Student: Cole Lamers **Date:** 06/29/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 9.1 Matrices and Systems of Equations

Use Gaussian elimination to find the complete solution to the following system of equations, or show that none exists.

$$\begin{cases} x+y-z=12 \\ -x-2y+4z=-26 \\ 2x-3y+z=14 \end{cases}$$

To solve a system of linear equations using Gaussian elimination with back-substitution, apply row operations to the augmented matrix of the system to obtain a matrix that is in row-echelon form. Gauss-Jordan elimination results in a matrix that is in reduced row-echelon form. We will use Gaussian elimination with back-substitution for this exercise.

First, write the augmented matrix that represents this system.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ -1 & -2 & 4 & -26 \\ 2 & -3 & 1 & 14 \end{array}\right]$$

The first step requires getting 1 in the upper left-hand corner of the matrix. Notice that this entry is already 1. We will use this 1 to get 0s below it.

The row operation R_1+R_2 will result in a 0 in row 2, column 1. Applying the row operation R_1+R_2 results in the matrix below.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ -1 & -2 & 4 & -26 \\ 2 & -3 & 1 & 14 \end{array}\right] \to \left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & -1 & 3 & -14 \\ 2 & -3 & 1 & 14 \end{array}\right] \qquad R_1+R_2$$

The next step requires getting a 0 in row 3, column 1. This can be accomplished by multiplying row 1 by -2 and adding the result to row 3. Applying the row operation $-2R_1+R_3$ results in the matrix below.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & -1 & 3 & -14 \\ 2 & -3 & 1 & 14 \end{array}\right] \to \left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & -1 & 3 & -14 \\ 0 & -5 & 3 & -10 \end{array}\right] \qquad -2R_1+R_3$$

The next step is to obtain a 1 in row 2, column 2. This can be accomplished by multiplying row 2 by -1. Applying the row operation $-R_2$ results in the matrix below.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & -1 & 3 & -14 \\ 0 & -5 & 3 & -10 \end{array}\right] \to \left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & 1 & -3 & 14 \\ 0 & -5 & 3 & -10 \end{array}\right] \qquad -R_2$$

The next step is to obtain a 0 in row 3, column 2. This can be accomplished by multiplying row 2 by 5 and adding the result to row 3. Applying the row operation $5R_2+R_3$ results in the matrix below.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & 1 & -3 & 14 \\ 0 & -5 & 3 & -10 \end{array}\right] \to \left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & 1 & -3 & 14 \\ 0 & 0 & -12 & 60 \end{array}\right] \qquad 5R_2+R_3$$

Finally, we need to obtain a 1 in row 3, column 3. This can be accomplished by multiplying row 3 by $-\frac{1}{12}$. Applying the row operation $-\frac{1}{12}R_3$ results in the matrix below.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & 1 & -3 & 14 \\ 0 & 0 & -12 & 60 \end{array}\right] \to \left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & 1 & -3 & 14 \\ 0 & 0 & 1 & -5 \end{array}\right] \qquad -\frac{1}{12}R_3$$

We now have the matrix in row-echelon form, with 1s down the diagonal and 0s below the 1s. Now we will write the system of linear equations corresponding to the matrix and use back-substitution to find the system's solution.

$$\left[\begin{array}{rrr|r} 1 & 1 & -1 & 12 \\ 0 & 1 & -3 & 14 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

The system of equations is:

$$\begin{cases} x + y - z = 12 \\ \quad\; y - 3z = 14 \\ \qquad\quad z = -5 \end{cases} \qquad \left[\begin{array}{ccc|c} 1 & 1 & -1 & 12 \\ 0 & 1 & -3 & 14 \\ 0 & 0 & 1 & -5 \end{array}\right]$$

We read from the third equation that the value of z is -5. We will use this value in the first and second equations to solve for x and y.

To find y, we back-substitute -5 for z in the second equation.

$y - 3z = 14$	Equation 2
$y - 3(-5) = 14$	Substitute -5 for z.
$y + 15 = 14$	Multiply.
$y = -1$	Subtract 15 from both sides.

Finally, back-substitute -1 for y and -5 for z in the first equation.

$x + y - z = 12$	Equation 1
$x + (-1) - (-5) = 12$	Substitute -1 for y and -5 for z.
$x + 4 = 12$	Simplify and add.
$x = 8$	Subtract 4 from both sides.

The solution set for the original system is $\{(8, -1, -5)\}$. We can verify the solution by checking to see that it satisfies all three equations in the given system.

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Solve the following system of equations by using Gaussian elimination.

$$\begin{cases} 3x + 2y + 4z = 37 & (1) \\ 2x - y + z = 7 & (2) \\ 6x + 5y - z = 33 & (3) \end{cases}$$

To solve the given system of linear equations by Gaussian elimination, first write the augmented matrix of the system.

$$A = \left[\begin{array}{ccc|c} 3 & 2 & 4 & 37 \\ 2 & -1 & 1 & 7 \\ 6 & 5 & -1 & 33 \end{array}\right]$$

Now use elementary row operations to transform the augmented matrix into row-echelon form. To produce a 1 in the (1,1) position, add -1 times the 2nd row to the 1st row.

$$\left[\begin{array}{ccc|c} 3 & 2 & 4 & 37 \\ 2 & -1 & 1 & 7 \\ 6 & 5 & -1 & 33 \end{array}\right] \xrightarrow{-1R_2 + R_1 \to R_1} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 2 & -1 & 1 & 7 \\ 6 & 5 & -1 & 33 \end{array}\right]$$

Add -2 times the 1st row to the 2nd row. This operation produces a 0 in the (2,1) position.

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 2 & -1 & 1 & 7 \\ 6 & 5 & -1 & 33 \end{array}\right] \xrightarrow{-2R_1 + R_2 \to R_2} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & -7 & -5 & -53 \\ 6 & 5 & -1 & 33 \end{array}\right]$$

Add -6 times the 1st row to the 3rd row. This operation produces a 0 in the (3,1) position.

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & -7 & -5 & -53 \\ 6 & 5 & -1 & 33 \end{array}\right] \xrightarrow{-6R_1 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & -7 & -5 & -53 \\ 0 & -13 & -19 & -147 \end{array}\right]$$

Now multiply the 2nd row by $-\frac{1}{7}$ to produce a 1 in the (2,2) position.

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & -7 & -5 & -53 \\ 0 & -13 & -19 & -147 \end{array}\right] \xrightarrow{-\frac{1}{7}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & 1 & \frac{5}{7} & \frac{53}{7} \\ 0 & -13 & -19 & -147 \end{array}\right]$$

Next add 13 times the 2nd row to the 3rd row. This operation produces a 0 in the (3,2) position.

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & 1 & \frac{5}{7} & \frac{53}{7} \\ 0 & -13 & -19 & -147 \end{array}\right] \xrightarrow{13R_2 + R_3 \to R_3} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & 1 & \frac{5}{7} & \frac{53}{7} \\ 0 & 0 & -\frac{68}{7} & -\frac{340}{7} \end{array}\right]$$

Finally, multiply the 3rd row by $-\frac{7}{68}$ to produce a 1 in the (3,3) position.

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & 1 & \frac{5}{7} & \frac{53}{7} \\ 0 & 0 & -\frac{68}{7} & -\frac{340}{7} \end{array}\right] \xrightarrow{-\frac{7}{68}R_3} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 30 \\ 0 & 1 & \frac{5}{7} & \frac{53}{7} \\ 0 & 0 & 1 & 5 \end{array}\right]$$

Write the system of linear equations that corresponds to the last matrix in the previous step.

$$\begin{cases} x + 3y + 3z = 30 & (4) \\ y + \frac{5}{7}z = \frac{53}{7} & (5) \\ z = 5 & (6) \end{cases}$$

To solve this system, note that equation (6) gives the value of z. Back-substitute z = 5 in equation (5) to find y.

$$y + \frac{5}{7}z = \frac{53}{7}$$

$$y + \frac{5}{7}(5) = \frac{53}{7} \qquad \text{Replace z with 5.}$$

$$y + \frac{25}{7} = \frac{53}{7} \qquad \text{Simplify.}$$

$$y = \frac{28}{7} \qquad \text{Solve for y.}$$

$$y = 4 \qquad \text{Simplify.}$$

Back-substitute the values of y and z in equation (4) to find x.

$$x + 3y + 3z = 30$$

$$x + 3(4) + 3(5) = 30 \qquad \text{Replace y with 4 and z with 5.}$$

$$x + 27 = 30 \qquad \text{Simplify.}$$

$$x = 3 \qquad \text{Solve for x.}$$

Check the solution by substituting 3 for x, 4 for y, and 5 for z in the original system of equations and verifying that true statements result. First check equation (1).

$$3x + 2y + 4z = 37$$

$$3(3) + 2(4) + 4(5) \stackrel{?}{=} 37$$

$$9 + 8 + 20 \stackrel{?}{=} 37$$

$$37 = 37 \checkmark$$

Check equation (2).

$$2x - y + z = 7$$

$$2(3) - 4 + 5 \stackrel{?}{=} 7$$

$$6 - 4 + 5 \stackrel{?}{=} 7$$

$$7 = 7 \checkmark$$

Finally, check equation (3).

$$6x + 5y - z = 33$$

$$6(3) + 5(4) - 5 \stackrel{?}{=} 33$$

$$18 + 20 - 5 \stackrel{?}{=} 33$$

$$33 = 33 \checkmark$$

Therefore, the solution set is {(3,4,5)}.

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Solve the system of equations by using row operations.

$$\begin{cases} 4 \cdot 3^x - 5 \cdot 5^y + 7^z = -10 \\ 3^x + 4 \cdot 5^y - 3 \cdot 7^z = 14 \\ 3 \cdot 3^x - 3 \cdot 5^y + 5 \cdot 7^z = 9 \end{cases}$$

Let $u = 3^x$, $v = 5^y$, and $w = 7^z$. Rewrite the system of equations by substituting u, v, and w.

$$\begin{cases} 4 \cdot 3^x - 5 \cdot 5^y + 7^z = -10 \\ 3^x + 4 \cdot 5^y - 3 \cdot 7^z = 14 \\ 3 \cdot 3^x - 3 \cdot 5^y + 5 \cdot 7^z = 9 \end{cases} \xrightarrow{u = 3^x,\ v = 5^y,\ \text{and } w = 7^z} \begin{cases} 4u - 5v + w = -10 \\ u + 4v - 3w = 14 \\ 3u - 3v + 5w = 9 \end{cases}$$

Write an augmented matrix for the given system of equations.

$$\begin{cases} 4u - 5v + w = -10 \\ u + 4v - 3w = 14 \\ 3u - 3v + 5w = 9 \end{cases} \qquad \text{Augmented matrix A} = \left[\begin{array}{rrr|r} 4 & -5 & 1 & -10 \\ 1 & 4 & -3 & 14 \\ 3 & -3 & 5 & 9 \end{array}\right]$$

To get 1 at (1,1) position in the augmented matrix, use the row operation $\frac{1}{4}$R1.

Perform the row operation $\frac{1}{4}$R1.

$$\left[\begin{array}{rrr|r} 4 & -5 & 1 & -10 \\ 1 & 4 & -3 & 14 \\ 3 & -3 & 5 & 9 \end{array}\right] \xrightarrow{\frac{1}{4}R1} \left[\begin{array}{rrr|r} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 1 & 4 & -3 & 14 \\ 3 & -3 & 5 & 9 \end{array}\right]$$

To get 0 at (2,1) position in the resulting matrix, use the row operation $-1 \cdot R1 + R2 \rightarrow R2$.

Perform the row operation $-1 \cdot R1 + R2 \rightarrow R2$.

$$\left[\begin{array}{rrr|r} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 1 & 4 & -3 & 14 \\ 3 & -3 & 5 & 9 \end{array}\right] \xrightarrow{-1 \cdot R1 + R2 \rightarrow R2} \left[\begin{array}{rrr|r} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & \frac{21}{4} & -\frac{13}{4} & \frac{33}{2} \\ 3 & -3 & 5 & 9 \end{array}\right]$$

To get 0 at (3,1) position in the resulting matrix, use the row operation $-3 \cdot R1 + R3 \rightarrow R3$.

Perform the row operation $-3 \cdot R1 + R3 \rightarrow R3$.

$$\left[\begin{array}{rrr|r} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & \frac{21}{4} & -\frac{13}{4} & \frac{33}{2} \\ 3 & -3 & 5 & 9 \end{array}\right] \xrightarrow{-3 \cdot R1 + R3 \rightarrow R3} \left[\begin{array}{rrr|r} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & \frac{21}{4} & -\frac{13}{4} & \frac{33}{2} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array}\right]$$

To get 1 at (2,2) position in the resulting matrix, use the row operation $\frac{4}{21} \cdot R2$.

Perform the row operation $\frac{4}{21} \cdot R2$.

$$\left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & \frac{21}{4} & -\frac{13}{4} & \frac{33}{2} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array}\right] \xrightarrow{\frac{4}{21} \cdot R2} \left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array}\right]$$

To get 0 at (1,2) position in the resulting matrix, use the row operation $\frac{5}{4} \cdot R2 + R1 \rightarrow R1$.

Perform the row operation $\frac{5}{4} \cdot R2 + R1 \rightarrow R1$.

$$\left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array}\right] \xrightarrow{\frac{5}{4} \cdot R2 + R1 \rightarrow R1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array}\right]$$

To get 0 at (3,2) position in the resulting matrix, use the row operation $-\frac{3}{4} \cdot R2 + R3 \rightarrow R3$.

Perform the row operation $-\frac{3}{4} \cdot R2 + R3 \rightarrow R3$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array}\right] \xrightarrow{-\frac{3}{4} \cdot R2 + R3 \rightarrow R3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & \frac{33}{7} & \frac{99}{7} \end{array}\right]$$

To get 1 at (3,3) position in the resulting matrix, use the row operation $\frac{7}{33}R3$.

Perform the row operation $\frac{7}{33}R3$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & \frac{33}{7} & \frac{99}{7} \end{array}\right] \xrightarrow{\frac{7}{33}R3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & 1 & 3 \end{array}\right]$$

To get 0 at (1,3) position in the resulting matrix, use the row operation $\frac{11}{21} \cdot R3 + R1 \rightarrow R1$.

Perform the row operation $\frac{11}{21} \cdot R3 + R1 \rightarrow R1$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{\frac{11}{21} \cdot R3 + R1 \rightarrow R1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & 1 & 3 \end{array}\right]$$

To get 0 at (2,3) position in the resulting matrix, use the row operation $\frac{13}{21} \cdot R3 + R2 \rightarrow R2$.

Perform the row operation $\frac{13}{21} \cdot R3 + R2 \rightarrow R2$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{\frac{13}{21} \cdot R3 + R2 \to R2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Write the corresponding system of equations for the last augmented matrix $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array}\right]$.

$$\begin{cases} u = 3 \\ v = 5 \\ w = 3 \end{cases}$$

Back substitute 3^x for u, 5^y for v, and 7^z for w in the system.

$$\begin{cases} 3^x = 3 \\ 5^y = 5 \\ 7^z = 3 \end{cases}$$

Solve each equation to evaluate x, y, and z. First solve for x.

$$\begin{aligned} 3^x &= 3 \\ \ln 3^x &= \ln 3 \\ x \ln 3 &= \ln 3 \\ x &= 1 \end{aligned}$$

Now, solve for y.

$$\begin{aligned} 5^y &= 5 \\ \ln 5^y &= \ln 5 \\ y \ln 5 &= \ln 5 \\ y &= 1 \end{aligned}$$

Finally, solve for z.

$$\begin{aligned} 7^z &= 3 \\ \ln 7^z &= \ln 3 \\ z \ln 7 &= \ln 3 \\ z &= \frac{\ln 3}{\ln 7} \end{aligned}$$

Therefore, the solution set is $\left\{\left(1, 1, \frac{\ln 3}{\ln 7}\right)\right\}$.

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Given the following matrices, state whether, in general, $(A+B)^2 \neq A^2 + 2AB + B^2$ is true or false.

$$A = \begin{bmatrix} 6 & 3 \\ 7 & 2 \end{bmatrix} \quad B = \begin{bmatrix} -8 & 0 \\ 1 & -6 \end{bmatrix}$$

To check whether $(A+B)^2 \neq A^2 + 2AB + B^2$ is a true statement, first calculate $(A+B)^2$. Begin by finding A + B.

Since A and B have the same order, the addition A + B is defined.

Use the definition of matrix addition to complete the matrix sum below.

$$\begin{aligned} A + B &= \begin{bmatrix} 6 & 3 \\ 7 & 2 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ 1 & -6 \end{bmatrix} \\ &= \begin{bmatrix} 6+(-8) & 3+(0) \\ 7+(1) & 2+(-6) \end{bmatrix} \end{aligned}$$

Simplify the matrix sum.

$$A + B = \begin{bmatrix} -2 & 3 \\ 8 & -4 \end{bmatrix}$$

Finding $(A+B)^2$ is the same as finding the product $(A+B)(A+B)$. First check whether the product $(A+B)(A+B)$ is defined.

In order to define the product RS of two matrices R and S, the number of columns of R must be equal to the number of rows of S. If R is an $m \times p$ matrix and S is a $p \times n$ matrix, then the product RS is an $m \times n$ matrix.

Recall that we already calculated that $A + B = \begin{bmatrix} -2 & 3 \\ 8 & -4 \end{bmatrix}$. Since the number of columns in A + B, 2, is equal to the number of rows in A + B, 2, the product $(A+B)^2 = (A+B)(A+B)$ is defined.

Therefore, the order of $(A+B)^2 = (A+B)(A+B)$ is 2×2.

Let $R = \left[r_{ij}\right]$ be an $m \times p$ matrix and $S = \left[s_{ij}\right]$ be a $p \times n$ matrix. Then the product RS is the $m \times n$ matrix $T = \left[t_{ij}\right]$, where the entry t_{ij} of T is obtained by matrix multiplying the ith row of R by the jth column of S. The definition of the product RS says that $t_{ij} = r_{i1}s_{1j} + r_{i2}s_{2j} + \cdots + r_{ip}s_{pj}$.

To begin computing the matrix product, multiply the first row of A + B by the first column of A + B.

$$\begin{aligned} (A+B)^2 &= \begin{bmatrix} -2 & 3 \\ 8 & -4 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 8 & -4 \end{bmatrix} \\ &= \begin{bmatrix} (-2)(-2)+(3)(8) & * \\ * & * \end{bmatrix} \end{aligned}$$

Next, multiply the first row of A + B by the second column of A + B.

$$\begin{aligned} (A+B)^2 &= \begin{bmatrix} -2 & 3 \\ 8 & -4 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 8 & -4 \end{bmatrix} \\ &= \begin{bmatrix} (-2)(-2)+(3)(8) & (-2)(3)+(3)(-4) \\ * & * \end{bmatrix} \end{aligned}$$

Note that the remaining entries of the product matrix have been completed.

$$(A+B)^2 = \begin{bmatrix} (-2)(-2)+(3)(8) & (-2)(3)+(3)(-4) \\ (8)(-2)+(-4)(8) & (8)(3)+(-4)(-4) \end{bmatrix}$$

Simplify the entries of the matrix.

$$(A+B)^2 = \begin{bmatrix} 28 & -18 \\ -48 & 40 \end{bmatrix}$$

Now to find $A^2 + 2AB + B^2$, find each of the matrix A^2, 2AB, and B^2, if possible. Begin by finding A^2. Finding A^2 is the same as finding the product AA. First check whether the product AA is defined.

Recall that in order to define the product RS of two matrices R and S, the number of columns of R must be equal to the number of rows of S. Since the number of columns in A, 2, is equal to the number of rows in A, 2, the product $A^2 = AA$ is defined.

Since the number of columns in B, 2, is equal to the number of rows in B, 2, the product $B^2 = BB$ is defined.

Recall that, if $R = \left[r_{ij}\right]$ is an $m \times p$ matrix and $S = \left[s_{ij}\right]$ is a $p \times n$ matrix, then the product RS is the $m \times n$ matrix $T = \left[t_{ij}\right]$, where the entry t_{ij} of T is obtained by matrix multiplying the ith row of R by the jth column of S. The definition of the product RS says that $t_{ij} = r_{i1}s_{1j} + r_{i2}s_{2j} + \cdots + r_{ip}s_{pj}$.

To begin computing the matrix product, multiply the first row of A by the first column of A.

$$\begin{aligned} A^2 &= \begin{bmatrix} 6 & 3 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ 7 & 2 \end{bmatrix} \\ &= \begin{bmatrix} (6)(6)+(3)(7) & * \\ * & * \end{bmatrix} \end{aligned}$$

Next, multiply the first row of A by the second column of A.

$$\begin{aligned} A^2 &= \begin{bmatrix} 6 & 3 \\ 7 & 2 \end{bmatrix}\begin{bmatrix} 6 & 3 \\ 7 & 2 \end{bmatrix} \\ &= \begin{bmatrix} (6)(6)+(3)(7) & (6)(3)+(3)(2) \\ * & * \end{bmatrix} \end{aligned}$$

Note that the remaining entries of the product matrix have been completed.

$$A^2 = \begin{bmatrix} (6)(6)+(3)(7) & (6)(3)+(3)(2) \\ (7)(6)+(2)(7) & (7)(3)+(2)(2) \end{bmatrix}$$

Simplify the entries of the matrix.

$$A^2 = \begin{bmatrix} 57 & 24 \\ 56 & 25 \end{bmatrix}$$

Now find the product $B^2 = BB$. To begin computing the matrix product, multiply the first row of B by the first column of B.

$$\begin{aligned} B^2 &= \begin{bmatrix} -8 & 0 \\ 1 & -6 \end{bmatrix}\begin{bmatrix} -8 & 0 \\ 1 & -6 \end{bmatrix} \\ &= \begin{bmatrix} (-8)(-8)+(0)(1) & * \\ * & * \end{bmatrix} \end{aligned}$$

Next, multiply the first row of B by the second column of B.

$$\begin{aligned} B^2 &= \begin{bmatrix} -8 & 0 \\ 1 & -6 \end{bmatrix}\begin{bmatrix} -8 & 0 \\ 1 & -6 \end{bmatrix} \\ &= \begin{bmatrix} (-8)(-8)+(0)(1) & (-8)(0)+(0)(-6) \\ * & * \end{bmatrix} \end{aligned}$$

Note that the remaining entries of the product matrix have been completed.

$$B^2 = \begin{bmatrix} (-8)(-8)+(0)(1) & (-8)(0)+(0)(-6) \\ (1)(-8)+(-6)(1) & (1)(0)+(-6)(-6) \end{bmatrix}$$

Simplify the entries of the matrix.

$$B^2 = \begin{bmatrix} 64 & 0 \\ -14 & 36 \end{bmatrix}$$

Next step is to find 2AB. First check whether the product AB is defined. Since the number of columns in A, 2, is equal to the number of rows in B, 2, the product AB is defined.

Now find the product AB. To begin computing the matrix product, multiply the first row of A by the first column of B.

$$AB = \begin{bmatrix} 6 & 3 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} -8 & 0 \\ 1 & -6 \end{bmatrix}$$
$$= \begin{bmatrix} (6)(-8)+(3)(1) & * \\ * & * \end{bmatrix}$$

Next, multiply the first row of AB by the second column of AB.

$$AB = \begin{bmatrix} 6 & 3 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} -8 & 0 \\ 1 & -6 \end{bmatrix}$$
$$= \begin{bmatrix} (6)(-8)+(3)(1) & (6)(0)+(3)(-6) \\ * & * \end{bmatrix}$$

Note that the remaining entries of the product matrix have been completed.

$$AB = \begin{bmatrix} (6)(-8)+(3)(1) & (6)(0)+(3)(-6) \\ (7)(-8)+(2)(1) & (7)(0)+(2)(-6) \end{bmatrix}$$

Simplify the entries of the matrix.

$$AB = \begin{bmatrix} -45 & -18 \\ -54 & -12 \end{bmatrix}$$

Let $R = \left[r_{ij}\right]$ be an $m \times n$ matrix, and let c be a real number. Then the scalar product of R and c is denoted by cR and is defined by the following.

$$cR = \left[cr_{ij}\right]$$

Find 2AB. Multiply each element of the matrix by 2.

$$2AB = \begin{bmatrix} -45 & -18 \\ -54 & -12 \end{bmatrix}$$
$$= \begin{bmatrix} -90 & -36 \\ -108 & -24 \end{bmatrix}$$

Recall that $A^2 = \begin{bmatrix} 57 & 24 \\ 56 & 25 \end{bmatrix}$, $B^2 = \begin{bmatrix} 64 & 0 \\ -14 & 36 \end{bmatrix}$, and $2AB = \begin{bmatrix} -90 & -36 \\ -108 & -24 \end{bmatrix}$. To check whether $(A+B)^2 \neq A^2 + 2AB + B^2$ is a true statement, first find $A^2 + 2AB + B^2$, if possible. Since A^2, B^2, and 2AB have the same order, the addition $A^2 + 2AB + B^2$ is defined.

Find $A^2 + 2AB + B^2$. First find the addition $A^2 + 2AB$.

$$A^2 + 2AB + B^2 = \begin{bmatrix} 57 & 24 \\ 56 & 25 \end{bmatrix} + \begin{bmatrix} -90 & -36 \\ -108 & -24 \end{bmatrix} + \begin{bmatrix} 64 & 0 \\ -14 & 36 \end{bmatrix}$$
$$= \begin{bmatrix} -33 & -12 \\ -52 & 1 \end{bmatrix} + \begin{bmatrix} 64 & 0 \\ -14 & 36 \end{bmatrix}$$
$$= \begin{bmatrix} 31 & -12 \\ -66 & 37 \end{bmatrix} \qquad \text{Add.}$$

Two matrices R and S are equal if they have the same order and each entry of matrix R is equal to the corresponding entry of S.

Recall that $(A+B)^2 = \begin{bmatrix} 28 & -18 \\ -48 & 40 \end{bmatrix}$ and $A^2 + 2AB + B^2 = \begin{bmatrix} 31 & -12 \\ -66 & 37 \end{bmatrix}$. Since the corresponding entries of two matrices $(A+B)^2$ and $A^2 + 2AB + B^2$ are not the same, $(A+B)^2 \neq A^2 + 2AB + B^2$ is a true statement.

Student: Cole Lamers **Date:** 06/29/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 9.3 Matrix Inverse

An investor inherited \$80,000, and she split it into three investments. Part of the money she invested in a treasury bill that yields 2% annual interest, part she invested in bonds with an annual yield of 5%, and the rest she invested in a mutual fund. In 2008, when the mutual fund lost 13%, her net income from all three investments was \$1090. In 2009, when the mutual fund gained 10%, her net income from all three investments was \$4080. How much money did the investor put into each investment?

Let x represent the amount invested in the treasury bill, y represent the amount invested in bonds, and z represent the amount invested in the mutual fund.

The investor inherited \$80,000 and invested it in the treasury bill, bonds, and the mutual fund. Equate the total amount invested, the sum of x, y, and z, to 80,000.

$$x + y + z = 80{,}000 \qquad (1)$$

In 2008, she received 2% annual interest on the amount invested in the treasury bill and 5% annual interest on the amount invested in bonds. She lost 13% of the amount invested in the mutual fund. Write the expressions for income received from each type of investment.

Type of investment	Treasury bill	Bonds	Mutual fund
Income received in 2008	0.02x	0.05y	− 0.13z

Equate the sum of the expressions of incomes received from each type of investment in 2008 to the net income 1090. Multiply both sides by 100, so that the coefficients of the variables are integers.

$$0.02x + 0.05y - 0.13z = 1090$$
$$2x + 5y - 13z = 109{,}000 \qquad (2)$$

In 2009, she received 2% annual interest on the amount invested in the treasury bill and 5% annual interest on the amount invested in bonds. She gained 10% of the amount invested in the mutual fund. Write expressions for the income received from each type of investment.

Type of investment	Treasury bill	Bonds	Mutual fund
Income received in 2009	0.02x	0.05y	0.1z

Equate the sum of the expressions of incomes received from each type of investment in 2009 to the net income 4080. Multiply both sides by 100, so that the coefficients of the variables are integers.

$$0.02x + 0.05y + 0.1z = 4080$$
$$2x + 5y + 10z = 408{,}000 \qquad (3)$$

To find the amount invested in each type of investment, solve the following system of equations.

$$\begin{cases} x + y + z = 80{,}000 & (1) \\ 2x + 5y - 13z = 109{,}000 & (2) \\ 2x + 5y + 10z = 408{,}000 & (3) \end{cases}$$

Solving the above system of linear equations amounts to solving the following matrix equation of the form AX = B.

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 5 & -13 \\ 2 & 5 & 10 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 80{,}000 \\ 109{,}000 \\ 408{,}000 \end{bmatrix}$$

If A is a square matrix and A is invertible, then the matrix equation AX = B has a unique solution, $X = A^{-1}B$.

To find A^{-1}, start with the matrix $\left[\, A \,\middle|\, I \,\right]$.

$$\left[\, A \,\middle|\, I \,\right] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 5 & -13 & 0 & 1 & 0 \\ 2 & 5 & 10 & 0 & 0 & 1 \end{array}\right]$$

Apply a sequence of row operations that transforms A into I. First, add − 2 times the first row to the second row and add − 2 times the first row to the third row.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 5 & -13 & 0 & 1 & 0 \\ 2 & 5 & 10 & 0 & 0 & 1 \end{array}\right] \xrightarrow[-2R_1 + R_2 \to R_3]{-2R_1 + R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & -15 & -2 & 1 & 0 \\ 0 & 3 & 8 & -2 & 0 & 1 \end{array}\right]$$

Multiply the second row by $\frac{1}{3}$.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 3 & -15 & -2 & 1 & 0 \\ 0 & 3 & 8 & -2 & 0 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -5 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 3 & 8 & -2 & 0 & 1 \end{array}\right]$$

Add -3 times the second row to the third row.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -5 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 3 & 8 & -2 & 0 & 1 \end{array}\right] \xrightarrow{-3R_2 + R_3 \to R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -5 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 23 & 0 & -1 & 1 \end{array}\right]$$

Now multiply the third row by $\frac{1}{23}$.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -5 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 23 & 0 & -1 & 1 \end{array}\right] \xrightarrow{\frac{1}{23}R_3} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -5 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{23} & \frac{1}{23} \end{array}\right]$$

Add 5 times the third row to the second row and add -1 times the third row to the first row.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -5 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 1 & 0 & -\frac{1}{23} & \frac{1}{23} \end{array}\right] \xrightarrow[-R_3 + R_1 \to R_1]{5R_3 + R_2 \to R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & \frac{1}{23} & -\frac{1}{23} \\ 0 & 1 & 0 & -\frac{2}{3} & \frac{8}{69} & \frac{5}{23} \\ 0 & 0 & 1 & 0 & -\frac{1}{23} & \frac{1}{23} \end{array}\right]$$

Finally, add -1 times the second row to the third row.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & \frac{1}{23} & -\frac{1}{23} \\ 0 & 1 & 0 & -\frac{2}{3} & \frac{8}{69} & \frac{5}{23} \\ 0 & 0 & 1 & 0 & -\frac{1}{23} & \frac{1}{23} \end{array}\right] \xrightarrow{-R_2 + R_1 \to R_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & \frac{5}{3} & -\frac{5}{69} & -\frac{6}{23} \\ 0 & 1 & 0 & -\frac{2}{3} & \frac{8}{69} & \frac{5}{23} \\ 0 & 0 & 1 & 0 & -\frac{1}{23} & \frac{1}{23} \end{array}\right]$$

The matrix on the right side of the augmented matrix is the inverse matrix A^{-1}.

$$A^{-1} = \begin{bmatrix} \frac{5}{3} & -\frac{5}{69} & -\frac{6}{23} \\ -\frac{2}{3} & \frac{8}{69} & \frac{5}{23} \\ 0 & -\frac{1}{23} & \frac{1}{23} \end{bmatrix}$$

Rewrite the inverse using the least common denominator of all the entries, so that the entries in the matrix are integers. This facilitates the matrix multiplication used in next step.

$$A^{-1} = \frac{1}{69}\begin{bmatrix} 115 & -5 & -18 \\ -46 & 8 & 15 \\ 0 & -3 & 3 \end{bmatrix}$$

The solution of the system of equations is $A^{-1}B$. Find the product $A^{-1}B$.

$$X = A^{-1}B$$

$$= \frac{1}{69}\begin{bmatrix} 115 & -5 & -18 \\ -46 & 8 & 15 \\ 0 & -3 & 3 \end{bmatrix}\begin{bmatrix} 80{,}000 \\ 109{,}000 \\ 408{,}000 \end{bmatrix} \qquad \text{Substitute for } A^{-1} \text{ and } B.$$

$$X = \frac{1}{69}\begin{bmatrix} 115(80{,}000) - 5(109{,}000) - 18(408{,}000) \\ -46(80{,}000) + 8(109{,}000) + 15(408{,}000) \\ -3(109{,}000) + 3(408{,}000) \end{bmatrix} \quad \text{Multiply.}$$

$$= \frac{1}{69}\begin{bmatrix} 1311000 \\ 3312000 \\ 3312000 \end{bmatrix}$$

Finally, multiply each element of the matrix by $\frac{1}{69}$.

$$X = \frac{1}{69}\begin{bmatrix} 1311000 \\ 3312000 \\ 3312000 \end{bmatrix}$$

$$= \begin{bmatrix} 19{,}000 \\ 48{,}000 \\ 13{,}000 \end{bmatrix}$$

The values of x, y, and z are as shown below.

$x = 19{,}000$ $y = 48{,}000$ $z = 13{,}000$

Check the solution by substituting the answers into the original equations to verify that true statements result.

Check for equation (1).

$$x + y + z = 80{,}000 \qquad (1)$$
$$19{,}000 + 13{,}000 + 48{,}000 \stackrel{?}{=} 80{,}000$$
$$80{,}000 = 80{,}000 \quad \checkmark$$

Check for equation (2).

$$2x + 5y - 13z = 109{,}000 \qquad (2)$$
$$2(19{,}000) + 5(13{,}000) - 13(48{,}000) \stackrel{?}{=} 109{,}000$$
$$109{,}000 = 109{,}000 \quad \checkmark$$

Check for equation (3).

$$2x + 5y + 10z = 408{,}000 \qquad (3)$$
$$2(19{,}000) + 5(13{,}000) + 10(48{,}000) \stackrel{?}{=} 408{,}000$$
$$408{,}000 = 408{,}000 \quad \checkmark$$

Therefore, the investor invested \$19,000 in the treasury bill, \$48,000 in bonds, and \$13,000 in the mutual fund.

Student: Cole Lamers Date: 06/29/19	Instructor: Kelly Galarneau Course: CA&T Internet (70263) Galarneau	Assignment: 9.3 Matrix Inverse

Determine whether B is the inverse of A by computing AB and BA.

$$A = \begin{bmatrix} -2 & 1 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}, B = \frac{1}{9}\begin{bmatrix} -2 & 8 & 5 \\ 1 & -4 & 2 \\ 1 & 5 & 2 \end{bmatrix}$$

Let A be an $n \times n$ matrix and let I be the $n \times n$ identity matrix. If there is an $n \times n$ matrix B such that AB = I and BA = I, then B is called the inverse of A.

To determine whether B is the inverse of A, begin by computing AB. Note that the matrix B has a scalar $\frac{1}{9}$. Use the associative property of scalar multiplication.

Let A and B be matrices and let c be a scalar. Assume that the product is defined. Then the associative property of scalar multiplication is given below.

$$c(AB) = (cA)B = A(cB)$$

Apply the associative property of scalar multiplication to the matrices A and B.

$$AB = \begin{bmatrix} -2 & 1 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}\left(\frac{1}{9}\begin{bmatrix} -2 & 8 & 5 \\ 1 & -4 & 2 \\ 1 & 5 & 2 \end{bmatrix}\right)$$

$$= \frac{1}{9}\begin{bmatrix} -2 & 1 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} -2 & 8 & 5 \\ 1 & -4 & 2 \\ 1 & 5 & 2 \end{bmatrix}$$

Recall the matrix multiplication. Let $A = [a_{ij}]$ be an $m \times p$ matrix and $B = [b_{ij}]$ be a $p \times n$ matrix. Then the product AB is the $m \times n$ matrix $C = [c_{ij}]$, where the entry c_{ij} of C is obtained by matrix multiplying the i^{th} row of A by the j^{th} column of B.

The definition of product AB is given below.

$$c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \ldots + a_{ip}b_{pj}$$

The matrix AB which is a product of matrix A and matrix B will be a 3×3 matrix.

Multiply the matrices.

$$AB = \frac{1}{9}\begin{bmatrix} -2 & 1 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}\begin{bmatrix} -2 & 8 & 5 \\ 1 & -4 & 2 \\ 1 & 5 & 2 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} (-2)(-2)+(1)(1)+(4)(1) & (-2)(8)+(1)(-4)+(4)(5) & (-2)(5)+(1)(2)+(4)(2) \\ & & \\ & & \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} (-2)(-2)+(1)(1)+(4)(1) & (-2)(8)+(1)(-4)+(4)(5) & (-2)(5)+(1)(2)+(4)(2) \\ (0)(-2)+(-1)(1)+(1)(1) & (0)(8)+(-1)(-4)+(1)(5) & (0)(5)+(-1)(2)+(1)(2) \\ & & \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} (-2)(-2)+(1)(1)+(4)(1) & (-2)(8)+(1)(-4)+(4)(5) & (-2)(5)+(1)(2)+(4)(2) \\ (0)(-2)+(-1)(1)+(1)(1) & (0)(8)+(-1)(-4)+(1)(5) & (0)(5)+(-1)(2)+(1)(2) \\ (1)(-2)+(2)(1)+(0)(1) & (1)(8)+(2)(-4)+(0)(5) & (1)(5)+(2)(2)+(0)(2) \end{bmatrix}$$

Simplify.

$$AB = \frac{1}{9}\begin{bmatrix} -2(-2)+1(1)+4(1) & -2(8)+1(-4)+4(5) & -2(5)+1(2)+4(2) \\ 0(-2)-1(1)+1(1) & 0(8)-1(-4)+1(5) & 0(5)-1(2)+1(2) \\ 1(-2)+2(1)+0(1) & 1(8)+2(-4)+0(5) & 1(5)+2(2)+0(2) \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Hence, $AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Now compute BA. Note that the matrix B has a scalar $\frac{1}{9}$. Apply the associative property of scalar multiplication to the matrices A and B.

$$BA = \left(\frac{1}{9}\begin{bmatrix} -2 & 8 & 5 \\ 1 & -4 & 2 \\ 1 & 5 & 2 \end{bmatrix}\right)\begin{bmatrix} -2 & 1 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} -2 & 8 & 5 \\ 1 & -4 & 2 \\ 1 & 5 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

Multiply the matrices.

$$BA = \frac{1}{9}\begin{bmatrix} -2 & 8 & 5 \\ 1 & -4 & 2 \\ 1 & 5 & 2 \end{bmatrix}\begin{bmatrix} -2 & 1 & 4 \\ 0 & -1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} (-2)(-2)+(8)(0)+(5)(1) & (-2)(1)+(8)(-1)+(5)(2) & (-2)(4)+(8)(1)+(5)(0) \\ & & \\ & & \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} (-2)(-2)+(8)(0)+(5)(1) & (-2)(1)+(8)(-1)+(5)(2) & (-2)(4)+(8)(1)+(5)(0) \\ (1)(-2)+(-4)(0)+(2)(1) & (1)(1)+(-4)(-1)+(2)(2) & (1)(4)+(-4)(1)+(2)(0) \\ & & \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} (-2)(-2)+(8)(0)+(5)(1) & (-2)(1)+(8)(-1)+(5)(2) & (-2)(4)+(8)(1)+(5)(0) \\ (1)(-2)+(-4)(0)+(2)(1) & (1)(1)+(-4)(-1)+(2)(2) & (1)(4)+(-4)(1)+(2)(0) \\ (1)(-2)+(5)(0)+(2)(1) & (1)(1)+(5)(-1)+(2)(2) & (1)(4)+(5)(1)+(2)(0) \end{bmatrix}$$

Simplify.

$$BA = \frac{1}{9}\begin{bmatrix} (-2)(-2)+(8)(0)+(5)(1) & (-2)(1)+(8)(-1)+(5)(2) & (-2)(4)+(8)(1)+(5)(0) \\ (1)(-2)+(-4)(0)+(2)(1) & (1)(1)+(-4)(-1)+(2)(2) & (1)(4)+(-4)(1)+(2)(0) \\ (1)(-2)+(5)(0)+(2)(1) & (1)(1)+(5)(-1)+(2)(2) & (1)(4)+(5)(1)+(2)(0) \end{bmatrix}$$
$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Hence, $BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Therefore, B is the inverse of A since AB = I and BA = I, where I is an identity matrix.

Student: Cole Lamers **Date:** 06/29/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 9.3 Matrix Inverse

Find the inverse of the following matrix A, if possible. Check that $AA^{-1} = I$.

$$A = \begin{bmatrix} 3 & -2 \\ -6 & 4 \end{bmatrix}$$

The first step in finding the inverse of matrix A, A^{-1}, is to set up the augmented matrix [A | I], where I is the identity matrix of the same order as A.

The elements of the augmented matrix [A | I] are $\left[\begin{array}{cc|cc} 3 & -2 & 1 & 0 \\ -6 & 4 & 0 & 1 \end{array}\right]$.

You must use row operations to convert the left side of the augmented matrix to the identity matrix. To obtain a 1 in the first row of the first column multiply the first row by $\frac{1}{3}$.

The augmented matrix becomes $\left[\begin{array}{cc|cc} 1 & -\frac{2}{3} & \frac{1}{3} & 0 \\ -6 & 4 & 0 & 1 \end{array}\right]$.

In order to get a 0 in the second row of the first column you must multiply the first row by 6 and add it to the second row.

The resulting matrix is $\left[\begin{array}{cc|cc} 1 & -\frac{2}{3} & \frac{1}{3} & 0 \\ 0 & 0 & 2 & 1 \end{array}\right]$.

You've taken legitimate steps, but now both elements on the left side of the second row are 0. This means that there is no inverse for this matrix.