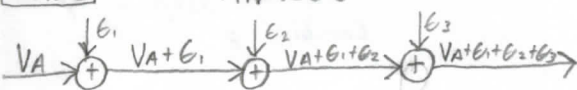
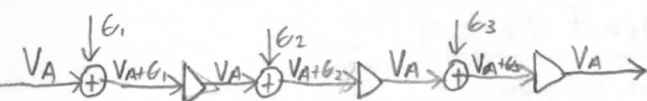


Chapter 1: Digital Abstraction

Noise



ANALOG (Noise accumulates)



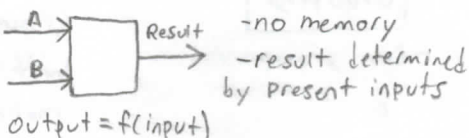
DIGITAL

Buffer: determines if it's in a 0 or 1 range and makes it a pristine 0 or 1. Effectively scrubbing noise.

Digital Signals

- N elements requires $\log_2 N$ bits, i.e. 8 colours needs $\log_2 8 = 3$ bits

Combinational Logic Circuit



Verilog - Verilog is a hardware description language **HDL** **NOT** code/software
- Quartus does not run verilog it runs the logic gates made from the verilog

Sequential Logic Circuit

- Has memory

Chapter 3: Boolean Algebra

Axioms

$$\begin{aligned} 1 \wedge x &= x & 0 \wedge x &= 0 \\ 1 \vee x &= 1 & 0 \vee x &= x \\ \bar{0} &= 1 & \bar{1} &= 0 \end{aligned}$$

Equivilantly
 $\rightarrow 0 = 1 \quad \rightarrow 1 = 0$

Properties

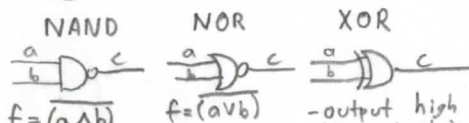
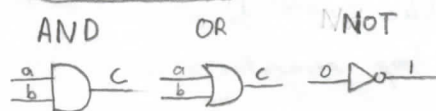
Commutative: $x \wedge y = y \wedge x$
Associative: $x \wedge (y \wedge z) = (x \wedge y) \wedge z$
Distributive: $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$
Idempotence: $x \wedge x = x$; $x \vee x = x$
Complementation: $x \wedge \bar{x} = 0$; $x \vee \bar{x} = 1$

Absorption: $x \wedge (x \vee y) = x$

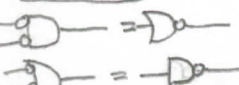
Combining: $(x \wedge y) \vee (x \wedge \bar{y}) = x$

De Morgan's: $\overline{(x \wedge y)} = \bar{x} \vee \bar{y}$

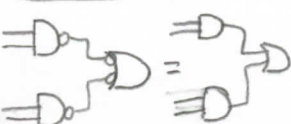
Logic Gates



De Morgan



Bubble Rule



Truth Tables

a	b	$a \wedge b$	$a \vee b$	a	\bar{a}
0	0	0	0	0	1
1	0	0	1	1	0
0	1	0	1		
1	1	1	1		

Dual Functions

- Switch everything

$$f(x, y) = (x \wedge 1) \vee (0 \vee \bar{y})$$

$$f^0(x, y) = (x \vee 0) \wedge (1 \wedge \bar{y})$$

Duality

- You can change every $\wedge \rightarrow \vee$ and $\vee \rightarrow \wedge$ and $0 \rightarrow 1$ and $1 \rightarrow 0$ and everything holds

Normal Form

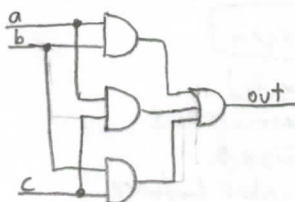
a	b	c	q
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$f(a, b, c) = (\bar{a} \wedge \bar{b} \wedge c) \vee (a \wedge \bar{b} \wedge c) \vee (a \wedge b \wedge \bar{c}) \vee (a \wedge b \wedge c)$$

Verilog

AND OR NOT XOR
 $\& \quad | \quad \sim \quad \wedge$

```
module Majority(a, b, c, out);
    input a, b, c;
    output out;
    assign out = (a & b) | (a & c) | (b & c);
endmodule
```



Chapter 6: Combinational Logic

CL

closed

- closed under acyclic composition
- you can add them up and its still combinational as long as you don't make loops

- no memory

Binary

$2^n = \text{combos}$

where n is bits

$2^3 = 8$

$2^4 = 16$

Karnaugh Map

	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	1	0	0
10	0	0	1	0

Grouping

- can group in

8

4

2

- must choose biggest and work down

Prime implicant

- unique
- an implicant that shares no bits
- cannot be made larger

Essential Prime implicant

- an implicant with a 1 only covered once

Minterm

- Product term

$$f = \sum_{dcba} m(1, 2, 3, 5, 7, 11, 13)$$

- must have min 4 & if dcba

Normal Form

- Sum of the product terms

Implicant

- a minterm could have value 0
- an implicant could have less terms by combining

Don't Cares

- allows you to cover a larger area with X's and thus make less implicants

Chapter 7 Verilog

"To describe combinational logic to be synthesized we only use assign and case statements"

Structure

```
module Count(in, out);
  input [3:0] in;
  output [out];
  <Module Body (logic)>
endmodule
```

Wire

- outputs are a wire by default
- Assigned by an ASSIGN statement
- Used to connect modules

Reg

- If assigned in a case statement or an always block
- Not a register

Case Statement

- specifies a truth table
- must have all possibilities covered or include a default

Always Block

```
always@(in) begin
  end
always@(*) begin
  end
```

begin/end

- like {} in C (Brackets)

Assign

- computed whenever RHS changes.
- Describes hardware

Testbench

Initial → like an always block
 #100 → wait 100ps
 \$display("like printf");
 repeat → loop generator

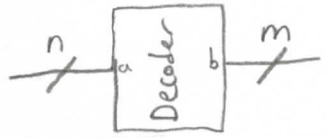
===
 !==

Decoder

- Converts one type of symbol to a different type

In general:

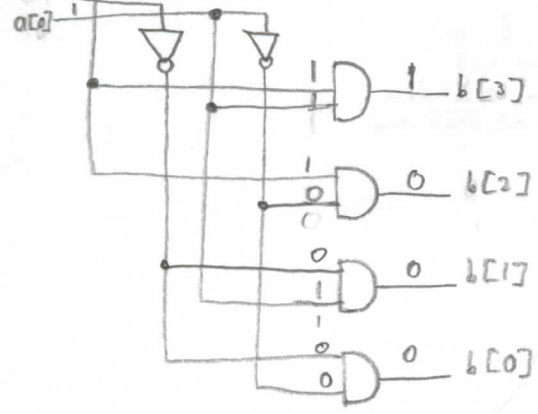
Decoder: Binary \rightarrow one-hot
Encoder: one-hot \rightarrow binary



Verilog (Decoder)

```
// n  $\rightarrow$  m decoder
// a - binary input (n bits wide)
// b - one-hot output (m bits wide)
module Dec(a,b);
    parameter n=2;
    parameter m=4;
    input [n-1:0] a;
    output [m-1:0] b;
    assign b = 1<<a;
endmodule
```

Gates (2 \rightarrow 4 Decoder)



Key Building Blocks

- ① Decoder
- ② Encoder
- ③ Multiplexer
- ④ Arbiter
- ⑤ Comparator
- ⑥ Read only memories

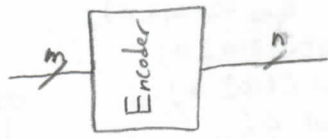
Parameter

- Can override default values

```
Dec #(3,8) dec38(a,b);
```

Encoder

- Converts one-hot to binary



```
module Enc42(a,b);
```

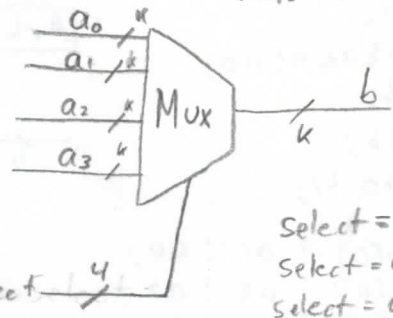
```
    input [3:0] a;
    output [1:0] b;
```

```
    assign b = {a[3] | a[2], a[3] | a[1]};
```

```
endmodule
```

Multiplexer

- Chooses what input becomes the output



4 input k bit mux, one-hot select

Select = 0001	b = a ₀
Select = 0010	b = a ₁
Select = 0100	b = a ₂
Select = 1000	b = a ₃

```
module Mux4(a3,a2,a1,a0,s,b);
```

```
    parameter k=1;
    input [k-1:0] a0,a1,a2,a3;
    input [3:0] s;
    output [k-1:0] b;
    wire [k-1:0] b;
```

```
    assign b = ({k{s[0]}} & a0 |
                {k{s[1]}} & a1 |
                {k{s[2]}} & a2 |
                {k{s[3]}} & a3);
```

```
endmodule
```

module Prime(in, isprime);

```
    input [2:0] in;
```

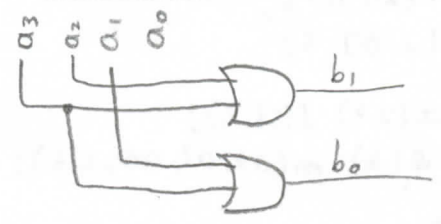
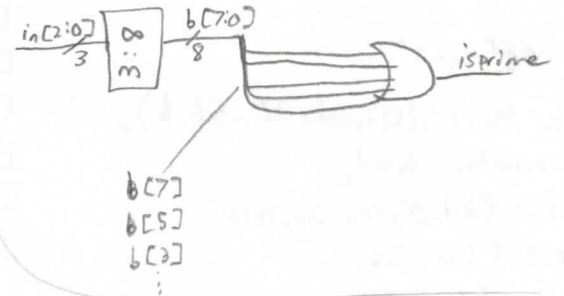
```
    output isprime;
```

```
    wire [7:0] b;
```

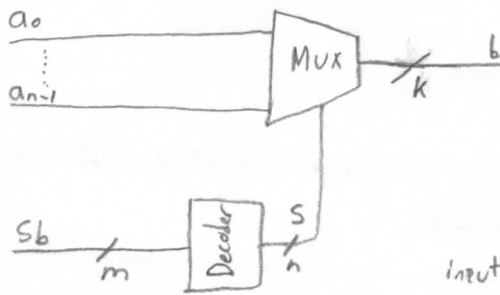
```
    wire isprime = b[1] | b[2] | b[3] | b[4] | b[5] | b[6] | b[7];
```

```
    Dec #(3,8) d(in,b);
```

```
endmodule
```



Binary-Select Mux



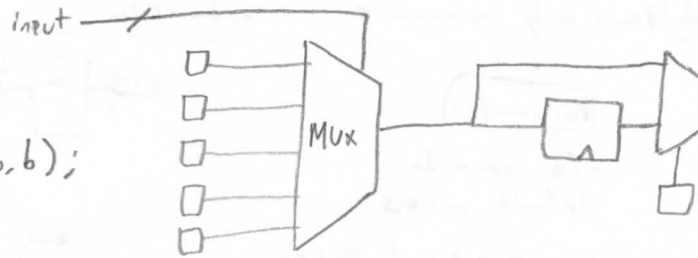
Shannon Expansion

$$F(x_0, x_1, x_2) = (\bar{x}_0 \wedge F(0, x_1, x_2)) \vee (x_0 \wedge F(1, x_1, x_2))$$

-used to simplify muxes to smaller muxes

-Can be done with any X input

FPGA Logic Block



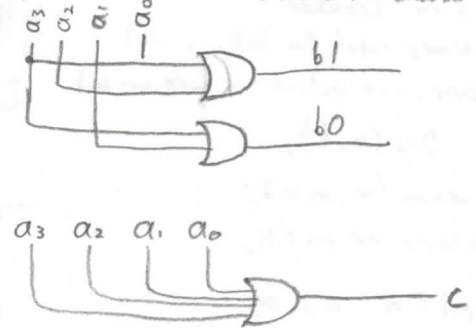
lookup table

Flip Flop

*C values = Blue

*D values = Red

From large encoder example



4:2 Encoder

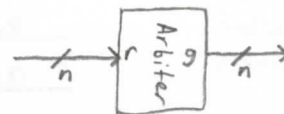
```
module Enc42(a,b,c);
  input [3:0] a;
  output [1:0] b;
  output c;
  wire [1:0] b;
  wire c;
```

```
  assign b[1] = a[3] | a[2];
```

```
  assign b[0] = a[3] | a[1];
```

```
  assign c = 1a; // c = a[0] | a[1] | a[2] | a[3];
endmodule
```

Arbiter



-Finds first 1 in r starting at LSB and outputs a 1-hot-code as the position of that one

-Less efficient

```
module Muxb3(a2,a1,a0,sb,b);
```

```
  parameter k=1;
```

```
  input [k-1:0] a2,a1,a0;
```

```
  input [1:0] sb;
```

```
  output [k-1:0] b;
```

```
  wire [2:0] s;
```

```
  Dec # (2,3) d(sb,s);
```

```
  Mux3 # (k) m(a2,a1,a0,s,b);
```

```
endmodule
```

Factoring

4:1 Mux Binary Select

Built out of 2 smaller Muxes

```
module Mux4b(a3,a2,a1,a0,sb,b);
```

```
  parameter k=1;
```

```
  input [k-1:0] a3,a2,a1,a0;
```

```
  input [1:0] sb;
```

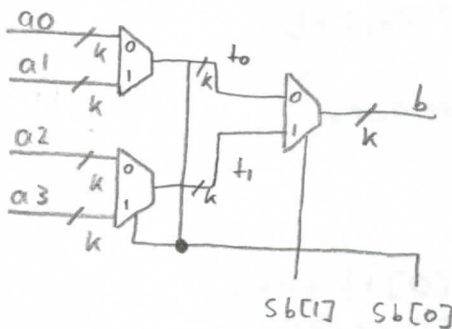
```
  output [k-1:0] b;
```

```
  wire [k-1:0] t0,t1;
```

```
  assign t0 = sb[0] ? a1 : a0;
```

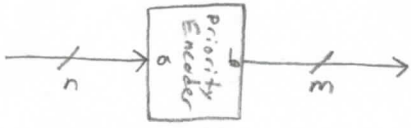
```
  assign t1 = sb[0] ? a3 : a2;
```

```
  assign b = sb[1] ? t0 : t1;
```

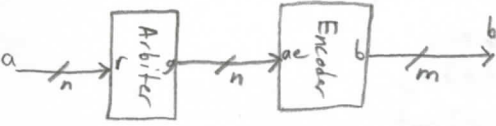


Priority Encoder

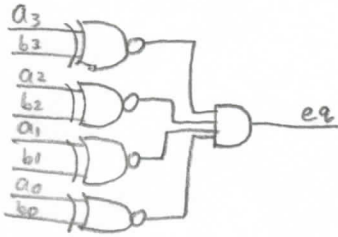
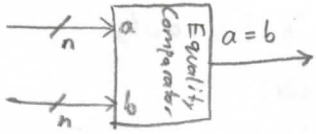
- Finds the first occurrence of a 1 in the input bus starting at the LSB and outputs its position in binary



```
module PriorityEncoder83(r, b);
    input [7:0] r;
    output [2:0] b;
    wire [7:0] g;
    Arb #18 a(r, g);
    Enc83 e(g, b);
endmodule
```



Equality Comparator



```
module EqComp(a, b, eq);
    parameter k=8;
    input [k-1:0] a, b;
    output eq;
    wire eq;

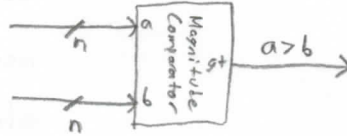
    assign eq = (a==b);
endmodule
```

Read only Memory (ROM)

```
module ROM(read-address, dout);
    parameter data-width = 32;
    parameter addr-width = 4;
    parameter filename = "data.txt";
    input [addr-width-1:0] read-address;
    output [data-width-1:0] dout;
    reg [data-width-1:0] mem[2**addr-width-1:0];

    initial $readmemb(filename, mem);
    assign dout = mem(read-address);
endmodule
```

Magnitude Comparator

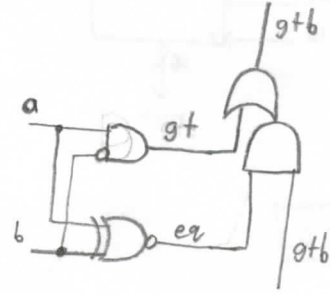


```
module MagComp(a, b, gt);
    parameter k=8;
    input [k-1:0] a, b;
    output gt;

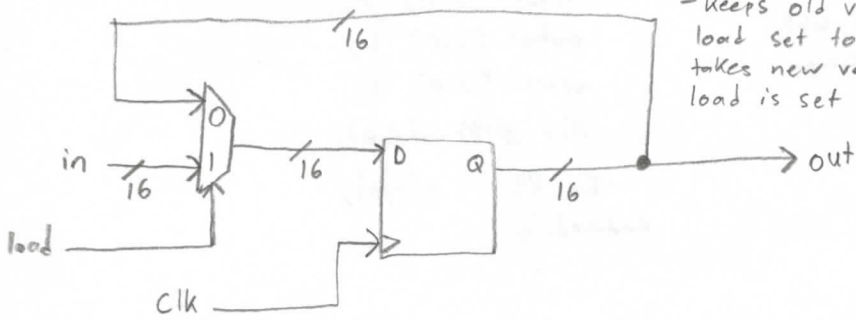
    wire [k-1:0] eqi = a ~ ^ b;
    wire [k-1:0] gti = a & ~ b;
    wire [k:0] gtb = {((eqi[k-1:0] & gti[k-1:0]) | gti[k-1:0]), 1'b0};
    wire gt = gtb[k];
endmodule
```

```
module MagComp(a, b, gt);
    parameter k=8;
    input [k-1:0] a, b;
    output gt;
    Wire gt = (a>b);
endmodule
```

$a > b$ iff $a[i] > b[i]$ AND $a[j] = b[j]$ for $j > i$



Register With load enable



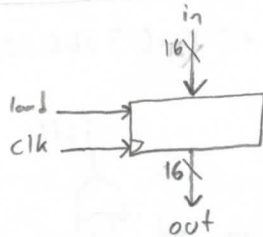
- 16 Flip Flops
- Remembers as long as load set to 0
- Keeps old value if load set to 0 and takes new value if load is set to 1

```

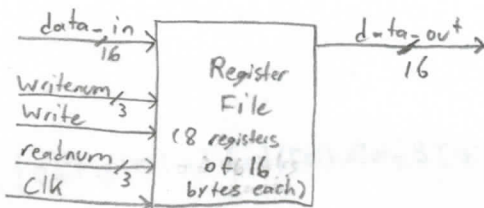
module vDFFE(clk, en, in, out);
    parameter n = 1; // Width
    input clk, en;
    input [n-1:0] in;
    output [n-1:0] out;
    reg [n-1:0] next_out;

    assign next_out = en ? in : out;

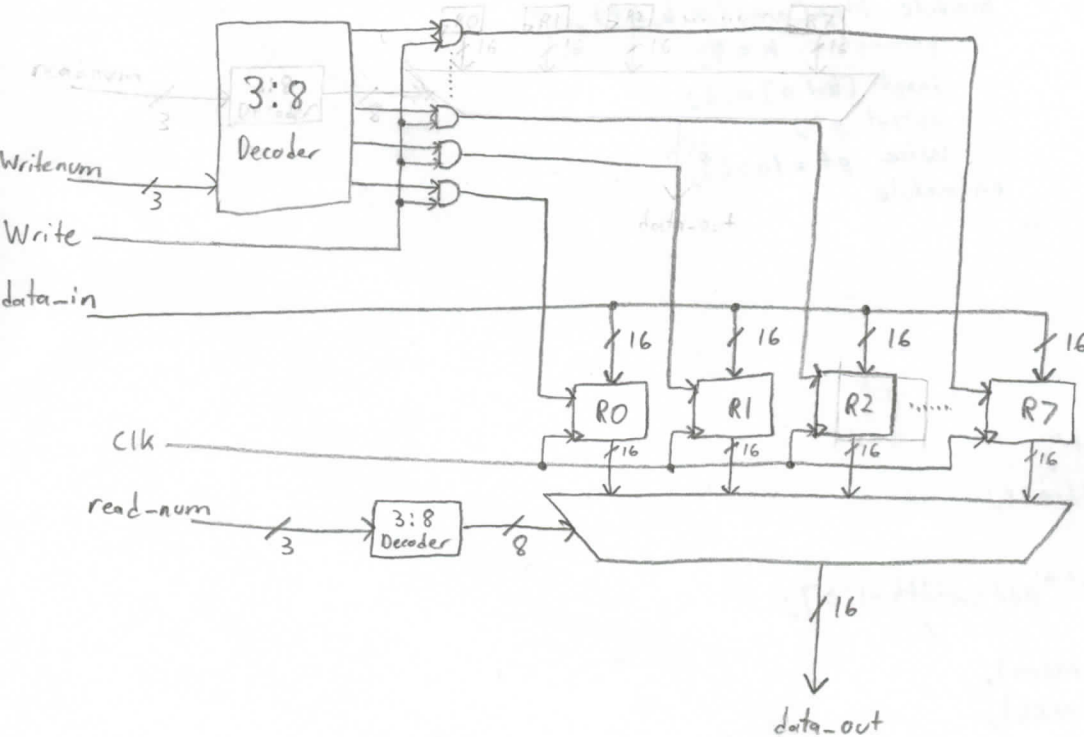
    always @(posedge clk)
        out = next_out;
endmodule
    
```



Register File



- data-in, data-out is values
- writenum, readnum are addresses



ARM

Assembly Syntax

Levels of Representation

High-Level

temp = V[i];
temp++;

Assembly

LDR R5, [R2]
LDR R6, [R2, #4]

Machine language

1100 0101
1111 1000

Control signals

ALUop[0:3] ← f(instr.Rop[25:20])

Ex C: $a = b + c + d + e;$

ARM: ADD a, b, c // add b and c and place in a
ADD a, a, d
ADD a, a, e

Ex C: $f = (g + h) - (i + j);$

ARM: ADD t0, g, h
ADD t1, i, j
SUB f, t0, t1 // $f = t0 - t1$

Types of Memory

- 1) DRAM (Dynamic)
- 2) SRAM (static)
- 3) Flash memory (holds value even when powered off)

Reading from Memory

LDR R5, [R2, #8] address
// R2 contains <val> 1000, takes that and stores it in R5

#(<val>) is the offset to get to the right address

Bytes

- int takes up 32 bit 'word' broken up into 4-8 bit bytes

0000 0000 0000 1010

- ARM puts LSB first in 32 bit 'word' "little endian"
- Could also put MSB first "big endian"

Address

Data

16	
12	
8	10
4	
0	

Writing to Memory

STR r5, [r3, #48]

// stores the value in r5 at that address

Immediate operands

ADD r3, r3, #4
// r3 = r3 + 4

bits

Instruction Field

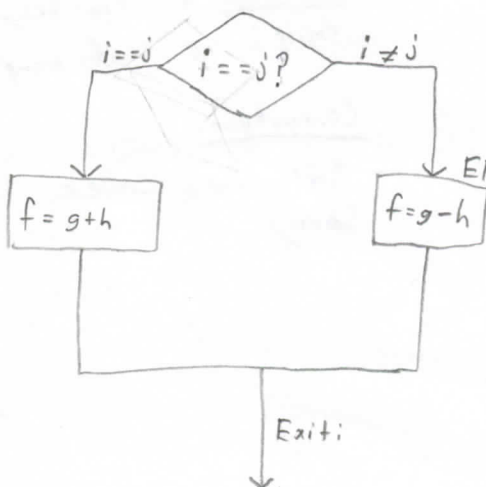
Cond	F	I	Opcode	S	Rn	Rd	Operand 2
4	2	1	4	1	4	4	12

Logical Operations

AND r5, r1, r2 // reg r5 = reg r1 & reg r2
ORR r5, r1, r2 // reg r5 = reg r1 | reg r2
MVN r5, r2 // reg r5 = ~reg r2
MOV r6, r5 // reg r6 = reg r5

Branches

if (i == j)
f = g + h;
else f = g - h;



Loops

C Code { while (save[i] == k)
i += 1;

CMP R3, R4
BNE Else // Branch not equal, go to Else:
ADD r0, r2, r2 // skipped if i ≠ j
B Exit
Else: SUB r0, r2, r2
Exit:

ARM Assembly

Loop: ADD r12, r6, r3, LSL #2
// r12 = address of save[i]
LDR r0, [r12, #0]
// Temp reg r0 = save[i]
CMP r0, r5
BNE Exit
ADD r3, r3, #1
B Loop
Exit:

Conditional Execution

- Can eliminate branches

```

CMP R3, R4
BNE Else
ADD R0, R2, R2
B Exit

```

Else: SUB R0, R2, R2

Exit:

```

CMP R3, R4
ADDEQ R0, R2, R2
SUBNE R0, R2, R2

```

Hexadecimal

D ₁₀	H ₁₆	Hexadecimal → Decimal:
0	0	
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	
9	9	
10	a	
11	b	
12	c	
13	d	
14	e	
15	f	

Hexadecimal → Decimal:

(A B 0 9)₁₆ = (43785)₁₀

↓ ↓ ↓ ↓
 $16^3 16^2 16^1 16^0$

$$10 \cdot 16^3 + 11 \cdot 16^2 + 0 \cdot 16 + 9 \cdot 1 = 43,785$$

Decimal → Hexadecimal:

(479)₁₀ = (1DF)₁₆

$$479 \div 16 = 29 \text{ R } 15 \rightarrow F$$

$$29 \div 16 = 1 \text{ R } 13 \rightarrow D$$

$$1 \div 16 = 0 \text{ R } 1 \rightarrow 1$$

Step 1

- ARM uses R0-R3 for the first 4 parameters passed

- More than 4 parameters requires using memory

main:

```

MOV R0, #1
MOV R1, #5
MOV R2, #9
MOV R3, #20

```

Step 2

- ARM uses branch and link BL to transfer control to subroutine
 - Places address to return home to in LR (R14)

BL leaf-function

Step 5

- ARM puts the result to be returned in R0.

*Note this overwrites parameter 1 placed in R0. This is ok because we will save a copy of it on the stack

MOV R0, R4

Indirect Branches

Bx Rm

MOV PC, Rm

Function Call in ARM

Steps:

- ① Put parameters where function can find them
- ② Transfer control to the function
- * ③ Acquire storage for function
- ④ Perform task
- ⑤ Put result where calling function can access it
- ⑥ Return control to the point of origin (function might be called from many places)

C Code to Execute

```

/* main.c */
extern int leaf-example(int i, int j, int k, int l);
void main() {
    int result = leaf-example(1, 5, 9, 20);
    if (result > 10)
        ;
}

/* leaf.c */
int leaf-example(int g, int h, int i, int j) {
    int f;
    f = (g+h) - (i+j);
    return f;
}

```

Terminology

Caller: makes the call i.e. main

Callee: function being called i.e. leaf-example

Convention:

- ARM uses Callee Saving

Step 6

MOV PC, LR

- PC (R15) in ARM
 - value in LR is PC address at BL+4