



80 Pages
27.6 cm x 21.2 cm

Ruled 7 mm • Ligné 7 mm

EXERCISE BOOK CAHIER D'EXERCICES

NAME/NOM Cole Shanks

SUBJECT/SUJET ELEC 202



ASSEMBLED IN CANADA WITH IMPORTED MATERIALS
ASSEMBLÉ AU CANADA AVEC DES MATIÈRES IMPORTÉES

12107

ELEC 202 LEC 1

1/02/19

LEC

Ohm's Law $GV = I$ where $G = \frac{1}{R}$ (conductance)
* in Siemens

Complex numbers

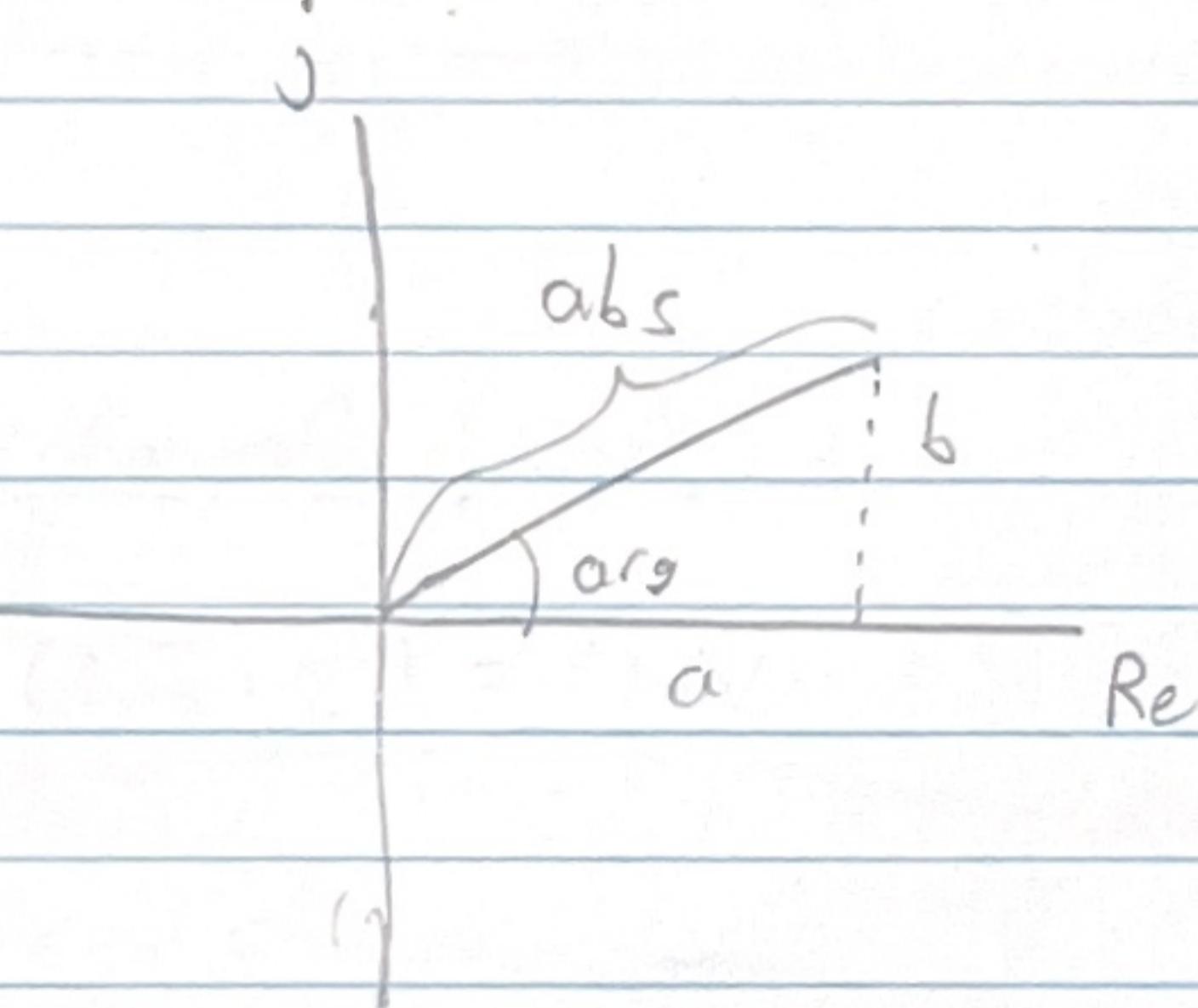
$$(3 + j4)(5 - j7) = 43 - j$$

Rectangular form :

$$(3, 4)(5, -7) = (43, -1)$$

Polar Form :

$$abs[\arg^\circ]$$



$$V(t) = 30 \cos(377t + 25^\circ)$$

$$\omega = 2\pi f$$

$$V = 30 \angle 25^\circ$$

1/07/18

LEC

Laplace Domain

Laplace

$$f(t) \longleftrightarrow F(s)$$

$$u(t) \longleftrightarrow \frac{1}{s}$$

$$e^{-at} u(t) \longleftrightarrow \frac{1}{s+a}$$

$$t u(t) \longleftrightarrow \frac{1}{s^2}$$

$$\delta(t) \longleftrightarrow 1$$

For the inductor

Time Domain

$$V(t) = L \frac{di}{dt}$$

Laplace Domain

$$V(s) = L s \cdot I(s)$$

Impedance (Z) of the inductor
in Ohms

For the capacitor

Time Domain

$$I(t) = C \frac{dV}{dt}$$

Laplace Domain

$$I(s) = C s \cdot V(s)$$

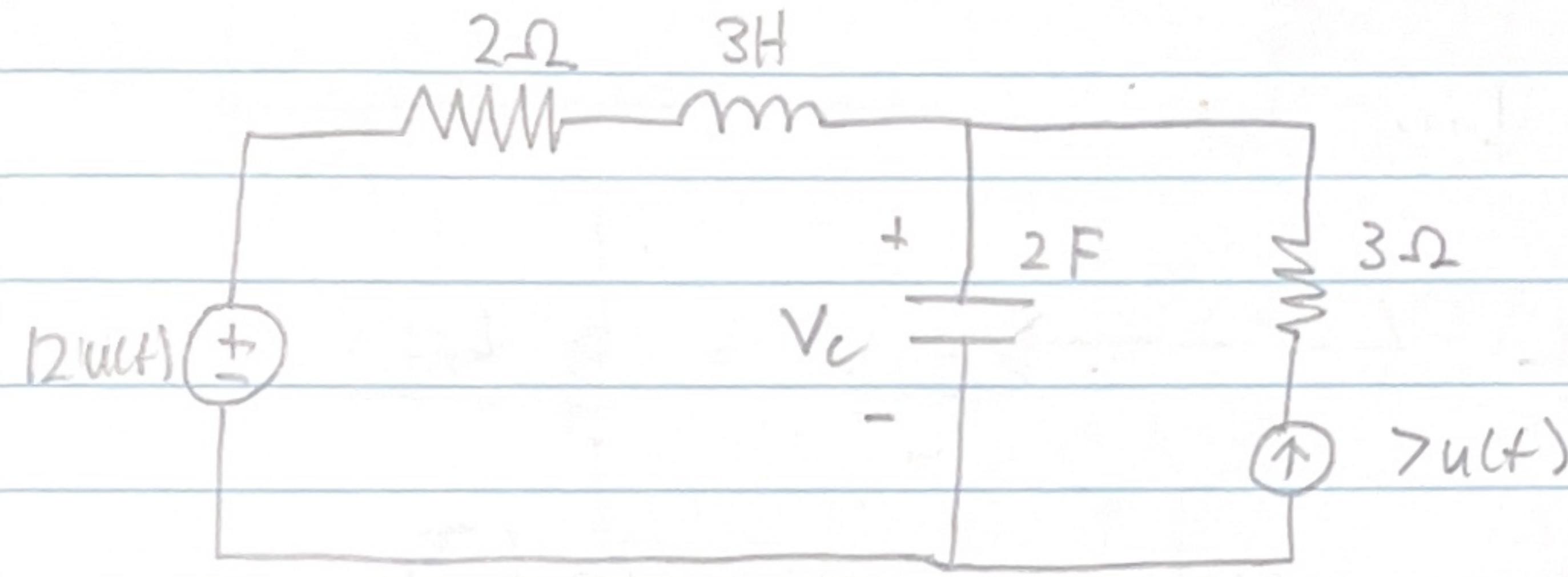
$$V(s) = \frac{1}{Cs} \cdot I(s)$$

Impedance (Z) of the
capacitor in Ohms

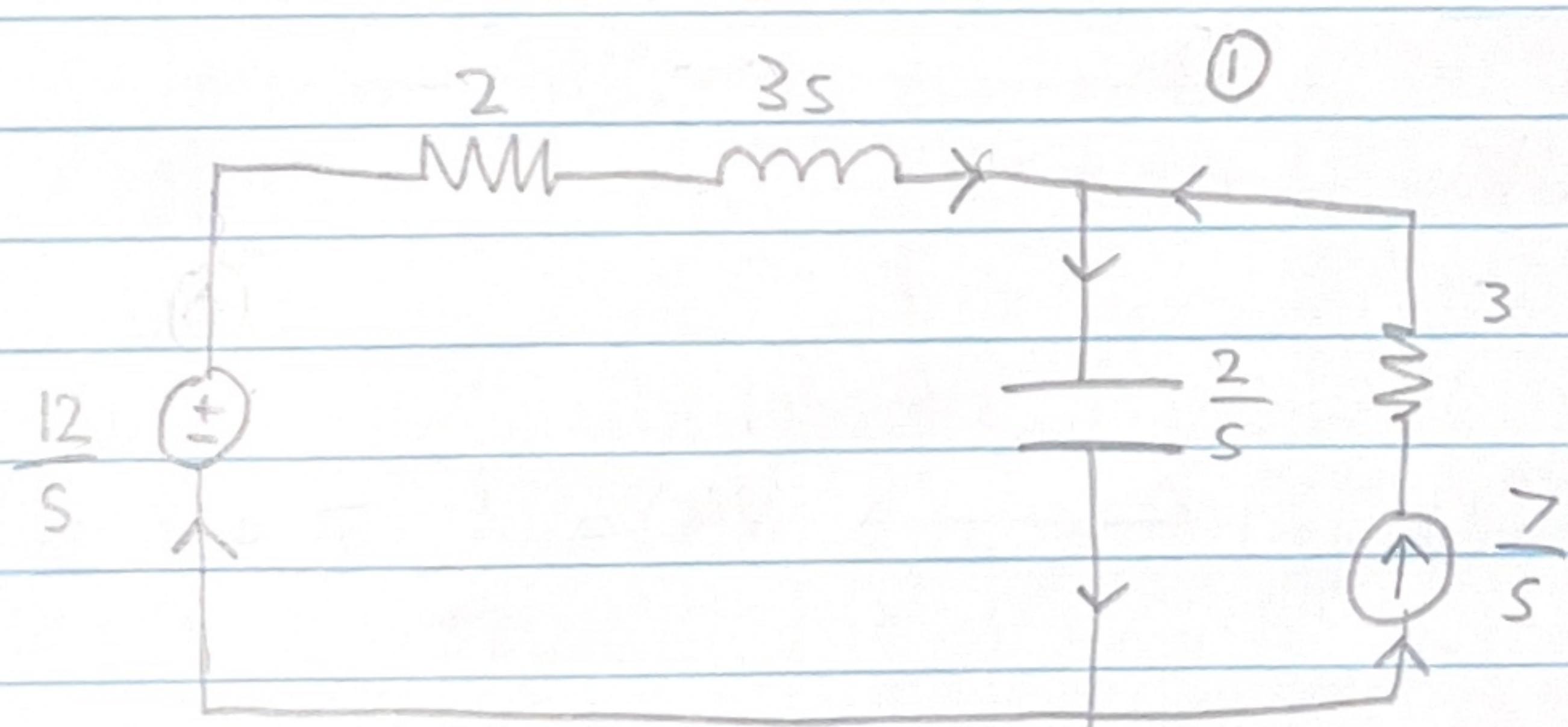
MNA in Laplace Domain



$$I(s) = V_3 - V_1$$



Laplace Domain



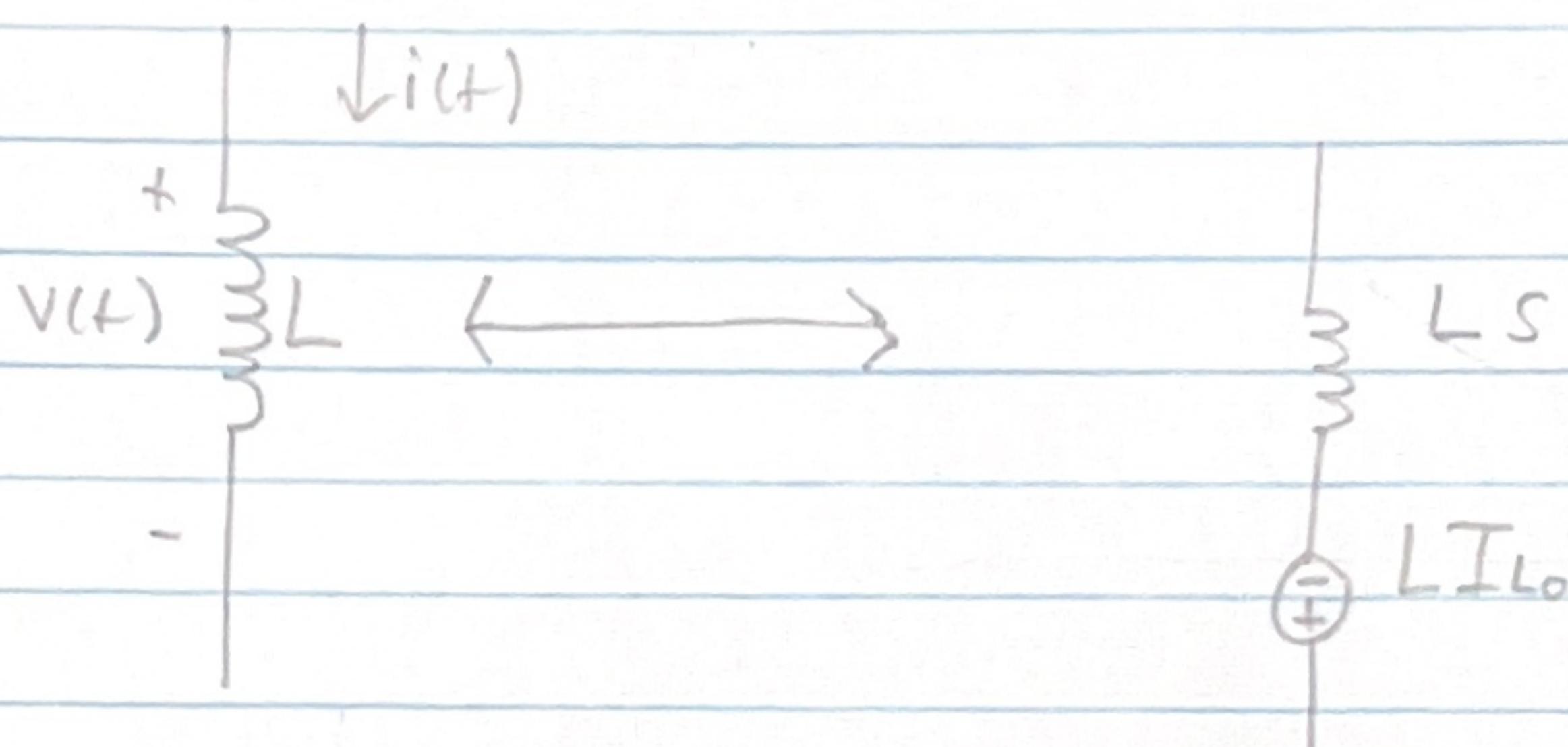
$$KCL_1: \frac{7}{s} + \frac{\frac{12}{s} - V_1}{2+3s} = \frac{V_1 \cdot s}{2}$$

$$V_1 = \frac{42s + 52}{3s^3 + 2s^2 + 2s}$$

Accounting For IC

Inductors:

$$V(t) = L \frac{di}{dt} \longleftrightarrow V(s) = LS I(s) - L I_{L0}$$



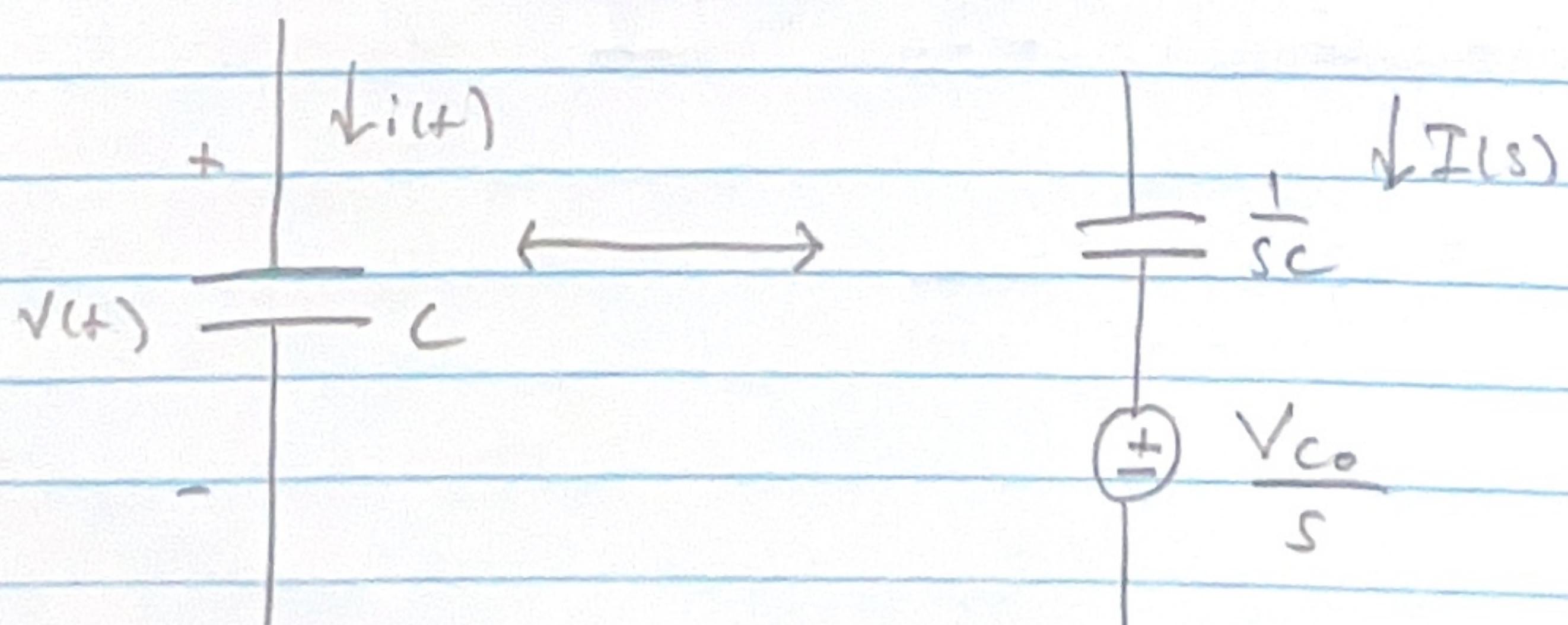
Capacitors:

$$i(t) = C \frac{dV}{dt}$$

$$V(t) = \frac{1}{C} \int i(t) dt \longleftrightarrow V(s) = \frac{1}{sC} I + \frac{1}{sC} V_{co}$$

initial voltage

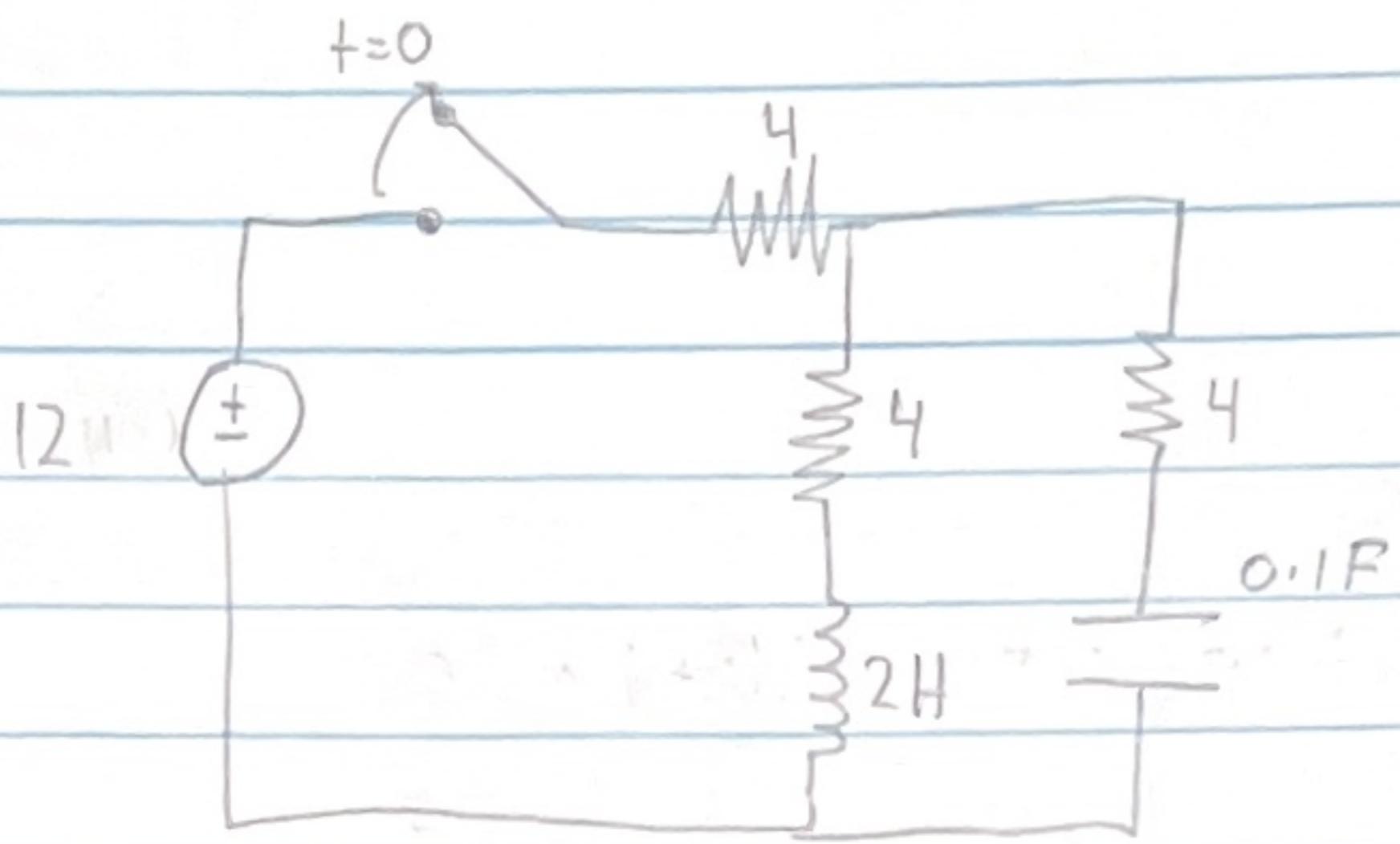
$$V(s) = \frac{1}{sC} I + \frac{V_{co}}{s}$$



Laplace Transform

11/09/19

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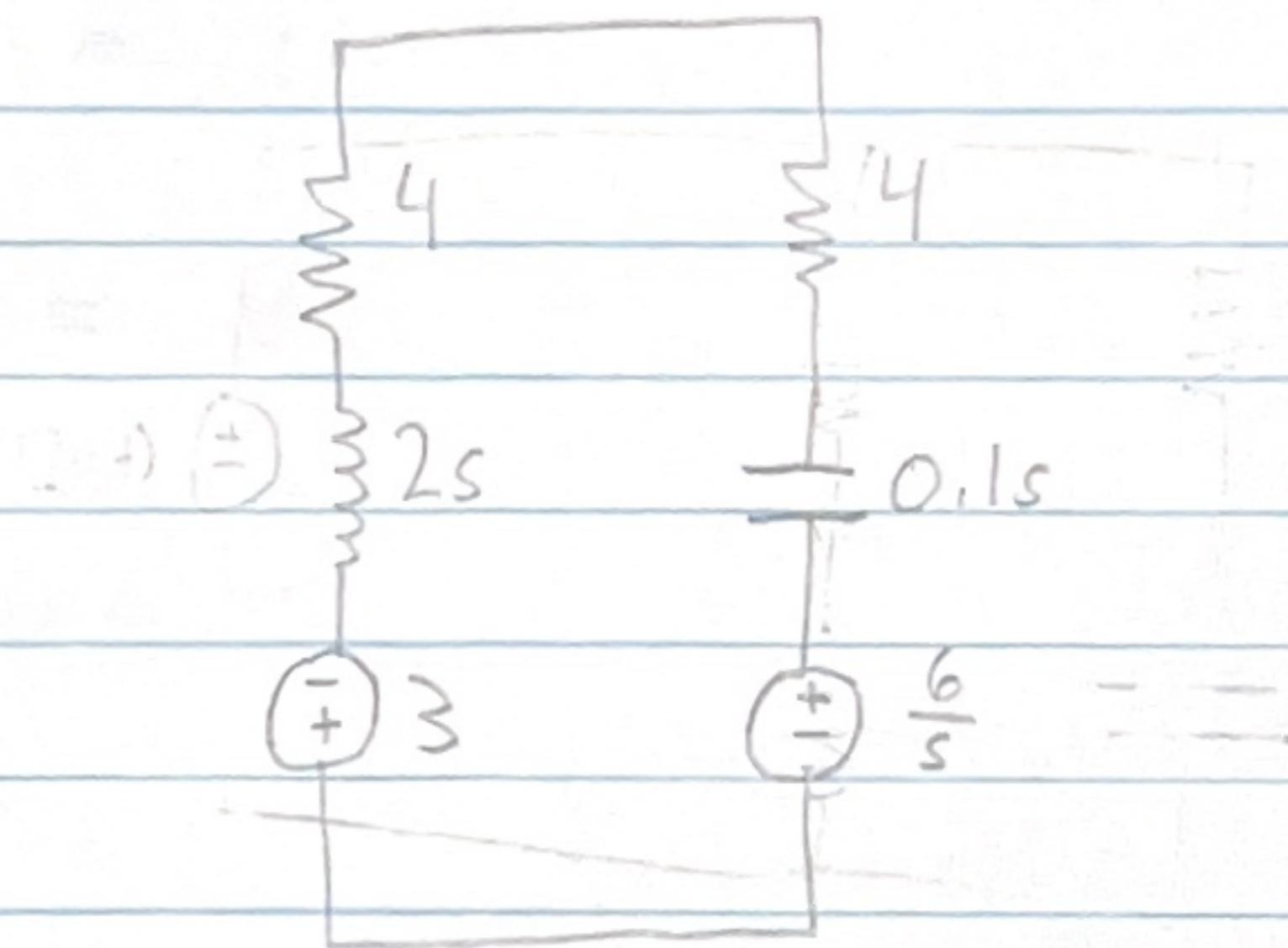
DC.SS Findings:

$$I_L = 1.5A$$

$$V_C = 6V$$

$t > 0$

Laplace Domain



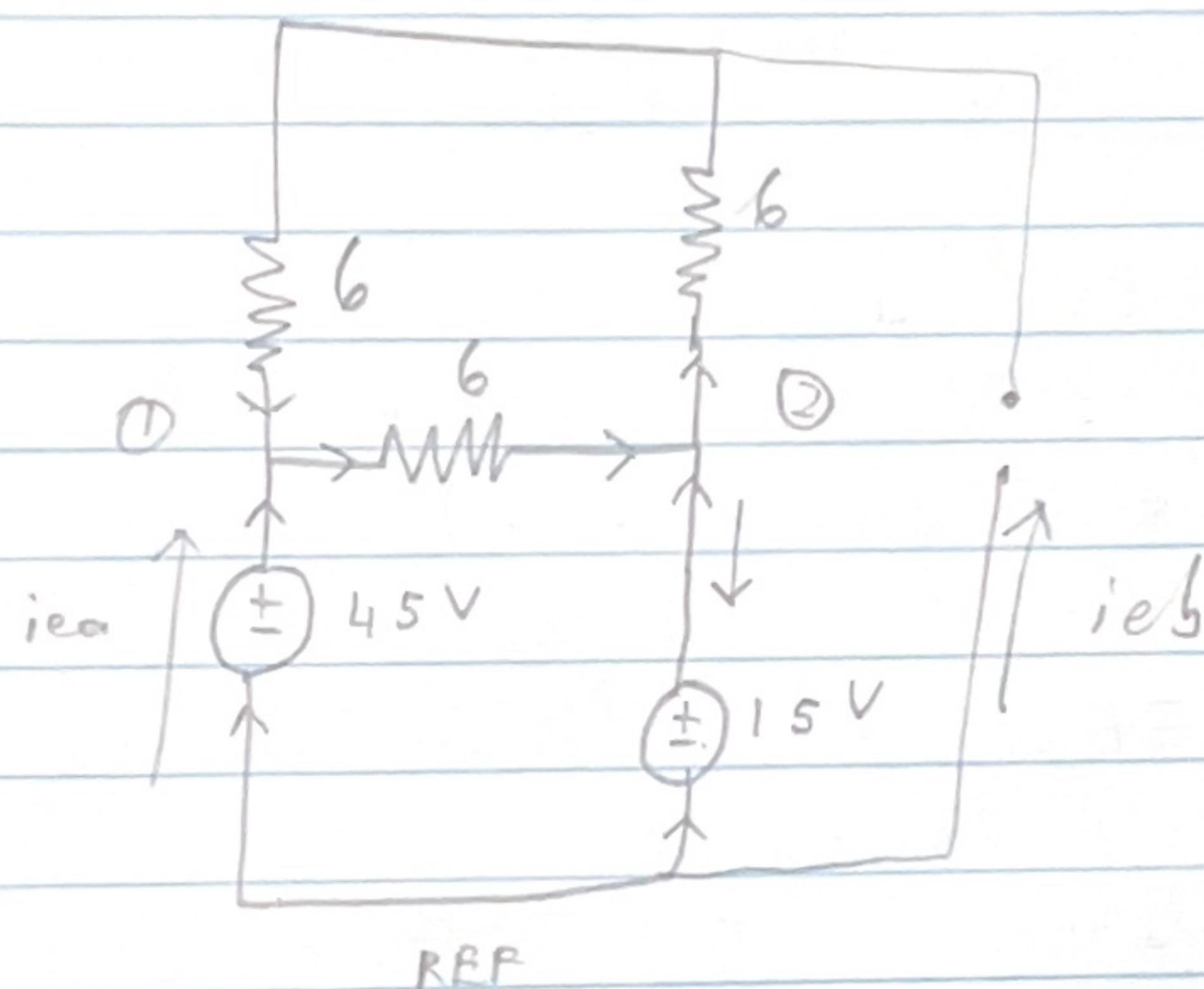
$$I_L = \frac{3 + \frac{6}{5}}{4 + 4 + 2s + 0.1s} = \frac{3}{2} \cos(t) e^{-2t}$$

Charles Proteus Steinmetz

2nd Order Circuits

of reactive elements (capacitors / inductors) gives order

$$I = \frac{15 - 45}{12} = V_C = 30V$$



$$V_1: i_{ea} + \frac{V_2 - V_1}{12} = \frac{V_1 - V_2}{6}$$

$$V_2: \frac{V_1 - V_2}{6} + i_{eb} = \frac{V_2 - V_1}{12}$$

$$EV1_1: 0 + 45 = V_1$$

$$EV1_2: 0 + 15 = V_2$$

$$V_2: \frac{45 - 15}{6} + i_{eb} = \frac{15 - 45}{12}$$

$$\therefore V_1 =$$

$$V_2 =$$

$$EVL :$$

$$V_2:$$

$$V_2: \frac{45 - 15}{6} = \frac{45 - 15}{12} -$$

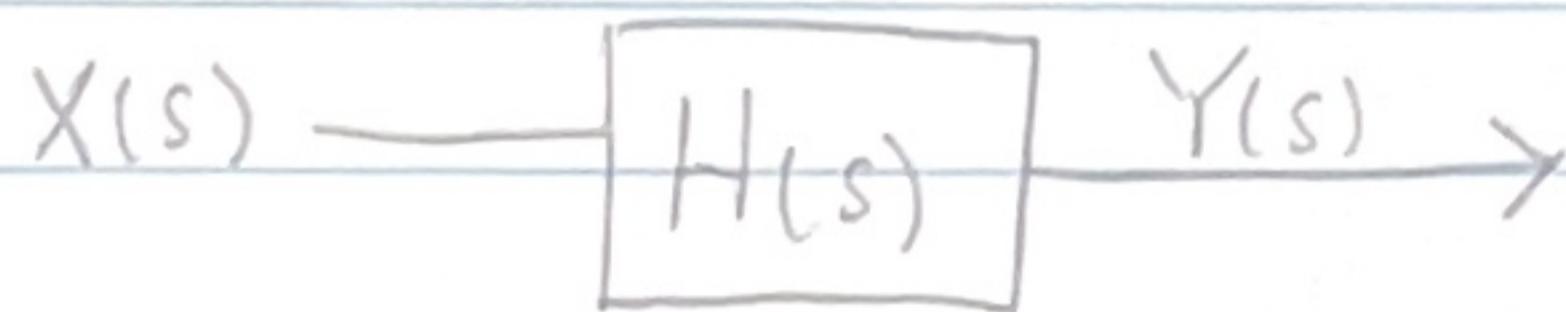
Frequency Response and Bode Plots

AC steady state

$$v(t) = 9 \cos(300t + 45^\circ) \longleftrightarrow \bar{v} = \frac{9}{\sqrt{2}} \angle 45^\circ$$

$$\begin{aligned}\bar{s} &= \rho + j\omega \\ &= \bar{v} \cdot \bar{i} \\ &= s \angle \theta\end{aligned}$$

Transfer Function



$$Y(s) = H(s) X(s)$$

* kill internal sources, kill I^cs → Find $\frac{Y(s)}{X(s)} = H(s)$

Decibels from Transfer Function (α_{dB})

$$\alpha_{dB} = 20 \log_{10} |H(s)| \Big|_{s=j\omega}$$

zeroes

$$H(s) = \frac{a_n s (s+z_1)(s+z_2)\dots(s+z_n)}{(s+p_1)(s+p_2)\dots(s+p_n)}$$

poles

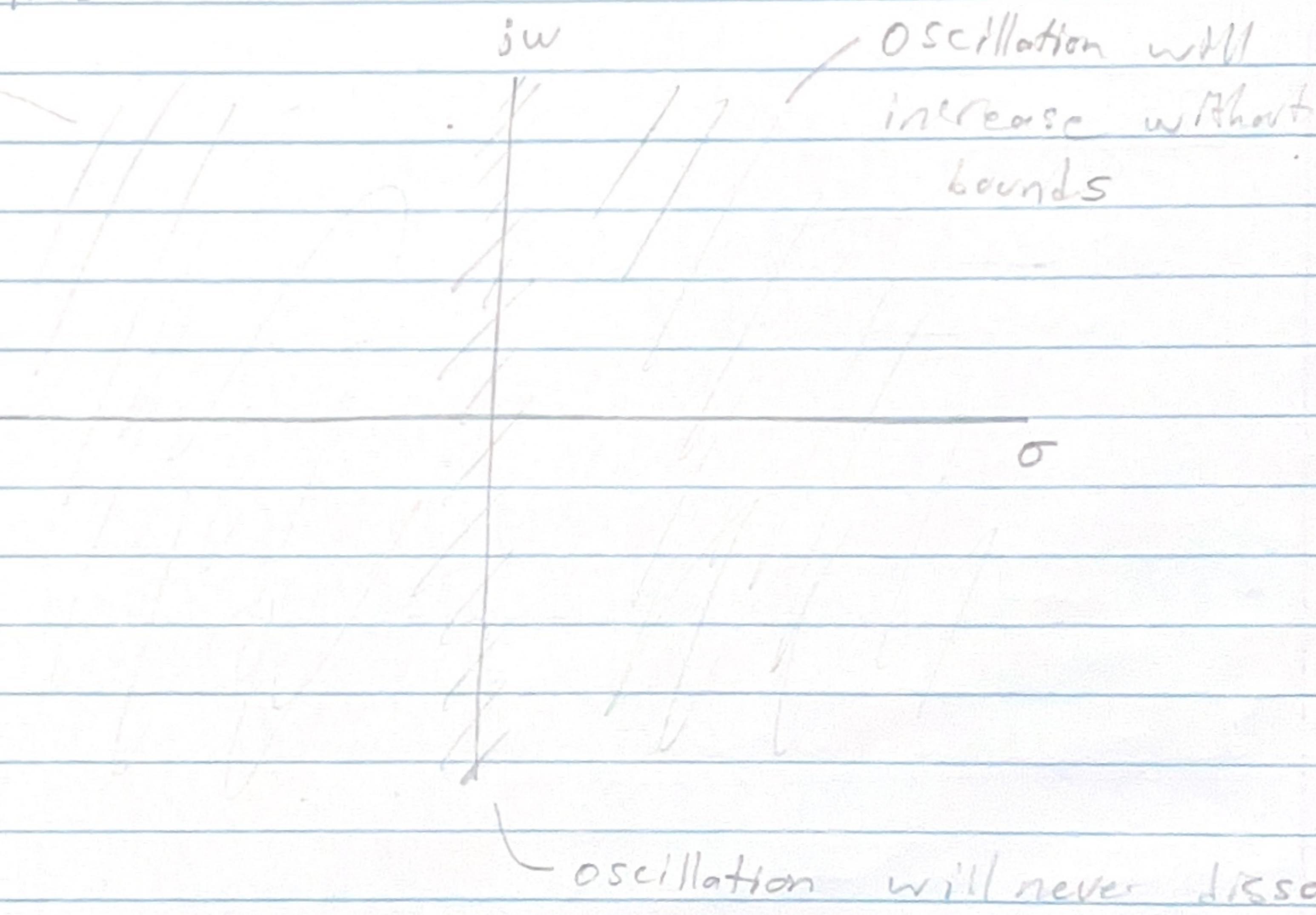
Stability of Poles

$$h(t) = k_1 e^{\sigma_1 t} e^{j\omega_1 t} + \dots + k_n e^{\sigma_n t} e^{j\omega_n t}$$

To be stable all $\sigma_n = 0$

* That means all poles (complex numbers) are on the ~~right of~~ j axis in complex plane

The oscillation will eventually die off.

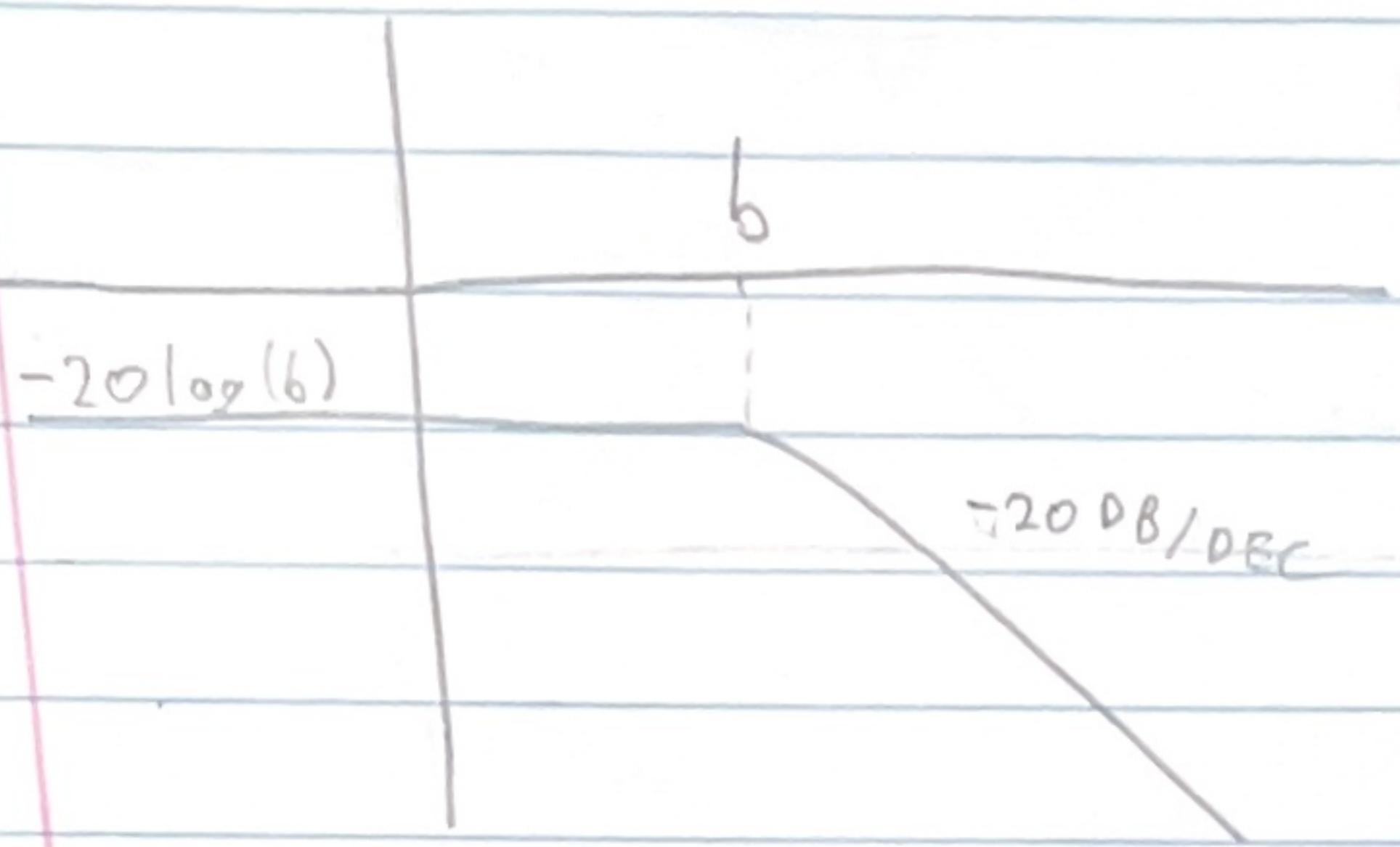


$$|H(s)| = \frac{|a_n| |s| |s+z_1| \dots |s+z_n|}{|s+p_1| \dots |s+p_n|}$$

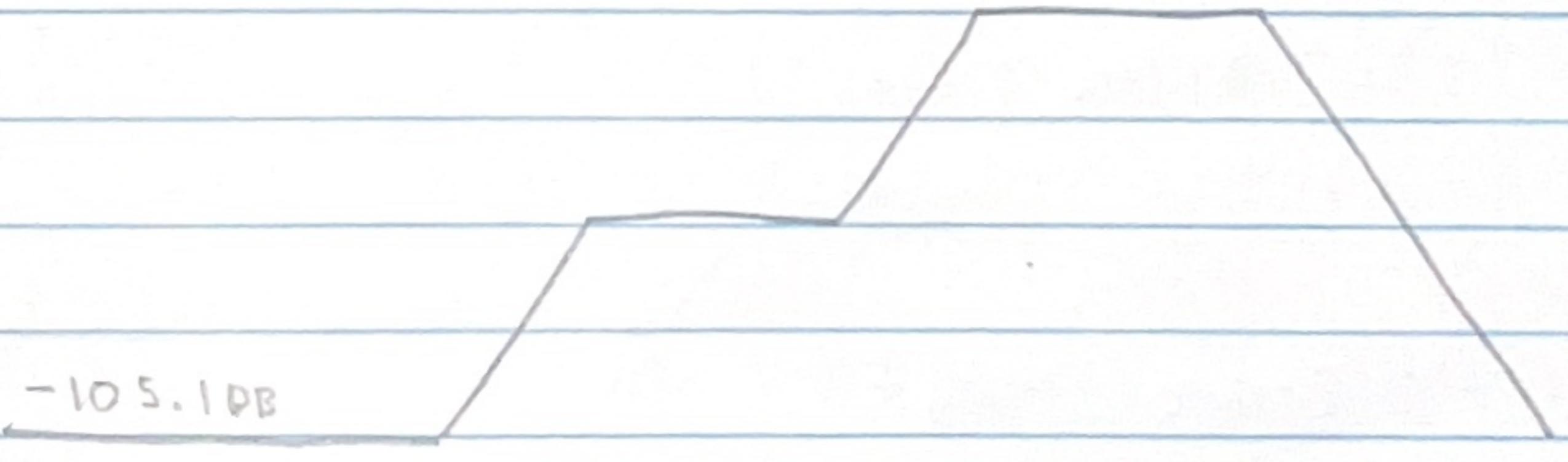
$$H_{OB} = 20 \log |a_n| + 20 \log |w| + 20 \log \sqrt{w^2 + z_1^2} - 20 \log \sqrt{w^2 + p_1^2} \dots$$

Bode Plots

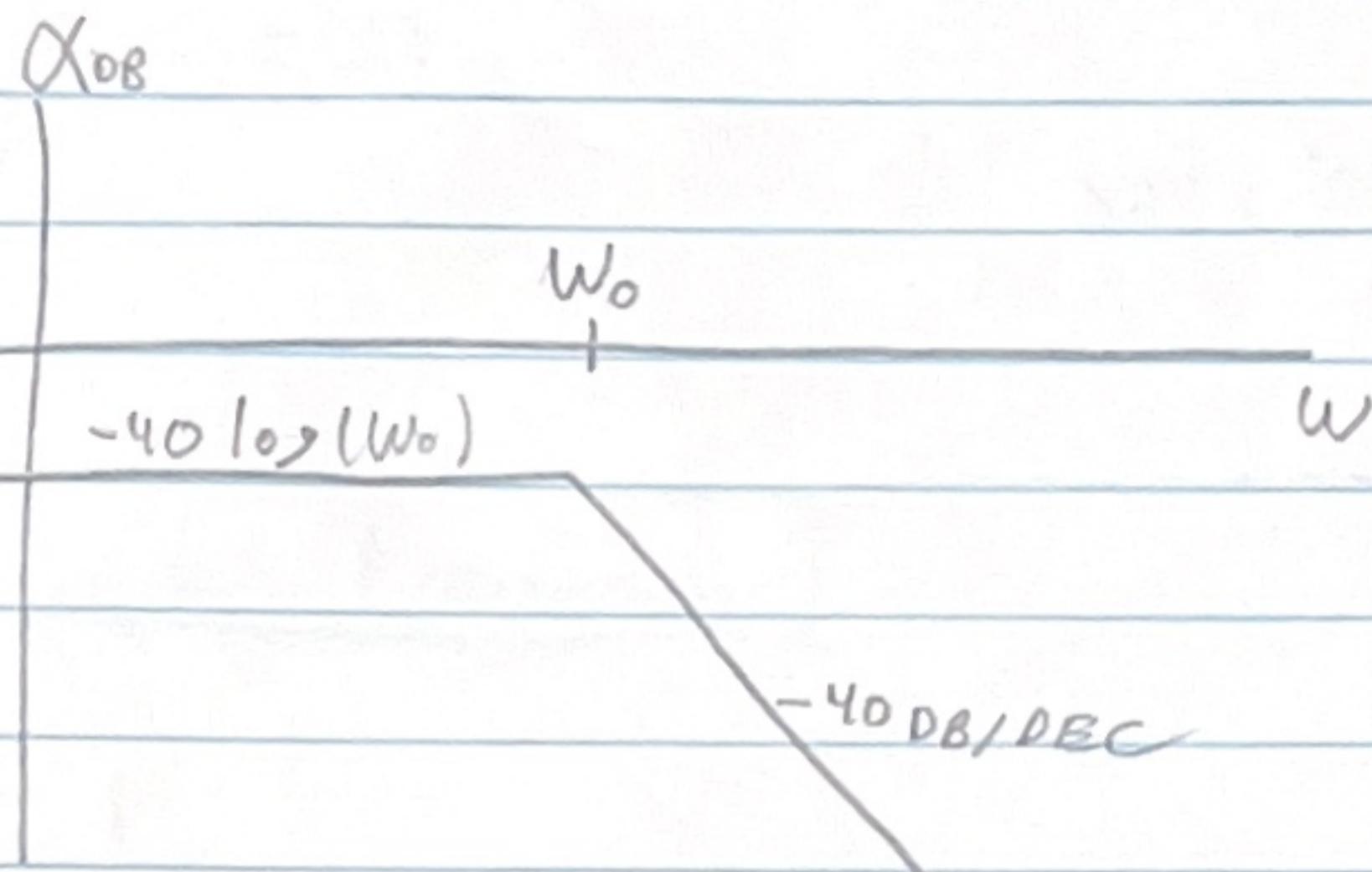
$$H(s) = \frac{1}{(s+6)}$$



$$H(s) = \frac{(s+20)(s+500)}{(s+150)(s+1200)(s+10,000)}$$



$$H(s) = \frac{1}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$$



Phase

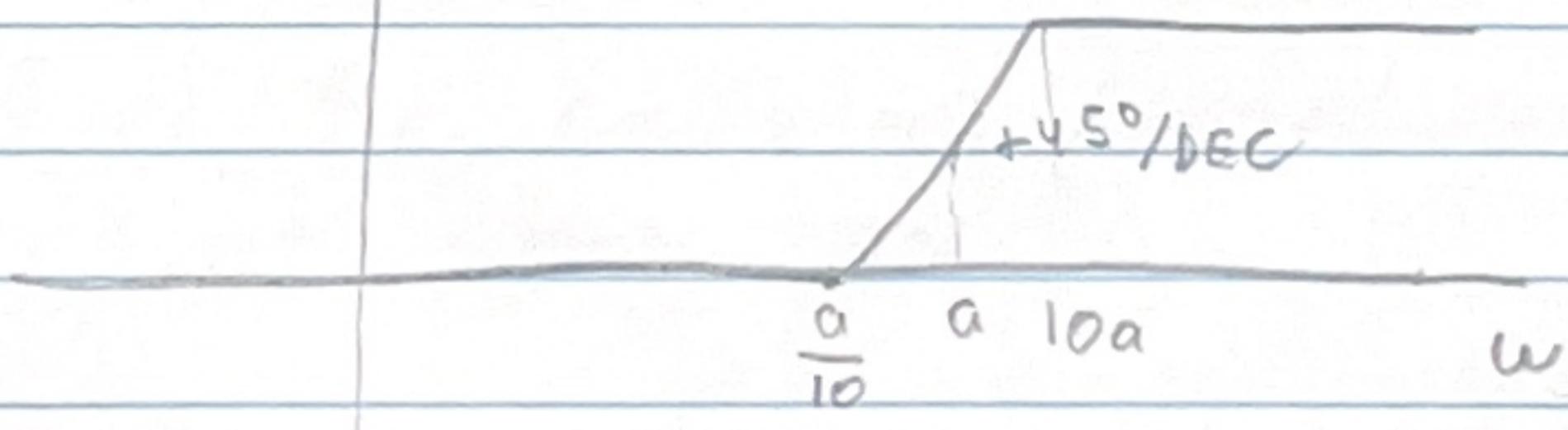
$$H(s) = (s + a)$$

$$H(s) = jw \quad (\text{shift } x\text{-axis up } 90^\circ)$$

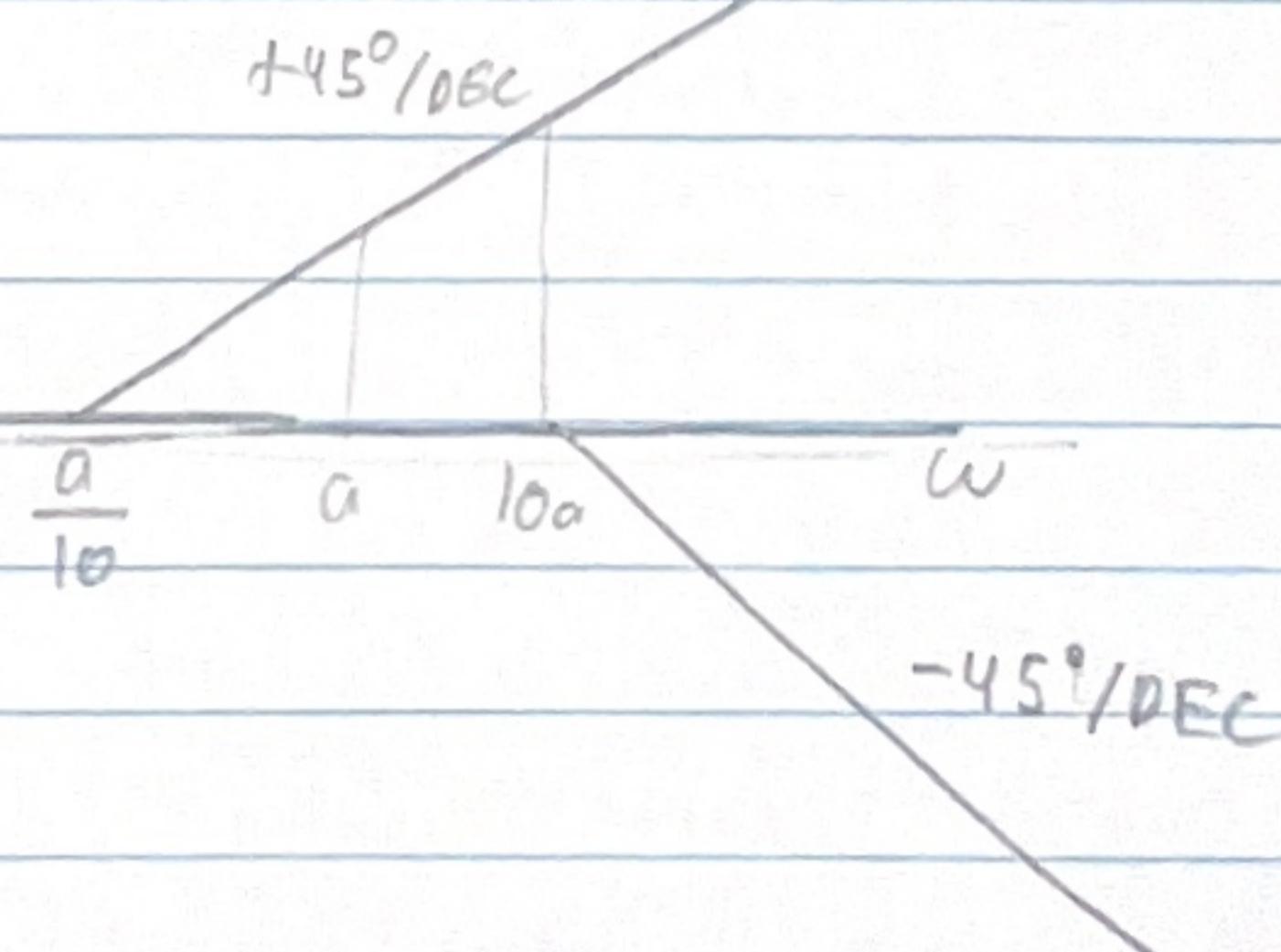
$$H(s) = 5s(s+a)$$

→ no effect

$\phi(^{\circ})$



ϕ



Resonance

1/28/19

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$$H(s) = \frac{Y(s)}{X(s)}$$

$$\alpha|_{s=jw} \text{ in dB}$$

$$\phi|_{s=jw} \text{ in } ^\circ$$

Resonance Frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

Quality Factor

$$Q = \frac{\omega_0}{BW}$$

* Another way to see it:

Find the equivalent impedance Z of the circuit where $s = j\omega$

Find the frequency (ω) where the angle of the impedance is zero, ie $\text{Im}(Z(\omega)) = 0$ then that ω is the resonance frequency

Filters

1/30/19

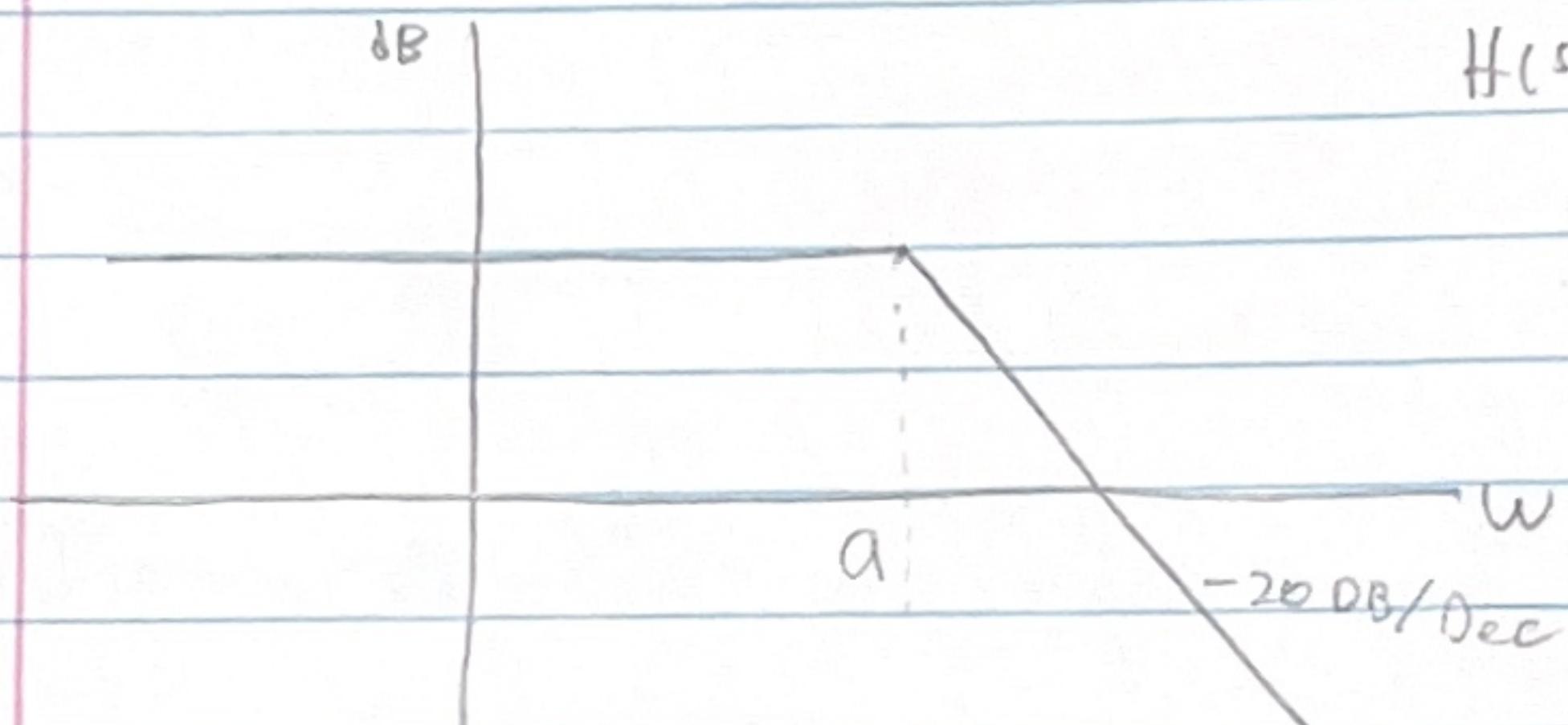
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Pass band $\rightarrow W_p = \text{cutoff frequency}$

Filters \rightarrow 4 types

* Edges of the filter are the poles of the function

$$H(s) = \frac{k}{(s+a)}$$

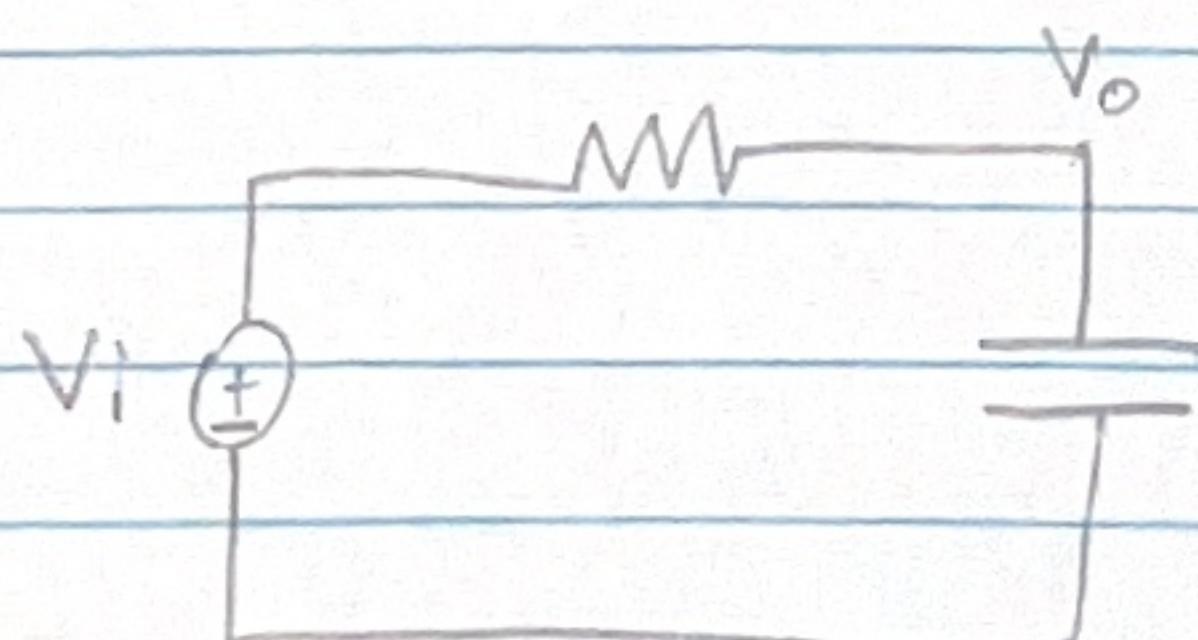


Approximated $a = \text{Pole}$
(ω_{cutoff})

$$\frac{3s - 1}{10,000}$$

$$H(s) = \frac{25000}{s + 6500}$$

$$\alpha_{\text{dB}} = \frac{25000}{\sqrt{(10^x)^2 + 6500^2}} \rightarrow \text{calculator}$$



$$kcl: 0 + V_i - V_o = V_o \cdot C \cdot s$$

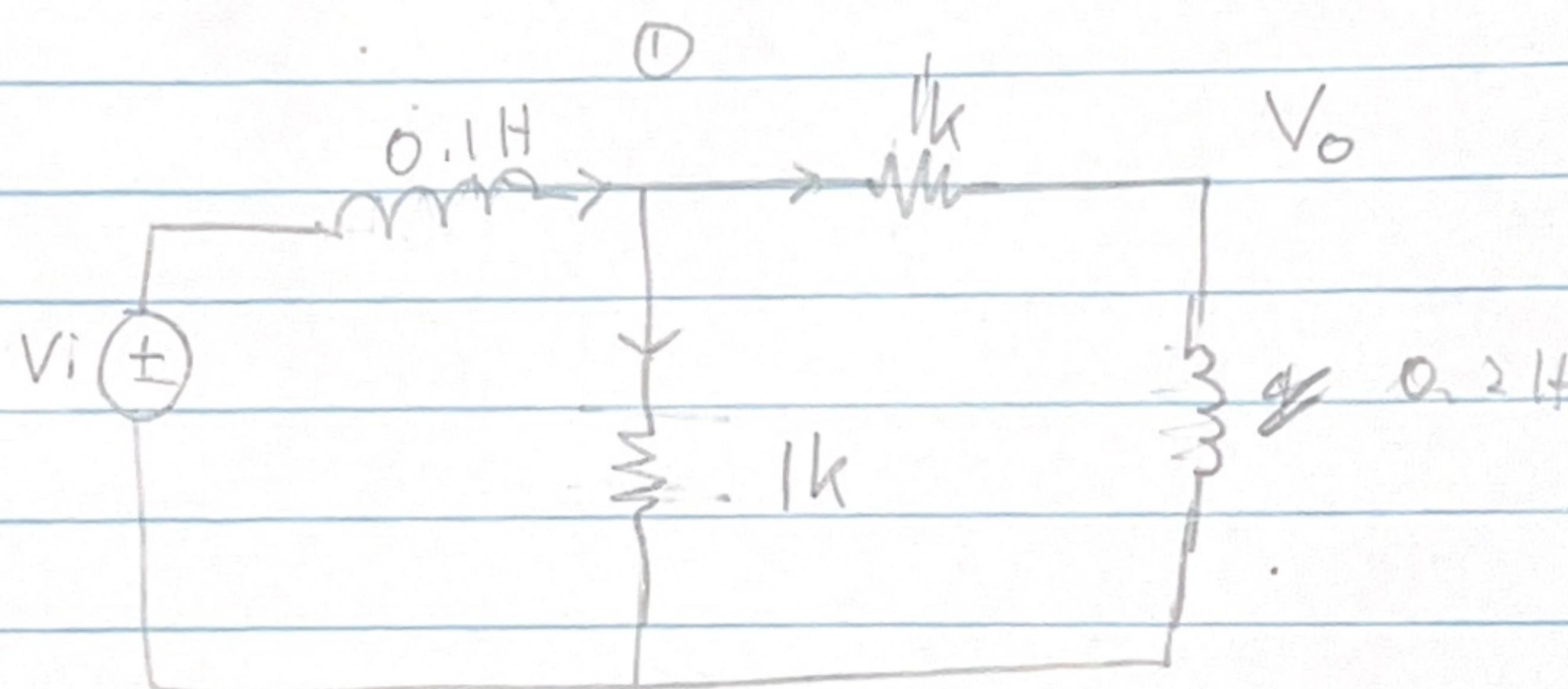
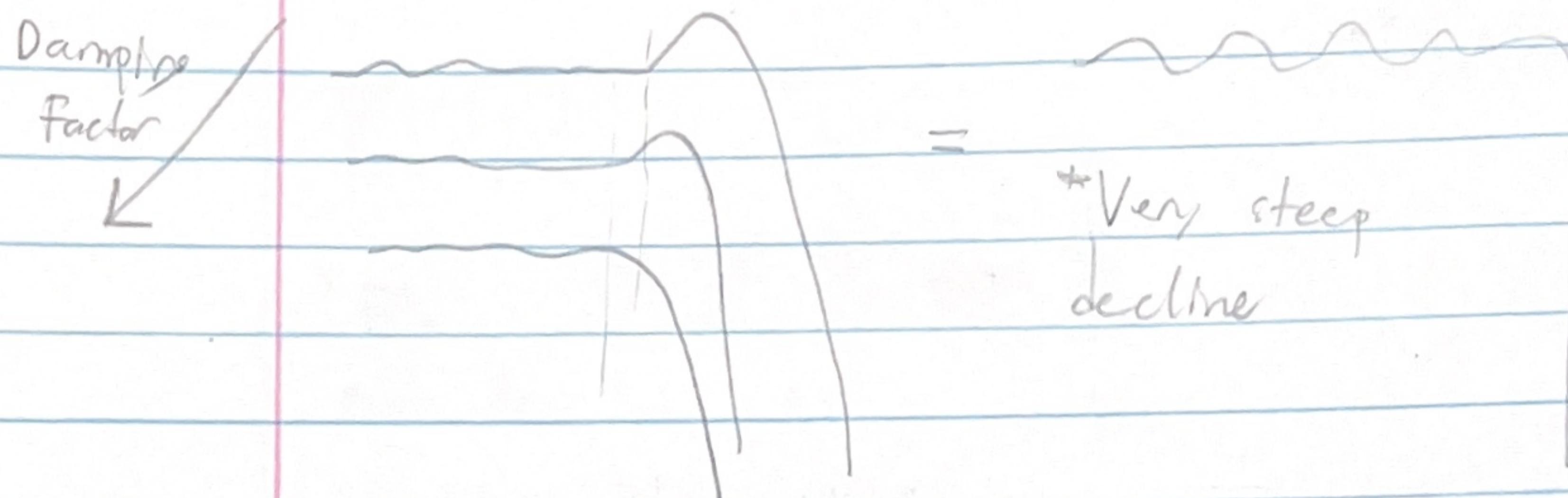
$$\frac{V_o}{V_i} =$$

Filters

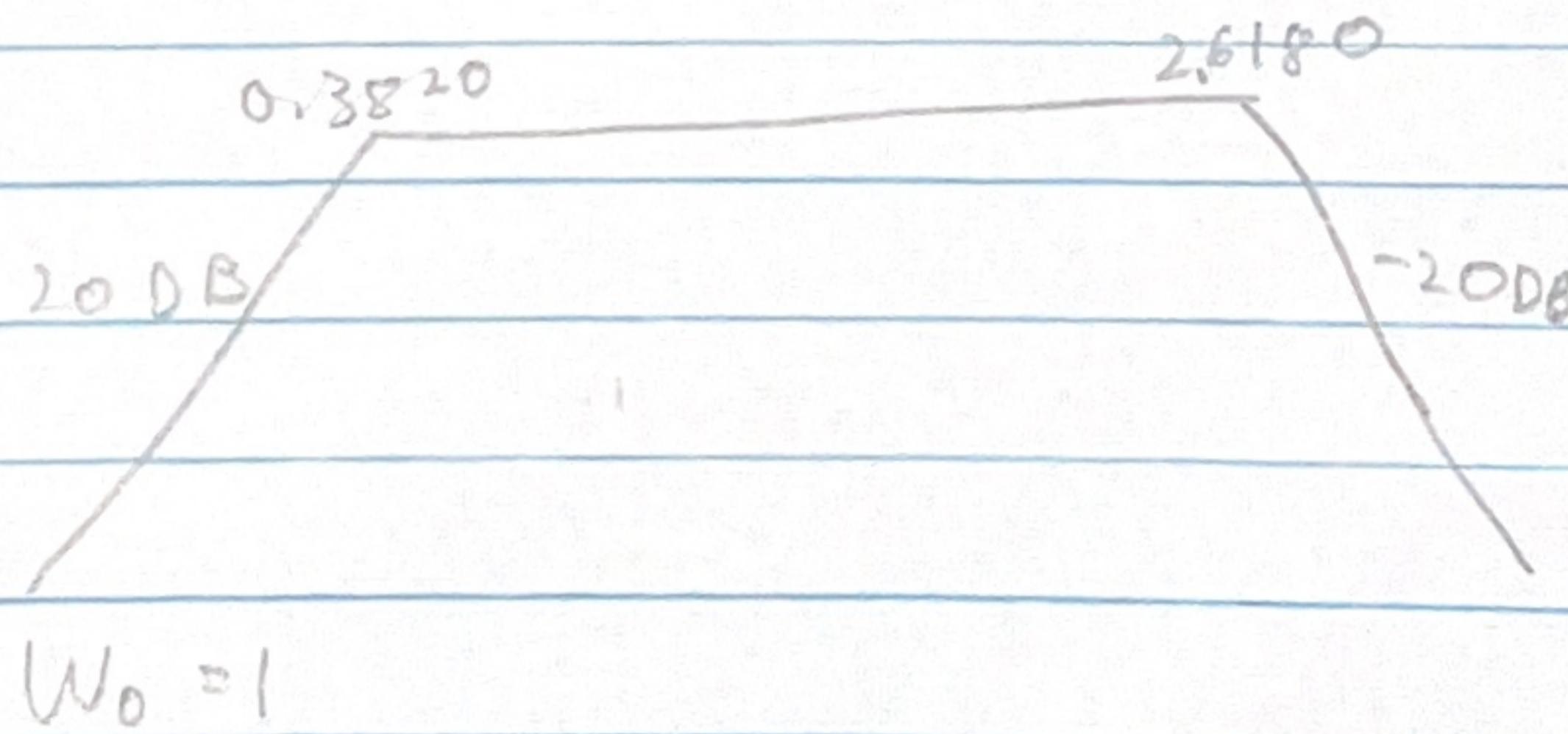
2/4/19

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Butterworth Filters (Chelgchen **)



$$H(s) = \frac{s}{s^2 + 3s + 1}$$



$$\omega_0 = 1$$

$$KCL: V_i - V_1 = \frac{V_1}{0.1s} + \frac{V_1 - V_o}{1k}$$

$$BW = \omega_2 - \omega_1$$

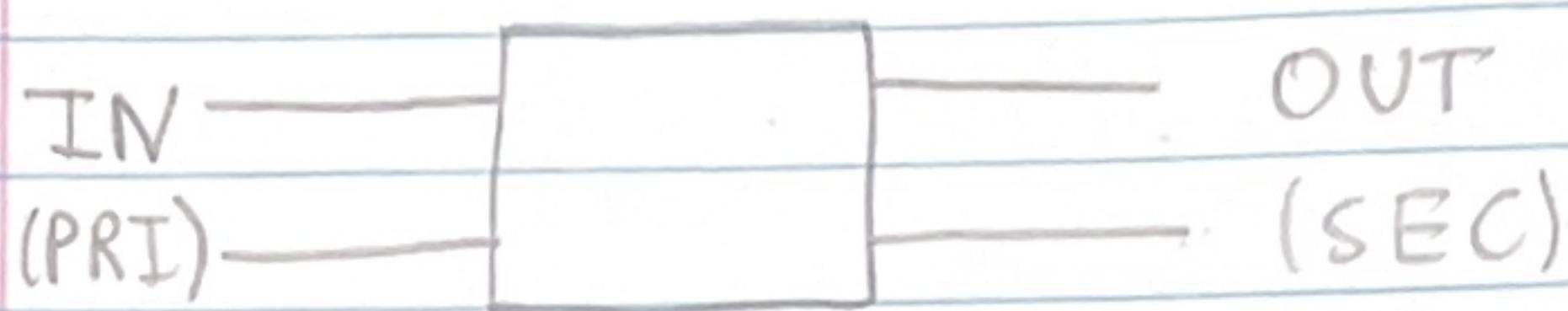
$$EQ_2: \frac{V_1 - V_o}{1k} = \frac{V_o - 0}{s \cdot 0.2}$$

Two-Port Networks

2/6/19

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Ideal Transformer



Step down

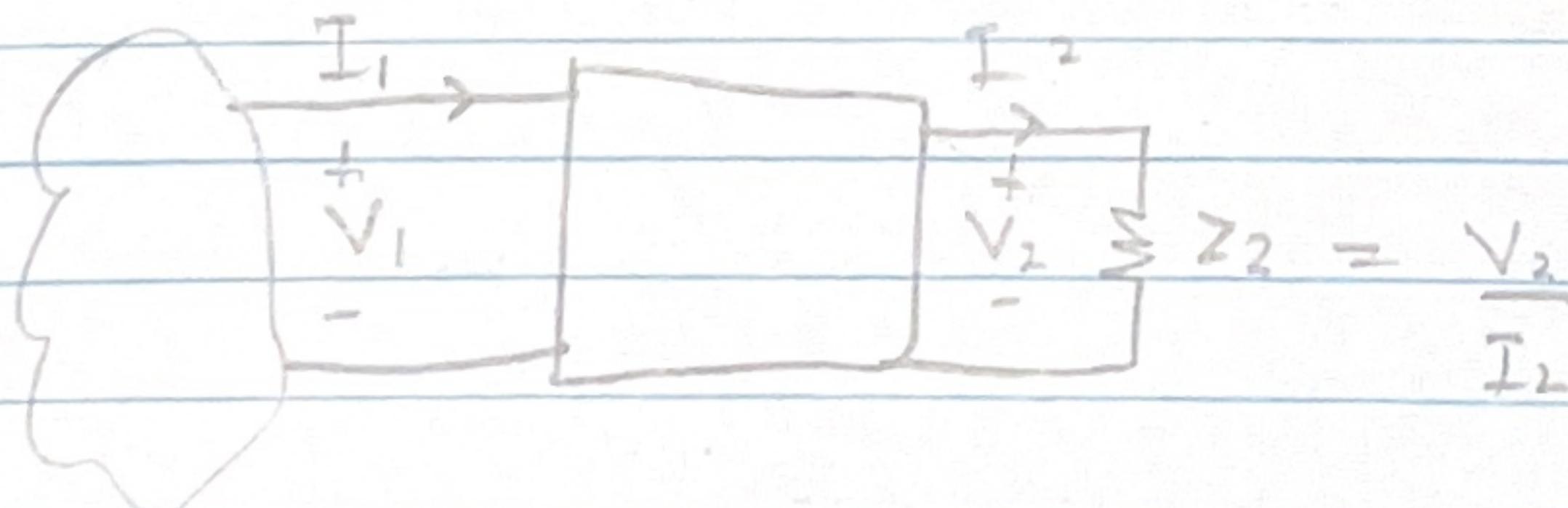


$$V_1 = aV_2$$

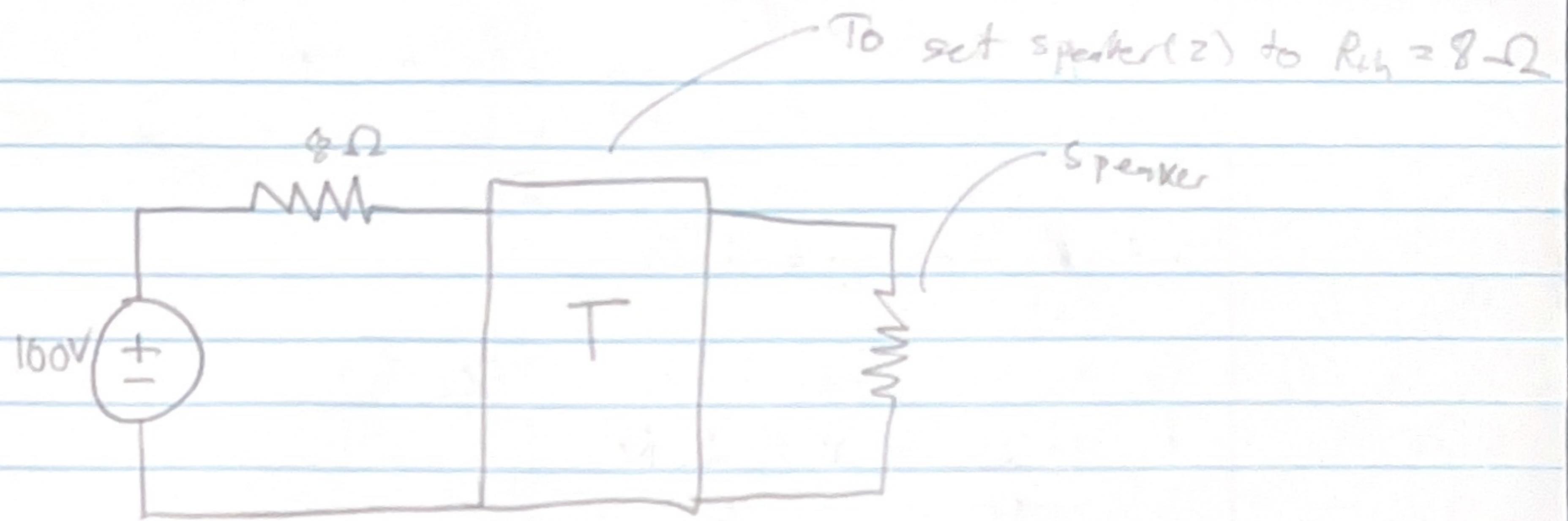
Transformation ratio

$$I_2 = aI_1$$

$$P_1 = V_1 I_1 = aV_2 \frac{I_2}{a} = V_2 I_2 = P_2$$



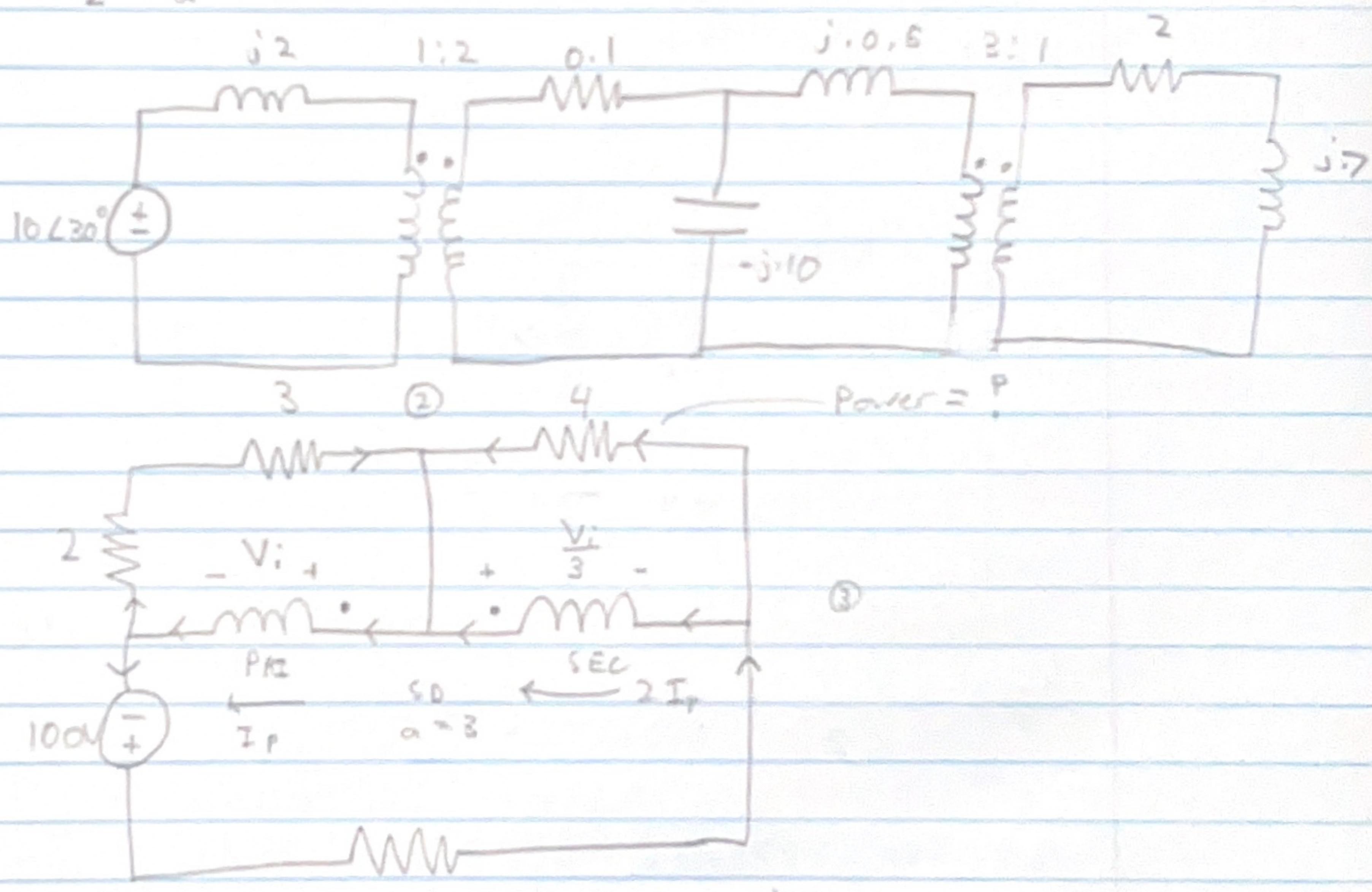
$$Z_1 = \frac{V_1}{I_1} = \frac{aV_2}{\frac{I_2}{a}} = a^2 Z_2$$



LV \rightarrow HV :

- $V_s \cdot a$
- $I_s \div a$
- $Z \cdot a^2$

Tutorial 1



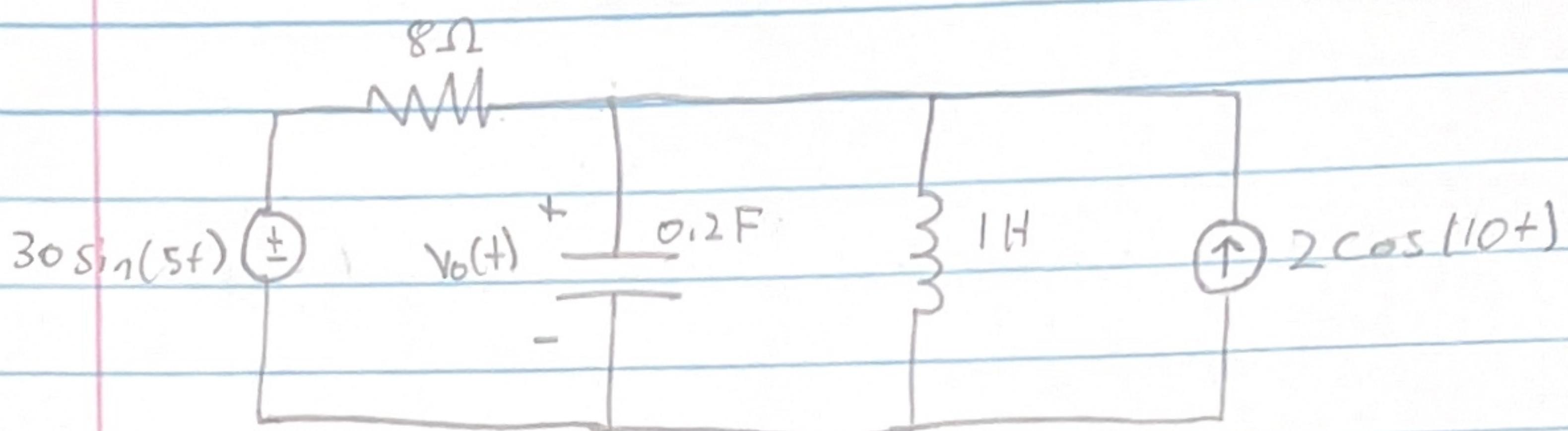
$$kcl_2: I_p = \frac{0 - V_2}{5} + \frac{0 + 100 - V_3}{5}$$

$$\text{kcl: } V_1 + 100 - V_3 = 2I_p + \frac{V_3 - V_2}{4}$$

$$EvL: V_2 - V_1 = V_1$$

$$kcl_2: \frac{V_1 - V_2}{5} + \frac{V_3 - V_2}{4} + 2I_p = I_p$$

SS 2.13

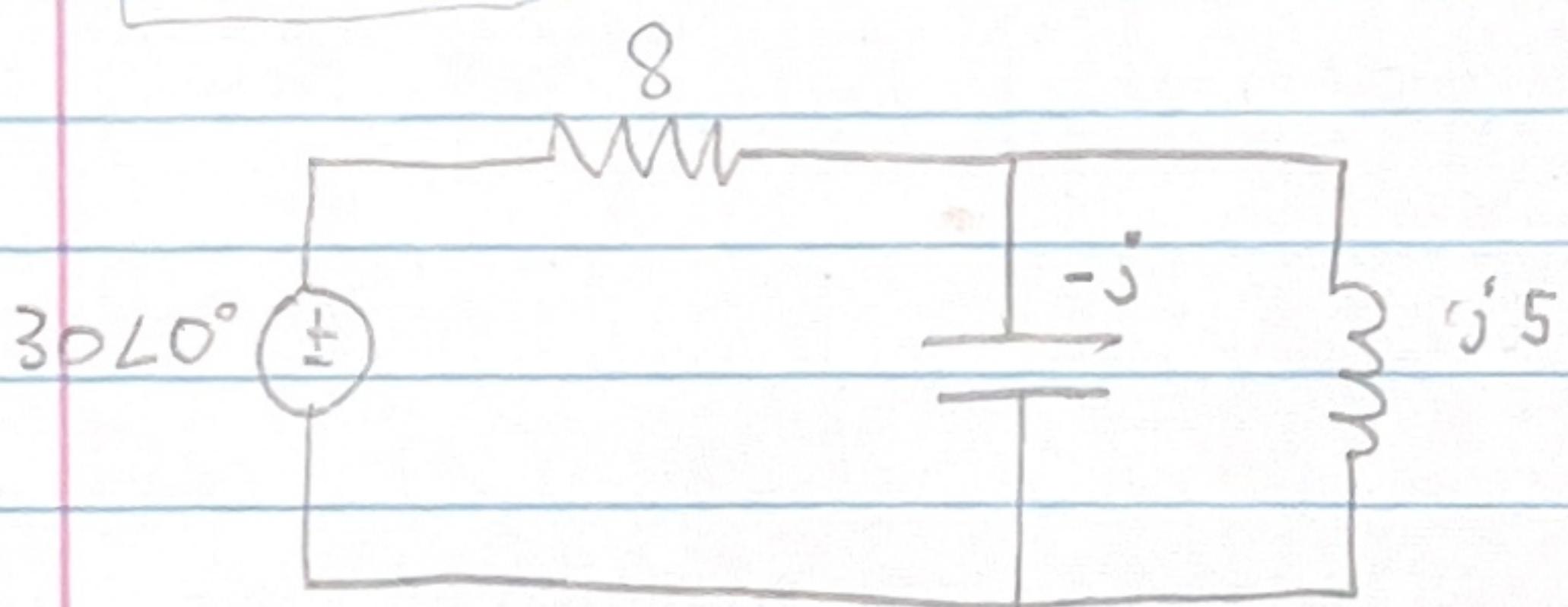


Superposition $\rightarrow V_o = V_o' + V_o''$

V_o' (voltage source)

V_o'' (current source)

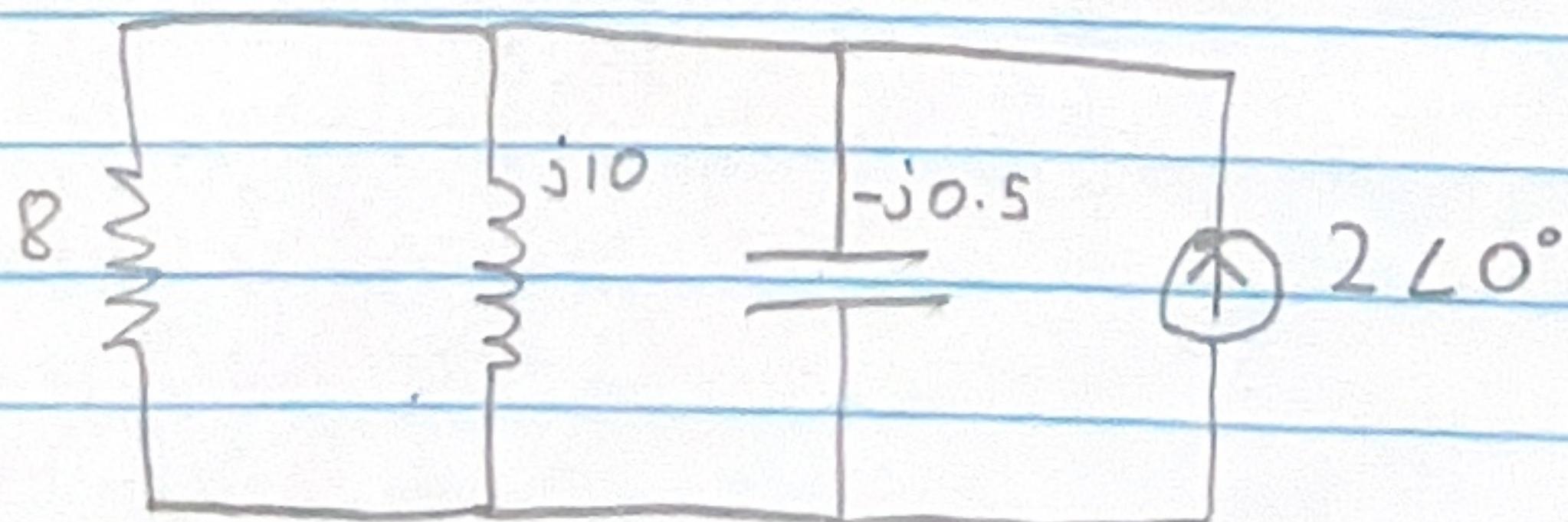
for V_o'



$$V_o' = \frac{-j/0.5}{8 + (-j/0.5)} \cdot 30 = 4.631 \angle -81.12^\circ$$

$$V_o'(t) = 4.631 \sin(5t - 81.12^\circ) V$$

For V_o'' $2 \cos(10t) \rightarrow 2 \angle 0^\circ$



$$V_o'' = \frac{-j0.5}{8/j10 + (-j0.5)} \cdot 2 \cdot (-j0.5) = 1.051 \angle -86.24^\circ$$

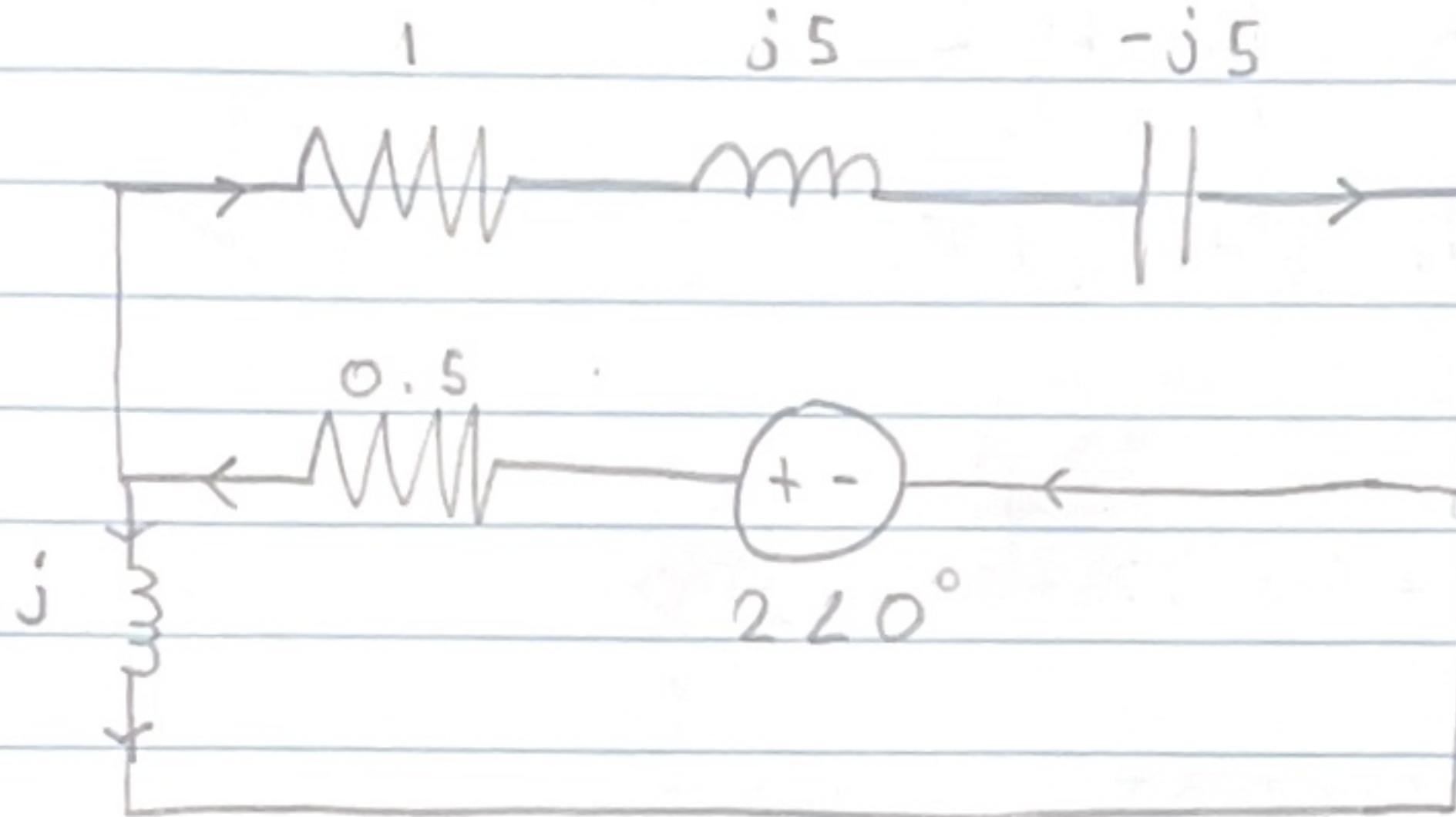
$$V_o''(t) = 1.051 \cos(10t - 86.24^\circ) V$$

Superposition

1.33

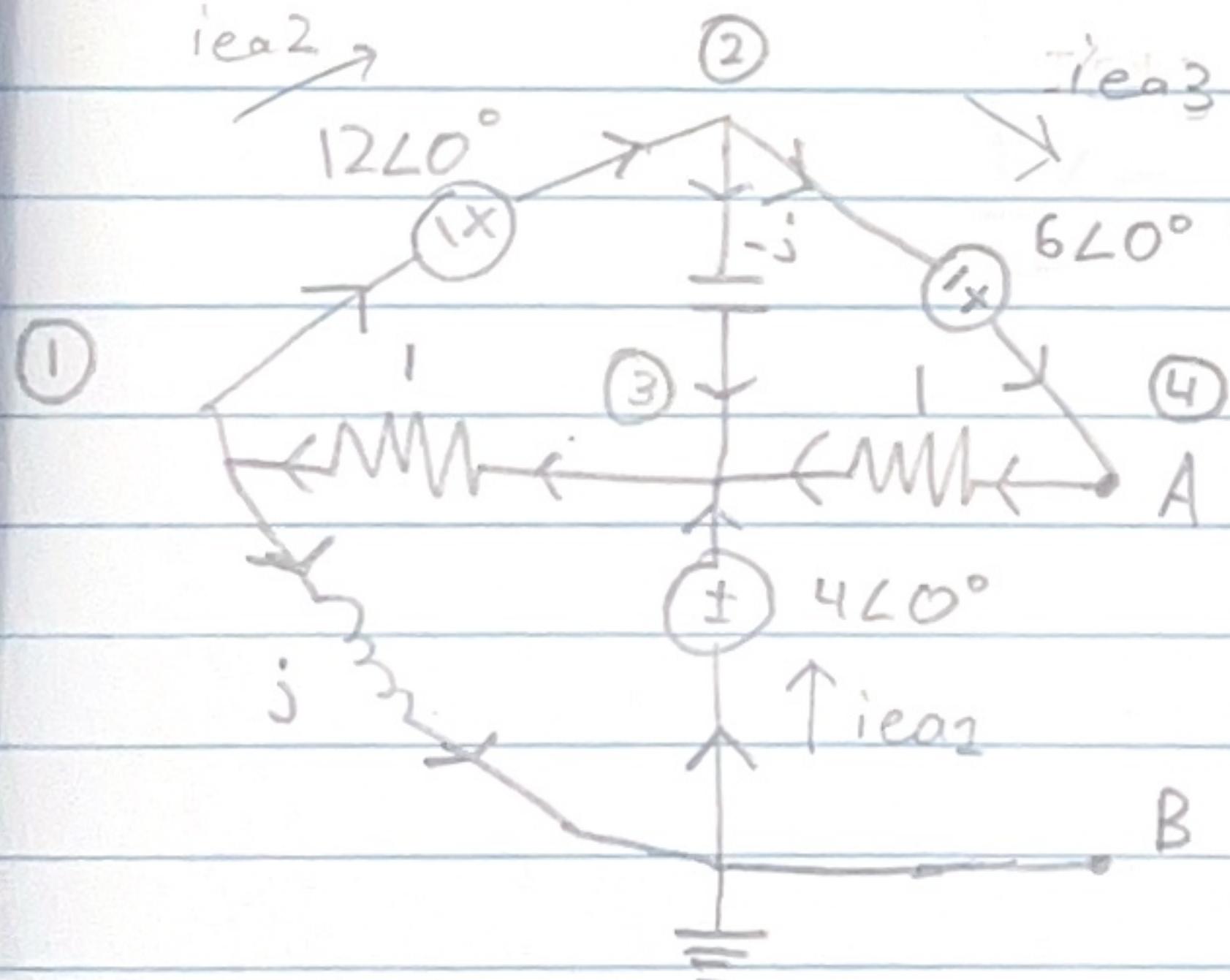
1.7

$$V_o(t) = 4.631 \sin(5t - 81.12^\circ) + 1.051 \cos(10t - 86.24^\circ)$$



REF

$$V_o(t) = 3.92 \cos(1000t - 71.31^\circ) V$$



$$\text{EVL1: } V_1 + 12 = V_2$$

$$\text{EVL2: } 0 + 4 = V_3$$

$$\text{EVL3: } V_2 + 6 = V_4$$

$$\text{KCL}_1: (V_3 - V_1) = \frac{V_1 + ie_{a2}}{j}$$

$$\text{KCL}_2: ie_{a2} = \frac{(V_2 - V_3)}{-j} + ie_{a3}$$

$$\text{KCL}_3: (V_4 - V_3) + \frac{(V_2 - V_3)}{-j} + ie_{a1} = (V_3 - V_1)$$

$$\text{KCL}_4: ie_{a3} = (V_4 - V_3) \quad V_4 = 13 - j4$$

* TH2 $V_{TH} = 13 - j4$

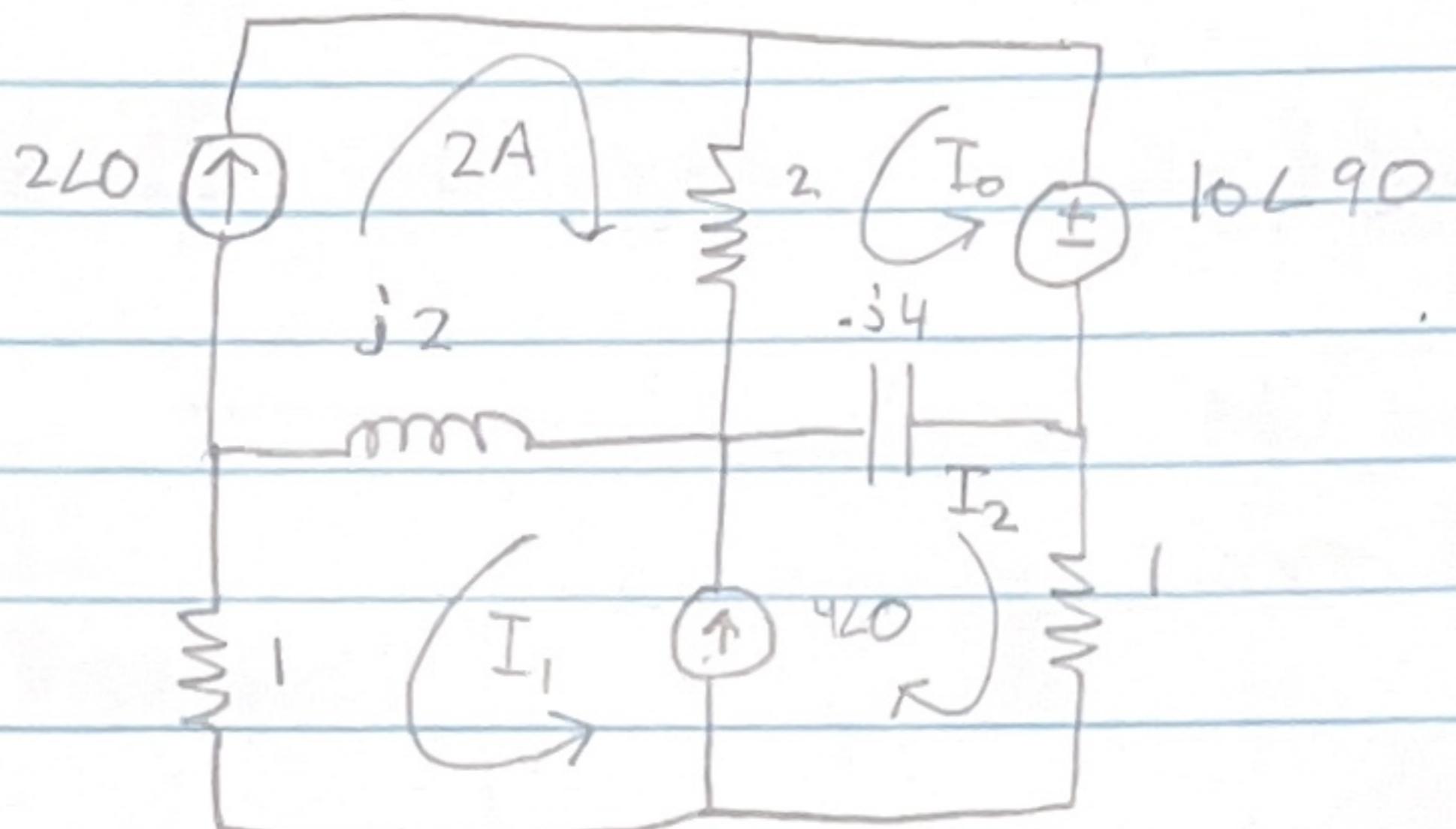
Sinusoidal Steady State

2/4/19

LEC

SS 3.6

Mesh Analysis:



$$\text{KVL}_1: (2-j4)I_o + (-j4)I_2 + 2 \cdot 2 = j \cdot 10$$

$$(\text{Bottom Rect}) \text{KVL}_2: -(-j4)I_o + (1+j2)I_1 - (1-j4)I_2 + j2 \cdot 2 = 0$$

$$\text{KVL}_3: I_1 + I_2 = 4$$

Sol'n:

$$I_o = 3.35 \angle 174.3^\circ A$$

$$I_1 = 3.02 \angle -6.34^\circ A$$

$$I_2 = 1.054 \angle 18.43^\circ A$$

AC Power

3/6/19

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[SS 4.2]

"Power is the time rate of expending or absorbing energy"

Positive Power \rightarrow element is absorbing power

Negative Power \rightarrow element is supplying power

[SS 4.3]

$$P(t) = V(t)i(t) = V_m \cos(\omega t + \theta_v) I_m \cos(\omega t + \theta_i)$$

Trig Identity

$$= \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

[SS 4.5]

$$\frac{1}{T} \int_0^T P(t) dt = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

① P_{avg} is not time dependent (it is constant)

② When $\theta_v = \theta_i \rightarrow$ it is a purely resistive load

resulting in $P = \frac{1}{2} V_m I_m$

③ When $\theta_v - \theta_i = \pm 90^\circ \rightarrow P = 0$ this is a purely reactive load

AC Power Analysis

3/11/19

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SS 5.2

Average Power:

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \underbrace{V_{rms} I_{rms}}_{\text{Apparent power}} \cos(\theta_v - \theta_i) \quad \underbrace{\text{Power factor}}$$

SS 5.3

- Apparent power is $S = V_{rms} I_{rms}$

- Power factor $pf = \frac{P}{S} = \cos(\theta_v - \theta_i)$

- Power factor angle $= (\theta_v - \theta_i)$, or the angle of \underline{Z}_{eq}

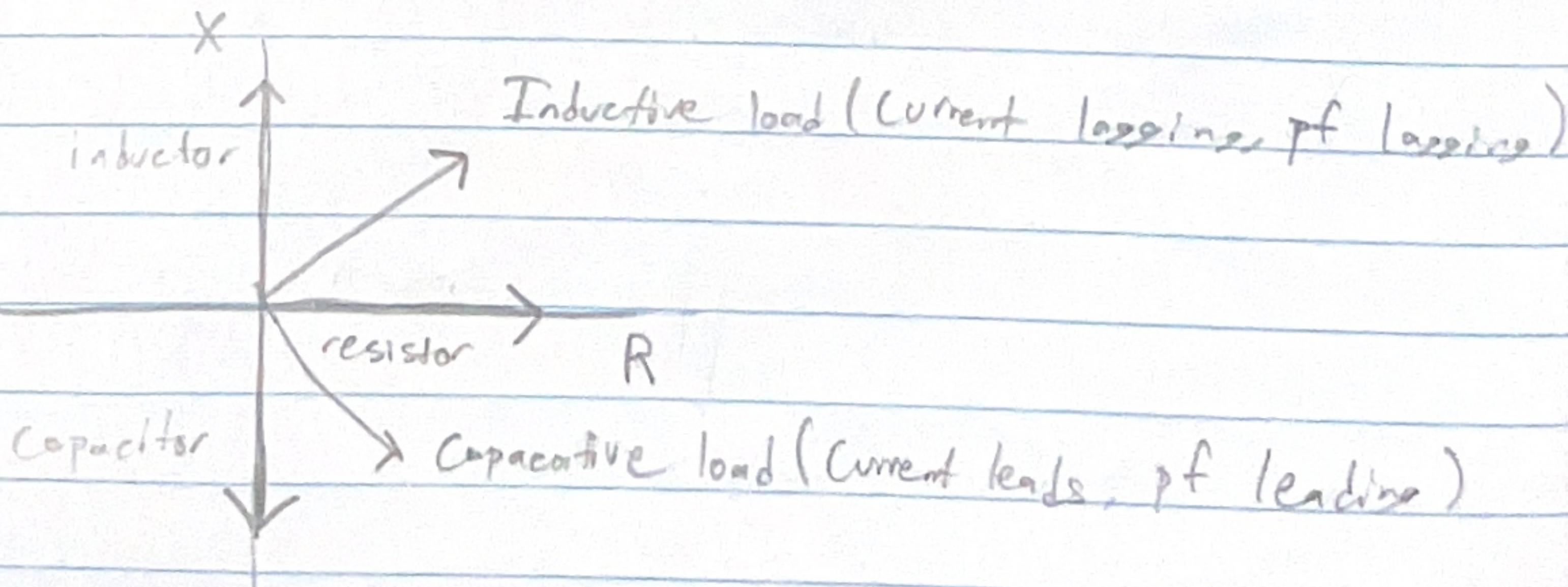
SS 5.4

$$pf = \cos(\theta_v - \theta_i)$$

$$V = V_m \angle \theta_v$$

$$I = I_m \angle \theta_i \quad \text{Resistance}$$

$$\underline{Z} = \frac{V_m}{I_m} \angle \theta_v - \theta_i = R + jX \quad \text{reactance}$$



$$P_{avg} = V_{rms} I_{rms} \cdot (pf)$$

Three Phase

3/18/19

LEC

SS 7.6

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ \quad \text{For abc sequence}$$

$$V_{cn} = V_p \angle -240^\circ$$

$$= V_p \angle 120^\circ$$

SS 7.9

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ$$

$$= V_p \angle 120^\circ$$

phase
voltages

$$V_{ab} = V_{an} - V_{bn}$$

$$= V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$= V_p \left(1 + \frac{1}{2} + j \frac{\sqrt{3}}{2} \right)$$

$$= \sqrt{3} V_p \angle 30^\circ$$

Line

voltages

$$V_{bc} = V_{bn} - V_{cn} = \sqrt{3} V_p \angle -90^\circ$$

$$V_{ca} = \sqrt{3} V_p \angle -210^\circ$$

$$V_p = |V_{an}| = |V_{bn}| = |V_{cn}|$$

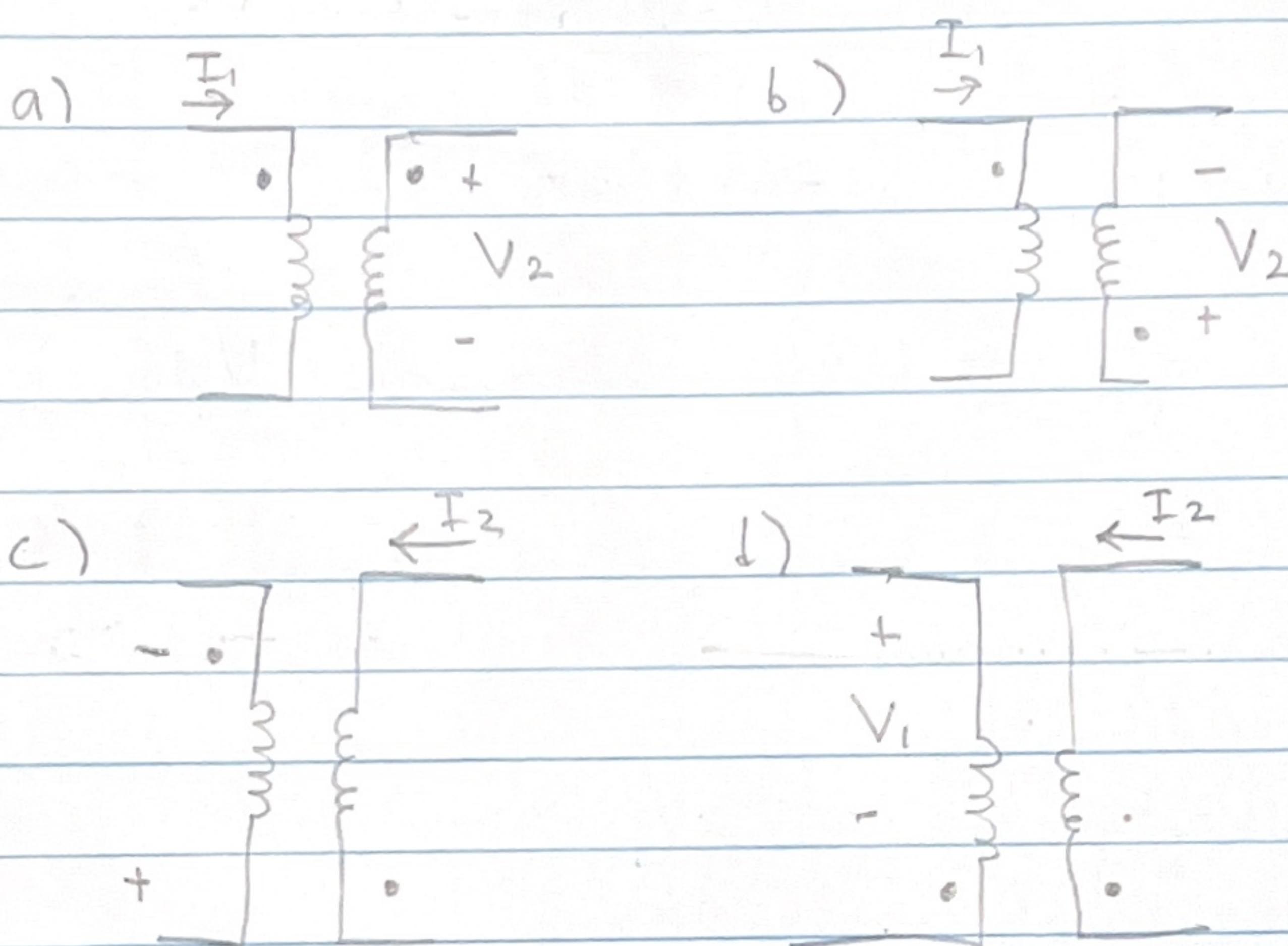
$$V_L = |V_{ab}| = |V_{bc}| = |V_{ca}|$$

$$= \sqrt{3} V_p$$

SS 8.5

- RHR
- Winding orientation affects mutual induction
- dot convention

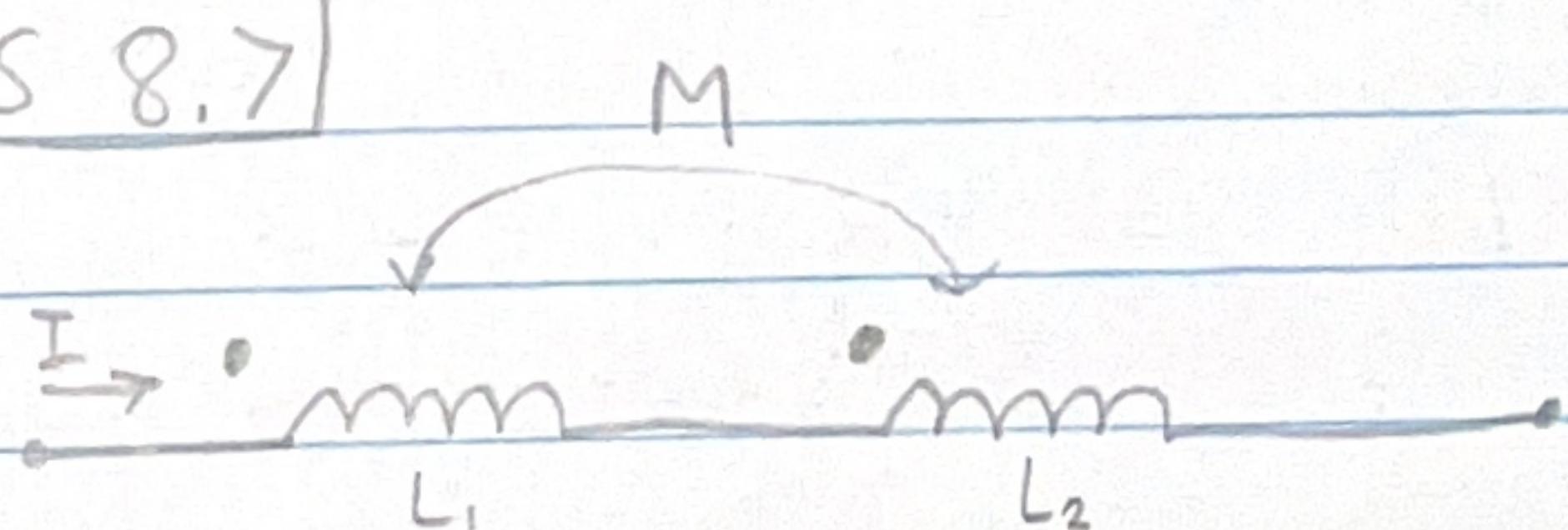
SS 8.6



"Self inductance induced voltage follows direction of current"

"Dot convention only applies to mutual induction"

SS 8.7



In Phasor Domain:

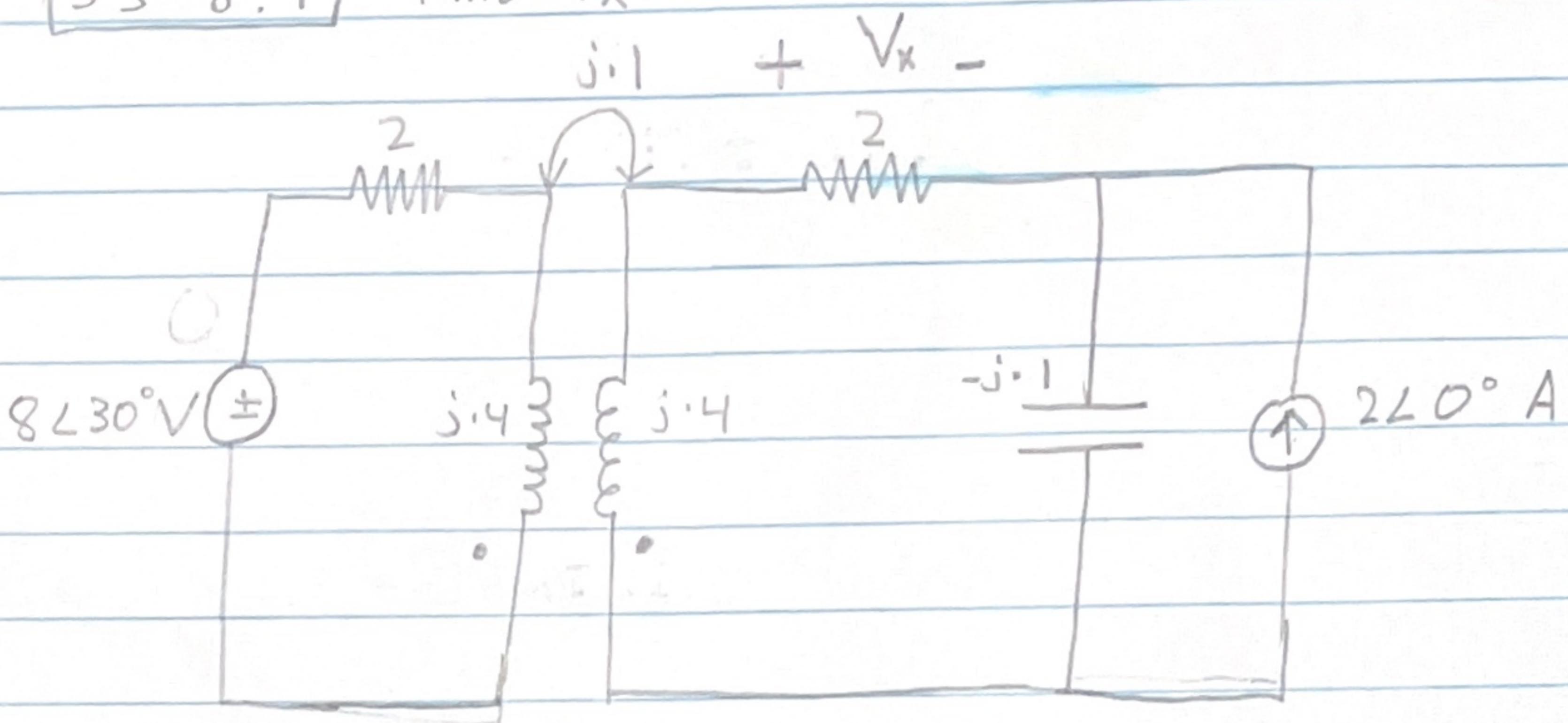
$$Z_L = j\omega L$$

$$V = j\omega L \cdot I \quad (\text{Self inductance})$$

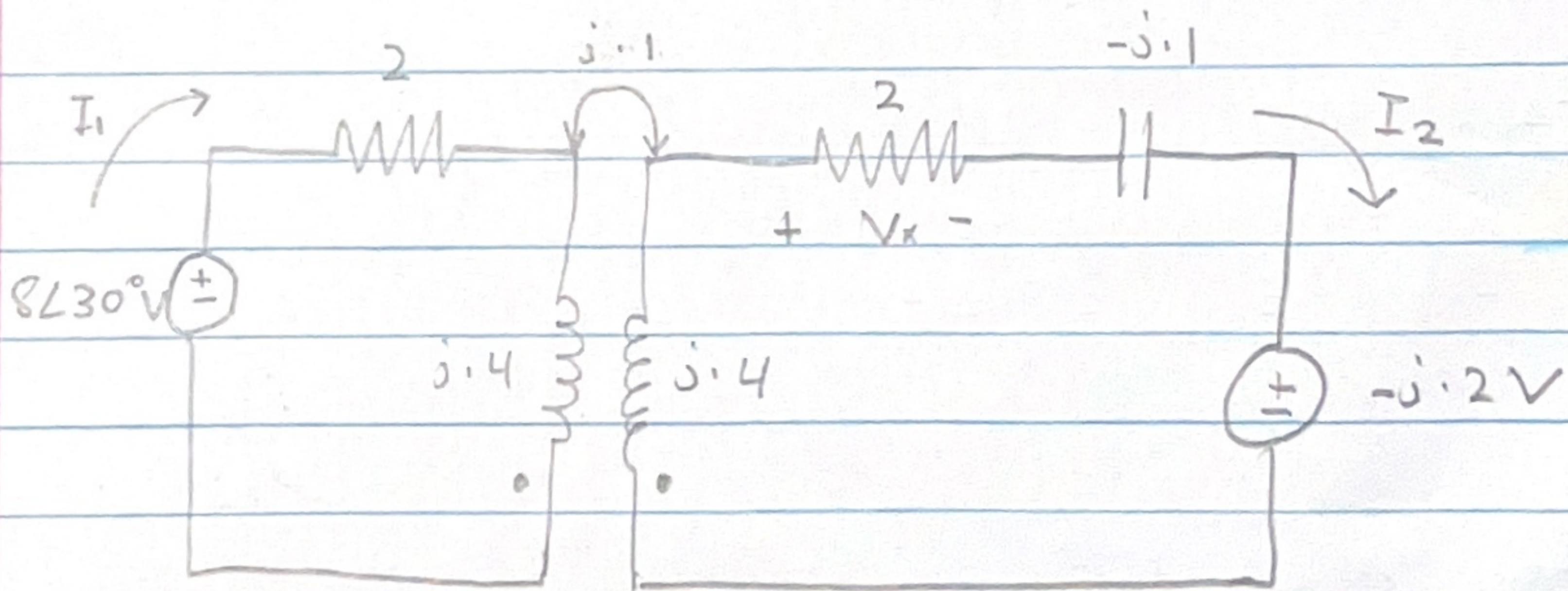
$$Z_M = j\omega M$$

$$V_M = j\omega M \cdot I \quad (\text{Mutual Inductance})$$

SS 8.9 Find V_x



↓ Source Transformation



$$KVL_1: 8\angle 30^\circ = 2I_1 + j \cdot 4 I_1 - j \cdot 1 \cdot I_2$$

$$KVL_2: j \cdot 4 \cdot I_2 + (2 - j \cdot 1) I_2 - j \cdot 2 - j \cdot 1 \cdot I_1 = 0$$

$$\rightarrow I_2 = 1.03 \angle 21.12^\circ A$$

$$V_x = 2I_2 = 2.074 \angle 21.12^\circ V$$