Bayes Theorem Example

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Example Problem

Imagine that we have a disease that is preseint in 1 in a million people in the general population. We have developed a test for this disease that is 99.9% correct in returning a positive result when someone is truly infected. However, it has a false positive rate of 5%. Lets use Bayes Theorem to calculate the probility that a random individual chosen from the population that tests positive actually has the disease.

Bayes Theorem

We will use *hyp* to represent our hypothesis that someone has the disease and we will use *post* to represent a positive test. Also remember that p is read as probability and the | character is read as given that. So the equation $P(hyp|post) = \frac{P(post|hyp)P(hyp)}{P(post)}$ would be read as the probability that a person has the disease given that they tested positive is equal to the probability that you get a positive test given that someone has the disease times the probability of someone having the disease (this is the prior) divided by the probability of getting a positive test. Remember that there are two kinds of positive tests: true positives and false positives and both must be accounted for.

Lets use our quickly building R skills to work out this example question in R. I will use the variables posthyp, hyp, and post to hold the probabilities of each of the 3 values on the right side of Bayes Theorem

```r
# first the probability of getting positive test given the test subject is infected
posthyp <- .999

# then the probability of a random person being infected
hyp <- 1/1000000

# the the probability of a positive test
false.pos <- .05 * 999999/1000000
true.pos <- .999 * 1/1000000
post <- false.pos + true.pos

# now we combine these in Bayes Theorem
prob.having.disease.if.test.positive <- posthyp * hyp / post
```

The chance that a random person who tests positive has the disease is 1.9979621×10^{-5}.