Student: Cole Lamers **Date:** 07/20/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.5 Trigonometric Equations I and II

Solve the equation.

$$2\,\mathbf{sin}^2\theta + 3\,\mathbf{sin}\,\theta + 1 = 0$$

Many trigonometric equations can be solved by applying techniques that we already know, such as factoring.

This equation is a quadratic equation (in $\mathbf{sin}\,\theta$) that can be factored.

$$\begin{aligned} 2\,\mathbf{sin}^2\theta + 3\,\mathbf{sin}\,\theta + 1 &= 0 \\ (2\,\mathbf{sin}\,\theta + 1)(\mathbf{sin}\,\theta + 1) &= 0 \end{aligned}$$

Apply the zero-product property. Solve for $\mathbf{sin}\,\theta$.

$$(2\,\mathbf{sin}\,\theta + 1)(\mathbf{sin}\,\theta + 1) = 0$$

$$\begin{array}{ccc} 2\,\mathbf{sin}\,\theta + 1 = 0 & \text{or} & \mathbf{sin}\,\theta + 1 = 0 \\ \mathbf{sin}\,\theta = -\dfrac{1}{2} & \text{or} & \mathbf{sin}\,\theta = -1 \end{array}$$

Solve each equation in the interval $0 \le \theta < 2\pi$.

$$\begin{aligned} \mathbf{sin}\,\theta &= -\frac{1}{2} \\ \theta &= \frac{7\pi}{6}, \frac{11\pi}{6} \\ \mathbf{sin}\,\theta &= -1 \\ \theta &= \frac{3\pi}{2} \end{aligned}$$

Therefore, the solution set is $\left\{\frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}\right\}$.

Student: Cole Lamers **Date:** 07/20/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.5 Trigonometric Equations I and II

Find all solutions of the equation in the interval $[0, 2\pi)$.

$$\sqrt{3}\,\mathbf{tan}\left(x - \frac{\pi}{10}\right) - 1 = 0$$

Begin by replacing $\left(x - \frac{\pi}{10}\right)$ by some unknown θ.

$$\begin{aligned}\sqrt{3}\,\mathbf{tan}\left(x - \frac{\pi}{10}\right) - 1 &= 0\\ \sqrt{3}\,\mathbf{tan}\,\theta - 1 &= 0\end{aligned}$$

Next, solve the equation for **tan** θ.

$$\begin{aligned}\sqrt{3}\,\mathbf{tan}\,\theta - 1 &= 0\\ \mathbf{tan}\,\theta &= \frac{\sqrt{3}}{3}\end{aligned}$$

Now, find the exact angle of θ for which the tangent value equates to $\frac{\sqrt{3}}{3}$.

$$\mathbf{tan}\,\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

The tangent function has positive values in quadrants I and III. The solutions of the equation $\mathbf{tan}\,\theta = \frac{\sqrt{3}}{3}$ in $[0, 2\pi)$ are shown below.

$$\theta = \frac{\pi}{6} \qquad \text{or} \qquad \begin{aligned}\theta &= \pi + \frac{\pi}{6}\\ \theta &= \frac{7\pi}{6}\end{aligned}$$

Finally, back-substiute $\left(x - \frac{\pi}{10}\right)$ for θ in the solutions found in the previous step and solve for x.

$$\begin{aligned}\left(x - \frac{\pi}{10}\right) &= \frac{\pi}{6}\\ x &= \frac{4\pi}{15}\end{aligned} \qquad \text{or} \qquad \begin{aligned}\left(x - \frac{\pi}{10}\right) &= \frac{7\pi}{6}\\ x &= \frac{19\pi}{15}\end{aligned}$$

Student: Cole Lamers **Date:** 07/20/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.5 Trigonometric Equations I and II

Find all solutions of the equation in the interval $[0^\circ, 360^\circ)$. Round your answers to the nearest tenth of a degree.

$$\cos\theta = 0.9$$

Use the function $\cos^{-1}$ to find the reference angle θ' for any angle θ with $\cos\theta = 0.9$.

$$\cos\theta' = |\cos\theta| = 0.9$$

Solve for θ', where $0^\circ < \theta' < 90^\circ$. Use a calculator in Degree mode.

$$\begin{aligned}\theta' &= \cos^{-1}(0.9)\\ \theta' &\approx 25.8^\circ\end{aligned}$$

Now, the cosine functions has positive values in quadrants I and IV. Therefore, the solutions of the equation are solved below.

$$x \approx 25.8^\circ \quad \text{or} \quad \begin{aligned}x &\approx 360^\circ - 25.8^\circ\\ &= 334.2^\circ\end{aligned}$$

Find all solutions of $\sin x = 0.3$. Express solutions in degrees rounded to the nearest tenth of a degree.

$\sin x = 0.3$

$x' = \sin^{-1}(0.3)$

$x' \approx 17.5^\circ$

Sol in $[0, 360^\circ)$

QI QII

$x \approx 17.5^\circ$ $x \approx 180^\circ - 17.5^\circ$

$x \approx 162.5^\circ$

$x \approx 17.5^\circ + 360^\circ n$, n is any integer

$x \approx 162.5^\circ + 360^\circ n$

Find all solutions of $\cot x = -3.5$ in the interval $[0, 2\pi)$. Round solutions to four decimal places.

QII , QIV

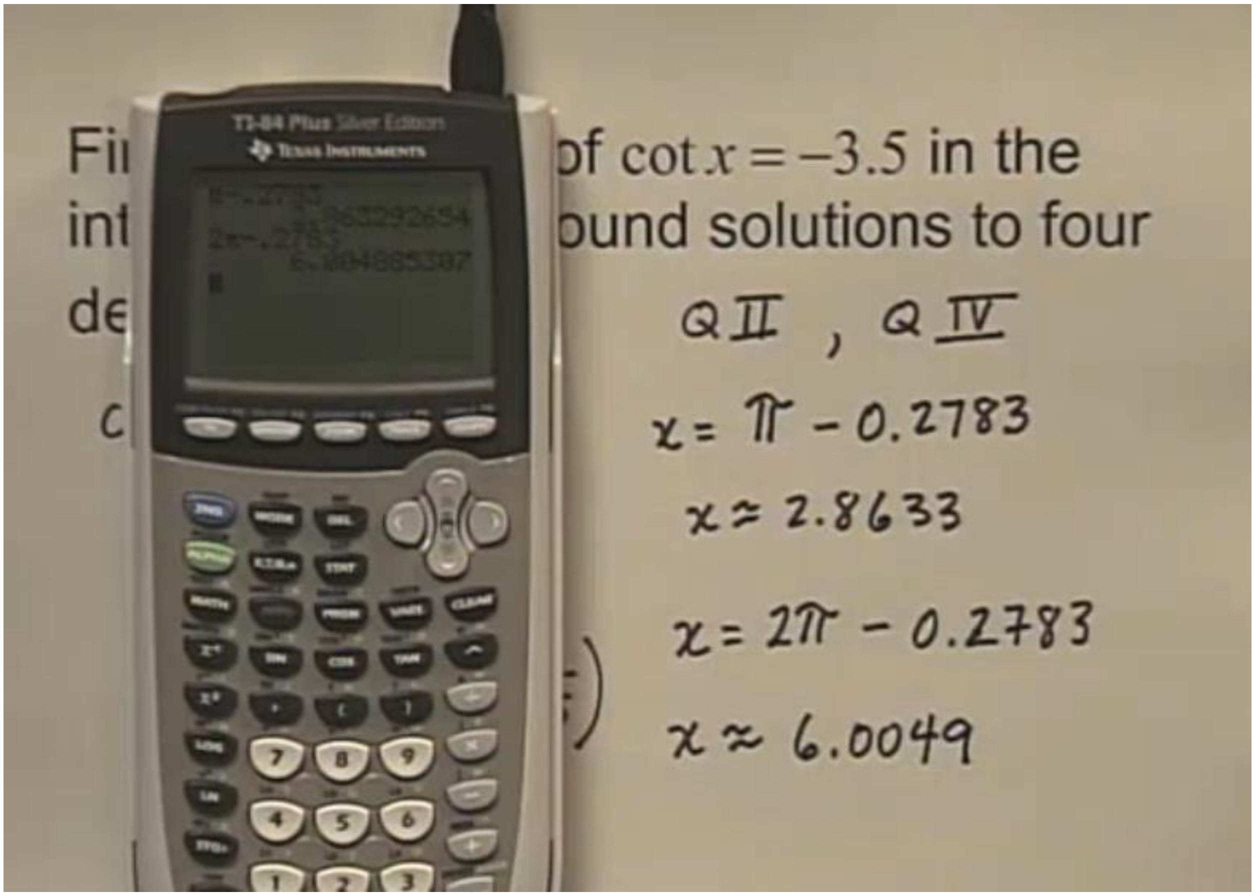
$\cot x = -3.5$

$\tan x = -\frac{1}{3.5}$

$x' = \tan^{-1}\left(+\frac{1}{3.5}\right)$

$x' = 0.2783$

$x = \pi - 0.$


TI-84 Plus Silver Edition
Texas Instruments
π-.2783
2.863292654
2π-.2783
6.004885307
Fi
of $\cot x = -3.5$ in the
int
ound solutions to four
de
QII , QIV
$x = \pi - 0.2783$
$x \approx 2.8633$
$x = 2\pi - 0.2783$
$x \approx 6.0049$

Student: Cole Lamers **Date:** 07/20/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.5 Trigonometric Equations I and II

Use trigonometric identities to solve the equation in the interval $[0, 2\pi)$.

$$6\cos^2\theta - 3\sin\theta - 3 = 0$$

When a trigonometric equation contains more than one trigonometric function, identities sometimes can be used to obtain an equivalent equation that contains only one function.

The equation in its present form contains sines and cosines. However, a form of the Pythagorean Identity, $\cos^2\theta = 1 - \sin^2\theta$, can be used to transform the equation into an equivalent equation that contains only sines.

$$\begin{aligned} 6\cos^2\theta - 3\sin\theta - 3 &= 0 \\ 6\left(1 - \sin^2\theta\right) - 3\sin\theta - 3 &= 0 \end{aligned}$$

Combine the like terms and simplify.

$$\begin{aligned} 6\left(1 - \sin^2\theta\right) - 3\sin\theta - 3 &= 0 \\ 6 - 6\sin^2\theta - 3\sin\theta - 3 &= 0 \\ -6\sin^2\theta - 3\sin\theta + 3 &= 0 \\ -2\sin^2\theta - \sin\theta + 1 &= 0 \qquad \text{Divide each side by 3.} \end{aligned}$$

The resultant equation $-2\sin^2\theta - \sin\theta + 1 = 0$ can be written as $2\sin^2\theta + \sin\theta - 1 = 0$ by multiplying both sides by -1.

Factor.

$$\begin{aligned} 2\sin^2\theta + \sin\theta - 1 &= 0 \\ (2\sin\theta - 1)(\sin\theta + 1) &= 0 \end{aligned}$$

Apply the zero-product property. Solve for $\sin\theta$.

$$2\sin\theta - 1 = 0 \qquad \text{or} \qquad \sin\theta + 1 = 0$$

$$\sin\theta = \frac{1}{2} \qquad\qquad \sin\theta = -1$$

Solve the equation $\sin\theta = \frac{1}{2}$ in the interval $[0, 2\pi)$.

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$$

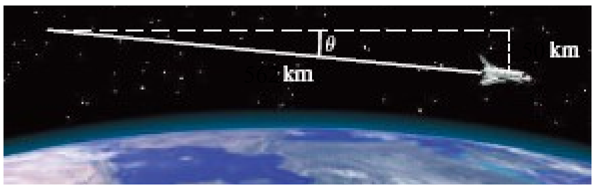
Solve the equation $\sin\theta = -1$ in the interval $[0, 2\pi)$.

$$\theta = \frac{3\pi}{2}$$

Therefore, the solutions are $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

Student: Cole Lamers **Date:** 07/20/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.5 Trigonometric Equations I and II

When a spacecraft reenters the atmosphere, the angle of reentry off the horizontal must be between 5.1° and 7.1°. Under 5.1°, the craft would "skip" back into space, and over 7.1°, the acceleration forces would be too high and the craft would crash. On reentry, a spacecraft descends 50 kilometers vertically while traveling 562 kilometers. Find the angle of reentry off the horizontal to the nearest tenth of a degree.



Note that if P(x,y) is any point on the terminal ray of an angle in standard position (other than the origin) and if $r = \sqrt{x^2 + y^2}$, then $\mathbf{sin}\,\theta = \frac{y}{r}$.

Let the position of the spacecraft be denoted by P(x,y) and the point where it starts its descent be referred to as the origin.

The value of y is -50.

The value of r is 562.

Using the values of y and r, write $\mathbf{sin}\,\theta$.

$$\mathbf{sin}\,\theta = \frac{-50}{562}$$

Use the function $\mathbf{sin}^{-1}$ to find the angle θ.

$$\theta = \mathbf{sin}^{-1}\left(\frac{-50}{562}\right)$$
$$\approx -5.1^\circ$$

Note that the negative sign of the angle indicates that it is an angle of depression. Therefore, the angle of reentry off the horizontal to the nearest tenth of a degree of the spacecraft is approximately 5.1°.