Student: Cole Lamers **Date:** 07/23/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.3 Area of Polygons Using Trigonometry

Find the area of the triangle ABC.

$A = 50.6^\circ \quad B = 32.1^\circ \quad c = 27.5\text{ m}$

The given information is two angles and one side length. Use the information to find a second side length, to use in an area formula.

First, find the measure of angle C. Recall that all the angles of a triangle add to 180°, so $C = 180^\circ - A - B$. Substitute the values for A and B.

$$C = 180^\circ - 50.6^\circ - 32.1^\circ$$
$$C = 97.3^\circ$$

Now that side c and angles B and C are known, use the law of sines to find side b.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

Substitute the known values in the formula.

$$\frac{b}{\sin 32.1^\circ} = \frac{27.5}{\sin 97.3^\circ}$$

Solve this equation for b.

$$b = \sin 32.1^\circ \cdot \frac{27.5}{\sin 97.3^\circ} \approx 14.733$$

The length of side b is 14.733 m (rounded to the nearest thousandth).

Since you now know angle A and sides b and c, use the following formula to find the area.

$$\text{Area} = \frac{1}{2} bc \sin A$$

Substituting the values into the area formula gives the following.

$$\text{Area} = \frac{1}{2}(14.733)(27.5) \sin 50.6^\circ$$

$$= 156.5\text{ m}^2 \text{ (rounded to the nearest tenth)}$$

Student: Cole Lamers **Date:** 07/23/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.3 Area of Polygons Using Trigonometry

Find the area of AAS triangle ABC.

$b = 15.6 \text{ yd}, \;\; A = 67^\circ, \;\; B = 34^\circ$

First, find the third angle C by the angle sum formula. Recall that the sum of all the angles in a triangle is 180°.

$$C = 180^\circ - A - B$$

$$C = 180^\circ - 67^\circ - 34^\circ$$

$$C = 79^\circ$$

The area K of a triangle ABC with sides a, b, and c is given by the following formulas.

$$K = \frac{a^2 \sin B \sin C}{2 \sin A} \qquad K = \frac{b^2 \sin C \sin A}{2 \sin B} \qquad K = \frac{c^2 \sin A \sin B}{2 \sin C}$$

Because side b is given, use the area formula for an AAS triangle that contains side b. Use the following formula.

$$K = \frac{b^2 \sin C \sin A}{2 \sin B}$$

Substitute the given values in the formula and calculate the area. Round to the nearest tenth.

$$\begin{aligned} K &= \frac{b^2 \sin C \sin A}{2 \sin B} \\ &= \frac{(15.6)^2 \sin 79^\circ \sin 67^\circ}{2 \sin 34^\circ} && \text{Substitute.} \\ &\approx 196.6 && \text{Simplify.} \end{aligned}$$

Therefore, the area of the triangle is approximately 196.6 square yards.

Student: Cole Lamers **Date:** 07/23/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.3 Area of Polygons Using Trigonometry

Find the area K of the triangle specified below.

$a = 5, \ b = 7, \ c = 10$

We use the following theorem:

Heron's Formula

The area K of a triangle with sides a, b, and c is

$$K = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$.

First calculate the value of s as specified in the theorem.

$$\begin{aligned} s &= \frac{1}{2}(a+b+c) && \\ &= \frac{1}{2}(5+7+10) && a=5, \ b=7, \ c=10. \\ &= 11.0 && \text{Simplify.} \end{aligned}$$

So $s = 11.0$.

Now use Heron's Formula to calculate the area.

$$\begin{aligned} K &= \sqrt{s(s-a)(s-b)(s-c)} && \text{Heron's Formula.} \\ &= \sqrt{11.0(11.0-5)(11.0-7)(11.0-10)} && a=5, \ b=7, \ c=10, \ s=11.0. \\ &\approx 16.25 && \text{Simplify and round to two decimal places.} \end{aligned}$$

The area K of the triangle is about 16.25 square units.

Student: Cole Lamers **Date:** 07/23/19	**Instructor:** Kelly Galarneau **Course:** CA&T Internet (70263) Galarneau	**Assignment:** 7.3 Area of Polygons Using Trigonometry

Find an angle θ between the sides a and b of a triangle ABC with the given area K.

$K = 10,\ a = 4,\ b = 5\sqrt{2}$

Let θ be the angle between the sides of lengths a and b.

The area of triangle ABC with sides a,b and angle θ is given by $K = \frac{1}{2}ab\ \sin\theta$.

Substitute $K = 10$, $a = 4$, $b = 5\sqrt{2}$ in $K = \frac{1}{2}ab\ \sin\theta$.

$$K = \frac{1}{2}ab\ \sin\theta$$
$$10 = \frac{1}{2}(4)\left(5\sqrt{2}\right)\ \sin\theta$$

Solve for $\sin\theta$ and simplify.

$$10 = \frac{1}{2}(4)\left(5\sqrt{2}\right)\ \sin\theta$$
$$\sin\theta = \frac{\sqrt{2}}{2}$$

Solve for θ.

$$\sin\theta = \frac{\sqrt{2}}{2}$$
$$\theta = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$
$$= 45^\circ$$

The angle between a and b is $\theta = 45^\circ$. However, because $\sin(180^\circ - \theta) = \sin\theta$, another angle satisfying the above conditions can be found by substituing $\theta = 45^\circ$ in $\sin(180^\circ - \theta) = \sin\theta$. Substitute $\theta = 45^\circ$ in $\sin(180^\circ - \theta) = \sin\theta$ and find the second value for θ.

$$\sin(180^\circ - \theta) = \sin\theta$$
$$\sin(180^\circ - 45^\circ) = \sin 45^\circ$$
$$\sin(135^\circ) = \sin 45^\circ = \frac{\sqrt{2}}{2}$$

Therefore, the angle between sides of lengths a and b can be 45° and 135°.