Unit 2 – Uniformly Accelerated Post-Lecture

$\bar{v}$ = average velocity

v = final velocity

v_0 = initial velocity

a = acceleration

t = time

Δx = change in distance (x-axis)

Δy = change in distance (y-axis)

Δt = change in time

g = acceleration due to gravity

Common Equations

$$\bar{v} = \frac{v_0 + v}{2} \qquad \bar{v} \equiv \frac{\Delta x}{\Delta t} \qquad v = v_0 + at$$

$$v_y = v_{0y} + gt$$

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x\Delta x \qquad \Delta x = v_{0x}t + \tfrac{1}{2}a_x t^2$$

A moving car’s brakes are applied. 5.42 seconds are required for it to come to rest. If the car’s average velocity is 26.3 km/hr, what was the initial velocity in m/s?

Organize Data and Convert to k·m·s

$x = ?$

$a = ?$

$t = ?$

$v = ?$

$v_o = ?$

$\bar{v} = ?$

$$\Delta x = ?$$

$$a = ?$$

$$t = 5.42\ \text{s}$$

$$v = 0\ \text{m/s}$$

$$v_o = ?$$

$$\bar{v} = \frac{26.3\ \cancel{\text{km}}}{\cancel{\text{hr}}} \cdot \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \cdot \frac{1\ \cancel{\text{hr}}}{3600\ \text{s}} = 7.31\ \text{m/s}$$

$$\bar{v} = \frac{v + v_o}{2} \qquad \therefore \; 2\bar{v} = v + v_o$$

$$\therefore \; 2\bar{v} - v = v_o$$

$$\therefore \; 2(7.31 \text{ m/s}) - 0 \text{ m/s} = \boxed{14.6 \text{ m/s}}$$

How far did the car travel during this time frame?

$$x = \bar{v}t \qquad \therefore \quad x = (7.31\ \text{m}/\cancel{\text{s}})(5.42\ \cancel{\text{s}}) = \boxed{39.6\ \text{m}}$$

A car traveling 35.0 mph applies its brakes . It requires a stopping distance of 50.0 ft. What is the rate of its acceleration? What is the time required to perform this maneuver?

$$\Delta x = 50.0\ \cancel{\text{ft}} \cdot \frac{1\ \text{m}}{3.281\ \cancel{\text{ft}}} = 15.2\ \text{m}$$

$$a = ?$$

$$t = ?$$

$$v = 0\ \text{m/s}$$

$$v_o = \frac{35.0\ \cancel{\text{mi}}}{1\ \cancel{\text{hr}}} \cdot \frac{1609\ \text{m}}{1\ \cancel{\text{mi}}} \cdot \frac{1\ \cancel{\text{hr}}}{3600\ \text{s}} = 15.6\ \text{m/s}$$

$$\bar{v} = ?$$

$$2ax = v^2 - v_o^2$$

$$\therefore \quad a = \frac{v^2 - v_o^2}{2x} = \frac{(0 \text{ m/s})^2 - (15.6 \text{ m/s})^2}{2(15.2 \text{ m})}$$

$$\therefore \quad a = \boxed{-8.01 \text{ m/s}^2}$$

**When an object slows down, a negative acceleration is being applied.

$$v = v_0 + at \qquad \therefore \ t = \frac{v - v_o}{a}$$

$$\therefore \ t = \frac{v - v_o}{a} = \frac{0 \text{ m/s} - 15.6 \text{ m/s}}{-8.01 \text{ m/s}^2}$$

$$\therefore \ t = \boxed{1.95 \text{ s}}$$

A highway speed limit is 55.0 mph. Assume that applying the brakes allows a car to decelerate uniformly at $-6.50\ m/s^2$. How far will the car travel before coming to rest? If the initial speed was 65.0 mph, how far would it travel before stopping?

$$x = ?$$

$$a = -6.50 \text{ m/s}^2$$

$$t = ?$$

$$v = 0 \text{ m/s}$$

$$v_o = \frac{55.0 \cancel{\text{mi}}}{1 \cancel{\text{hr}}} \cdot \frac{1609 \text{ m}}{1 \cancel{\text{mi}}} \cdot \frac{1 \cancel{\text{hr}}}{3600 \text{ s}} = 24.6 \text{ m/s}$$

$$\bar{v} = ?$$

$$2ax = v^2 - v_o^2$$

$$\therefore \quad x = \frac{v^2 - v_o^2}{2a} = \frac{(0 \text{ m/s})^2 - (24.6 \text{ m/s})^2}{2(-6.50 \text{ m/s}^2)}$$

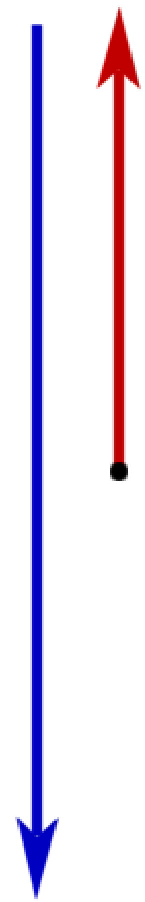
$$\therefore \quad x = \boxed{46.6 \text{ m}}$$

$$2ax = v^2 - v_o^2$$

$$\therefore \quad x = \frac{v^2 - v_o^2}{2a} = \frac{(0 \text{ m/s})^2 - (29.1 \text{ m/s})^2}{2(-6.50 \text{ m/s}^2)}$$

$$\therefore \quad x = \boxed{65.1 \text{ m}}$$

A passenger in a helicopter drops an object while he is ascending vertically at a constant speed of 5.65 m/s. If the object hits the ground 6.78 s later, how high was the helicopter when he initially dropped the object?



The object is initially going upward at the point of release (+y direction).

At the point of release, the object immediately begins to slow down due to gravity (neg. acceleration).

$$y = v_{o_y} t + \frac{1}{2} g t^2$$

$$= (+5.65 \text{ m/s})(6.78 \text{ s}) + (0.500)\left(-9.80 \text{ m/s}^2\right)(6.78 \text{ s})^2$$

$$= -187 \text{ m}$$

The object drops this distance below the point of release (y = 0).

Therefore, the helicopter is 187 m above the ground at release.

$$\boxed{187 \text{ m}}$$

If an object is dropped from the top of the Empire State Building (1250 ft), how fast will it be traveling just before impacting the ground? State the answer in ft/s and mph.

$$y = -1250\ \cancel{\text{ft}} \cdot \frac{1\ \text{m}}{3.281\ \cancel{\text{ft}}} = -381\ \text{m}$$

$$g = -9.80\ \text{m/s}^2$$

$$t = ?$$

$$v = ?$$

$$v_o = 0\ \text{m/s}$$

$$\bar{v} = ?$$

$$v^2 - v_o^2 = 2gy$$

$$\therefore \quad v = \sqrt{2gy + v_o^2}$$

$$\therefore \quad v = \sqrt{2(-9.80 \text{ m/s}^2)(-381 \text{ m}) + (0 \text{ m/s})^2}$$

$$\therefore \quad v = 86.4 \text{ m/s}$$

$$\frac{86.4\ \cancel{\mathrm{m}}}{\mathrm{s}} \cdot \frac{3.281\ \mathrm{ft}}{1\ \cancel{\mathrm{m}}} = \boxed{283\ \mathrm{ft/s}}$$

$$\frac{86.4\ \cancel{\mathrm{m}}}{\cancel{\mathrm{s}}} \cdot \frac{1\ \mathrm{mi}}{1609\ \cancel{\mathrm{m}}} \cdot \frac{3600\ \cancel{\mathrm{s}}}{1\ \mathrm{hr}} = \boxed{193\ \mathrm{mph}}$$

- A race car starting from rest accelerates at a constant rate of 5.00 m/s^2. **(a)** What is the velocity (in m/s) of the car after it has traveled 1.00 x 10^2 ft? **(b)** How much time has elapsed (in seconds)? Calculate the average velocity (in m/s) using **(c)** equation 2.2 and **(d)** equation 2.7

$$x = 10\bar{0}\ \cancel{\text{ft}} \cdot \frac{1\ \text{m}}{3.28\ \cancel{\text{ft}}} = 30.5\ \text{m}$$

$$a = +5.00\ \text{m/s}^2$$

$$t =$$

$$v_0 = \cancel{0}\ \text{m/s}$$

$$v =$$

$$\bar{v} =$$

$$
\begin{aligned}
v^2 &= v_0{}^2 + 2a\Delta x \\
\therefore \quad v &= \sqrt{v_0{}^2 + 2a\Delta x} \\
&= \sqrt{\left(0\ \text{m/s}\right)^2 + 2\left(+5.00\ \text{m/s}^2\right)\left(30.5\ \text{m}\right)} \\
&= \boxed{17.5\ \text{m/s}}
\end{aligned}
$$

$$v = v_0 + at$$

$$\therefore \quad t = \frac{v - v_0}{a} = \frac{17.5 \text{ m/s} - 0 \text{ m/s}}{+5.00 \text{ m/s}^2} = \boxed{3.50 \text{ s}}$$

$$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{30.5 \text{ m} - 0 \text{ m}}{3.50 \text{ s}} = \boxed{8.71 \text{ m/s}}$$

$$\bar{v} = \frac{v_0 + v}{2} = \frac{0 \text{ m/s} + 17.5 \text{ m/s}}{2} = \boxed{8.75 \text{ m/s}}$$

- A car traveling at a constant speed of 24.0 m/s passes a trooper hidden behind a billboard (*as in lecture Figure 2.17*). One second after the speeding car passes the billboard, the trooper sets off in chase with a constant acceleration of 3.00 m/s^2. **(a)** How long (in seconds) does it take the trooper to overtake the speeding car? **(b)** How fast (in m/s) is the trooper going at that time?

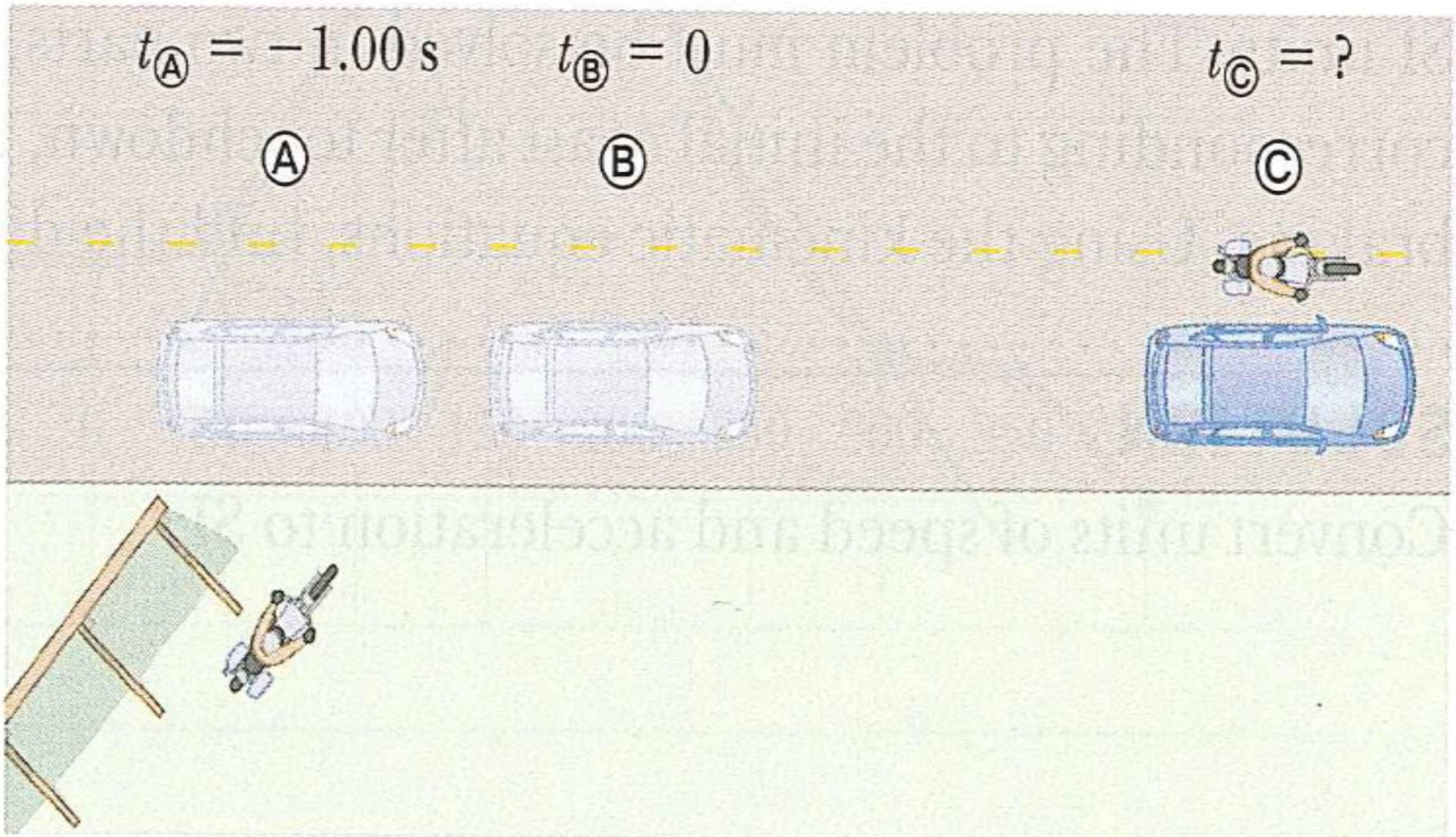


Figure 2.17

Photos/Illustrations courtesy of *College Physics*, 9th edition.

car:

$x = 24.0$ m (after 1.00 s)

$a = \cancel{0}$ m/s^2

$t = ?$

$v_0 = 24.0$ m/s

$v =$

$\bar{v} = 24.0$ m/s

trooper:

$x = \cancel{0}$ m (at exactly 1.00 s)

$a = +3.00$ m/s^2

$t = ?$

$v_0 = \cancel{0}$ m/s

$v =$

$\bar{v} =$

$$\Delta x_{car} = x_{car} - x_{0_{car}} = v_0 t + \frac{1}{2} a t^2$$

$$= x_{car} - 24.0\text{ m} = (24.0\text{ m/s})t + \frac{1}{2}\left(0\text{ m/s}^2\right)t^2$$

$$x_{car} = (24.0\text{ m/s})t + 24.0\text{ m}$$

$$\Delta x_{trooper} = x_{trooper} - x_{0_{trooper}} = v_0 t + \frac{1}{2} a t^2$$

$$= x_{trooper} - 0 \text{ m} = \left(0 \text{ m/s}\right) t + \frac{1}{2}\left(+3.00 \text{ m/s}^2\right) t^2$$

$$x_{trooper} = \frac{1}{2}\left(+3.00 \text{ m/s}^2\right) t^2$$

$$x_{car} = x_{trooper}$$

$$\therefore \quad (24.0\text{ m/s})t + 24.0\text{ m} = \frac{1}{2}\left(+3.00\text{ m/s}^2\right)t^2$$

$$\therefore \quad 0 = \left(+1.50\text{ m/s}^2\right)t^2 - (24.0\text{ m/s})t - 24.0\text{ m}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-24.0) \pm \sqrt{(-24.0)^2 - 4(1.50)(-24.0)}}{2(1.50)}$$

$$t = \frac{+(24.0) \pm 26.8}{3.00} = 16.9$$

$$\therefore \quad t = \boxed{16.9\text{ s}}$$

$$v = v_0 + at$$

$$= 0 \text{ m/s} + (+3.00 \text{ m/s}^2)(16.9 \text{ s})$$

$$v = \boxed{50.7 \text{ m/s}}$$

- A jetliner lands at a speed of 1.60×10^2 mi/h and decelerates at the rate of 10.0 mi/h/s. If the plane travels at a constant speed of 1.60×10^2 mi/h for 1.00 s after landing before applying the brakes, what is the **(a)** total displacement (in meters) of the aircraft between touchdown on the runway and coming to rest?

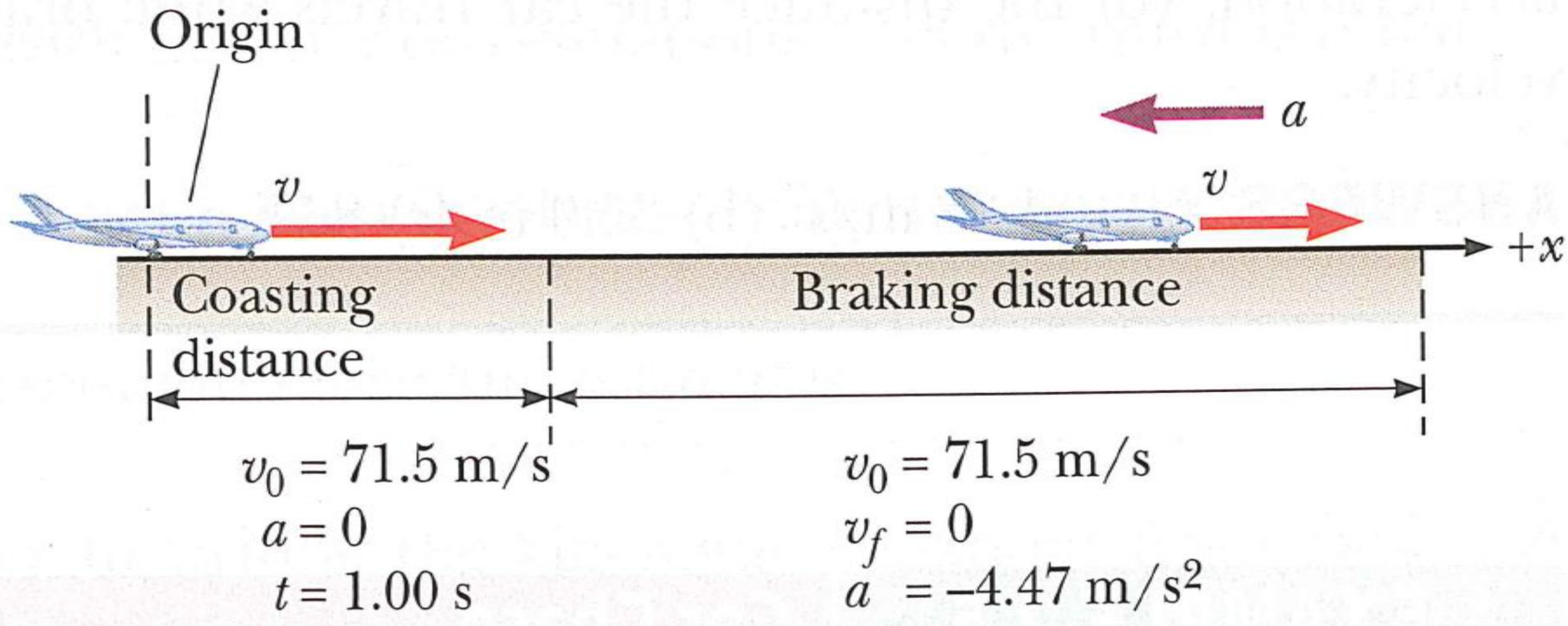


Figure 2.18

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$x =$$

$$a = \left(\frac{-10.0\ \cancel{\text{mi}}/\cancel{\text{h}}}{\text{s}}\right)\left(\frac{0.447\ \text{m/s}}{1.00\ \cancel{\text{mi}}/\cancel{\text{h}}}\right) = -4.47\ \text{m/s}^2$$

$$t =$$

$$v_0 = \frac{160\ \cancel{\text{mi}}}{\cancel{\text{h}}} \cdot \frac{1609\ \text{m}}{1\ \cancel{\text{mi}}} \cdot \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} = 71.5\ \text{m/s}$$

$$v = \cancel{0}\ \text{m/s}$$

$$\bar{v} =$$

$$\Delta x_{coasting} = v_0 t + \frac{1}{2} a t^2$$

$$= (71.5 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(0 \text{ m/s}^2)(1.00 \text{ s})^2 = 71.5 \text{ m}$$

$$v^2 = v_0^{\ 2} + 2a\Delta x_{braking}$$

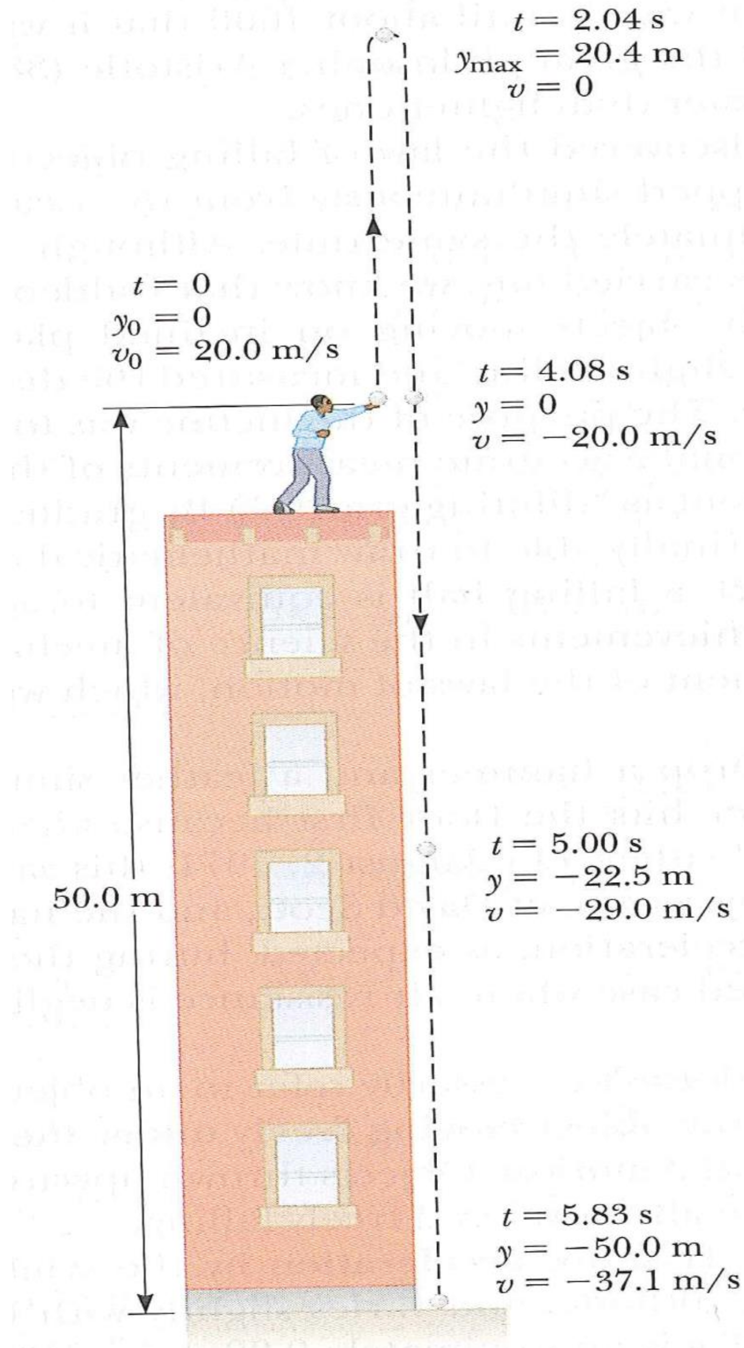
$$\therefore \Delta x_{braking} = \frac{v^2 - v_0^{\ 2}}{2a} = \frac{(0 \text{ m/s})^2 - (71.5 \text{ m/s})^2}{2(-4.47 \text{ m/s}^2)} = 572 \text{ m}$$

$$\Sigma x = \Delta x_{coasting} + \Delta x_{braking} = 71.5 \text{ m} + 572 \text{ m} = \boxed{644 \text{ m}}$$

(The even/odd rule was initiated)

- A ball is thrown from the top edge of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. The ball just misses the edge of the roof on its way down (*as shown similarly in lecture Figure 2.20*). Determine **(a)** the time needed for the ball to reach its maximum height (m), **(b)** the maximum height (m), **(c)** the time (s) needed for the ball to return to the height from which it was thrown and the **(d)** the velocity (m/s) of the ball at this point, **(e)** the time (s) needed for the ball to reach the ground, and **(f)** the velocity (m/s) and **(g)** position (m) of the ball after a total of 5.00 seconds (Σt = 5.00 s) have elapsed since the initial release.

Figure 2.20



Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$v = v_0 + at = 0 \text{ m/s} = (+20.0 \text{ m/s}) + (-9.80 \text{ m/s}^2)(t)$$

$$\therefore \; t = \boxed{2.04 \text{ s}}$$

$$\Delta y = y - y_0 = v_0 t + \frac{1}{2}at^2$$

$$= y - (0 \text{ m}) = (+20.0 \text{ m/s})(2.04 \text{ s}) + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)(2.04 \text{ s})^2$$

$$= \boxed{20.4 \text{ m}}$$

$$\Delta y = y - y_0 = v_0 t + \frac{1}{2} a t^2$$

$$= 0\text{ m} = \left(+20.0\text{ m/s}\right)\left(t\right) + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(t\right)^2$$

$$= 0\text{ m} = \left(+20.0\text{ m/s}\right)\left(t\right) + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(t\right)^2$$

$$\therefore \quad \frac{\left(+20.0\text{ m/s}\right)\left(\cancel{t}\right)}{\cancel{t}} + \frac{\frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(t\right)^{\cancel{2}}}{\cancel{t}} = \frac{0}{t} = 0$$

$$= 0\text{ m} = 20.0\text{ m/s} + \left(-4.90\text{ m/s}^2\right)t$$

$$\therefore \quad -20.0\text{ m/s} = \left(-4.90\text{ m/s}^2\right)t$$

$$\therefore \quad t = \frac{-20.0\text{ m/s}}{-4.90\text{ m/s}^2} = \boxed{4.08\text{ s}}$$

$$v = v_0 + at$$

$$= \left(+20.0\ \text{m/s}\right) + \left(-9.80\ \text{m/s}^2\right)\left(4.08\ \text{s}\right) = \boxed{-20.0\ \text{m/s}}$$

$$\Delta y = y - y_0 = v_0 t + \frac{1}{2} a t^2$$

$$= \left(-50.0\text{ m}\right) - \left(0\text{ m}\right) = \left(+20.0\text{ m/s}\right)\left(t\right) + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(t\right)^2$$

$$\therefore\ -50.0\text{ m} = \left(+20.0\text{ m/s}\right)\left(t\right) + \left(-4.90\text{ m/s}^2\right)\left(t\right)^2$$

$$\therefore\ 0 = \left(-4.90\text{ m/s}^2\right)\left(t\right)^2 + \left(20.0\text{ m/s}\right)\left(t\right) + 50.0\text{ m}$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\left(20.0\right) \pm \sqrt{\left(20.0\right)^2 - 4\left(-4.90\right)50.0}}{2\left(-4.90\right)}$$

$$= \frac{-20.0 \pm 37.1}{-9.80} = 5.83 \qquad \therefore\ t = \boxed{5.83\text{ s}}$$

$$v = v_0 + at$$

$$= (+20.0\text{ m/s}) + (-9.80\text{ m/s}^2)(5.00\text{ s}) = \boxed{-29.0\text{ m/s}}$$

$$\Delta y = y - y_0 = v_0 t + \frac{1}{2}at^2$$

$$= (+20.0\text{ m/s})(5.00\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(5.00\text{ s})^2 = \boxed{-22.5\text{ m}}$$