Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.5 The Dot Product

Find the dot product of **u** and **v**.

$\mathbf{u} = 3\mathbf{i} - 3\mathbf{j}$ and $\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$

The dot product of two vectors $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is given by $u_1 v_1 + u_2 v_2$. Thus, you multiply the first components of the vectors, and add that product to the product of the second components of the vectors.

The first components of the vectors **u** and **v** are 3 and -4. The second components are -3 and 5. Multiply each pair and add the products to get the dot product.

$$\mathbf{u} \cdot \mathbf{v} = 3(-4) + (-3)5$$
$$\mathbf{u} \cdot \mathbf{v} = -27$$

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Find the dot product $\mathbf{u} \cdot \mathbf{v}$.

$\mathbf{u} = 6\mathbf{i} - 4\mathbf{j} \qquad \mathbf{v} = 5\mathbf{j}$

Note that the vector $\mathbf{v} = 5\mathbf{j}$ can be rewritten as $\mathbf{v} = 0\mathbf{i} + 5\mathbf{j}$.

First, rewrite **u** and **v** as position vectors.

$\mathbf{u} = 6\mathbf{i} - 4\mathbf{j} = \langle 6, -4 \rangle$

$\mathbf{v} = 0\mathbf{i} + 5\mathbf{j} = \langle 0, 5 \rangle$

For two vectors $\mathbf{v} = \langle v_1, v_2 \rangle$ and $\mathbf{w} = \langle w_1, w_2 \rangle$, the dot product of **v** and **w**, denoted $\mathbf{v} \cdot \mathbf{w}$, is defined as follows.

$$\mathbf{v} \cdot \mathbf{w} = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle = v_1 w_1 + v_2 w_2$$

Now, use the definition of dot product to find $\mathbf{u} \cdot \mathbf{v}$.

$$\begin{aligned} \mathbf{u} \cdot \mathbf{v} &= (6\mathbf{i} - 4\mathbf{j}) \cdot (0\mathbf{i} + 5\mathbf{j}) \\ &= \langle 6, -4 \rangle \cdot \langle 0, 5 \rangle \\ &= (6)(0) + (-4)(5) \\ &= -20 \end{aligned}$$

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Find **u** • **v**, where θ is the angle between the vectors **u** and **v**.

$$\|\mathbf{u}\| = 5, \ \|\mathbf{v}\| = 10, \ \theta = \frac{\pi}{6}$$

Let θ $(0 \le \theta \le \pi)$ be the angle between two nonzero vectors **v** and **w**. Then the dot product is related to this angle as shown below.

$$\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \ \|\mathbf{w}\| \ \cos \theta$$

Substitute the given values into the formula for the dot product.

$$\mathbf{u} \cdot \mathbf{v} = (5)\ (10) \cos\left(\frac{\pi}{6}\right)$$

Use a calculator to evaluate the dot product.

$$\mathbf{u} \cdot \mathbf{v} \approx 43.3$$

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Find $\mathbf{u} \cdot \mathbf{v}$, where θ is the angle between the vectors **u** and **v**.

$\|\mathbf{u}\| = 5$, $\|\mathbf{v}\| = 10$, $\theta = 150^\circ$

Let θ $(0 \le \theta \le 180^\circ)$ be the angle between two nonzero vectors **v** and **w**. The dot product is related to this angle as shown below.

$$\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \ \|\mathbf{w}\| \ \cos \theta$$

Substitute the given values into the formula for the dot product.

$$\mathbf{u} \cdot \mathbf{v} = (5)(10) \cos (150^\circ)$$

Use a calculator to evaluate the dot product.

$$\mathbf{u} \cdot \mathbf{v} \approx -43.3$$

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Find the angle between the vectors **v** and **w**.

$\|\mathbf{v}\| = \sqrt{2}$, $\|\mathbf{w}\| = \sqrt{13}$, and $\mathbf{v} \cdot \mathbf{w} = \sqrt{13}$

If θ, $0^\circ \le \theta \le 180^\circ$, is the angle between two nonzero vectors **v** and **w**, then $\mathbf{v} \cdot \mathbf{w} = \|\mathbf{v}\| \, \|\mathbf{w}\| \cos \theta$ or $\cos \theta = \dfrac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\| \, \|\mathbf{w}\|}$.

Substitute the value of $\mathbf{v} \cdot \mathbf{w}$, $\|\mathbf{v}\|$, and $\|\mathbf{w}\|$ in the formula for the angle between two vectors.

$$\begin{aligned} \cos \theta &= \frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\| \, \|\mathbf{w}\|} \\ &= \frac{\sqrt{13}}{\sqrt{2} \cdot \sqrt{13}} && \text{Substitute.} \\ &= \frac{1}{\sqrt{2}} && \text{Simplify.} \end{aligned}$$

Find θ.

$$\begin{aligned} \cos \theta &= \frac{1}{\sqrt{2}} \\ \theta &= \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \\ &= 45^\circ \end{aligned}$$

Therefore, the angle between the vectors **v** and **w** is 45°.

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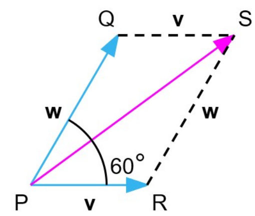
Let **v** and **w** be two vectors in the plane of magnitudes 8 and 12, respectively. The angle between **v** and **w** is 60°. Find $\|\mathbf{v}+\mathbf{w}\|$.

Draw the vectors **v**, **w**, and **v** + **w**. The three vectors form a triangle PRS. Since PQRS is a parallelogram, then angle R is supplementary to angle QPR.



The sum of the measures of two supplementary angles is 180°. Therefore, the measure of angle R is $180^\circ - 60^\circ = 120^\circ$.

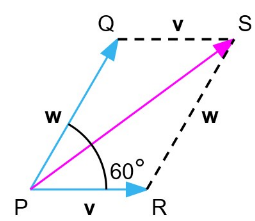
Now two sides and the included angle are known in the triangle PRS. In order to find the length of the side $\|\mathbf{v}+\mathbf{w}\|$, the law of cosines must be used.

Let A, B, and C denote the measure of the angles of a triangle ABC with opposite sides of lengths a, b, and c. The law of cosines can be written as shown below.

$$a^2 = b^2 + c^2 - 2bc\,\mathbf{cos}\,A$$

Since the angle R is opposite of the side whose length is $\|\mathbf{v}+\mathbf{w}\|$, the correct form of the law of cosines is as shown.

$$\|\mathbf{v}+\mathbf{w}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - 2\,\|\mathbf{v}\|\,\|\mathbf{w}\|\,\mathbf{cos}\,R$$



Substitute the known values into the law of cosines. Evaluate the right side using a calculator.

$$\begin{aligned}\|\mathbf{v}+\mathbf{w}\|^2 &= \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - 2\,\|\mathbf{v}\|\,\|\mathbf{w}\|\,\mathbf{cos}\,R \\ &= (8)^2 + (12)^2 - 2(8)(12)\,\mathbf{cos}\,(120^\circ) \\ &= 304\end{aligned}$$

Take the positive square root to find the length or magnitude of **v** + **w**.

$$\begin{aligned}\|\mathbf{v}+\mathbf{w}\|^2 &= 304 \\ \|\mathbf{v}+\mathbf{w}\| &\approx 17.4\end{aligned}$$

Therefore, the magnitude of **v** + **w** is $\|\mathbf{v}+\mathbf{w}\| \approx 17.4$.

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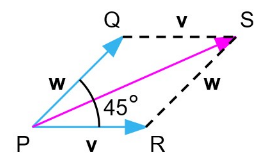
Let **v** and **w** be two vectors in the plane of magnitudes 8 and 9, respectively. The angle between **v** and **w** is 45°. Find the angle between **v** + **w** and **w**.

Draw the vectors **v**, **w**, and **v** + **w**. The three vectors form a triangle PQS. Since PQRS is a parallelogram, then angle Q is supplementary to angle QPR.



The sum of the measures of two supplementary angles is 180°. Therefore, the measure of angle Q is $180^\circ - 45^\circ = 135^\circ$.

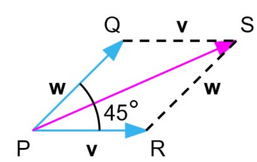
That means that two sides and the included angle are known for the triangle PQS. However, that is not enough information to directly find the measure of the angle between **v** + **w** and **w**.

First, use the law of cosines to find the length of **v** + **w**. Let A, B, and C denote the measure of the angles of a triangle ABC with opposite sides of lengths a, b, and c. The law of cosines can be written as shown below.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Since the angle Q is opposite of the side whose length is $\|\mathbf{v}+\mathbf{w}\|$, the correct form of the law of cosines is as shown.

$$\|\mathbf{v}+\mathbf{w}\|^2 = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - 2\,\|\mathbf{v}\|\,\|\mathbf{w}\| \cos Q$$



Substitute the known values into the law of cosines. Evaluate the right side using a calculator.

$$\begin{aligned} \|\mathbf{v}+\mathbf{w}\|^2 &= \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - 2\,\|\mathbf{v}\|\,\|\mathbf{w}\| \cos Q \\ &= (8)^2 + (9)^2 - 2(8)(9) \cos (135^\circ) \\ &\approx 246.8234 \end{aligned}$$

Take the positive square root to find the length or magnitude of **v** + **w**.

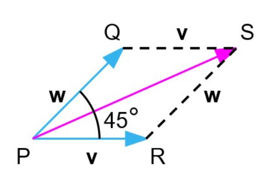
$$\begin{aligned} \|\mathbf{v}+\mathbf{w}\|^2 &\approx 246.8234 \\ \|\mathbf{v}+\mathbf{w}\| &\approx 15.7106 \end{aligned}$$

Now use the law of sines to find the measure of angle P. In any triangle ABC, with sides of length a, b, and c, the following is true.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

The side opposite angle P corresponds to vector **v**.

$$\frac{\sin P}{\|\mathbf{v}\|} = \frac{\sin Q}{\|\mathbf{v}+\mathbf{w}\|}$$



Substitute the known values into the law of sines and solve for **sin** P.

$$\begin{aligned} \frac{\sin P}{\|\mathbf{v}\|} &= \frac{\sin Q}{\|\mathbf{v}+\mathbf{w}\|} \\ \frac{\sin P}{8} &\approx \frac{\sin (135^\circ)}{15.7106} \\ \sin P &\approx 0.3601 \end{aligned}$$

Finally, solve for the angle.

$$\sin P \approx 0.3601$$
$$P \approx 21.1^\circ$$

Therefore, the angle between the vectors $\mathbf{v} + \mathbf{w}$ and $\mathbf{w}$ is about 21.1°.

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A vector **F** represents a force that has a magnitude of 20 pounds, and $\frac{2\pi}{3}$ is the angle for its direction. Find the work done by the force in moving an object from the origin to the point $(-7,4)$. Distance is measured in feet.

The work done W done by a constant force **F** in moving an object from a point P to a point Q is defined by $\mathbf{F} \cdot \overrightarrow{PQ}$.

First write **F** in terms of its magnitude $\|\mathbf{F}\|$ and direction angle θ. To write **F** in terms of its magnitude and direction, use the formula, $\mathbf{F} = \|\mathbf{F}\|(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$.

Substitute $\|\mathbf{F}\| = 20$ and $\theta = \frac{2\pi}{3}$ into $\|\mathbf{F}\|(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})$.

$$\begin{aligned}\mathbf{F} &= \|\mathbf{F}\|(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})\\ &= 20\left(\cos\frac{2\pi}{3}\mathbf{i} + \sin\frac{2\pi}{3}\mathbf{j}\right)\\ &= 20\left(-\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) && \text{Evaluate } \cos\frac{2\pi}{3} \text{ and } \sin\frac{2\pi}{3}.\end{aligned}$$

Use the distributive property and simplify.

$$\begin{aligned}\mathbf{F} &= 20\left(-\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right)\\ &= -10\mathbf{i} + 10\sqrt{3}\mathbf{j} && \text{Simplify.}\end{aligned}$$

Next, find the vector $\overrightarrow{PQ}$. Note that the vector $\overrightarrow{PQ}$ with initial point $P(x_1, y_1)$ and terminal point $Q(x_1x_2)$ is equal to the position vector $\mathbf{w} = \langle x_2 - x_1, y_2 - y_1\rangle$.

Determine the initial point P and terminal point Q of the vector $\overrightarrow{PQ}$.

$P(x_1, y_1) = (0,0) \qquad Q(x_1x_2) = (-7,4)$

Now write the vector $\overrightarrow{PQ}$ as a position vector **w**.

$$\begin{aligned}\mathbf{w} &= \langle x_2 - x_1, y_2 - y_1\rangle\\ &= \langle -7-0, 4-0\rangle\\ &= \langle -7,4\rangle && \text{Subtract.}\end{aligned}$$

To compute the dot product of **F** and $\overrightarrow{PQ}$, first rewrite the force vector $\mathbf{F} = -10\mathbf{i} + 10\sqrt{3}\mathbf{j}$ as a position vector **v**.

$$\begin{aligned}\mathbf{F} &= -10\mathbf{i} + 10\sqrt{3}\mathbf{j}\\ \mathbf{v} &= \langle -10, 10\sqrt{3}\rangle.\end{aligned}$$

Find the work done using the formula $W = \mathbf{F} \cdot \overrightarrow{PQ} = \mathbf{v} \cdot \mathbf{w}$. Note that the dot product of the vector **v** and **w** is defined as $\mathbf{v} \cdot \mathbf{w} = \langle v_1, v_2\rangle \cdot \langle w_1, w_2\rangle = v_1w_1 + v_2w_2$.

Compute the dot product, rounding to the nearest tenth.

$$\begin{aligned}\mathbf{F} \cdot \overrightarrow{PQ} &= \mathbf{v} \cdot \mathbf{w}\\ &= \langle -10, 10\sqrt{3}\rangle \cdot \langle -7,4\rangle\\ &= (-10)(-7) + \left(10\sqrt{3}\right)(4)\\ &= 139.3 && \text{Simplify.}\end{aligned}$$

Therefore, the work done is 139.3 foot-pounds.