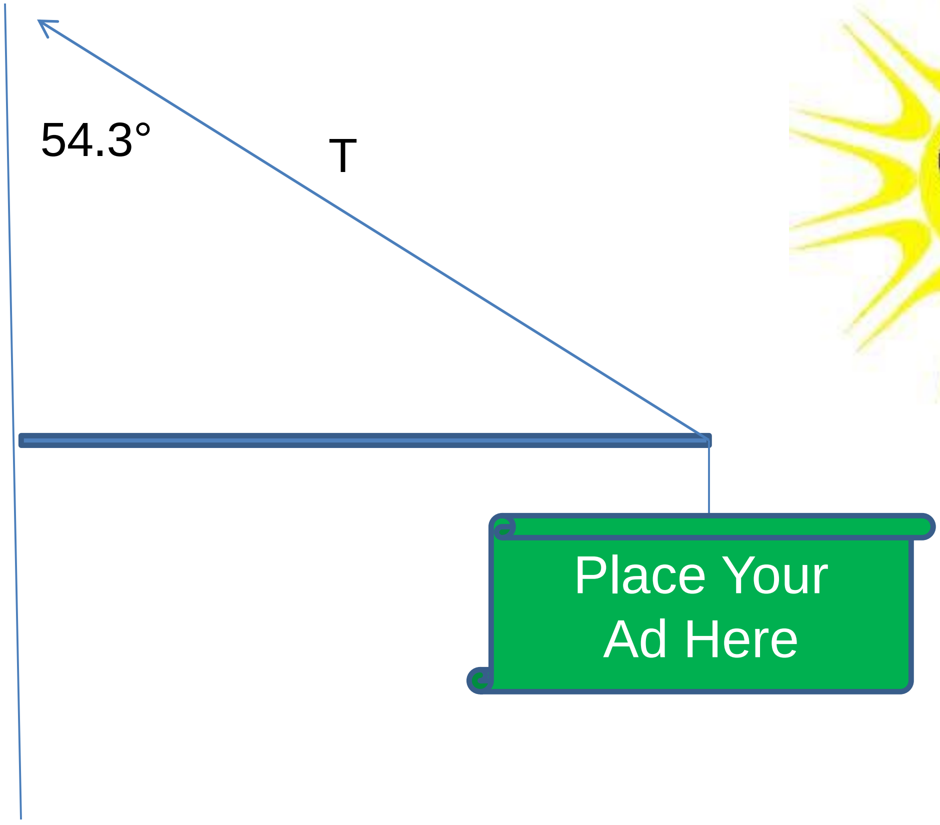
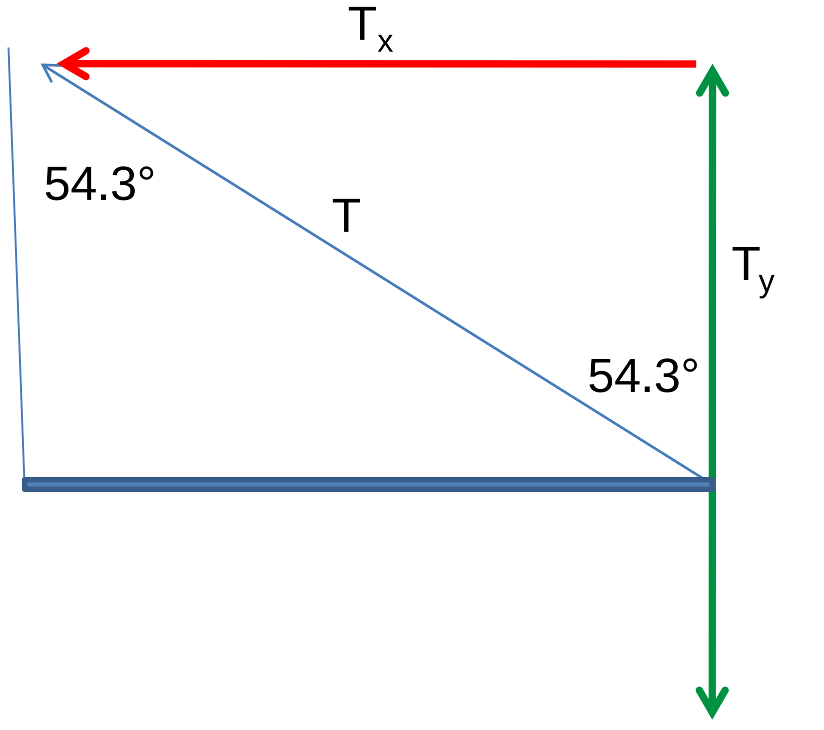
Unit 3 – Forces in Equilibrium
Post-Lecture

The First Condition of Equilibrium

- The net force acting on an object is zero.
- $\Sigma F_x = 0$
- $\Sigma F_y = 0$
- $\Sigma F_{net} = 0$

The weight of a sign is 335 lbs, what is the tension in the support cable?





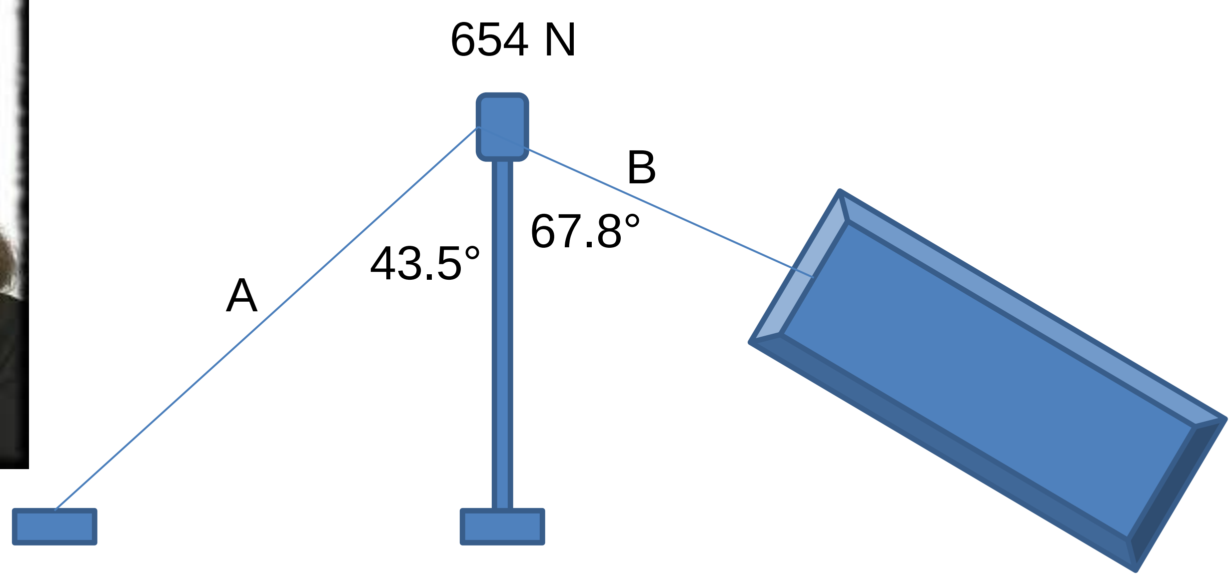
$$w = mg = 335\ \cancel{\text{lb}}\text{s} \cdot \left(\frac{4.448\ \text{N}}{1\ \cancel{\text{lb}}} \right) = -1490\ \text{N}$$

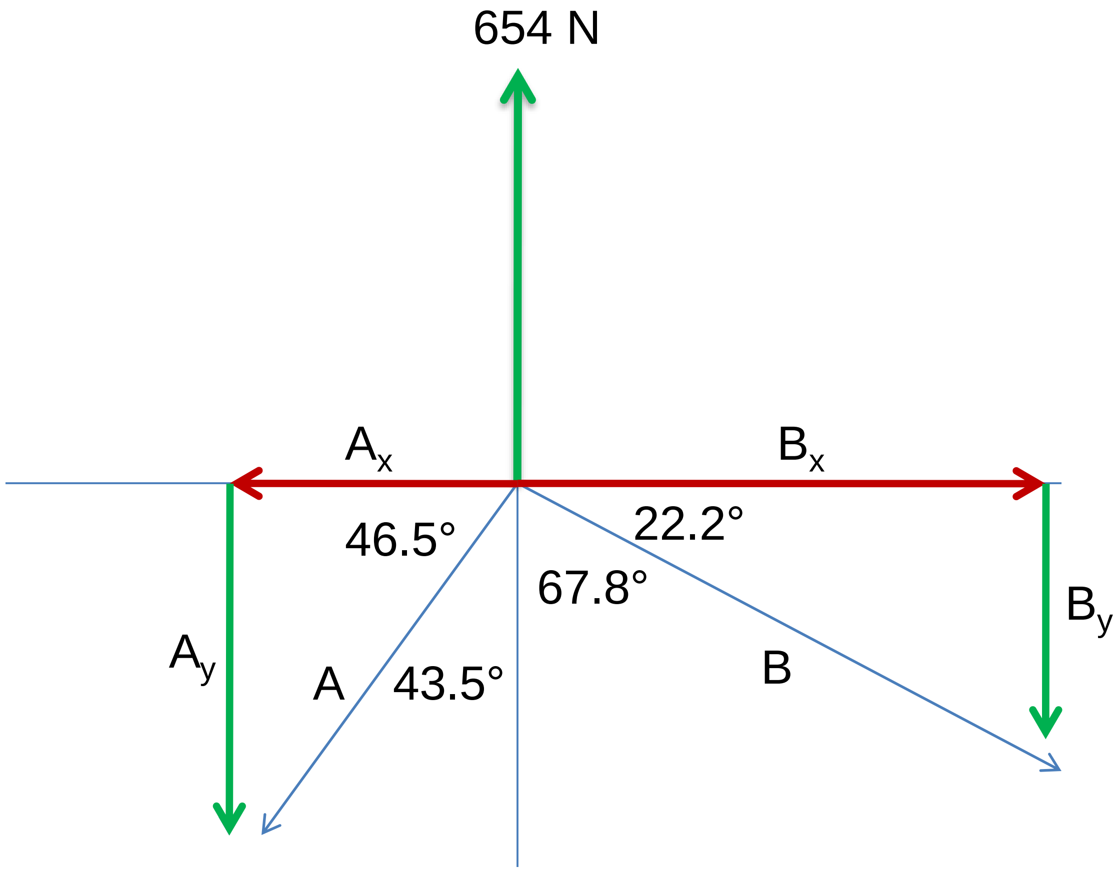
$$\Sigma F_y = \mathrm{T}_y + (-\ 1490\ \mathrm{N}) = 0$$

$$\therefore \quad \mathrm{T}_y = 1490\ \mathrm{N}$$

$$\mathrm{T} = \left(\frac{\mathrm{T}_y}{\cos\theta} \right) = \left(\frac{1490\ \mathrm{N}}{\cos 54.3^\circ} \right) = \boxed{2550\ \mathrm{N}}$$

The hydraulic ram applies 654 N of force vertically. What is the tension in the chains “A” and “B” if the system is in equilibrium?





654 N
A_x
B_x
46.5°
22.2°
67.8°
A_y
A
43.5°
B
B_y

$$\Sigma F_x = \mathrm{B}_x - \mathrm{A}_x = 0$$

$$\therefore \left(\cos 22.2°\right)(\mathrm{B}) - \left(\cos 46.5°\right)(\mathrm{A}) = 0$$

$$\therefore \mathrm{B} = \frac{\left(\cos 46.5°\right)(\mathrm{A})}{\left(\cos 22.2°\right)} = 0.743 \cdot (\mathrm{A})$$

$$\Sigma F_y = 654\text{ N} - \text{A}_y - \text{B}_y = 0$$

$$\therefore\ 654\text{ N} - \left[\left(\sin 46.5°\right)\left(\text{A}\right)\right] - \left[\left(\sin 22.2°\right)\left(\text{B}\right)\right] = 0$$

$$\therefore\ 654\text{ N} - \left[\left(\sin 46.5°\right)\left(\text{A}\right)\right] - \left[\left(\sin 22.2°\right)\left(0.743\cdot(\text{A})\right)\right] = 0$$

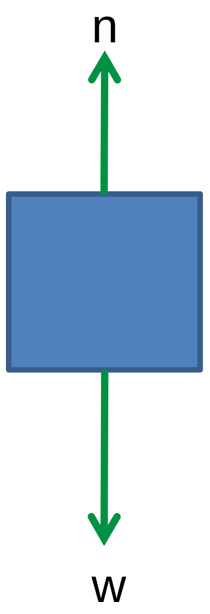
$$\therefore\ 654\text{ N} - \left[0.725\left(\text{A}\right)\right] - \left[0.281\left(\text{A}\right)\right] = 0$$

$$\therefore\ 654\text{ N} - 1.006(A) = 0$$

$$\therefore\ 654\text{ N} = 1.006(A) \qquad \therefore\ A = \boxed{65\bar{0}\text{ N}}$$

$$B = 0.743(A) = 0.743(65\bar{0}\text{ N}) = \boxed{483\text{ N}}$$

For rubber against dry concrete, $\mu_s = 0.752$ and $\mu_k = 0.536$. Determine the force needed to start moving a 1350 lb vehicle. Determine the force needed to maintain constant velocity.



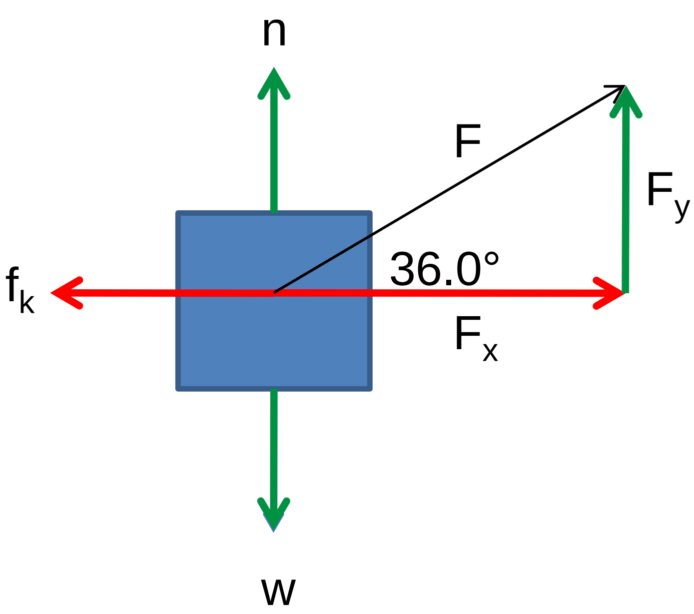
$$\Sigma F_y = n - w = 0$$

$$\therefore \; n = w = 1350 \text{ l}\cancel{\text{b}}\text{s} \cdot \left(\frac{4.448 \text{ N}}{1 \text{ l}\cancel{\text{b}}} \right) = 60\bar{0}0 \text{ N}$$

$$f_s = \mu_s n = (0.752)\left(60\bar{0}0 \text{ N}\right) = \boxed{4510 \text{ N}}$$

$$f_k = \mu_k n = (0.536)\left(60\bar{0}0 \text{ N}\right) = \boxed{3220 \text{ N}}$$

What force is needed to pull a 327 N wagon across a horizontal surface? Assume the pulling force is applied at an angle of 36.0° above the horizontal and the coefficient of friction is 0.423



$$\Sigma F_y = n + F_y - \mathrm{w} = 0$$

$$\therefore\ n = 327\ \mathrm{N} - F_y$$

$$\therefore\ n = 327\ \mathrm{N} - (\sin\ 36.0°)(F)$$

$$\therefore\ n = 327\ \mathrm{N} - (0.588)(F)$$

$$\Sigma F_x = F_x - f_k = 0$$

$$\therefore\ F_x = f_k$$

$$\therefore\ (\cos 36.0°)(F) = \mu_k n$$

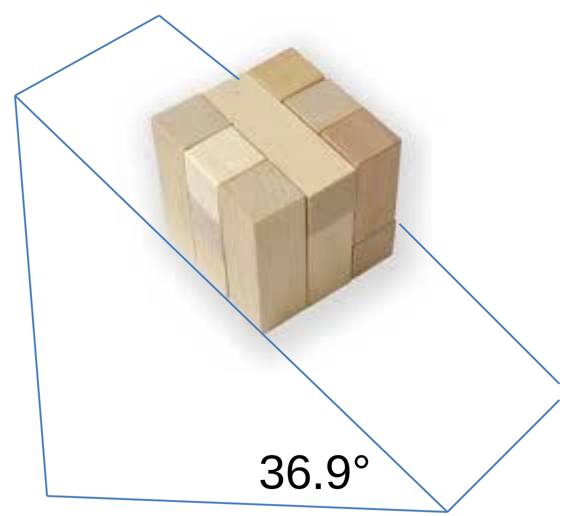
$$\therefore\ 0.809(F) = (0.423)\left[327\text{ N} - (0.588)(F)\right]$$

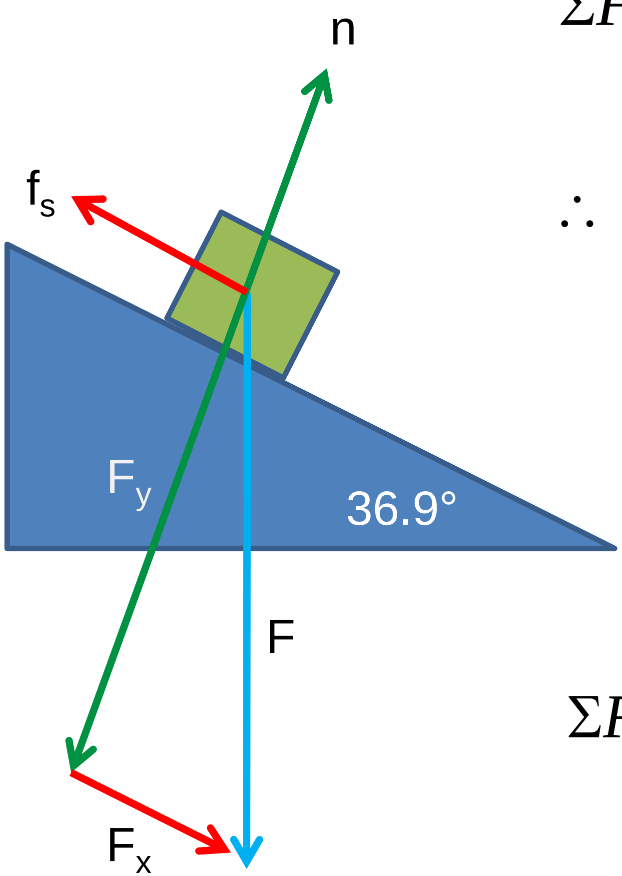
$$\therefore\ 0.809(F) = 138\text{ N} - (0.249)(F)$$

$$\therefore\ 1.058(F) = 138\text{ N}$$

$$\therefore\ F = \boxed{13\overline{0}\text{ N}}$$

A 264 N block rests on a 36.9 ° incline. What is the static frictional force? What is the normal force? What is the coefficient of static friction?





$$\Sigma F_y = n - F_y = 0$$

$$\therefore\ n = F_y = (\cos\ 36.9°)(264\ \text{N}) = \boxed{211\ \text{N}}$$

$$\Sigma F_x = F_x - f_s = 0$$

$$\therefore\ f_s = F_x = (\sin\ 36.9°)(264\ \text{N}) = \boxed{159\ \text{N}}$$

$$f_s = \mu_s n \qquad \therefore \; \mu_s = \frac{f_s}{n}$$

$$\therefore \; \mu_s = \frac{159\ \cancel{\mathrm{N}}}{211\ \cancel{\mathrm{N}}} = \boxed{0.754}$$

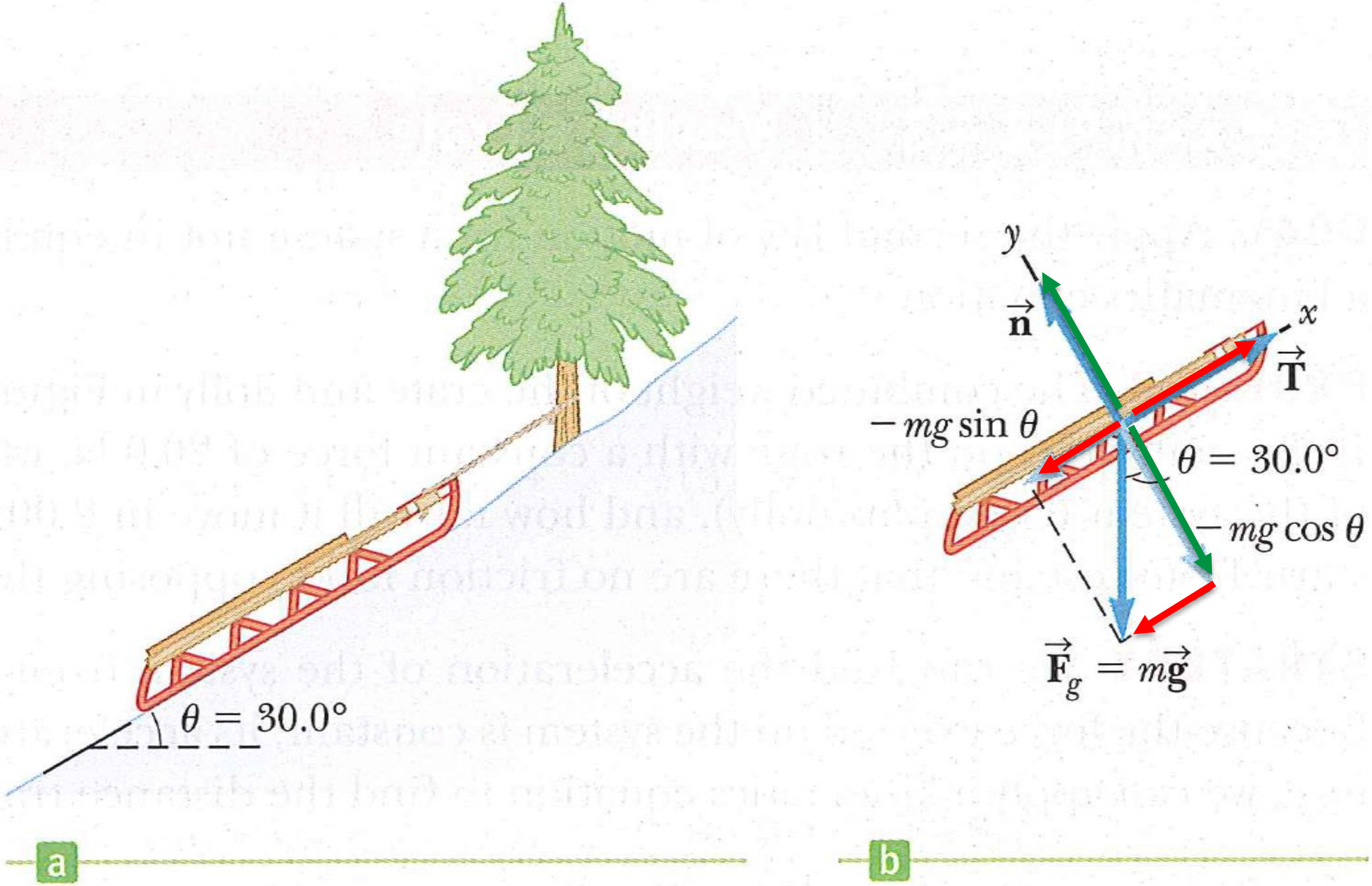


Figure 4.15

Photos/Illustrations courtesy of *College Physics*, 9th edition.

- A sled is tied to a tree on a frictionless, snow-covered hill (*as shown similarly in lecture Figure 4.15*). If the sled weighs 77.0 N,
- **(a)** find the magnitude of the tension force **T** (N) exerted by the rope on the sled and
- **(b)** that of the normal force **n** (N) exerted by the hill on the sled.

$$\Sigma\vec{F} = \vec{T} + \vec{n} + \vec{F}_g = 0$$

$$\Sigma F_x = T + 0 - mg(\sin\theta) = T - (77.0\text{ N})(\sin 30.0°) = 0$$

$$\therefore\ T = \boxed{38.5\text{ N}}$$

$$\Sigma F_y = 0 + n - mg(\cos\theta) = n - (77.0\text{ N})(\cos 30.0°) = 0$$

$$\therefore\ n = \boxed{66.7\text{ N}}$$

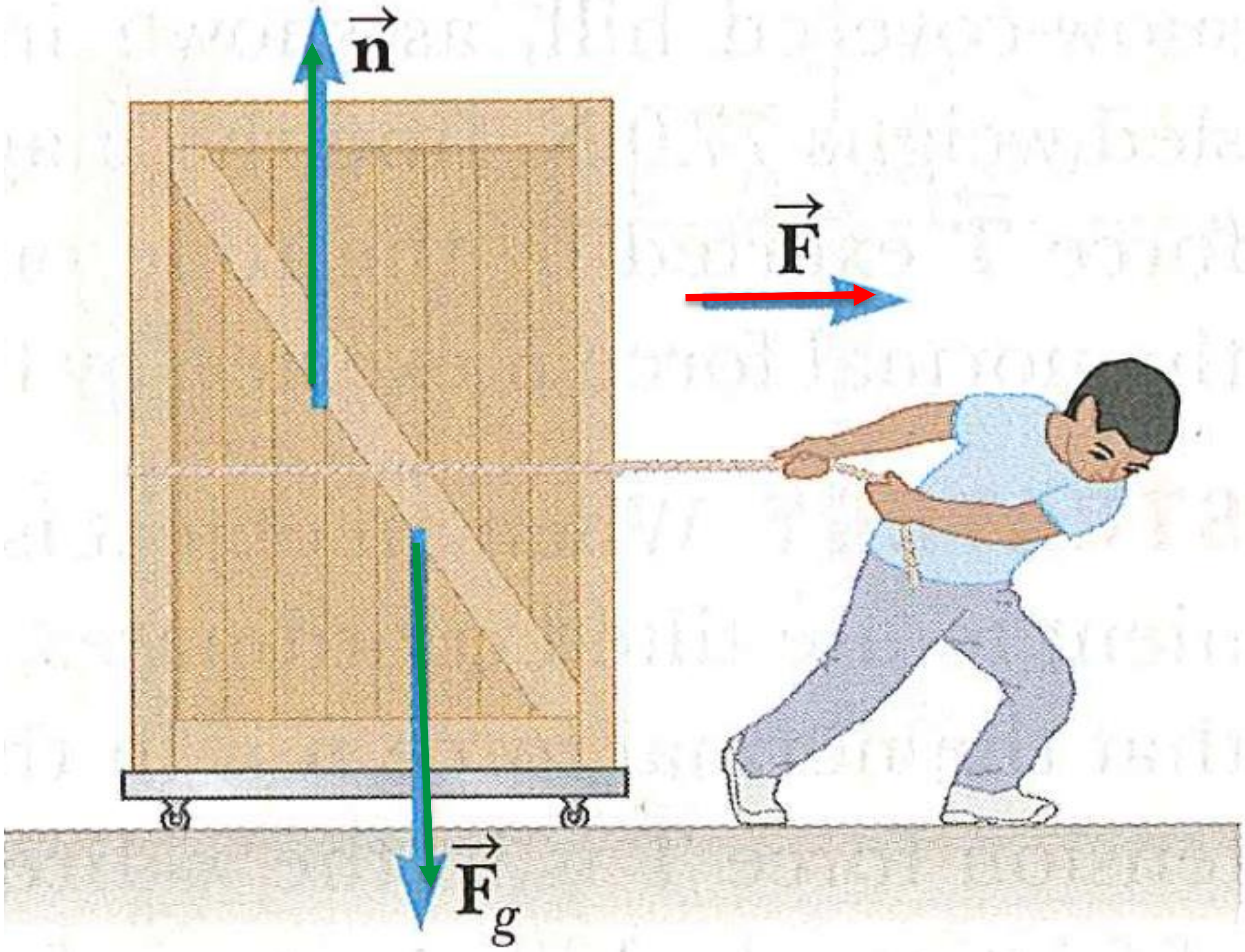


Figure 4.17

Photos/Illustrations courtesy of *College Physics*, 9th edition.

- The combined weight of the crate and dolly (*as in lecture Figure 4.17*) is 300. N. If the man pulls on the rope with a constant force of 20.0 N,
- **(a)** what is the acceleration (m/s^2) of the system (crate plus dolly), and
- **(b)** how far (m) will it move in 2.00 s? Assume the system starts from rest and that there are no friction forces opposing the motion.

$$m = \frac{w}{g} = \frac{30\bar{0} \text{ N}}{9.80 \text{ m/s}^2} = 30.6 \text{ kg}$$

$$a_x = \frac{F_x}{m} = \frac{20.0 \text{ N}}{30.6 \text{ kg}} = \boxed{0.654 \text{ m/s}^2}$$

$$\Delta x = \tfrac{1}{2} a_x t^2 = \tfrac{1}{2}\left(0.654 \text{ m/s}^2\right)\left(2.00 \text{ s}\right)^2 = \boxed{1.31 \text{ m}}$$

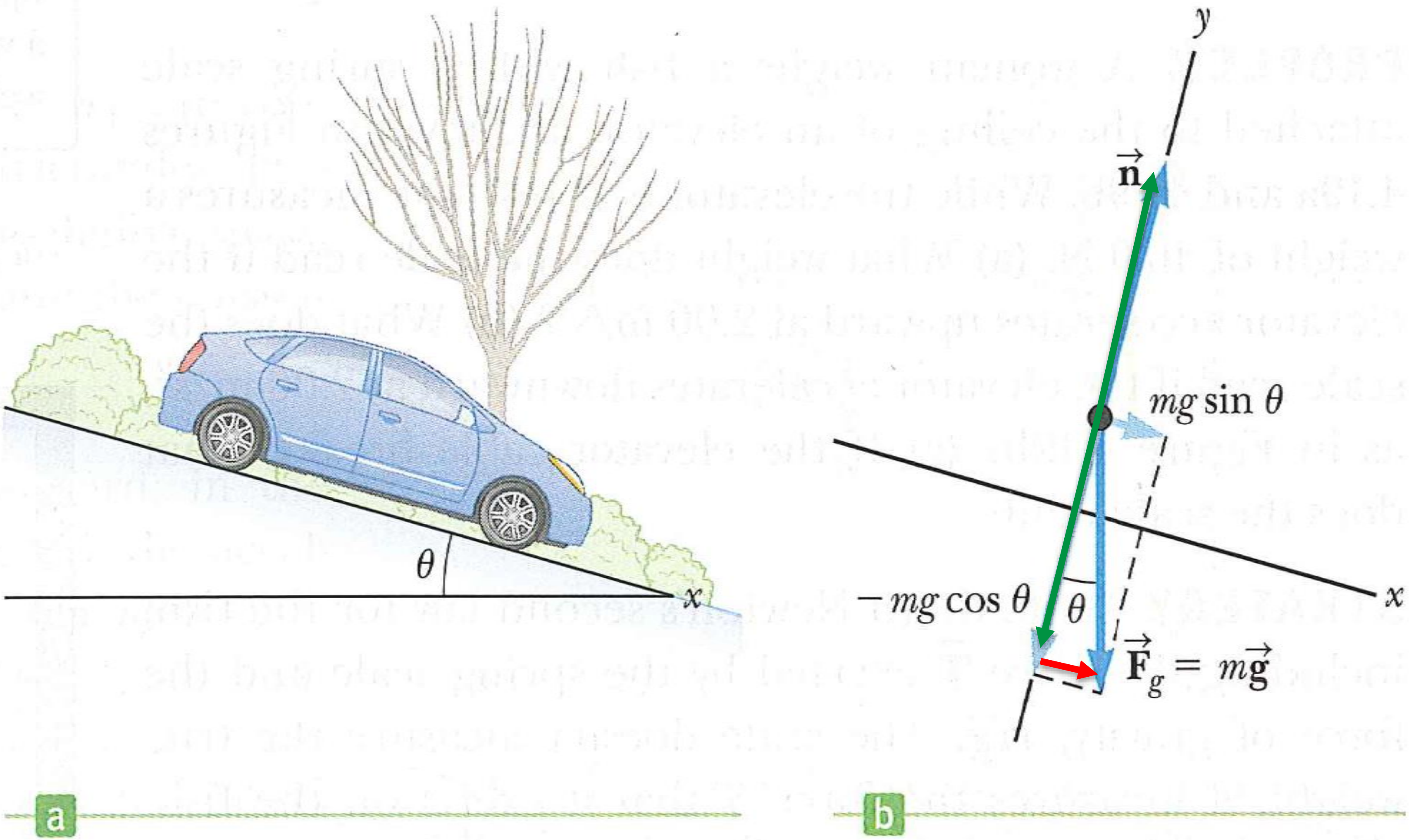


Figure 4.18

Photos/Illustrations courtesy of *College Physics*, 9th edition.

- A car of mass *m* is on an icy driveway inclined at an angle $\theta = 20.0°$ (*as in lecture Figure 4.18*).
- **(a)** Determine the acceleration (m/s^2) of the car assuming the incline is frictionless. If the length of the driveway is 25.0 m and the car starts from rest at the top,
- **(b)** how long (s) does it take to travel to the bottom?
- **(c)** What is the car's speed (m/s) at the bottom?

$$m\vec{a} = \Sigma\vec{F} = \vec{F}_g + n$$

$$ma_x = \Sigma F_x = mg\left(\sin\theta\right)$$

$$\therefore \quad a_x = g\left(\sin\theta\right) = \left(9.80 \text{ m/s}^2\right)\left(\sin 20.0°\right) = \boxed{3.35 \text{ m/s}^2}$$

$$\Delta x = v_o t + ½ a t^2 = \left(0\ \text{m/s}\right)t + ½\left(3.35\ \text{m/s}^2\right)t^2 = 25.0\ \text{m}$$

$$\therefore\ t = \boxed{3.86\ \text{s}}$$

$$v_x = v_{0x} + a_x t = \left(0\ \text{m/s}\right) + \left(3.35\ \text{m/s}^2\right)\left(3.86\ \text{s}\right)$$

$$\therefore\ v_x = \boxed{12.9\ \text{m/s}}$$

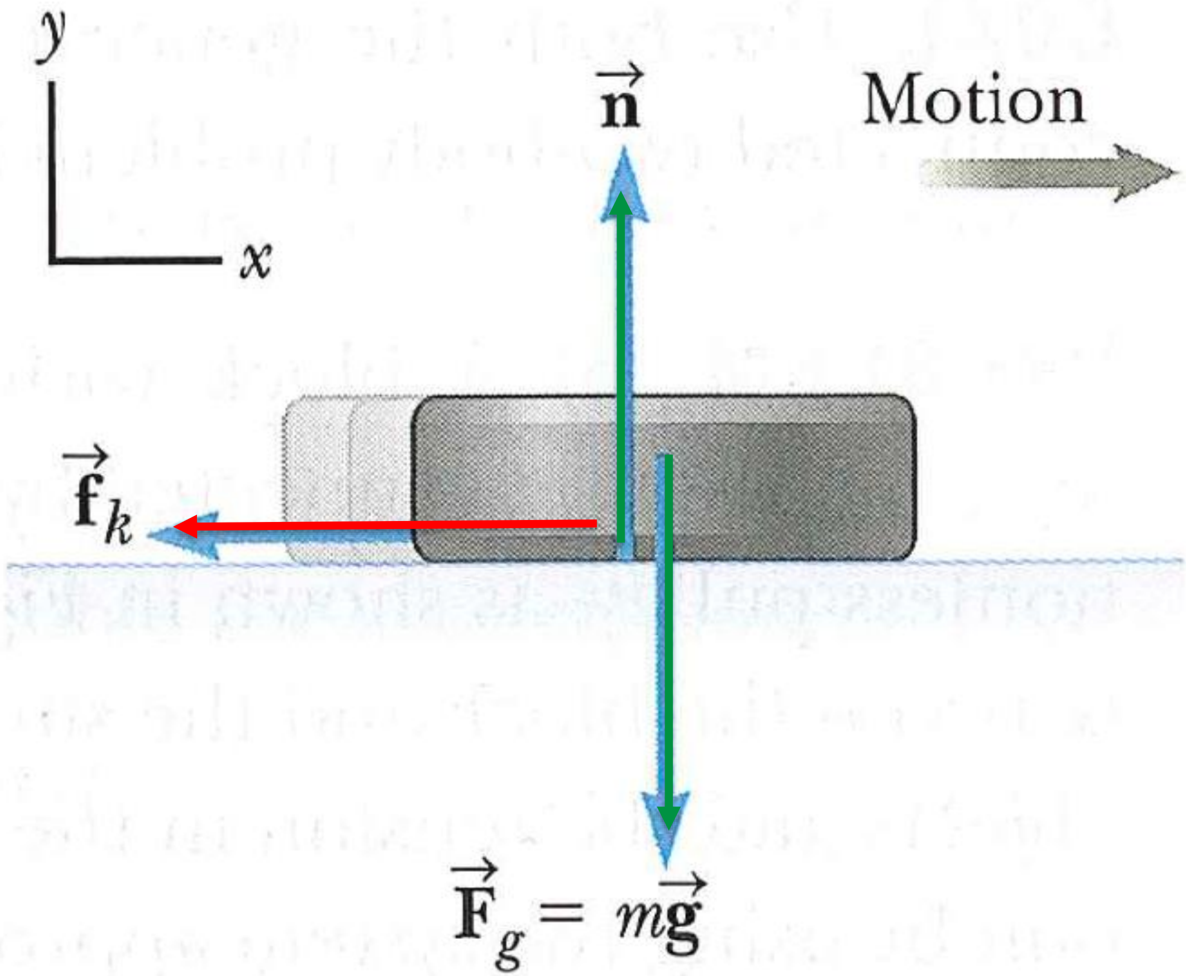


Figure 4.24

Photos/Illustrations courtesy of *College Physics*, 9th edition.

- The hockey puck (*as in lecture Figure 4.24*), struck by a hockey stick, is given an initial speed of 20.0 m/s on a frozen pond. The puck remains on the ice and slides 120. m, slowing down steadily until it comes to rest. **(a)** Determine the coefficient of kinetic friction between the puck and the ice.
- If the puck is given an initial speed of 20.0 m/s and has a coefficient of kinetic friction of 0.195, **(b)** how far (m) will it travel before coming to rest?

$$v^2 = v_0^2 + 2a\Delta x$$

$$a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{\left(0 \text{ m/s}\right)^2 - \left(20.0 \text{ m/s}\right)^2}{2\left(12\overline{0} \text{ m}\right)} = -1.67 \text{ m/s}^2$$

$$\Sigma F_y = n - F_g = n - mg = 0$$

$$\therefore \; n = mg$$

$$f_k = \mu_k n = \mu_k mg$$

$$ma = \Sigma F_x = -f_k = -\mu_k mg$$

$$\mu_k = -\frac{a}{g} = -\frac{\left(-1.67 \text{ m/s}^2\right)}{\left(9.80 \text{ m/s}^2\right)} = \boxed{0.170}$$

$$a = -\mu_k g = -(0.195)(9.80 \text{ m/s}^2) = -1.91 \text{ m/s}^2$$

$$\Delta x = \frac{v^2 - v_0{}^2}{2a} = \frac{(0 \text{ m/s})^2 - (20.0 \text{ m/s})^2}{2(-1.91 \text{ m/s}^2)} = \boxed{105 \text{ m}}$$

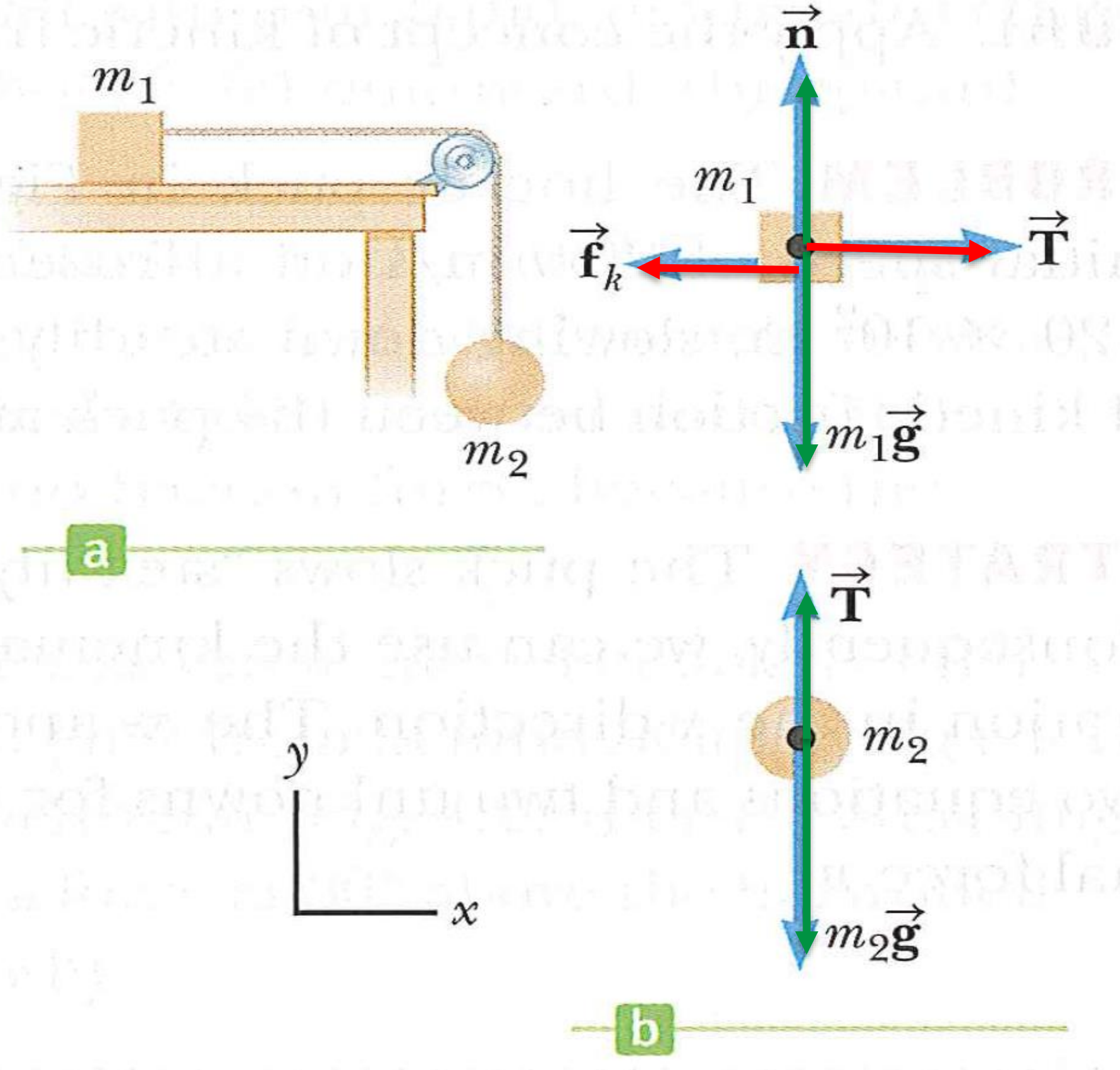


Figure 4.25

Photos/Illustrations courtesy of *College Physics*, 9th edition.

- A block with mass m_1 = 4.00 kg and a ball with mass m_2 = 7.00 kg are connected by a light string that passes over a frictionless pulley (*as in lecture Figure 4.25*). The coefficient of kinetic friction between the block and the surface is 0.300.
- **(a)** Find the acceleration of the two objects and
- **(b)** the tension in the string.

ALTERNATE APPROACH RECOMMENDED

$$\Sigma F_x = T_1 - f_k = m_1 a_1 \qquad \Sigma F_y = n - m_1 g = 0 \qquad \therefore\ n = m_1 g$$

$$\therefore\ T_1 - \mu_k m_1 g = m_1 a_1 \qquad \therefore\ T_1 = m_1 a_1 + \mu_k m_1 g$$

a_2 has the same magnitude as a_1 but is directed along the $-y$ axis.

$$\therefore\ a_2 = \left(-a_1\right)$$

$$\Sigma F_y = T_2 - m_2 \mathrm{g} = m_2 a_2 = m_2\left(-a_1\right) \qquad \therefore\ T_2 = m_2\left(-a_1\right) + m_2 \mathrm{g}$$

$$T_1 = T_2$$

$$\therefore \quad m_1 a_1 + \mu_k m_1 g = m_2(-a_1) + m_2 \mathrm{g}$$

$$\therefore \quad m_1 a_1 - m_2(-a_1) = m_2 \mathrm{g} - \mu_k m_1 g$$

$$\therefore \quad a_1(m_1 + m_2) = m_2 \mathrm{g} - \mu_k m_1 g$$

$$\therefore \quad a_1 = \frac{m_2 \mathrm{g} - \mu_k m_1 g}{(m_1 + m_2)}$$

$$= \frac{(7.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) - (0.300)(4.00\ \mathrm{kg})(9.80\ \mathrm{m/s^2})}{(4.00\ \mathrm{kg} + 7.00\ \mathrm{kg})}$$

$$\therefore \quad a_1 = \boxed{5.17\ \mathrm{m/s^2}}$$

$$T_1 = m_1 a_1 + \mu_k m_1 g$$

$$= (4.00\ \text{kg})(5.17\ \text{m/s}^2) + (0.300)(4.00\ \text{kg})(9.80\ \text{m/s}^2)$$

$$= \boxed{32.4\ \text{N}}$$

ALTERNATE APPROACH

$$\Sigma F_{y_1} = n - m_1 g = 0 \qquad \therefore \quad n = m_1 g = (4.00 \text{ kg}) g = 39.2 \text{ N}$$

$$\Sigma F_{x_1} = T - f_k = m_1 a$$

$$= T - \mu_k n = T - (0.300)(39.2 \text{ N}) = (4.00 \text{ kg}) a$$

$$\therefore \quad T = (4.00 \text{ kg}) a + (11.8 \text{ N})$$

ALTERNATE APPROACH

** Recognize that the Tension and the Acceleration are the same for both objects.

$$\Sigma F_{y_2} = T - m_2 g = m_2(-a)$$

$$= T - (7.00 \text{ kg})g = (7.00 \text{ kg})(-a)$$

$$= T - (68.6 \text{ N}) = (7.00 \text{ kg})(-a)$$

$$\therefore \quad T = -(7.00 \text{ kg})(a) + (68.6 \text{ N})$$

ALTERNATE APPROACH

** Recognize that the Tension and the Acceleration are the same for both objects.

$$T = (4.00 \text{ kg})a + (11.8 \text{ N})$$

$$T = -(7.00 \text{ kg})(a) + (68.6 \text{ N})$$

$$\therefore \quad -(7.00 \text{ kg})(a) + (68.6 \text{ N}) = (4.00 \text{ kg})a + (11.8 \text{ N})$$

$$\therefore \quad (56.8 \text{ N}) = (11.00 \text{ kg})a \qquad a = \boxed{5.16 \text{ m/s}^2}$$

$$T = -(7.00 \text{ kg})(5.16 \text{ m/s}^2) + (68.6 \text{ N}) = \boxed{32.5 \text{ N}}$$