$

What number must the numerator and the denominator be multiplied by so that the numerator is equal to the argument of  $\sin$ ?

11

Multiplying the numerator and the denominator of the function in the liith by 11 gives the following liith

$$\lim_{x \rightarrow 0} \frac{x \csc 11x}{\cos 8x} = \lim_{x \rightarrow 0} \frac{11x}{11 \sin 11x}$$

Notice that the liith is almost of the form

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta}$$

except the  $\sin$  is in the denominator and the  $x$ -term is in the numerator.

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**Assignment:** 2.4 One Sided Lits

Use the relation  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$  to determine the limit shown below.

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin 25\theta}$$

Recall the half-angle formula for cosine.

$$\cos x = 1 - 2 \sin^2 \left( \frac{x}{2} \right)$$

Use the half-angle formula for cosine to express  $1 - \cos \theta$  in terms of sines.

$$\frac{1 - \cos \theta}{\sin 25\theta} = \frac{2 \sin^2 \left( \frac{\theta}{2} \right)}{\sin 25\theta}$$

The given expression has now been written in the form  $\frac{\text{numerator}}{\text{denominator}}$  where numerator  $= 2 \sin^2 \left( \frac{\theta}{2} \right)$  and denominator  $= \sin 25\theta$

Rewrite the denominator,  $\sin 25\theta$  to get an equivalent expression in the form  $\frac{\sin x}{x}$  where  $x = 25\theta$

$$\sin 25\theta = (25\theta) \cdot \left( \frac{\sin 25\theta}{25\theta} \right)$$

Split the numerator,  $2 \sin^2 \left( \frac{\theta}{2} \right)$  into factors and, in same manner as seen in the previous step, multiply one of the sine factors by  $\frac{\theta}{\theta}$ .

$$2 \sin^2 \left( \frac{\theta}{2} \right) = 2 \cdot \frac{2}{\theta} \cdot \sin \left( \frac{\theta}{2} \right) \cdot \sin \left( \frac{\theta}{2} \right)$$

Express the result in terms of expressions of the form  $\frac{\sin x}{x}$

$$\frac{\text{numerator}}{\text{denominator}} = \frac{2 \cdot \frac{\theta}{2} \cdot \left( \frac{\sin \frac{\theta}{2}}{\frac{\theta}{2}} \right) \cdot \sin \frac{\theta}{2}}{25\theta \left( \frac{\sin 25\theta}{25\theta} \right)}$$

Simplify this expression.

The right-hand limit  $\lim_{x \rightarrow 2^+} f(x)$  is 2, and the left hand limit  $\lim_{x \rightarrow 2^-} f(x) = 4$ .

Because the right-hand and left-hand limits are not equal,  $\lim_{x \rightarrow 2} f(x)$  does not exist.

c. Find  $\lim_{x \rightarrow 5^+} f(x)$  and  $\lim_{x \rightarrow 5^-} f(x)$ .

Begin by determining the right-hand limit  $\lim_{x \rightarrow 5^+} f(x)$ .

For this limit  $x$  approaches 5. Because  $x$  is greater than 5, use the expression  $\frac{x}{2} + 1$  to calculate  $\lim_{x \rightarrow 5^+} f(x)$  and  $\lim_{x \rightarrow 5^-} f(x)$ .

Determine the value of  $\lim_{x \rightarrow 5^+} \left( \frac{x}{2} + 1 \right)$  by evaluating  $\frac{x}{2} + 1$  for  $x = 5$ .

$$\lim_{x \rightarrow 5^+} \left( \frac{x}{2} + 1 \right) = \frac{(5)}{2} + 1 = 3.5$$

Therefore,  $\lim_{x \rightarrow 5^+} f(x) = 3.5$ . Next, find the value of  $\lim_{x \rightarrow 5^-} f(x)$ .

Determine the value of  $\lim_{x \rightarrow 5^-} \left( \frac{x}{2} + 1 \right)$  by evaluating  $\frac{x}{2} + 1$  for  $x = 5$ .

$$\lim_{x \rightarrow 5^-} \left( \frac{x}{2} + 1 \right) = \frac{(5)}{2} + 1 = 3.5$$

Therefore,  $\lim_{x \rightarrow 5^-} f(x) = 3.5$ .

d. Does  $\lim_{x \rightarrow 5} f(x)$  exist? If so, what is it? If not, why not?

Recall that a function  $f(x)$  has a limit as  $x$  approaches  $c$  if and only if it has left-hand and right-hand limits there and these one-sided limits are equal.

The right-hand limit  $\lim_{x \rightarrow 5^+} f(x)$  and the left-hand limit  $\lim_{x \rightarrow 5^-} f(x)$  are both 3.5.

Therefore,  $\lim_{x \rightarrow 5} f(x)$  exists because both the right-hand and left-hand limits exist and are equal.

a. Find  $\lim_{x \rightarrow 4^+} f(x)$  and  $\lim_{x \rightarrow 4^-} f(x)$ .

Begin by determining the right-hand limit  $\lim_{x \rightarrow 4^+} f(x)$

If  $f(x)$  is defined on the interval  $(c, b)$ , where  $c < b$ , and approaches arbitrarily close to  $L$  as  $x$  approaches  $c$  from within the interval, then  $f$  has right-hand limit  $L$  at  $c$ , denoted  $\lim_{x \rightarrow c^+} f(x) = L$

Which expression should be used to calculate  $\lim_{x \rightarrow 4^+} f(x)$ ?

- x + 2
- 6 - x

Determine the value of  $\lim_{x \rightarrow 4^+} (x + 2)$ . Select the correct choice below and fill in any answer boxes in your choice.

- A.  $\lim_{x \rightarrow 4^+} (x + 2) =$   (Simplify your answer.)

- B. The limit does not exist.

Therefore,  $\lim_{x \rightarrow 4^+} f(x) = 6$ . Next, find the value of  $\lim_{x \rightarrow 4^-} f(x)$ .

If  $f(x)$  is defined on the interval  $(a, c)$ , where  $a < c$ , and approaches arbitrarily close to  $M$  as  $x$  approaches  $c$  from within the interval, then  $f$  has left-hand limit  $M$  at  $c$ , denoted  $\lim_{x \rightarrow c^-} f(x) = M$

Which expression should be used to calculate  $\lim_{x \rightarrow 4^-} f(x)$ ?

- 6 - x
- x + 2

Determine the value of  $\lim_{x \rightarrow 4^-} (6 - x)$ . Select the correct choice below and fill in any answer boxes in your choice.

- A.  $\lim_{x \rightarrow 4^-} (6 - x) =$   (Simplify your answer.)

- B. The limit does not exist.

Therefore,  $\lim_{x \rightarrow 4^-} f(x) = 2$ .

b. Does  $\lim_{x \rightarrow 4} f(x)$  exist? If so, what is it? If not, why not?

A function  $f(x)$  has a limit as  $x$  approaches  $c$  if and only if it has left-hand and right-hand limits there and these one-sided limits are equal.

Does  $\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^-} f(x)$ ?

- No
- Yes

Write  $\frac{2x}{\sin 2x}$  as  $\frac{1}{\frac{\sin 2x}{2x}}$ .

$$\frac{1}{2} \lim_{x \rightarrow 0} \frac{2x}{\sin 2x} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{1}{\frac{\sin 2x}{2x}}$$

Evaluate  $\lim_{x \rightarrow 0} \frac{1}{\frac{\sin 2x}{2x}}$  by using the quotient rule and the fact  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$ .

$$\frac{1}{2} \lim_{x \rightarrow 0} \frac{1}{\frac{\sin 2x}{2x}} = \frac{1}{2} \cdot 1$$

Simplify.

$$\frac{1}{2} \lim_{x \rightarrow 0} \frac{1}{\frac{\sin 2x}{2x}} = \frac{1}{2}$$

Therefore,  $\lim_{x \rightarrow 0} \frac{x \csc 2x}{\cos 7x} = \frac{1}{2}$ .

Apply the constant multiple rule.

$$\lim_{x \rightarrow 0} \frac{11x}{11 \sin 11x} = \frac{1}{11} \lim_{x \rightarrow 0} \frac{11x}{\sin 11x}$$

Write  $\frac{11x}{\sin 11x}$  as  $\frac{\sin 11x}{11x}$ .

$$\frac{1}{11} \lim_{x \rightarrow 0} \frac{11x}{\sin 11x} = \frac{1}{11} \lim_{x \rightarrow 0} \frac{1}{\frac{\sin 11x}{11x}}$$

Evaluate  $\lim_{x \rightarrow 0} \frac{1}{\frac{\sin 11x}{11x}}$ .

$$\frac{1}{11} \lim_{x \rightarrow 0} \frac{1}{\frac{\sin 11x}{11x}} = \frac{1}{11} \cdot \boxed{1} \quad (\text{Type an integer or a simplified fraction.})$$

Simplify.

$$\frac{1}{11} \lim_{x \rightarrow 0} \frac{1}{\frac{\sin 11x}{11x}} = \boxed{\frac{1}{11}}$$

(Simplify your answer.)

$$\text{Therefore, } \lim_{x \rightarrow 0} \frac{x \csc 11x}{\cos 8x} = \frac{1}{11}.$$

$$\frac{\text{numerator}}{\text{denominator}} = \frac{1}{25} \cdot \frac{\left( \frac{\sin \frac{\theta}{2}}{\frac{\theta}{2}} \right) \cdot \sin \frac{\theta}{2}}{\left( \frac{\sin 25\theta}{25\theta} \right)}$$

Write the limit of this expression as a product and quotient of limits.

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin 25\theta} = \frac{1}{25} \cdot \frac{\lim_{\theta \rightarrow 0} \left( \frac{\sin \frac{\theta}{2}}{\frac{\theta}{2}} \right) \cdot \lim_{\theta \rightarrow 0} \sin \frac{\theta}{2}}{\lim_{\theta \rightarrow 0} \left( \frac{\sin 25\theta}{25\theta} \right)}$$

Finally, evaluate the limits using the fact that  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} \neq 0$  and  $\lim_{\theta \rightarrow 0} \sin \theta = 0$ .

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin 25\theta} = \frac{1}{25} \cdot \frac{1 \cdot 0}{1}$$

Simplifying gives  $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin 25\theta} = 0$ .

Does  $\lim_{x \rightarrow 4} f(x)$  exist?

x → 4

- No
- Yes

Because the right-hand and left-hand limits are not equal,  $\lim_{x \rightarrow 4} f(x)$  does not exist.

c. Find  $\lim_{x \rightarrow 6^+} f(x)$  and  $\lim_{x \rightarrow 6^-} f(x)$ .

Begin by determining the right-hand limit  $\lim_{x \rightarrow 6^+} f(x)$

Which expression should be used to calculate  $\lim_{x \rightarrow 6^+} f(x)$ ?

- x + 2
- 6 - x

Determine the value of  $\lim_{x \rightarrow 6^+} (x + 2)$ . Select the correct choice below and fill in any answer boxes in your choice.

- A.  $\lim_{x \rightarrow 6^+} (x + 2) =$   (Simplify your answer.)

- B. The limit does not exist.

Therefore,  $\lim_{x \rightarrow 6^+} f(x) = 8$ . Next, find the value of  $\lim_{x \rightarrow 6^-} f(x)$ .

Which expression should be used to calculate  $\lim_{x \rightarrow 6^-} f(x)$ ?

- 6 - x
- x + 2

Determine the value of  $\lim_{x \rightarrow 6^-} (x + 2)$ . Select the correct choice below and fill in any answer boxes in your choice.

- A.  $\lim_{x \rightarrow 6^-} (x + 2) =$   (Simplify your answer.)

- B. The limit does not exist.

Therefore,  $\lim_{x \rightarrow 6^-} f(x) = 8$ .

d. Does  $\lim_{x \rightarrow 6} f(x)$  exist? If so, what is it? If not, why not?

Recall that a function  $f(x)$  has a limit as  $x$  approaches  $c$  if and only if it has left-hand and right-hand limits there and these one-sided limits are equal.

Does  $\lim_{x \rightarrow 6^+} f(x) = \lim_{x \rightarrow 6^-} f(x)$ ?

Yes No

Does  $\lim_{x \rightarrow 6} f(x)$  exist?

 Yes No

Therefore,  $\lim_{x \rightarrow 6} f(x)$  exists because both the right-hand and left-hand limits exist and are equal.

YOU ANSWERED:  $6 - x$

B.

$x + 2$

B.

Yes

$6 - x$

B.

$6 - x$

B.

No

No