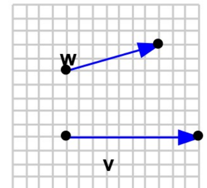
Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.4 Vectors

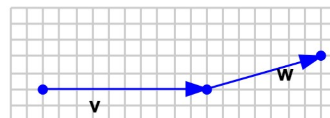
Use the vectors in the figure at the right to graph the following vector.

v + **w**



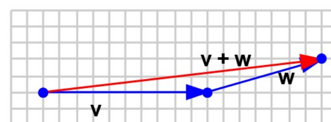
First, position the vectors **v** and **w** so that the terminal point of **v** coincides with the initial point of **w**.

The arrow head on a vector points towards the terminal point of that vector. Thus, the arrow head on **v** points at the initial point of **w**, as shown in the graph on the right.



Now graph the sum **v** + **w**. The vector **v** + **w** is the unique vector whose initial point coincides with the initial point of **v** and whose terminal point coincides with the terminal point of **w**.

The graph of **v** + **w** is shown on the right.



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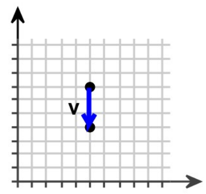
Use the vector in the figure at the right to graph the following vector.

2**v**



If α is a scalar and **v** is a vector, the scalar product of α**v** is defined as follows:

1. If $\alpha > 0$, the product α**v** is the vector whose magnitude is α times the magnitude of **v** and whose direction is the same as **v**.

2. If $\alpha < 0$, the product α**v** is the vector whose magnitude is $|\alpha|$ times the magnitude of **v** and whose direction is opposite that of **v**.

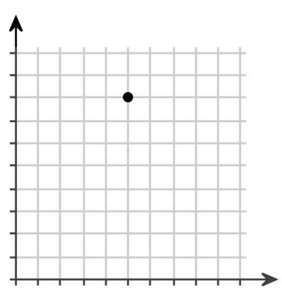
3. If $\alpha = 0$ or **v** = $\vec{0}$, then α**v** = $\vec{0}$.

In this case, α is positive so the vector 2**v** is the vector whose magnitude is 2 times the magnitude and whose direction is the same as our original vector.

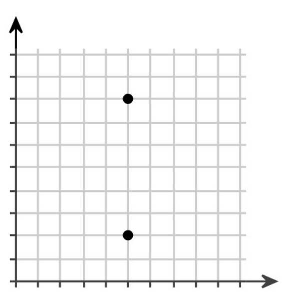
Looking at our original vector **v**, we observe that from its initial point, the vector moves 3 units down. Our new vector must move 2 times as much as the original.

Thus, the new vector moves $2 \cdot 3 = 6$ units down.

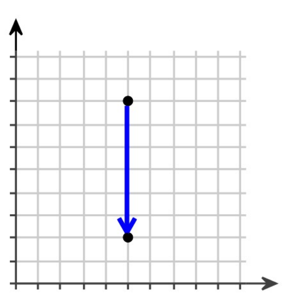
We begin plotting our new vector by placing the initial point on the grid so that there is room to plot a point 6 units down from the initial point.



Next, we plot our terminal point by placing it 6 units down from the initial point.



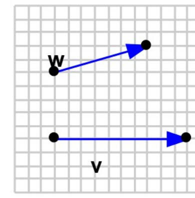
Finally, we connect the initial point to the terminal point with a line segment and add an arrowhead.



Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.4 Vectors

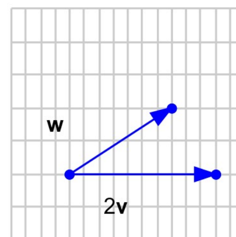
Use the vectors in the figure at the right to graph the following vector.

2**v** − **w**



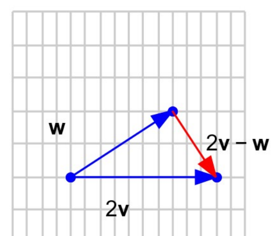
First, position the vectors 2**v** and **w** so that the initial point of 2**v** coincides with the initial point of **w**.

The graph is shown on the right.



Now graph the difference 2**v** − **w**. The vector 2**v** − **w** is the unique vector whose initial point coincides with the terminal point of **w** and whose terminal point coincides with the terminal point of 2**v**.

The graph of 2**v** − **w** is shown on the right.



Student: Cole Lamers **Date:** 07/27/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.4 Vectors

The vector **v** has initial point P and terminal point Q. Write **v** as a position vector.

P(4,7), Q(1,8)

The vector $\overrightarrow{PQ}$ with initial point $P(x_1,y_1)$ and terminal point $Q(x_2,y_2)$ is equal to the position vector shown below.

$\mathbf{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$

The initial point is P(4,7). Identify x_1 and y_1 from this point.

$x_1 = 4$
$y_1 = 7$

The terminal point is Q(1,8). Identify x_2 and y_2 from the terminal point.

$x_2 = 1$
$y_2 = 8$

Substitute the values for x_1, y_1, x_2, and y_2 into the position vector representation for $\overrightarrow{PQ}$.

$$\begin{aligned} \mathbf{v} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 1 - 4, 8 - 7 \rangle \end{aligned}$$

Subtract to find the first component of the position vector.

$$\begin{aligned} \mathbf{v} &= \langle 1 - 4, 8 - 7 \rangle \\ &= \langle -3, 8 - 7 \rangle \end{aligned}$$

Then subtract to find the second component of the position vector.

$$\begin{aligned} \mathbf{v} &= \langle -3, 8 - 7 \rangle \\ &= \langle -3, 1 \rangle \end{aligned}$$

Therefore, the position vector is $\mathbf{v} = \langle -3, 1 \rangle$.

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For the points A(5,6), B(9,8), C(6,7), and D(10,9), determine whether the vectors $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equivalent. [Hint: Write $\overrightarrow{AB}$ and $\overrightarrow{CD}$ as position vectors.]

To determine whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are equivalent, check whether their position vectors are equal.

The vector $\overrightarrow{PQ}$ with initial point $P(x_1,y_1)$ and terminal point $Q(x_2,y_2)$ is equal to the position vector $\mathbf{w} = \langle x_2 - x_1, y_2 - y_1 \rangle$.

In the notation $\overrightarrow{AB}$, A is the initial point and B is the terminal point.

The vector $\overrightarrow{AB}$ has the initial point (5,6) and the terminal point (9,8).

Represent $\overrightarrow{AB}$ as a position vector **v**.

$$\begin{aligned} \mathbf{v} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 9-5, 8-6 \rangle && \text{Substitute values.} \\ &= \langle 4,2 \rangle && \text{Simplify.} \end{aligned}$$

Now determine the position vector for $\overrightarrow{CD}$.

The vector $\overrightarrow{CD}$ has the initial point (6,7) and the terminal point (10,9).

Represent $\overrightarrow{CD}$ as a position vector **w**.

$$\begin{aligned} \mathbf{w} &= \langle x_2 - x_1, y_2 - y_1 \rangle \\ &= \langle 10-6, 9-7 \rangle && \text{Substitute values.} \\ &= \langle 4,2 \rangle && \text{Simplify.} \end{aligned}$$

Since the position vector of $\overrightarrow{AB}$, $\langle 4,2 \rangle$ is equal to the position vector of $\overrightarrow{CD}$, $\langle 4,2 \rangle$ the two vectors are equivalent.

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Let $\mathbf{v} = \langle -4,8\rangle$. Find $\|\mathbf{v}\|$.

The magnitude of a position vector $\mathbf{v} = \langle v_1, v_2\rangle$ is $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$.

Substitute the values of v_1 and v_2 into the formula for $\|\mathbf{v}\|$.

$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{v_1^2 + v_2^2}\\ &= \sqrt{(-4)^2 + (8)^2}\end{aligned}$$

Simplify the radical.

$$\begin{aligned}\|\mathbf{v}\| &= \sqrt{(-4)^2 + (8)^2}\\ &= \sqrt{80}\\ &= 4\sqrt{5}\end{aligned}$$

The magnitude of the position vector $v = \langle -4,8\rangle$ is $||v|| = 4\sqrt{5}$.

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Let $\mathbf{u} = 8\mathbf{i} - 6\mathbf{j}$ and $\mathbf{v} = -5\mathbf{i} + 8\mathbf{j}$. Find the vector $\mathbf{u} + \mathbf{v}$.

If $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$, then $\mathbf{u} + \mathbf{v} = \left(u_1 + v_1\right)\mathbf{i} + \left(u_2 + v_2\right)\mathbf{j}$.

Use the formula $\mathbf{u} + \mathbf{v} = \left(u_1 + v_1\right)\mathbf{i} + \left(u_2 + v_2\right)\mathbf{j}$ to find the vector $\mathbf{u} + \mathbf{v}$ for the given vectors $\mathbf{u} = 8\mathbf{i} - 6\mathbf{j}$ and $\mathbf{v} = -5\mathbf{i} + 8\mathbf{j}$.

$\mathbf{u} + \mathbf{v} = (8\mathbf{i} - 6\mathbf{j}) + (-5\mathbf{i} + 8\mathbf{j})$	These are the given vectors.
$= (8 + (-5))\mathbf{i} + (-6 + 8)\mathbf{j}$	Add the horizontal components and add the vertical components.
$= 3\mathbf{i} + 2\mathbf{j}$	Simplify.

Therefore, $\mathbf{u} + \mathbf{v} = 3\mathbf{i} + 2\mathbf{j}$.

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Let $\mathbf{v} = -3\mathbf{i} + 11\mathbf{j}$ and $\mathbf{w} = -\mathbf{i} - 10\mathbf{j}$. Find $2\mathbf{v} - 3\mathbf{w}$.

Remember if $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ and k is a real number, then the scalar multiplication of the vector **u** and the scalar k is $k\mathbf{u} = \left(ku_1\right)\mathbf{i} + \left(ku_2\right)\mathbf{j}$. Also, if $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$, then $\mathbf{u} - \mathbf{v} = \left(u_1 - v_1\right)\mathbf{i} + \left(u_2 - v_2\right)\mathbf{j}$.

To find $2\mathbf{v} - 3\mathbf{w}$, first perform each scalar multiplication. Then combine the horizontal and the vertical components to complete the vector subtraction.

$2\mathbf{v} - 3\mathbf{w} = 2(-3\mathbf{i} + 11\mathbf{j}) - 3(-\mathbf{i} - 10\mathbf{j})$	Substitute $-3\mathbf{i} + 11\mathbf{j}$ and $-\mathbf{i} - 10\mathbf{j}$ for **v** and **w**.
$= -6\mathbf{i} + 22\mathbf{j} + 3\mathbf{i} + 30\mathbf{j}$	Perform each scalar multiplication.
$= (-6 + 3)\mathbf{i} + (22 + 30)\mathbf{j}$	Combine the horizontal and the vertical components.
$= -3\mathbf{i} + 52\mathbf{j}$	Simplify.

Therefore, $2\mathbf{v} - 3\mathbf{w} = -3\mathbf{i} + 52\mathbf{j}$.

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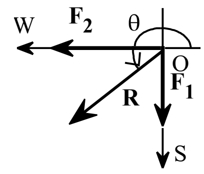
Find the magnitude and bearing of the resultant **R** of two forces $\mathbf{F_1}$ and $\mathbf{F_2}$, where $\mathbf{F_1}$ is a force of 12 pounds acting due south and $\mathbf{F_2}$ is a force of 28 pounds acting due west.

Let θ be the direction angle of **R**. Note that the direction angle differs with the bearing. Direction angle is the angle of the vector with the positive x-axis. A bearing is the measure of an acute angle from due north or due south. That is, either with positive or negative y-axis.

First set up the coordinate system with the negative y-axis pointing southward and the forces $\mathbf{F_1}$ and $\mathbf{F_2}$ shown along with the resultant vector **R**.



Express the vectors $\mathbf{F_1}$ and $\mathbf{F_2}$ in i, j form. Note that $\mathbf{F_1}$ is a force of 12 pounds acting due south (the negative y-axis).

The standard unit vector that lies along the negative y-axis is $-\mathbf{j}$.

Thus, $\mathbf{F_1} = -12\mathbf{j}$.

Note that $\mathbf{F_2}$ is a force of 28 pounds acting due west (the negative x-axis).

The standard unit vector that lies along the negative x-axis is $-\mathbf{i}$.

Thus, $\mathbf{F_2} = -28\mathbf{i}$.

Now add vectors $\mathbf{F_1}$ and $\mathbf{F_2}$ to find the resultant vector **R**. Substitute the values of $\mathbf{F_1}$ and $\mathbf{F_2}$, and simplify.

$$\begin{aligned}\mathbf{R} &= \mathbf{F_1} + \mathbf{F_2}\\ &= -12\mathbf{j} + (-28\mathbf{i})\\ &= -28\mathbf{i} - 12\mathbf{j} && \text{Write the } \mathbf{i}\text{-component first.}\end{aligned}$$

Next find the magnitude of **R**.

The magnitude of the vector $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j}$ is given by $\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2}$.

Find the magnitude of $\mathbf{R} = -28\mathbf{i} - 12\mathbf{j}$, using the definition of magnitude. Round to one decimal place.

$$\begin{aligned}\|\mathbf{R}\| &= \sqrt{(-28)^2 + (-12)^2}\\ \|\mathbf{R}\| &= \sqrt{784 + 144}\\ \|\mathbf{R}\| &\approx 30.5 && \text{Use a calculator to simplify and approximate.}\end{aligned}$$

The magnitude of the resultant **R** is approximately 30.5 pounds.

Now find the bearing of the resultant **R**. Start by finding the reference angle θ' for $\mathbf{R} = -28\mathbf{i} - 12\mathbf{j}$. Note that for the vector $\mathbf{r} = r_1\mathbf{i} + r_2\mathbf{j}$, the reference angle $\theta' = \left|\tan^{-1}\left(\frac{r_2}{r_1}\right)\right|$.

$$\begin{aligned}\theta' &= \left|\tan^{-1}\left(\frac{r_2}{r_1}\right)\right|\\ \theta' &= \left|\tan^{-1}\left(\frac{-12}{-28}\right)\right|\\ \theta' &= \left|\tan^{-1}\left(\frac{12}{28}\right)\right|\\ \theta' &\approx 23.2^\circ && \text{Use a calculator. The answer is rounded to nearest tenth.}\end{aligned}$$

Thus, the reference angle $\theta' = 23.2^\circ$.

Since the terminal point for **R**, $(-28, -12)$, lies in the third quadrant, the direction angle for **R** is, $\theta = 180^\circ + \theta'$.

Determine θ.

$$\begin{aligned}\theta &= 180^\circ + \theta'\\ \theta &= 180^\circ + 23.2^\circ && \text{Substitute } \theta' = 23.2^\circ.\\ \theta &\approx 203.2^\circ && \text{Add.}\end{aligned}$$

Note that bearing is the measure of an acute angle from due north or due south. For **R**, the bearing is with respect to due south.

Thus, the angle between **R** and the negative y-axis (the south direction) is $270^\circ - 203.2^\circ = 66.8^\circ$.

Therefore, **R** is a force of approximately 30.5 pounds with bearing of S 66.8° W.