

STAT-321

Sample Space (S)

"List of possible outcomes for a random experiment where the outcome is unknown"

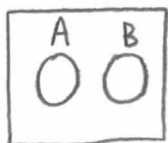
Complement: (NOT)

"The outcomes in the sample space not covered by an event"

$$S^c = \phi$$

Mutually Exclusive:

$$A \cap B = \phi$$



Axioms of Probability:

- ① $0 \leq P(E) \leq 1$
Event
Probability function
- ② $P(S) = 1$
sample space

- ③ For a sequence of mutually exclusive events,

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3)$$

Combinatorial Formula: ($r \leq n$)

"The number of distinct combinations of size r that can be made from n elements"

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(order doesn't matter, just unique sets)

Event: (A, B, C, \dots)

"Subset of a Sample Space"

$$A = \{2, 4, 6\}$$

$$B = [0, 60]$$

Null set: (ϕ)

"An event with no outcome"

ex: $E = \text{roll a number } > \text{ on a die}$

$$E = \phi$$

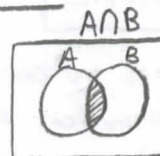
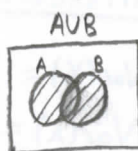
Union: (OR)

$$A \cup B$$

Intersection: (AND)

$$A \cap B$$

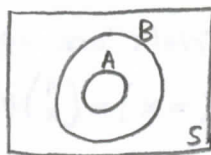
Venn Diagram:



Contained Event:

$$A \subset B \text{ 'or' } B \supset A$$

"Little guy eats the big guy"



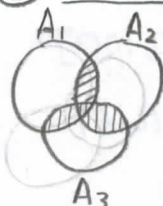
Probability Rules:

- ① $P(E^c) = 1 - P(E)$
- ② If $E \subset F$, $P(E) \leq P(F)$
- ③ $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

- ④ If all outcomes are equally likely

$$P(E) = \frac{\text{\# of outcomes in } E}{\text{\# of outcomes in } S}$$

Inclusion - Exclusion formula:

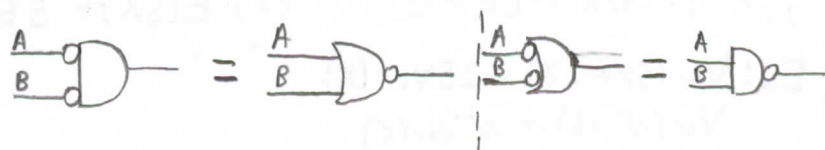


$$\begin{aligned} P(A_1 \cup A_2 \cup A_3) &= P(A_1) + P(A_2) + P(A_3) \\ &\quad - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) \\ &\quad + P(A_1 \cap A_2 \cap A_3) \end{aligned}$$

DeMorgan's Law

$$P(A^c \cap B^c) = P((A \cup B)^c)$$

$$P(A^c \cup B^c) = P((A \cap B)^c)$$



Conditional Probability

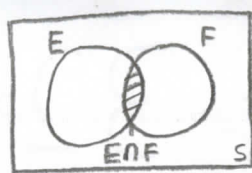
$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

"The probability that event E occurs given the fact that event F has already occurred"

Independence of Events:

E and F are independent if, $P(E|F) = P(E)$, i.e. "whether F has occurred has no effect on E"

If independent, ..., $P(E \cap F) = P(E) \cdot P(F)$



Bayes Theorem:

$$P(E|F) = \frac{P(F|E) \cdot P(E)}{P(F)}$$

Conditional Independence: (For E and F)

- $P(E|F \cap G) = P(E|G)$
- $P(F|E \cap G) = P(F|G)$
- $P(E \cap F|G) = P(E|G) \cdot P(F|G)$

Mean: (Expected Value) $E(X) = \mu$

"Tossing a Weighted Die"

X	1	2	3	4	5	6
p(x)	0.1	0.1	0.1	0.1	0.1	0.5

$$\text{Expected Value} = \sum X P(X)$$

$$0.1(1+2+3+4+5) + 0.5(6) = 4.5$$

Discrete Random Variable

"Random variable that maps outcomes is a sample space. Numerical values are given to events in the sample space"

$$X = \begin{cases} 0 & \text{- Tails} \\ 1 & \text{- Heads} \end{cases} \quad X = \{0, 1\}$$

Moment Generating Function: $M_X(t)$

$$M_X(t) = E(e^{tx})$$

-Characterizes the distribution of a random variable X

Poisson Process:

$$X \sim \text{Poisson}(\lambda)$$

Continuous Random Variable

-Takes on continuous values

$$X = [0, 100]$$

Cumulative Distribution Function: (cdf)

$$F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt \text{ for all } x$$

$$F(x) = \int_{-\infty}^x f(t) dt \quad \text{--- pdf}$$

Variance: $\text{Var}(X) = \sigma^2$

$$\text{Var}(X) = E(X^2) - [E(X)]^2$$

$$E_X: \text{Var}(5X+27) = 25\text{Var}(X)$$

$$\text{Var}(aX+b) = a^2\text{Var}(X)$$

Variance: $\text{Var}(X)$

$$\textcircled{1} \text{Var}(X) = \sum (x - E(X))^2 P(X)$$

$$\textcircled{2} \text{Var}(X) = E(X^2) - [E(X)]^2$$

Standard Deviation: $SD(X)$

$$SD(X) = \sqrt{\text{Var}(X)}$$

Binomial Random Variable:

① Trial results in Success/Failure

② $X \sim \text{Bin}(n, P)$

③ Trials are independent of one another

$$P(X=x) = \binom{n}{x} P^x (1-P)^{n-x} \quad \text{[just a Histogram]}$$

Probability Density function: (pdf)

$$f(x) \geq 0 \text{ for all } x$$

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad \text{[Area under graph]}$$

$$f(x) = F'(x)$$

Expected Value: $E(X)$

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

$$\textcircled{1} E(X+Y) = E(X) + E(Y)$$

$$\textcircled{2} E(5X) = 5E(X)$$

pdf, cdf Relationship

What is $P(X_1 \leq X \leq X_2)$?

$$P = \int_{X_1}^{X_2} f(x) dx$$

$$P = F_X(X_2) - F_X(X_1)$$

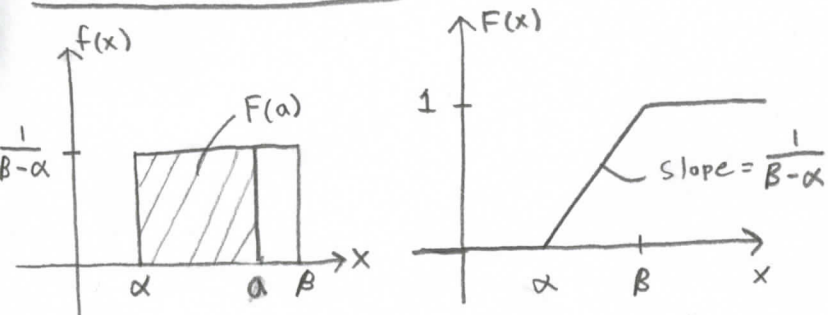
Common Continuous Distributions

Remember $f(x) = F'(x)$

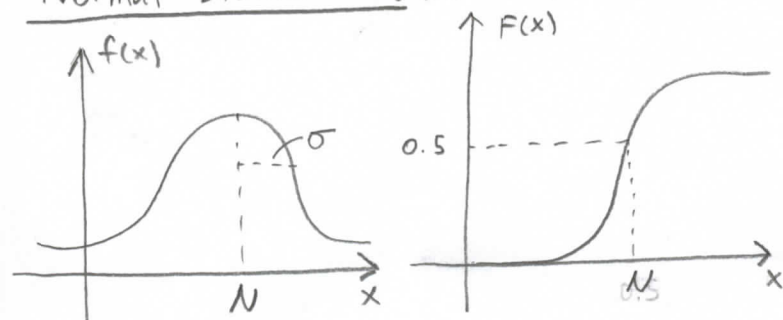
$$f(x) = \text{pdf}(X)$$

$$F(x) = \text{cdf}(X)$$

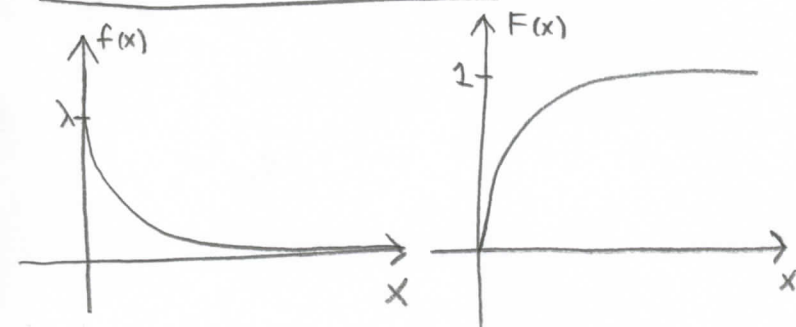
Uniform Distribution: $X \sim U(\alpha, \beta)$



Normal Distribution: $X \sim N(\mu, \sigma^2)$



Exponential Distribution: $X \sim \text{Exp}(\lambda)$



Expectations of functions:

$$E(g(X, Y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f(x, y) dx dy$$

Correlation:

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X) \text{Var}(Y)}}$$

Failure Rate:

$$\lambda(x) = \lim_{\delta \rightarrow 0} \frac{P(X \leq x + \delta | X > x)}{\delta}$$

Jointly distributed random Variables

probability density function: $\text{pdf}(x, y)$

- Continuous random variables X, Y

$$P(X \in A, Y \in B) = \iint_{A \times B} f(x, y) dy dx$$

Cumulative distribution function: cdf

$$F_{X,Y}(x, y) = P(X \leq x, Y \leq y)$$

$$= \int_{-\infty}^y \int_{-\infty}^x f(x, y) dx dy$$

Marginal PDF's:

$$\text{For } X: f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

$$\text{For } Y: f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

Covariance:

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$$

properties:

$$\textcircled{1} \text{Cov}(X, X) = \text{Var}(X)$$

$$\textcircled{2} \text{Cov}(aX, bY) = ab \text{Cov}(X, Y)$$

Central limit thm: (CLT)

- The distribution Z converges to $N(0, 1)$ as $n \rightarrow \infty$

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \quad \text{where } \bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}$$