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Assignment: 9.1 Matrices and Systems
of Equations

Solve the system of equations by using row operations.

$$\left\{ \begin{array}{l} 4 \cdot 3^x - 5 \cdot 5^y + 7^z = -10 \\ 3^x + 4 \cdot 5^y - 3 \cdot 7^z = 14 \\ 3 \cdot 3^x - 3 \cdot 5^y + 5 \cdot 7^z = 9 \end{array} \right.$$

Let $u = 3^x$, $v = 5^y$, and $w = 7^z$. Rewrite the system of equations by substituting u , v , and w .

$$\left\{ \begin{array}{l} 4 \cdot 3^x - 5 \cdot 5^y + 7^z = -10 \\ 3^x + 4 \cdot 5^y - 3 \cdot 7^z = 14 \\ 3 \cdot 3^x - 3 \cdot 5^y + 5 \cdot 7^z = 9 \end{array} \right. \xrightarrow{u = 3^x, v = 5^y, \text{ and } w = 7^z} \left\{ \begin{array}{l} 4u - 5v + w = -10 \\ u + 4v - 3w = 14 \\ 3u - 3v + 5w = 9 \end{array} \right.$$

Write an augmented matrix for the given system of equations.

$$\left\{ \begin{array}{l} 4u - 5v + w = -10 \\ u + 4v - 3w = 14 \\ 3u - 3v + 5w = 9 \end{array} \right. \quad \text{Augmented matrix } A = \left[\begin{array}{ccc|c} 4 & -5 & 1 & -10 \\ 1 & 4 & -3 & 14 \\ 3 & -3 & 5 & 9 \end{array} \right]$$

To get 1 at (1,1) position in the augmented matrix, use the row operation $\frac{1}{4}R1$.

Perform the row operation $\frac{1}{4}R1$.

$$\left[\begin{array}{ccc|c} 4 & -5 & 1 & -10 \\ 1 & 4 & -3 & 14 \\ 3 & -3 & 5 & 9 \end{array} \right] \xrightarrow{\frac{1}{4}R1} \left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 1 & 4 & -3 & 14 \\ 3 & -3 & 5 & 9 \end{array} \right]$$

To get 0 at (2,1) position in the resulting matrix, use the row operation $-1 \cdot R1 + R2 \rightarrow R2$.

Perform the row operation $-1 \cdot R1 + R2 \rightarrow R2$.

$$\left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 1 & 4 & -3 & 14 \\ 3 & -3 & 5 & 9 \end{array} \right] \xrightarrow{-1 \cdot R1 + R2 \rightarrow R2} \left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & \frac{21}{4} & -\frac{13}{4} & \frac{33}{2} \\ 3 & -3 & 5 & 9 \end{array} \right]$$

To get 0 at (3,1) position in the resulting matrix, use the row operation $-3 \cdot R1 + R3 \rightarrow R3$.

Perform the row operation $-3 \cdot R1 + R3 \rightarrow R3$.

$$\left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & \frac{21}{4} & -\frac{13}{4} & \frac{33}{2} \\ 3 & -3 & 5 & 9 \end{array} \right] \xrightarrow{-3 \cdot R1 + R3 \rightarrow R3} \left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & \frac{21}{4} & -\frac{13}{4} & \frac{33}{2} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array} \right]$$

To get 1 at (2,2) position in the resulting matrix, use the row operation $\frac{4}{21} \cdot R2$.

Perform the row operation $\frac{4}{21} \cdot R2$.

$$\left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & \frac{21}{4} & -\frac{13}{4} & \frac{33}{2} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array} \right] \xrightarrow{\frac{4}{21} \cdot R_2} \left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array} \right]$$

To get 0 at (1,2) position in the resulting matrix, use the row operation $\frac{5}{4} \cdot R_2 + R_1 \rightarrow R_1$.

Perform the row operation $\frac{5}{4} \cdot R_2 + R_1 \rightarrow R_1$.

$$\left[\begin{array}{ccc|c} 1 & -\frac{5}{4} & \frac{1}{4} & -\frac{5}{2} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array} \right] \xrightarrow{\frac{5}{4} \cdot R_2 + R_1 \rightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array} \right]$$

To get 0 at (3,2) position in the resulting matrix, use the row operation $-\frac{3}{4} \cdot R_2 + R_3 \rightarrow R_3$.

Perform the row operation $-\frac{3}{4} \cdot R_2 + R_3 \rightarrow R_3$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & \frac{3}{4} & \frac{17}{4} & \frac{33}{2} \end{array} \right] \xrightarrow{-\frac{3}{4} \cdot R_2 + R_3 \rightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & \frac{33}{7} & \frac{99}{7} \end{array} \right]$$

To get 1 at (3,3) position in the resulting matrix, use the row operation $\frac{7}{33}R_3$.

Perform the row operation $\frac{7}{33}R_3$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & \frac{33}{7} & \frac{99}{7} \end{array} \right] \xrightarrow{\frac{7}{33}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & 1 & 3 \end{array} \right]$$

To get 0 at (1,3) position in the resulting matrix, use the row operation $\frac{11}{21} \cdot R_3 + R_1 \rightarrow R_1$.

Perform the row operation $\frac{11}{21} \cdot R_3 + R_1 \rightarrow R_1$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{11}{21} & \frac{10}{7} \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{\frac{11}{21} \cdot R_3 + R_1 \rightarrow R_1} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & 1 & 3 \end{array} \right]$$

To get 0 at (2,3) position in the resulting matrix, use the row operation $\frac{13}{21} \cdot R_3 + R_2 \rightarrow R_2$.

Perform the row operation $\frac{13}{21} \cdot R_3 + R_2 \rightarrow R_2$.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & -\frac{13}{21} & \frac{22}{7} \\ 0 & 0 & 1 & 3 \end{array} \right] \xrightarrow{\frac{13}{21} \cdot R3 + R2 \rightarrow R2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

Write the corresponding system of equations for the last augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right].$$

$$\left\{ \begin{array}{l} u = 3 \\ v = 5 \\ w = 3 \end{array} \right.$$

Back substitute 3^x for u , 5^y for v , and 7^z for w in the system.

$$\left\{ \begin{array}{l} 3^x = 3 \\ 5^y = 5 \\ 7^z = 3 \end{array} \right.$$

Solve each equation to evaluate x , y , and z . First solve for x .

$$3^x = 3$$

$$\ln 3^x = \ln 3$$

$$x \ln 3 = \ln 3$$

$$x = 1$$

Now, solve for y .

$$5^y = 5$$

$$\ln 5^y = \ln 5$$

$$y \ln 5 = \ln 5$$

$$y = 1$$

Finally, solve for z .

$$7^z = 3$$

$$\ln 7^z = \ln 3$$

$$z \ln 7 = \ln 3$$

$$z = \frac{\ln 3}{\ln 7}$$

Therefore, the solution set is $\left\{ \left(1, 1, \frac{\ln 3}{\ln 7} \right) \right\}$.