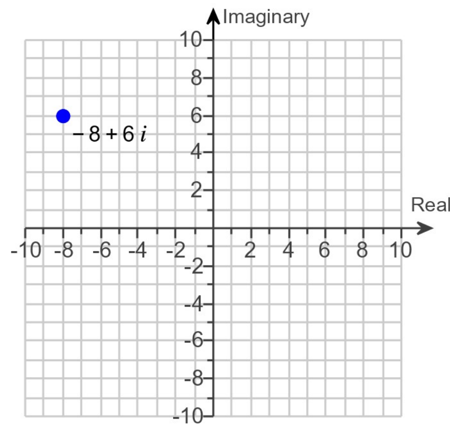
Student: Cole Lamers
Date: 07/27/19

Instructor: Kelly Galarneau
Course: CA&T Internet (70263) Galarneau

Assignment: 7.7 Polar Form of Complex Numbers; DeMoivre's The

Plot the complex number and find its absolute value.

$-8+6i$



A complex number a + b*i* is plotted in the same way as (a,b) is plotted in the rectangular coordinate system, using the horizontal axis of the coordinate plane as the real axis, and the vertical axis as the imaginary axis.

For the complex number a + b*i*, the real part is a. The number -8 corresponds to a in the complex number $-8+6i$.

For the complex number a + b*i*, the imaginary part is b. The number 6 corresponds to b in the complex number $-8+6i$.

Hence, this complex number corresponds to the point $(-8,6)$. The ordered pair $(-8,6)$ is plotted on the plane to the right.

The absolute value of a complex number is found using the formula below.

$$|a+bi| = \sqrt{a^2+b^2}$$

Substitute the values of a and b and simplify under the radical.

$$\begin{aligned} |a+bi| &= \sqrt{a^2+b^2} \\ |-8+6i| &= \sqrt{(-8)^2+(6)^2} \\ &= \sqrt{64+36} \\ &= \sqrt{100} \end{aligned}$$

Now simplify the radical.

$$\sqrt{100} = 10$$

Thus, the absolute value of $-8+6i$ is equal to 10.

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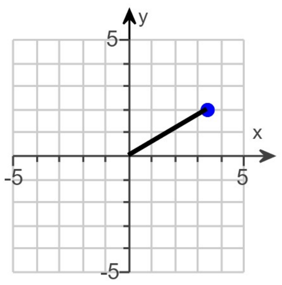
Write the complex number in polar form. Express the argument θ in degrees, with $0 \le \theta < 360°$.

$2\sqrt{3} + 2i$

If $z = a + bi$ is a complex number, then the polar form of z is given by $z = r(\textbf{cos}\,\theta + i\,\textbf{sin}\,\theta)$ where $r = \sqrt{a^2 + b^2}$ and θ is an angle in standard position whose terminal side contains the point (a,b).

First, locate $2\sqrt{3} + 2i$ in the complex plane.



Find r, using $a = 2\sqrt{3}$ and $b = 2$.

$r = \sqrt{a^2 + b^2}$

$= \sqrt{\left(2\sqrt{3}\right)^2 + (2)^2}$

$= 4$

To find θ, find an angle in standard position whose terminal side contains the point $\left(2\sqrt{3},2\right)$.

Substitute the values of a and r into $a = r\,\textbf{cos}\,\theta$ and solve for $\textbf{cos}\,\theta$.

$$a = r\,\textbf{cos}\,\theta$$
$$2\sqrt{3} = 4\,\textbf{cos}\,\theta$$
$$\textbf{cos}\,\theta = \frac{\sqrt{3}}{2}$$

The smallest nonnegative solution to this equation where the terminal side of θ goes through $\left(2\sqrt{3},2\right)$ is $\theta = 30°$.

The polar form is shown below.

$2\sqrt{3} + 2i = = 4(\textbf{cos}\,30° + i\,\textbf{sin}\,30°)$

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Write the following complex number in polar form. Express the argument θ in degrees, with $0^\circ \le \theta < 360^\circ$.

$4 - 4i$

The complex number $z = 4 - 4i$ is in rectangular form $z = a + bi$ with $a = 4$ and $b = -4$.

By definition, the polar form of z is $r(\cos\theta + i\sin\theta)$. Determine the value for r, the modulus, and the value for θ, the argument.

First, use $r = \sqrt{a^2 + b^2}$ with $a = 4$ and $b = -4$. to find r.

$r = \sqrt{a^2 + b^2}$	This is the formula for the modulus.
$= \sqrt{(4)^2 + (-4)^2}$	Substitute $a = 4$ and $b = -4$.
$r = 4\sqrt{2}$	Simplify.

Next, use $\tan\theta = \frac{b}{a}$ with $a = 4$ and $b = -4$ to find θ.

$\tan\theta = \frac{b}{a}$	This is the formula for the tangent.
$= \frac{-4}{4}$	Substitute $a = 4$ and $b = -4$.
$= -1$	Simplify the right side.

The tangent equals -1 at 135° and 315°.

Since $4 - 4i$ is in quadrant IV, the correct angle is 315°. Therefore, the polar form of $z = 4 - 4i$ is $z = 4\sqrt{2}(\cos 315^\circ + i\sin 315^\circ)$.

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Write the following complex number in rectangular form.

$$20(\cos 60^\circ + i \sin 60^\circ)$$

A complex number z written in the form $z = a + bi$ is said to be in rectangular form. The point (a,b) has polar coordinates (r,θ), where $r = \sqrt{a^2 + b^2}$, $a = r\cos\theta$, and $b = r\sin\theta$. Thus, the complex number $z = a + bi$ can be written in the form shown below.

$$z = r\cos\theta + (r\sin\theta)i = r(\cos\theta + i\sin\theta)$$

First, apply the distributive property to distribute the 20 to each term in the parentheses.

$$20(\cos 60^\circ + i \sin 60^\circ) = 20\cos 60^\circ + 20i\sin 60^\circ$$

Notice that $a = 20\cos 60^\circ$ and $b = 20\sin 60^\circ$. Find the values of a and b by simplifying the trigonometric terms.

Determine the value of $\cos 60^\circ$.

$$\begin{aligned} z &= 20\cos 60^\circ + 20i\sin 60^\circ \\ &= 20\left(\frac{1}{2}\right) + 20i\sin 60^\circ \end{aligned}$$

Now perform the multiplication and simplify the first term.

$$\begin{aligned} z &= 20\left(\frac{1}{2}\right) + 20i\sin 60^\circ \\ &= 10 + 20i\sin 60^\circ \end{aligned}$$

Determine the value of $\sin 60^\circ$.

$$\begin{aligned} z &= 10 + 20i\sin 60^\circ \\ &= 10 + 20i\left(\frac{\sqrt{3}}{2}\right) \end{aligned}$$

Now perform the multiplication and simplify the second term.

$$\begin{aligned} z &= 10 + 20i\left(\frac{\sqrt{3}}{2}\right) \\ &= 10 + 10\sqrt{3}\,i \end{aligned}$$

Therefore, $20(\cos 60^\circ + i\sin 60^\circ) = 10 + 10\sqrt{3}\,i$.

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Write the complex number in rectangular form.

$$6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$$

The complex number $6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right)$ is written in **polar form**. It can be converted to **rectangular**, or **Cartesian**, **form** by simplifying it to the sum of its real component, x, and its imaginary component, y.

Find the missing trigonometric values.

$$6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) = 6[(0) + (-1)i]$$

Now simplify the expression to find the rectangular form.

$$\begin{aligned} 6\left(\cos\frac{3\pi}{2} + i\sin\frac{3\pi}{2}\right) &= 6(0 - i) \\ &= -6i \end{aligned}$$

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Find $z_1 z_2$ and $\frac{z_1}{z_2}$ for $z_1 = 3(\mathbf{cos}\ 35^\circ + i\ \mathbf{sin}\ 35^\circ)$, $z_2 = 7(\mathbf{cos}\ 18^\circ + i\ \mathbf{sin}\ 18^\circ)$. Write each answer in polar form.

Note that when finding the product or quotient of complex numbers, it is useful to have them written in polar form so that their magnitudes and arguments can be used to find the magnitude and argument of the result.

The given expressions are in polar form because in polar form, the complex number is written as $z = r(\mathbf{cos}\ \theta + i\ \mathbf{sin}\ \theta)$.

To find the product of two complex numbers $z_1 = r_1\left(\mathbf{cos}\ \theta_1 + i\ \mathbf{sin}\ \theta_1\right)$ and $z_2 = r_2\left(\mathbf{cos}\ \theta_2 + i\ \mathbf{sin}\ \theta_2\right)$, use the formula $z_1 z_2 = r_1 r_2\left[\mathbf{cos}\left(\theta_1 + \theta_2\right) + i\ \mathbf{sin}\left(\theta_1 + \theta_2\right)\right]$.

Find the product. Multiply the moduli and add the arguments.

$$\begin{aligned} z_1 z_2 &= 3(\mathbf{cos}\ 35^\circ + i\ \mathbf{sin}\ 35^\circ) \cdot 7(\mathbf{cos}\ 18^\circ + i\ \mathbf{sin}\ 18^\circ) \\ &= 3 \cdot 7[\mathbf{cos}\ (35^\circ + 18^\circ) + i\ \mathbf{sin}\ (35^\circ + 18^\circ)] \\ &= 21[\mathbf{cos}\ 53^\circ + i\ \mathbf{sin}\ 35^\circ] \end{aligned}$$

Therefore, $z_1 z_2 = 21[\mathbf{cos}\ 53^\circ + i\ \mathbf{sin}\ 35^\circ]$.

To find the quotient of two complex numbers $z_1 = r_1\left(\mathbf{cos}\ \theta_1 + i\ \mathbf{sin}\ \theta_1\right)$ and $z_2 = r_2\left(\mathbf{cos}\ \theta_2 + i\ \mathbf{sin}\ \theta_2\right)$, use the formula $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\mathbf{cos}\left(\theta_1 - \theta_2\right) + i\ \mathbf{sin}\left(\theta_1 - \theta_2\right)\right]$, $z_2 \neq 0$.

Find the quotient. Divide the moduli and subtract arguments.

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{3(\mathbf{cos}\ 35^\circ + i\ \mathbf{sin}\ 35^\circ)}{7(\mathbf{cos}\ 18^\circ + i\ \mathbf{sin}\ 18^\circ)} \\ &= \frac{3}{7}[\mathbf{cos}\ (35^\circ - 18^\circ) + i\ \mathbf{sin}\ (35^\circ - 18^\circ)] \\ &= \frac{3}{7}[\mathbf{cos}\ 17^\circ + i\ \mathbf{sin}\ 17^\circ] \end{aligned}$$

Therefore, $\frac{z_1}{z_2} = \frac{3}{7}[\mathbf{cos}\ 17^\circ + i\ \mathbf{sin}\ 17^\circ]$.

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Find $z_1 z_2$ and $\frac{z_1}{z_2}$ for $z_1 = 5 + 5i$, $z_2 = 5 - 5i$. Write each answer in polar form.

Note that when finding the product or quotient of complex numbers, it is useful to have them written in polar form so that their magnitudes and arguments can be used to find the magnitude and argument of the result.

Since the given expression are not in form $z = r(\cos\theta + i\sin\theta)$, first write them in the polar form.

To find the polar form of $z_1 = 5 + 5i$, find its modulus r_1 and argument θ_1.

Note that for the complex number of the form $z = a + bi$, the modulus is $r = \sqrt{a^2 + b^2}$. Find the modulus for $z_1 = 5 + 5i$.

$$r_1 = \sqrt{a^2 + b^2}$$
$$r_1 = \sqrt{(5)^2 + (5)^2}$$
$$r_1 = 5\sqrt{2}$$

Now find its argument. Use $\tan\theta = \frac{b}{a}$ to find the possible values of θ.

$$\tan\theta = \frac{b}{a}$$
$$\tan\theta = \frac{5}{5} \qquad \text{Substitute the values of a and b.}$$
$$\tan\theta = 1 \qquad \text{Simplify.}$$

To find the angle θ, first find the reference angle θ'. Note that the reference angle θ' is given by $\theta' = \left|\tan^{-1}\left(\frac{b}{a}\right)\right|$.

$$\theta' = \left|\tan^{-1}\left(\frac{b}{a}\right)\right|$$
$$\theta' = \left|\tan^{-1}(1)\right|$$
$$\theta' = 45^\circ$$

Now choose θ in the quadrant in which the point (a,b) lies.

The point (a,b) = (5,5) lies in quadrant I and in that quadrant, $\tan\theta$ is positive.

Since θ lies in quadrant I, $\theta = \theta'$. Thus, $\theta_1 = 45^\circ$.

Thus, the polar form of z_1 is $z_1 = 5\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)$.

Similarly, for the polar form of $z_2 = 5 - 5i$, find the modulus r_2 and the argument θ_2.

Find the magnitude for $z_2 = 5 - 5i$.

$$r_2 = \sqrt{a^2 + b^2}$$
$$r_2 = \sqrt{(5)^2 + (-5)^2}$$
$$r_2 = 5\sqrt{2}$$

Now find the argument for $z_2 = 5 - 5i$. Use $\tan\theta = \frac{b}{a}$ to find the possible values of θ.

$$\tan\theta = \frac{b}{a}$$
$$\tan\theta = \frac{-5}{5} \qquad \text{Substitute the values of a and b.}$$
$$\tan\theta = -1 \qquad \text{Simplify.}$$

To find the angle θ, first find the reference angle θ'. Note that the reference angle θ' is given by $\theta' = \left|\tan^{-1}\left(\frac{b}{a}\right)\right|$.

$$\theta' = \left|\tan^{-1}\left(\frac{b}{a}\right)\right|$$
$$\theta' = \left|\tan^{-1}(-1)\right|$$
$$\theta' = 45^\circ$$

Now choose θ in the quadrant in which (a,b) lies.

The point $(a,b) = (5, -5)$ lies in quadrant IV and in that quadrant, $\tan\theta$ is negative.

Since θ lies in quadrant IV, $\theta = 360^\circ - \theta'$. Thus, $\theta_2 = 360^\circ - 45^\circ = 315^\circ$.

Thus, the polar form of z_2 is $z_2 = 5\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)$.

To find the product of two complex numbers in polar form, $z = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, use the formula $z_1 z_2 = r_1 r_2\left[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)\right]$.

Find the product. Multiply the moduli and add the arguments.

$$\begin{aligned} z_1 z_2 &= 5\sqrt{2}[\cos 45^\circ + i \sin 45^\circ] \cdot 5\sqrt{2}[\cos 315^\circ + i\sin 315^\circ] \\ &= 5\sqrt{2} \cdot 5\sqrt{2}[\cos(45^\circ + 315^\circ) + i\sin(45^\circ + 315^\circ)] \\ &= 50[\cos 360^\circ + i \sin 360^\circ] \qquad \text{Simplify.} \end{aligned}$$

Express the argument θ of $z_1 z_2$ in degrees, $0° \le \theta < 360°$.

$$z_1 z_2 = 50[\cos 0^\circ + i \sin 0^\circ]$$

To find the quotient of two complex numbers in polar form, $z = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, use the formula $\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$, $z_2 \neq 0$.

Find the quotient. Divide the moduli and subtract the arguments.

$$\begin{aligned} \frac{z_1}{z_2} &= \frac{5\sqrt{2}(\cos 45^\circ + i\sin 45^\circ)}{5\sqrt{2}(\cos 315^\circ + i \sin 315^\circ)} \\ &= \frac{5\sqrt{2}}{5\sqrt{2}}[\cos(45^\circ - 315^\circ) + i\sin(45^\circ - 315^\circ)] \\ &= 1[\cos(-270^\circ) + i\sin(-270^\circ)] \qquad \text{Simplify.} \end{aligned}$$

Express the argument θ of $\frac{z_1}{z_2}$ in degrees, $0° \le \theta < 360°$.

$$\frac{z_1}{z_2} = \cos 90^\circ + i \sin 90^\circ.$$

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Use DeMoivre's theorem to compute the following power in polar form, with θ in degrees, with $0 \le \theta < 360^\circ$

$$\left(-1 - i\sqrt{3}\right)^{14}(-5 - 5i)^{-8}$$

First, convert each of the complex numbers into polar form.

By definition, the polar form of z is $r(\cos\theta + i\sin\theta)$. Determine the value for r, the modulus, and the value for θ, the argument.

Convert the first complex number into polar form. The complex number $\left(-1 - i\sqrt{3}\right)^{14}$ is in rectangular form, $z = (a + bi)^n$ with $a = -1$ and $b = -\sqrt{3}$. Use $r = \sqrt{a^2 + b^2}$ to find r.

$$\begin{aligned} r &= \sqrt{a^2 + b^2} \\ &= \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} && \text{Substitute } a = -1 \text{ and } b = -\sqrt{3} \\ &= 2 && \text{Simplify.} \end{aligned}$$

Next, substitute $a = -1$ and $b = -\sqrt{3}$ to find θ.

$$\begin{aligned} \tan\theta &= \frac{b}{a} \\ &= \sqrt{3} && \text{Simplify.} \end{aligned}$$

The tangent equals $\sqrt{3}$ at $\theta = 60^\circ$ and $\theta = 240^\circ$.

Since $\left(-1 - i\sqrt{3}\right)^{14}$ will be in quadrant III, the complex number $\left(-1 - i\sqrt{3}\right)^{14}$ is at $\theta = 240^\circ$.

Therefore, $z = \left(-1 - i\sqrt{3}\right)^{14}$ can be written as $z = (2(\cos 240^\circ + i\sin 240^\circ))^{14}$.

To evaluate $(2(\cos 240^\circ + i\sin 240^\circ))^{14}$, use DeMoivre's theorem that states that if $z = r(\cos\theta + i\sin\theta)$ is a complex number in polar form then for any integer n, $z^n = r^n(\cos n\theta + i\sin n\theta)$.

Evaluate the first complex number.

$$\begin{aligned} z &= (2(\cos 240^\circ + i\sin 240^\circ))^{14} \\ &= 16{,}384(\cos 3360^\circ + i\sin 3360^\circ) \end{aligned}$$

Thus, $16{,}384(\cos 3360^\circ + i\sin 3360^\circ)$ is the polar form of the complex number $\left(-1 - i\sqrt{3}\right)^{14}$.

Now, convert the second complex number into polar form. The complex number $(-5 - 5i)^{-8}$ is in rectangular form, $z = (a + bi)^n$ with $a = -5$ and $b = -5$. Use $r = \sqrt{a^2 + b^2}$ to find r.

$$\begin{aligned} r &= \sqrt{a^2 + b^2} \\ &= \sqrt{(-5)^2 + (-5)^2} && \text{Substitute } a = -5 \text{ and } b = -5 \\ &= 5\sqrt{2} && \text{Simplify.} \end{aligned}$$

Next, substitute $a = -5$ and $b = -5$ to find θ.

$$\begin{aligned} \tan\theta &= \frac{b}{a} \\ &= 1 && \text{Simplify.} \end{aligned}$$

The tangent equals 1 at $\theta = 45^\circ$ and $\theta = 225^\circ$.

Since $(-5 - 5i)^{-8}$ will be in quadrant III, the complex number $(-5 - 5i)^{-8}$ is at $\theta = 225^\circ$.

Therefore, $z = (-5 - 5i)^{-8}$ can be written as $z = \left(5\sqrt{2}(\cos 225^\circ + i\sin 225^\circ)\right)^{-8}$.

To evaluate $z = \left(5\sqrt{2}(\cos 225^\circ + i\sin 225^\circ)\right)^{-8}$, use DeMoivre's theorem stated above.

Evaluate the second complex number.

$$\begin{aligned} z &= \left(5\sqrt{2}(\cos 225^\circ + i\sin 225^\circ)\right)^{-8} \\ &= \frac{1}{6{,}250{,}000}(\cos(-1800^\circ) + i\sin(-1800^\circ)) \end{aligned}$$

Thus, $\frac{1}{6{,}250{,}000}(\cos(-1800^\circ) + i\sin(-1800^\circ))$ is the polar form of the complex number $(-5 - 5i)^{-8}$.

Now, perform the multiplication. To multiply complex numbers, multiply the moduli and add the arguments. First multiply the moduli.

$$r = 16{,}384 \cdot \frac{1}{6{,}250{,}000} = \frac{1024}{390{,}625}$$

Next, add the arguments.

$$\theta = 3360 + (-1800) = 1560^\circ$$

The angle 1560° is coterminal with 120°, which is between 0° and 360°. Therefore, the polar form of the product is $\frac{1024}{390{,}625}(\cos 120^\circ + i \sin 120^\circ)$.

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Use DeMoivre's theorem to compute the following power in polar form, with θ in degrees, and $0 \le \theta < 360^\circ$.

$$\left(\sin\frac{7\pi}{6} + i\cos\frac{7\pi}{6}\right)^{-4}$$

First consider $z = \sin\frac{7\pi}{6} + i\cos\frac{7\pi}{6}$. Rewrite $\sin\frac{7\pi}{6} + i\cos\frac{7\pi}{6}$ with the angles in degrees.

$$\sin\frac{7\pi}{6} + i\cos\frac{7\pi}{6} = \sin 210^\circ + i\cos 210^\circ$$

DeMoivre's theorem states that if $z = r(\cos\theta + i\sin\theta)$ is a complex number in polar form, then for any integer n, $z^n = r^n(\cos n\theta + i\sin n\theta)$.

To apply DeMoivre's theorem to $\sin 210^\circ + i\cos 210^\circ$, write it in the form $r(\cos\theta + i\sin\theta)$. To rewrite in the form $r(\cos\theta + i\sin\theta)$, recall that $\sin\theta = \cos(90 - \theta)$.

$$\begin{aligned} z &= \sin 210^\circ + i\cos 210^\circ \\ &= 1[\cos(-120)^\circ + i\sin(-120)^\circ] \end{aligned}$$

To find z^{-4}, apply DeMoivre's theorem to $z = 1[\cos(-120)^\circ + i\sin(-120)^\circ]$.

$$z^{-4} = 1^{-4}\{\cos[-4(-120)]^\circ + i\sin[-4(-120)]^\circ\}$$

Simplify.

$$\begin{aligned} z^{-4} &= 1^{-4}\{\cos[-4(-120)]^\circ + i\sin[-4(-120)]^\circ\} \\ &= 1(\cos 480^\circ + i\sin 480^\circ) \end{aligned}$$

Thus, $z^{-4} = \cos 480^\circ + i\sin 480^\circ$.

The angle between 0° and 360° that is coterminal with 480° is 120°.

Therefore, $\left(\sin\frac{7\pi}{6} + i\cos\frac{7\pi}{6}\right)^{-4} = \cos 120^\circ + i\sin 120^\circ$.