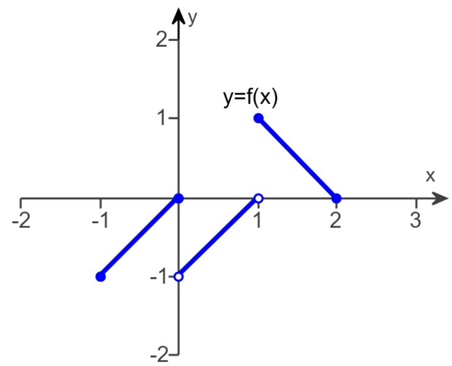
Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



The limit $\lim_{x \to x_0} f(x)$ exists only if the values of $f(x)$ are close to a fixed value L whenever x is close to x_0 (on either side of x_0).

There are three main cases where a function does not have a limit, namely, if it jumps, if its absolute value grows too large, or if it oscillates too much.

In this example, $y = f(x)$ does not grow very large (the range is $[-1,1]$) and does not oscillate, so it is only necessary to determine if it jumps.

Remember that a single point will not prevent a limit from existing. A function is only said to jump if there are two disconnected segments at a point.

a. $\lim_{x \to 0} f(x)$ exists.

Since $f(x)$ jumps at $x = 0$, $\lim_{x \to 0} f(x)$ does not exist.

b. $\lim_{x \to 0} f(x) = 0$.

Since $f(x)$ does not approach 0 as x approaches 0, $\lim_{x \to 0} f(x) \neq 0$.

c. $\lim_{x \to 0} f(x) = -1$.

Since $f(x)$ does not approach -1 as x approaches 0, $\lim_{x \to 0} f(x) \neq -1$.

d. $\lim_{x \to 1} f(x) = 1$.

Since $f(x)$ does not approach 1 as x approaches 1, $\lim_{x \to 1} f(x) \neq 1$.

e. $\lim_{x \to 1} f(x) = 0$.

Since $f(x)$ does not approach 0 as x approaches 1, $\lim_{x \to 1} f(x) \neq 0$.

f. $\lim_{x \to x_0} f(x)$ exists at every point x_0 in $(-1,1)$.

Since $f(x)$ jumps at $x = 0$, $\lim_{x \to x_0} f(x)$ does not exist at every point x_0 in $(-1,1)$.

THEOREM 1—Limit Laws If L, M, c, and k are real numbers and

$$\lim_{x \to c} f(x) = L \qquad \text{and} \qquad \lim_{x \to c} g(x) = M, \quad \text{then}$$

1. *Sum Rule:* $\lim_{x \to c} (f(x) + g(x)) = L + M$
2. *Difference Rule:* $\lim_{x \to c} (f(x) - g(x)) = L - M$
3. *Constant Multiple Rule:* $\lim_{x \to c} (k \cdot f(x)) = k \cdot L$
4. *Product Rule:* $\lim_{x \to c} (f(x) \cdot g(x)) = L \cdot M$
5. *Quotient Rule:* $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \quad M \neq 0$
6. *Power Rule:* $\lim_{x \to c} [f(x)]^n = L^n$, n a positive integer
7. *Root Rule:* $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$, n a positive integer

(If n is even, we assume that $\lim_{x \to c} f(x) = L > 0$.)

THEOREM 2—Limits of Polynomials

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$, then

$$\lim_{x \to c} P(x) = P(c) = a_n c^n + a_{n-1} c^{n-1} + \cdots + a_0.$$

THEOREM 3—Limits of Rational Functions

If $P(x)$ and $Q(x)$ are polynomials and $Q(c) \neq 0$, then

$$\lim_{x \to c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}.$$

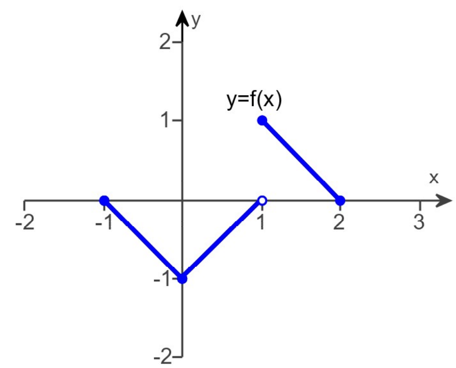
Identifying Common Factors

It can be shown that if $Q(x)$ is a polynomial and $Q(c) = 0$, then $(x - c)$ is a factor of $Q(x)$. Thus, if the numerator and denominator of a rational function of x are both zero at $x = c$, they have $(x - c)$ as a common factor.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

2.

Which of the following statements about the function $y = f(x)$ graphed here are true, and which are false?



a. $\lim_{x \to 0} f(x)$ exists.

False

True

b. $\lim_{x \to 0} f(x) = 0$.

False

True

c. $\lim_{x \to 0} f(x) = -1$.

True

False

d. $\lim_{x \to 1} f(x) = 1$.

False

True

e. $\lim_{x \to 1} f(x) = 0$.

True

False

f. $\lim_{x \to x_0} f(x)$ exists at every point x_0 in $(-1,1)$.

True

False

YOU ANSWERED: False

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Find the limit as x approaches -3 for the function $f(x) = 4x + 7$.

The $\lim_{x \to -3} (4x + 7)$ has the form $\lim_{x \to x_0} (f(x) + g(x))$.

Apply the Sum Rule.

$$\lim_{x \to x_0} (f(x) + g(x)) = \lim_{x \to x_0} f(x) + \lim_{x \to x_0} g(x)$$

Substitute $f(x) = 4x$ and $g(x) = 7$.

$$\lim_{x \to -3} (4x + 7) = \lim_{x \to -3} (4x) + \lim_{x \to -3} (7)$$

Apply the Constant Multiple Rule to $\lim_{x \to -3} (4x)$.

$$\lim_{x \to -3} (4x) = 4 \lim_{x \to -3} (x)$$

Apply the Limit of the Identity Function Rule to $\lim_{x \to -3} (x)$.

$$\lim_{x \to -3} (x) = -3$$

Thus, $4 \lim_{x \to -3} (x) = 4(-3) = -12$.

Apply the Limit of a Constant Function Rule to $\lim_{x \to -3} (7)$.

$$\lim_{x \to -3} (7) = 7$$

Thus, $4 \lim_{x \to -3} (x) + \lim_{x \to -3} (7) = -5$.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Find the limit as t approaches 5 for the function $f(t) = 3(t-2)(t-4)$.

The $\lim\limits_{t \to 5} (3(t-2)(t-4))$ has the form $\lim\limits_{t \to t_0} [k \cdot f(t) \cdot g(t)]$.

Apply the Constant Multiple Rule.

$$\lim_{t \to t_0} [k \cdot f(t) \cdot g(t)] = k \cdot \lim_{t \to t_0} [f(t) \cdot g(t)]$$

Apply the Product Rule.

$$k \cdot \lim_{t \to t_0} [f(t) \cdot g(t)] = k \cdot \lim_{t \to t_0} f(t) \cdot \lim_{t \to t_0} g(t)$$

For this exercise, $k = 3$, $t_0 = 5$, $f(t) = t - 2$, and $g(t) = t - 4$.

Find the limits of f(t) and g(t).

$$\lim_{t \to 5} f(t) = 3, \quad \lim_{t \to 5} g(t) = 1$$

Putting all the parts together, $\lim\limits_{t \to 5} (3(t-2)(t-4)) = 9$.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Evaluate the following limit.

$$\lim_{x \to -5} \left(4x^3 - 3x^2 + 6x + 4\right)$$

The limit of a polynomial is found by direct substitution. That is, $\lim_{x \to a} p(x) = p(a)$, where p is a polynomial and a is a constant.

Substitute -5 for x in the polynomial.

$$\lim_{x \to -5} \left(4x^3 - 3x^2 + 6x + 4\right) = 4(-5)^3 - 3(-5)^2 + 6(-5) + 4$$

Simplify the expression to evaluate the limit.

$$\begin{aligned} \lim_{x \to -5} \left(4x^3 - 3x^2 + 6x + 4\right) &= 4(-5)^3 - 3(-5)^2 + 6(-5) + 4 \\ &= -601 \end{aligned}$$

Therefore, $\lim_{x \to -5} \left(4x^3 - 3x^2 + 6x + 4\right) = -601$.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Find the following limit.

$$\lim_{x \to 7} \frac{x+3}{x^2+6x+9}$$

To evaluate the limit of a rational function as x approaches a point c at which the denominator is not zero, substitute c for x in the formula for the function.

If P(x) and Q(x) are polynomials and $Q(c) \neq 0$, then the limit of the rational function as x approaches c is the value of the function at $x = c$.

$$\lim_{x \to c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}$$

For the given rational function, the denominator is not equal to zero when $x = 7$.

Since the denominator is not equal to zero when $x = 7$, use the theorem from above to evaluate the limit. Substitute 7 for x in the function, then simplify.

$$\lim_{x \to 7} \frac{x+3}{x^2+6x+9} = \frac{7+3}{7^2+6(7)+9} \qquad \text{Substitute.}$$

$$= \frac{10}{7^2+6(7)+9} \qquad \text{Simplify the numerator.}$$

$$= \frac{10}{100} \qquad \text{Simplify the denominator.}$$

Simplify the resulting fraction.

$$\frac{10}{100} = \frac{1}{10}$$

Thus, $\lim_{x \to 7} \frac{x+3}{x^2+6x+9} = \frac{1}{10}$.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Find $\lim_{x \to 6} \frac{x-6}{x^2-36}$.

Before trying to determine the limit, factor the denominator and simplify the resulting expression.

$x^2 - 36 = (x+6)(x-6)$

Since $x^2 - 36 = (x+6)(x-6)$, $\frac{x-6}{x^2-36} = \frac{x-6}{(x+6)(x-6)} = \frac{1}{x+6}$.

Since $\frac{1}{x+6}$ is equivalent to the original function, $\frac{x-6}{x^2-36}$, except perhaps at $x = 6$, $\lim_{x \to 6} \frac{x-6}{x^2-36} = \lim_{x \to 6} \frac{1}{x+6} = \frac{1}{12}$.

Thus, $\lim_{x \to 6} \frac{x-6}{x^2-36} = \frac{1}{12}$.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Find the limit.

$$\lim_{x \to 10} \frac{x^2 - 5x - 50}{x - 10}$$

Recall the quotient rule for limits, where $\lim\limits_{x \to c} f(x) = L$ and $\lim\limits_{x \to c} g(x) = M$.

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M},\ M \neq 0$$

Notice that the quotient rule cannot be applied directly to $\lim\limits_{x \to 10} \frac{x^2 - 5x - 50}{x - 10}$ since the value of the denominator is 0 when $x = 10$. However, the limit can be found after factoring the polynomial in the numerator and dividing out the common factor. Factor the numerator.

$$\lim_{x \to 10} \frac{x^2 - 5x - 50}{x - 10} = \lim_{x \to 10} \frac{(x - 10)(x + 5)}{x - 10}$$

Divide out any common terms and rewrite the expression.

$$\lim_{x \to 10} \frac{\overset{1}{\cancel{(x - 10)}}(x + 5)}{\underset{1}{\cancel{(x - 10)}}} = \lim_{x \to 10} (x + 5)$$

Recall the polynomial rule for limits. If $p(x)$ defines a polynomial function, then $\lim\limits_{x \to a} p(x) = p(a)$.

Apply the polynomial rule to the limit. Evaluate $p(x)$ for $x = 10$.

$$\begin{aligned} p(x) &= x + 5 \\ p(10) &= (10) + 5 \end{aligned}$$

Now simplify the expression.

$$10 + 5 = 15$$

Therefore, $\lim\limits_{x \to 10} \frac{x^2 - 5x - 50}{x - 10} = 15$.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Find $\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16}$.

Before trying to determine the limit, rationalize the numerator and simplify.

$$\frac{\sqrt{x} - 4}{x - 16} \cdot \frac{\sqrt{x} + 4}{\sqrt{x} + 4} = \frac{x - (4)^2}{(x - 16)\left(\sqrt{x} + 4\right)} = \frac{x - 16}{(x - 16)\left(\sqrt{x} + 4\right)} = \frac{1}{\sqrt{x} + 4}$$

Substituting the rationalized and simplified form gives the following.

$$\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} = \lim_{x \to 16} \frac{1}{\sqrt{x} + 4} = \frac{1}{8}$$

$$\lim_{h\to 0} \frac{(\sqrt{3h+1}-1)}{h} \cdot \frac{(\sqrt{3h+1}+1)}{(\sqrt{3h+1}+1)}$$

$$\lim_{h\to 0} \frac{3\cancel{h}\cancel{+1}\cancel{-1}}{\cancel{h}(\sqrt{3h+1}+1)} = \lim_{h\to 0} \frac{3}{\sqrt{3h+1}+1}$$

$$= \frac{3}{\sqrt{3\cdot 0+1}+1} = \frac{3}{\sqrt{1}+1} = \frac{3}{2}$$

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Limits of the form $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ occur frequently in calculus. Evaluate this limit for the given value of x and function f.

$f(x) = x^2, \quad x = 7$

First rewrite the limit in terms of the given function f.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

Now substitute 7 for x in the function and simplify.

$$\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{(7+h)^2 - 7^2}{h}$$

$$= \lim_{h \to 0} \frac{49 + 14h + h^2 - 49}{h} \qquad \text{Evaluate each term in the numerator.}$$

$$= \lim_{h \to 0} \frac{h(14+h)}{h} \qquad \text{Simplify and factor the numerator.}$$

$$= \lim_{h \to 0} (14+h) \qquad \text{Simplify.}$$

To evaluate the limit of a polynomial function as x approaches c, substitute c for x in the formula for the function.

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0$, then the limit as x approaches c is the value of the polynomial function at $x = c$.

$$\lim_{x \to c} P(x) = P(c) = a_n c^n + a_{n-1} c^{n-1} + \ldots + a_0$$

Use the theorem given above to evaluate the limit.

$$\lim_{h \to 0} (14+h) = 14$$

The value of the limit is 14.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

Find $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ for the given function and value of x.

$f(x) = 3x - 5$, $x = 8$

To find the limit, begin by finding $f(x+h)$. Substitute $x + h$ for each x in $f(x)$.

$f(x+h) = 3(x+h) - 5$

Next, substitute the functions $f(x+h)$ and $f(x)$ into $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h}$ and then simplify.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h) - 5 - (3x - 5)}{h} = \lim_{h \to 0} \frac{3h}{h}$$

Simplify the quotient.

$$\lim_{h \to 0} \frac{3h}{h} = \lim_{h \to 0} 3$$

Find the limit.

$$\lim_{h \to 0} 3 = 3$$

Thus, $\lim\limits_{h \to 0} \frac{f(x+h) - f(x)}{h} = 3$.

Student: Cole Lamers **Date:** 08/30/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 2.2 Limit of a Function and Limit Laws

If $\sqrt{11-8x^2} \le f(x) \le \sqrt{11-x^2}$ for $-1 \le x \le 1$, find $\lim_{x\to 0} f(x)$.

The sandwich theorem allows you to calculate the limit of a function f whose values fall between the values of two other functions g and h that have the same limit L at a point c.

Suppose that $g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself. If $\lim_{x\to c} g(x)$ and $\lim_{x\to c} h(x)$ are both equal to L, then $\lim_{x\to c} f(x)$ is also equal to L.

Begin by finding $\lim_{x\to 0} \sqrt{11-8x^2}$.

$$\lim_{x\to 0} \sqrt{11-8x^2} = \sqrt{11-8(0)^2} = \sqrt{11}$$

Next, find $\lim_{x\to 0} \sqrt{11-x^2}$.

$$\lim_{x\to 0} \sqrt{11-x^2} = \sqrt{11-(0)^2} = \sqrt{11}$$

Since $\lim_{x\to 0} \sqrt{11-8x^2} = \sqrt{11}$ and $\lim_{x\to 0} \sqrt{11-x^2} = \sqrt{11}$, the sandwich theorem implies that $\lim_{x\to 0} f(x) = \sqrt{11}$.