

Student: Cole Lamers
Date: 10/02/19

Instructor: Viktoriya Shcherban
Course: Calc 1 11:30 AM / Internet
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Assignment: 5.3 The Definite Integral
(Set 2)

Evaluate the integral $\int_a^{14a} x \, dx$.

The definite integral $\int_a^b x \, dx = \frac{b^2}{2} - \frac{a^2}{2}$.

So, $\int_a^{14a} x \, dx = \frac{(14a)^2}{2} - \frac{a^2}{2}$.

Thus, $\int_a^{14a} x \, dx = \frac{195}{2}a^2$.

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Evaluate the integral $\int_8^4 3 \, dx$.

For any constant c , $\int_a^b c \, dx = c(b - a)$.

So, $\int_8^4 3 \, dx = 3(4 - 8)$.

Thus, $\int_8^4 3 \, dx = -12$.

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Evaluate the integral $\int_1^4 (2t - 3)dt$.

First apply the Difference rule for definite integrals.

$$\int_1^4 (2t - 3)dt = \int_1^4 2t dt - \int_1^4 3 dt$$

Then, apply the Constant Multiple rule.

$$\int_1^4 2t dt - \int_1^4 3 dt = 2 \int_1^4 t dt - \int_1^4 3 dt$$

$$2 \int_1^4 t dt = 2 \left(\frac{4^2}{2} - \frac{1^2}{2} \right) = 15$$

$$\int_1^4 3 dt = 3(4 - 1) = 9$$

$$\text{So, } \int_1^4 (2t - 3)dt = 15 - 9 = 6.$$

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Use a definite integral to find the area of the region between the given curve and the x-axis on the interval $[0, b]$.

$$y = 6x^2$$

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the area under the curve $y = f(x)$ over $[a, b]$ is the integral of f from a to b .

$$A = \int_a^b f(x) dx$$

First, write the definite integral that corresponds to the area.

$$A = \int_0^b 6x^2 dx$$

Use the rules satisfied by definite integrals to rewrite the given integral in terms of simpler integrals. Try to write the integrand x^2 so the rule below can be used.

$$\int_a^b x^2 dx = \frac{b^3}{3} - \frac{a^3}{3}$$

Notice that the given integration rule is for $f(x) = x^2$. Therefore, use the constant multiple rule shown below to rewrite the integral so the integrand is x^2 .

$$\int_a^b kf(x) dx = k \int_a^b f(x) dx$$

For the integral $\int_0^b 6x^2 dx$, the value of the constant, k , is 6.

Therefore, the integral can be rewritten as shown.

$$\int_0^b 6x^2 dx = 6 \int_0^b x^2 dx$$

Now use the integration rule for x^2 to evaluate the integral. Substitute $a = 0$ and $b = b$ into the formula for the integral and simplify.

$$\begin{aligned} 6 \int_0^b x^2 dx &= 6 \left(\frac{b^3}{3} - \frac{0^3}{3} \right) \\ &= 6 \left(\frac{b^3}{3} - \frac{0^3}{3} \right) \\ &= 2b^3 \end{aligned}$$

Therefore, the area between the curve and the x-axis over the interval $[0, b]$ is $2b^3$.

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Assignment: 5.3 The Definite Integral
(Set 2)

Use a definite integral to find the area of the region between the curve $y = 9x$ and the x-axis on the interval $[0, b]$.

The area between the curve $y = f(x)$ and the x-axis over $[a, b]$ is defined to be the definite integral of f from a to b . In symbolic

terms, $A = \int_a^b f(x) dx$.

In this situation, $f(x) = 9x$ and $[a, b]$ is $[0, b]$.

So the area, A , is $A = \int_0^b 9x dx$.

Apply the constant multiple rule.

$$\int_0^b 9x dx = 9 \int_0^b x dx$$

Finally, evaluate $9 \int_0^b x dx$.

$$9 \int_0^b x dx = 9 \left(\frac{b^2}{2} - \frac{0^2}{2} \right) = \frac{9}{2} b^2$$

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Assignment: 5.3 The Definite Integral
(Set 2)

Find the average value of the function $f(x) = x^2 - 5$ on $[0, \sqrt{10}]$.

The average value of $f(x)$ on $[a,b]$ is defined as $\frac{1}{b-a} \int_a^b f(x) dx$.

So, the average value of $f(x) = x^2 - 5$ on $[0, \sqrt{10}]$ is $\frac{1}{\sqrt{10}-0} \int_0^{\sqrt{10}} (x^2 - 5) dx$.

Applying the Difference and Constant Multiple rules, $\frac{1}{\sqrt{10}-0} \int_a^b [x^2 - 5] dx = \frac{1}{\sqrt{10}} \left(\int_0^{\sqrt{10}} x^2 dx - 5 \int_0^{\sqrt{10}} dx \right)$.

$$\int_0^{\sqrt{10}} x^2 dx = \frac{10^{3/2}}{3}$$

Since $\int_0^{\sqrt{10}} dx = \sqrt{10}$, the following is true.

$$\begin{aligned} \frac{1}{\sqrt{10}} \left(\int_0^{\sqrt{10}} x^2 dx - 5 \int_0^{\sqrt{10}} dx \right) &= \frac{1}{\sqrt{10}} \left(\frac{10^{3/2}}{3} - 5\sqrt{10} \right) \\ &= \frac{10}{3} - 5 \\ &= -\frac{5}{3} \end{aligned}$$

So, the average value of $f(x) = x^2 - 5$ on $[0, \sqrt{10}]$ is $-\frac{5}{3}$.

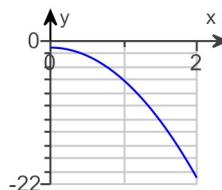
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Assignment: 5.3 The Definite Integral
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Graph the function $f(x) = -5x^2 - 1$ on $[0, 2]$ and find its average value over the interval.

The graph of the function on the interval $[0, 2]$ is shown to the right.



If f is integrable on $[a, b]$, then its average or mean value on $[a, b]$ is given by the definite integral below.

$$\text{av}(f) = \frac{1}{b-a} \int_a^b f(x) dx$$

The lower limit of integration, a , is 0 and the upper limit of integration, b , is 2.

Therefore, the average is $\frac{1}{2} \int_0^2 (-5x^2 - 1) dx$. Use the sum and difference rule as well as the constant multiple rule to rewrite the expression.

$$\frac{1}{2} \int_0^2 (-5x^2 - 1) dx = -\frac{5}{2} \int_0^2 x^2 dx - \frac{1}{2} \int_0^2 1 dx$$

The remaining integrals can be evaluated using the integration rules below.

$$\int_a^b c dx = c(b-a) \quad \int_a^b x^2 dx = \frac{b^3}{3} - \frac{a^3}{3}$$

Substitute $a = 0$ and $b = 2$ into the formulas for the integrals and simplify.

$$\begin{aligned} -\frac{5}{2} \int_0^2 x^2 dx - \frac{1}{2} \int_0^2 1 dx &= -\frac{5}{2} \left(\frac{2^3}{3} - \frac{0^3}{3} \right) - \frac{1}{2}(2-0) \\ &= -\frac{20}{3} - 1 \\ &= -\frac{23}{3} \end{aligned}$$

The average value of the function $f(x) = -5x^2 - 1$ on the interval $[0, 2]$ is $-\frac{23}{3}$.