College Algebra with Trig Formula Sheet

Linear Equation:

Slope-Intercept Form:

$$y = mx + b$$

Point-Slope Form:

$$y - y_1 = m(x - x_1)$$

Slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Difference of Squares:

$$a^2 - b^2 = (a - b)(a + b)$$

Perfect Squares:

$$(\boldsymbol{a} + \boldsymbol{b})^2 = \boldsymbol{a}^2 + 2\boldsymbol{ab} + \boldsymbol{b}^2$$

$$(\boldsymbol{a} - \boldsymbol{b})^2 = \boldsymbol{a}^2 - 2\boldsymbol{ab} + \boldsymbol{b}^2$$

Sum of Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Difference of Cubes:

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Test for Symmetry:

Symmetry with respect to $y-$**axis if** $f(x, y) = f(-x, y)$

Symmetry with respect to $x-$**axis if** $f(x, y) = f(x, -y)$

Symmetry with respect to origin if $f(x, y) = f(-x, -y)$

Graph Transformations:

Vertical and Horizontal Shifts: Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

Vertical and Horizontal Stretching and Reflecting: Suppose $c > 1$. To obtain the graph of

$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c (multiply y by c, and keep x unchanged)

$y = \frac{1}{c} f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c (multiply x by $\frac{1}{c}$, and keep y unchanged)

$y = f(\frac{x}{c})$, stretch the graph of $y = f(x)$ horizontally by a factor of c

$y = -f(x)$, reflect the graph of $y = f(x)$ about the $x - axis$

$y = f(-x)$, reflect the graph of $y = f(x)$ about the $y - axis$

College Algebra with Trig Formula Sheet

Laws of Exponents: If a and b are positive real numbers, m and n are any real numbers then:

Product of Powers: $b^n b^m = b^{n+m}$

Power of a Product: $(ab)^n = a^n b^n$

Quotient of Powers: $\dfrac{a^n}{a^m} = a^{n-m}$

Number raised to zero: $b^0 = 1$

Power of a Power: $(b^n)^m = b^{nm}$

Power of Quotient: $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Number to a negative exponent: $b^{-m} = \dfrac{1}{b^m}$

Fraction to negative exponent: $\left(\dfrac{a}{b}\right)^{-m} = \left(\dfrac{b}{a}\right)^m$

Radical Conversion: $\sqrt[m]{a^n} = a^{\frac{n}{m}} = \left(\sqrt[m]{a}\right)^n$

Product Rule: $\sqrt[m]{ab} = \sqrt[m]{a}\sqrt[m]{b}$

Quotient Rule: $\sqrt[m]{\dfrac{a}{b}} = \dfrac{\sqrt[m]{a}}{\sqrt[m]{b}}$

Root of a Root: $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$

If $b > 0$, $b \neq 1$, and m and n are real numbers then $b^m = b^n$ if and only if $m = n$.

Properties of Logarithms:

$\log_a 1 = 0;$

$\log_a a = 1;$

$\log_a (a^k) = k;$

$a^{\log_a t} = t$

$\log_a MN = \log_a M + \log_a N$

$\log_a \dfrac{M}{N} = \log_a M - \log_a N$

$\log_a M^k = k \log_a M$

$\log_a (\dfrac{1}{M}) = -\log_a M$

$\log_a M = \dfrac{\log_b M}{\log_b a}$ **(Change of base formula)**

$y = \log_b x$ if and only if $x = b^y$

$\log_e M = \ln M$

$\log_{10} M = \log M$

Conic Sections: *Center* at (h,k)

<u>Circle</u>: $(x-h)^2 + (y-k)^2 = r^2$

<u>Parabola</u>: ***H*:** $(y-k)^2 = 4p(x-h)$

***V*:** $(x-h)^2 = 4p(y-k)$

<u>Ellipse</u>:

***H*:** $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

***V*:** $\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1$

$b^2 = a^2 - c^2$, $a > b > 0$

Eccentricity: e = c/a

<u>Hyperbola</u>:

***H*:** $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

Asymptotes: $y = \pm\dfrac{b}{a}x$ ***(if center at the origin)***

***V*:** $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

Asymptotes: $y = \pm\dfrac{a}{b}x$ ***(if center at the origin)***

$b^2 = c^2 - a^2$, $a,b > 0$

College Algebra with Trig Formula Sheet

Sequences and Series:

Arithmetic Sequence: for $n \geq 1$

$d = a_{n+1} - a_n$

$a_{n+1} = a_n + d$

$a_n = a_1 + (n-1)d$

$S_n = n\left(\dfrac{a_1 + a_n}{2}\right)$

$S_n = n\left(\dfrac{2a_1 + (n-1)d}{2}\right)$

Geometric Sequence: for $n \geq 1$

$\dfrac{a_{n+1}}{a_n} = r$

$a_{n+1} = r \cdot a_n$

$a_n = a_1 \cdot r^{n-1}$

$S_n = \sum_{i=1}^{n} a_1 \cdot r^{i-1} = \dfrac{a_1(1-r^n)}{1-r}, r \neq 1$

$S_\infty = \sum_{i=1}^{\infty} a_1 \cdot r^{i-1} = \dfrac{a_1}{1-r}, |r| < 1$

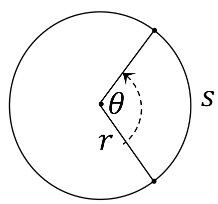
The Binomial Theorem:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots + b^n$$

Arc Length:

$s = \frac{\theta}{360°}(2\pi r)$, θ measured in degrees.

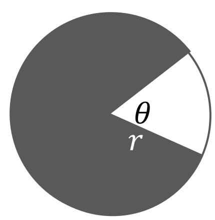
$s = r\theta$, θ measured in radians.



Sector:

Area of a sector $= \frac{\theta}{360°}(\pi r^2)$, θ measured in degrees.

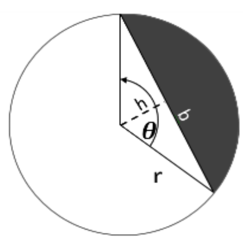
Area of a sector $= \frac{\theta r^2}{2}$, θ measured in radians.



Segment:

Area of Segment $= \frac{\theta}{360°}(\pi r^2) - \frac{1}{2}bh$,

θ measured in degrees.



Linear and Angular Speed:

Angular Speed: $\boldsymbol{w = \frac{\theta}{t}}$, θ measured in radians

Linear Speed: $\boldsymbol{v = \frac{s}{t}}$, s is arc length

Relation between angular and linear speed $\boldsymbol{v = rw}$, r is radius of circle

Graphs of Trigonometric Functions: $\mathbf{y = a\cos[b(x-c)] + d}$ or $\mathbf{y = a\sin[b(x-c)] + d}$

Amplitude: $|a|$ Period: $\frac{2\pi}{b}$ Phase shift: $-\frac{c}{b}$

College Algebra with Trig Formula Sheet

Right Triangle Trigonometry ($r = 1$):

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = y \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{y} \qquad sec(\theta) = \frac{1}{cos(\theta)} \qquad tan(\theta) = \frac{sin(\theta)}{cos(\theta)}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = x \qquad \sec\theta = \frac{hypotenuse}{\text{adjacent}} = \frac{1}{x} \qquad csc(\theta) = \frac{1}{sin(\theta)} \qquad cot(\theta) = \frac{cos(\theta)}{sin(\theta)}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{y}{x} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{x}{y} \qquad cot(\theta) = \frac{1}{tan(\theta)} \qquad tan(\theta)cot(\theta) = 1$$

Co-function Identities:

$$sin(\theta) = cos(\frac{\pi}{2} - \theta) \qquad tan(\theta) = cot(\frac{\pi}{2} - \theta) \qquad sec(\theta) = csc(\frac{\pi}{2} - \theta)$$

$$cos(\theta) = sin(\frac{\pi}{2} - \theta) \qquad cot(\theta) = tan(\frac{\pi}{2} - \theta) \qquad csc(\theta) = sec(\frac{\pi}{2} - \theta)$$

Co-terminal Angles:

$sin(\theta) = sin(\theta + 2\pi n)$ $\quad csc(\theta) = csc(\theta + 2\pi n)$

$cos(\theta) = cos(\theta + 2\pi n)$ $\quad sec(\theta) = sec(\theta + 2\pi n)$

$tan(\theta) = tan(\theta + \pi n)$ $\quad cot(\theta) = cot(\theta + \pi n)$

Reference Angles: let θ' be reference angle for θ

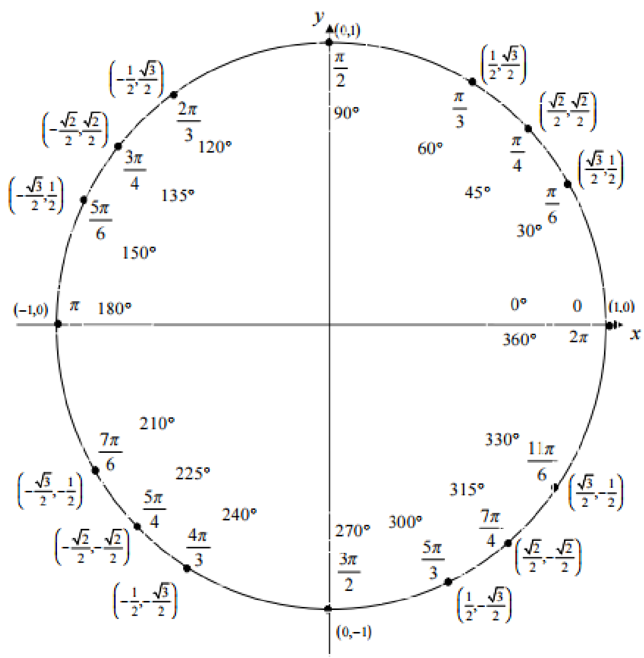
$\theta' = \theta, in\ I\ quadrant$

$\theta' = \pi - \theta\ in\ II\ quadrant$

$\theta' = \theta - \pi\ in\ III\ quadrant$

$\theta' = 2\pi - \theta\ in\ IV\ quadrant$

Unit Circle



Even/Odd Formulas:

$sin(-\theta) = -sin(\theta)$ $\quad csc(-\theta) = -csc(\theta)$

$cos(-\theta) = cos(\theta)$ $\quad sec(-\theta) = sec(\theta)$

$tan(-\theta) = -tan(\theta)$ $\quad cot(-\theta) = -cot(\theta)$

Inverse Trig Functions:

$y = sin^{-1}x\ is\ equivalent\ to\ x = sin(y)$

$y = cos^{-1}x\ is\ equivalent\ to\ x = cos(y)$

$y = tan^{-1}x\ is\ equivalent\ to\ x = tan(y)$

Basic Trigonometric Identities:

Pythagorean Identity:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \sin^2\theta = 1 - \cos^2\theta \qquad \cos^2\theta = 1 - \sin^2\theta$$

$$1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \csc^2\theta$$

Sum and Difference Formulas:

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta) \qquad \cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta) \qquad \cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

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College Algebra with Trig Formula Sheet

Reduction Formula:

If (a, b) is any point on the terminal side of an angle θ (radians) in standard position, then for any real number x,

$$a\sin x + b\cos x = \sqrt{a^2+b^2}\,\sin(x+\theta),\ \cos\theta = \frac{a}{\sqrt{a^2+b^2}},\ \sin\theta = \frac{b}{\sqrt{a^2+b^2}}$$

Double-Angle Formulas:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) \qquad \cos(2\theta) = 2\cos^2(\theta) - 1 \qquad \cos(2\theta) = 1 - 2\sin^2(\theta)$$

$$\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$$

Half-Angle Formula:

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}} \qquad \cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}} \qquad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

Power-Reducing Formulas:

$$\sin^2\mathrm{x} = \frac{1-\cos(2\mathrm{x})}{2} \qquad \cos^2\mathrm{x} = \frac{1+\cos(2\mathrm{x})}{2} \qquad \tan^2\mathrm{x} = \frac{1-\cos(2\mathrm{x})}{1+\cos(2\mathrm{x})}$$

Product-to-Sum Formulas:

$$\cos(x)\cos(y) = \frac{1}{2}\,[\cos(x-y)+\cos(x+y)]$$

$$\sin(x)\sin(y) = \frac{1}{2}\,[\cos(x-y)-\cos(x+y)]$$

$$\sin(x)\cos(y) = \frac{1}{2}\,[\sin(x+y)+\sin(x-y)]$$

$$\cos(x)\sin(y) = \frac{1}{2}\,[\sin(x+y)-\sin(x-y)]$$

Sum-to-Product Formulas:

$$\cos(\mathrm{x})+\cos(\mathrm{y}) = 2\cos\left(\frac{\mathrm{x}+\mathrm{y}}{2}\right)\cos\left(\frac{\mathrm{x}-\mathrm{y}}{2}\right)$$

$$\cos(\mathrm{x})-\cos(\mathrm{y}) = -2\sin\left(\frac{\mathrm{x}+\mathrm{y}}{2}\right)\sin\left(\frac{\mathrm{x}-\mathrm{y}}{2}\right)$$

$$\sin(\mathrm{x})+\sin(\mathrm{y}) = 2\sin\left(\frac{\mathrm{x}+\mathrm{y}}{2}\right)\cos\left(\frac{\mathrm{x}-\mathrm{y}}{2}\right)$$

$$\sin(\mathrm{x})-\sin(\mathrm{y}) = 2\sin\left(\frac{\mathrm{x}-\mathrm{y}}{2}\right)\cos\left(\frac{\mathrm{x}+\mathrm{y}}{2}\right)$$

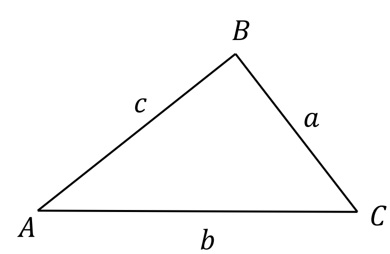
Oblique Triangle Trigonometry:

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C \quad \text{or} \quad \cos C = \frac{a^2+b^2-c^2}{2ab}$$



College Algebra with Trig Formula Sheet

Other Triangle Area Formulas:

$A = \frac{1}{2} ab \sin C$

Heron's Formula:

$A = \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{1}{2}(a+b+c)$

Complex Numbers and Polar Form

$x = rcos(\theta)$

$y = rsin(\theta)$

$r^2 = x^2 + y^2$

$\tan(\theta) = \frac{y}{x}$

$x + yj = r[\cos(\theta) + jsin(\theta)]$

$r\angle\theta = r[\cos(\theta) + jsin(\theta)]$

Inscribed and Circumscribed Regular Polygons and Circles:

Degrees per angle of a regular polygon $= \frac{180°(\text{number of sides}-2)}{\text{number of sides}}$

Triangle –Circle Relationships:

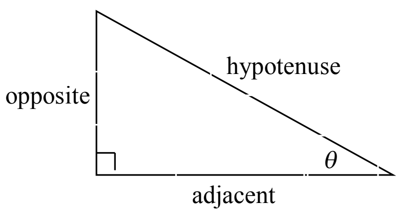
Radius of Circle $= \frac{2}{3}$(Height of Triangle)

Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition
For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.

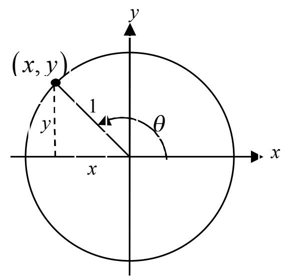


$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ $\quad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$

$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\quad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$

$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ $\quad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$

Unit circle definition
For this definition θ is any angle.



$\sin\theta = \frac{y}{1} = y$ $\quad \csc\theta = \frac{1}{y}$

$\cos\theta = \frac{x}{1} = x$ $\quad \sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

Facts and Properties

Domain
The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, $\quad \theta$ can be any angle
$\cos\theta$, $\quad \theta$ can be any angle
$\tan\theta$, $\quad \theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$
$\csc\theta$, $\quad \theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$
$\sec\theta$, $\quad \theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$
$\cot\theta$, $\quad \theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

Range
The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1$ $\quad \csc\theta \ge 1$ and $\csc\theta \le -1$
$-1 \le \cos\theta \le 1$ $\quad \sec\theta \ge 1$ and $\sec\theta \le -1$
$-\infty < \tan\theta < \infty$ $\quad -\infty < \cot\theta < \infty$

Period
The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$

$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$

$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$

Formulas and Identities

Tangent and Cotangent Identities

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

Reciprocal Identities

$\csc\theta = \frac{1}{\sin\theta}$ $\quad \sin\theta = \frac{1}{\csc\theta}$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \cos\theta = \frac{1}{\sec\theta}$

$\cot\theta = \frac{1}{\tan\theta}$ $\quad \tan\theta = \frac{1}{\cot\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

Even/Odd Formulas

$\sin(-\theta) = -\sin\theta$ $\quad \csc(-\theta) = -\csc\theta$

$\cos(-\theta) = \cos\theta$ $\quad \sec(-\theta) = \sec\theta$

$\tan(-\theta) = -\tan\theta$ $\quad \cot(-\theta) = -\cot\theta$

Periodic Formulas
If n is an integer.

$\sin(\theta + 2\pi n) = \sin\theta$ $\quad \csc(\theta + 2\pi n) = \csc\theta$

$\cos(\theta + 2\pi n) = \cos\theta$ $\quad \sec(\theta + 2\pi n) = \sec\theta$

$\tan(\theta + \pi n) = \tan\theta$ $\quad \cot(\theta + \pi n) = \cot\theta$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas
If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas **(alternate form)**

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$ $\quad \sin^2\theta = \frac{1}{2}(1 - \cos(2\theta))$

$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}$ $\quad \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$ $\quad \tan^2\theta = \frac{1 - \cos(2\theta)}{1 + \cos(2\theta)}$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha + \beta}{2}\right)\sin\left(\frac{\alpha - \beta}{2}\right)$$

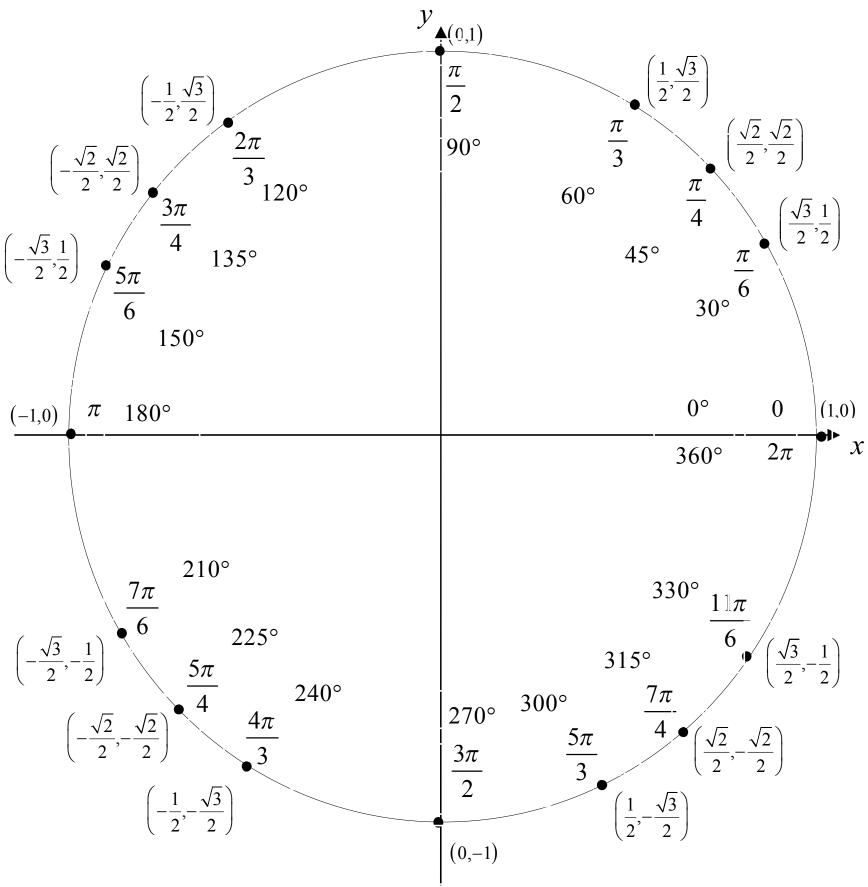
Cofunction Formulas

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ $\quad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$

$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$ $\quad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$

$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ $\quad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$

Unit Circle



For any ordered pair on the unit circle (x,y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right)=\frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right)=-\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y=\sin^{-1}x$ is equivalent to $x=\sin y$

$y=\cos^{-1}x$ is equivalent to $x=\cos y$

$y=\tan^{-1}x$ is equivalent to $x=\tan y$

Domain and Range

Function	Domain	Range
$y=\sin^{-1}x$	$-1\le x\le 1$	$-\frac{\pi}{2}\le y\le\frac{\pi}{2}$
$y=\cos^{-1}x$	$-1\le x\le 1$	$0\le y\le\pi$
$y=\tan^{-1}x$	$-\infty< x<\infty$	$-\frac{\pi}{2}< y<\frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right)=x \qquad \cos^{-1}\left(\cos(\theta)\right)=\theta$

$\sin\left(\sin^{-1}(x)\right)=x \qquad \sin^{-1}\left(\sin(\theta)\right)=\theta$

$\tan\left(\tan^{-1}(x)\right)=x \qquad \tan^{-1}\left(\tan(\theta)\right)=\theta$

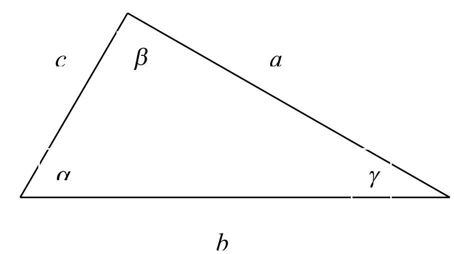
Alternate Notation

$\sin^{-1}x=\arcsin x$

$\cos^{-1}x=\arccos x$

$\tan^{-1}x=\arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a}=\frac{\sin\beta}{b}=\frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2=b^2+c^2-2bc\cos\alpha$$
$$b^2=a^2+c^2-2ac\cos\beta$$
$$c^2=a^2+b^2-2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c}=\frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b}=\frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$
$$\frac{b-c}{b+c}=\frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$
$$\frac{a-c}{a+c}=\frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

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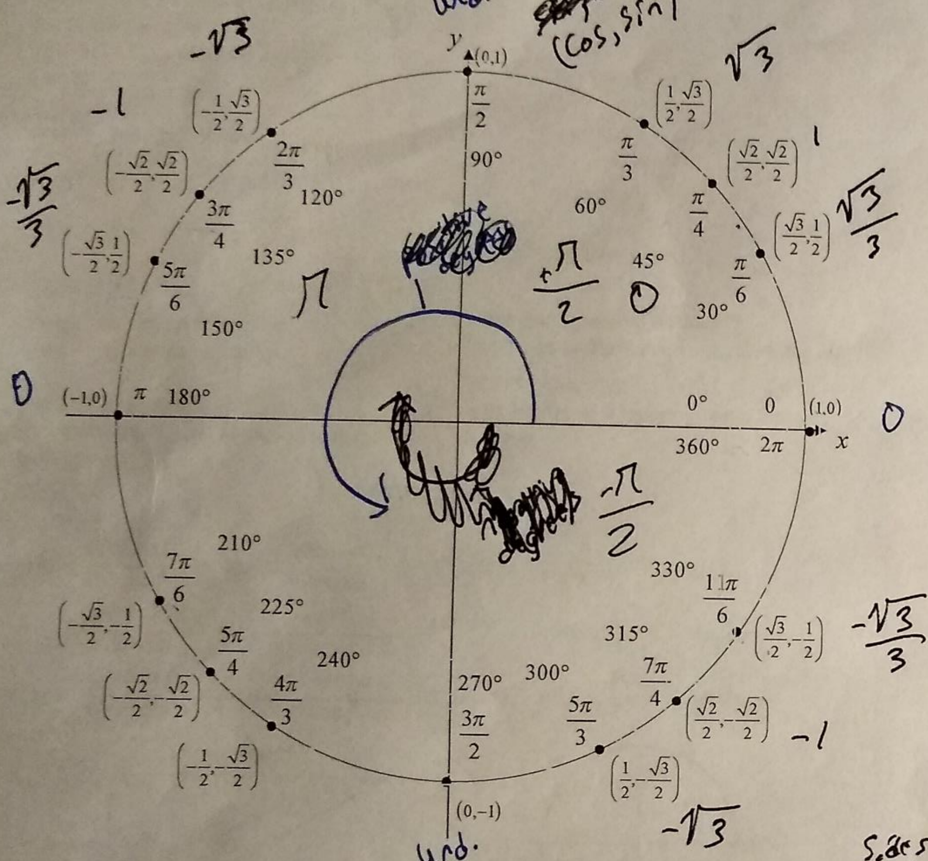
$csc = \frac{1}{\sin}$
$sec = \frac{1}{\cos x}$
$cot = \frac{x}{y}$

~~Secant~~
$\sec\frac{5\pi}{6} = \frac{1}{-\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

AAS Triangle $= \frac{a^2 \sin B \sin C}{2 \sin A}$

SAS Triangle $= \frac{1}{2} ac \sin B$ = area

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}\left(\tan(\theta)\right) = \theta$

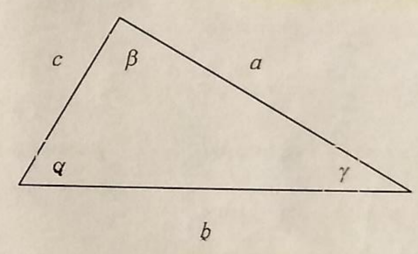
Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$ ← angles ← sides

Sides **Law of Cosines** ← angles

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$b^2 = a^2 + c^2 - 2ac\cos\beta$

$c^2 = a^2 + b^2 - 2ab\cos\gamma$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

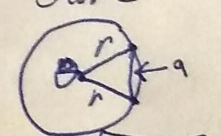
Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

Chord



$a^2 = r^2(2 - 2\cos\theta)$

AAA Tri
Heron's Formula $= K = \sqrt{s(s-a)(s-b)(s-c)}$

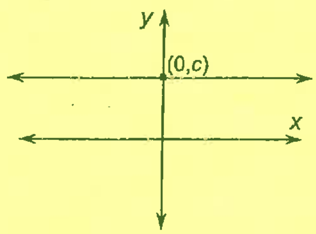
$s = \frac{1}{2}(a+b+c)$

SSS equation

$A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$ swap variables as needed

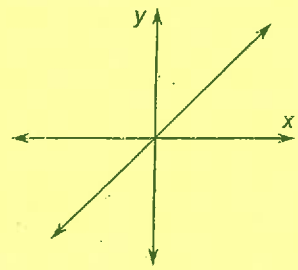
A Library of Basic Functions

Constant Function $f(x)=c$



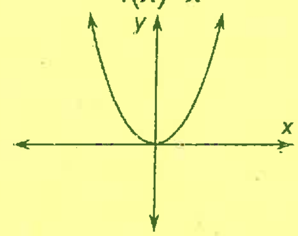
Domain: $(-\infty, \infty)$
Range: $\{c\}$
Constant on $(-\infty, \infty)$
Even function

Identity Function $f(x)=x$



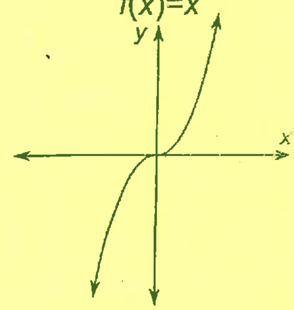
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Odd function

Squaring Function $f(x)=x^2$



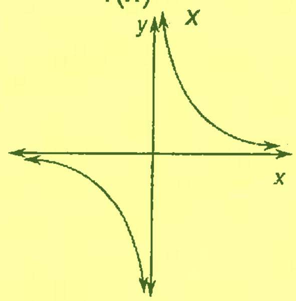
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function

Cubing Function $f(x)=x^3$



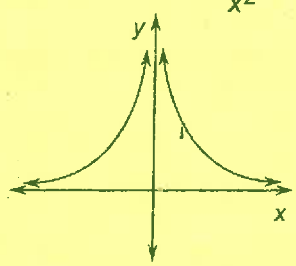
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Odd function

Reciprocal Function $f(x)=\frac{1}{x}$



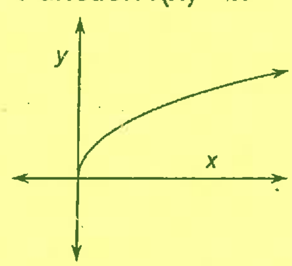
Domain: $(-\infty, 0)\cup(0, \infty)$
Range: $(-\infty, 0)\cup(0, \infty)$
Decreasing on $(-\infty,0)\cup(0, \infty)$
Odd function

Reciprocal Square Function $f(x)=\frac{1}{x^2}$



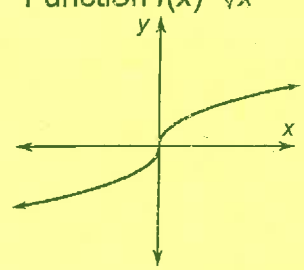
Domain: $(-\infty, 0)\cup(0, \infty)$
Range: $(0, \infty)$
Increasing on $(-\infty, 0)$
Decreasing on $(0, \infty)$
Even function

Square Root Function $f(x)=\sqrt{x}$



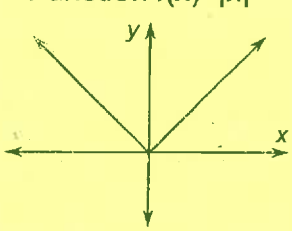
Domain: $[0, \infty)$
Range: $[0, \infty)$
Increasing on $(0, \infty)$
Neither even nor odd

Cube Root Function $f(x)=\sqrt[3]{x}$



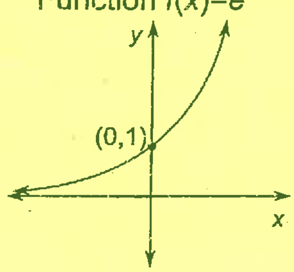
Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Increasing on $(-\infty, \infty)$
Odd function

Absolute Value Function $f(x)=|x|$



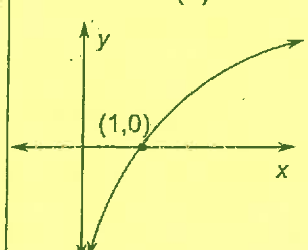
Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function

Natural Exponential Function $f(x)=e^x$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Increasing on $(-\infty, \infty)$
Neither even nor odd

Natural Logarithmic Function $f(x)= \ln x$



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Increasing on $(0, \infty)$
Neither even nor odd

Graph Transformations

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

Vertical and Horizontal Stretching and Reflecting Suppose $c > 1$. To obtain the graph of

$y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c

$y = \frac{1}{c}f(x)$, shrink the graph of $y = f(x)$ vertically by a factor of c

$y = f(cx)$, shrink the graph of $y = f(x)$ horizontally by a factor of c

$y = f(\frac{x}{c})$, stretch the graph of $y = f(x)$ horizontally by a factor of c

$y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis

$y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis