

Student: Cole Lamers
Date: 09/22/19

Instructor: Viktoriya Shcherban
Course: Calc 1 11:30 AM / Internet
(81749&81750) Shcherban

Assignment: 4.3 Monotonic Functions
and the First Derivative T

1. Answer the following questions about the function whose derivative is $f'(x) = x(x - 5)$.

- a. What are the critical points of f ?
- b. On what open intervals is f increasing or decreasing?
- c. At what points, if any, does f assume local maximum and minimum values?

a. Find the critical points, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☒ A. The critical point(s) of f is/are $x =$.
(Simplify your answer. Use a comma to separate answers as needed.)
- ☐ B. The function f has no critical points.

b. Determine where f is increasing and decreasing. Select the correct choice below and fill in the answer box to complete your choice.

(Type your answer in interval notation. Use a comma to separate answers as needed.)

- ☒ A. The function f is increasing on the open interval(s) , and decreasing on the open interval(s) .
- ☐ B. The function f is decreasing on the open interval(s) , and never increasing.
- ☐ C. The function f is increasing on the open interval(s) , and never decreasing.

c. Determine the local maximum/maxima, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☒ A. The function f has a local maximum at $x =$.
(Simplify your answer. Use a comma to separate answers as needed.)
- ☐ B. There is no local maximum.

Determine the local minimum/minima, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. There is no local minimum.
- ☒ B. The function f has a local minimum at $x =$.
(Simplify your answer. Use a comma to separate answers as needed.)

2. Answer the following questions about the function whose derivative is $f'(x) = (x - 6)^2(x + 8)$.

- What are the critical points of f ?
 - On what open intervals is f increasing or decreasing?
 - At what points, if any, does f assume local maximum and minimum values?
-

a. Find the critical points, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☒ A. The critical point(s) of f is/are $x =$.
(Simplify your answer. Use a comma to separate answers as needed.)
- ☐ B. The function f has no critical points.

b. Determine where f is increasing and decreasing. Select the correct choice below and fill in the answer box to complete your choice.

(Type your answer in interval notation. Use a comma to separate answers as needed.)

- ☒ A. The function is increasing on the open interval(s) , and decreasing on the open interval(s) .
- ☐ B. The function f is decreasing on the open interval(s) , and never increasing.
- ☐ C. The function f is increasing on the open interval(s) , and never decreasing.

c. Determine the local maximum/maxima, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☐ A. $x =$.
(Simplify your answer. Use a comma to separate answers as needed.)
- ☒ B. There is no local maximum.

Determine the local minimum/minima, if any. Select the correct choice below and, if necessary, fill in the answer box to complete your choice.

- ☒ A. $x =$.
(Simplify your answer. Use a comma to separate answers as needed.)
- ☐ B. There is no local minimum.
-

3.

Answer the following questions about the function whose derivative is $f'(x) = \frac{x^2(x-4)}{x+5}$, $x \neq -5$.

- a. What are the critical points of f ?
- b. On what open intervals is f increasing or decreasing?
- c. At what points, if any, does f assume local maximum and minimum values?

(a) What are the critical points of f ? Select the correct choice below and fill in the answer box within your choice.

- ☒ A. $x =$ (Use comma to separate answers as needed)
- ☐ B. The function f has no critical points.

(b) On what open intervals is f increasing?

- ☐ A. $(-\infty - 5)$ and $(-5, 0)$
- ☒ B. $(-\infty - 5)$ and $(4, \infty)$
- ☐ C. $(-5, 0)$ and $(0, 4)$
- ☐ D. The function f is not increasing.

On what open intervals is f decreasing?

- ☐ A. $(-\infty - 5)$ and $(4, \infty)$
- ☒ B. $(-5, 0)$ and $(0, 4)$
- ☐ C. $(0, 4)$ and $(4, \infty)$
- ☐ D. The function is not decreasing.

(c) At what points, if any, does f assume local maximum values?

- ☐ A. $x =$ (Use comma to separate answers as needed.)
- ☒ B. There is no local maximum.

At what points, if any, does f assume local minimum values?

- ☒ A. $x =$ (Use comma to separate answers as needed.)
- ☐ B. There is no local minimum.
-

4. Answer the questions below about the function whose derivative is $f'(x) = \frac{(x-2)(x+8)}{(x+1)(x-3)}$, $x \neq -1, 3$.

- a. What are the critical points of f ?
 b. On what open intervals is f increasing or decreasing?
 c. At what points, if any, does f assume local maximum and minimum values?

a. What are the critical points of f ? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ☒ A. $x =$ (Use comma to separate answers as needed)
☐ B. The function f has no critical points.

b. On what open intervals is f increasing? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ☒ A. The function f is increasing on the interval(s) .
 (Type your answer in interval notation. Use a comma to separate answers as needed.)
☐ B. The function f is not increasing anywhere.

On what open intervals is f decreasing? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ☒ A. The function f is decreasing on the interval(s) .
 (Type your answer in interval notation. Use a comma to separate answers as needed.)
☐ B. The function f is not decreasing anywhere.

c. At what points, if any, does f assume local maximum values? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ☒ A. $x =$ (Use comma to separate answers as needed)
☐ B. There is no local maximum.

At what points, if any, does f assume local minimum values? Select the correct choice below and, if necessary, fill in the answer box within your choice.

- ☐ A. $x =$ (Use comma to separate answers as needed)
☒ B. There is no local minimum.

5. Answer the following questions about the function whose derivative is given below.

(a) What are the critical points of f ?

(b) On what open intervals is f increasing or decreasing?

(c) At what points, if any, does f assume local maximum and minimum values?

$$f'(x) = (\sin x - 1)(2 \cos x + \sqrt{3}), \quad 0 \leq x \leq 2\pi$$

(a) What are the critical points of f ?

$$\frac{7\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$$

(Use a comma to separate answers as needed.)

(b) On what open intervals is f increasing?

- ☐ A. $\left(0, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{5\pi}{6}\right), \left(\frac{7\pi}{6}, 2\pi\right)$
- ☒ B. $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$
- ☐ C. $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$
- ☐ D. There are no intervals where f is increasing.

On what open intervals is f decreasing?

- ☐ A. $\left(\frac{\pi}{2}, \frac{5\pi}{6}\right)$
- ☒ B. $\left(0, \frac{\pi}{2}\right), \left(\frac{\pi}{2}, \frac{5\pi}{6}\right), \left(\frac{7\pi}{6}, 2\pi\right)$
- ☐ C. $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$
- ☐ D. There are no intervals where f is decreasing.

(c) At what points, if any, does f assume local maximum values?

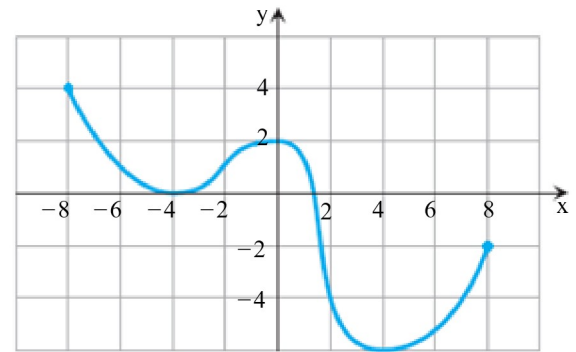
- ☐ A. $x = 0, 2\pi$
- ☐ B. $x = \frac{5\pi}{6}, 2\pi$
- ☒ C. $x = 0, \frac{7\pi}{6}$
- ☐ D. There are no local maxima.

At what points, if any, does f assume local minimum values?

- ☒ A. $x = \frac{5\pi}{6}, 2\pi$
- ☐ B. $x = 0, 2\pi$
- ☐ C. $x = 0, \frac{7\pi}{6}$

6. (a) Find the open intervals on which the function shown in the graph is increasing and decreasing.

(b) Identify the function's local and absolute extreme values, if any, saying where they occur.



(a) On what open interval(s), if any, is the function increasing? Select the correct choice below and fill in any answer boxes within your choice.

- ☒ A. The function is increasing on the open interval(s) $(-4, 0), (4, 8)$.
(Type your answer in interval notation. Use a comma to separate answers as needed.)
- ☐ B. The function is never increasing.

On what open interval(s), if any, is the function decreasing? Select the correct choice below and fill in any answer boxes within your choice.

- ☒ A. The function is decreasing on the open interval(s) $(-8, -4), (0, 4)$.
(Type your answer in interval notation. Use a comma to separate answers as needed.)
- ☐ B. The function is never decreasing.

(b) If the function has an absolute maximum, where does it occur? Select the correct choice below and fill in any answer boxes within your choice.

- ☒ A. An absolute maximum occurs at the point(s) $(-8, 4)$.
(Type an ordered pair. Use a comma to separate answers as needed.)
- ☐ B. The function has no absolute maximum.

If the function has other local maxima, where do they occur? Since a list of local maxima automatically includes the absolute maximum, do not include the absolute maximum in the list of local maxima. Select the correct choice below and fill in any answer boxes within your choice.

- ☒ A. A local maximum occurs at the point(s) $(0, 2), (8, -2)$.
(Type an ordered pair. Use a comma to separate answers as needed.)
- ☐ B. The function has no local maximum that is not an absolute maximum.

If the function has an absolute minimum, where does it occur? Select the correct choice below and fill in any answer boxes within your choice.

- ☒ A. An absolute minimum occurs at the point(s) $(4, -6)$.
(Type an ordered pair. Use a comma to separate answers as needed.)
- ☐ B. The function has no absolute minimum.

If the function has other local minima, where do they occur? Since a list of local minima automatically includes the absolute minimum, do not include the absolute minimum in the list of local minima. Select the correct choice below and fill in any answer boxes within your choice.

- ☒ A. (Type an ordered pair. Use a comma to separate answers as needed.)

7. **a.** Find the open intervals on which the function is increasing and decreasing.
b. Identify the function's local and absolute extreme values, if any, saying where they occur.

$$g(t) = 3t^2 - 5t - 4$$

a. Find the open intervals on which the function is increasing. Select the correct choice below and fill in any answer boxes within your choice.

- ☒ **A.** The function is increasing on $\left(\frac{5}{6}, \infty\right)$.

(Use interval notation. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

- ☐ **B.** There are no intervals on which the function is increasing.

Find the open intervals on which the function is decreasing. Select the correct choice below and fill in any answer boxes within your choice.

- ☒ **A.** The function is decreasing on $\left(-\infty, \frac{5}{6}\right)$.

(Use interval notation. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

- ☐ **B.** There are no intervals on which the function is decreasing.

b. Identify the function's local extreme values, if any.

- ☐ **A.** The function g has no local extrema.
- ☒ **B.** The function g has a local minimum at $t = \frac{5}{6}$.
- ☐ **C.** The function g has a local maximum at $t = \frac{5}{6}$.

Identify the function's absolute extreme values, if any.

- ☒ **A.** The function g has an absolute minimum at $t = \frac{5}{6}$.
- ☐ **B.** The function g has an absolute maximum at $t = \frac{5}{6}$.
- ☐ **C.** The function g has no absolute extrema.
- ☐ **D.** The function g has a local minimum at $t = \frac{5}{6}$, but not an absolute minimum.

8. (a) Find the open intervals on which the function $f(\theta) = \theta^2 + 2\theta^3$ is increasing and decreasing.
 (b) Identify the function's local and absolute extreme values, if any, saying where they occur.

(a) On what open interval(s), if any, is the function increasing? Select the correct choice below and fill in any answer boxes within your choice.

☒ A. $\left(-\infty, -\frac{1}{3}\right), (0, \infty)$

(Type your answer in interval notation. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

On what open interval(s), if any, is the function decreasing? Select the correct choice below and fill in any answer boxes within your choice.

☒ A. $\left(-\frac{1}{3}, 0\right)$

(Type your answer in interval notation. Use integers or fractions for any numbers in the expression. Use a comma to separate answers as needed.)

☐ B. The function is never decreasing.

(b) What are the function's absolute maximum and local maximum, and where do they occur? Select the correct choice below and fill in any answer boxes within your choice.

(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

☐ A. The absolute and local maximum are both 0 and occur at $x =$ _____.

☒ B. There is no absolute maximum. The local maximum is $\frac{1}{27}$ and occurs at $x =$ $-\frac{1}{3}$.

☐ C. The absolute and local maximum are both $\frac{1}{27}$ and occur at $x =$ _____.

☐ D. There is no absolute maximum and no local maximum.

What are the function's absolute minimum and local minimum, and where do they occur? Select the correct choice below and fill in any answer boxes within your choice.

(Use a comma to separate answers as needed. Type an integer or a simplified fraction.)

☐ A. The absolute and local minimum are both 0 and occur at $x =$ _____.

☒ B. There is no absolute minimum. The local minimum is 0 and occurs at $x =$ 0.

☐ C. The absolute and local minimum are both $-\frac{1}{3}$ and occur at $x =$ _____.

☐ D. There is no absolute minimum and no local minimum.

9. **a.** Find the open intervals on which the function is increasing and decreasing.
b. Identify the function's local and absolute extreme values, if any, saying where they occur.

$$f(x) = x^4 - 32x^2 + 256$$

a. Find the open intervals on which the function is increasing and decreasing.

- ☐ **A.** The function f is increasing on the subintervals $(-\infty, -4)$, $(0, 4)$ and decreasing on the subintervals $(-4, 0)$, $(4, \infty)$.
☒ **B.** The function f is increasing on the subintervals $(-\infty, -4)$, $(4, \infty)$ and decreasing on the subintervals $(-4, 0)$, $(0, 4)$.

b. Identify the function's local extreme values, if any.

- ☐ **A.** The function f has a local maximum at $x = -4$ and $x = 4$, and it has a local minimum at $x = 0$.
☒ **B.** The function f has a local minimum at $x = -4$ and $x = 4$, and it has a local maximum at $x = 0$.
☐ **C.** The function f has no local extrema.

Identify the function's absolute extreme values, if any.

- ☐ **A.** The function f has no absolute extrema.
☐ **B.** The function f has an absolute maximum at $x = -4$ and $x = 4$ and no absolute minimum.
☐ **C.** The function f has an absolute minimum at $x = -4$ and $x = 4$ and an absolute maximum at $x = 0$.
☒ **D.** The function f has an absolute minimum at $x = -4$ and $x = 4$ and no absolute maximum.

10. For the function $f(x) = 5x + 4x^2$ on the interval $-\infty < x \leq 1$, identify the function's local extreme values in the given domain, and say where they are assumed. Which of the extreme values, if any, are absolute?

Choose the correct answer regarding local extreme values.

- ☒ **A.** f has a local minimum at $x = -\frac{5}{8}$ and a local maximum at $x = 1$.
☐ **B.** f has a local minimum at $x = -\frac{5}{8}$ but no local maximum.
☐ **C.** f has no local extrema.
☐ **D.** f has a local maximum at $x = -\frac{5}{8}$ but no local minimum.

Choose the correct answer regarding absolute extreme values.

- ☐ **A.** The function has an absolute maximum at $x = -\frac{5}{8}$ but no absolute minimum.
☒ **B.** The function has an absolute minimum at $x = -\frac{5}{8}$ but no absolute maximum.
☐ **C.** The function has no absolute extrema.
☐ **D.** The function has a local minimum at $x = -\frac{5}{8}$ and a local minimum at $x = 1$.

11. a. Identify the function's local extreme values in the given domain, and say where they occur.
 b. Which of the extreme values, if any, are absolute?
 c. Support your findings with a graphing calculator or computer grapher.

$$f(x) = -x^2 + 4x + 6, \quad -2 \leq x < \infty$$

a. Identify the local extrema. Select the correct choice below and fill in any answer boxes within your choice. (Simplify your answer. Type an ordered pair. Use a comma to separate answers as needed.)

- ☒ A. The local minima are and the local maxima are .
- ☐ B. The local minima are and there are no local maxima.
- ☐ C. The local maxima are and there are no local minima.
- ☐ D. There are no local extrema.

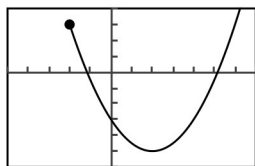
b. Which of the extreme values, if any, are absolute?

(Simplify your answer. Use a comma to separate answers as needed.)

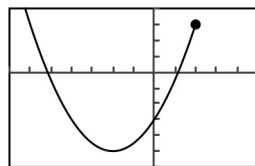
- ☐ A. The absolute minimum is at $x =$. The absolute maximum is at $x =$.
- ☒ B. The absolute maximum is at $x =$. There is no absolute minimum.
- ☐ C. The absolute minimum is at $x =$. There is no absolute maximum.
- ☐ D. There are no absolute extrema.

c. Choose the correct graph of the function below.

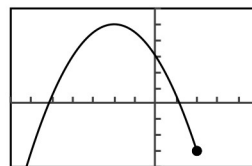
- ☐ A. ☐ B. ☐ C. ☒ D.



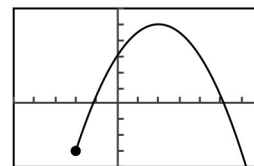
$[-5, 7, 1]$ by $[-12, 8, 2]$



$[-7, 5, 1]$ by $[-12, 8, 2]$



$[-7, 5, 1]$ by $[-8, 12, 2]$



$[-5, 7, 1]$ by $[-8, 12, 2]$

12. For the function $f(t) = 15t - 5t^3$ on the domain $-3 \leq t < \infty$, do the following.

- Identify the function's extreme values in the given domain.
- Which of the extreme values, if any, are absolute?
- Support your findings with a graphing calculator or computer grapher.

a. Select the correct choice below and fill in any answer boxes in your choice.

- ☒ **A.** The function has local maxima at the point(s) $(1, 10), (-3, 90)$.
(Type an ordered pair. Use a comma to separate answers as needed.)
- ☐ **B.** There are no local maxima.

Select the correct choice below and fill in any answer boxes in your choice.

- ☒ **A.** The function has local minima at the point(s) $(-1, -10)$.
(Type an ordered pair. Use a comma to separate answers as needed.)

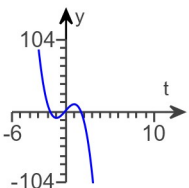
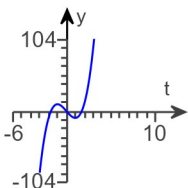
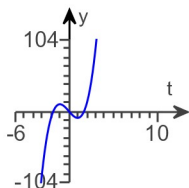
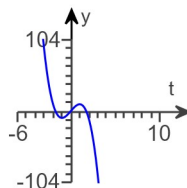
b. Select the correct choice below and fill in any answer boxes in your choice.

- ☒ **A.** The function has absolute maxima at the point(s) $(-3, 90)$.
(Type an ordered pair. Use a comma to separate answers as needed.)
- ☐ **B.** There are no absolute maxima.

Select the correct choice below and fill in any answer boxes in your choice.

- ☐ **A.** The function has absolute minima at the point(s) _____.
(Type an ordered pair. Use a comma to separate answers as needed.)
- ☒ **B.** There are no absolute minima.

c. Choose the correct graph below that represents f .

- ☒ **A.** 
- ☐ **B.** 
- ☐ **C.** 
- ☐ **D.** 

13.

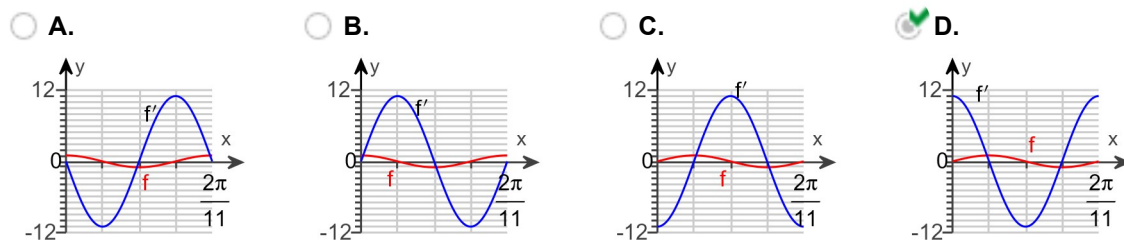
- (a) Find the local extrema of the function $f(x) = \sin 11x$ on the interval $0 \leq x \leq \frac{2\pi}{11}$, and say where they occur.
- (b) Graph the function and its derivative together. Comment on the behavior of f in relation to the signs and values of f' .

a) Choose the correct local extrema of the given function below.

- ☐ A. Local maximum is 1 at $x = \frac{3\pi}{22}$. Local minimum is 0 at $x = 0$.
- Local maximum is 0 at $x = \frac{2\pi}{11}$. Local minimum is -1 at $x = \frac{\pi}{22}$.

- ☐ B. Local maximum is 0 at $x = \frac{\pi}{22}$. Local minimum is -1 at 0.
- Local maximum is 1 at $x = \frac{2\pi}{11}$. Local minimum is 0 at $x = \frac{3\pi}{22}$.

(b) Graph the function and its derivative together. Choose the correct graph below.



Comment on the behavior of f in relation to the signs and values of f' . Choose the correct comment below.

- ☐ A. Moving across the interval from left to right, f' changes from positive to negative at $x = \frac{3\pi}{22}$, so f has a local maximum at $x = \frac{3\pi}{22}$. Also f' changes from negative to positive at $x = \frac{\pi}{22}$, so f has a local minimum at $x = \frac{\pi}{22}$.
- ☐ B. Moving across the interval from left to right, f' changes from positive to negative at $x = \frac{\pi}{22}$, so f has a local minimum at $x = \frac{\pi}{22}$. Also f' changes from negative to positive at $x = \frac{3\pi}{22}$, so f has a local maximum at $x = \frac{3\pi}{22}$.
- ☒ C. Moving across the interval from left to right, f' changes from positive to negative at $x = \frac{\pi}{22}$, so f has a local maximum at $x = \frac{\pi}{22}$. Also f' changes from negative to positive at $x = \frac{3\pi}{22}$, so f has a local minimum at $x = \frac{3\pi}{22}$.