Probability and Bayes Theorem

Biology 683

Lecture 3

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Last week

1. What is the difference in a population and a sample?
2. What is the difference in a parameter and a statistic?
3. How does the central limit theorem help us?

Today

1. Probability

2. Introduction to Bayes

Mutually exclusive events

What is the probability of drawing either an ace or a king from a deck cards?

addition rule: if two events are mutually exclusive then the probability that either will occur is just the sum of the individual probabilities

$$\Pr(A\ or\ B) = \Pr(A) + \Pr(B)$$

$$\Pr(ace\ or\ king) = 15\% = \frac{4}{52} + \frac{4}{52}$$

assumes standard deck with no jokers

Independent events

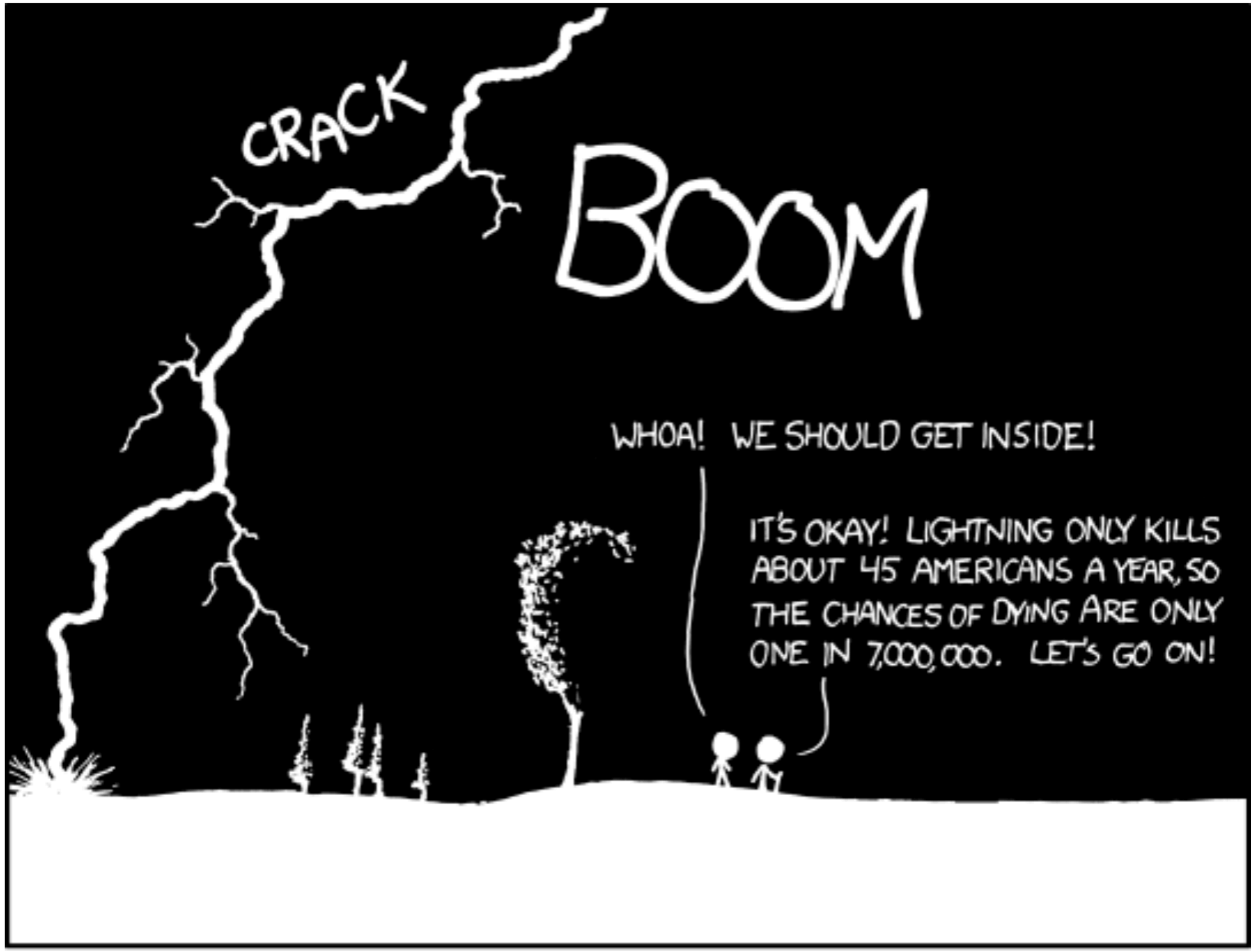
What is the probability of rolling a 1 and then a 2 on a pair of dice?

multiplication rule: if two events are independent of one another then the probability of both occurring is just the product of the individual probabilities.

$$\Pr(A\ then\ B) = \Pr(A) \times \Pr(B)$$

$$\Pr(1\ then\ 2) = 3\% = \frac{1}{6} \times \frac{1}{6}$$

Setting matters



THE ANNUAL DEATH RATE AMONG PEOPLE WHO KNOW THAT STATISTIC IS ONE IN SIX.

Non-independence

non-independent events- conditional probability: is the probability of an event given that another event has already occurred.

For instance, the probability of surviving the sinking of the Titanic was very different than the probability of surviving if you were a female

	died	lived
male	1364	367
female	126	344

Conditional probability

$$\Pr(survivial) = 32\% = \frac{711}{2201}$$

$$\Pr(survivial \mid female) = 73\% = \frac{344}{470}$$

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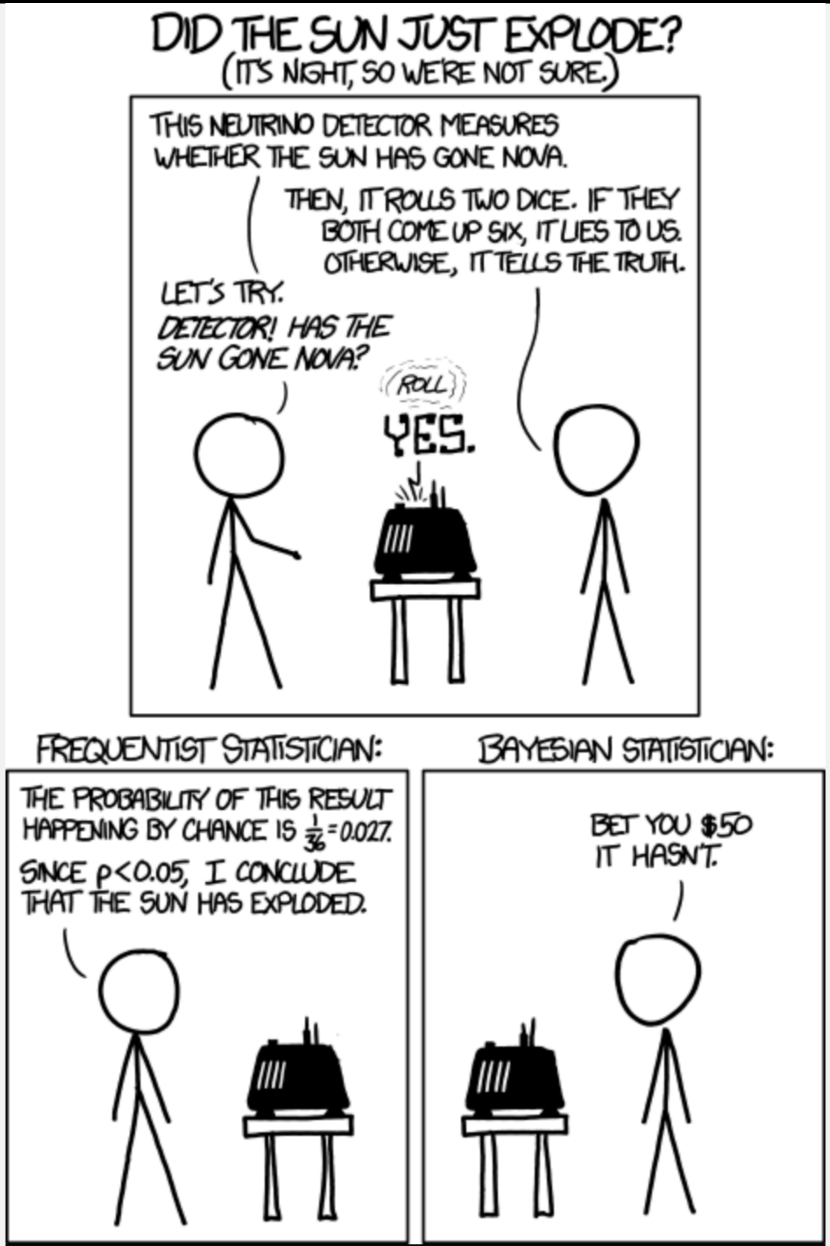
Frequentist and Bayesian



Frequentist approach

You have a null hypothesis and calculate the probability of observing your data under that hypothesis.

Null: The sun has not exploded

$$\Pr(det\ yes \mid sun\ is\ ok) = 0.027 = \frac{1}{6} \times \frac{1}{6}$$

this is less than the typical ∝ level of 0.05 so we reject the null that the sun is ok.

Bayesian approach

$$\Pr(A|B) = \frac{\Pr(B|A) \times \Pr(A)}{\Pr(B)}$$

$$\Pr(nova|det\ yes) = \frac{\Pr(det\ yes|nova) \times \Pr(nova)}{\Pr(\det yes)}$$

$$\Pr(nova|det\ yes) = 0.00000000037\ \frac{1 \times 0.000000001}{0.027}$$

Bayesian approach

Imagine a genetic mutation present in 1 person per 100,000. If you take a test that correctly returns a positive result 99.9% of the time when someone is infected but has a false positive rate of 0.5%.

How likely are you to have the disease?

Should you be concerned?

Bayes' theorem provides a natural way to think about this.

Bayesian approach

$$\Pr(inf|pos.test) = \frac{\Pr(pos.test|inf)\times\Pr(inf)}{\Pr(pos.test)}$$

$$\Pr(inf|pos.test) = 0.001 = \frac{0.999\times0.00001}{0.00501}$$

Which means that you have only a 0.1% chance of having the disease even if you test positive.

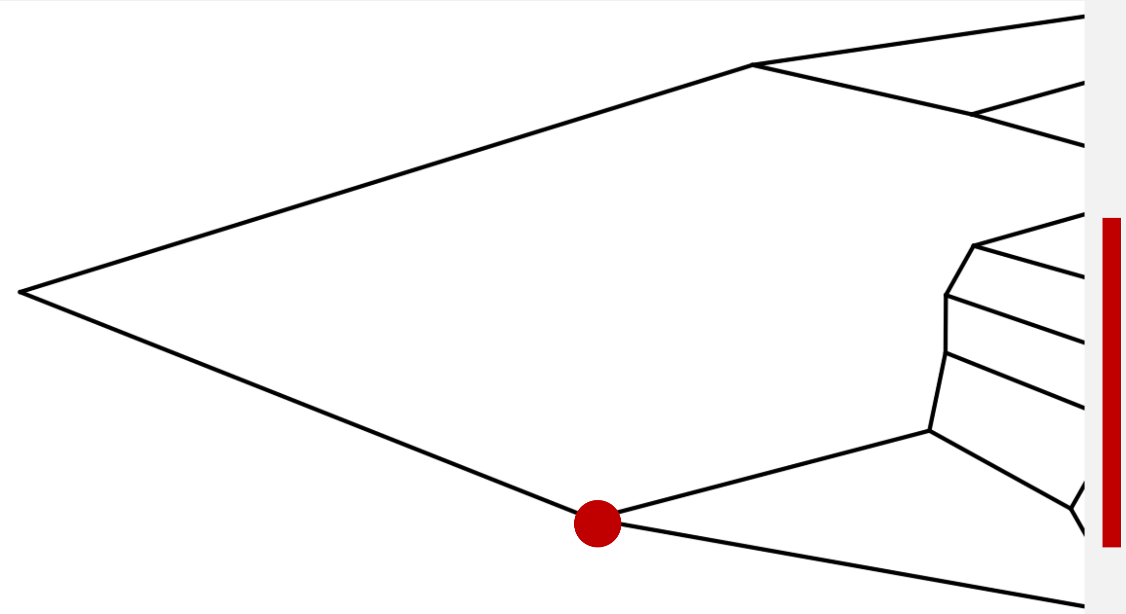
What if your doctor noticed a symptom that made them give you this test what changes?

Priors

$$\Pr(A|B) = \frac{\Pr(B|A) \times \boldsymbol{\Pr(A)}}{\Pr(B)}$$

Pr(A) this is the prior. It is powerful because it allows you to incorporate previous knowledge into your analysis, but it can lead to very bad inference if you are careless.

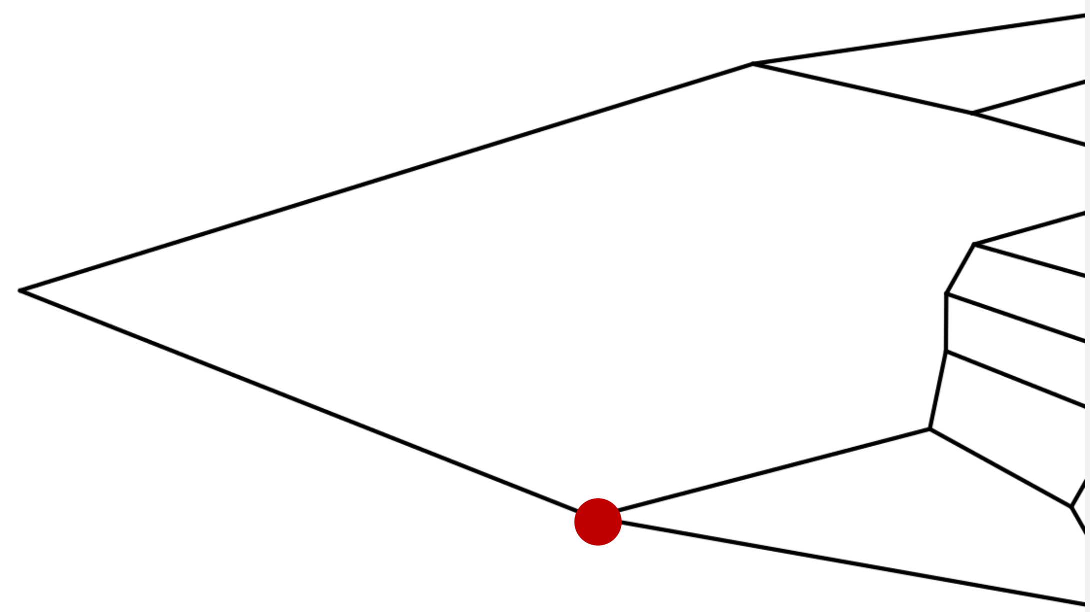
Fossils as Priors

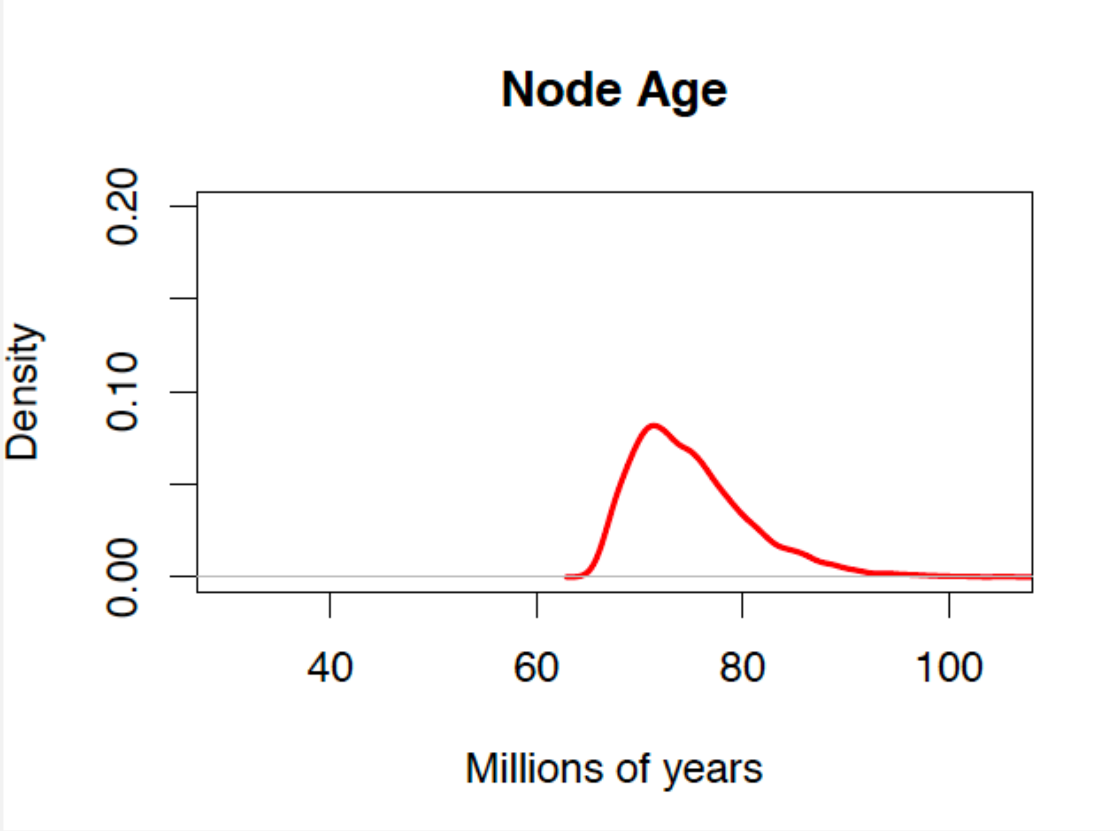


When we use DNA to infer the relationship of extant species we can use fossils to tell us how long ago a group of organisms existed.

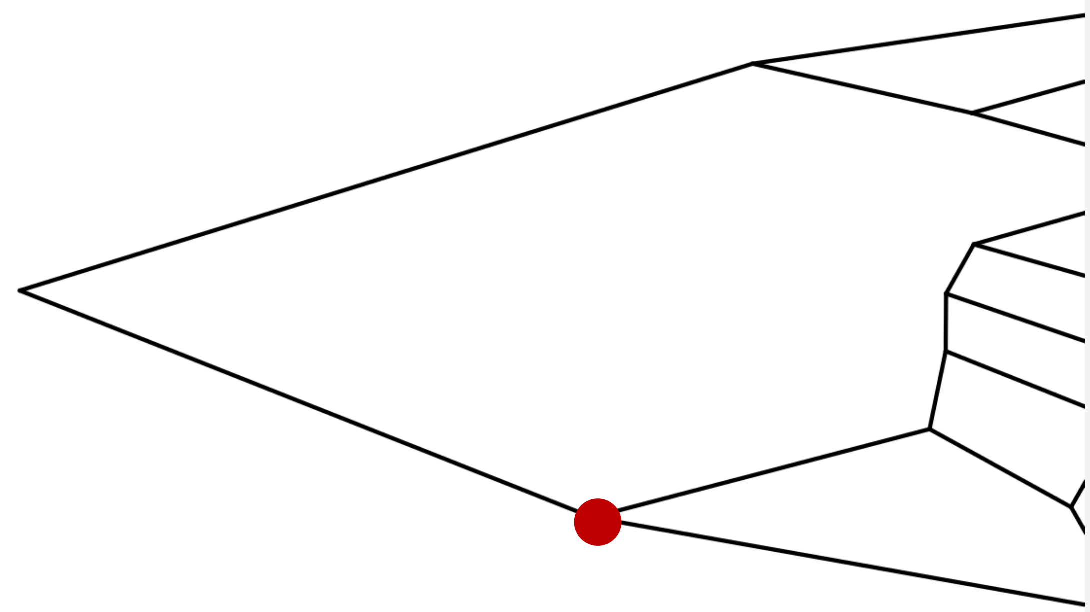
If my fossil 65 MY old then the node in red must be at least 65 MY old as well.

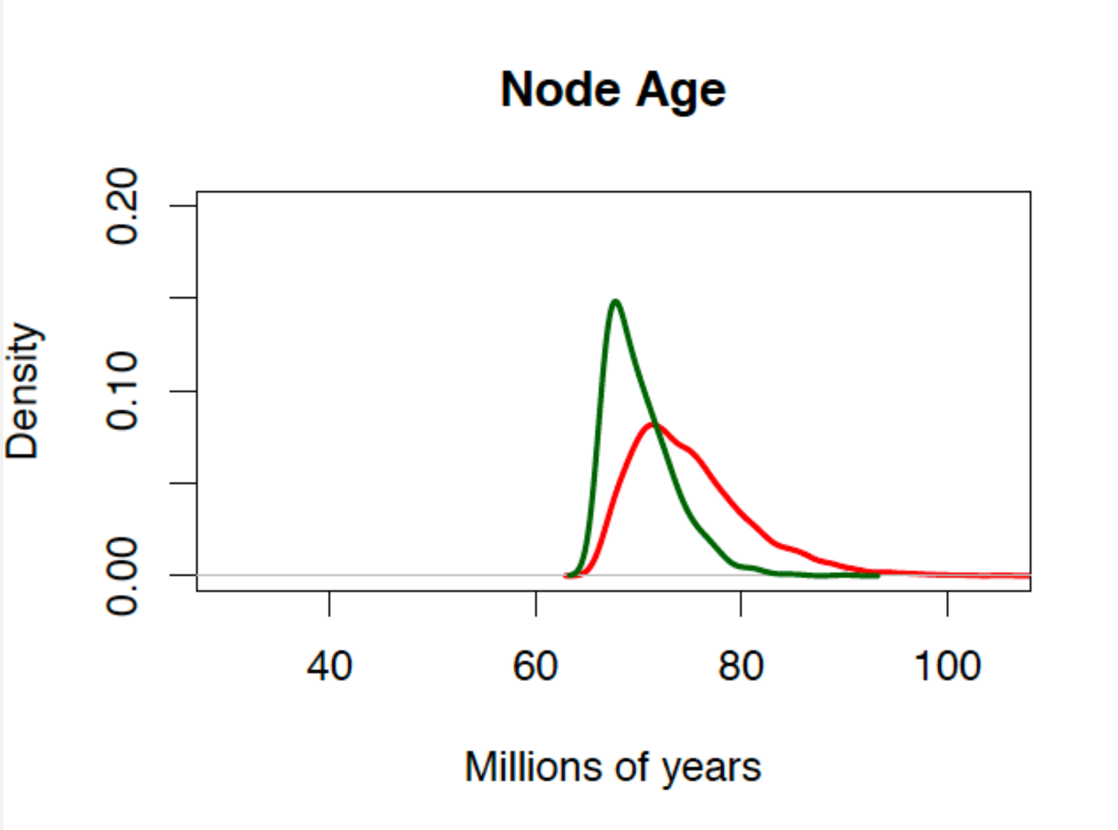
Fossils as Priors



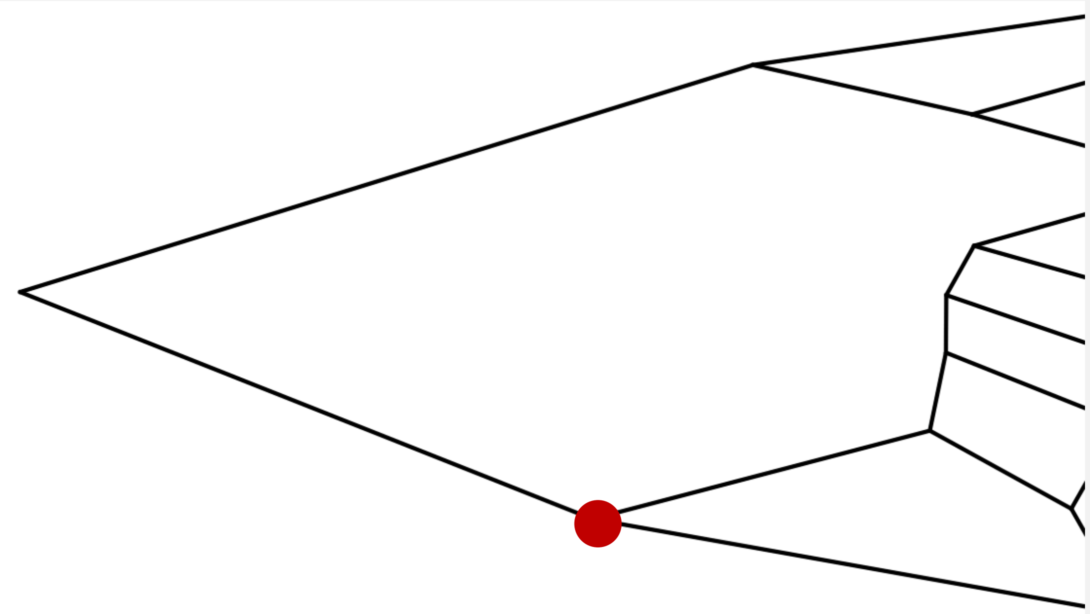


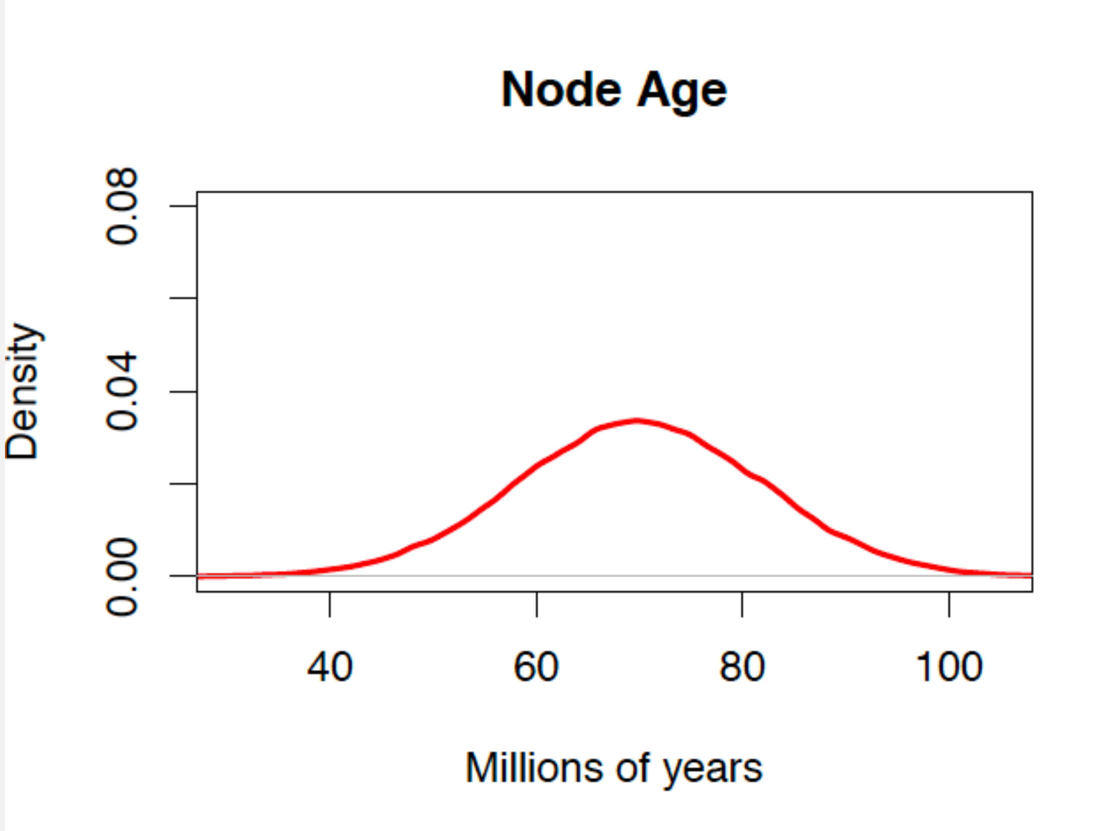
Fossils as Priors



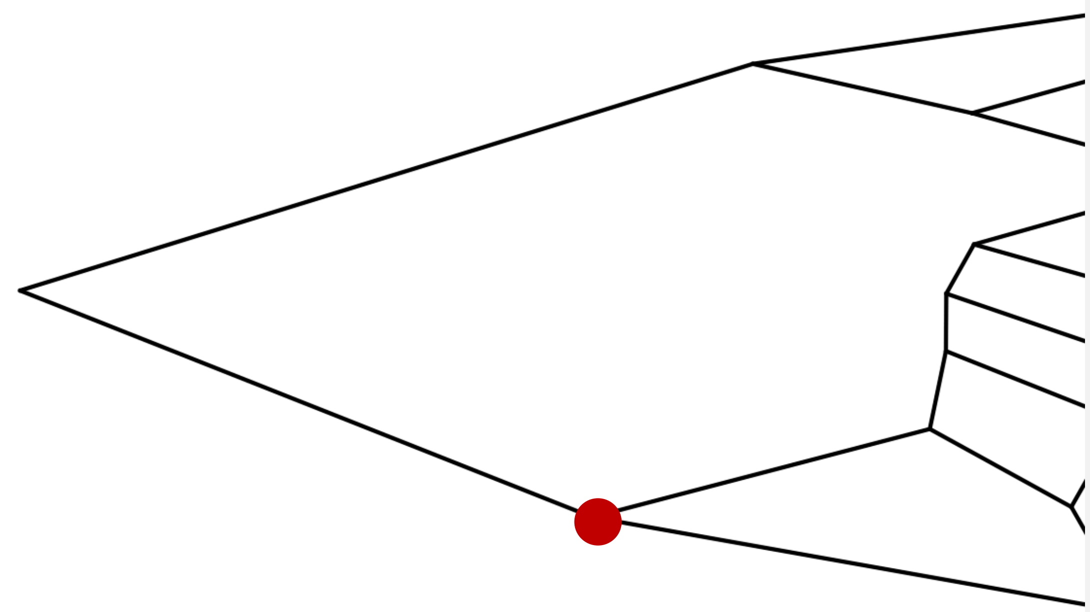


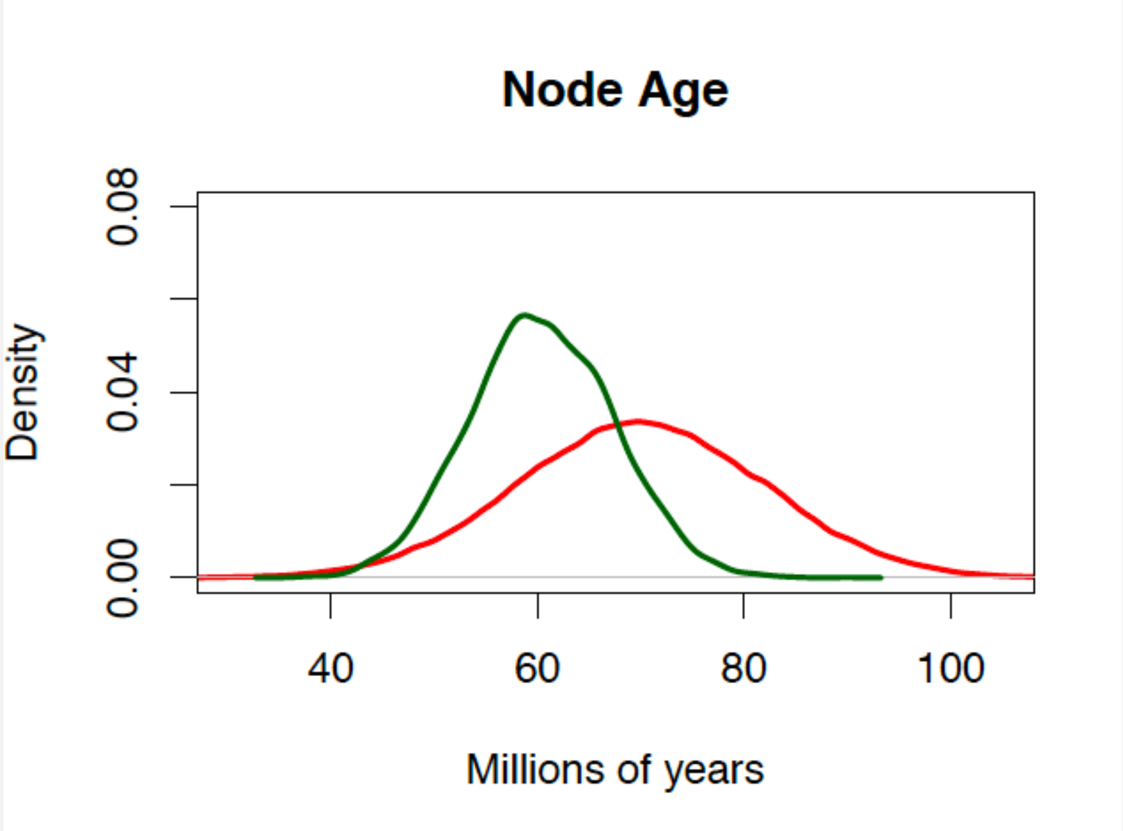
Fossils as Priors





Fossils as Priors

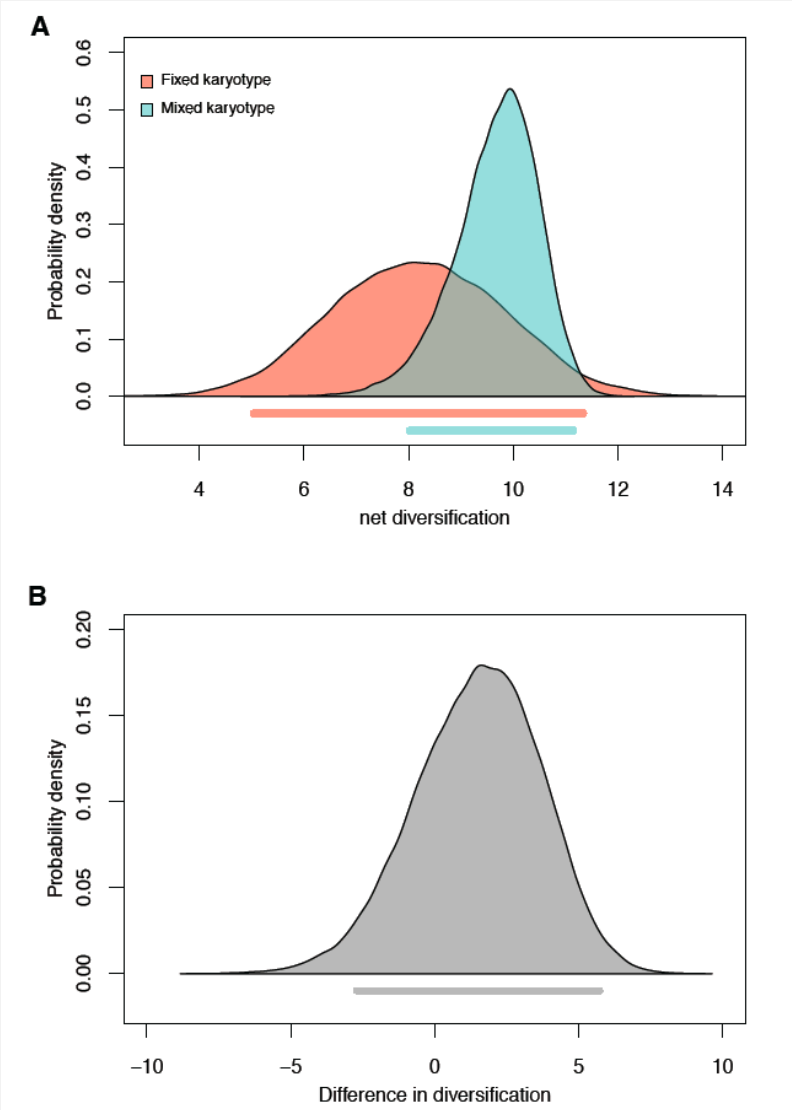




The rise of Bayesian approaches

Since the 1990s Bayesian approaches have come to the forefront in phylogenetics, population genetics, genomics, ecology, and other fields. This largely because we now have the computational power to use MCMCs to sample posterior distributions...

The power of Bayesian approaches



With a Bayesian approach we can take into account all types of uncertainty and be more conservative.

- Phylogenetic
- Model selection uncertainty
- Parameter value uncertainty
- Uncertainty in measurements

For Thursday

Read chapter WS 5 and chapter 2 of McElreath

Bring laptop to class!

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