

Homework 3 P1 Cole Rottenberg

MLE Rule

$$P(B_0|A_0) = 0.5, P(B_1|A_0) = 0.25,$$

$$P(B_2|A_0) = 0.25, P(B_0|A_1) = 0.1$$

$$P(B_1|A_1) = 0.3, P(B_2|A_1) = 0.6$$

$$\begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \begin{matrix} \rightarrow 1 \\ \rightarrow 1 \end{matrix}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$0.6 \quad 0.55 \quad 0.85$$

so our MLE is $[0 \ 1 \ 1]$

Given B_0 assume A_0

Given B_1 assume A_1

Given B_2 assume A_1

MAP rule: $P(A_i|B_j) = \frac{P(B_j|A_i)P(A_i)}{\sum_i P(B_j|A_i)P(A_i)}$

$$P(A_0|B_0) = \frac{P(B_0|A_0)P(A_0)}{P(B_0|A_0)P(A_0) + P(B_0|A_1)P(A_1)}$$

$$\frac{0.5 \cdot 0.6}{(0.5 \cdot 0.6) + (0.1)(0.4)} = 0.882$$

so

$$P(A_1|B_0) = 0.118 \quad \text{Given } B_0, \hat{A}_0$$

Homework 3 P1

Cole Rottenberg

MLE Rule

$$P(B_0|A_0) = 0.5, P(B_1|A_0) = 0.25,$$

$$P(B_2|A_0) = 0.25, P(B_0|A_1) = 0.1$$

$$P(B_1|A_1) = 0.3, P(B_2|A_1) = 0.6$$

$$\begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.1 & 0.3 & 0.6 \end{bmatrix} \begin{matrix} \rightarrow 1 \\ \rightarrow 1 \end{matrix}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$= 0.6 \quad 0.55 \quad 0.85$$

So our MLE is $[0 \ 1 \ 1]$

Given B_0 assume A_0

Given B_1 assume A_1

Given B_2 assume A_1

MAP rule: $P(A_i|B_j) = \frac{P(B_j|A_i)P(A_i)}{\sum_i P(B_j|A_i)P(A_i)}$

$$P(A_0|B_0) = \frac{P(B_0|A_0)P(A_0)}{P(B_0|A_0)P(A_0) + P(B_0|A_1)P(A_1)}$$

$$\frac{0.5 \cdot 0.6}{(0.5 \cdot 0.6) + (0.1)(0.4)} = 0.882$$

so

$$P(A_1|B_0) = 0.118 \quad \text{Given } B_0, \hat{A}_0$$

$$P(A_0|B_1) = \frac{P(B_1|A_0)P(A_0)}{P(B_1|A_0)P(A_0) + P(B_1|A_1)P(A_1)}$$

$$= \frac{0.25 \cdot 0.6}{0.25 \cdot 0.6 + 0.3 \cdot 0.4} = 0.556$$

$P(A_1|B_1) = 0.444$, so given B_1 , \hat{A}_0 is our choice

$$P(A_0|B_2) = \frac{P(B_2|A_0)P(A_0)}{P(B_2|A_0)P(A_0) + P(B_2|A_1)P(A_1)}$$

$$= \frac{0.25 \cdot 0.6}{0.25 \cdot 0.6 + 0.6 \cdot 0.4} = 0.385$$

$P(A_1|B_2) = 0.615$, so given B_2 , we chose \hat{A}_1

$[0 \ 0 \ 1]$ as choices

2). Error of both?

MLE

$$P(B_j) = \sum_{i \in \{0,1\}} P(B_j|A_i)P(A_i)$$

$$P(B_0) = 0.5 \cdot 0.6 + 0.1 \cdot 0.4 = 0.34$$

$$P(B_1) = 0.27$$

$$P(B_2) = 0.39$$

$$P(B) = [0.34, 0.27, 0.39]$$

$$P(E) = 0.34 \cdot 0.04 + 0.27 \cdot 0.15 + 0.39 \cdot 0.15$$

$$P_{\text{error}} = \sum [P(A_j) \times P(B_i | A_j) \text{ where } j \text{ is not the decision}]$$

$$P(A_1) \cdot P(B_0 | A_1) = 0.04$$

$$P(A_0) \cdot P(B_1 | A_0) = 0.15$$

$$P(A_0) \cdot P(B_2 | A_0) = 0.15$$

$$P_{\text{error}} = 0.34 \cdot 0.04 + 0.27 \cdot 0.15 + 0.39 \cdot 0.15$$

for MLE

$$= 0.113 \text{ for MLE}$$

for MAP

$$P_{\text{error}} = 0$$

$$P(A_1) \cdot P(B_0 | A_1) = 0.04$$

$$P(A_1) \cdot P(B_1 | A_1) = 0.12$$

$$P(A_0) \cdot P(B_2 | A_0) = 0.15$$

$$P_{\text{error}} = 0.34 \times 0.04 + 0.27 \times 0.12 + 0.39 \times 0.15$$

for MAP

$$= 0.1045, .105$$

for MAP P_{error}

3) rule 3: ~~0.5~~ $0.5 \cdot P_{\text{error for MLE}} + \dots$

$0.5 \cdot P_{\text{error for MAP}}$

≈ 0.109