Student: Cole Lamers **Date:** 07/10/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 5.4 Graphs of the Sine and Cosine Functions

Sketch the graph of the given equation over the interval $[-2\pi,2\pi]$.

$y = \frac{6}{5}\cos x$

To graph an equation of the form $y = a\cos b(x - c)$, first find the amplitude, period, and phase shift given by the equation, where the amplitude is $|a|$, the period is $\frac{2\pi}{b}$, and the phase shift is c.

The amplitude of $y = \frac{6}{5}\cos x$ is $|a| = \frac{6}{5}$.

The period of $y = \frac{6}{5}\cos x$ is $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$.

The phase shift of $y = \frac{6}{5}\cos x$ is $c = 0$.

Begin by graphing one complete cycle. The starting point for the cycle is $x = c$, that is, $x = 0$. The interval over which one complete cycle occurs is $\left[c, c + \frac{2\pi}{b}\right]$. For this problem, the one complete cycle interval is $[0,2\pi]$.

Divide this interval into four equal parts, each of length $\frac{1}{4}(\text{period}) = \frac{1}{4}\left(\frac{2\pi}{b}\right)$. This requires 5 points—a starting point c, $c + \frac{1}{4}\left(\frac{2\pi}{b}\right)$, $c + \frac{1}{2}\left(\frac{2\pi}{b}\right)$, $c + \frac{3}{4}\left(\frac{2\pi}{b}\right)$, and $c + \frac{2\pi}{b}$.

To find the values of x at these five points, substitute $c = 0$ and $b = 1$ into the formula for each general point and simplify. Calculate the first three values of x.

$$\begin{aligned} c &= 0 \\ c + \frac{1}{4}\left(\frac{2\pi}{b}\right) &= 0 + \frac{1}{4}\left(\frac{2\pi}{1}\right) = \frac{\pi}{2} \\ c + \frac{1}{2}\left(\frac{2\pi}{b}\right) &= 0 + \frac{1}{2}\left(\frac{2\pi}{1}\right) = \pi \end{aligned}$$

Calculate the fourth value of x.

$$c + \frac{3}{4}\left(\frac{2\pi}{b}\right) = 0 + \frac{3}{4}\left(\frac{2\pi}{1}\right) = \frac{3\pi}{2}$$

Calculate the fifth value of x.

$$c + \frac{2\pi}{b} = 0 + \frac{2\pi}{1} = 2\pi$$

If $a > 0$, for $y = a\cos b(x - c)$, sketch one cycle starting at (c,a) through the points $\left(c + \frac{\pi}{2b}, 0\right)$, $\left(c + \frac{\pi}{b}, -a\right)$, $\left(c + \frac{3\pi}{2b}, 0\right)$, and $\left(c + \frac{2\pi}{b}, a\right)$. If $a < 0$, reflect the graph of $y = |a|\cos b(x - c)$ in the x-axis.

For this problem, a is $\frac{6}{5}$, and, therefore, greater than zero.

Use the x-values calculated previously to form the 5 points for one cycle of the graph. The five points are shown below.

$$\left(0,\frac{6}{5}\right), \left(\frac{\pi}{2},0\right), \left(\pi,-\frac{6}{5}\right), \left(\frac{3\pi}{2},0\right), \left(2\pi,\frac{6}{5}\right)$$

Use these five points to sketch one cycle and extend the cycle over the interval $[-2\pi,2\pi]$.

The graph of $y = \frac{6}{5}\cos x$ is shown below. Notice that the graph contains the five points $\left(0,\frac{6}{5}\right)$, $\left(\frac{\pi}{2},0\right)$, $\left(\pi,-\frac{6}{5}\right)$, $\left(\frac{3\pi}{2},0\right)$, and $\left(2\pi,\frac{6}{5}\right)$.

Student: Cole Lamers **Date:** 07/10/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 5.4 Graphs of the Sine and Cosine Functions

Sketch the graph of the given equation over the interval $[-2\pi, 2\pi]$.

$y = \mathbf{cos}\,(9x)$

To draw the graph of a sinusoidal function, first identify the amplitude and period of the given function. If $\omega > 0$, the amplitude and period of a function of the form $y = A\,\mathbf{cos}\,(\omega x)$ are shown below.

Amplitude = $|A|$ Period = $T = \dfrac{2\pi}{\omega}$

Determine the value that corresponds to A in the function $y = \mathbf{cos}\,(9x)$. Notice that A is the coefficient of the trigonometric function.

$A = 1$

Because the amplitude is $|1| = 1$, the graph of $y = \mathbf{cos}\,(9x)$ will lie between -1 and 1 on the y-axis.

Determine the value that corresponds to ω in the function $y = \mathbf{cos}\,(9x)$. The coefficient of x in the trigonometric function is ω.

$\omega = 9$

Use ω to determine the period.

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{9} \qquad \text{Substitute.}$$

The graph of a sinusoidal function has five key points per period. These points are the maximum values, minimum values, and x-intercepts. Use the period and A to find the coordinates of these five points for the function $y = \mathbf{cos}\,(9x)$.

Because the period is $\frac{2\pi}{9}$, one cycle will start at 0 and end at $x = \frac{2\pi}{9}$. Divide the interval $\left[0, \frac{2\pi}{9}\right]$ into four subintervals of equal length. Determine the length of each subinterval.

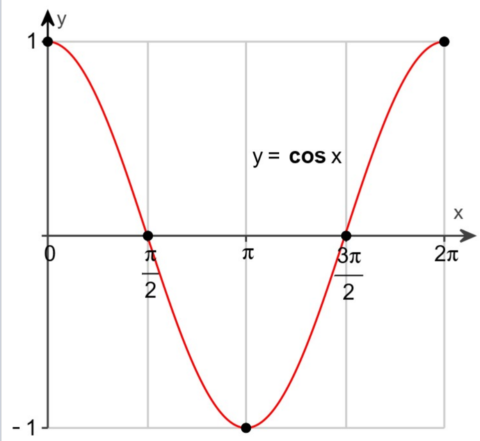
$$\frac{2\pi}{9} \div 4 = \frac{\pi}{18}$$

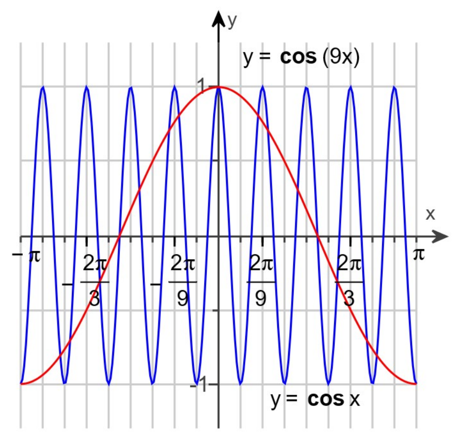
The endpoints of the subintervals are $0, \frac{\pi}{18}, \frac{\pi}{9}, \frac{\pi}{6}$, and $\frac{2\pi}{9}$. These are the x-coordinates of the five key points of $y = \mathbf{cos}\,(9x)$ that correspond to the five key points for $y = \mathbf{cos}\,x$.

The y-coordinates of the five key points for the function $y = \mathbf{cos}\,(9x)$ are the y-coordinates of the five key points for $y = \mathbf{cos}\,x$ (shown on the right) multiplied by A, 1. Determine the y-coordinates of these points for the function $y = \mathbf{cos}\,(9x)$.

$$(0,1), \left(\frac{\pi}{18}, 0\right), \left(\frac{\pi}{9}, -1\right), \left(\frac{\pi}{6}, 0\right), \left(\frac{2\pi}{9}, 1\right)$$

A graph of the function $y = \mathbf{cos}\,(9x)$ is shown on the right. Notice that the graph passes through the five key points and that it is the graph of $y = \mathbf{cos}\,x$ compressed horizontally by a factor of 9.

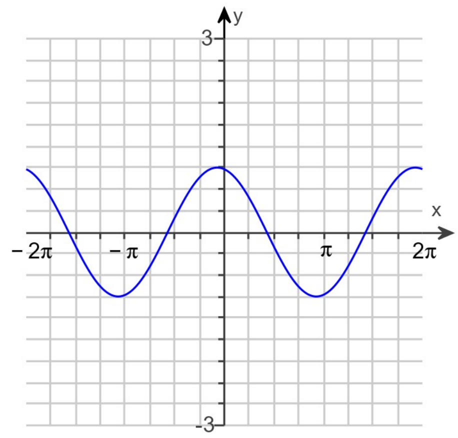




Student: Cole Lamers **Date:** 07/10/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 5.4 Graphs of the Sine and Cosine Functions

Sketch the graph of the given equation over the interval $[-2\pi, 2\pi]$.

$y = \cos\left(x + \frac{\pi}{14}\right)$



To graph an equation of the form $y = a\cos[b(x - c)]$, first find the amplitude, period, and phase shift given by the equation, where the amplitude is $|a|$, the period is $\frac{2\pi}{b}$, and the phase shift is c.

The amplitude of $y = \cos\left(x + \frac{\pi}{14}\right)$ is $|a| = 1$.

The period of $y = \cos\left(x + \frac{\pi}{14}\right)$ is $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$.

The phase shift of $y = \cos\left(x + \frac{\pi}{14}\right)$ is $c = -\frac{\pi}{14}$.

Begin graphing one complete cycle. The starting point for the cycle is $x = c$, that is, $x = -\frac{\pi}{14}$. The interval over which one complete cycle occurs is $\left[c, c + \frac{2\pi}{b}\right]$. For this problem, the one complete cycle interval is $\left[-\frac{\pi}{14}, \frac{27\pi}{14}\right]$.

Divide this interval into four equal parts, each of length $\frac{1}{4}(\text{period}) = \frac{1}{4}\left(\frac{2\pi}{b}\right)$. This gives the x-coordinates for the five key points, a starting point c, $c + \frac{1}{4}\left(\frac{2\pi}{b}\right)$, $c + \frac{1}{2}\left(\frac{2\pi}{b}\right)$, $c + \frac{3}{4}\left(\frac{2\pi}{b}\right)$, and $c + \frac{2\pi}{b}$.

To find the values of x at these five points, substitute $c = -\frac{\pi}{14}$ and $b = 1$ into the formula for each general point and simplify.

Calculate the first three values of x.

$$c = -\frac{\pi}{14}$$
$$c + \frac{1}{4}\left(\frac{2\pi}{b}\right) = -\frac{\pi}{14} + \frac{1}{4}\left(\frac{2\pi}{1}\right) = \frac{3\pi}{7}$$
$$c + \frac{1}{2}\left(\frac{2\pi}{b}\right) = -\frac{\pi}{14} + \frac{1}{2}\left(\frac{2\pi}{1}\right) = \frac{13\pi}{14}$$

Calculate the fourth value of x.

$$c + \frac{3}{4}\left(\frac{2\pi}{b}\right) = -\frac{\pi}{14} + \frac{3}{4}\left(\frac{2\pi}{1}\right) = \frac{10\pi}{7}$$

Calculate the fifth value of x.

$$c + \frac{2\pi}{b} = -\frac{\pi}{14} + \frac{2\pi}{1} = \frac{27\pi}{14}$$

If $a > 0$, for $y = a\cos[b(x - c)]$, sketch one cycle starting at (c, a) through the points $\left(c + \frac{\pi}{2b}, 0\right)$, $\left(c + \frac{\pi}{b}, -a\right)$, $\left(c + \frac{3\pi}{2b}, 0\right)$, and $\left(c + \frac{2\pi}{b}, a\right)$. If $a < 0$, reflect the graph of $y = |a|\cos[b(x - c)]$ in the x-axis.

For this problem, a is 1, and, therefore, greater than zero. Use the x-values calculated previously to form the 5 points for one cycle of the graph.The five points are shown below.

(π) (3π) (13π) (10π) (27π)

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Write the following function in the form y = a **sin** b(x − c). Find the period and phase shift.

$$y = -\frac{7}{4}\sin(12x - \pi)$$

Compare the given equation to the form y = a **sin** b(x − c).

Use the distributive property to write the given equation in the form y = a **sin** b(x − c).

$$y = -\frac{7}{4}\sin(12x - \pi) \qquad \text{Given equation.}$$

$$y = -\frac{7}{4}\sin 12\left(x - \frac{\pi}{12}\right) \qquad \text{Apply the distributive property.}$$

The period of a function of the form y = a **sin** b(x − c) equals $\frac{2\pi}{b}$.

The value of b is 12.

Substitute the value of b into the expression of the period and simplify.

$$\text{period} = \frac{2\pi}{b}$$

$$= \frac{2\pi}{12} \qquad \text{Substitute } b = 12.$$

$$= \frac{\pi}{6} \qquad \text{Simplify.}$$

The phase shift of a function of the form y = a **sin** b(x − c) equals c.

The value of c in the equation $y = -\frac{7}{4}\sin 12\left(x - \frac{\pi}{12}\right)$ is $\frac{\pi}{12}$.

Thus, the given equation written in the form y = a **sin** b(x − c) is $y = -\frac{7}{4}\sin 12\left(x - \frac{\pi}{12}\right)$, where the period is $\frac{\pi}{6}$ and the phase shift is $\frac{\pi}{12}$.

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Write the following function in the form $y = a\,\textbf{sin}\,[b(x - c)]$. Find the period and phase shift.

$$y = 7\,\textbf{sin}\,(4\pi x + 12)$$

Compare the given equation to the form $y = a\,\textbf{sin}\,[b(x - c)]$.

Notice that the constant term in the specified form is subtracted from x while the constant term in the given equation is added to x. Change the addition to subtraction. Then factor out the coefficient of x.

$$y = 7\,\textbf{sin}\,(4\pi x + 12)$$

$$y = 7\,\textbf{sin}\,(4\pi x - (-12)) \qquad \text{Rewrite the constant term.}$$

$$y = 7\,\textbf{sin}\left[4\pi\left(x - \left(-\frac{3}{\pi}\right)\right)\right] \qquad \text{Factor out } 4\pi.$$

The period of a function of the form $y = a\,\textbf{sin}\,[b(x - c)]$ equals $\frac{2\pi}{b}$.

The value of b in the equation $y = 7\,\textbf{sin}\left[4\pi\left(x - \left(-\frac{3}{\pi}\right)\right)\right]$ is 4π.

Substitute the value of b into the expression for the period and simplify.

$$\frac{2\pi}{b} = \frac{2\pi}{4\pi} \qquad \text{Substitute } b = 4\pi.$$

$$= \frac{1}{2} \qquad \text{Simplify.}$$

Therefore, the period of the function is $\frac{1}{2}$.

The phase shift of a function of the form $y = a\,\textbf{sin}\,[b(x - c)]$ equals c.

The value of c in the equation $y = 7\,\textbf{sin}\left[4\pi\left(x - \left(-\frac{3}{\pi}\right)\right)\right]$ is $-\frac{3}{\pi}$.

Therefore, the phase shift is $-\frac{3}{\pi}$.

The function $y = 7\,\textbf{sin}\,(4\pi x + 12)$ is rewritten in the form $y = a\,\textbf{sin}\,[b(x - c)]$ as $y = 7\,\textbf{sin}\left[4\pi\left(x - \left(-\frac{3}{\pi}\right)\right)\right]$. Its period is $\frac{1}{2}$ and its phase shift is $-\frac{3}{\pi}$.

Graph the following equation over the interval $[0, 2\pi]$.

$$5\cos\left(-3x+\frac{\pi}{2}\right)+5$$

Write the expression $5\cos\left(-3x+\frac{\pi}{2}\right)+5$ in the form $a\cos(x-c)$.

$$5\cos\left(-3x+\frac{\pi}{2}\right)+5=5\cos\left[-3\left(x-\frac{\pi}{6}\right)\right]+5$$

Since $\cos(-x)=\cos x$, the graph of $y=5\cos\left[-3\left(x-\frac{\pi}{6}\right)\right]+5$ is the same as the graph of $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]+5$.

Note that the function $y=a\cos(x-c)+d$ has a vertical shift of d.

The vertical shift of $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]+5$ is 5.

To get the graph of $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]+5$, shift the graph of $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ up 5 units.

To graph the function $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$, find the amplitude, period, and phase shift. Note that the function $y=a\cos[b(x-c)]$ has an amplitude $|a|$, period $\frac{2\pi}{b}$, and phase shift c.

The amplitude of $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ is 5.

The period of $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ is $\frac{2\pi}{3}$.

The phase shift of $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ is $\frac{\pi}{6}$.

The phase shift c is the amount by which the graph of $y=5\cos 3x$ is shifted horizontally.

Since $c>0$, shift the graph of $y=5\cos 3x$ to the right.

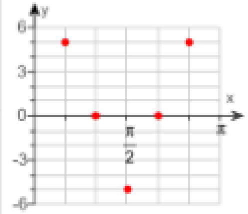
Begin the cycle at $x=\frac{\pi}{6}$. One complete cycle occurs over the interval of $\left[c,c+\frac{2\pi}{b}\right]$. The function $5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ completes one cycle over the interval $\left[\frac{\pi}{6},\frac{5\pi}{6}\right]$.

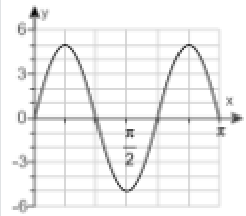
Divide this interval into four equal parts, each of length $\frac{1}{4}(\text{period})=\frac{1}{4}\left(\frac{2\pi}{3}\right)=\frac{\pi}{6}$. This requires 5 points, a starting point c, $c+\frac{1}{4}\left(\frac{2\pi}{3}\right)$, $c+\frac{1}{2}\left(\frac{2\pi}{3}\right)$, $c+\frac{3}{4}\left(\frac{2\pi}{3}\right)$, and $c+\frac{2\pi}{3}$.

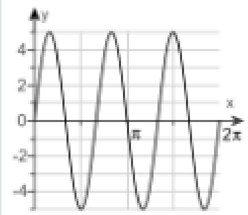
To find the x-coordinates of the five key points, substitute $c=\frac{\pi}{6}$ into the formula for each x-coordinate and simplify.

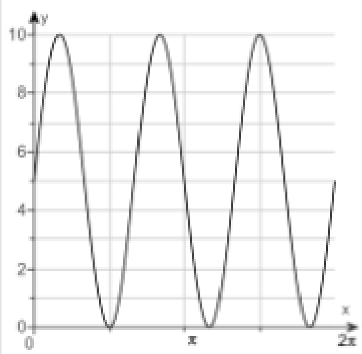
$$c+\frac{1}{4}\left(\frac{2\pi}{3}\right)=\frac{\pi}{6}+\frac{1}{4}\left(\frac{2\pi}{3}\right)=\frac{\pi}{3}$$

$$c+\frac{1}{2}\left(\frac{2\pi}{3}\right)=\frac{\pi}{6}+\frac{1}{2}\left(\frac{2\pi}{3}\right)=\frac{\pi}{2}$$









$$c+\frac{2\pi}{3}=\frac{\pi}{6}+\frac{2\pi}{3}=\frac{5\pi}{6}$$

Evaluate the function $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ at each x-coordinate to find the y-coordinates of the five key points.

The five key points of the graph are $\left(\frac{\pi}{6},5\right)$, $\left(\frac{\pi}{3},0\right)$, $\left(\frac{\pi}{2},-5\right)$, $\left(\frac{2\pi}{3},0\right)$, and $\left(\frac{5\pi}{6},5\right)$

The five key points of the graph are shown to the right.

The graph of the function $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ is shown to the right.

The graph of the function $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ over the interval $[0, 2\pi]$ is shown to the right.

To graph $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]+5$, shift the graph of $y=5\cos\left[3\left(x-\frac{\pi}{6}\right)\right]$ up 5 units.

Therefore, the graph of the function $y=5\cos\left[-3\left(x-\frac{\pi}{6}\right)\right]+5$ over the interval $[0, 2\pi]$ is shown to the right.

Question is complete.