Student: Cole Lamers **Date:** 10/01/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.1-5.2 Area, Sigma Notation and Limits of Finite

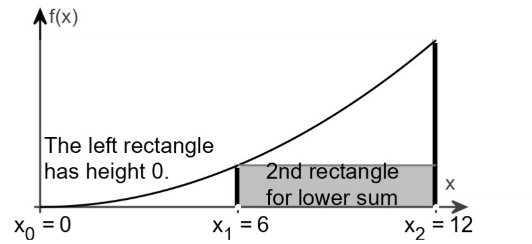
Use finite approximation to estimate the area under the graph of $f(x) = 5x^2$ and above the graph of $f(x) = 0$ from $x_0 = 0$ to $x_2 = 12$ using

i) a lower sum with two rectangles of equal width.
ii) a lower sum with four rectangles of equal width.
iii) an upper sum with two rectangles of equal width.
iv) an upper sum with four rectangles of equal width.

To find a two-rectangle estimate, upper or lower, begin by dividing the interval $x_0 = 0$ to $x_2 = 12$ into two equal parts. The left half of the interval is 0 to 6. The right half is 6 to 12.

For the lower sum estimate find f(0) and f(6), f(0) = 0, f(6) = 180.

Approximate the area under f(x) with two rectangles: one with the width, $x_1 - x_0$, times the height, $f(x_0)$; the other with the width, $x_2 - x_1$, times the height $f(x_1)$.



Estimate the area using the lower sum with two rectangles.

$$\begin{aligned} A &= (x_1 - x_0) f(x_0) + (x_2 - x_1) f(x_1) \\ &= 1080 \text{ square units} \end{aligned}$$

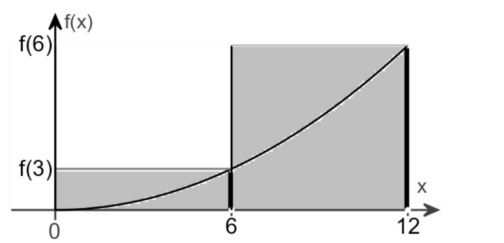
For a four-rectangle lower estimate, divide the interval 0 to 12 into four equal parts. Each of the four parts is 3 units long.

The area is approximated with four rectangles of width 3. The height of the first rectangle is $f(x_0)$, the height of the second is $f(x_1)$, the height of the third is $f(x_2)$, and the height of the fourth is $f(x_3)$.

Estimate the area using the lower sum with four rectangles.

$$\begin{aligned} A &= (x_1 - x_0) f(x_0) + (x_2 - x_1) f(x_1) + (x_3 - x_2) f(x_2) + (x_4 - x_3) f(x_3) \\ &= 1890 \text{ square units} \end{aligned}$$

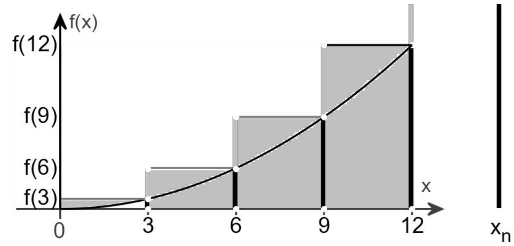
A two-rectangle upper estimate is calculated in a similar fashion, the difference being that the heights of the rectangles are at the right of the width intervals.



Estimate the area using the upper sum with two rectangles.

$$\begin{aligned} A &= (x_1 - x_0) f(x_1) + (x_2 - x_1) f(x_2) \\ &= 5400 \text{ square units} \end{aligned}$$

The four-rectangle upper estimate is the sum of the rectangles shown.



Estimate the area using the lower sum with four rectangles.

$$\begin{aligned} A &= (x_1 - x_0)f(x_1) + (x_2 - x_1)f(x_2) + (x_3 - x_2)f(x_3) + (x_4 - x_3)f(x_4) \\ &= 4050 \text{ square units} \end{aligned}$$

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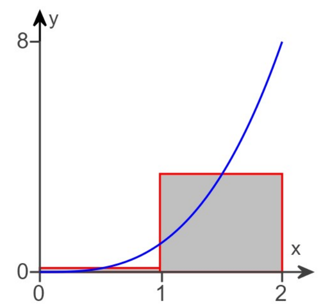
Using rectangles whose height is given by the value of the function at the midpoint of the rectangle's base, estimate the area under the graph using first two and then four rectangles.

$f(x) = x^3$ between $x = 0$ and $x = 2$

The area of a region with a curved boundary can be approximated by summing the areas of a collection of rectangles. Using more rectangles can increase the accuracy of the approximation.

First, approximate the area under f using two rectangles. To do so, the interval [0,2] must be partitioned into two subintervals of equal length. The subintervals are [0,1] and [1,2].

Now, estimate the area under f by finding the area of the two rectangles with bases defined by the subintervals and heights defined by the value of f at the midpoints of the subintervals.



The midpoint of the first subinterval, [0,1], is $\frac{1}{2}$.

Find the height of the first rectangle, $f\left(\frac{1}{2}\right)$.

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3$$
$$= \frac{1}{8}$$

The base of the first rectangle is the length of the first subinterval, 1.

The first rectangle has a height of $\frac{1}{8}$ and a base of 1. Find its area. Remember that the area of a rectangle is its base length times its height.

$$\text{Area} = \frac{1}{8} \cdot 1 = \frac{1}{8}$$

The midpoint of the second subinterval, [1,2], is $\frac{3}{2}$.

Find the height of the second rectangle, $f\left(\frac{3}{2}\right)$.

$$f\left(\frac{3}{2}\right) = \left(\frac{3}{2}\right)^3$$
$$= \frac{27}{8}$$

The second rectangle has a height of $\frac{27}{8}$ and a base of 1.

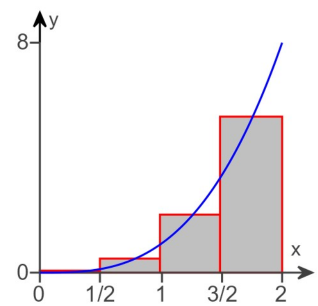
Area = $\frac{27}{8} \cdot 1 = \frac{27}{8}$

The first rectangle has an area of $\frac{1}{8}$ and the second rectangle has an area of $\frac{27}{8}$. The area under f can be approximated by the sum of the areas of the rectangles.

Area = $\frac{1}{8} + \frac{27}{8} = \frac{7}{2}$

Next, approximate the area under f using four rectangles. To do so, the interval [0,2] must be partitioned into four subintervals of equal length. The subintervals are $\left[0,\frac{1}{2}\right]$, $\left[\frac{1}{2},1\right]$, $\left[1,\frac{3}{2}\right]$, and $\left[\frac{3}{2},2\right]$.

Now, estimate the area under f by finding the area of the four rectangles with bases defined by the subintervals and heights defined by the value of f at the midpoints of the subintervals.



The midpoint of the first subinterval, $\left[0,\frac{1}{2}\right]$, is $\frac{1}{4}$.

Find the height of the first rectangle, $f\left(\frac{1}{4}\right)$.

$$f\left(\frac{1}{4}\right) = \left(\frac{1}{4}\right)^3$$
$$= \frac{1}{64}$$

The base of the rectangle is the length of the subinterval, $\frac{1}{2}$.

The first rectangle has a height of $\frac{1}{64}$ and a base of $\frac{1}{2}$.

Area = $\frac{1}{64} \cdot \frac{1}{2} = \frac{1}{128}$

The midpoint of the second subinterval, $\left[\frac{1}{2},1\right]$, is $\frac{3}{4}$.

Find the height of the second rectangle, $f\left(\frac{3}{4}\right)$.

$$f\left(\frac{3}{4}\right) = \left(\frac{3}{4}\right)^3$$
$$= \frac{27}{64}$$

The second rectangle has a height of $\frac{27}{64}$ and a base of $\frac{1}{2}$.

Area $= \frac{27}{64} \cdot \frac{1}{2} = \frac{27}{128}$

The midpoint of the third subinterval, $\left[1,\frac{3}{2}\right]$, is $\frac{5}{4}$.

Find the height of the third rectangle, $f\left(\frac{5}{4}\right)$.

$$f\left(\frac{5}{4}\right) = \left(\frac{5}{4}\right)^3$$
$$= \frac{125}{64}$$

The third rectangle has a height of $\frac{125}{64}$ and a base of $\frac{1}{2}$.

Area $= \frac{125}{64} \cdot \frac{1}{2} = \frac{125}{128}$

The midpoint of the fourth subinterval, $\left[\frac{3}{2},2\right]$, is $\frac{7}{4}$.

Find the height of the fourth rectangle, $f\left(\frac{7}{4}\right)$.

$$f\left(\frac{7}{4}\right) = \left(\frac{7}{4}\right)^3$$
$$= \frac{343}{64}$$

The fourth rectangle has a height of $\frac{343}{64}$ and a base of $\frac{1}{2}$.

Area $= \frac{343}{64} \cdot \frac{1}{2} = \frac{343}{128}$

The four rectangles have areas of $\frac{1}{128}$, $\frac{27}{128}$, $\frac{125}{128}$, and $\frac{343}{128}$, respectively. The area under f can be approximated by the sum of the areas of the rectangles.

Area $= \frac{1}{128} + \frac{27}{128} + \frac{125}{128} + \frac{343}{128} = \frac{31}{8}$

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Write the sum without sigma notation. Then evaluate.

$$\sum_{k=1}^{6} \cos k\pi$$

Recall that sigma notation can be used to write a sum with many terms in the compact form shown below. Note that the index of summation k indicates where the sum begins (at the number below the $\sum$ symbol) and where it ends (at the number above $\sum$).

$$\sum_{k=1}^{n} a_k = a_1 + a_2 + a_3 + \cdots + a_{n-1} + a_n$$

Now write out the sum with one term for each value of k.

$$\sum_{k=1}^{6} \cos k\pi = \cos(1\pi) + \cos(2\pi) + \cos(3\pi) + \cos(4\pi) + \cos(5\pi) + \cos(6\pi)$$

Next simplify each term.

$$\sum_{k=1}^{6} \cos k\pi = -1 + 1 + (-1) + 1 + (-1) + 1$$

Evaluate the sum.

$$\sum_{k=1}^{6} \cos k\pi = 0$$

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Express the sum in sigma notation.

$$\frac{4}{3}+\frac{4}{9}+\frac{4}{27}+\frac{4}{81}+\frac{4}{243}+\frac{4}{729}+\frac{4}{2187}$$

The Greek letter Σ (capital sigma, corresponding to the letter S), stands for "sum." The index of the summation k shows where the sum begins (at the number below the Σ symbol) and where it ends (at the number above Σ).

While any letter can be used to denote the index, for this exercise, use k. A visual representation of sigma notation is shown below.

The index k ends at k = n.

The summation symbol (Greek letter sigma) $\sum_{k=1}^{n} a_k$ a_k is a formula for the kth term.

The index k starts at k = 1.

The formula for generating the terms changes with the lower limit of summation, but the terms generated remain the same. For this exercise, use k = 1.

Since the index begins at k = 1 and there are seven terms in the summation, the upper index value will be 7.

Comparing the denominator of each term to the corresponding index value, k, the kth denominator is 3^k.

The formula for the kth term is $a_k = \frac{4}{3^k}$.

Therefore, in sigma notation, the sum is $\sum_{k=1}^{7} \frac{4}{3^k}$.

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If $\sum_{k=1}^{n} a_k = 5$ and $\sum_{k=1}^{n} b_k = 18$, find the following values.

$$\sum_{k=1}^{n} 4a_k, \quad \sum_{k=1}^{n} \frac{b_k}{18}, \quad \sum_{k=1}^{n} \left(a_k + b_k\right), \quad \sum_{k=1}^{n} \left(a_k - b_k\right), \quad \sum_{k=1}^{n} \left(b_k - 7a_k\right)$$

There are several algebra rules for finite sums that can be applied to simplify expressions in sigma notation.

Since the Constant Multiple Rule states $\sum_{k=1}^{n} ca_k = c\sum_{k=1}^{n} a_k$, $\sum_{k=1}^{n} 4a_k = 4\sum_{k=1}^{n} a_k$.

Since $\sum_{k=1}^{n} a_k = 5$, $\sum_{k=1}^{n} 4a_k = 4\sum_{k=1}^{n} a_k = 20$.

Because $\sum_{k=1}^{n} \frac{b_k}{18} = \sum_{k=1}^{n} \left(\frac{1}{18}\right) b_k = \frac{1}{18}\sum_{k=1}^{n} b_k$, the Constant Multiple Rule applies and $\sum_{k=1}^{n} \frac{b_k}{18} = \frac{1}{18} \cdot 18 = 1$.

The Sum Rule states $\sum_{k=1}^{n} \left(a_k + b_k\right) = \sum_{k=1}^{n} \left(a_k\right) + \sum_{k=1}^{n} \left(b_k\right)$.

So, $\sum_{k=1}^{n} \left(a_k + b_k\right) = 23$.

The Difference Rule is similar to the Sum Rule.

$$\sum_{k=1}^{n} \left(a_k - b_k\right) = \sum_{k=1}^{n} \left(a_k\right) - \sum_{k=1}^{n} \left(b_k\right) = -13.$$

First the Difference Rule and then the Constant Multiple Rule can be applied to $\sum_{k=1}^{n} \left(b_k - 7a_k\right)$.

$$\sum_{k=1}^{n} \left(b_k - 7a_k\right) = \sum_{k=1}^{n} \left(b_k\right) - \sum_{k=1}^{n} \left(7a_k\right) = \sum_{k=1}^{n} \left(b_k\right) - 7\sum_{k=1}^{n} \left(a_k\right) = -17.$$

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Evaluate the following sums.

a. $\sum_{k=1}^{18} k$ **b.** $\sum_{k=1}^{18} k^2$ **c.** $\sum_{k=1}^{18} k^3$

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a. Evaluate the sum $\sum_{k=1}^{18} k$.

The Greek letter Σ (capital sigma, corresponding to the letter S), stands for "sum." The index of the summation k shows where the sum begins (at the number below the Σ symbol) and where it ends (at the number above Σ).

To find the sum of the first n integers, use the formula $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$, where n is the upper index value of the sum written in summation notation.

For $\sum_{k=1}^{18} k$, the value of n, the upper index value of the sum, is 18.

Substitute 18 for n in the formula for the sum of the first n integers, and simplify.

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{18} k = \frac{18(18+1)}{2} \qquad \text{Substitute.}$$

$$= 171 \qquad \text{Simplify.}$$

b. Evaluate the sum $\sum_{k=1}^{18} k^2$.

To find the sum of the first n squares, use the formula $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$, where n is the upper index value of the sum written in summation notation.

Similar to part (a), the value of n, the upper index value of the sum, is 18.

Substitute 18 for n in the formula for the sum of the first n squares, and simplify.

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{18} k^2 = \frac{18(18+1)(2(18)+1)}{6} \qquad \text{Substitute.}$$

$$= 2109 \qquad \text{Simplify.}$$

c. Evaluate the sum $\sum_{k=1}^{18} k^3$.

To find the sum of the first n cubes, use the formula $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$, where n is the upper index value of the sum written in summation notation.

Similar to part (a) and part (b), the value of n, the upper index value of the sum, is 18.

Substitute 18 for n in the formula for the sum of the first n cubes, and simplify.

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{k=1}^{18} k^3 = \left(\frac{18(18+1)}{2}\right)^2 \qquad \text{Substitute.}$$

$$= 29{,}241 \qquad \text{Simplify.}$$

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Graph the function $f(x) = \sin x$ over the interval $[-\pi, \pi]$. Partition the interval into four subintervals of equal length. Then add to your sketch the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$, given that c_k is the **(a)** left-hand endpoint, **(b)** right-hand endpoint, **(c)** midpoint of the kth subinterval.

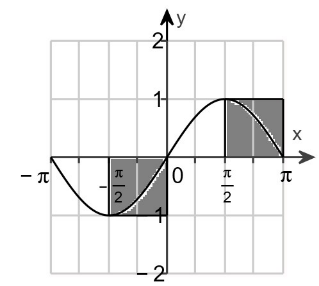
(a) Start by subdividing $[-\pi, \pi]$ into four equal width subintervals. Since all the four rectangles have equal widths, the width of each rectangle is, $\Delta x = \frac{\pi}{2}$. The subintervals are $\left[-\pi, -\frac{\pi}{2}\right]$, $\left[-\frac{\pi}{2}, 0\right]$, $\left[0, \frac{\pi}{2}\right]$, and $\left[\frac{\pi}{2}, \pi\right]$.

Construct rectangles whose height is the value of the function at the subinterval's left-hand endpoint c_k.

$f(c_1) = \sin(-\pi) = 0$ $\qquad f(c_2) = \sin\left(-\frac{\pi}{2}\right) = -1$

$f(c_3) = \sin 0 = 0$ $\qquad f(c_4) = \sin \frac{\pi}{2} = 1$

The graph of the given function with the rectangles associated with the Riemann sum is shown to the right.

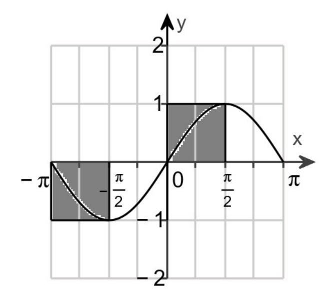


(b) Construct rectangles whose height is the value of the function at the subinterval's right-hand endpoint c_k.

$f(c_1) = \sin\left(-\frac{\pi}{2}\right) = -1$ $\qquad f(c_2) = \sin 0 = 0$

$f(c_3) = \sin \frac{\pi}{2} = 1$ $\qquad f(c_4) = \sin \pi = 0$

The graph of the given function with the rectangles associated with the Riemann sum is shown to the right.

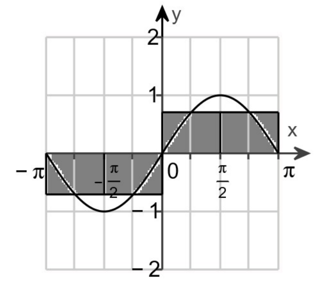


(c) Construct rectangles whose height is the value of the function at the subinterval's midpoint c_k.

$f(c_1) = \sin\left(-\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ $\qquad f(c_2) = \sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

$f(c_3) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ $\qquad f(c_4) = \sin \frac{3\pi}{4} = \frac{1}{\sqrt{2}}$

The graph of the given function with the rectangles associated with the Riemann sum is shown to the right.



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Graph the function f(x) = **cos** x on the interval $[-\pi,\pi]$, showing the addition of the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$ given that c_k is the left endpoint of the kth subinterval.

The interval $-\pi$ to π is divided into four equal subintervals. The subintervals are $\left[-\pi,-\frac{\pi}{2}\right]$, $\left[-\frac{\pi}{2},0\right]$, $\left[0,\frac{\pi}{2}\right]$, and $\left[\frac{\pi}{2},\pi\right]$.

Each subinterval is the base of one of the four rectangles used to approximate the area under the curve.

The heights of the rectangles are the function values at the left endpoints of the intervals. For the given conditions, the height of a rectangle is either 0 or 1.

The height of the left-most rectangle is $f(-\pi)$, which is -1. Notice that the height is still 1, but because the graph is below the axis, the value of $f(-\pi)$ is negative.

The height of the second rectangle counting from the left is $f\left(-\frac{\pi}{2}\right)$, which is 0.

The height of the third rectangle counting from the left is f(0), which is 1.

The height of the right-most rectangle is $f\left(\frac{\pi}{2}\right)$, which is 0.

The graph that meets the specified condition is below.