| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Determine the number of triangles that can be drawn with the given data.

$a = 45$, $b = 75$, $A = 30^\circ$

Since this is the ambiguous case (SSA), there may be 0, 1 or 2 triangles with the given data. Since $A < 90^\circ$, it is acute, and there are four possibilities.

| Case | Condition on a | Number of triangles |
|---|---|---|
| 1 | $a < b \sin A$ | None |
| 2 | $a = b \sin A$ | One right triangle |
| 3 | $b \sin A < a < b$ | Two |
| 4 | $a \geq b$ | One |

Notice that $A = 30^\circ$ is an acute angle, and $a = 45 < 75 = b$. Now calculate $b \sin A$.

$$\begin{aligned} b \sin A &= 75 \sin(30^\circ) \\ &\approx 37.5 \end{aligned}$$

Now, $b \sin A < a < b$. So by case 3 of the table, two triangles can be drawn with the given measurements.

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|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Determine the number of triangles that can be drawn with the given data

$b = 14$, $c = 8$, $B = 162^\circ$

Since this is the ambiguous case (SSA), there may be 0, 1 or 2 triangles with the given parts. Since $B > 90^\circ$, it is obtuse, and there are only two possibilities.

| Case | Condition on b | Number of triangles |
|---|---|---|
| 1 | $b \le c$ | None |
| 2 | $b > c$ | One |

Therefore, only one triangle can be drawn with the given measurements.

| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Given $B = 45^\circ$, $C = 105^\circ$, and $a = 14$, find the exact value of b in triangle ABC.

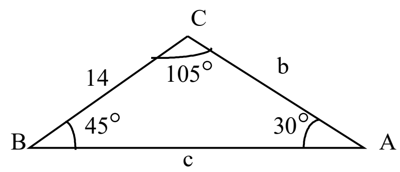
Note that the measures of two of the angles are known. Begin by finding the measure of the third angle. Recall that the sum of the measures of the angles of a triangle is $180^\circ$.

Find the measure of the third angle by subtracting the measures of the known angles from 180°.

$$A + B + C = 180^\circ$$
$$A = 180^\circ - B - C$$
$$A = 180^\circ - 45^\circ - 105^\circ \quad \text{Substitute.}$$
$$A = 30^\circ \quad \text{Simplify.}$$

Make a chart of the parts of the triangle, including the known and the unknown parts. Sketch the triangle as shown below.

| | |
|---|---|
| $A = 30^\circ$ | $a = 14$ |
| $B = 45^\circ$ | $b = ?$ |
| $C = 105^\circ$ | |




Next, use the Law of Sines, $\frac{b}{\sin B} = \frac{a}{\sin A}$, to find the required length, b.

Substitute the values of side a and angles A and B. Solve for side b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$
$$\frac{b}{\sin 45^\circ} = \frac{14}{\sin 30^\circ} \quad \text{Substitute.}$$
$$b = \frac{14 \sin 45^\circ}{\sin 30^\circ} \quad \text{Multiply by } \sin 45^\circ.$$
$$= \frac{14\left(\frac{1}{\sqrt{2}}\right)}{\frac{1}{2}} \quad \text{Evaluate.}$$
$$= 14\sqrt{2} \quad \text{Simplify.}$$

Therefore, in triangle ABC, the exact value of b is $14\sqrt{2}$.

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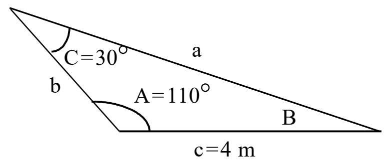
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Solve the triangle shown to the right.




To solve the triangle, find the measure of the third angle by subtracting the measures of the known angles from 180°.

$A + B + C = 180^\circ$ The sum of the angles of a triangle is equal to 180°.
$110^\circ + B + 30^\circ = 180^\circ$ Substitute the known values.

Solve the equation for B.

$110^\circ + B + 30^\circ = 180^\circ$
$B + 140^\circ = 180^\circ$ Add.
$B = 40^\circ$ Simplify.

The Law of Sines states that if A, B, and C are the measures of the angles of a triangle, and a, b, and c are the lengths of the sides opposite these angles, then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Next apply the Law of Sines. Select two ratios from the Law of Sines in which three of the four quantities are known. Solve for the fourth quantity. Use $\frac{a}{\sin A} = \frac{c}{\sin C}$ to find a.

Substitute the known values in $\frac{a}{\sin A} = \frac{c}{\sin C}$.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 110^\circ} = \frac{4}{\sin 30^\circ}$$

Solve for a. Round the final answer to the nearest hundredth.

$$\frac{a}{\sin 110^\circ} = \frac{4}{\sin 30^\circ}$$
$$a = \frac{4 \cdot \sin 110^\circ}{\sin 30^\circ}$$
$$a \approx 7.52$$

Substitute the known values in $\frac{b}{\sin B} = \frac{c}{\sin C}$.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 40^\circ} = \frac{4}{\sin 30^\circ}$$

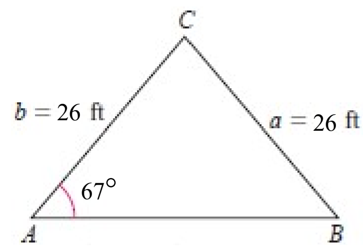
Solve for b. Round the final answer to the nearest hundredth.

$$\frac{b}{\sin 40^\circ} = \frac{4}{\sin 30^\circ}$$
$$b = \frac{4 \cdot \sin 40^\circ}{\sin 30^\circ}$$
$$b \approx 5.14$$

Therefore, $B = 40^\circ$, $a \approx 7.52$ m, and $b \approx 5.14$ m.

| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Solve the triangle ABC.




Make a chart of the six parts, the known and unknown parts.

| | |
|---|---|
| $A = 67^\circ$ | $a = 26$ |
| $B = ?$ | $b = 26$ |
| $C = ?$ | $c = ?$ |

First, take the known measures of the triangle and use the law of sines to find angle B.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Substitute the known values and solve for **sin** B, rounding to four decimal places.

$$\frac{26}{\sin 67^\circ} = \frac{26}{\sin B}$$

$$\sin B = \frac{26 \sin 67^\circ}{26}$$

$$\sin B \approx 0.9205$$

Since **sin** B is between 0 and 1, there are two possible values of B, rounding to one decimal place.

$$B = \sin^{-1}(0.9205) \approx 67^\circ$$

Two possible values of B are as follows.

$$B_1 \approx 67^\circ$$

$$B_2 \approx 180^\circ - 67^\circ = 113^\circ$$

Now, if $A + B_1 \geq 180^\circ$ there is no triangle. If $\sin B \neq 1$, with $A + B_2 < 180^\circ$, then there are two triangles. Otherwise, there is only one triangle. Find $A + B_1$.

$$\begin{aligned} A + B_1 &= 67^\circ + 67^\circ \\ &= 134^\circ \end{aligned}$$

Find $A + B_2$.

$$\begin{aligned} A + B_2 &= 67^\circ + 113^\circ \\ &= 180^\circ \end{aligned}$$

Since $A + B_2 = 180^\circ \geq 180^\circ$, there is no triangle with vertex $B_2$. Therefore, there is only one triangle and angle $B_1 \approx 67^\circ$.

To find C, recall that the sum of all the angles in a triangle is $180^\circ$.

$$C = 180^\circ - A - B_1$$
$$C = 180^\circ - 67^\circ - 67^\circ$$
$$C = 46^\circ$$

Now find the length of side c using the law of sines.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

Substitute the known values and solve for c, rounding to one decimal place.

$$\frac{c}{\sin 46^\circ} = \frac{26}{\sin 67^\circ}$$

$$c = \frac{26 \sin 46^\circ}{\sin 67^\circ}$$

$$c \approx 20.3$$

Therefore, the completed chart is shown below.

| | |
|---|---|
| $A = 67^\circ$ | $a = 26$ |
| $B \approx 67^\circ$ | $b = 26$ |
| $C \approx 46^\circ$ | $c \approx 20.3$ |

| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Solve the SSA triangle. Indicate whether the given measurements result in no triangle, one triangle, or two triangles. Solve each resulting triangle. Round each answer to the nearest tenth.

$A = 43^\circ \quad a = 27 \quad b = 21$

First, make a chart of the given information.

| | |
|---|---|
| $A = 43^\circ$ | $a = 27$ |
| $B = ?$ | $b = 21$ |
| $C = ?$ | $c = ?$ |

Since the known measures are SSA, there may be 0, 1, or 2 triangles with the given measures. Find the number of possible triangles.

In this case, $\theta = A = 43^\circ$ is acute and the opposite side, $a = 27$, is greater than the adjacent side, $b = 21$.

Since the opposite side is greater than the adjacent side, finding the altitude is not necessary. Use the values of the adjacent side, $b = 21$, and the opposite side, $a = 27$, to determine the number of triangles that can be formed. There is only one triangle which has the given side lengths and angle measure.

Now use the Law of Sines to determine the measure of angle B. Use the equation $\frac{a}{\sin A} = \frac{b}{\sin B}$.

Substitute the values of sides a and b and the measure of angle A. Simplify.

$$\frac{27}{\sin 43^\circ} = \frac{21}{\sin B}$$
$$\sin B = \frac{21 \sin 43^\circ}{27}$$
$$\sin B \approx 0.5304$$

Find the measure of angle B.

$$B = \sin^{-1}(0.5304)$$
$$\approx 32^\circ$$

Use the given angle, $A = 43^\circ$, and the angle found above, $B = 32^\circ$, and the fact that the sum of the measures of the angles in a triangle is $180^\circ$ to find the measure of the third angle, C.

$$C = 180^\circ - A - B$$
$$\approx 180^\circ - 43^\circ - 32^\circ$$
$$\approx 105^\circ$$

Now, find the length of side c using the Law of Sines.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

Substitute the values of angle A, angle C, and side a, then simplify.

$$\frac{c}{\sin 105^\circ} = \frac{27}{\sin 43^\circ}$$
$$c \approx \frac{27 \sin 105^\circ}{\sin 43^\circ}$$
$$\approx 38.2$$

Therefore, the measurements of the solved triangle are shown below.

| | |
|---|---|
| $A = 43^\circ$ | $a = 27$ |
| $B \approx 32^\circ$ | $b = 21$ |
| $C \approx 105^\circ$ | $c \approx 38.2$ |

| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Solve the following SSA triangle. Indicate whether the given measurements result in no triangle, one triangle, or two triangles. Solve each resulting triangle. Round each answer to the nearest tenth.

$B = 48^\circ$, $b = 11$, $c = 15$

First, make a chart of the six parts. Identify $\theta$, opposite, and adjacent sides.

| | |
|---|---|
| A = ? | a = ? |
| $B = 48^\circ$ | b = 11 |
| C = ? | c = 15 |

This is an ambiguous case (SSA) and there may be 0, 1 or 2 triangles with the given parts. Now find the number of triangles.

In this case, $\theta = B = 48^\circ$ is acute and the adjacent side c is greater than the opposite side b.

To determine the number of possible triangles, find the altitude. The altitude can be found using the formula altitude = (adjacent side)**sin** $\theta$. Use a calculator to evaluate and round the answer to two decimal places as needed.

$$\begin{aligned} \text{altitude} &= (\text{adjacent side})\sin\theta \\ &= 15\sin 48 && \text{Substitute.} \\ &\approx 11.15 && \text{Evaluate.} \end{aligned}$$

Using the lengths of the adjacent side c = 15 and that of the altitude h = 11.15, determine the number of triangles that can be formed.

Because the length of the opposite side is less than the altitude, the is no triangle with the given measurements.

| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Solve the SSA triangle. Indicate whether the given measurements result in no triangle, one triangle, or two triangles. Solve each resulting triangle. Round each answer to the nearest tenth.

$B = 49^\circ \quad b = 37 \quad c = 44$

First, make a chart of the given information.

| | |
|---|---|
| A = ? | a = ? |
| $B = 49^\circ$ | b = 37 |
| C = ? | c = 44 |

Since the known measures are SSA, there may be 0, 1, or 2 triangles with the given measures. Find the number of possible triangles.

In this case $\theta = B = 49^\circ$ is acute and the adjacent side, c = 44, is greater than the opposite side, b = 37.

Since c, the side adjacent to the given angle, is greater than b, the side opposite the given angle, use the altitude to determine the number of possible triangles. To find the altitude, use the formula altitude = (adjacent side)**sin** $\theta$. Use a calculator to evaluate and round the answer to one decimal place.

$$\begin{aligned} \text{altitude} &= (\text{adjacent side})\sin\theta \\ &= 44 \bullet \sin(49) && \text{Substitute.} \\ &\approx 33.2 && \text{Evaluate.} \end{aligned}$$

Use the given lengths of the adjacent side, c = 44, the opposite side, b = 37, and the length of the altitude found above, h = 33.2, to determine the number of possible triangles.

Because altitude < opposite side < adjacent side, there there are two possible triangles having the given measures.

Now, use the Law of Sines to determine the measure of angle C. Use the equation $\frac{b}{\sin B} = \frac{c}{\sin C}$.

Substitute the values for angle B and sides b and c into $\frac{b}{\sin B} = \frac{c}{\sin C}$. Simplify.

$$\frac{37}{\sin 49^\circ} = \frac{44}{\sin C}$$
$$\sin C = \frac{44 \sin 49^\circ}{37}$$
$$\sin C \approx 0.8975$$

Because there are two solutions, find the two angles $C_1$ and $C_2$.

$$\begin{aligned} C_1 &\approx \sin^{-1}(0.8975) \\ &\approx 63.8^\circ \end{aligned}$$

To find the other angle, subtract $C_1 \approx 63.8^\circ$ from $180^\circ$.

$$\begin{aligned} C_2 &\approx 180^\circ - 63.8^\circ \\ &= 116.2^\circ \end{aligned}$$

Use the given angle, $B = 49^\circ$, and the angle found above, $C_1 = 63.8^\circ$, to find the measure of the third angle $\angle BAC_1$. Recall that the sum of the measures of the angles of a triangle is $180^\circ$.

$$\begin{aligned} \angle BAC_1 &\approx 180^\circ - 49^\circ - 63.8^\circ \\ &= 67.2^\circ \end{aligned}$$

Now, find the length of side $a_1$ using the Law of Sines.

$$\frac{a_1}{\sin \angle BAC_1} = \frac{b}{\sin B}$$

Substitute the measures of angle B and $\angle BAC_1$ and the length of side b. Simplify.

$$\frac{a_1}{\sin 67.2^\circ} = \frac{37}{\sin 49^\circ}$$
$$a_1 \approx \frac{37 \sin 67.2^\circ}{\sin 49^\circ}$$
$$\approx 45.2$$

Use the given angle, $B = 49^\circ$, and the angle found above, $C_2 = 116.2^\circ$, to find the measure of the third angle, $\angle BAC_2$. Recall that the sum of the measures of the angles of a triangle is $180^\circ$.

$$\angle BAC_2 \approx 180^\circ - 49^\circ - 116.2^\circ$$
$$= 14.8^\circ$$

Now, find the length of side $a_2$ using the Law of Sines.

$$\frac{a_2}{\sin \angle BAC_2} = \frac{b}{\sin B}$$

Substitute the known values and solve for $a_2$.

$$\frac{a_2}{\sin 14.8^\circ} = \frac{37}{\sin 49^\circ}$$
$$a_2 \approx \frac{37 \sin 14.8^\circ}{\sin 49^\circ}$$
$$\approx 12.5$$

Therefore, the two possible solved triangles are shown in the tables below.

**$\Delta BAC_1$**

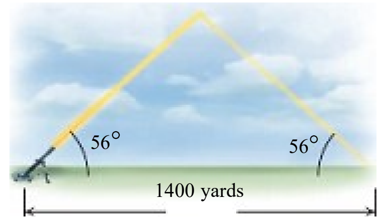
| | |
|---|---|
| $\angle BAC_1 = 67.2^\circ$ | $a_1 \approx 45.2$ |
| $B = 49^\circ$ | $b = 37$ |
| $C_1 \approx 63.8^\circ$ | $c = 44$ |

**$\Delta BAC_2$**

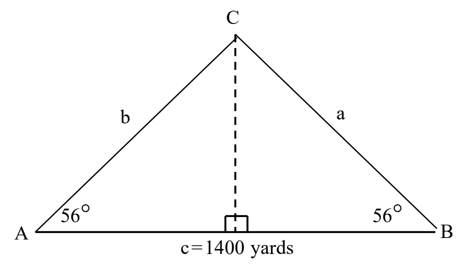
| | |
|---|---|
| $\angle BAC_2 = 14.8^\circ$ | $a_2 \approx 12.5$ |
| $B = 49^\circ$ | $b = 37$ |
| $C_2 \approx 116.2^\circ$ | $c = 44$ |

| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

A laser beam with an angle of elevation of $56^\circ$ is reflected by a target and is received 1400 yards from the point of origin. Assume that the trajectory of the beam forms (approximately) an isosceles triangle.
**a**. Find the total distance the beam travels.
**b**. What is the height of the target?




**a.** Begin by setting up a triangle representing the situation and labeling each of the sides and angles.




In order to find the total distance the beam travels, use the Law of Sines and solve the triangle. First determine the value of the missing angle, C.

$$\begin{aligned} C &= 180^\circ - A - B \\ &= 180^\circ - 56^\circ - 56^\circ \\ &= 68^\circ \end{aligned}$$

The Law of Sines states that in any triangle ABC, with sides of length a, b, and c, the following is true.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Use the proportion $\frac{\sin A}{a} = \frac{\sin C}{c}$ and solve for a.

$$\frac{\sin 56^\circ}{a} = \frac{\sin 68^\circ}{1400} \qquad \text{Substitute values.}$$

$$\frac{1400 \cdot \sin 56^\circ}{\sin 68^\circ} = a \qquad \text{Solve for a.}$$

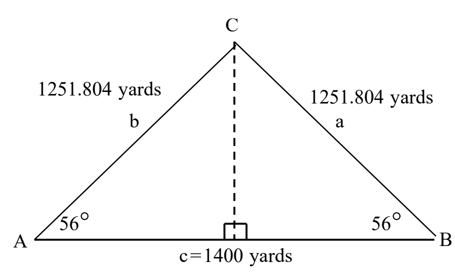
$$1251.804 \approx a \qquad \text{Use a calculator.}$$

Due to the symmetry of an isosceles triangle, the value of b is equal to the value of a.

$$b \approx 1251.804$$

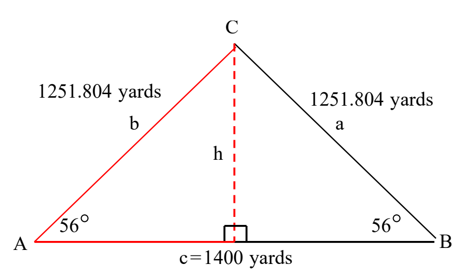
The completed triangle is shown. The total distance the beam travels is the sum of the lengths of sides a and b.

$$1251.804 \text{ yds} + 1251.804 \text{ yds} \approx 2504 \text{ yds}$$




The total distance the beam travels is 2504 yards.

**b.** Use the Law of Sines again to determine the maximum height of the target, h. We limit our focus to the right triangle shown in red.




Applying the Law of Sines to the right triangle obtains the following proportion.

$$\frac{\sin 90^\circ}{1251.804} = \frac{\sin 56^\circ}{h}$$

Solve for h.

$$\frac{\sin 90^\circ}{1251.804} = \frac{\sin 56^\circ}{h}$$

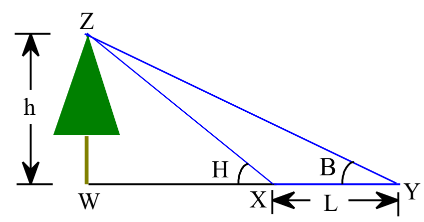
$$h = \frac{1251.804 \cdot \sin 56^\circ}{\sin 90^\circ}$$

$$h \approx 1038$$

Thus, the height of the target at its highest point is approximately 1038 yards.

| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

Pat needs to determine the height of a tree before cutting it down to be sure that it will not fall on a nearby fence. The angle of elevation of the tree from one position on a flat path from the tree is $H = 50^\circ$, and from a second position $L = 20$ feet farther along this path it is $B = 40^\circ$. What is the height of the tree?




The height h of the tree is the vertical side of the right triangle ZWX. The angle opposite the vertical side is angle H in the diagram. Before we can determine h we need to find the length of the side ZX in triangle XYZ. We can obtain this side by solving triangle XYZ.

We solve an ASA triangle using the Law of Sines.

Law of Sines

For a triangle with sides a, b, c and opposite angles A, B, C respectively,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

First we need to find the angles ZXY ( = A) and XZY ( = C), given the values for angles H and B.

$$A = 180^\circ - H = 180^\circ - 50^\circ = 130^\circ$$

Now, we find the value of angle C. We use the fact that the sum of the angles of any triangle equals $180^\circ$.

$$C = 180^\circ - A - B = 180^\circ - 130^\circ - 40^\circ = 10^\circ$$

We can now determine side b using the Law of Sines. Side b represents the distance from X to Z. The distance L corresponds to side c.

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin 40^\circ}{b} = \frac{\sin 10^\circ}{20}$$

$$b \approx 74.03 \text{ ft}$$

We can use the Law of Sines once again, this time to find h, using information for triangle ZWX. h is the side opposite angle H, while side b of triangle XYZ is the side opposite the $90^\circ$ angle.

$$\frac{\sin H}{h} = \frac{\sin 90^\circ}{b}$$

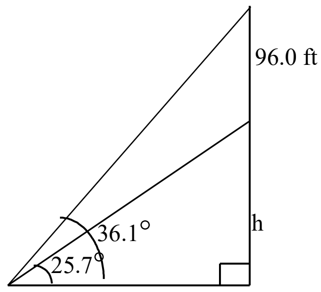
$$\frac{\sin 50^\circ}{h} = \frac{\sin 90^\circ}{74.03}$$

$$h \approx 56.7 \text{ ft}$$

| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

A flagpole 96.0 ft tall is on the top of a building. From a point on level ground, the angle of elevation of the top of the flagpole is 36.1°, while the angle of elevation of the bottom of the flagpole is 25.7°. Find the height of the building.

Let h represent the height of the building. A diagram showing all of the information is on the right. (The diagram is not drawn to scale.) It is possible to use the law of sines to find h, but a few angle and side measures must be determined first. Recall the law of sines formula for a triangle ABC with sides of length a, b, and c.



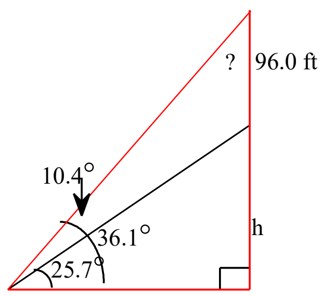

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

First, calculate the difference between the two angles of elevation.

$$36.1^\circ - 25.7^\circ = 10.4^\circ$$

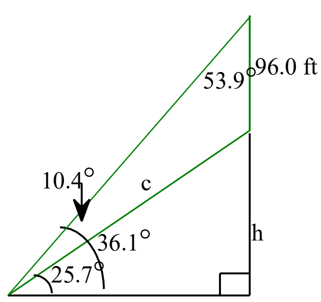
Next, find the measure of the angle at the top of the flagpole.

Note that the other two angles in the large triangle have measures of 36.1° and 90°, and the three angles must sum to 180°.




$$180^\circ - 36.1^\circ - 90^\circ = 53.9^\circ$$

Let c represent the distance from the point to the bottom of the flagpole. Use the law of sines with the upper triangle to find c.

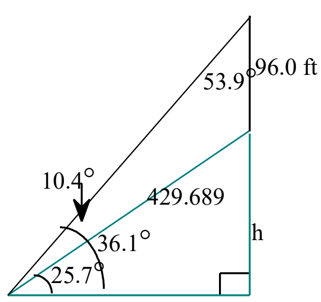



$$\frac{c}{\sin 53.9^\circ} = \frac{96.0}{\sin 10.4^\circ}$$ Substitute the known values.

$$c = \frac{96.0(\sin 53.9^\circ)}{\sin 10.4^\circ}$$ Multiply both sides by $\sin 53.9^\circ$.

$$c \approx 429.689 \text{ ft}$$ Use a calculator.

Use the value of c and the law of sines in the lower triangle to find the height of the building, h.




$$\frac{h}{\sin 25.7^\circ} = \frac{429.689}{\sin 90^\circ}$$ Substitute the known values.

$$h = \frac{429.689(\sin 25.7^\circ)}{\sin 90^\circ}$$ Multiply both sides by $\sin 25.7^\circ$.

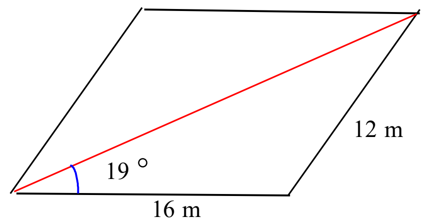
$$h \approx 186 \text{ ft}$$ Use a calculator.

The building is about 186 feet tall.

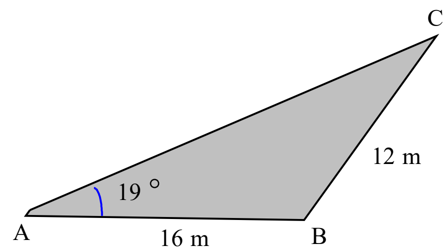
| | | |
|---|---|---|
| **Student:** Cole Lamers<br>**Date:** 07/21/19 | **Instructor:** Kelly Galarneau<br>**Course:** CA&T Internet (70263) Galarneau | **Assignment:** 7.1 The Law of Sines |

The sides of a parallelogram are 16 m and 12 m, and the longer diagonal makes an angle of $19^\circ$ with the longer side. Find the length of the longer diagonal.

Use the given information and draw the parallelogram. The sides of a parallelogram are 16 m and 12 m, and the longer diagonal makes an angle of $19^\circ$ with the longer side. The figure is shown to the right.




Recall that the parallelogram is made up of two congruent triangles. Therefore, consider the shaded triangle. To find the length of the longest diagonal, find the length of side b as shown in figure.




Apply the law of sines to find angle C, rounding to four decimal places.

$$\frac{\sin C}{c} = \frac{\sin A}{a} \qquad \text{The law of sines.}$$

$$\sin C = \frac{c \sin A}{a} \qquad \text{Multiply both sides by c.}$$

$$\sin C = \frac{16 \sin 19^\circ}{12} \qquad \text{Substitute values.}$$

$$\sin C \approx 0.4341 \qquad \text{Use a calculator.}$$

Since $\sin C$ is between 0 and 1, there are two possible values of C, rounding to one decimal palce.

$$C = \sin^{-1}(0.4341) = 25.7^\circ$$

Two possible values of C are as follows.

$$C_1 \approx 25.7^\circ$$

$$C_2 \approx 180^\circ - 25.7^\circ = 154.3^\circ$$

Now, if $A + C_1 \geq 180^\circ$ there is no triangle. If $\sin C \neq 1$, with $A + C_2 < 180^\circ$, then there are two triangles. Otherwise, there is only one triangle. Find $A + C_{1.}$

$$\begin{aligned} A + C_1 &= 19^\circ + 25.7^\circ \\ &= 44.7^\circ \end{aligned}$$

Find $A + C_2$.

$$\begin{aligned} A + C_2 &= 19^\circ + 154.3^\circ \\ &= 173.3^\circ \end{aligned}$$

Since $A + C_1 = 44.7^\circ < 180^\circ$ and $A + C_2 = 173.3^\circ < 180^\circ$, there are two triangles with vertex $C_1$ and $C_2$.

Now find the third angle of the triangle. Recall that the sum of all the angles in a triangle is $180^\circ$.

$$\angle ABC_1 = 180^\circ - 19^\circ - 25.7^\circ = 135.3^\circ$$

$$\angle ABC_2 = 180^\circ - 19^\circ - 154.3^\circ = 6.7^\circ$$

Now, find the length of side $b_1$ using the law of sines.

$$\frac{b_1}{\sin \angle ABC_1} = \frac{a}{\sin A}$$

Substitute the known values and solve for the length of side $b_1$, rounding to one decimal place.

$$\frac{b_1}{\sin \angle ABC_1} = \frac{a}{\sin A}$$

$$b_1 = \frac{a \sin \angle ABC_1}{\sin A}$$

$$b_1 \approx \frac{12 \sin 135.3^\circ}{\sin 19^\circ}$$

$$b_1 \approx 25.9$$

Now, find the length of side $b_2$ using the law of sines.

$$\frac{b_2}{\sin \angle ABC_2} = \frac{a}{\sin A}$$

Substitute the known values and solve for the length of side $b_2$, rounding to one decimal place.

$$\frac{b_2}{\sin \angle ABC_2} = \frac{a}{\sin A}$$

$$b_2 = \frac{a \sin \angle ABC_2}{\sin A}$$

$$b_2 \approx \frac{12 \sin 6.7^\circ}{\sin 19^\circ}$$

$$b_2 \approx 4.3$$

The length of side b is either 4.3 m or 25.9 m. Therefore the length of longest diagonal is 25.9 m.