Homework 5 Cole Kottenberg

* Problem 1 $\mu_c = \frac{1}{N}\sum_{i=1}^{N}(X_i + c)$ $c \in \mathbb{R}$

1. The bias will be $b_\mu(\hat{\mu}_c) = c$

2. $\sigma_X^2 = \frac{1}{N}\sum_{i=1}^{N}(X-\mu_x)^2$, so w/ c as a bias to the estimated mean we have $(X-(\mu_c+c))^2$ so ~~$\sigma_c^2 \neq c^2 = Var[X]$~~ however the variance will stay the same as. $(\mu+c, \frac{\sigma^2}{N})$

3. So the normal distro is $(\mu+c, \sigma^2)$

4. $\hat{\mu}_\lambda = \frac{\lambda}{N}\cdot\sum_{i=1}^{N} X_i$, $\lambda > 0$, so $b_\mu(\hat{\mu}_\lambda) =$ $\mu_x \cdot (\lambda - 1) \rightarrow \lambda\mu - \mu = b_\mu$ or $\ominus$

5. $Var[\hat{\mu}_\lambda] = \lambda^2\sigma^2$ of the variance.

6. Minimizing the MSE of $\hat{\mu}_\lambda$ for estimating true mean. Is the optimal $>$ or $<$ than 1?
   * The λ that minimizes the MSE
   $MSE = Var[\hat{\mu}_\lambda] + (b(\mu_\lambda))^2 = \lambda^2\sigma^2 + (\mu(\lambda-1))^2$
   so $\frac{d}{d\lambda}MSE = 0$ will find the minimal value of the MSE, so this λ will be less than 1.

$2\lambda\sigma^2 + 2((\lambda-1)\mu)\cdot(\mu) = 0$

7. The MSE will be less than the unbiased estimator w/o the λ.

Problem 2

$\sigma_x^2 = 22$, $\sigma_y^2 = 39$, $\bar{x} = 191$, $\bar{y} = 185$, $n = 20$

1. 95% C.I. $Z = 1.96$, $\bar{x} \pm Z \cdot \frac{\sigma_x}{\sqrt{n_x}}$

X: (188.94, 193.06)

y: (182.26, 187.74)

2. The C.I. do not overlap suggesting a difference in lifespans.

$$Z = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\sigma_x^2}{20} + \frac{\sigma_y^2}{20}}} = 3.436 > 1.96.$$

Prob. 3.1

1. $v_1 = \begin{bmatrix}1\\2\\0\end{bmatrix}, v_2 = \begin{bmatrix}2\\3\\0\end{bmatrix}$ $\quad RREF\begin{bmatrix}1&2&0\\2&3&0\\0&0&0\end{bmatrix} = \begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}$

   so both are linearly independant

2. The basis of v_1, v_2 is $\mathbb{R}^2 = \left\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\0\end{bmatrix}\right\}$

3. $\text{span}(v_1, v_2) = \left\{\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\3\\0\end{bmatrix}\right\}$ as they are l.i.

4. The dimension of v_1 and v_2 is 2

5. V_3 must bee linearly indipendant of v_1 and v_2 to add to the dimensionality

Prob 3.2

$V = \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}$

so $\text{rref}\begin{bmatrix}1&1&1&2&0\\0&1&1&3&0\\0&0&1&4&0\end{bmatrix} = \begin{bmatrix}1&0&0&-1&0\\0&1&0&-1&0\\0&0&1&4&0\end{bmatrix}$ $\quad x_1 \Rightarrow x_4$ $\quad \cancel{x_2 \Rightarrow}$

~ l.d.

v.s.

$\text{rref}\begin{bmatrix}1&1&1&0\\0&1&1&0\\0&0&1&0\end{bmatrix} = \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\end{bmatrix}$ so all ace l.i.

so $4V_3 - V_2 - V_1 = V_4 \ldots$

Prob 3.3

with each vector being 3x1 and 6 vectors, the max # of dimensions is 3, so 3 l.i. vectors.

Prob 3.4

1. because the max dimensionality is 3, and there are 4 vectors in a space.
2. There is a scalar that can transform $v_1 \to v_2$ or $v_2 \to v_1$.
3. a scalar $k=0$ can turn v_1 into the 0 vector thr a linear combination.

Prob 4

1. False, depending on the offset of the linear regression model, v can be < 0,
2. True. $y_2 = v_2$ for all x_2
3. True. As the $\frac{dy}{dx} = -0.8$ because there is no change other than y_6 and x_6.

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