Student: Cole Lamers **Date:** 07/17/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.3 Double-Angle and Half-Angle Formulas

1. Complete the following statement.

The double-angle identity for $\sin 2x$ is $\sin 2x =$ ________.

$\sin 2x = 2\sin x \cos x$

2. Complete the following statement.

In the double-angle identity $\cos 2x = \cos^2 x - \sin^2 x$, replace $\cos^2 x$ with $1 - \sin^2 x$ to obtain a double-angle identity $\cos 2x =$ _______ in terms of $\sin^2 x$. Solve this identity for $\sin^2 x$ to obtain the power-reducing identity $\sin^2 x =$ _______.

$\cos 2x = 1 - 2\sin^2 x$

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

3. Complete the following statement.

The identity for $\cos 2x$ in terms of $\cos^2 x$ is $\cos 2x =$ _______. Solve this identity for $\cos^2 x$ to obtain the power-reducing identity $\cos^2 x =$ _______.

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \dfrac{1 + \cos 2x}{2}$

4. State whether the following statement is true or false.

$$\frac{1}{2}\tan 2x = \tan x$$

Choose the correct answer below.

- **A.** False, because $\frac{1}{2}\tan 2x = \tan x$.
- **B.** False, because the double-angle formula for tangent is not applied correctly.
- **C.** False, because $\frac{1}{2}\tan 2x = \tan^2 x$.
- **D.** True, because the double-angle formula for tangent is applied correctly.

5. Use a double-angle formula to find the exact value of the given expression.

$1 - 2\sin^2 67.5°$

$1 - 2\sin^2 67.5° = \dfrac{-\sqrt{2}}{2}$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

6. Use a double-angle formula to find the exact value of the given expression.

$2\cos^2 157.5° - 1$

$2\cos^2 157.5° - 1 = \dfrac{\sqrt{2}}{2}$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

7. Use a double-angle formula to find the exact value of the given expression.

$2\cos^2\frac{11\pi}{12} - 1$

$2\cos^2\frac{11\pi}{12} - 1 = \frac{\sqrt{3}}{2}$

(Simplify your answer. Type an exact answer, using radicals as needed. Use integers or fractions for any numbers in the expression.)

8. Establish the identity.

$\cos^4\theta - \sin^4\theta = \cos(2\theta)$

Choose the sequence of steps below that verifies the identity.

- A. $\cos^4\theta - \sin^4\theta = (1 - \sin^2\theta)(1 - \cos^2\theta) = 1 \cdot \cos(2\theta) = \cos(2\theta)$
- B. $\cos^4\theta - \sin^4\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta) = 1 \cdot \cos(2\theta) = \cos(2\theta)$
- C. $\cos^4\theta - \sin^4\theta = (\cos^2\theta + \sin^2\theta)(\cos^2\theta + \sin^2\theta) = 1 \cdot \cos(2\theta) = \cos(2\theta)$
- D. $\cos^4\theta - \sin^4\theta = (\cos^2\theta - \sin^2\theta)^2 = 1 \cdot \cos(2\theta) = \cos(2\theta)$

9. Use the power-reducing identities to rewrite the expression to one that contains a single trigonometric function of power 1.

$4\sin 10x \cos 10x\left(1 - 2\sin^2 10x\right)$

$4\sin 10x \cos 10x\left(1 - 2\sin^2 10x\right) = \sin 40x$ (Simplify your answer.)

10. Use the half-angle formulas to find the exact value of the trigonometric function $\tan\left(-\frac{7\pi}{8}\right)$.

Choose the exact value of the trigonometric function $\tan\left(-\frac{7\pi}{8}\right)$ below.

- A. $\sqrt{2} + 1$
- B. $-\sqrt{2} - 1$
- C. $2 - \sqrt{3}$
- D. $\sqrt{2} - 1$

11. Given that $\cos\alpha = \frac{2}{5}$ and $0^\circ < \alpha < 90^\circ$, determine the exact values of $\sin\frac{\alpha}{2}$, $\cos\frac{\alpha}{2}$ and $\tan\frac{\alpha}{2}$. Be sure to rationalize your answers.

$\sin\frac{\alpha}{2} = \frac{\sqrt{30}}{10}$ (Type an exact answer, using radicals as needed.)

$\cos\frac{\alpha}{2} = \frac{\sqrt{70}}{10}$ (Type an exact answer, using radicals as needed.)

$\tan\frac{\alpha}{2} = \frac{\sqrt{21}}{7}$ (Type an exact answer, using radicals as needed.)

12. Verify the following identity.

$$7\cos^2\left(\frac{x}{2}\right) - 7\cos(x) = 7\sin^2\left(\frac{x}{2}\right)$$

Write the second term on the left side of the identity in terms of trigonometric functions of $\frac{x}{2}$, rather than x, so that it matches the other terms. What type of formula should be used to do this?

- **A.** Double-angle formula
- **B.** Half-angle formula
- **C.** Power-reducing formula

Which of the following equations is the correct type of formula and is the best choice for making the left side look like the right side?

- **A.** $\cos(x) = 1 - 2\sin^2\left(\frac{x}{2}\right)$
- **B.** $\cos(x) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$
- **C.** $\cos(x) = 2\cos^2\left(\frac{x}{2}\right) - 1$
- **D.** $\cos(x) = 2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)$

Substitute the correct formula from the previous step and simplify the left side by combining like terms.

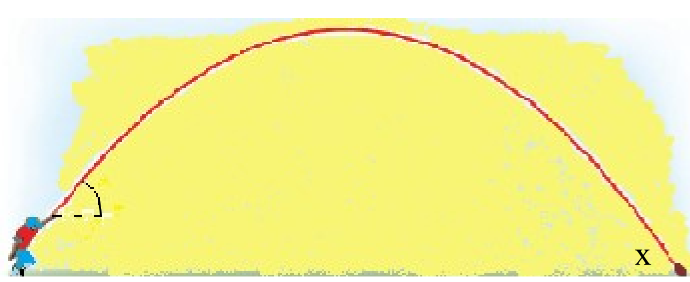
$7\sin^2\left(\frac{x}{2}\right)$

(Simplify your answer. Use integers or fractions for any numbers in the expression.)

The expression from the previous step then simplifies to $7\sin^2\left(\frac{x}{2}\right)$ using what?

- **A.** Applying a Pythagorean identity to the left side gives $7\sin^2\left(\frac{x}{2}\right)$.
- **B.** Applying an Even-Odd identity to the left side gives $7\sin^2\left(\frac{x}{2}\right)$.
- **C.** Applying a Reciprocal identity to the left side gives $7\sin^2\left(\frac{x}{2}\right)$.
- **D.** The answer from the previous step verifies the identity.

13. A quarterback throws a ball with an initial velocity of v_0 feet per second at an angle θ with the horizontal. The horizontal distance x in feet the ball is thrown is modeled by the equation $\frac{v_0^2}{16}\sin\theta\cos\theta$. For a fixed v_0, use a double-angle identity to find the angle that produces the maximum distance x.



The angle that produces the maximum distance x is $\frac{\pi}{4}$.

(Type your answer in radians.)

14. Find the exact value of the expression

$$\sin\frac{\pi}{4}\cos\frac{7\pi}{12} + \cos\frac{\pi}{4}\sin\frac{7\pi}{12}$$

$\sin\frac{\pi}{4}\cos\frac{7\pi}{12} + \cos\frac{\pi}{4}\sin\frac{7\pi}{12} = \frac{1}{2}$

(Simplify your answer, including any radicals. Use integers or fractions for any numbers in the expression.)

15. Watch the video and then solve the problem given below.

Click here to watch the video.[1]

If $\mathbf{sin}\,\theta = -\frac{12}{13}$ and θ is in quadrant IV, find the exact value of $\mathbf{cos}\,(2\theta)$.

$\mathbf{cos}\,(2\theta) = -\frac{119}{169}$ (Type an integer or a fraction.)

1: http://mediaplayer.pearsoncmg.com/assets/Lr0Aakph4XH8_e_RXGqAYL1lLOOBnlNZ?clip=1

16. Watch the video and then solve the problem given below.

Click here to watch the video.[2]

Find the exact value of the expression $\mathbf{cos}^2\left(\frac{5\pi}{6}\right) - \mathbf{sin}^2\left(\frac{5\pi}{6}\right)$.

$\mathbf{cos}^2\left(\frac{5\pi}{6}\right) - \mathbf{sin}^2\left(\frac{5\pi}{6}\right) = \frac{1}{2}$ (Type an integer or a fraction.)

2: http://mediaplayer.pearsoncmg.com/assets/Lr0Aakph4XH8_e_RXGqAYL1lLOOBnlNZ?clip=2