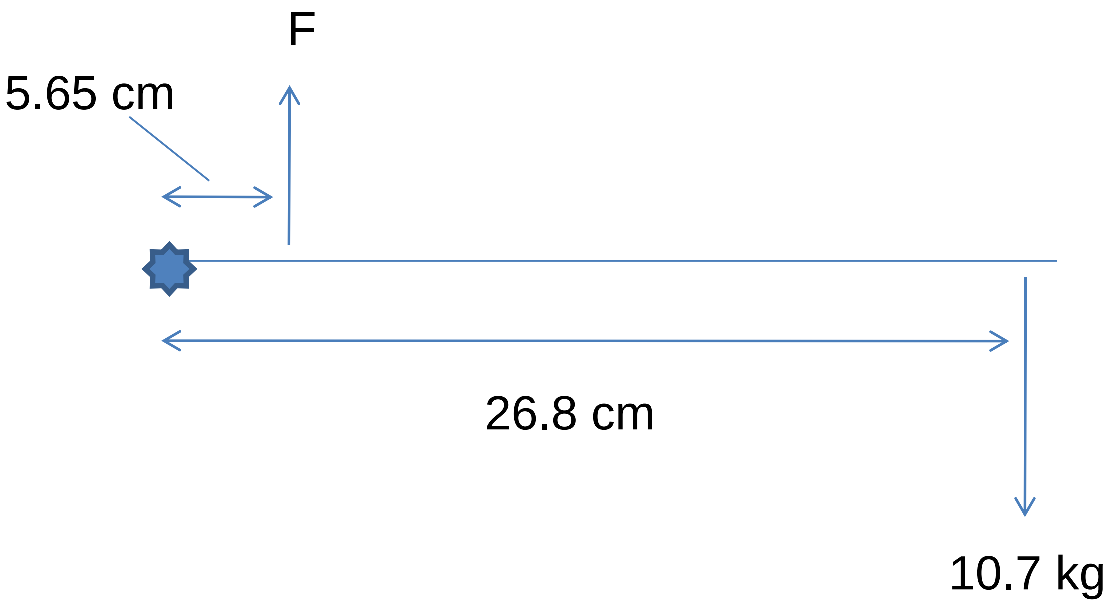
Unit 3 – Rotational Equilibrium Post-Lecture

If a board has a rope attached 5.65 cm from its hinge, what force is needed to support a 10.7 kg mass that is 26.8 cm from the hinge?



$$\Sigma\tau = Fd$$

$$\tau_{ccw} = +\text{ Torque}$$

$$\tau_{cw} = -\text{ Torque}$$

1st Condition of Equilibrium: $\Sigma F = 0$

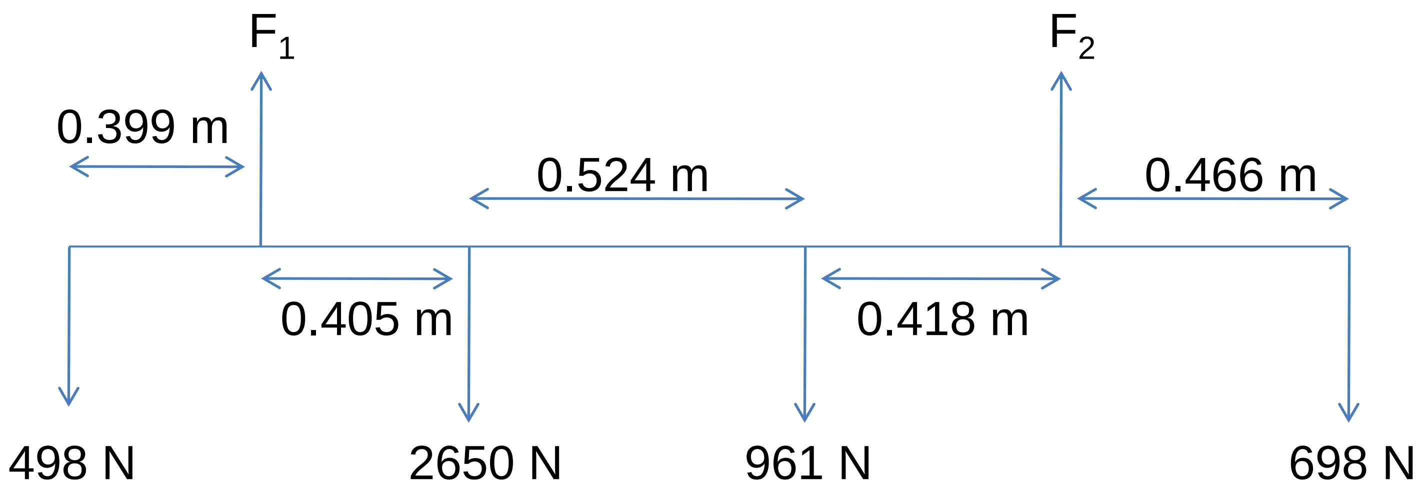
2nd Condition of Equilibrium: $\Sigma\tau = 0$

$$W = mg = (10.7\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 105\ \text{N}$$

$$\Sigma\tau = F\left(0.0565\ \text{m}\right) - \left(105\ \text{N}\right)\left(0.268\ \text{m}\right) = 0$$

$$\therefore\quad F = \boxed{498\ \text{N}}$$

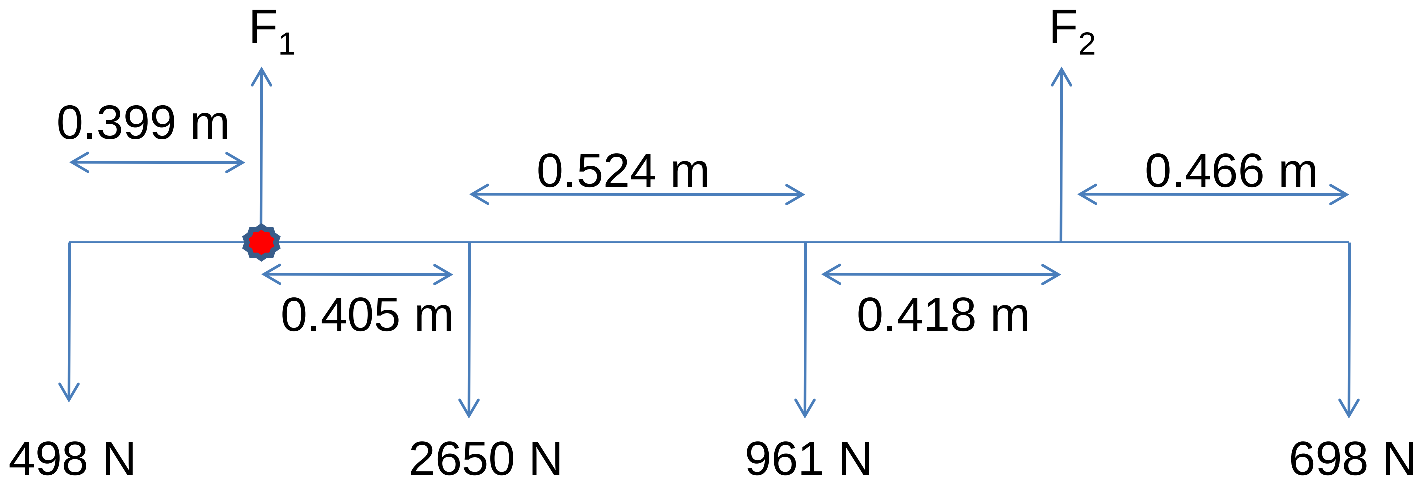
What support forces are required to maintain equilibrium for the structure shown? (Neglect the weight of the horizontal bar.)



$$\Sigma F_y = F_1 + F_2 - (498\text{ N}) - (2650\text{ N}) - (961\text{ N}) - (698\text{ N}) = 0$$

$$\Sigma F_y = F_1 + F_2 - (4810\text{ N}) = 0 \qquad \therefore\ F_1 = (4810\text{ N}) - F_2$$

What support forces are required to maintain equilibrium for the structure shown? (Neglect the weight of the horizontal bar.)



$$\Sigma\tau = \left[(498\ \mathrm{N})(0.399\ \mathrm{m})\right] + \left[(\mathrm{F}_2)(1.347\ \mathrm{m})\right] - \left[(2650\ \mathrm{N})(0.405\ \mathrm{m})\right] - \left[(961\ \mathrm{N})(0.929\ \mathrm{m})\right] - \left[(698\ \mathrm{N})(1.813\ \mathrm{m})\right] = 0$$

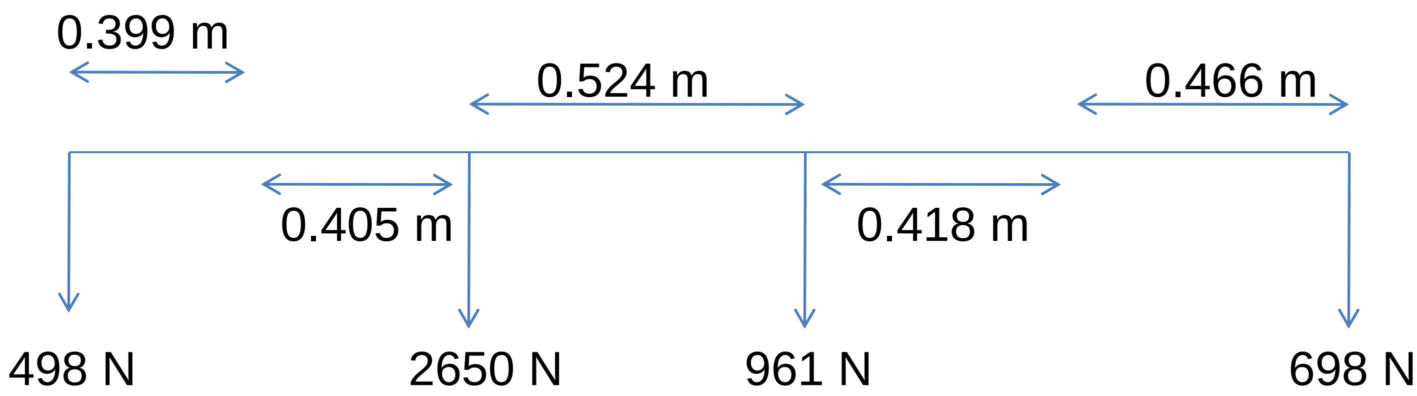
$$\Sigma\tau = (199\ \mathrm{N\cdot m}) + \left[(\mathrm{F}_2)(1.347\ \mathrm{m})\right] - (1070\ \mathrm{N\cdot m}) - (893\ \mathrm{N\cdot m}) - (1270\ \mathrm{N\cdot m}) = 0$$

$$\Sigma\tau = \left[(\mathrm{F}_2)(1.347\ \mathrm{m})\right] - (3030\ \mathrm{N\cdot m}) = 0$$

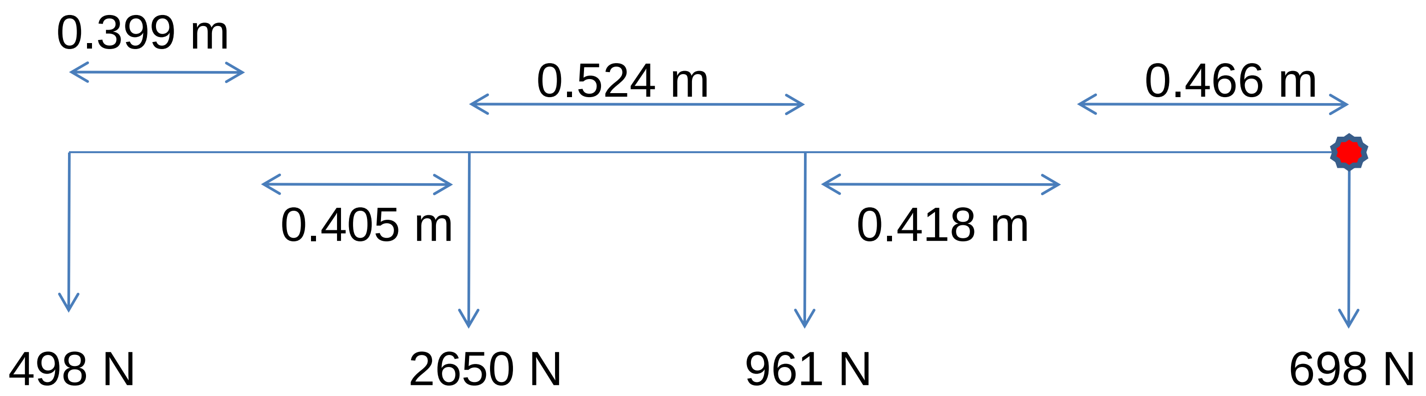
$$\therefore \quad \left[(\mathrm{F}_2)(1.347\ \mathrm{m})\right] = (3030\ \mathrm{N\cdot m})$$

$$\therefore \quad \mathrm{F}_2 = \boxed{2250\ \mathrm{N}} \qquad F_1 = (4810\ \mathrm{N}) - F_2 \qquad \therefore\ F_1 = \boxed{2560\ \mathrm{N}}$$

Determine the center of gravity measured from the right end of this diagram.



Determine the center of gravity measured from the right end of this diagram.



$$\Sigma F_y = \text{n} - (498\text{ N}) - (2650\text{ N}) - (961\text{ N}) - (698\text{ N}) = 0$$

$$\therefore \qquad \text{n} = 4810\text{ N}$$

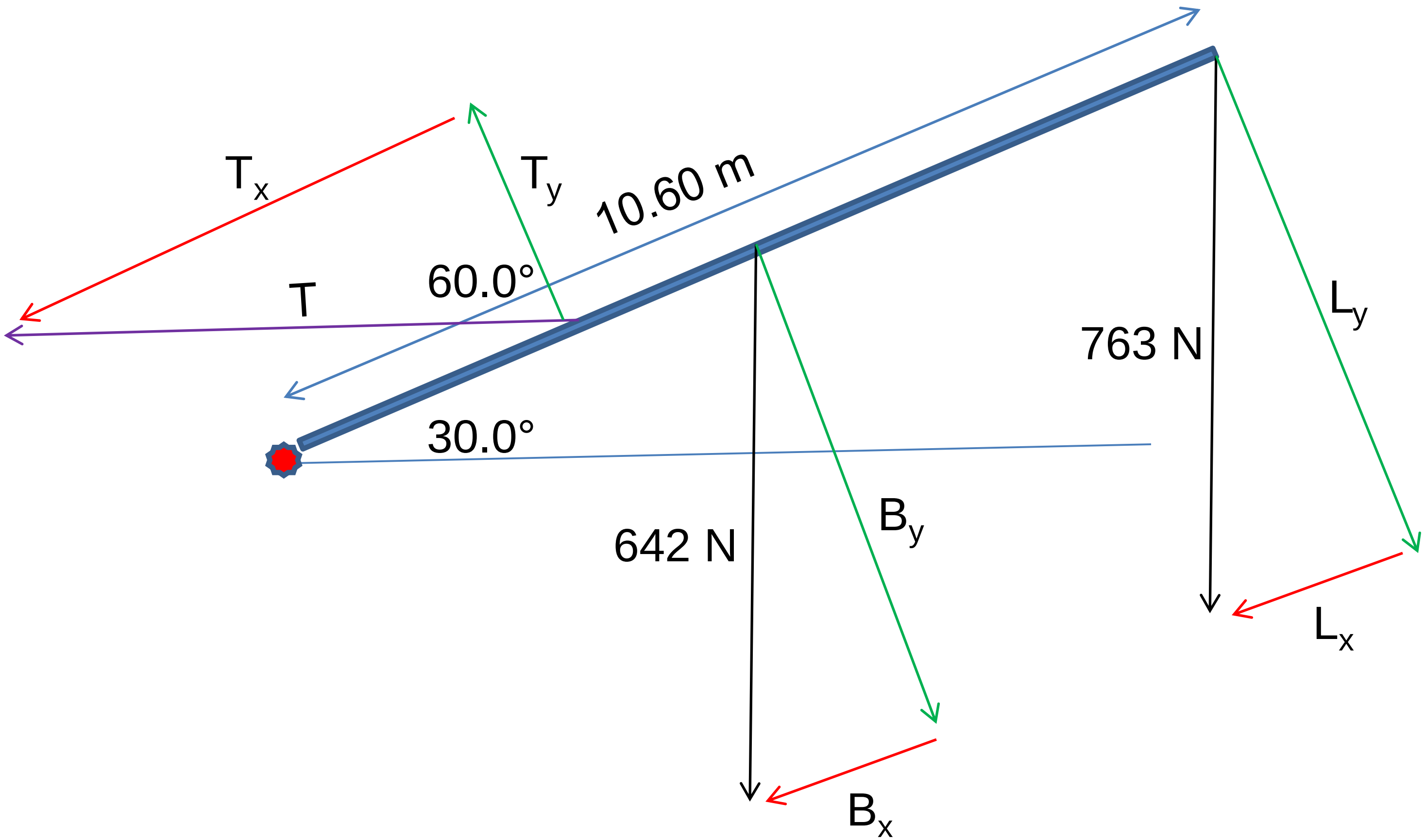
$$\Sigma\tau = \left[(498\text{ N})(2.212\text{ m})\right] + \left[(2650\text{ N})(1.408\text{ m})\right] + \left[(961\text{ N})(0.884\text{ m})\right] - \left[(4810\text{ N})(x)\right] = 0$$

$$\Sigma\tau = (11\bar{0}0\text{ N}\cdot\text{m}) + (3730\text{ N}\cdot\text{m}) + (849\text{ N}\cdot\text{m}) - \left[(4810\text{ N})(x)\right] = 0$$

$$\Sigma\tau = (5680\text{ N}\cdot\text{m}) - \left[(4810\text{ N})(x)\right] = 0$$

$$\therefore \left[(4810\text{ N})(x)\right] = (5680\text{ N}\cdot\text{m}) \qquad \therefore\ x = \boxed{1.18\text{ m}}$$

A uniform 642 N boom, 10.60 m long, is pivoting on a pin at the floor. It supports a 763 N load. How strong must the horizontal tie rope, 3.10 m from the bottom, be to hold the system at a 30.0° angle?



T_x
T_y
10.60 m
60.0°
T
L_y
763 N
30.0°
B_y
642 N
L_x
B_x

$$\Sigma\tau = \left[\left(\cos 60.0°\right)\left(\text{T}\right)\right]\cdot\left[3.10\text{ m}\right] - \left[\left(\cos 30.0°\right)\left(642\text{ N}\right)\right]\cdot\left[5.30\text{ m}\right] -$$

$$\left[\left(\cos 30.0°\right)\left(763\text{ N}\right)\right]\cdot\left[10.60\text{ m}\right] = 0$$

$$\Sigma\tau = \left[\left(1.55\text{ m}\right)\left(\text{T}\right)\right] - \left[2950\text{ N}\cdot\text{m}\right] - \left[70\bar{0}0\text{ N}\cdot\text{m}\right] = 0$$

$$\Sigma\tau = \left[\left(1.55\text{ m}\right)\left(\text{T}\right)\right] - \left[9950\text{ N}\cdot\text{m}\right] = 0$$

$$\therefore\ \left[\left(1.55\text{ m}\right)\left(\text{T}\right)\right] = \left[9950\text{ N}\cdot\text{m}\right]$$

$$\therefore\ \text{T} = \boxed{6420\text{ N}}$$

Two disgruntled business people are trying to use a revolving door (*as in lecture Figure 8.3*). The woman on the left exerts a force of 625 N perpendicular to the door and 1.20 m from the hub's center, while the man on the right exerts a force of 850. N perpendicular to the door and 0.800 m from the hub's center.

(a) Find the net torque on the revolving door.

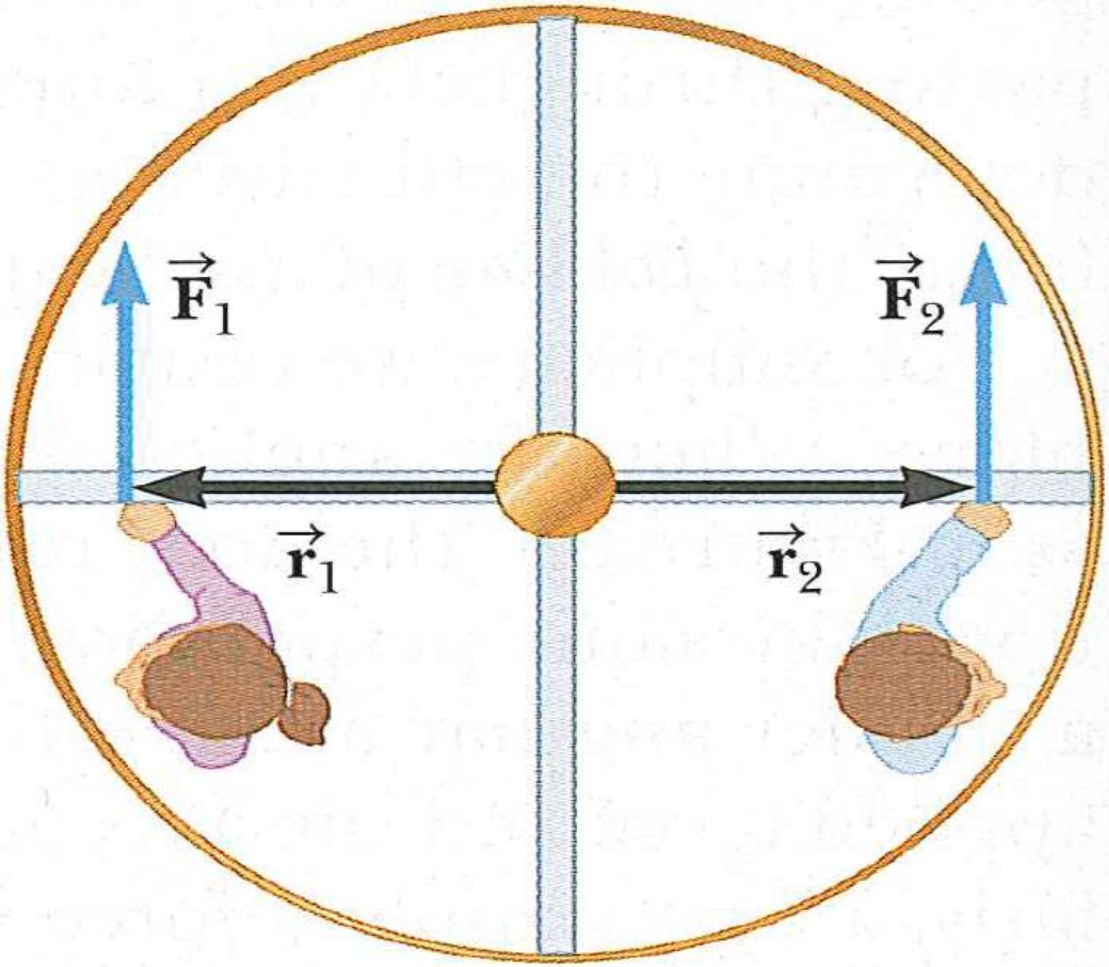


Figure 8.3

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$\tau_2 = r_2 F_2 = (0.800\ \text{m})(85\bar{0}\ \text{N}) = 68\bar{0}\ \text{N}\cdot\text{m}$$

$$\tau_1 = -r_1 F_1 = -(1.20\ \text{m})(625\ \text{N}) = -75\bar{0}\ \text{N}\cdot\text{m}$$

$$\tau_{net} = \tau_1 + \tau_2 = (-75\bar{0}\ \text{N}\cdot\text{m}) + (68\bar{0}\ \text{N}\cdot\text{m})$$

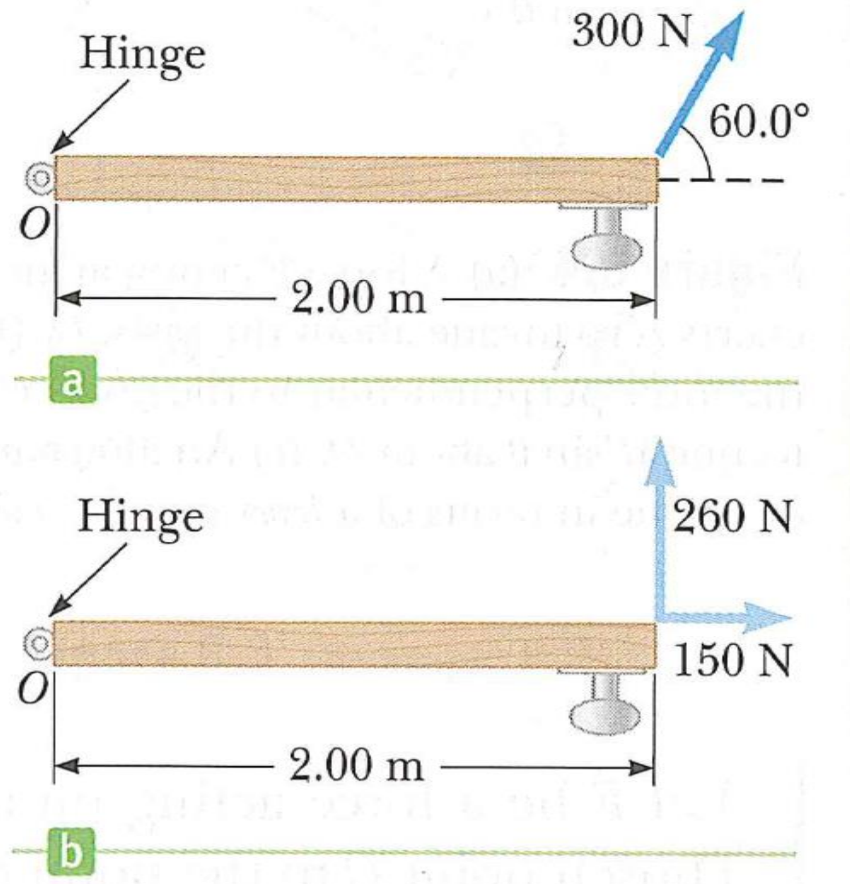
$$\tau_{net} = \boxed{-7\bar{0}\ \text{N}\cdot\text{m}}$$

A man applies a force of F = 300. N at an angle of 60.0° to the door (*as shown similarly in lecture Figure 8.7*), 2.00 m from the hinges.

(a) Find the torque on the door, choosing the position of the hinges as the axis of rotation. Suppose a wedge is placed 1.50 m from the hinges on the other side of the door.

(b) What minimum force must the wedge exert so that the force applied in part (a) won't open the door?

Figure 8.7



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$$\tau_F = rF(\sin\theta) = (2.00\text{ m})(30\bar{0}\text{ N})(\sin 60.0°) = \boxed{520\text{ N}\cdot\text{m}}$$

$$\Sigma\tau = \tau_{hinge} + \tau_{wedge} + \tau_F = 0$$

$$= 0 + -F_{wedge}(1.50\text{ m}) + 520\text{ N}\cdot\text{m} = 0$$

$$\therefore F_{wedge} = \boxed{347\text{ N}}$$

A woman of mass m = 55.0 kg sits on the left end of a seesaw – a plank of length L = 4.00 m, pivoted in the middle (*as shown similarly in lecture Figure 8.8*). First compute the torques on the seesaw about an axis that passes through the pivot point.

(a) Where should a man of mass M = 75.0 kg sit if the system (seesaw plus man and woman) is to be balanced?

(b) Find the normal force exerted by the pivot if the plank has a mass of m_{pl} = 12.0 kg.

(c) Repeat part (a), but this time compute the torques about an axis through the left end of the plank.

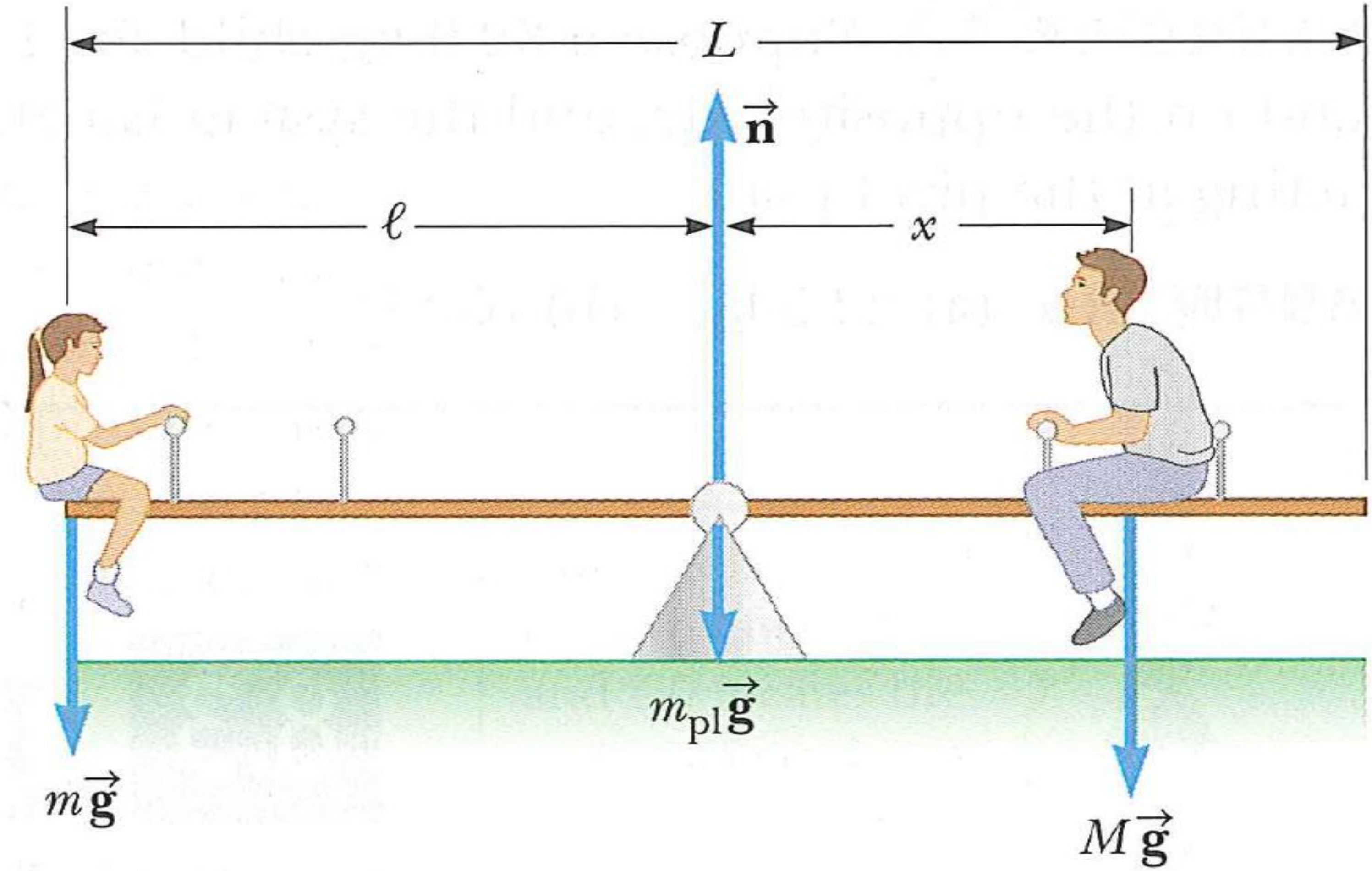


Figure 8.8

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$\tau_{normal} + \tau_{plank} + \tau_{man} + \tau_{woman} = 0$$

$$0 + 0 + (75.0\text{ kg})gx + (55.0\text{ kg})g(2.00\text{ m}) = 0$$

$$\therefore\ x = \boxed{1.47\text{ m}}$$

$$n - Mg - mg - m_{pl}g = 0$$

$$\mathrm{n} = \left(M + m + m_{pl}\right)g = \left(75.0\text{ kg} + 55.0\text{ kg} + 12.0\text{ kg}\right)g = \boxed{1390\text{ N}}$$

$$\tau_{normal} + \tau_{plank} + \tau_{man} + \tau_{woman} = 0$$

$$n(2.00\ \mathrm{m}) - m_{pl}g(2.00\ \mathrm{m}) - Mg(2.00\ \mathrm{m} + x) + mg(0\ \mathrm{m}) = 0$$

$$(1390\ \mathrm{N})(2.00\ \mathrm{m}) - (12.0\ \mathrm{kg})g(2.00\ \mathrm{m}) - (75.0\ \mathrm{kg})g(2.00\ \mathrm{m} + x) + (55.0\ \mathrm{kg})g(0\ \mathrm{m}) = 0$$

$$(2780\ \mathrm{N\cdot m}) - (235\ \mathrm{N\cdot m}) - (1470\ \mathrm{N\cdot m}) - (735\ \mathrm{kg})x + (0\ \mathrm{N\cdot m}) = 0$$

$$(2780\ \mathrm{N\cdot m}) - (235\ \mathrm{N\cdot m}) - (1470\ \mathrm{N\cdot m}) = (735\ \mathrm{kg})x$$

$$\therefore\ x = \boxed{1.46\ \mathrm{m}}$$

Three objects are located in a coordinate system (*as shown similarly in lecture Figure 8.11a*).

(a) Find the center of gravity.

(b) How does the answer change if the object on the left is displaced upward by 1.00 m and the right is displaced downward by 0.500 m (*as shown similarly in lecture Figure 8.11b*).

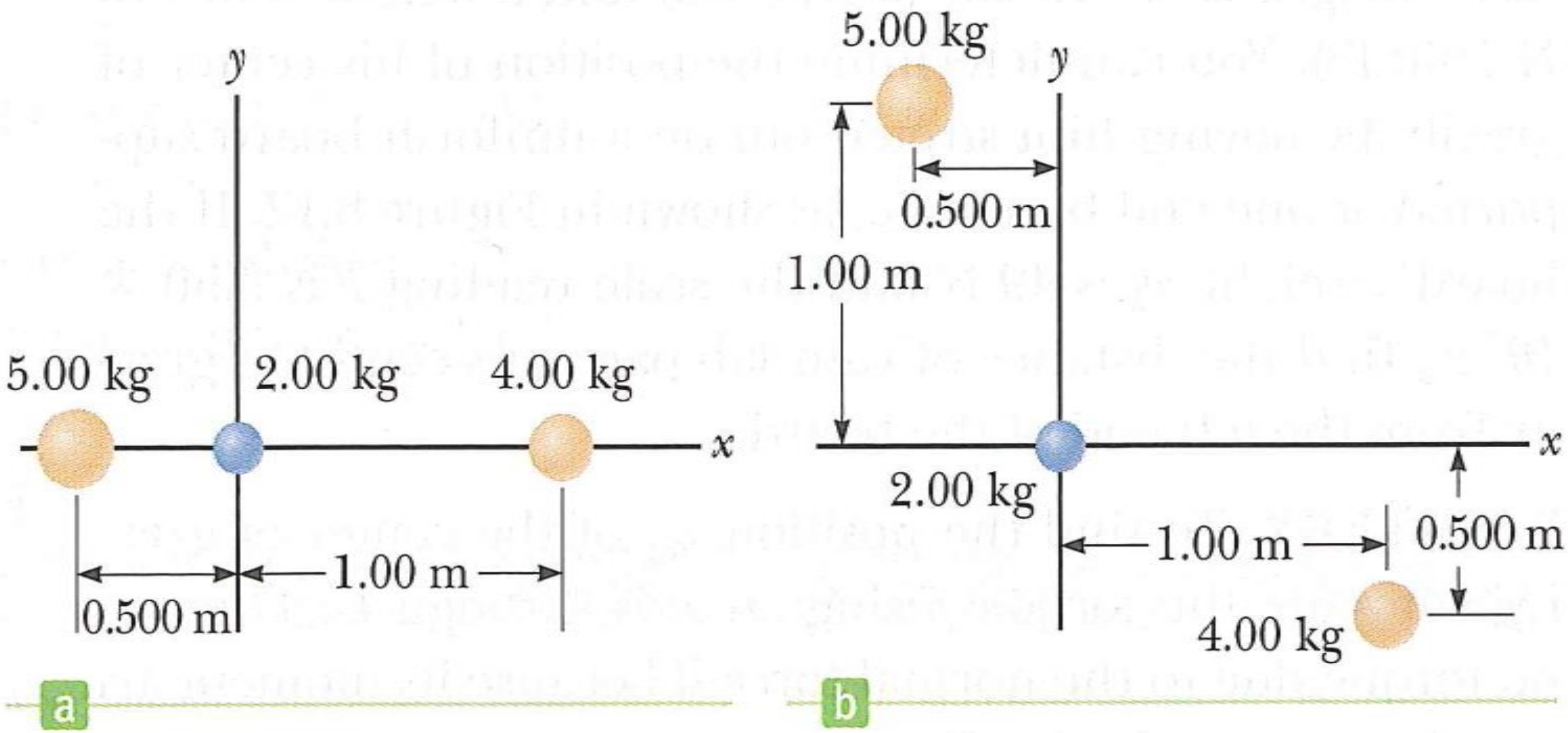


Figure 8.11

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(5.00\text{ kg})(-0.500\text{ m}) + (2.00\text{ kg})(0\text{ m}) + (4.00\text{ kg})(1.00\text{ m})}{(5.00\text{ kg} + 2.00\text{ kg} + 4.00\text{ kg})}$$

$$= \boxed{0.136\text{ m}}$$

$$y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{(5.00\text{ kg})(1.00\text{ m}) + (2.00\text{ kg})(0\text{ m}) + (4.00\text{ kg})(-0.500\text{ m})}{(5.00\text{ kg} + 2.00\text{ kg} + 4.00\text{ kg})}$$

$$= \boxed{0.273\text{ m}}$$

A 50.0 N bowling ball is held in a person's hand with the forearm horizontal (*as shown similarly in lecture Figure 8.13*). The bicep muscle is attached 0.0300 m from the joint, and the ball is 0.350 m from the joint.

(a) Find the upward force **F** exerted by the biceps on the forearm (the ulna) and

(b) the downward force **R** exerted by the humerus on the forearm, acting at the joint. Neglect the weight of the forearm and slight deviation from the vertical of the biceps.

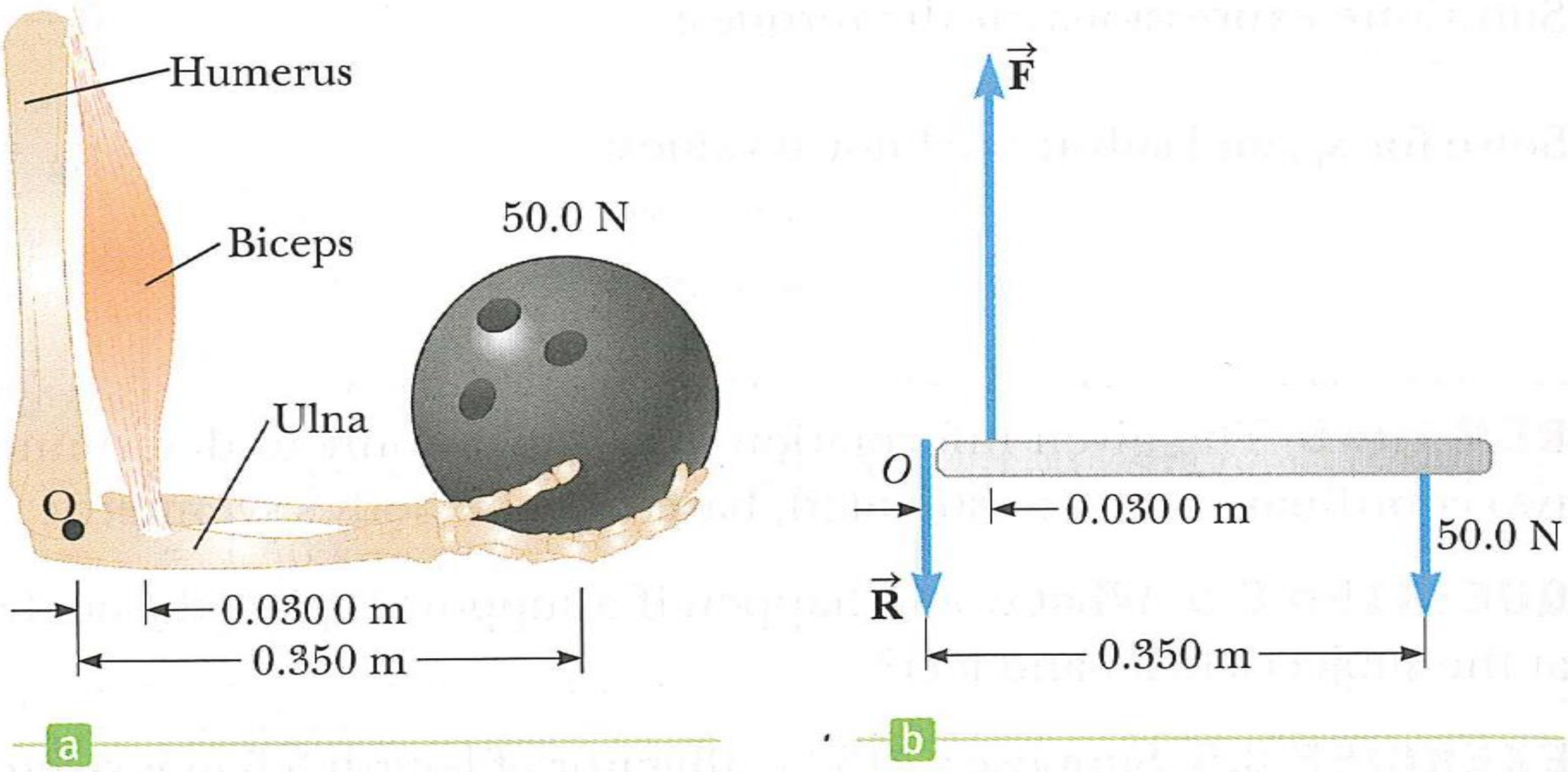


Figure 8.13

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$\Sigma \tau_i = \tau_R + \tau_F + \tau_{BB} = 0$$

$$R(0) + F(0.0300\text{ m}) - (50.0\text{ N})(0.350\text{ m}) = 0$$

$$\therefore F = \boxed{583\text{ N}}$$

$$\Sigma F_y = F - R - (50.0\ \text{N}) = 0$$

$$\therefore\ R = F - (50.0\ \text{N}) = (583\ \text{N}) - (50.0\ \text{N})$$

$$\therefore\ R = \boxed{533\ \text{N}}$$

A uniform ladder 10.0 m long and weighing 50.0 N rests against a smooth vertical wall (*as shown similarly in lecture Figure 8.14*). If the ladder is just on the verge of slipping when it makes a 50.0° angle with the ground,

(a) find the coefficient of static friction between the ladder and ground.

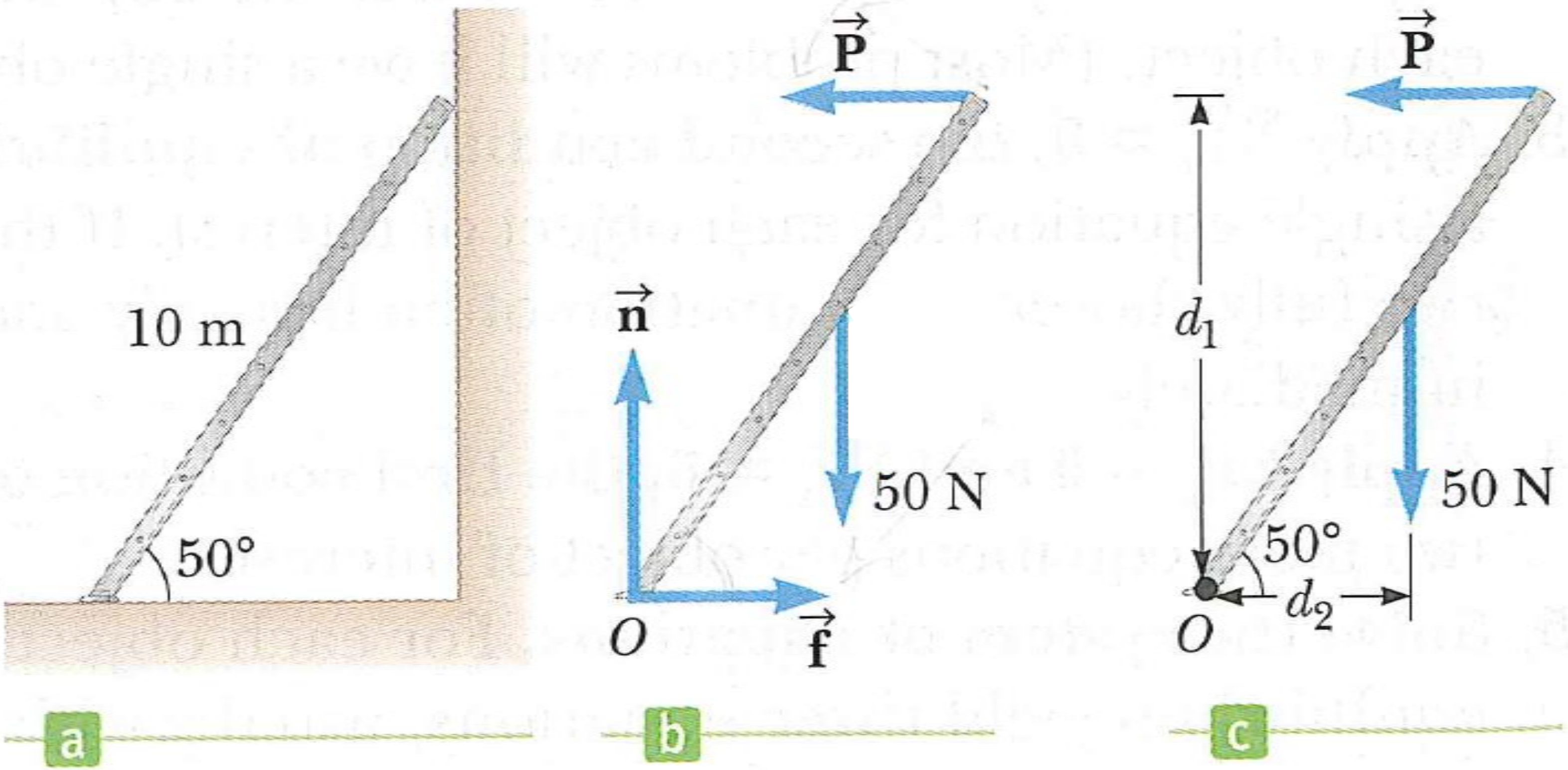


Figure 8.14

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$\Sigma F_x = f - P = 0 \qquad \therefore \; f = P$$

$$\Sigma F_y = n - (50.0 \text{ N}) = 0 \qquad \therefore \; n = 50.0 \text{ N}$$

$$\Sigma \tau_i = \tau_f + \tau_n + \tau_{grav} + \tau_p = 0$$

$$0 = 0 + 0 - (50.0 \text{ N})(\cos\ 50.0°)(5.00 \text{ m}) + (P)(\cos\ 40.0°)(10.0 \text{ m})$$

$$\therefore \; P = 21.0 \text{ N}$$

$$21.0 \text{ N} = f = f_{s\ (\text{max})} = \mu_s n = \mu_s (50.0 \text{ N})$$

$$\therefore \; \mu_s = \frac{21.0 \text{ N}}{50.0 \text{ N}} = \boxed{0.420}$$

A uniform horizontal beam 5.00 m long and weighing 300. N is attached to a wall by a pin connection that allows the beam to rotate. Its far end is supported by a cable that makes an angle of 53.0° with the horizontal (*as shown similarly in lecture Figure 8.15*). If a person weighing 600. N stands 1.50 m from the wall,

(a) find the tension **T** in the cable and

(b) the components of force **R** exerted by the wall on the beam.

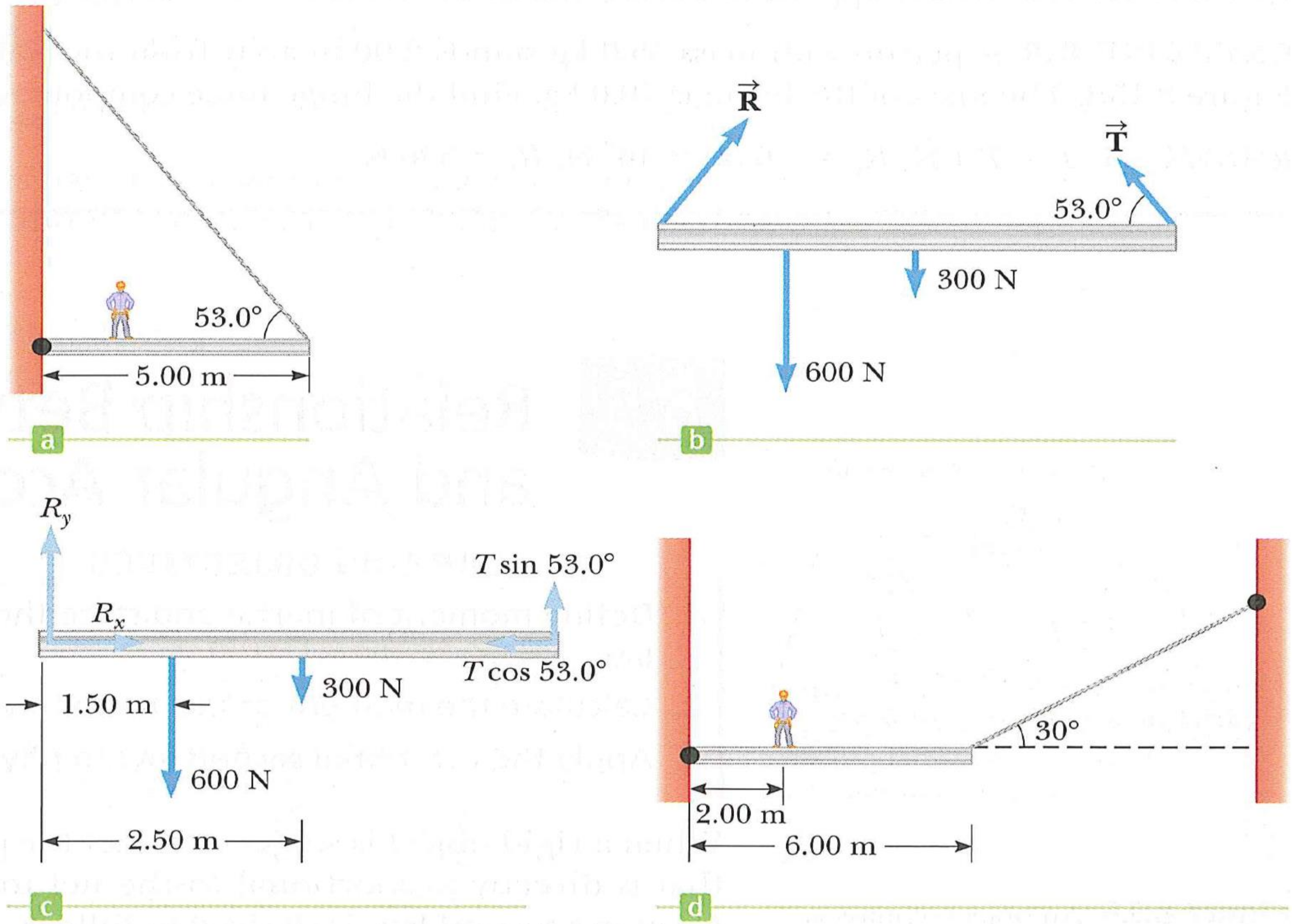


Figure 8.15

Photos/Illustrations courtesy of *College Physics*, 9th edition.

$$\Sigma \tau_i = \tau_R + \tau_B + \tau_M + \tau_T = 0$$

$$0 = 0 - w_B(2.50 \text{ m}) - w_M(1.50 \text{ m}) + (T)(\cos 37.0°)(5.00 \text{ m})$$

$$0 = 0 - (30\overline{0} \text{ N})(2.50 \text{ m}) - (60\overline{0} \text{ N})(1.50 \text{ m}) + (T)(\cos 37.0°)(5.00 \text{ m})$$

$$\therefore T = \boxed{413 \text{ N}}$$

$$\Sigma F_x = R_x - T(\sin 37.0°) = 0$$

$$\Sigma F_x = R_x - (413 \text{ N})(\sin 37.0°) = 0$$

$$\therefore R_x = \boxed{249 \text{ N}}$$

$$\Sigma F_y = R_y - w_b - w_m + T(\sin 48.0°) = 0$$

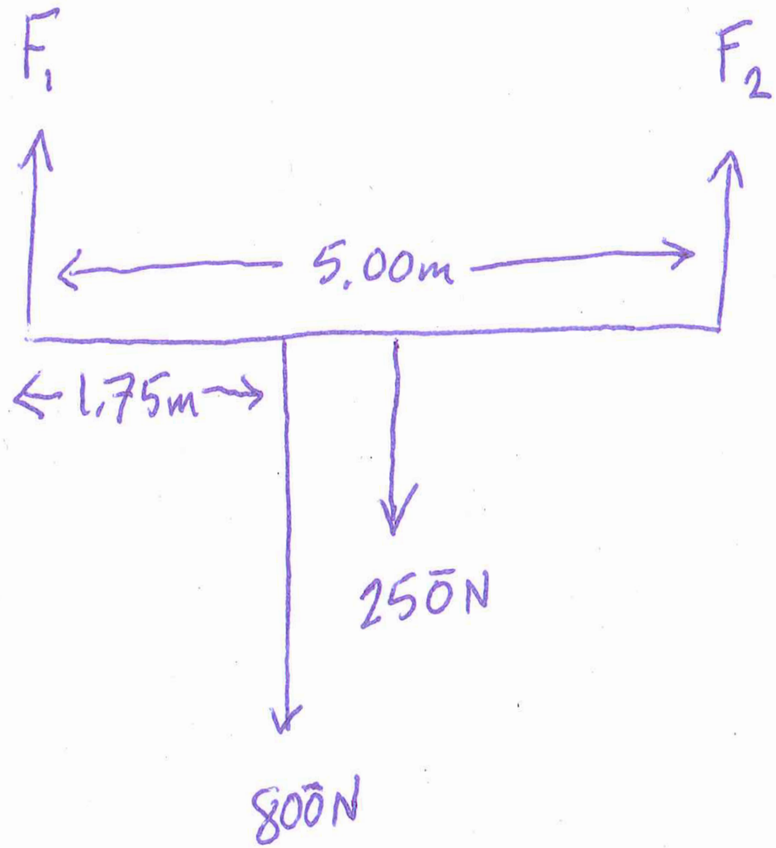
$$\Sigma F_y = R_y - (30\bar{0}\text{ N}) - (60\bar{0}\text{ N}) + (413\text{ N})(\cos 37.0°) = 0$$

$$\therefore R_y = \boxed{57\bar{0}\text{ N}}$$

A painter weighing 800. N stands on a uniform plank 5.00 m long, which is supported at each end by a step ladder. The plank weighs 250. N. If the painter stands 1.75 m from the left end of the plank,

(a) what upward force is exerted by the step ladder on the right side

(b) and on the left side?



F1
F2
5.00m
1.75m
250̄N
800̄N

$$\Sigma\tau_i = F_2(5.00\text{ m}) - (250\text{ N})(2.50\text{ m}) - (80\bar{0}\text{ N})(1.75\text{ m}) = 0$$

$$\therefore\ F_2 = \boxed{405\text{ N}}$$

$$\Sigma F_y = F_1 + F_2 - \left(80\overline{0}\text{ N}\right) - \left(25\overline{0}\text{ N}\right) = 0$$

$$\therefore\ F_1 = \boxed{645\text{ N}}$$