Cole Rottenberg HW1 EEL3850

Problem 1.

1. $P(A \cup B) = 1 - P(\overline{A \cup B}) = 0.58$
2. $P(A \cap B) = P(A \cup B) - P(A) - P(B) = -0.12?$
3. $P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.58?$
4. $P(A \cap B \cap C) = P(A) \times P(B) \times P(C)$
   $= 0.012$

5. $P(A \cap B) = P(A) \times P(B)$ proves s.i.
   but $-0.12 \neq 0.12$, so no, not s.i.

6. mut. ex means $P(A \cap B) = 0$, but $P(A \cap B) = -0.12$
   so not mut. ex

Problem 2

1. Knowing the $P(\Omega) = 1$, we can instead look at the probability of not rolling a 6 all 3 rolls then subtract 1

$1 - \left(\frac{5}{6}\right)^3 = 0.42$

2. Now the same logic, $1 - \left(\frac{5}{6}\right)^8 = 0.77$

3. $0.9 = 1 - \left(\frac{5}{6}\right)^x$, $0.1 = \left(\frac{5}{6}\right)^x$

$\ln(0.1) = x \ln\left(\frac{5}{6}\right)$

$x = \frac{\ln(0.1)}{\ln\left(\frac{5}{6}\right)} = 12.63 \rightarrow$ so 13 rolls

4. $p = 1 - \left(\frac{5}{6}\right)^x$, $1 - p = \left(\frac{5}{6}\right)^x$

$\frac{\ln(1-P)}{\ln\left(\frac{5}{6}\right)} = x$

Problem 3

1. $P(\text{Positive} \mid \text{No Disease}) = 0.2$

   1. is false

2. $P(\text{Negative} \mid \text{Disease}) = 1 - 0.9 = 0.1$

   2. is false

3. Bayes' Theorem $P(A \mid B) = \dfrac{P(B \mid A) \cdot P(A)}{P(B)}$

$P(D \mid +) > P(D \mid -)$

known

$$P(D \mid +) = \frac{P(+ \mid D) \cdot P(D)}{P(+)}$$

$$P(+) = P(+ \mid D) \cdot P(D) + P(+ \mid N) \cdot P(N)$$

$$0.9 \times 0.01 + 0.2 \cdot 0.99$$

$$= 0.207$$

so $P(D \mid +) = \dfrac{0.90 \times 0.01}{0.207} = 0.0435$

and $P(D \mid -) = 0.9565$

so 3 is true

Problem 4

1. $P(D) = P(D|A) \cdot P_A + P(D|B)P_B + P(D|C)P_C$

$0.001 \cdot \frac{1}{3} + 0.005 \cdot \frac{1}{3} + 0.01 \cdot \frac{1}{3}$

$P(D) = 0.005\overline{3}$

2. Using Bayes' Theorem

$- P(A|D) = \dfrac{P(D|A) \cdot P_A}{P(D)}$

$= \dfrac{0.001 \times \frac{1}{3}}{0.005\overline{3}} = P(A|D) = 0.0625$

so $P(B|D) = 0.313$

$P(C|D) = 0.625$

3. Now using the same formulas as before

$P(D) = P(D|A)P_A + P(D|B)P_B + P(D|C)P_C$

$0.001 \times 0.1 + 0.005 \times 0.2 + 0.01 \times 0.7$

$P(D) = 0.0081$

4. $P(A|D) = \dfrac{P(D|A) \cdot P_A}{P(D)} = \dfrac{0.001 \times 0.1}{0.0081} = 0.0123$

$P(B|D) = 0.123$

$P(C|D) = 0.864$

Problem 5

C=has has cancer $\qquad$ $P(C)$

P = has elevated PSA levels

$$P(P|\bar{C}) = 0.135$$
$$P(P|C) = 0.268$$

1. $P(C|P) = \dfrac{P(P|C)\cdot P(C)}{P(P)}$

$$P(P) = P(P|C)\cdot P(C) + P(P|\bar{C}) \times P(\bar{C})$$
$$0.268 \times .8 + 0.135 \times 0.2 = 0.2414$$

$$\frac{0.268\cdot 0.8}{0.241} = 0.888$$

2. $P(C) = 0.8$ $\quad P(C|\bar{P})$ $\qquad P(\bar{P}) = 1 - P(P)$

$= 0.7586$

$$= \frac{P(\bar{P}|C) \times P(C)}{P(\bar{P})}$$

$$\frac{(1-0.268)(0.8)}{(1-0.2414)} = 0.772$$

3. $P(C) = 0.2$

$$P(C|P) = \frac{P(P|C)\,P(C)}{P(P)}$$

$$P(P) = P(P|C)\,P(C) + P(P|\bar{C})\,P(\bar{C})$$
$$= 0.1616$$

$$P(C|P) = 0.331$$

4. $P(C|\bar{P})$, $P(C) = 0.2$

$$P(C|\bar{P}) = \frac{P(\bar{P}|C)\,P(C)}{P(\bar{P})}$$

$$P(\bar{P}) = P(\bar{P}|C)\,P(C) + P(\bar{P}|\bar{C})\,P(\bar{C})$$
$$= 0.8384$$

$$= \frac{(1 - P(P|C))(0.2)}{P(\bar{P})}$$

$$= 0.175$$

Problem 6

10 books

1. 3 ways, $k=3$ $n=10$

$$\binom{10}{3} = \frac{10!}{3!(10-3)!} = 120 \text{ ways}$$

2. 3 p+s books × 2 lin alg × 3 Culn.

18 ways he cleans

$$P = 18/120 = 0.15$$