Module 1

Objective Function for Vanilla Linear Regression:

$$J(\omega) = \frac{1}{2}\|t - X\omega\|_2^2 \tag{1}$$

We solve for the optimal ω by taking the derivative of the objective function with respect to ω and setting it to zero:

$$\frac{\partial J(\omega)}{\partial \omega} = 0, (X^T X)^{-1} X^T t = \omega \tag{2}$$

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} t = \begin{bmatrix} t_1 \\ t_2 \\ \vdots \\ t_n \end{bmatrix} \tag{3}$$

$$(X^T X + \lambda I)^{-1} X^T t = \omega \tag{4}$$

Experimental Design and Analysis

Basis Functions

$$\phi(x) = \begin{bmatrix} \phi_0(x) \\ \phi_1(x) \\ \vdots \\ \phi_M(x) \end{bmatrix} \tag{5}$$

$$\phi_j(x) = \exp\left(-\frac{(x-\mu_j)^2}{2s^2}\right) \tag{6}$$

$$\phi_j(x) = x^j \tag{7}$$

Model Selection

$$\min_{\omega} \frac{1}{2}\|t - X\omega\|_2^2 + \lambda\|\omega\|_1 \tag{8}$$

$$\min_{\omega} \frac{1}{2}\|t - X\omega\|_2^2 + \lambda_1\|\omega\|_1 + \lambda_2\|\omega\|_2^2, \quad \lambda_1 + \lambda_2 = 1 \tag{9}$$

Metrics of Regression

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(t_i - \hat{t}_i)^2 \tag{10}$$

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|t_i - \hat{t}_i| \tag{11}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(t_i - \hat{t}_i)^2}{\sum_{i=1}^{n}(t_i - \bar{t})^2}, \quad \bar{t} = \frac{1}{n}\sum_{i=1}^{n} t_i \tag{12}$$

$$R^2 = 1 - \frac{\|t - X\omega\|_2^2}{\|t - \bar{t}\|_2^2} \tag{13}$$

Cross-Validation