

Constants: (Elementary Charges)

$k = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$

$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$

$c = 3 \times 10^8 \text{ m/s}$

$e = 1.6 \times 10^{-19} \text{ C}$

$m_p = 1.67 \times 10^{-27} \text{ kg}$

$m_e = 9.11 \times 10^{-31} \text{ kg}$

$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$

$h = 6.63 \times 10^{-34} \text{ J}\cdot\text{s}$

Prefixes:

$P = 10^{11}$ $f = 10^{-15}$

$T = 10^{12}$ $r = 10^{-9}$

$G = 10^9$ $n = 10^{-9}$

$M = 10^6$ $\mu = 10^{-6}$

$k = 10^3$ $m = 10^{-3}$

$h = 10^2$ $c = 10^{-2}$

Geometry: Cylinder

Sphere: $A = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

Cone: $A = \pi r(r + \sqrt{h^2 + r^2})$ $V = \frac{1}{3}\pi r^2 h$

Circle: $A = \pi r^2$ $C = 2\pi r$

Triangle: $A = \frac{1}{2}bh$

Right Triangle: $a^2 + b^2 = c^2$

Pythagorean Theorem: $a^2 + b^2 = c^2$

Trigonometry

$\sin A = \frac{a}{c}$ $\cos A = \frac{b}{c}$ $\tan A = \frac{a}{b}$

$\sin^2 \theta + \cos^2 \theta = 1$

$\sin(2\theta) = 2 \sin \theta \cos \theta$

27 Neutron: $n = e \Delta t$, $i = n e A v_d$

where v_d = drift speed

A = area

i = electrons per second

Conventional Current: $I = \frac{Q}{\Delta t}$

Current Density: $I = J \cdot A$

Conductivity: $\sigma = \frac{n e^2 \tau}{m}$

Resistivity: $\rho = \frac{1}{\sigma} = \frac{m}{n e^2 \tau}$

28 DC Circuits

Power: $P = IV$ $P_{\text{bat}} = EV$ $P_r = IV = I^2 R = \frac{V^2}{R}$

Resistor Dissipate: $P_r = IV = I^2 R = \frac{V^2}{R}$

Equivalent Resistance: $R_{\text{eq}} = R_1 + R_2 + R_3$

$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)^{-1}$

Capacitors

Parallel: $C = \frac{Q}{\Delta V}$ $C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1}$

Series: $C_{\text{eq}} = C_1 + C_2 + C_3$

Dielectric: $C = k C_0$ where k is the dielectric constant

Energy: $U = \frac{1}{2} C (\Delta V)^2$

Density: $U = \frac{1}{2} k \epsilon_0 E^2$ where k is dielectric constant

Electric Field

$E = \frac{\Delta V}{d}$

RC Circuit

Charging Capacitor: $V_b = V_r + V_c$ $V_b = IR + \frac{Q}{C}$

Charge on Capacitor: $Q = C V_b [1 - e^{-t/RC}]$

Charging Current: $I = \frac{V_b}{R} e^{-t/RC}$

Discharging Capacitor: $V_c = \frac{Q}{C} = IR$

When $t = 0$: $Q = 0$ $V_c = 0$ $I = \frac{V_b}{R}$

When $t = \infty$: $Q = C V_b$ $V_c = V_b$ $I = 0$

22/23 Electric Field

Coulomb's Law: $F = k q_1 q_2 \frac{\hat{r}}{r^2}$

Superposition: $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \dots$

Electric Field of a point charge: $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$

Dipole Moment: $\vec{p} = q \vec{d}$

Dipole in an E field: $\tau = p E \sin \theta$

24 Gauss's Law

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\epsilon_0}$

Point Charge: $\vec{E} = \frac{kQ}{r^2} \hat{r}$

Conducting Sphere: $\vec{E} = \frac{kQ}{r^2} \hat{r}$ $\vec{E} = 0$ $r < R$

Infinite Line: $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 r}$

Uniform Cylinder of Charge: $\vec{E} = \frac{\lambda r}{2\pi\epsilon_0 R^2}$ $\vec{E} = \frac{\lambda}{2\pi\epsilon_0 R}$ $r > R$

Sheet of Charge: $\vec{E} = \frac{\sigma}{2\epsilon_0}$

Finite Line: $\vec{E} = \frac{k\lambda}{r} \left[\frac{b}{(r^2 + b^2)^{3/2}} + \frac{a}{(r^2 + a^2)^{3/2}} \right]$

Ring of Charge: $\vec{E} = \frac{k\lambda(2\pi R)(r)}{(r^2 + R^2)^{3/2}}$

Pipe of Charge: $\vec{E} = \frac{\rho(R^2 - r^2)}{2\epsilon_0}$

Uniform Slab of Charge: $\vec{E} = \frac{\rho d}{2\epsilon_0}$ outside $\vec{E} = \frac{\rho z}{\epsilon_0}$ inside

Finite Line: $\vec{E} = \frac{k\lambda L}{x(x-L)}$

25 Electric Potential

Point Charge: $V = \frac{kQ}{r}$

Superposition: $V = \frac{kQ_1}{r_1} + \frac{kQ_2}{r_2} + \frac{kQ_3}{r_3}$

Charge in potential: $V_a - V_b = kQ \left(\frac{1}{r_a} - \frac{1}{r_b} \right)$

Potential Energy: $U = qV$

Line of Charge: $V = k\lambda \ln \left[\frac{b + \sqrt{b^2 + L^2}}{-a + \sqrt{a^2 + L^2}} \right]$

Ring of Charge: $V = \frac{k\lambda 2\pi R}{r}$

Disk of Charge: $V = k\sigma 2\pi \left[\sqrt{z^2 + R^2} - z \right]$

Conducting Sphere: $V = \frac{kQ}{R}$ inside $V = \frac{kQ}{r}$ outside

Work: $W = q\Delta V$

Voltage from E field: $\Delta V = -\int \vec{E} \cdot d\vec{s}$

E field from Voltage: $\vec{E} = -\frac{dV}{dx} \hat{i} - \frac{dV}{dy} \hat{j} - \frac{dV}{dz} \hat{k}$

29/30 Magnetic Field

Lorentz Force Law: $\vec{F} = q\vec{v} \times \vec{B}$

Biot-Savart Law: $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \vec{r}}{r^3}$

Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I$

Long Straight Wire: $\vec{B} = \frac{\mu_0 I}{2\pi r}$

Center of a Current Loop: $\vec{B} = \frac{\mu_0 I}{2R}$

Cylinder Conductor: $\vec{B} = \frac{\mu_0 I}{2\pi R^2} r$ $r < R$ $\vec{B} = \frac{\mu_0 I}{2\pi R}$ $r > R$

Inside a long Solenoid: $\vec{B} = \mu_0 n I$

Force Between Wires: $F = \frac{\mu_0 I_1 I_2 L}{2\pi r}$

Cyclotron: $r = \frac{mv}{qB}$ $f = \frac{qB}{2\pi m}$

Magnetic Flux: $\Phi_B = \int \vec{B} \cdot d\vec{A}$

Magnetic Gauss Law: $\oint \vec{B} \cdot d\vec{A} = 0$

Share Uniform with hole: Finite Line: $\vec{E} = \frac{kQ}{r^2} \left(\frac{r^2 - a^2}{R^2 - a^2} \right)$

Faraday's Law

Two ways to induce current:

- Motional EMF**
 $\mathcal{E} = \frac{d\Phi_M}{dt}$
Voltage = $\frac{-N \Delta(BA)}{\Delta t}$
Induced Current: $I = \frac{\mathcal{E}}{R}$
- Changing Magnetic Field**
 $\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_M}{dt}$
 $\mathcal{E} = VLB$
 $\Delta V = VLB$

Inductors

Energy stored in \vec{B} Field

Solenoid
 $L = \frac{\mu_0 N^2 A}{l}$
Potential Difference: $V_L = -L \frac{dI}{dt}$
Energy density: $U_B = \frac{1}{2} \frac{B^2}{\mu_0}$

LC Circuit

Charge on the Capacitor:
 $Q(t) = Q_0 \cos \omega t$
Current through the inductor:
 $I = I_0 \sin \omega t = I_{max} \sin \omega t$
Angular frequency: $\omega = \sqrt{\frac{1}{LC}}$

LR Circuit "Discharge"

$\Delta V_R + \Delta V_L = 0$
 $I_0 = \frac{\Delta V_{bat}}{R}$
 $I = I_0 e^{-R/L t}$

32 AC Circuits

Root Mean Squared
 $I_{rms} = \frac{I_{max}}{\sqrt{2}}$
 $V_{rms} = \frac{V_{max}}{\sqrt{2}}$
 $\mathcal{E}_{rms} = \frac{\mathcal{E}_0}{\sqrt{2}}$

Resistor Circuit
 $V_{max} = \mathcal{E}_0$ (For this)
 $V_r = i_r R$
 $V_r = V_{max} \cos \omega t$
Instantaneous Current:
 $i_r = \frac{V_r}{R} = \frac{V_{max} \cos \omega t}{R}$
 $i_r = I_{max} \cos \omega t$
 $I_{max} = \frac{V_{max}}{R}$

Inductor Circuit
 $V_{max} = \mathcal{E}_0$ (For this)
 $V_L = i_L L$
 $V_L = V_{max} \cos \omega t$
Instantaneous Current:
 $i_L = \frac{V_L}{L} = \frac{V_{max} \cos \omega t}{L}$
 $i_L = I_{max} \cos(\omega t - \frac{\pi}{2})$
Inductive Reactance:
 $X_L = \omega L$

Capacitor Circuits
 $V_{max} = \mathcal{E}_0$ (For this)
 $V_C = i_C C$
 $V_C = V_{max} \cos \omega t$
Instantaneous Current:
 $i_C = \frac{V_C}{C} = \frac{V_{max} \cos \omega t}{C}$
 $i_C = I_{max} \cos(\omega t + \frac{\pi}{2})$
Capacitive Reactance:
 $X_C = \frac{1}{\omega C}$

RC Filter Circuit
 $\mathcal{E}_0^2 = (V_r)_{max}^2 + (V_C)_{max}^2$
 $I_{max} = \frac{V_{max}}{X_C}$

Out of Phase
Current leads Voltage by $\frac{\pi}{2}$

Capacitive Reactance
 $X_C = \frac{1}{\omega C}$
*Large at Low frequencies

Peak Values
 $I_{max} = \frac{\mathcal{E}_0}{\sqrt{R^2 + X_C^2}}$
Note: $(I_r)_{max} = (I_C)_{max} = I_{max}$

Crossover Frequency
 $f_c = \frac{\omega_c}{2\pi}$
We occurs when $\omega_c = \frac{1}{RC}$

Low Pass Filter
 $\omega < \omega_c$; Transmits
 $\omega > \omega_c$; Blocks
Let's Low frequencies pass

High Pass Filter
 $\omega > \omega_c$; Transmits
 $\omega < \omega_c$; Blocks
Let's high frequencies pass

Series RLC Circuit
 $\mathcal{E}_0^2 = (V_r)_{max}^2 + [(V_L)_{max} - (V_C)_{max}]^2$
if opposite, make ϕ negative
 $(V_L)_{max} > (V_C)_{max}$ (assumed by diagram)
Instantaneous Current: $i = I_{max} \cos(\omega t - \phi)$
 ϕ = Phase angle
Peak Current: $I_{max} = \frac{\mathcal{E}_0}{\sqrt{R^2 + (X_L - X_C)^2}}$
Peak Voltages:
 $(V_r)_{max} = (I_{max})R$
 $(V_L)_{max} = (I_{max})X_L$
 $(V_C)_{max} = (I_{max})X_C$

Impedance "measured in Ω 's"
 $Z = \sqrt{R^2 + (X_L - X_C)^2}$
SO... $I_{max} = \frac{\mathcal{E}_0}{Z}$

Phase Angle
 $\phi = \tan^{-1}(\frac{X_L - X_C}{R})$
 $\phi < 0$ $\phi = 0$ $\phi > 0$
Current Leads In phase E leads
 $\omega < \omega_0$ $\omega = \omega_0$ $\omega > \omega_0$

Resonance "where current is max"
occurs when:
 $X_L = X_C$
 $\omega_0 = \frac{1}{\sqrt{LC}}$ (resonance frequency)
 $Z = R$ at resonance
SO $I_{max} = \frac{\mathcal{E}_0}{R}$

Maxwell's equations in the vacuum:
 \vec{E} Gauss $\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$
 \vec{B} Gauss $\oint \vec{B} \cdot d\vec{A} = 0$
Faraday $\oint \vec{E} \cdot d\vec{r} = -\frac{d\Phi_B}{dt}$
Ampere $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{enc}$

Electromagnetic Waves

Wave equation
 $\frac{d^2 E}{dx^2} = \epsilon_0 \mu_0 \frac{d^2 E}{dt^2}$
 $\frac{d^2 B}{dx^2} = \epsilon_0 \mu_0 \frac{d^2 B}{dt^2}$

Polarization
Law of Malus:
 $I = I_0 \cos^2 \theta$
where I is the intensity of the wave. θ is the angle between the field and the polarizer axis

Wave speed
 $v = \frac{\omega}{k}$
 $E = cB$
 $v = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \frac{c}{\sqrt{\epsilon_r \mu_r}}$
 $k = \frac{2\pi}{\lambda}$

Connecting E and B
 $E_y(x,t) = E_0 \sin(kx - \omega t)$
 $B_z(x,t) = B_0 \sin(kx - \omega t)$
 $B_0 = \frac{E_0}{c}$

Energy and Intensity
Poynting Vector (\vec{S}):
 $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$
 $S_{max} = \frac{E_0^2}{\mu_0 c}$
Intensity (I) = $S_{avg} = \frac{1}{2} \frac{E_0^2}{\mu_0 c}$

Faraday's Law (x-y) Plane
 $\frac{d\Phi_B}{dt} = \frac{d}{dt} \int \vec{B} \cdot d\vec{A}$
 $\frac{dE}{dx} = -\frac{dB}{dt}$

Ampere's Law (x-z) Plane
 $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{enc}$
 $\frac{dB}{dx} = -\epsilon_0 \mu_0 \frac{dE}{dt}$

Transformer

$V_p = \mathcal{E} = -N_p \frac{\Delta \Phi}{\Delta t}$
 $V_s = \mathcal{E} = -N_s \frac{\Delta \Phi}{\Delta t}$
 $\frac{V_p}{N_p} = \frac{V_s}{N_s}$
Power: $P_p = V_p I_p = V_s I_s = P_s$
 $N_s > N_p$: Step up
 $N_p > N_s$: Step down

Angular
 $\omega = \frac{2\pi}{T} = 2\pi f$
 $f = \frac{1}{T}$
 $\lambda = \frac{c}{f}$

Amperes Law
 $\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{enc}$
• Do thin wire with current = I
• Thick shell, uniform charge density with current = $-2I$

Hall effect
 \vec{B} field out of page

Spinning charged disk
 $\Delta V_H = \frac{IB}{tne}$

Parallel RLC Circuit
 $\mathcal{E} = N \Delta \Phi / \Delta t$
 $I_r = \frac{V}{R}$
 $I_L = \frac{V}{X_L}$
 $I_C = \frac{V}{X_C}$
 $I_s = \sqrt{I_r^2 + (I_L - I_C)^2}$

Flip Coil
Flux before:
 $\Phi_B = -NB \cdot A$
Flux after:
 $\Phi_B = -NB \cdot A$
 $\Delta \Phi = -2NB \cdot A$
 $\Delta Q = \frac{2NB \cdot A}{R}$
 $B = \frac{VRC}{2NA}$

Rail Gun (Motional EMF)
"Work done by pulling or pushing exactly matches resistor dissipate"