Constants: (Elementary charges)

(c) Coulomb $= 6.25\times10^{18}$ e's

$k = 9\times10^{9}\ N\cdot m^2/C^2$

$\epsilon_0 = 8.85\times10^{-12}\ C^2/N m^2$ (Permitivity constant)

$c = 3\times10^{8}$ m/s (light speed)

$e = 1.6\times10^{-19}$ C

$m_p = \ldots 10^{-27}$ Kg (Proton)

$m_e = 9.11\times10^{-31}$ Kg (Electron)

1 Current and Resistance

Electron Current:

$N_e = i_e \Delta t$

where $\boxed{N_e}$ is the # of electrons that flow through the cross section during Δt and $\boxed{i_e}$ is the rate of the electron flow (Electrons per second)

$i_e = N_e \cdot A \cdot V_d$

where $\boxed{A}$ is the cross sectional area (πr^2) and $\boxed{V_d}$ is the Drift Speed

Conventional Current:

$I = e\cdot i_e,\quad I = \dfrac{Q}{\Delta t}$

Amp = Coulomb/sec

Current Density:

$I = J\cdot A$ where J = Current density and A = area

$J = I/A$

$J = \dfrac{N_e \cdot e^2\cdot \Upsilon}{m}\cdot \vec{E}$

$J = \sigma\cdot E$

Conductivity:

$\sigma = \dfrac{N_e\cdot e^2 \cdot \Upsilon}{m}$

where m is the mass of the electron

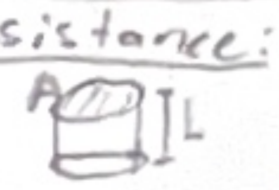
$E = \dfrac{I}{\sigma} = \dfrac{\Delta V}{\text{Length}}$

Resistivity: Ω

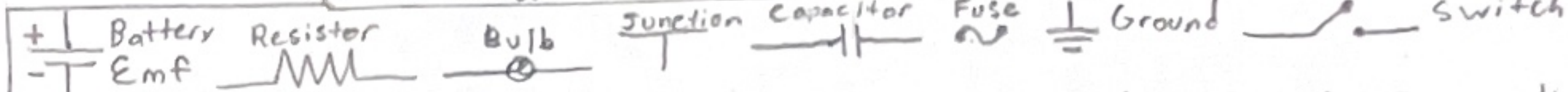
$\rho = \dfrac{1}{\sigma} = \dfrac{m}{N_e\cdot e^2\cdot \Upsilon}$

Wire Resistance:

$R = \dfrac{\rho\cdot L}{A}$



Battery Emf, Resistor, Bulb, Junction, Capacitor, Fuse, Ground, Switch



2 Circuits

Ohm's Law:

$V = IR$

Power (Watts):

$P = IV = \dfrac{\text{Energy}}{\Delta t}$

$= I^2 R$

$= \dfrac{V^2}{R}$

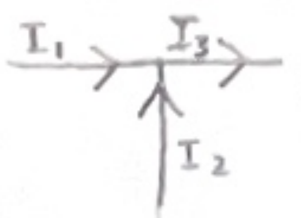
(Loop Law):

$\mathcal{E} - IR = 0$

$\therefore \mathcal{E} = IR$

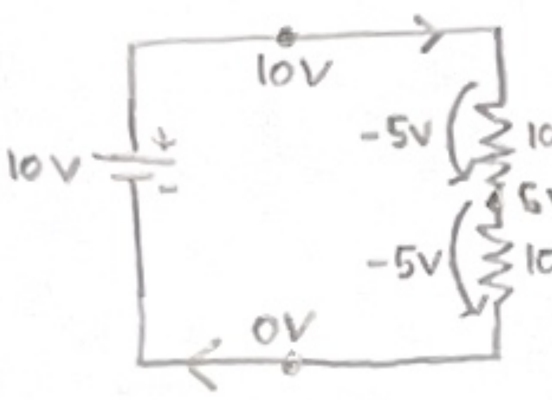
Kirchoff's Laws:

① Junction Law: $\sum I_{in} = \sum I_{out}$



$I_1 + I_2 = I_3$

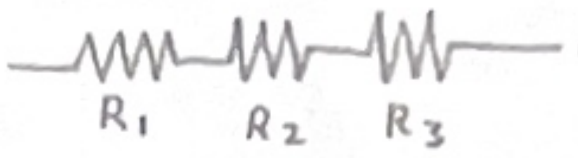
② Voltage Law: $\sum_{Loop} V_i = 0$



Battery (source)

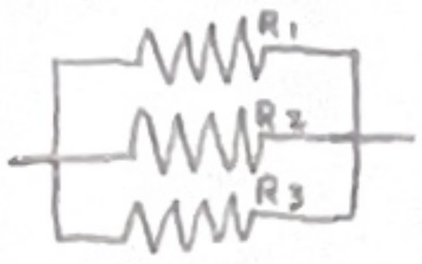
Resistor (load)

Resistance:



(Series)

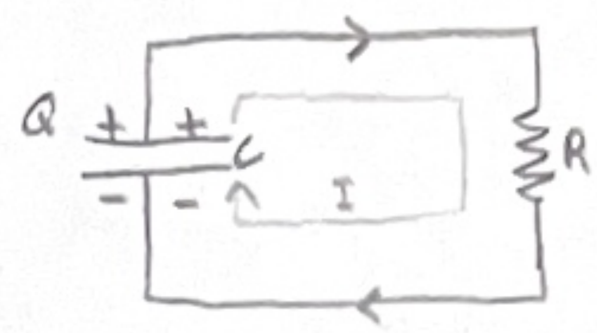
$R_{eq} = R_1 + R_2 + R_3 \ldots + R_n$



(Parallel)

$R_{eq} = \left(\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} \ldots + \dfrac{1}{R_n}\right)^{-1}$

RC Circuit:



$Q = Q_0 e^{-t/\Upsilon}$

where $\Upsilon = RC$ is the time constant

Battery with internal Resistance:

$\Delta V_r = \mathcal{E} - Ir$

Ammeter:

- Measures current
- Must be placed in series

Voltmeter:

- Measures potential difference
- Must be placed in parallel

Power Supplied by a battery:

$P = \mathcal{E} I$

Resistors lose energy at a rate of:

$P_r = IV = I^2 R = \dfrac{V^2}{R}$

Potential Difference:

$\Delta V_r = IR$

$\Delta V_r = \mathcal{E}$

3 Electric Charge/Forces

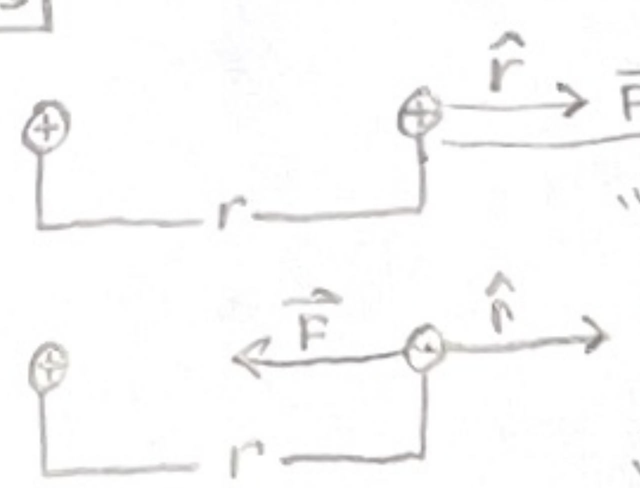
Elementary Charge: e

Proton => +e

Electron => -e

Coulomb's Law:

$\vec{F} = \dfrac{k q_1 q_2}{K r^2}$ *(Point charges only)

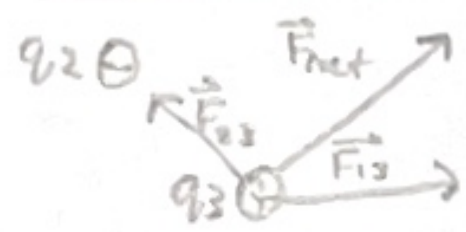


Electric Field (Point Charge):

$\vec{E} = \ldots \cdot \dfrac{q}{r^2}\hat{r}$

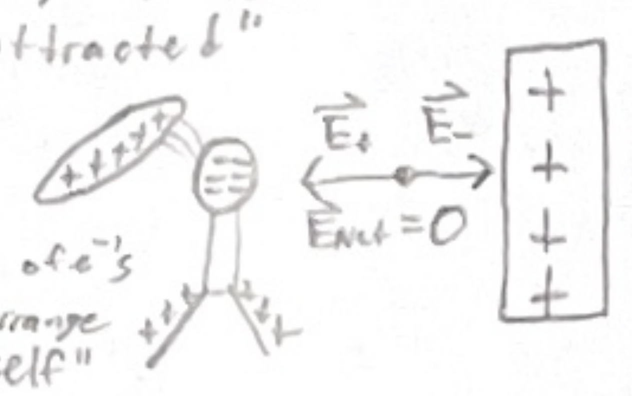
*$K = \dfrac{1}{4\pi\epsilon_0}$

Superposition:



Charge Polarization:

"Why are neutral objects attracted"

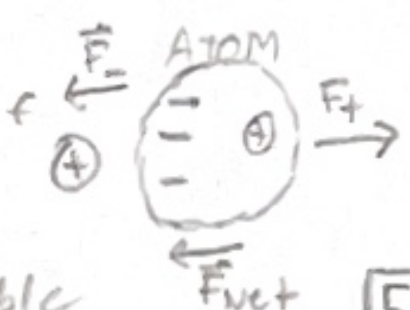


"Sea of e's rearrange itself"

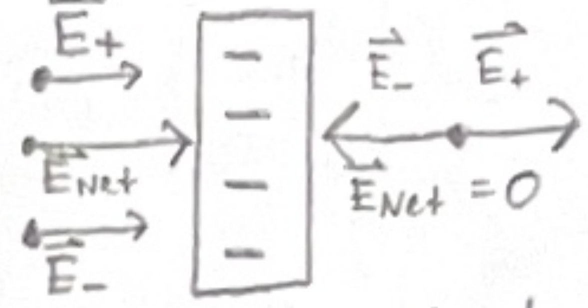
Electric Dipole:

"How can a charged comb pick up pieces of paper (insulator) if the charges inside the paper are immobile"

"Rearrange slightly within each atom"



Parallel-Plate Capacitor:



$\vec{E}_{\text{Capacitor}} = \begin{cases} \dfrac{Q}{\epsilon_0 A}; & \text{Inside} \\ \vec{0}; & \text{Outside} \end{cases}$

where A is the surface area of each electrode

Q is the charge on each electrode (Panel)

"The field strength inside the capacitor is the same at all points"

"The field outside is $\vec{0}$"

$\vec{E}_{\text{Dipole}} = \dfrac{\vec{P}}{4\pi\epsilon_0 r^3}$

4 Electric Fields

$\vec{E}_{NET} = \sum_{i=1}^{n} \vec{E}_i$

"Add up the fields produced by each charged object individualy"

Dipole:

$\vec{P} = q\cdot s$

$\vec{E}_{\text{Dipole}} \approx k\cdot\dfrac{2\vec{P}}{r^3}$

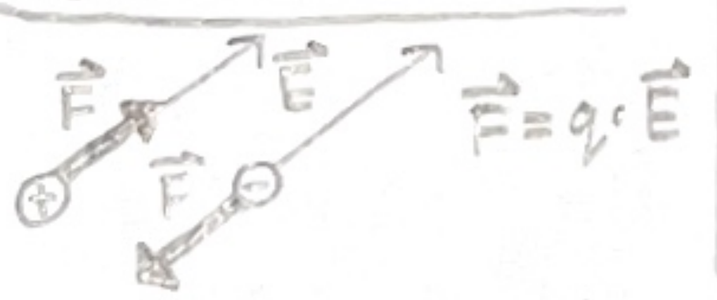
where r is halfway between the charges

$\Upsilon = \vec{E} q (s)\sin\theta$

"Torque maxed when $\theta = 90°$"

Dipole moment $= \|\vec{P}\|$

Motion of a charged particle in an electric field:



$\vec{F} = q\cdot\vec{E}$

If $\vec{F}$ is the only force on q it will accelerate by

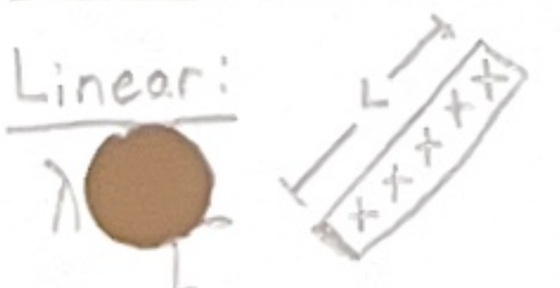
$\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{q}{m}\cdot\vec{E}$

where m is the mass

- The force on a positive q is in the direction of $\vec{E}$
- The force on a negative q is in the opposite direction of $\vec{E}$

Charge Density:

Linear:

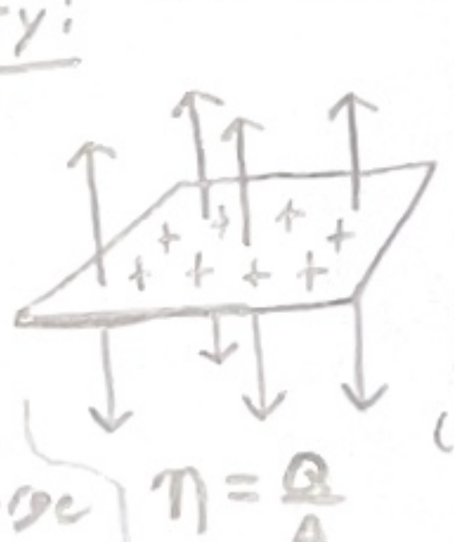


Where Q = Total charge, λ = Linear charge density

For very small lengths...

$Q = \lambda\cdot L$

Surface:



$\eta = \dfrac{Q}{A}$

where η = Surface Charge density per square metre (A)

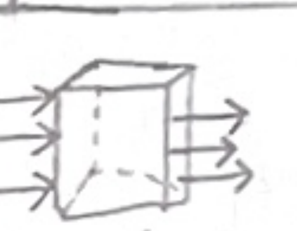
(small) $Q = \eta\Delta A$

Volume:

$\rho = \dfrac{Q}{V}$

(small) $Q = \rho V$

5 Guass's Law



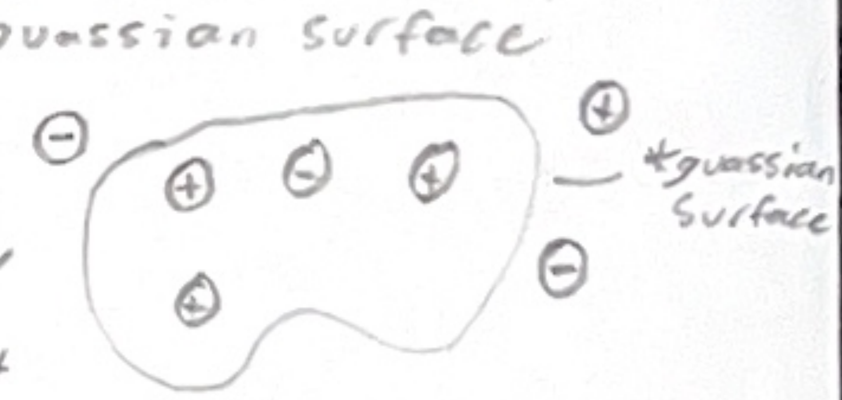
Flux $\Phi = \oint_{\text{Surface}} \vec{E}\cdot d\vec{A} = \dfrac{q_{enclosed}}{\epsilon_0}$

if $\Phi_{net} = 0$ then there is no charge enclosed in the guassian surface

① if $\vec{E} \parallel \vec{A}$; $\Phi = 0$

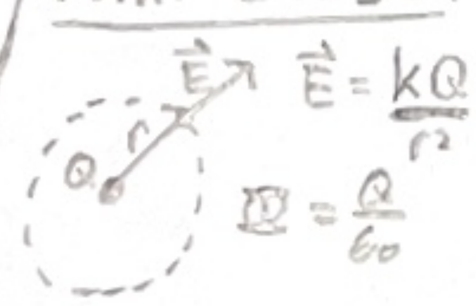
② if $\vec{E} \perp \vec{A}$; $\Phi = EA$

Charges outside may contribute to the $\vec{E}$ field but they don't contribute to the flux



*guassian Surface

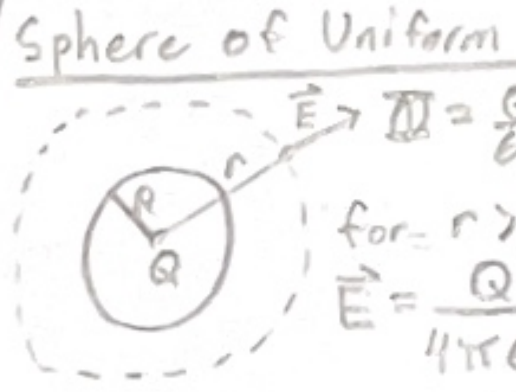
Point Charge:



$\vec{E} = \dfrac{kQ}{r^2}$

$\Phi = \dfrac{Q}{\epsilon_0}$

Sphere of Uniform Charge:



$\Phi = \dfrac{Q}{\epsilon_0}$

for $r > R$

$\vec{E} = \dfrac{Q}{4\pi\epsilon_0 r^2}$

for $r < R$

$\vec{E} = \dfrac{Qr}{4\pi\epsilon_0 R^3}$

Conducting:

$r > R$; Same

$r < R$; $\vec{E} = \vec{0}$

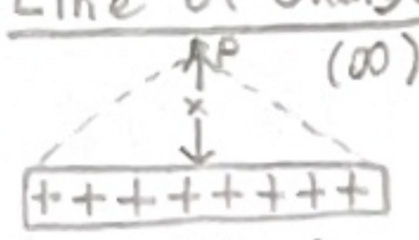
Inside a sphere of charge:

$\Phi = \vec{E}\,4\pi r^2$

$\Phi = \dfrac{Qr^3}{\epsilon_0 R^3}$

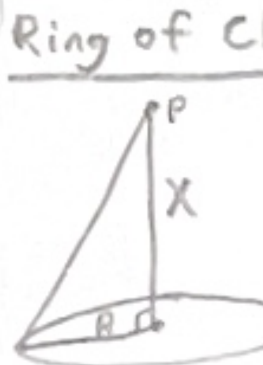
$\vec{E} = \dfrac{Qr}{4\pi\epsilon_0 R^3}$

Line of Charge: (∞)



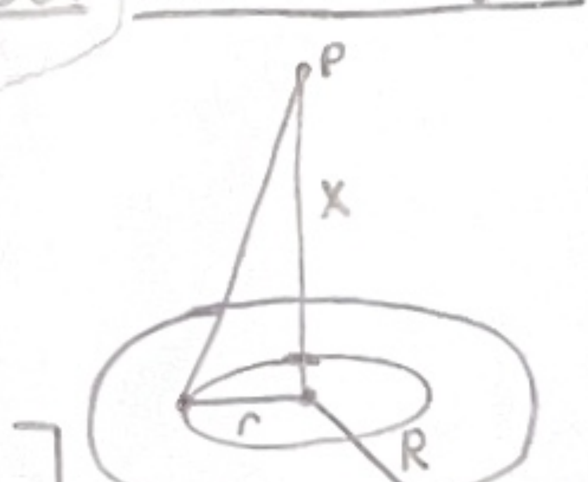
$\vec{E} = \dfrac{\lambda}{2\pi\epsilon_0 x}$

Ring of Charge:



$\vec{E} = \dfrac{(k)(\lambda)(2\pi)(R)(x)}{(x^2+R^2)^{3/2}}$

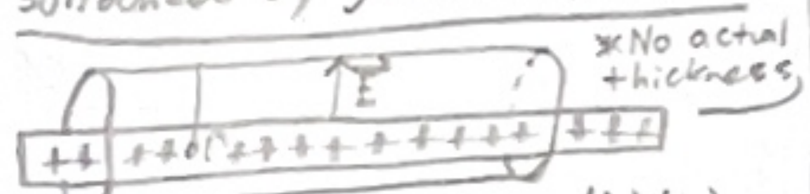
Disk of Charge:



$\vec{E} = (k)(\sigma)(2\pi)\left[1 - \dfrac{x}{\sqrt{x^2+R^2}}\right]$

*Area charge density

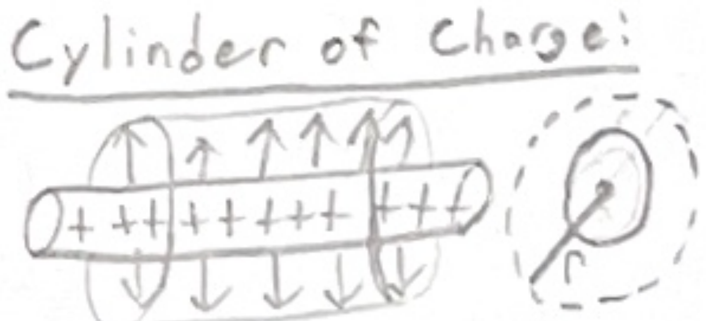
Infinite line of charge Surrounded by guassian Cylinder:



*No actual thickness

$\vec{E} = \dfrac{\lambda}{(2\pi)(r)(\epsilon_0)};\quad \Phi = \dfrac{(\lambda)(L)}{\epsilon_0}$

Cylinder of Charge:



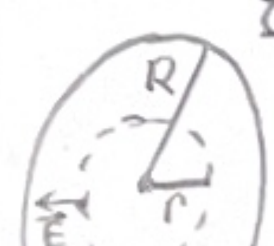
$\Phi = \dfrac{\lambda L}{\epsilon_0},\quad \vec{E} = \dfrac{\lambda}{(2\pi)(\epsilon_0)(r)}$

Conducting:

$r \geq R$ (Outside) "Same $\vec{E}$ field"

$r < R$; $\vec{E} = \vec{0}$

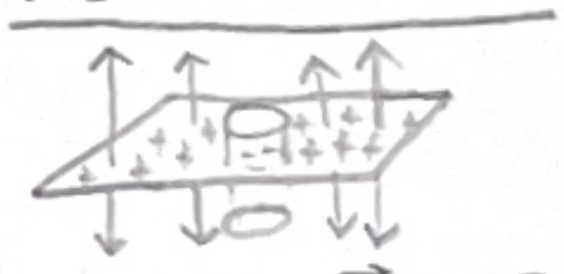
Inside a Cylinder:



$\Phi = \dfrac{\lambda L r^2}{\epsilon_0 R^2}$

$\vec{E} = \dfrac{\lambda r}{2\pi\epsilon_0 R^2}$

Sheet of charge:

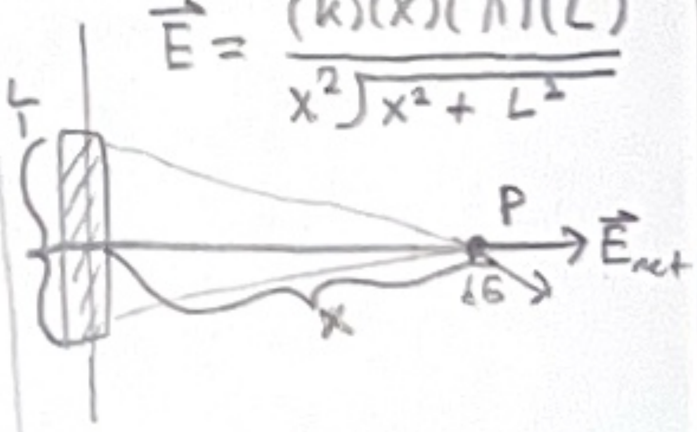


$\Phi = \dfrac{\sigma A}{\epsilon_0}\quad \vec{E} = \dfrac{\sigma}{2\epsilon_0}$

Parallel Plate, $\vec{E} = \dfrac{\sigma}{2\epsilon_0}$

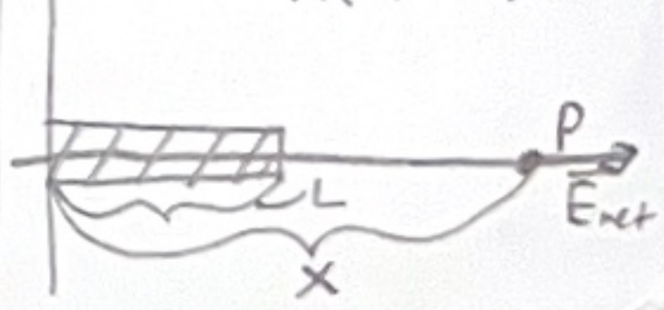
Finite Line of Charge:

$\vec{E} = \dfrac{(k)(x)(\lambda)(L)}{x^2\sqrt{x^2+L^2}}$



Line charge directly above:

$\vec{E} = \dfrac{(k)(\lambda)(L)}{x(x-L)}$

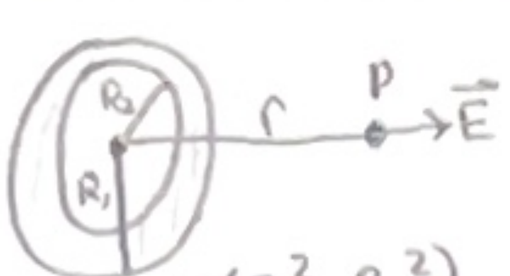


Non-Conducting Slab:

Outside: $\vec{E} = \frac{\rho d}{\epsilon_0}$

Inside: $\vec{E} = \frac{\rho z}{\epsilon_0}$

Uniform-Charge Pipe:



$\vec{E} = \frac{\rho(R_1^2 - R_2^2)}{2\epsilon_0 r}$

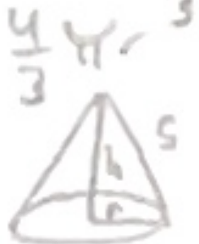
Prefixes:

$P = 10^{15}$	$c = 10^{-2}$
$T = 10^{12}$	$m = 10^{-3}$
$G = 10^{9}$	$\mu = 10^{-6}$
$M = 10^{6}$	$n = 10^{-9}$
$K = 10^{3}$	$p = 10^{-12}$
$h = 10^{2}$	$f = 10^{-15}$

Surface Area/Volume:

Sphere:
Area $= 4\pi r^2$
Volume $= \frac{4}{3}\pi r^3$

Cone:



Area $= \pi r s + \pi r^2$
Volume $= \frac{1}{3}\pi r^2 h$

Cylinder:

Area $= 2\pi r^2 + 2\pi r h$
Volume $= \pi r^2 h$

Prism:
Area $= bh + 2ls + lb$
Volume $= \frac{1}{2}(bh)L$

Ellipse:

Surface area $= \pi a b$

Proofs:

Ring of Charge:

$\vec{E} = \int_Q \frac{k\,dq}{r^2}$ $\quad * E\cos\theta$

$= \int_Q \frac{k\lambda\,ds}{(R^2+x^2)}$ $\quad * = E\left(\frac{adj}{hyp}\right)$

$= \frac{k\lambda}{(x^2+R^2)} \int_Q \frac{x}{\sqrt{R^2+x^2}}\,ds$

$= \frac{(k)(\lambda)(2\pi R)(x)}{(R^2+x^2)^{3/2}}$

Finite Line of Charge:

$\vec{E} = \left(\frac{kq}{r^2}\right)\cos\theta$

$= \left(\frac{kq}{r^2}\right)\left(\frac{x}{\sqrt{x^2+L^2}}\right)$

$= \int_{-a}^{a}\left(\frac{k\lambda\,dL}{(x^2+L^2)}\right)\left(\frac{x}{\sqrt{x^2+L^2}}\right)$

$= \frac{kx\lambda L}{x^2\sqrt{x^2+L^2}}$

Inside a Solid Insulating sphere:

$\oint_S \vec{E}\cdot d\vec{A} = \frac{q_{net}}{\epsilon_0}$

$\oint_S \vec{E}\cdot d\vec{A}\cos\theta = \frac{Qr^3/a^3}{\epsilon_0}$

$E\oint_S d\vec{A} = \frac{Qr^3}{\epsilon_0 a^3}$

$E(4\pi r^2) = \frac{Qr^3}{\epsilon_0 a^3}$

$\vec{E} = \frac{Qr}{4\pi\epsilon_0 a^3}$

Outside a Sphere:

$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

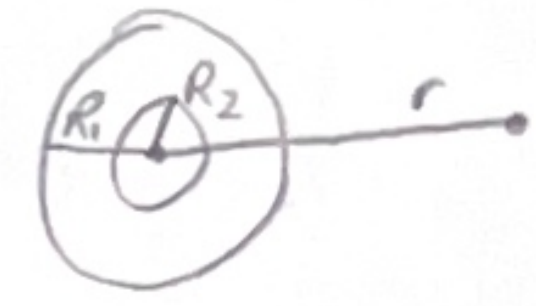
$\oint \vec{E}\cdot d\vec{A}\cos\theta = \frac{Q}{\epsilon_0}$

$E(4\pi r^2) = \frac{Q}{\epsilon_0}$

$E = \frac{Q}{4\pi\epsilon_0 r^2}$

$= \frac{kQ}{r^2}$

Pipe: (Infinitely long)



$E = \frac{Q}{A\epsilon_0}$ $\qquad V = \pi(R_2^2 - R_1^2)L$

$A = 2\pi r L$

$E = \frac{\pi\rho(R_1^2 - R_2^2)L}{2\pi L\epsilon_0} = \frac{\rho(R_1^2 - R_2^2)}{2r\epsilon_0}$

Infinite slab:

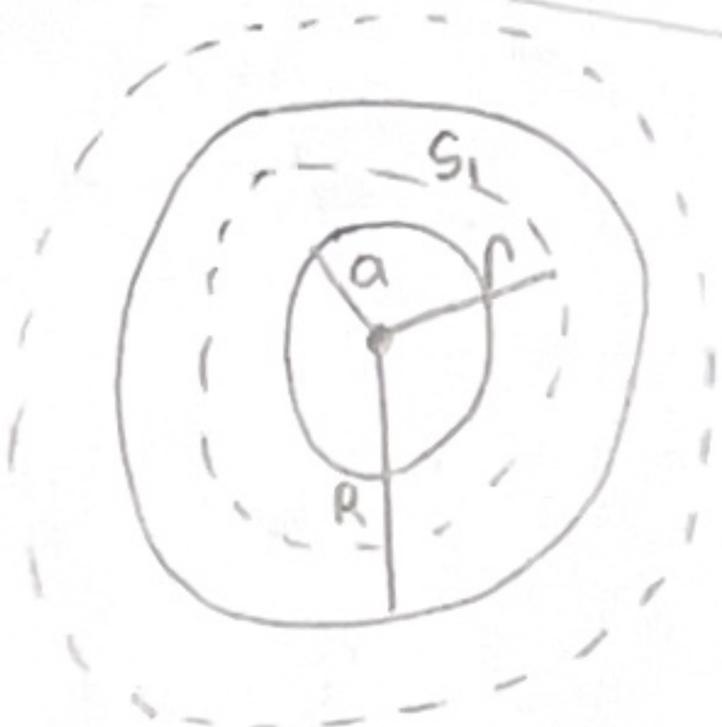
$Q_{enc} = \rho \times \text{Volume} = \rho A d$

$\int_{Total} \vec{E}\cdot d\vec{A} = \left(\int_{Base} \vec{E}\cdot d\vec{A} + \int_{laterals} \vec{E}\cdot d\vec{A} + \int_{Top} \vec{E}\cdot d\vec{A}\right)$

$EA = \frac{\rho A d}{\epsilon_0}$

$\vec{E} = \frac{\rho d}{\epsilon_0}$ $z \geq d$ $\qquad \vec{E} = \frac{\rho z}{\epsilon_0}$; (inside)

$\vec{E} = -\frac{\rho d}{\epsilon_0}$ $z \leq d$



$V_{TOT} = \frac{4}{3}\pi R^3 - \frac{4}{3}\pi a^3$

$= \frac{4}{3}\pi(R^3 - a^3)$

$Q\left(\frac{V_2}{V_{TOT}}\right) = q_{net}$

$V_1 = \frac{4}{3}\pi r^3 - \frac{4}{3}\pi a^3$

$= \frac{4}{3}\pi(r^3 - a^3)$

$\oint_{S_1} \vec{E}\cdot d\vec{A} = \frac{q_{net}}{\epsilon_0}$

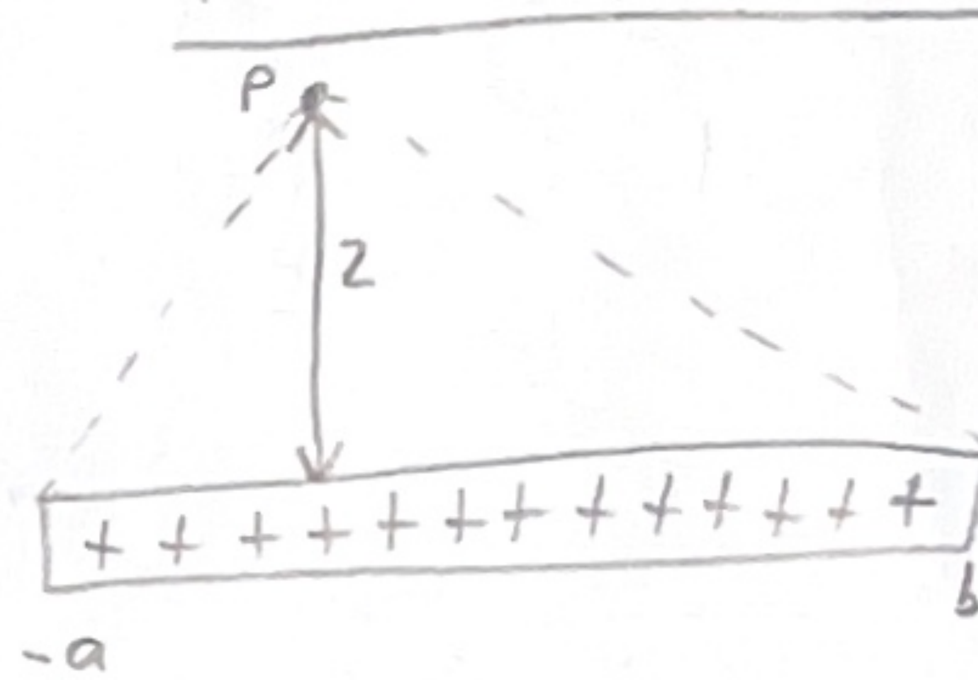
$= \oint_{S_1} E\,dA\cos\theta = \frac{1}{\epsilon_0}\left(\frac{\frac{4}{3}\pi(r^3-a^3)}{\frac{4}{3}\pi(R^3-a^3)}Q\right)$

$E\oint_{S_1} dA = \frac{Q}{\epsilon_0}\left(\frac{r^3-a^3}{R^3-a^3}\right)$

$E(4\pi r^2) = \frac{Q}{\epsilon_0}\left(\frac{r^3-a^3}{R^3-a^3}\right)$

$\vec{E} = \boxed{\frac{Q}{4\pi\epsilon_0}\cdot\frac{1}{r^2}\left(\frac{r^3-a^3}{R^3-a^3}\right)}$

Finite line of charge:



$E_z = \frac{k\lambda}{z}\left[\frac{b}{(z^2+b^2)^{1/2}} + \frac{a}{(z^2+a^2)^{1/2}}\right]$

We know Field due to a ring at origin is along the z-axis and $\vec{E} = \frac{kQz}{(z^2+a^2)^{3/2}}$

$\vec{E} = \int d\vec{E}$

$= \int_Q \frac{kz\,dq}{(z^2+r^2)^{3/2}}$

what is dq?

$\frac{2\pi r\,dr}{\pi R^2} = \frac{dq}{Q}$

$dq = \frac{2Qr\,dr}{R^2}$

$\vec{E} = \int_0^R \frac{kz\left(\frac{2Qr\,dr}{R^2}\right)}{(z^2+r^2)^{3/2}}$

$\vec{E} = \left(\frac{2kQz}{R^2}\right)\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$

$u = z^2 + r^2$, $2(r\,dr) = du$

$\frac{du}{dr} = 2r$, $r\,dr = \frac{du}{2}$

$\vec{E} = \frac{2kQz}{R^2}\int_{u_1}^{u_2} \frac{du/2}{u^{3/2}}$

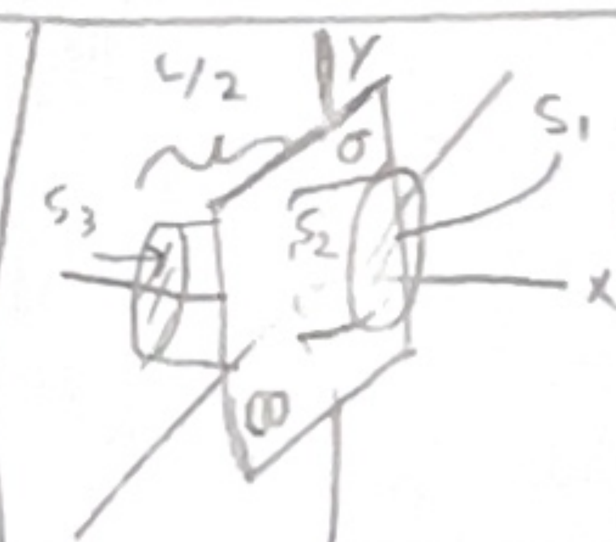
$\vec{E} = \left(\frac{kQz}{R^2}\right)\left(\frac{-1}{2}\right)u^{-1/2}\Big|_{u_1}^{u_2}$

$= -\left(\frac{kQz}{R^2}\right)\frac{1}{\sqrt{z^2+r^2}}\Big|_0^R$

$= \left(\frac{kQ}{2R^2}\right)$ [illegible] $\left(\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{\sqrt{z^2}}\right)$

$\vec{E} = \left(\frac{kQz}{2R^2}\right)\left(\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right)$

$\vec{E} = \left[\left(\frac{kQ}{2R^2}\right)\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\right]$



$\int_{S_1} \vec{E}\cdot d\vec{A} + \int_{S_2} \vec{E}\cdot d\vec{A} + \int_{S_3} \vec{E}\cdot d\vec{A} = \frac{\sigma A}{\epsilon_0}$

$2\int_{S_1} \vec{E}\,d\vec{A}\cos\theta = \frac{\sigma A}{\epsilon_0}$

$2E\int_{S_1} dA = \frac{\sigma A}{\epsilon_0}$

$2EA = \frac{\sigma A}{\epsilon_0}$

$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}}$