

---

# ELEC401: Analog CMOS Integrated Circuit Design

## Set 5

### Frequency Response of Amplifiers

Shahriar Mirabbasi  
Department of Electrical and Computer Engineering  
University of British Columbia  
shahriar@ece.ubc.ca

# Simple Pole

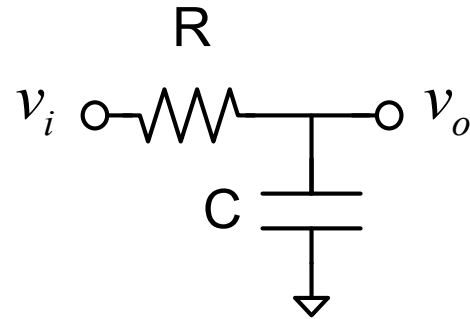
---

$$v_o / v_i = \frac{1/sC}{R + 1/sC}$$

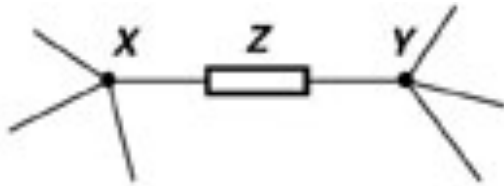
$$v_o / v_i = \frac{1}{sRC + 1}$$

$$\frac{v_o}{v_i}(f) = \frac{1}{1 + j2\pi fRC}$$

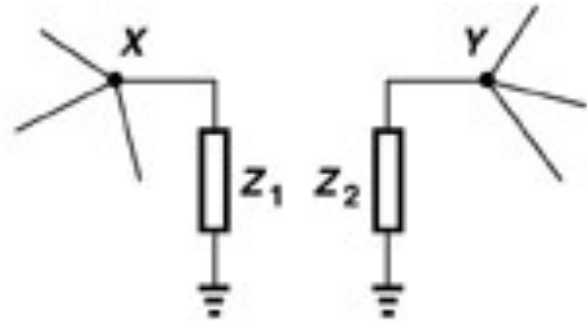
$$\frac{v_o}{v_i}(f) = \frac{1}{1 + j(\frac{f}{f_p})}, \quad f_p = \frac{1}{2\pi RC}$$



# Miller Effect



(a)



(b)

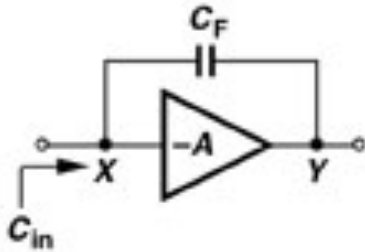
$$Z_1 = \frac{Z}{(1 - A_v)}$$

$$Z_2 = \frac{Z}{(1 - A_v^{-1})}$$

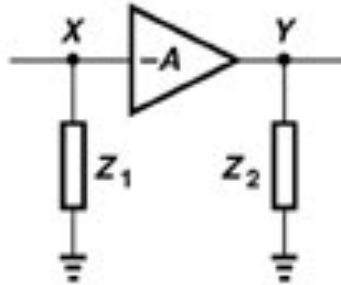
# Board Notes

---

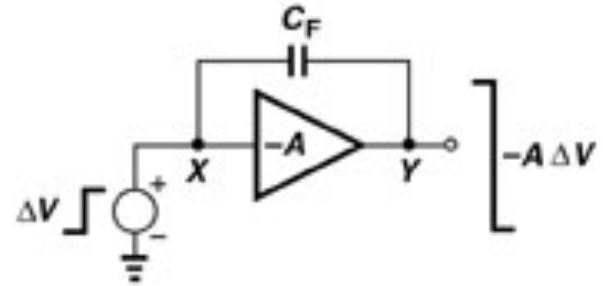
# Miller Capacitive Multiplication



(a)



(b)



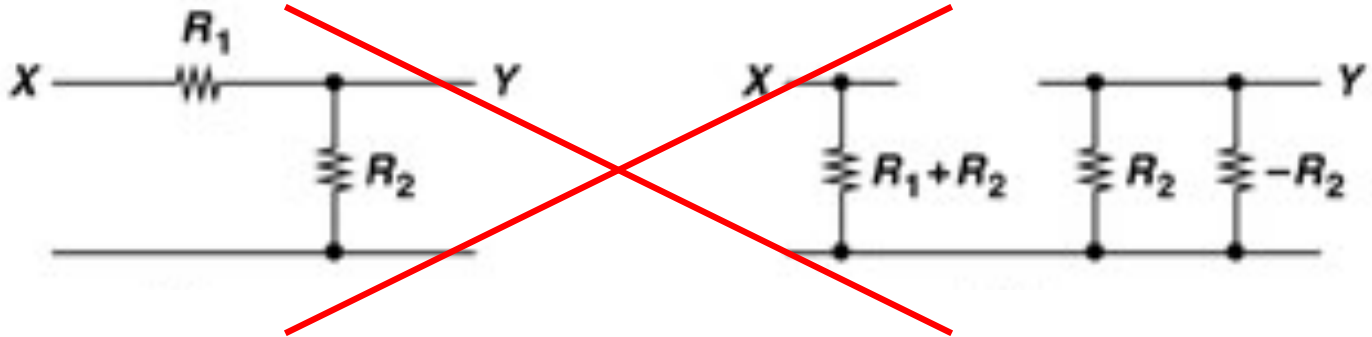
(c)

$$C_1 = C_F(1 - A_v)$$

$$C_2 = C_F(1 - A_v^{-1}) \approx C_F$$

# Applicability of Miller's Theorem

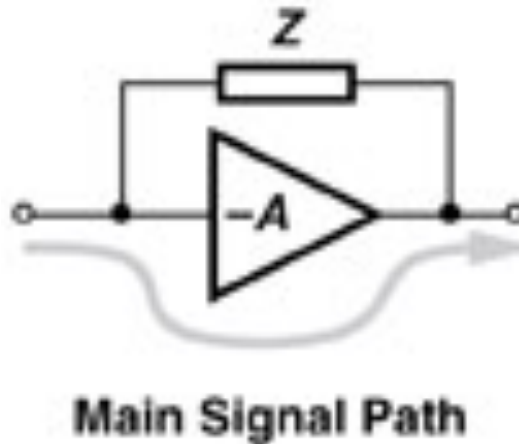
If the only signal path between X and Y is through impedance Z then Miller's theorem is typically not applicable.



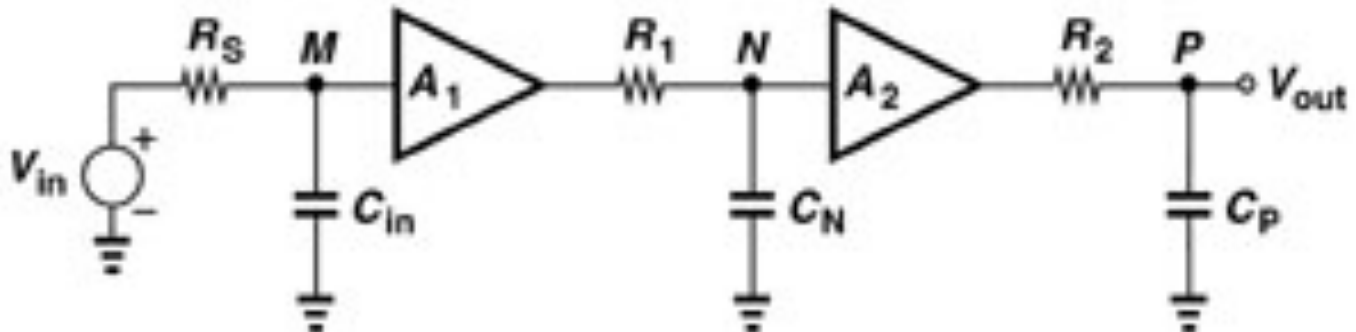
# Applicability of Miller's Theorem

---

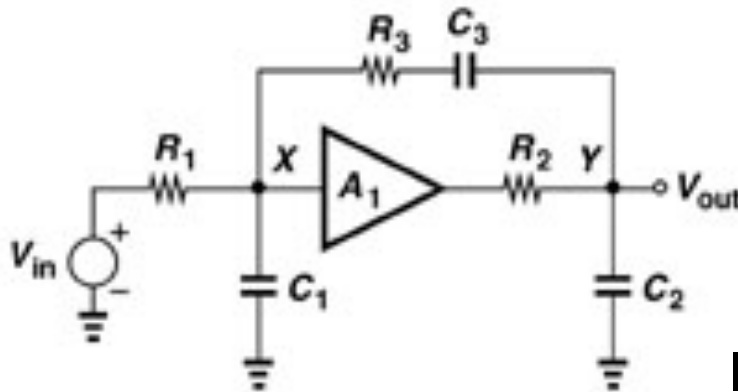
Miller's Theorem is typically useful in the cases where there is impedance in parallel with the main signal path.



# Poles and Nodes



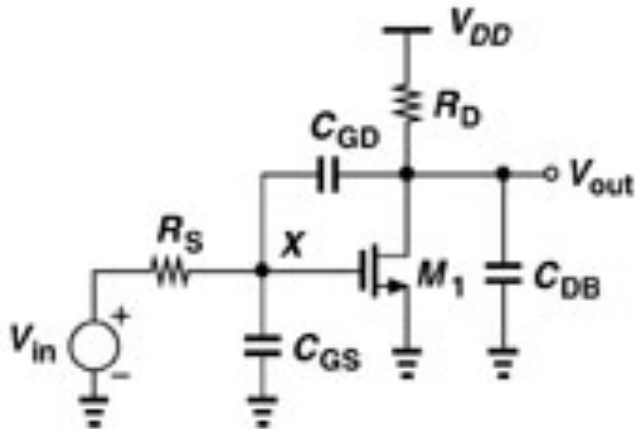
Non-Interacting Poles: One pole associated with each node



Interacting Poles



# Common Source

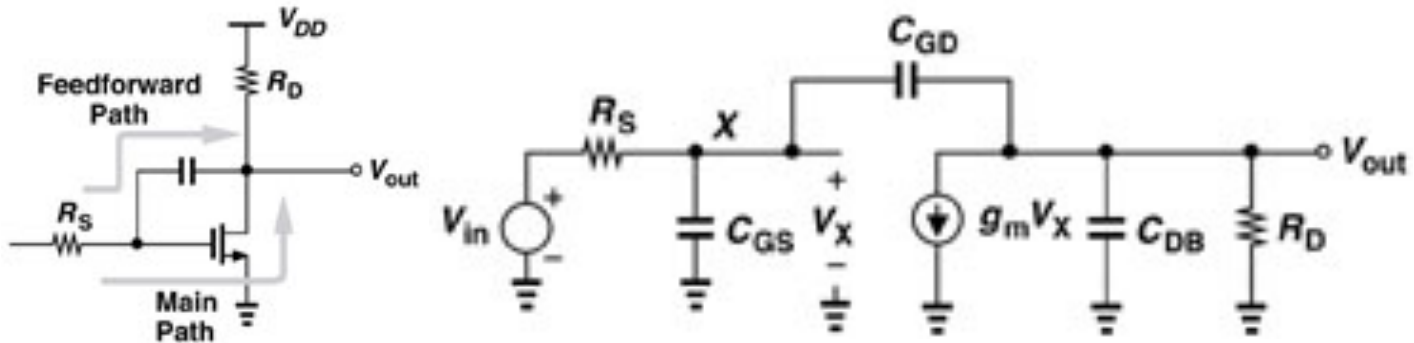


Neglecting input/output interaction,

$$f_{p,in} = \frac{1}{2\pi R_S [C_{GS} + (1 + g_m R_D) C_{GD}]}$$

$$f_{p,out} = \frac{1}{2\pi [(C_{GD} + C_{DB}) R_D]}$$

# Common Source



$$\frac{v_o}{v_i} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS} C_{GD} + C_{GS} C_{DB} + C_{GD} C_{DB}) + s [R_S (1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})] + 1}$$

$$\text{Assume } D = \left( \frac{s}{\omega_{p1}} + 1 \right) \left( \frac{s}{\omega_{p2}} + 1 \right) = \frac{s^2}{\omega_{p1} \omega_{p2}} + \frac{s}{\omega_{p1}} + 1, \quad \omega_{p2} \gg \omega_{p1}$$

$$f_{p,in} = \frac{1}{2\pi (R_S [C_{GS} + (1 + g_m R_D) C_{GD}] + R_D (C_{GD} + C_{DB}))}$$

# Common Source

---

$$f_{p,out} = \frac{R_S(1 + g_m R_D)C_{GD} + R_S C_{GS} + R_D(C_{GD} + C_{DB})}{2\pi R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})}$$

$$f_{p,out} \approx \frac{1}{2\pi R_D(C_{GD} + C_{DB})}, \text{ for large } C_{GS}$$

$$\begin{aligned} f_{p,out} &\approx \frac{g_m R_S R_D C_{GD}}{2\pi R_S R_D(C_{GS}C_{GD} + C_{GS}C_{DB} + C_{GD}C_{DB})} \\ &\approx \frac{gm}{2\pi(C_{GS} + C_{DB})}, \text{ for large } C_{GD} \end{aligned}$$

# Common Source

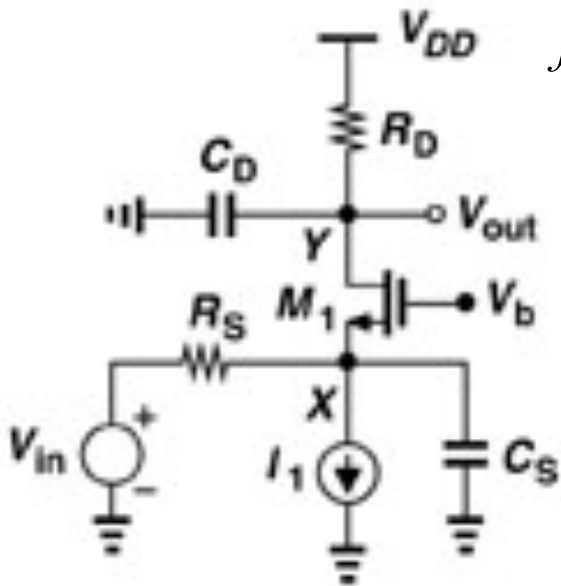
---

Right half plane zero, from the numerator of  $v_o/v_i$

$$\frac{v_o}{v_i} = \frac{(sC_{GD} - g_m)R_D}{s^2 R_S R_D (C_{GS} C_{GD} + C_{GS} C_{SB} + C_{GD} C_{DB}) + s [R_S (1 + g_m R_D) C_{GD} + R_S C_{GS} + R_D (C_{GD} + C_{DB})] + 1}$$

$$\frac{sC_{GD} - g_m}{\dots} \rightarrow f_z = \frac{+g_m}{2\pi C_{GD}}$$

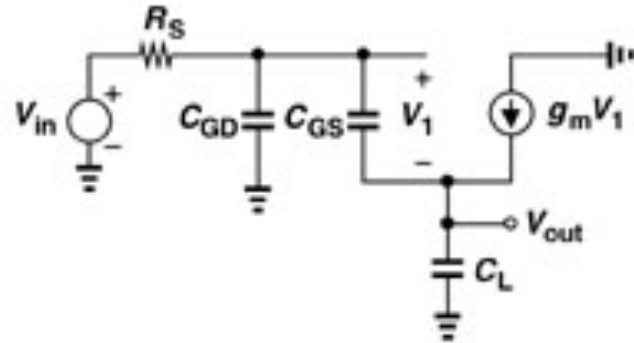
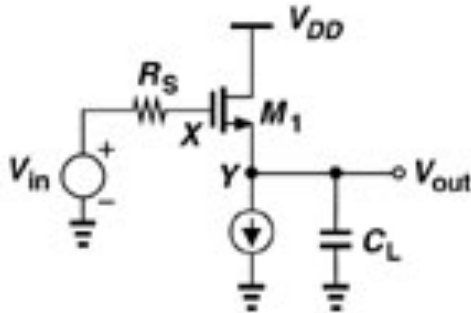
# Common Gate



$$f_{pX} = \frac{1}{2\pi \left[ (C_{GS} + C_{SB}) \left( R_S \parallel \left( \frac{1}{g_m + g_{mb}} \right) \right) \right]}$$

$$f_{pY} = \frac{1}{2\pi [(C_{GD} + C_{DB})R_D]}$$

# Source Follower (Common Drain)

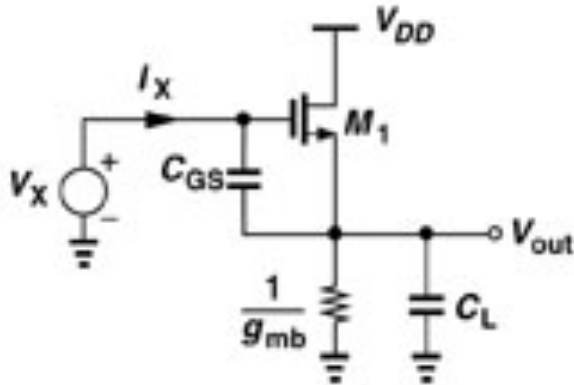


$$\frac{v_o}{v_i} = \frac{g_m + sC_{GS}}{s^2 R_S (C_{GS} C_L + C_{GS} C_{GD} + C_{GD} C_L) + s(g_m R_S C_{GD} + C_L + C_{GS}) + g_m}$$

$$f_{p1} \approx \frac{g_m}{2\pi(g_m R_S C_{GD} + C_L + C_{GS})}, \text{ assuming } f_{p2} \gg f_{p1}$$

$$= \frac{1}{2\pi \left( R_S C_{GD} + \frac{C_L + C_{GS}}{g_m} \right)}$$

# Source Follower Input Impedance



Neglecting  $C_{GD}$  ,

$$Z_{in} = \frac{1}{sC_{GS}} + \left( 1 + \frac{g_m}{sC_{GS}} \right) \frac{1}{g_{mb} + sC_L}$$

At low frequencies,  $g_{mb} \gg |sC_L|$

$$Z_{in} \approx \frac{1}{sC_{GS}} (1 + g_m / g_{mb}) + 1 / g_{mb}$$

$$\therefore C_{in} = C_{GS} g_{mb} / (g_m + g_{mb}) + C_{GD} \quad (\text{same as Miller})$$

# Source Follower

---

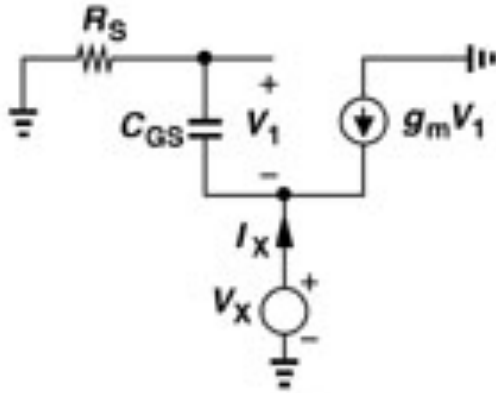
At high frequencies,  $g_{mb} \ll |sC_L|$

$$Z_{in} \approx \frac{1}{sC_{GS}} + \frac{1}{sC_L} + \frac{g_m}{s^2 C_{GS} C_L}$$

At high frequencies, overall input impedance includes  $C_{GD}$  in parallel with series combination of  $C_{GS}$  and  $C_L$  and a *negative* resistance equal to  $-g_m/(C_{GS}C_L\omega^2)$ .



# Source Follower Output Impedance

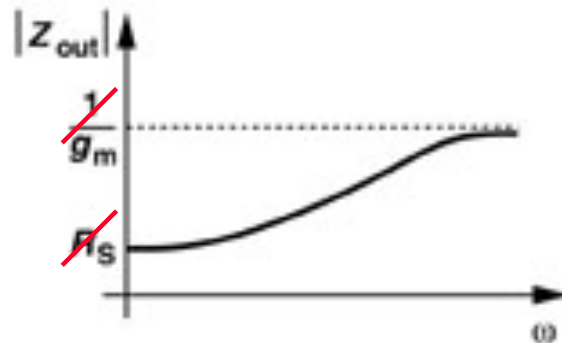
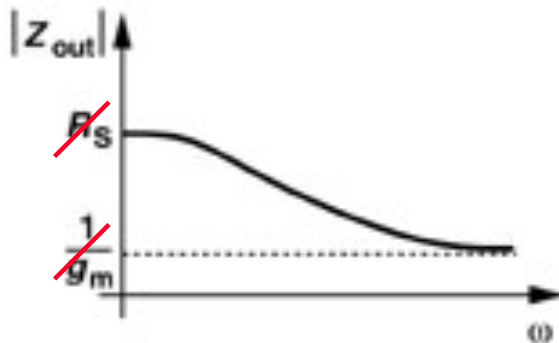


$$Z_{OUT} = V_X / I_X$$

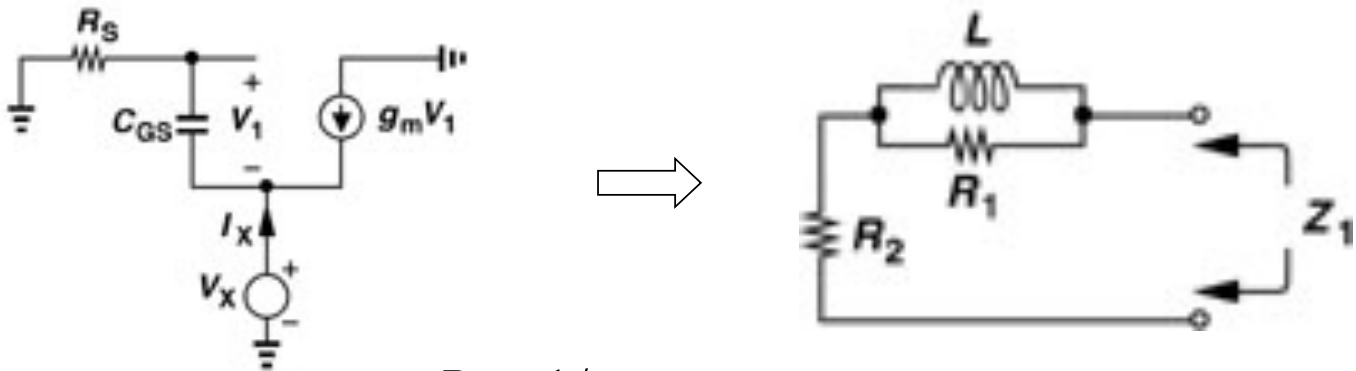
$$= \frac{sR_S C_{GS} + 1}{g_m + sC_{GS}}$$

$\approx 1/g_m$  , at low frequencies

$\approx R_S$  , at high frequencies



# Source Follower Output Impedance



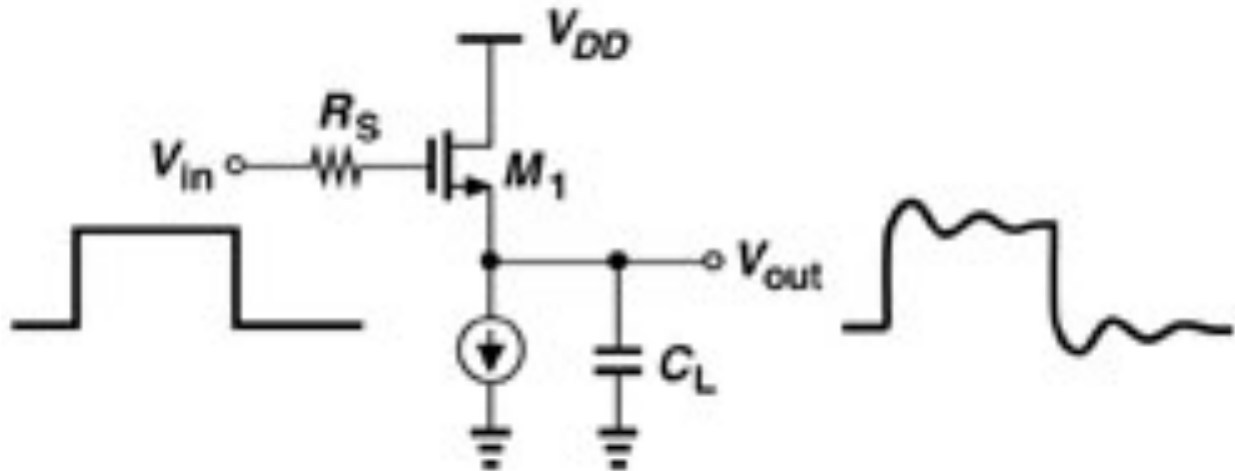
$$R_2 = 1/g_m$$

$$R_1 = R_S - 1/g_m$$

$$L = \frac{C_{GS}}{g_m} (R_S - 1/g_m)$$

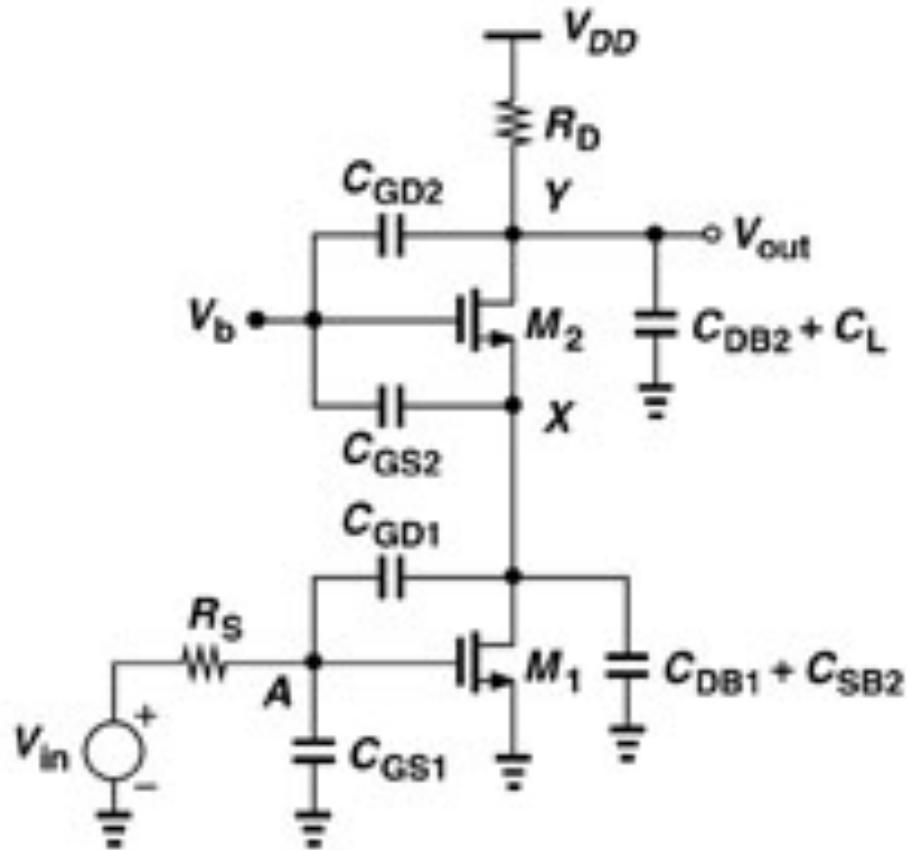
Output impedance inductance dependent on source impedance,  $R_S$ !

# Source Follower Ringing



Output ringing due to tuned circuit formed with  $C_L$  and inductive component of output impedance.

# Cascode Stage



# Cascode Stage

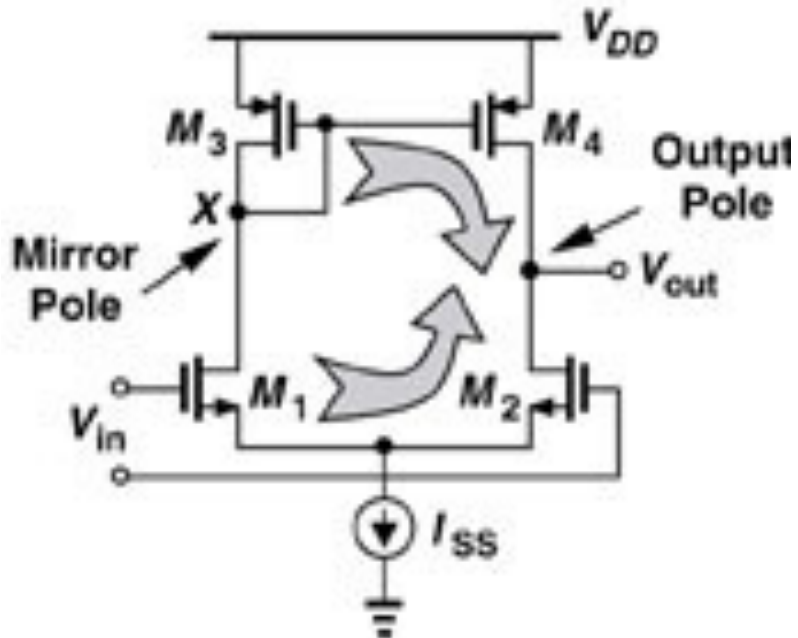
---

$$f_{pA} = \frac{1}{2\pi R_S \left[ C_{GS1} + C_{GD1} \left( 1 + \frac{g_{m1}}{g_{m2} + g_{mb2}} \right) \right]}$$

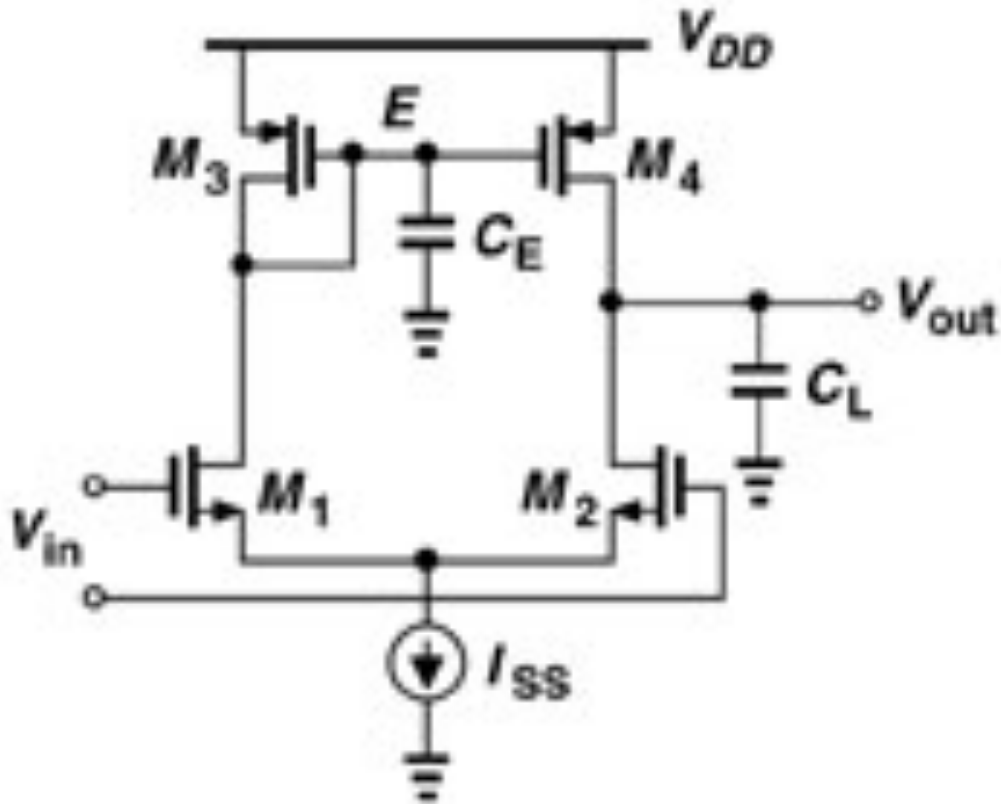
$$f_{pX} = \frac{g_{m2} + g_{mb2}}{2\pi (2C_{GD1} + C_{DB1} + C_{SB2} + C_{GS2})}$$

$$f_{pY} = \frac{1}{2\pi R_D (C_{DB2} + C_L + C_{GD2})}$$

# Differential Pair



# Differential Pair



# Differential Pair

---

$$f_{p1} \approx \frac{1}{2\pi(r_{oN} \parallel r_{oP})C_L}$$

$$f_{p2} = \frac{g_{mP}}{2\pi C_E}$$

$$f_Z = 2f_{p2} = \frac{2g_{mP}}{2\pi C_E}$$