Generate random numbers according to a given distribution

A commonly used technique is called the **Inverse transform** technique. let U be a uniform random variable in the range [0,1]. If $X = F^{-1}(U)$, then X is a random variable with CDF $F_X(x) = F$.

Explanation for the above result:

$F_X(a) = P(X \le a) = P(F^{-1}(U) \le a) = P(U \le F(a)) = F(a)$.

Therefore if we have a random number generator to generate numbers according to the uniform distribution, we can generate any random variable with a known distribution.

Apply to exponential random variable. If X is an exponential random variable, $F_X(x) = 1 - e^{-\lambda x}$. Therefore, for every number u generated with a uniform random number generator, compute x as $x = F^{-1}(u)$ where $F = F_X$. $F^{-1}(u) = -\frac{1}{\lambda}\ln(1-u)$.

The poisson_sp.m file uses the rand function call in matlab to generate uniformly distributed random numbers. It then uses this inverse transform technique to find exponentially distributed random numbers. See the lecture on SP for relation between exponential distribution and Poisson process.