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| **Student:** Cole Lamers<br>**Date:** 09/11/19 | **Instructor:** Viktoriya Shcherban<br>**Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban | **Assignment:** 3.7 Implicit Differentiation |

Use implicit differentiation to find $\frac{dy}{dx}$.

$2x^2y + 3xy^2 = -7$

To use implicit differentiation, first differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

Use the rules of differentiation.

$$\frac{d}{dx}(2x^2y) + \frac{d}{dx}(3xy^2) = \frac{d}{dx}(-7)$$

To find $\frac{d}{dx}(2x^2y)$, use implicit differentiation and the Derivative Product Rule.

$$\frac{d}{dx}(2x^2y) = 2x^2\frac{d}{dx}(y) + y\frac{d}{dx}(2x^2)$$
$$= 2x^2\frac{dy}{dx} + 4xy$$

To find $\frac{d}{dx}(3xy^2)$, again use implicit differentiation and the Derivative Product Rule.

$$\frac{d}{dx}(3xy^2) = 3x\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(3x)$$
$$= 6xy\frac{dy}{dx} + 3y^2$$

To find $\frac{d}{dx}(-7)$, use the Derivative of a Constant Function Rule.

$$\frac{d}{dx}(-7) = 0$$

Simplify.

$$\frac{d}{dx}(2x^2y) + \frac{d}{dx}(3xy^2) = \frac{d}{dx}(-7)$$
$$2x^2\frac{dy}{dx} + 4xy + 6xy\frac{dy}{dx} + 3y^2 = 0$$

Now, collect the terms with $\frac{dy}{dx}$ on one side of the equation.

$$2x^2\frac{dy}{dx} + 6xy\frac{dy}{dx} = -4xy - 3y^2$$

Finally, factor out $\frac{dy}{dx}$ and then solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx}(2x^2 + 6xy) = -4xy - 3y^2$$

$$\frac{dy}{dx} = \frac{-4xy - 3y^2}{2x^2 + 6xy}$$

Thus, using implicit differentiation, $\frac{dy}{dx} = \frac{-4xy - 3y^2}{2x^2 + 6xy}$.

$$2xy + y^2 = x + y$$

$$2y + 2x\frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$2x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} = 1 - 2y$$

$$(2x + 2y - 1)\frac{dy}{dx} = 1 - 2y$$

$$\frac{dy}{dx} = (1 - 2y)/(2x + 2y - 1)$$

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Use implicit differentiation to find dy / dx.

$$4xy + y^2 = 9x + y$$

In implicit differentiation, the first step is to differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

$$\begin{aligned} 4xy + y^2 &= 9x + y \\ \frac{d}{dx}(4xy) + \frac{d}{dx}\left(y^2\right) &= \frac{d}{dx}(9x) + \frac{d}{dx}(y) \end{aligned}$$

Determine the derivative of each expression. Treat xy as a product.

$$4\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + 2y\frac{dy}{dx} = 9 + \frac{dy}{dx}$$

Distribute and simplify.

$$4x\frac{dy}{dx} + 4y + 2y\frac{dy}{dx} = 9 + \frac{dy}{dx}$$

The next step is to collect the terms with dy / dx on one side of the equation. Collect the terms on the left side and factor out dy / dx.

$$(4x + 2y - 1)\frac{dy}{dx} + 4y = 9$$

Now solve for dy / dx. To do so, first combine all the other terms on the right side.

$$(4x + 2y - 1)\frac{dy}{dx} = 9 - 4y$$

Finally, solve for dy / dx by dividing.

$$\frac{dy}{dx} = \frac{9 - 4y}{4x + 2y - 1}$$

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Use implicit differentiation to find $\frac{dy}{dx}$.

$$4y^2 = \frac{3x-2}{3x+2}$$

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To use implicit differentiation, first differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

Use the rules of differentiation.

$$\frac{d}{dx}(4y^2) = \frac{d}{dx}\left(\frac{3x-2}{3x+2}\right)$$

To find $\frac{d}{dx}(4y^2)$, use implicit differentiation and the Power Rule for Positive Integers.

$$\frac{d}{dx}(4y^2) = 8y\frac{dy}{dx}$$

To find $\frac{d}{dx}\left(\frac{3x-2}{3x+2}\right)$, use the Derivative Quotient Rule.

$$\begin{aligned}\frac{d}{dx}\left(\frac{3x-2}{3x+2}\right) &= \frac{3(3x+2)-3(3x-2)}{(3x+2)^2}\\ &= \frac{12}{(3x+2)^2}\end{aligned}$$

The terms with $\frac{dy}{dx}$ are already collected on one side of the equation.

$$8y\frac{dy}{dx} = \frac{12}{(3x+2)^2}$$

Now, solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{3}{2y(3x+2)^2}$$

Thus, using implicit differentiation, $\frac{dy}{dx} = \frac{3}{2y(3x+2)^2}$.

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Find the slope of the curve at the given point.

$4y^7 + 9x^6 = 5y + 8x$ at $(1,1)$

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Use implicit differentiation to find the slope of the curve at a given point.

To use implicit differentiation, first differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

$$4y^7 + 9x^6 = 5y + 8x$$

$$\frac{d}{dx}(4y^7) + \frac{d}{dx}(9x^6) = \frac{d}{dx}(5y) + \frac{d}{dx}(8x)$$

To find $\frac{d}{dx}(4y^7)$ and $\frac{d}{dx}(9x^6)$, use implicit differentiation and the Power Rule for Positive Integers.

$$\frac{d}{dx}(4y^7) = 28y^6\frac{dy}{dx}, \ \frac{d}{dx}(9x^6) = 54x^5$$

To find $\frac{d}{dx}(5y)$ and $\frac{d}{dx}(8x)$, use implicit differentiation, the Constant Multiple Rule, and the Power Rule for Positive Integers.

$$\frac{d}{dx}(5y) = 5\frac{dy}{dx}, \ \frac{d}{dx}(8x) = 8$$

Simplify.

$$\frac{d}{dx}(4y^7) + \frac{d}{dx}(9x^6) = \frac{d}{dx}(5y) + \frac{d}{dx}(8x)$$

$$28y^6\frac{dy}{dx} + 54x^5 = 5\frac{dy}{dx} + 8$$

Now, collect the terms with $\frac{dy}{dx}$ on one side of the equation.

$$28y^6\frac{dy}{dx} - 5\frac{dy}{dx} = 8 - 54x^5$$

Solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{8 - 54x^5}{28y^6 - 5}$$

Now, evaluate $\frac{dy}{dx}$ at $(x,y) = (1,1)$.

$$\left.\frac{dy}{dx}\right|_{(1,1)} = \frac{8 - 54(1)^5}{28(1)^6 - 5} = -2$$

Thus, the slope of the curve $4y^7 + 9x^6 = 5y + 8x$ at $(1,1)$ is $-2$.

$$x^2y + xy^2 = 6$$

$$2xy + x^2\frac{dy}{dx} + 1\cdot y^2 + x\cdot 2y\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = -2xy - y^2$$

$$(x^2 + 2xy)\cdot\frac{dy}{dx} = -(2xy + y^2)$$

$$\frac{dy}{dx} = -\frac{2xy + y^2}{x^2 + 2xy}$$

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The given point is on the curve. Find the lines that are **(a)** tangent and **(b)** normal to the curve at the given point.

$x^2 + xy - y^2 = 11, (5,7)$

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Recall that the derivative dy / dx is the slope of the line that is tangent to the curve at point (x,y). To find the slope of the curve at (5,7), first use implicit differentiation to find a formula for dy / dx. In implicit differentiation, the first step is to differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

$$\begin{aligned} x^2 + xy - y^2 &= 11 \\ \frac{d}{dx}\left(x^2\right) + \frac{d}{dx}(xy) - \frac{d}{dx}\left(y^2\right) &= \frac{d}{dx}(11) \end{aligned}$$

Determine the derivative of each expression. Treat xy as a product.

$$2x + \left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) - 2y\frac{dy}{dx} = 0$$

The next step is to collect the terms with dy / dx on one side of the equation. Collect the terms on the left side and factor out dy / dx.

$$(x - 2y)\frac{dy}{dx} + 2x + y = 0$$

Now solve for dy / dx. To do so, first combine all the other terms on the right side.

$$(x - 2y)\frac{dy}{dx} = -2x - y$$

Finally, solve for dy / dx by dividing.

$$\frac{dy}{dx} = \frac{-2x - y}{x - 2y}$$

**(a)** Evaluate the derivative at (x,y) = (5,7).

$$\begin{aligned} \left.\frac{dy}{dx}\right|_{(5,7)} &= \left.\frac{-2x - y}{x - 2y}\right|_{(5,7)} \\ &= \frac{-2(5) - 7}{5 - 2(7)} \\ &= \frac{17}{9} \end{aligned}$$

The tangent at (5,7) is the line through (5,7) with slope $\frac{17}{9}$. Find this line by substituting the slope and the coordinates of the point into the point-slope form of the equation of a line.

$$y = y_1 + m\left(x - x_1\right)$$

$$y = 7 + \frac{17}{9}(x - 5) = \frac{17}{9}x - \frac{22}{9}$$

**(b)** The normal to the curve at (5,7) is the line perpendicular to the tangent at (5,7). Remember that if two nonvertical lines are perpendicular, then each slope is the negative reciprocal of the other. Thus, the slope of this line is $-\frac{9}{17}$.

Therefore, the normal to the curve at (5,7) is the line through (5,7) with slope $-\frac{9}{17}$. Find the equation of this line.

$$y = 7 - \frac{9}{17}(x - 5)$$

$$y = -\frac{9}{17}x + \frac{164}{17}$$

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The given point is on the curve. Find the lines that are **(a)** tangent and **(b)** normal to the curve at the given point.

$6x^2 + 8xy + 4y^2 + 17y - 6 = 0, \; (-1,0)$

Recall that the derivative dy / dx is the slope of the line that is tangent to the curve at point (x,y). To find the slope of the curve at ( − 1,0), first use implicit differentiation to find a formula for dy / dx. In implicit differentiation, the first step is to differentiate both sides of the equation with respect to x, treating y as a differentiable function of x.

$$6x^2 + 8xy + 4y^2 + 17y - 6 = 0$$
$$\frac{d}{dx}\left(6x^2\right) + \frac{d}{dx}(8xy) + \frac{d}{dx}\left(4y^2\right) + \frac{d}{dx}(17y) - \frac{d}{dx}(6) = \frac{d}{dx}(0)$$

Determine the derivative of each expression. Treat xy as a product.

$$12x + 8\left(x\frac{dy}{dx} + y\frac{dx}{dx}\right) + 8y\frac{dy}{dx} + 17\frac{dy}{dx} - 0 = 0$$

The next step is to collect the terms with dy / dx on one side of the equation. Collect the terms on the left side and factor out dy / dx.

$$(8x + 8y + 17)\frac{dy}{dx} + 12x + 8y = 0$$

Now solve for dy / dx. To do so, first combine all the other terms on the right side.

$$(8x + 8y + 17)\frac{dy}{dx} = -12x - 8y$$

Finally, solve for dy / dx by dividing.

$$\frac{dy}{dx} = \frac{-12x - 8y}{8x + 8y + 17}$$

**(a)** Evaluate the derivative at (x,y) = ( − 1,0).

$$\left.\frac{dy}{dx}\right|_{(-1,0)} = \left.\frac{-12x - 8y}{8x + 8y + 17}\right|_{(-1,0)}$$
$$= \frac{-12(-1) - 8(0)}{8(-1) + 8(0) + 17}$$
$$= \frac{4}{3}$$

The tangent at ( − 1,0) is the line through ( − 1,0) with slope $\frac{4}{3}$. Find this line by substituting the slope and the coordinates of the point into the point-slope form of the equation of a line.

$$y = y_1 + m\left(x - x_1\right)$$
$$y = 0 + \frac{4}{3}(x - (-1)) = \frac{4}{3}x + \frac{4}{3}$$

**(b)** The normal to the curve at $(-1,0)$ is the line perpendicular to the tangent at $(-1,0)$. Remember that if two nonvertical lines are perpendicular, then each slope is the negative reciprocal of the other. Thus, the slope of this line is $-\frac{3}{4}$.

Therefore, the normal to the curve at $(-1,0)$ is the line through $(-1,0)$ with slope $-\frac{3}{4}$. Find the equation of this line.

$$y = 0-\frac{3}{4}(x-(-1))$$

$$y = -\frac{3}{4}x-\frac{3}{4}$$