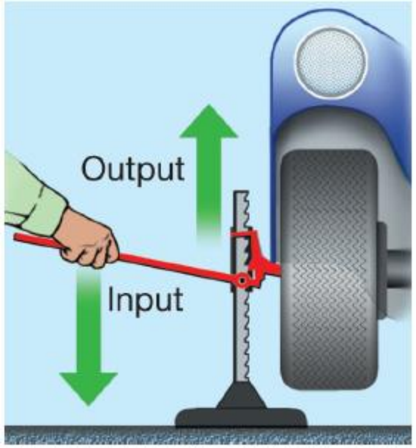
Unit 4 – Mechanical Advantage

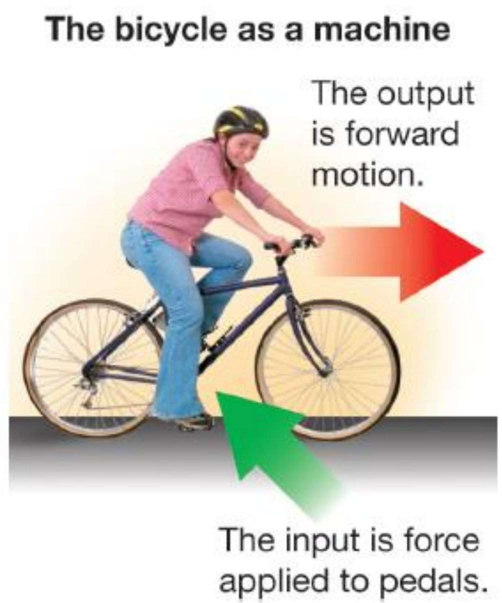
Machines and Mechanical Advantage

- The ability of humans to build buildings and move mountains began with our invention of *machines*.
- In physics the term "*simple machine*" means a machine that uses only the forces directly applied and accomplishes its task with a single motion.



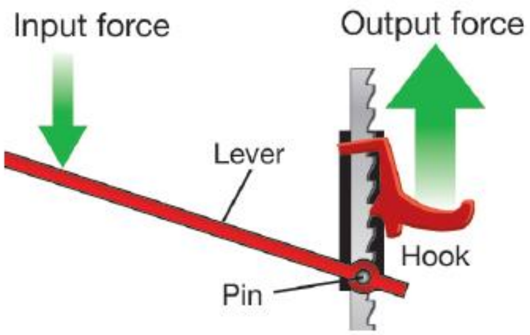
Machines

- The best way to analyze what a machine does is to think about the machine in terms of *input* and *output*.



Mechanical Advantage

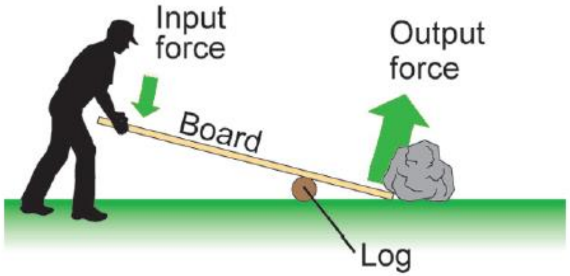
- *Mechanical advantage* is the ratio of output force to input force.
- For a typical automotive jack the mechanical advantage is 30 or more.
- A force of 100 newtons (22.5 pounds) applied to the input arm of the jack produces an outpu force of 3,000 newtons (675 pounds)— enoug to lift one corner of an automobile.



Mechanical Advantage

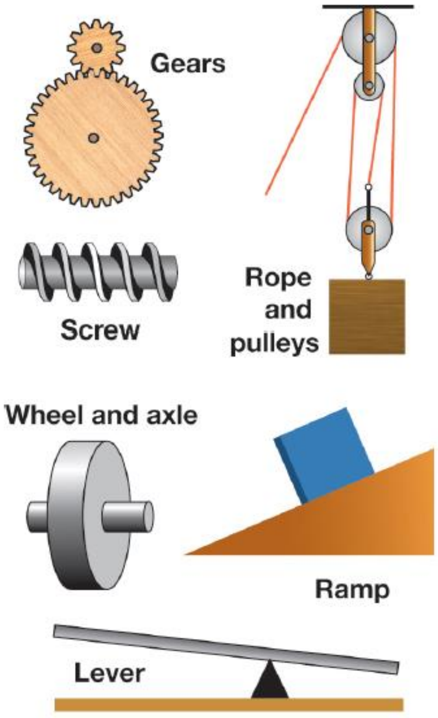
Mechanical advantage → $MA = \frac{F_o}{F_i}$ ← **Output force (N)**

Input force (N)



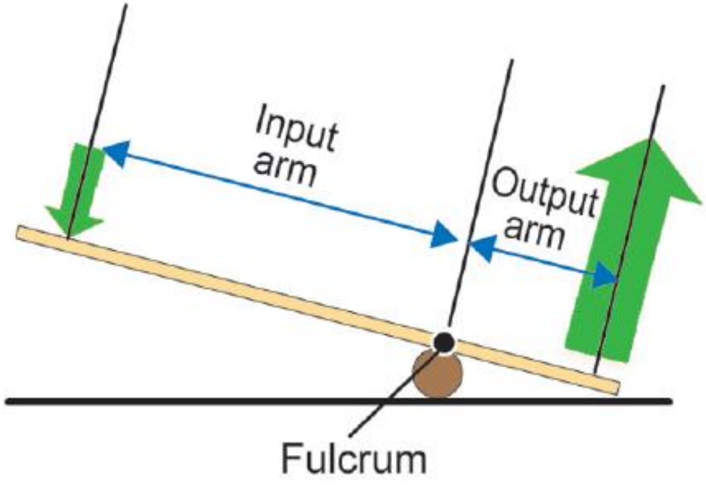
Types of simple machines

- The lever, wheel and axle, rope and pulleys, screw, ramp, and gears are the most common simple machines.
- Complex machines, combine many simple machines into *mechanical systems*.
- A *mechanical system* is an assembly of simple machines that work together to accomplish a task.



Mechanical Advantage of a Lever

- The essential features of a lever are the *input arm*, *output arm*, and *fulcrum*.

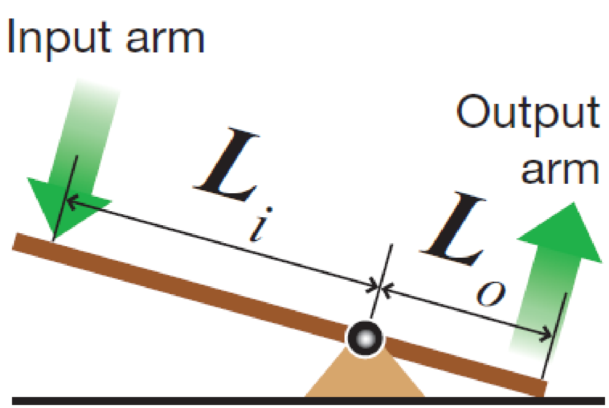


Levers

MECHANICAL ADVANTAGE OF A LEVER

$$MA_{lever} = \frac{L_i}{L_o}$$

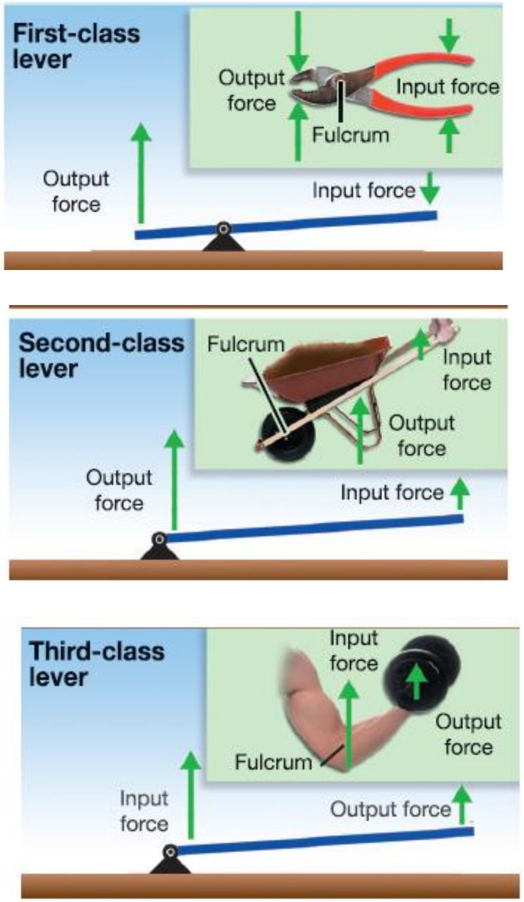
- *Mechanical advantage*: MA_{lever}
- L_i: *Length of input arm* (m)
- L_o: *Length of output arm* (m)



Slide courtesy of CPO Science

Three types of levers

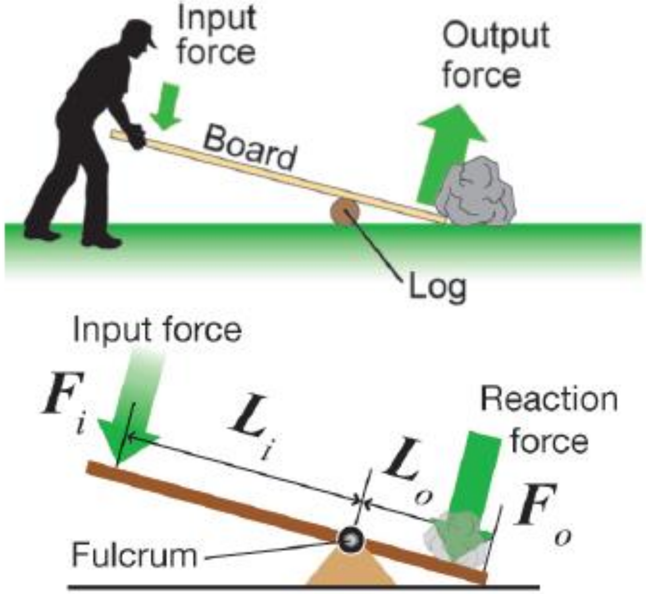
- The three types of levers are classified by the location of the input and output forces relative to the fulcrum:
  - first class lever
  - second class lever
  - third class lever



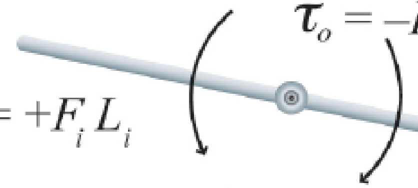
How a lever works

- A *lever* works by rotating about its fulcrum.
- The mechanical advantage can be deduced by calculating the *torques* created by the input and output forces.
- The input force creates a (positive) counterclockwise torque.
- The torque created by the reaction force is clockwise (negative).
- When the lever is in equilibrium the net torque must be zero.

Torque and mechanical advantage



Calculate torques



$$\tau_o = -F_o L_o$$

$$\tau_i = +F_i L_i$$

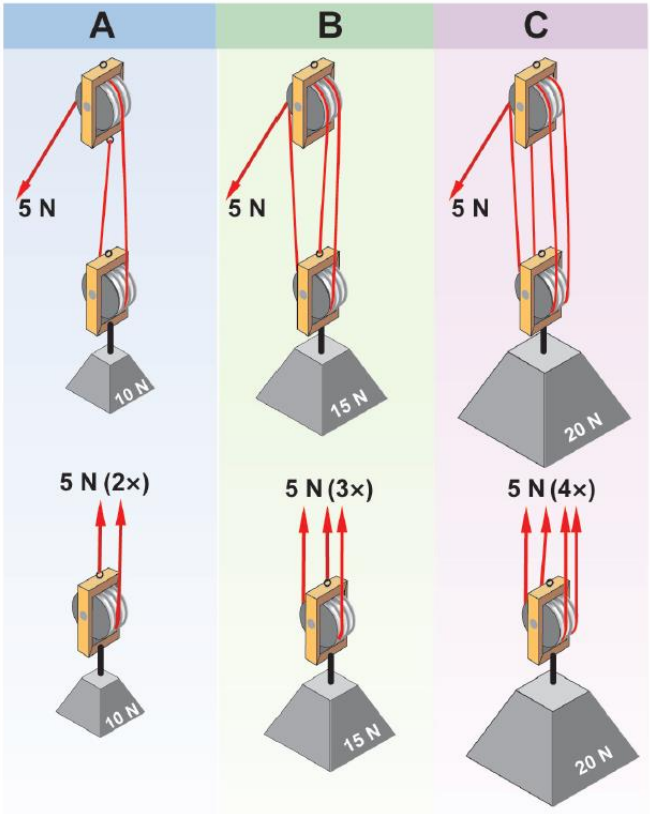
Set net torque = 0

$$F_i L_i - F_o L_o = 0 \rightarrow \frac{F_o}{F_i} = \frac{L_i}{L_o}$$

Mechanical advantage of ropes and pulleys

- A tension force is a pulling force acting along the direction of a rope or string.
- Ropes and strings carry tension forces throughout their length.
- If friction is small, the tension force in a rope is the same everywhere.
- If you were to cut a rope in tension and insert a force scale, the scale would measure the same force at any point along the rope.

Rope & Pulleys

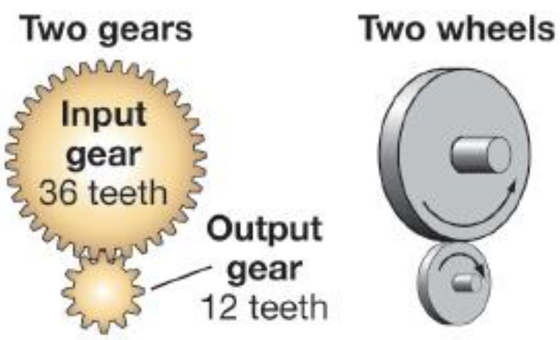


- The block-and-tackle machine is a simple machine using one rope and multiple pulleys.
- The *rope and pulleys* can be arranged to create different amounts of mechanical advantage.

	A	B	C
Input force	5 N	5 N	5 N
Output force	10 N	15 N	20 N
Mechanical advantage	2	3	4

Wheels, gears, & rotating machines

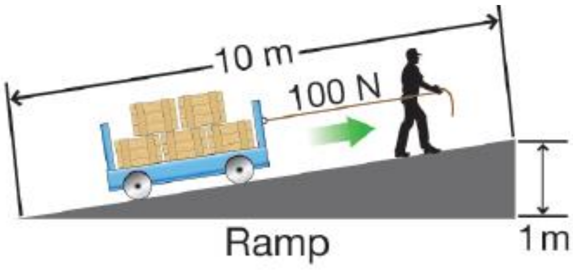
- *Wheels and axles* provide advantages.
- Friction occurs where the wheel and axle touch or where the wheel touches a surface.
- Rolling motion creates less wearing away of material compared with two surfaces sliding over each other.

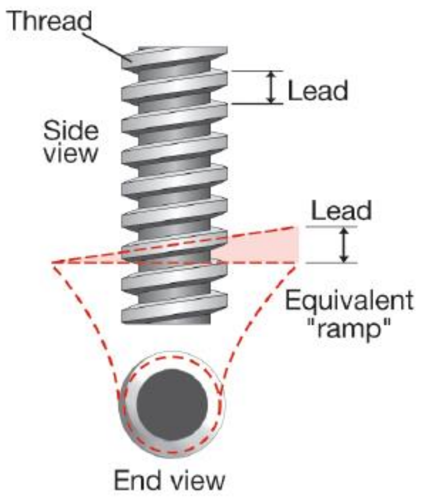


- **With *gears* the trade-off is made between torque and rotation speed.**
- **An output gear will turn with <u>more</u> torque when it rotates slower than the input gear.**

Ramps and Screws

- *Ramps* reduce input force by increasing the distance over which the input force needs to act.
- A *screw* is a simple machine that turns rotating motion into linear motion.
- A thread wraps around a screw at an angle, like the angle of a ramp.



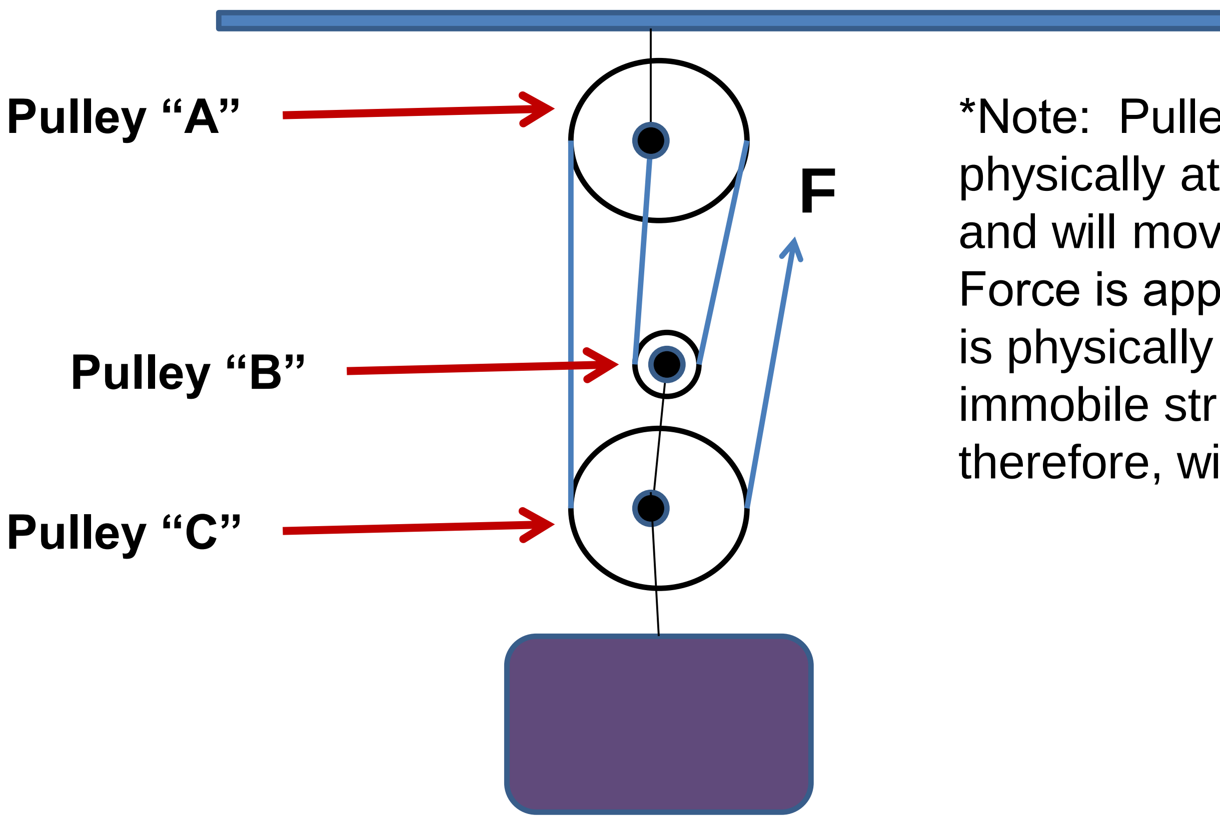


A can opener is an example of a first class lever. If the tip of the opener is 2.00 cm from the fulcrum and a lifting force of 35.0 N is applied 7.00 cm from the fulcrum, what is the penetrating force into the can?

$$\frac{F_{out}}{F_{in}} = \frac{r_{in}}{r_{out}} \qquad \therefore F_{out} = \frac{F_{in} \cdot r_{in}}{r_{out}}$$

$$\therefore F_{out} = \frac{(35.0\ \text{N})(7.00\ \cancel{\text{c}}\text{m})}{(2.00\ \cancel{\text{c}}\text{m})} = \boxed{122\ \text{N}}$$

The pulley system is used to lift a 192 lb load. If the effort force is 65.0 lbs, what is the actual mechanical advantage, ideal mechanical advantage, and efficiency?



*Note: Pulleys “B” & “C” are physically attached together and will move upward as Force is applied. Pulley “A” is physically attached to an immobile structure and, therefore, will not move.

$$\text{Mechanical Advantage} = M_A = \frac{F_{out}}{F_{in}}$$

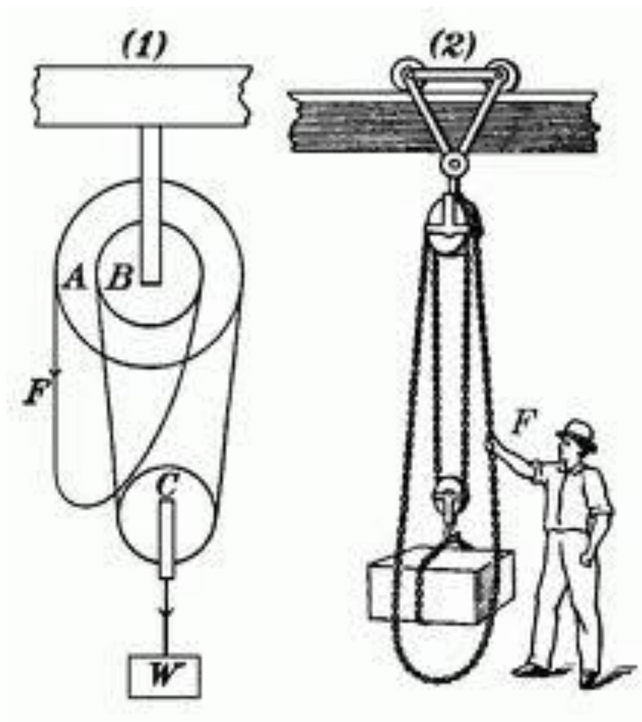
$$\text{Efficiency} = Eff = \frac{M_A}{M_I} \quad \text{Where } M_I = \text{Ideal Mechanical Advantage}$$

$$M_A = \frac{F_{out}}{F_{in}} = \frac{192 \cancel{\text{lbs}}}{65.0 \cancel{\text{lbs}}} = \boxed{2.95}$$

$$M_I = \text{Ideal Mechanical Advantage} = \text{Number of Support Strings} = \boxed{4}$$

$$Eff = \frac{M_A}{M_I} = \frac{2.95}{4} = 0.738 = \boxed{73.8\% \text{ efficiency}}$$

A chain hoist is used to lift a 850.0 N load. Determine the effort force needed if the diameter of the large sprocket is 4.50 cm, the small sprocket is 2.60 cm, and the efficiency is 75.0%.



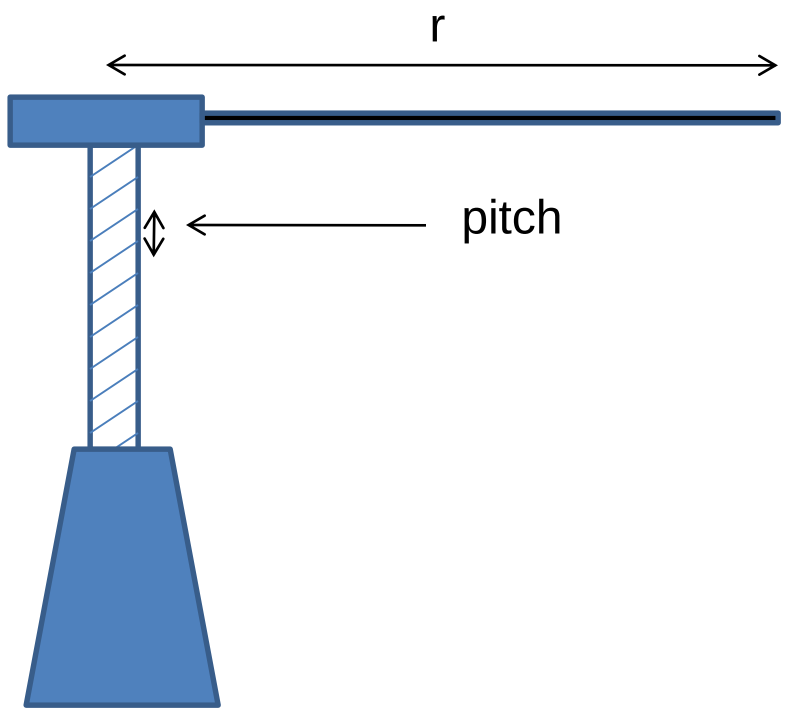
$$M_I = \frac{2R}{R - r} \qquad M_A = \frac{F_{out}}{F_{in}} \qquad Eff = \frac{M_A}{M_I}$$

$$M_I = \frac{2(2.25\ \text{cm})}{(2.25\ \text{cm} - 1.30\ \text{cm})} = 4.74$$

$$Eff = \frac{M_A}{M_I} \qquad \therefore M_A = (Eff)(M_I) = (0.750)(4.74) = 3.56$$

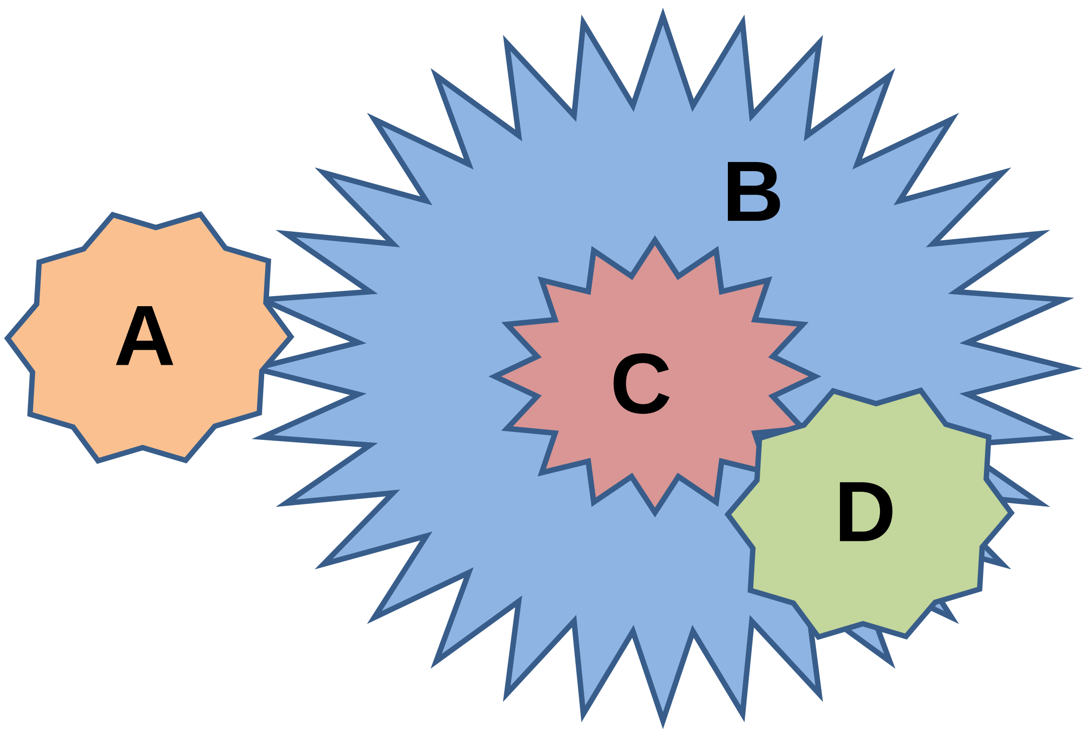
$$M_A = \frac{F_{out}}{F_{in}} \qquad \therefore F_{in} = \frac{F_{out}}{M_A} = \frac{850.0\ \text{N}}{3.56} = \boxed{239\ \text{N}}$$

A screwjack has a thread pitch of 1/32” and a handle length of 10.0“. What is the IMA?



$$M_I = \frac{2\pi R}{(pitch)} = \frac{2\pi(10.0\ \cancel{\text{in}})}{(1/32\ \cancel{\text{in}})} = \boxed{2\bar{0}00}$$

If gear “A” rotates at 145 rpm, what is the speed of gear “D”? (“A” has 10 teeth, “B” has 32 teeth, “C” has 16 teeth, and “D” has 10 teeth.) *Note: Gears “B” and “C” are on the same axle.



$$\frac{RPM_{out}}{RPM_{in}} = \frac{Teeth_{in}}{Teeth_{out}}$$

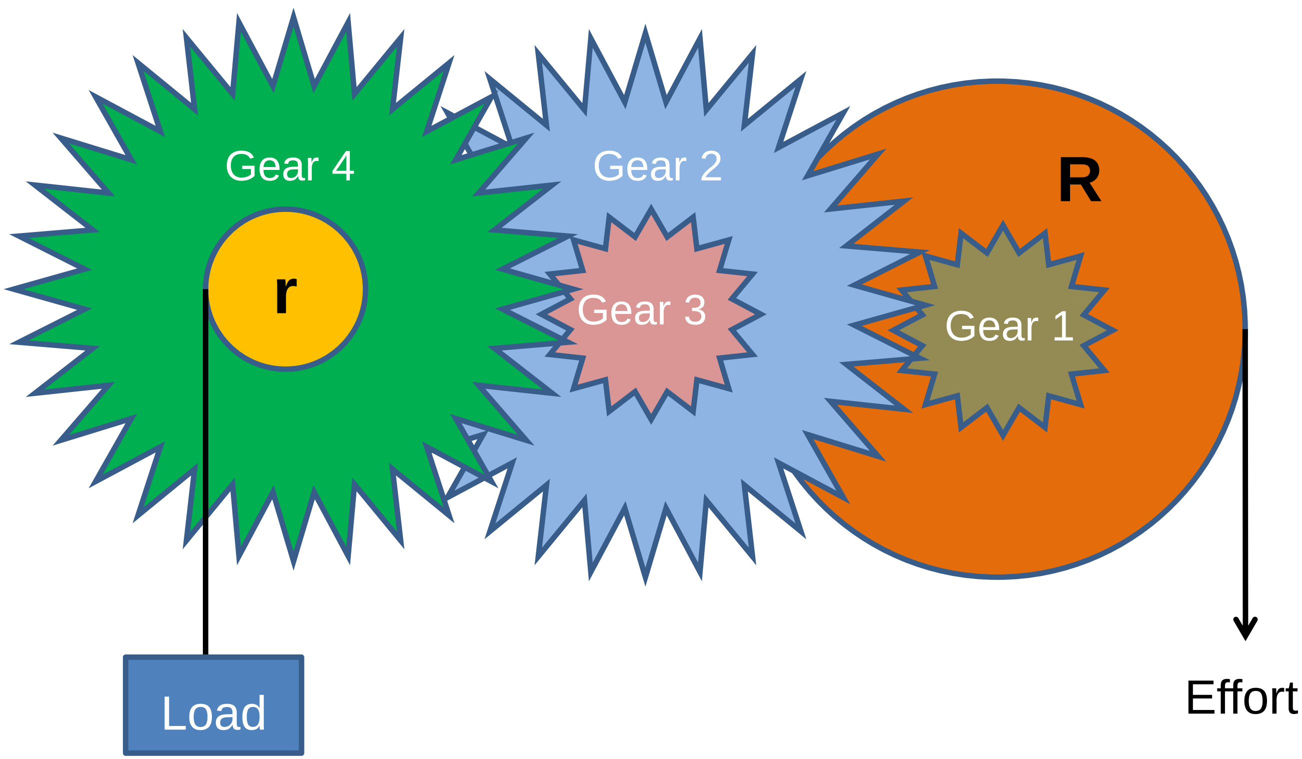
$$\frac{RPM_{B\,out}}{RPM_{A\,in}} = \frac{Teeth_{A\,in}}{Teeth_{B\,out}} = \frac{RPM_{B\,out}}{145\text{ rpm}} = \frac{10\text{ teeth}}{32\text{ teeth}} \quad \therefore RPM_{B\,out} = 45.3\text{ rpm}$$

$$RPM_{B\,in} = RPM_{C\,out} = 45.3\text{ rpm}$$

$$\frac{RPM_{D\,out}}{RPM_{C\,in}} = \frac{Teeth_{C\,in}}{Teeth_{D\,out}} = \frac{RPM_{D\,out}}{45.3\text{ rpm}} = \frac{16\text{ teeth}}{10\text{ teeth}} \quad \therefore RPM_{D\,out} = \boxed{72.5\text{ rpm}}$$

Ideal Mechanical Advantage

$$\mathrm{IMA} = \frac{R}{r} \cdot \frac{T_2}{T_1} \cdot \frac{T_4}{T_3}$$



Ideal Mechanical Advantage

Wheel & Axle System

$$IMA = \frac{D}{d}$$