Student: Cole Lamers **Date:** 07/20/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 6.2 Sum and Difference Formulas

Find the exact value of the expression.

$$\sin\frac{\pi}{4}\cos\frac{\pi}{2} + \cos\frac{\pi}{4}\sin\frac{\pi}{2}$$

First, note that the pattern matches the sum formula for sine.

$$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha+\beta)$$

Then, use the formula to rewrite the expression as the sine of a single angle.

$$\begin{aligned}\sin\frac{\pi}{4}\cos\frac{\pi}{2} + \cos\frac{\pi}{4}\sin\frac{\pi}{2} &= \sin\left(\frac{\pi}{4}+\frac{\pi}{2}\right)\\ &= \sin\frac{3\pi}{4}\end{aligned}$$

Lastly, evaluate the sine.

$$\begin{aligned}\sin\frac{\pi}{4}\cos\frac{\pi}{2} + \cos\frac{\pi}{4}\sin\frac{\pi}{2} &= \sin\frac{3\pi}{4}\\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

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Use an identity to find the exact value of the expression.

$$\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

To find the exact value of the expression, notice that it is the sine of a sum.

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

Here, $\alpha = \frac{\pi}{4}$ and $\beta = \frac{\pi}{6}$.

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta\\ \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right) &= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}\end{aligned}$$

Simplify.

$$\begin{aligned}\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right) &= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}\\ &= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}\end{aligned}$$

Multiply and simplify.

$$\begin{aligned}\sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right) &= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}\\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}\\ &= \frac{\sqrt{6}+\sqrt{2}}{4}\end{aligned}$$

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Find the exact value of the expression.

$$\tan\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$$

To find the exact value of the expression, notice that it is the tangent of a difference.

Here, $u = \frac{3\pi}{4}$ and $v = \frac{\pi}{6}$. Substitute these values in the difference identity for tangent.

$$\tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

$$\tan\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\frac{3\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{3\pi}{4}\tan\frac{\pi}{6}}$$

Substitute the exact trigonometric values for the given angles.

$$\tan\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\frac{3\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{3\pi}{4}\tan\frac{\pi}{6}}$$

$$= \frac{-1 - \frac{\sqrt{3}}{3}}{1 + \left(-1 \cdot \frac{\sqrt{3}}{3}\right)}$$

Simplify the numerator and denominator.

$$\tan\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) = \frac{-1 - \frac{\sqrt{3}}{3}}{1 + \left(-1 \cdot \frac{\sqrt{3}}{3}\right)}$$

$$= \frac{-3 - \sqrt{3}}{3 - \sqrt{3}}$$

Multiply by the conjugate of the denominator to get rid of the radical in the denominator.

$$\tan\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) = \frac{-3 - \sqrt{3}}{3 - \sqrt{3}} \cdot \frac{3 + \sqrt{3}}{3 + \sqrt{3}}$$

$$= \frac{\left(-3 - \sqrt{3}\right) \cdot \left(3 + \sqrt{3}\right)}{6}$$

Multiply the numerator and simplify.

$$\tan\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) = \frac{\left(-3 - \sqrt{3}\right) \cdot \left(3 + \sqrt{3}\right)}{6}$$

$$= -2 - \sqrt{3}$$

Thus, $\tan\left(\frac{3\pi}{4} - \frac{\pi}{6}\right) = -2 - \sqrt{3}$.

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Use a cosine sum or difference identity to find the exact value.

$$\cos\left(\frac{7\pi}{12}\right)$$

To use the sum or difference identity for cosine, write $\frac{7\pi}{12}$ as a sum or difference of angles whose cosine is known.

Notice that $\frac{7\pi}{12} = \frac{\pi}{4} + \frac{\pi}{3}$. Thus, use the cosine of a sum.

$$\begin{aligned}\cos\left(\frac{\pi}{4}+\frac{\pi}{3}\right) &= \cos\frac{\pi}{4}\cos\frac{\pi}{3} - \sin\frac{\pi}{4}\sin\frac{\pi}{3}\\ &= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}\\ &= \frac{\sqrt{2}-\sqrt{6}}{4}\end{aligned}$$