Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area B

Use the Substitution Formula to evaluate the integrals $\int_{0}^{9} \sqrt{y+16}\,dy$ and $\int_{-7}^{0} \sqrt{y+16}\,dy$.

If g' is continuous on the interval [a,b] and f is continuous on the range of g, then $\int_{a}^{b} f(g(x)) \cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Using the Substitution Formula, determine the function $u = g(y)$.

$u = y + 16$

Find du.

$du = dy$

Transform the limits for the first integral.

When $y = 9$, $u = 9 + 16 = 25$. When $y = 0$, $u = 0 + 16 = 16$.

Thus, the given integral $\int_{0}^{9} \sqrt{y+16}\,dy$ is equivalent to the integral $\int_{16}^{25} \sqrt{u}\,du$, where $u = y + 16$ and $du = dy$.

Evaluate the integral $\int_{16}^{25} \sqrt{u}\,du$.

$$\int_{16}^{25} \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big]_{16}^{25} = \frac{122}{3}$$

Transform the limits for the second integral.

When $y = 0$, $u = 0 + 16 = 16$. When $y = -7$, $u = 9$.

Thus, the given integral $\int_{-7}^{0} \sqrt{y+16}\,dy$ is equivalent to the integral $\int_{9}^{16} \sqrt{u}\,du$, where $u = y + 16$ and $du = dy$.

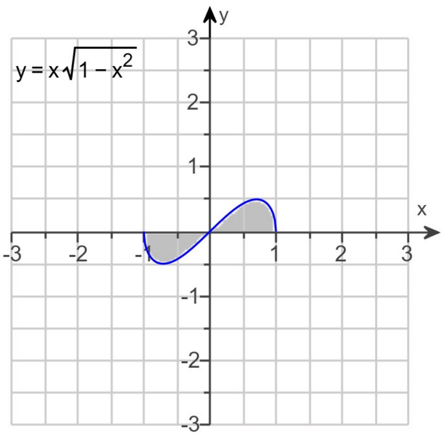
Evaluate the integral $\int_{9}^{16} \sqrt{u}\,du$.

$$\int_{9}^{16} \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big]_{9}^{16} = \frac{74}{3}$$

Thus, $\int_{0}^{9} \sqrt{y+16}\,dy = \frac{122}{3}$ and $\int_{-7}^{0} \sqrt{y+16}\,dy = \frac{74}{3}$.

Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area B

Find the total area of the shaded regions.



Examine the graph of y. Notice this graph is symmetric about the origin. Therefore, the shaded area to the right of the y-axis has the same amount of area as the shaded area to the left of the y-axis.

To find the total area, find the shaded area to the right of the y-axis and multiply by 2. That is, $A = 2\int_a^b h(x)dx$, where a is the lower limit of integration of the shaded area on the right side of the y-axis, and b is the upper limit.

The lower limit of integration of the shaded area on the right side of the y-axis is 0. The upper limit is 1.

The value of the shaded area on the right side of the y-axis is found by evaluating the following integral.

$$\int_0^1 x\sqrt{1-x^2}\,dx$$

If g' is continuous on the interval [a,b] and f is continuous on the range of g, then $\int_a^b f(g(x)) \cdot g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du$.

Using the substitution formula, determine the function $u = g(x)$ for the integral $\int_0^1 x\sqrt{1-x^2}\,dx$.

$u = 1 - x^2$

Find du.

$du = -2x\,dx$

Solve for x dx.

$x\,dx = -\frac{1}{2}\,du$

Now transform the limits of integration. Recall that $u = 1 - x^2$.

When $x = 0$, $u = 1$, and when $x = 1$, $u = 0$.

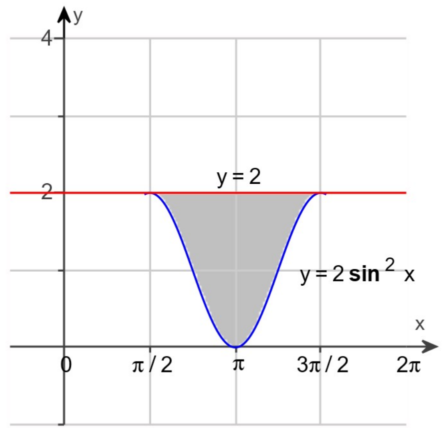
Thus, the integral $\int_0^1 x\sqrt{1-x^2}\,dx$ is equivalent to the integral

$\int_1^0 -\frac{1}{2}\sqrt{u}\,du$, where $u = 1 - x^2$ and $-\frac{1}{2}du = x\,dx$.

Evaluate the integral.

Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area B

Find the total area of the shaded region.



If f and g are continuous with $f(x) \geq g(x)$ throughout [a,b], then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]dx.$$

The upper boundary, f(x), is the curve $y = 2$.

The lower boundary, g(x), is the curve $y = 2\sin^2 x$.

The region runs from $x = \frac{\pi}{2}$ to $x = \frac{3\pi}{2}$.

Thus, the limits of integration are $a = \frac{\pi}{2}$ and $b = \frac{3\pi}{2}$.

Integrate to find the area of the shaded region.

$$A = \int_a^b [f(x) - g(x)]dx$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [(2) - (2\sin^2 x)]dx$$

$$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2(1 - \sin^2 x)dx$$

$$= 2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos^2 x \, dx$$

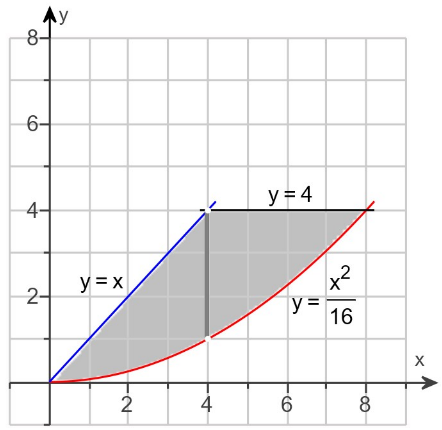
Use the trigonometric identity $\cos^2 x = \frac{1 + \cos(2x)}{2}$.

$$= 2\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{1 + \cos(2x)}{2}dx$$

$$= 2\left(\frac{x}{2} + \frac{\sin(2x)}{4}\right)\Bigg]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area

Find the total area of the shaded region.



If f and g are continuous with $f(x) \geq g(x)$ throughout [a,b], then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b, shown below.

$$A = \int_a^b [f(x) - g(x)]dx$$

When the formula for a bounding curve changes, the area integral changes to become the sum of integrals to match.

First, find the area of the shaded region to the left of vertical line.

The upper boundary, f(x), is the curve $y = x$.

The lower boundary, g(x), is the curve $y = \frac{x^2}{16}$.

The region runs from $x = 0$ to $x = 4$.

Thus, the limits of integration are $a = 0$ and $b = 4$.

Integrate.

$$A = \int_a^b [f(x) - g(x)]dx$$
$$= \int_0^4 \left[(x) - \left(\frac{x^2}{16}\right)\right]dx$$
$$= \left(\frac{x^2}{2} - \frac{x^3}{48}\right)\Bigg]_0^4$$
$$= \frac{20}{3}$$

Thus, the area of the shaded region to the left of the vertical line is $\frac{20}{3}$.

Now, find the area of the shaded region to the right of the vertical line.

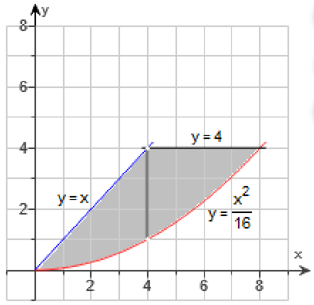
The upper boundary, f(x), is the curve $y = 4$. The lower boundary, g(x), is the curve $y = \frac{x^2}{16}$.

The region runs from $x = 4$ to $x = 8$. Thus, the limits of integration are $a = 4$ and $b = 8$.

Integrate.

$$A = \int_a^b [f(x) - g(x)]dx$$
$$= \int_4^8 \left[(4) - \left(\frac{x^2}{16}\right)\right]dx$$

Find the total area of the shaded region.



If f and g are continuous with $f(x) \ge g(x)$ throughout $[a,b]$, then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b, shown below.

$$A = \int_a^b [f(x) - g(x)]dx$$

When the formula for a bounding curve changes, the area integral changes to become the sum of integrals to match.

First, find the area of the shaded region to the left of vertical line.

The upper boundary, $f(x)$, is the curve $y = x$.

The lower boundary, $g(x)$, is the curve $y = \frac{x^2}{16}$.

The region runs from $x = 0$ to $x = 4$.

Thus, the limits of integration are $a = 0$ and $b = 4$.

Integrate.

$$A = \int_a^b [f(x) - g(x)]dx$$
$$= \int_0^4 \left[(x) - \left(\frac{x^2}{16}\right)\right]dx$$
$$= \left(\frac{x^2}{2} - \frac{x^3}{48}\right)\Bigg]_0^4$$
$$= \frac{20}{3}$$

Thus, the area of the shaded region to the left of the vertical line is $\frac{20}{3}$.

Now, find the area of the shaded region to the right of the vertical line.

The upper boundary, $f(x)$, is the curve $y = 4$. The lower boundary, $g(x)$, is the curve $y = \frac{x^2}{16}$.

The region runs from $x = 4$ to $x = 8$. Thus, the limits of integration are $a = 4$ and $b = 8$.

Integrate.

$$A = \int_a^b [f(x) - g(x)]dx$$
$$= \int_4^8 \left[(4) - \left(\frac{x^2}{16}\right)\right]dx$$
$$= \left(4x - \frac{x^3}{48}\right)\Bigg]_4^8$$
$$= \frac{20}{3}$$

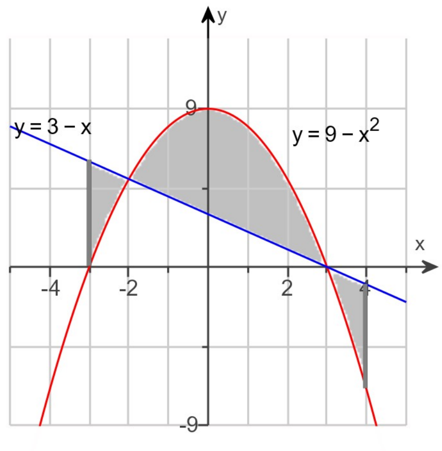
Thus, the area of the shaded region to the right of the vertical line is $\frac{20}{3}$.

The total area of the shaded region is the sum of the area of the region to the left of the vertical line and the area of the region to the right of the vertical line.

Thus, the total area of the shaded region is $\frac{20}{3} + \frac{20}{3} = \frac{40}{3}$.

Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area

Find the total area of the shaded regions.



If f and g are continuous with $f(x) \geq g(x)$ throughout [a,b], then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]dx.$$

First, find the area of the left-most shaded region.

The upper boundary, f(x), is the curve $y = 3 - x$.

The lower boundary, g(x), is the curve $y = 9 - x^2$.

The region runs from $x = -3$ to $x = -2$.

Thus, the limits of integration are $a = -3$ and $b = -2$.

Find the area of the left-most shaded region.

$$\begin{aligned} A &= \int_a^b [f(x) - g(x)]dx \\ &= \int_{-3}^{-2} [(3 - x) - (9 - x^2)]dx \\ &= \int_{-3}^{-2} (x^2 - x - 6)dx \\ &= \left.\left(\frac{x^3}{3} - \frac{x^2}{2} - 6x\right)\right]_{-3}^{-2} \\ &= \frac{17}{6} \qquad \text{Evaluate.} \end{aligned}$$

Thus, the area of the left-most shaded region is $\frac{17}{6}$.

Now, find the area of the middle shaded region.

The upper boundary, f(x), is the curve $y = 9 - x^2$.

The lower boundary, g(x), is the curve $y = 3 - x$.

The region runs from $x = -2$ to $x = 3$.

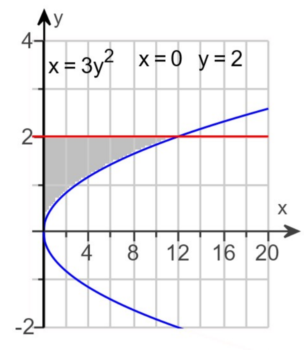
Thus, the limits of integration are $a = -2$ and $b = 3$.

Find the area left of the y-axis.

$$\begin{aligned} A &= \int_a^b [f(x) - g(x)]dx \\ &= \int_{-2}^{3} [(9 - x^2) - (3 - x)]dx \\ &= \int_{-2}^{3} (6 + x - x^2)dx \end{aligned}$$

Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area

Find the area of the region enclosed by the curves $x = 3y^2$, $x = 0$, and $y = 2$.



If f and g are continuous with $f(y) \ge g(y)$ throughout [c,d], then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from c to d is the integral of $(f - g)$ from c to d:

$$A = \int_c^d [f(y) - g(y)]dy.$$

A graph of the equations is shown to the right, with the enclosed region filled.

The right-hand boundary, f(y), is the curve $x = 3y^2$.

The lefthand boundary, g(y), is the curve $x = 0$.

The region runs from $y = 0$ to $y = 2$.

Thus, the limits of integration are $c = 0$ and $d = 2$.

Integrate.

$$A = \int_c^d [f(y) - g(y)]dy = \int_0^2 [(3y^2) - (0)]dy$$

$$= \left(y^3\right)\Big]_0^2$$

$$= 8$$

Thus, the area of the region enclosed by the curves is 8.

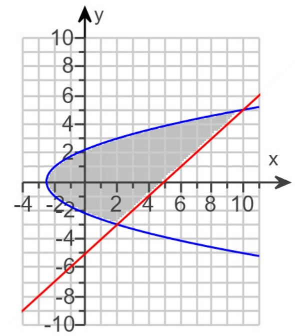
Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area

Find the area of the region enclosed by the curves $y^2 - 2x = 5$ and $x - y = 5$.

If f and g are continuous with $f(y) \geq g(y)$ throughout [c,d], then the area of the region between the curves $x = f(y)$ and $x = g(y)$ from c to d is the integral of $(f - g)$ from c to d:

$$A = \int_c^d [f(y) - g(y)]dy.$$

A graph of the equations is shown to the right, with the enclosed region filled.



The right-hand boundary, f(y), solved for x, is the curve $x = y + 5$.

The left-hand boundary, g(y), solved for x, is the curve $x = \frac{y^2 - 5}{2}$.

The boundary points of the shaded region occur where the upper boundary and the lower boundary intersect. Find the y-values of the points of intersection for the line $x = y + 5$ and the curve $x = \frac{y^2 - 5}{2}$.

The region runs from $y = -3$ to $y = 5$. Thus, the limits of integration are $c = -3$ and $d = 5$.

Find the enclosed area.

$$A = \int_c^d [f(y) - g(y)]dy = \int_{-3}^{5} \left[(y + 5) - \left(\frac{y^2 - 5}{2} \right) \right] dy$$

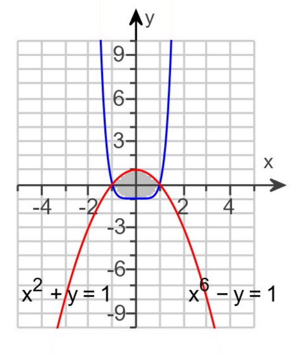
$$= \left[\left(-\frac{y^3}{6} + \frac{y^2}{2} + \frac{15y}{2} \right) \right]_{-3}^{5}$$

$$= \frac{128}{3}$$

Thus, the area of the region enclosed by the curves is $\frac{128}{3}$.

Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area

Find the area of the region enclosed by the curves $x^2 + y = 1$ and $x^6 - y = 1$.



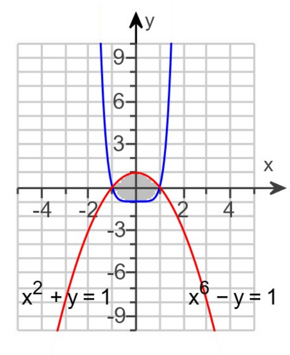
If f and g are continuous with $f(x) \geq g(x)$ throughout [a,b], then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]dx.$$

A graph of the equations is shown to the right, with the enclosed region filled.

The upper boundary, f(x), solved for y, is the curve $y = 1 - x^2$.

The lower boundary, g(x), solve for y, is the curve $y = x^6 - 1$.



The boundary points of the shaded region occur where the upper boundary and the lower boundary intersect. Find the x-values of the points of intersection for the lines $y = 1 - x^2$ and $y = x^6 - 1$. Examine the graph to the right of the shaded region. Notice the x-value of the intersection point to the left of the y-axis is -1 and the x-value of the intersection point to the right of the y-axis is 1.

The region runs from $x = -1$ to $x = 1$.

Thus, the limits of integration are $a = -1$ and $b = 1$.

Find the area enclosed by the curves.

$$A = \int_a^b [f(x) - g(x)]dx = \int_{-1}^{1} [(1 - x^2) - (x^6 - 1)]dx$$

$$= \int_{-1}^{1} (2 - x^2 - x^6)dx$$

$$= \left[\left(-\frac{x^7}{7} - \frac{x^3}{3} + 2x\right)\right]_{-1}^{1}$$

$$= \frac{64}{21}$$

Thus, the area of the region enclosed by the curves is $\frac{64}{21}$.

Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area

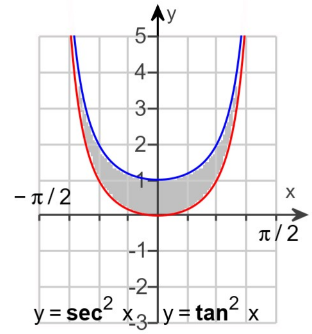
Find the area of the region enclosed by the following curves

$y = \mathbf{sec}^2\, x, \quad y = \mathbf{tan}^2\, x, \quad x = -\frac{\pi}{2}, \quad$ and $\quad x = \frac{\pi}{2}$

If f and g are continuous with $f(x) \geq g(x)$ throughout [a,b], then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b:

$$A = \int_a^b [f(x) - g(x)]dx.$$

A graph of the equations is shown to the right, with the enclosed region filled.



The upper boundary, f(x), is the curve $y = \mathbf{sec}^2\, x$.

The lower boundary, g(x), is the curve $y = \mathbf{tan}^2\, x$.

The region runs from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$.

Thus, the limits of integration are $a = -\frac{\pi}{2}$ and $b = \frac{\pi}{2}$.

Find the area enclosed by the curves.

$$A = \int_a^b [f(x) - g(x)]dx = \int_{-\pi/2}^{\pi/2} [(\mathbf{sec}^2\, x) - (\mathbf{tan}^2\, x)]dx$$

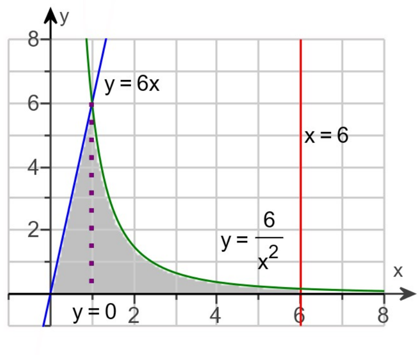
$$= (x)\Big]_{-\pi/2}^{\pi/2}$$

$$= \pi$$

Thus, the area of the region enclosed by the curves is π.

Student: Cole Lamers **Date:** 10/05/19	**Instructor:** Viktoriya Shcherban **Course:** Calc 1 11:30 AM / Internet (81749&81750) Shcherban	**Assignment:** 5.6 Definite Integral Substitutions and the Area

Find the area of the region in the first quadrant bounded by the line $y = 6x$, the line $x = 6$, the curve $y = \frac{6}{x^2}$, and the x-axis.



If f and g are continuous with $f(x) \geq g(x)$ throughout [a,b], then the area of the region between the curves $y = f(x)$ and $y = g(x)$ from a to b is the integral of $(f - g)$ from a to b.

$$A = \int_a^b [f(x) - g(x)]dx$$

A graph of the equations is shown to the left, with the enclosed region filled.

When the formula for a bounding curve changes, the area integral changes to become the sum of integrals to match.

First, find the area of the shaded region to the left of the dotted vertical line.

The upper boundary, f(x), is the curve $y = 6x$.

The lower boundary, g(x), is the curve $y = 0$.

Now determine the boundary points of the region to the left of the dotted vertical line. The lower boundary point occurs at $x = 0$, where the upper boundary line and lower boundary line intersect. The upper boundary point occurs where the dotted vertical line intersects the x-axis.

The region runs from $x = 0$ to $x = 1$.

Thus, the limits of integration are $a = 0$ and $b = 1$.

Find the area of the left shaded region.

$$\begin{aligned} A &= \int_a^b [f(x) - g(x)]dx \\ &= \int_0^1 [(6x) - (0)]dx \\ &= \left(3x^2\right)\Big]_0^1 \\ &= 3 \end{aligned}$$

The area of the shaded region to the left of the dotted vertical line is 3.

Now, find the area of the shaded region to the right of the dotted vertical line.

The upper boundary, f(x), is the curve $y = \frac{6}{x^2}$. The lower boundary, g(x), is the curve $y = 0$.

The region runs from $x = 1$ to $x = 6$. Thus, the limits of integration are $a = 1$ and $b = 6$.

Find the area of the right shaded region.

$$A = \int_a^b [f(x) - g(x)]dx$$