02_LASSO_example

June 25, 2019

1 Using the LASSO for inference and prediction

- Stepwise linear regression is time consuming the p-values are unstable and are pathdependent
- Would be nice to have the optimal subset of variables selected in an unbiased way
- Use LASSO/Ridge/ElasticNet for wide data
- Can use LASSO for unbiased feature selection
- LASSO uses regularization which penalizes high coefficients
- Lambda parameter analogy to garden hose most important features start to trickle through as you open the valve

```
In [1]: # Package "glmnet" contains the LASSO function
        # install.packages( 'qlmnet' )
        library( glmnet )
        # Package "mice" is for imputation of missing data
        #install.packages( 'mice' )
        library( mice )
        # Metapackage "tidyverse" imports libraries
        # for data manipulation (dplyr) and plotting (ggplot2)
        library( tidyverse )
        # Package "readxl" has the read_excel() function
        library( readxl )
        # Package "skimr" has excellent descriptive statistics
        # function skim_to_wide()
        library( skimr )
        # Package "GGally" has qqplot2-style scatterplot matrices
        library( GGally )
        # Package "ggfortify" has ggplot2-style regression diagnostic plots
        #install.packages( 'ggfortify' )
        library( ggfortify )
```

```
# Package tictoc has functions to time function calls
       library( tictoc )
        # Package "glmnetUtils" allows the use of R formulas for
        # specifying glmnet models (as opposed to converting to matrices)
        #library(devtools)
        #install github("hong-revo/glmnetUtils")
       library(glmnetUtils)
        # Package "broom" for tidy() function for pulling
        # regression coefficients from glmnet models
       library( broom )
        # Package "rsample" for train/test split utilities
       library( rsample )
Loading required package: Matrix
Loading required package: foreach
Loaded glmnet 2.0-18
Loading required package: lattice
Attaching package: mice
The following objects are masked from package:base:
    cbind, rbind
Registered S3 methods overwritten by 'ggplot2':
 method
                from
  [.quosures
                rlang
  c.quosures
                rlang
  print.quosures rlang
Registered S3 method overwritten by 'rvest':
 method
                   from
 read_xml.response xml2
 Attaching packages tidyverse 1.2.1
 ggplot2 3.1.1
                  purrr 0.3.2
tibble 2.1.1
                  dplyr
                           0.8.1
tidyr 0.8.3
                  stringr 1.4.0
 readr 1.3.1
                  forcats 0.4.0
Conflicts tidyverse conflicts()
 purrr::accumulate() masks foreach::accumulate()
 tidyr::complete() masks mice::complete()
 tidyr::expand()
                   masks Matrix::expand()
 dplyr::filter()
                   masks stats::filter()
 dplyr::lag()
                    masks stats::lag()
 purrr::when()
                   masks foreach::when()
```

```
Attaching package: skimr

The following object is masked from package:stats:

filter

Registered S3 method overwritten by 'GGally':
  method from
  +.gg ggplot2

Attaching package: GGally

The following object is masked from package:dplyr:
  nasa

Attaching package: glmnetUtils

The following objects are masked from package:glmnet:
  cv.glmnet, glmnet
```

2 Load data

- NYtowns dataset marketing data
- [Product] Penetration is target variable
- Many other variables, mostly demographic data

3 Exploratory: univariate

• Distribution of Penetration is right skew and non-negative; not a count, but more like a rate

variable	missing	complete	mean	sd	p0	p25
<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<ch< td=""></ch<>
AncArab	0	1006	0.28	0.43	0	0
AncCzech	0	1006	0.49	0.5	0	0.1
AncDanish	0	1006	0.37	0.61	0	0.1
AncDutch	0	1006	4.16	2.87	0	2.3
AncEnglish	0	1006	14.37	5.39	0	10.6
AncFrCanad	0	1006	2.07	2.35	0	0.8
AncFrench	0	1006	6.58	6.47	0	3
AncGerman	0	1006	19.89	8.34	0	14.8
AncGreek	0	1006	0.34	0.49	0	0
AncHungary	0	1006	0.61	0.67	0	0.2
AncIrish	0	1006	17.73	5.12	0	14.3
AncItalian	0	1006	11.23	7.11	0	6.12
AncLithu	0	1006	0.26	0.34	0	0
AncNorweg	0	1006	0.67	0.62	0	0.2
AncOthr	0	1006	11.62	8.24	1.6	7.6
AncPolish	0	1006	5.91	4.57	0	3.1
AncPortug	0	1006	0.15	0.25	0	0
AncRussian	0	1006	0.93	1.26	0	0.2
AncScot	0	1006	2.37	1.32	0	1.5
AncScotIre	0	1006	1.39	0.8	0	0.9
AncSlovak	0	1006	0.27	0.57	0	0
AncSubSah	0	1006	0.12	0.26	0	0
AncSwedish	0	1006	1.35	2.35	0	0.5
AncSwiss	0	1006	0.43	0.64	0	0.1
AncUkraine	0	1006	0.68	0.83	0	0.2
AncUS	0	1006	6.85	3.66	0	4.1
AncWelsh	0	1006	1.22	1.42	0	0.5
AncWIndian	0	1006	0.25	0.91	0	0
AreaLand	0	1006	1.2e+08	9.6e+07	230812	8.2e
AreaWater	0	1006	9e+06	5e+07	0	1587
BadKitchen	0	1006	0.69	1.35	0	0.1
BadPlumbing	0	1006	0.75	1.35	0	0.2
BoatRVVan	0	1006	0.45	1.14	0	0
BornAfrica	14	992	1.26	3.36	0	0
BornAsia	14	992	17.28	17.1	0	0
BornEurope	14	992	49.57	23.92	0	32.6
BornLatAmer	14	992	14.48	18.74	0	0
BornNorAmer	14	992	16.79	20.57	0	2.1
BornOceania	14	992	0.62	2.82	0	0
BuiltAfter40	0	1006	67.87	13.23	28.8	59.3
BuiltAfter60	0	1006	51.62	13.99	10.7	43.4
BuiltAfter70	0	1006	40.81	13.1	5.3	33.0
BuiltAfter80	0	1006	26.13	9.98	1.3	19.7
BuiltAfter90	0	1006	13.22	5.93	0.2	9.2
BuiltAfter95	0	1006	6.43	3.62	0	4.1
BuiltAfter99	0	1006	1.48	1.21	0	0.6
CitizenNative	0	1006	96.41	4.61	53.9	95.9
CitizenNat y	0	1006	1.97	2.12	0	0.7
CitizenNot	0	1006	1.62	2.75	0	0.7
CommuteAtHome	0	1006	4.45	2.75	0	2.6
CommuteAttionie	$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$	1006	4.43 25.91	6.03	6	∠.0

4 Optional: impute missing data

```
In [7]: data %>% is.na %>% sum
    190
In [8]: tic()
        imputation_model <- mice( data, method='cart', m=1, maxit=1 )
            toc()

iter imp variable
    1   1 GPGuardP BornEurope BornAsia BornAfrica BornOceania BornLatAmer BornNorAmer

Warning message:
Number of logged events: 23
27.761 sec elapsed

In [9]: #glimpse( imputation_model )
In [10]: imputed_data <- mice::complete( imputation_model )
In [11]: imputed_data %>% is.na %>% sum
    0
```

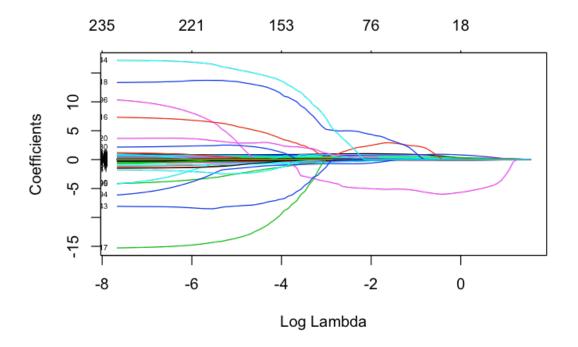
5 To evaluate predictions, split into train and test set

6 Perform LASSO

- Plain vanilla model (gaussian family w/identity link function)
 - Baseline: fitting linear model to poisson-style data
- Vary lambda regularization parameter, observe coefficient paths as lambda varies
 - By default, it will generate 100 different models with 100 different lambdas
 - A reasonable range of lambda vales are selected for you by default, also you can specify your own (or a range of lambdas)
- LASSO corresponds with glmnet with argument $\alpha = 1$ which is default
- glmnet standardized variables by default, so regression coefficients β_i is interpretable as relative importances

• Use dfmax argument to limit number of variables included in the model

```
In [13]: glmnet_lm_result <- glmnet( Penetration ~ ., data=train_data )
In [14]: print( glmnet_lm_result )
Call:
glmnet.formula(formula = Penetration ~ ., data = train_data)
Model fitting options:
    Sparse model matrix: FALSE
    Use model.frame: FALSE
    Alpha: 1
    Lambda summary:
    Min. 1st Qu. Median Mean 3rd Qu. Max.
0.000470 0.004705 0.047063 0.529144 0.470507 4.701200</pre>
```



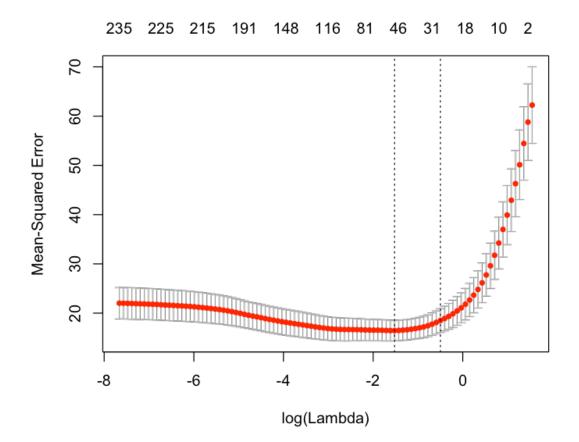
6.1 How many times does a given variable appear in a model?

```
In [16]: glmnet_lm_result %>%
                                                # Take the glmnet result object
             coef %>%
                                                # get all the coefficients for all 100 models (l
             t %>%
                                                # transpose so the variables are in the columns
             as.matrix \%>\% as.data.frame \%>\% # convert to something the tidyverse can manipul
             map int( ~ sum( . != 0 ) ) %>%
                                                # For each column, return a count
             sort( decreasing = TRUE ) %>%
                                                # sort from highest to lowest
             head( 40 ) %>%
                                                # show the top 20 used variables
             print
       (Intercept)
                                                                    ValueLT50K
                      HouseMultiFamily
                                                AncItalian
               100
                                     98
                                                         97
                                                                             95
       BadPlumbing
                            NoCashRent
                                              LessEq2Rooms
                                                                      EduHSDip
                                                                             91
                                             WorkClassSelf
    JobAgriculture MarFemaleDivorcees
                                                                  BuiltAfter90
                89
          AncSwiss
                                                  PopRural
                         CommuteAtHome
                                                                  JobConstruct
                85
                                     82
      BuiltAfter70
                               SchNurs
                                            JobOfficeSales
                                                                   EduColNoDeg
        BadKitchen CommuteAvgTravTime
                                            CostDivIncLT15
                                                                      VehicGT1
                                                                             78
   IndConstruction
                                                  IndAgric
                             Resid5Yrs
                                                                 PovIndSeniors
                77
                                     76
                                                         75
                                                                             75
                                            {\tt CostDivIncLT25}
      IncAvgSocSec
                                                                      MarWidow
                         IncRetirement
                                                         74
                74
                                     74
                                                                             72
     NativityOutUS
                          MortageLT500
                                                JobService
                                                                      IncSupSec
                72
                                     72
                                                         71
                                                                             71
          AreaLand
                               SchKind
                                                   AncOthr
                                                                IndPublicAdmin
                71
                                     70
                                                         70
                                                                             70
```

7 Which lambda gives the best model?

- BEST lambda is given by "lambda.min": the at which the minimal MSE is achieved
- Use cv.glmnet() Perform CROSS-VALIDATION LASSO
- Again, plain vanilla model (gaussian family w/identity link function)
- 10-fold cross validation to find optimal lambda parameter

Call: cv.glmnet.formula(formula = Penetration ~ ., data = train_data) Model fitting options: Sparse model matrix: FALSE Use model.frame: FALSE Number of crossvalidation folds: 10 Alpha: 1 Deviance-minimizing lambda: 0.2182104 (+1 SE): 0.6071833 In [19]: glmnet_cv_result\$lambda.min 0.218210370672902 In [20]: options(repr.plot.width=6, repr.plot.height=5) In [21]: log(glmnet_cv_result\$lambda.min) -1.52229567842249 In [22]: plot(glmnet_cv_result)



7.1 Given the best lambda, what are the regression coefficients?

- Remember, glmnet regression coefficients are standardized
 - Betas aren't in original units
 - Betas double as feature importances
 - To keep original units, use argument standardize=False in glmnet function call

	row	value
	<chr></chr>	<dbl></dbl>
	(Intercept)	-18.37737222
	MarFemaleDivorcees	-5.11554208
	CostDivIncLT25	3.24460585
	CostDivIncLT15	2.86530431
	AncSwiss	0.98236846
	BadPlumbing	0.80539507
	LessEq2Rooms	0.54193077
	BadKitchen	0.32900049
	JobAgriculture	0.32006860
	WorkClassSelf	0.19349322
	IncRatioM2F	-0.12500977
	VehicGT1	0.09359990
	SchNurs	-0.08918499
A df[,2]: 30 Œ 2	EduHSDip	0.08194796
A u1[,2]. 50 th 2	EduColNoDeg	-0.07702541
	IncSupSec	0.07538104
	NativityUS	0.07525924
	CommuteAvgTravTime	0.06972501
	IndConstruction	0.06312009
	CommuteAtHome	0.05860763
	BuiltAfter90	0.05768235
	NativityOutUS	-0.05674349
	IncRetirement	-0.05583241
	MarWidowedFemales	-0.05569634
	Resid5Yrs	0.05238722
	AncItalian	-0.05140025
	MarWidow	-0.05032510
	SchKind	-0.04929637
	MortgageLT700	0.04431094
	AncCzech	0.04281635

8 Use best lambda for prediction

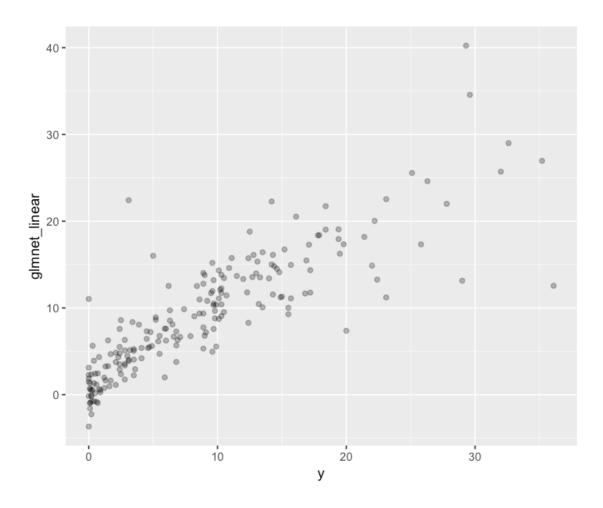
• For functions like predict(), arg s is where you input lambda

8.1 Model achieves training R-squared of 0.72

```
In [29]: summary( lm( glmnet_linear ~ y, all_pred_results) )
Call:
lm(formula = glmnet_linear ~ y, data = all_pred_results)
Residuals:
    Min
              1Q
                  Median
                                3Q
                                        Max
-17.4435 -2.0476 -0.1604
                            1.7866 17.6349
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 2.41597
                       0.39526
                                 6.112 5.09e-09 ***
            0.76420
                       0.03323 23.000 < 2e-16 ***
У
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1
Residual standard error: 3.717 on 199 degrees of freedom
Multiple R-squared: 0.7266, Adjusted R-squared: 0.7253
F-statistic:
              529 on 1 and 199 DF, p-value: < 2.2e-16
```

8.2 Make plot of predicted vs actual

• Note negative predictions!!!



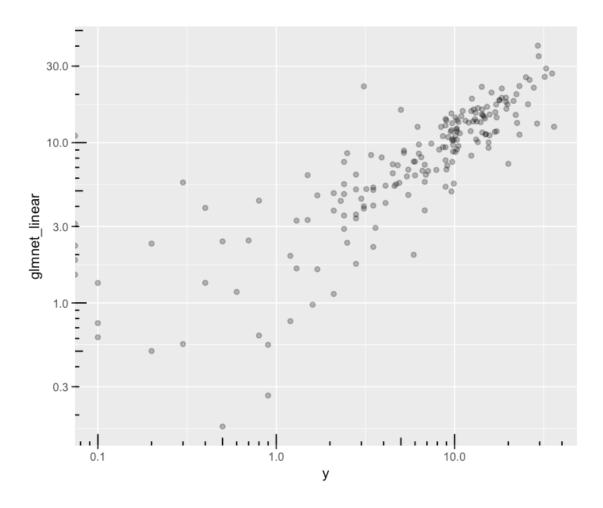
8.3 Make log-log plot of predicted vs actual

• ggplot throws warning messages because it can't handle log transformations of negative predictions - it drops them from the figure.

Warning message:

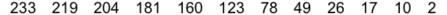
Transformation introduced infinite values in continuous x-axisWarning message in self\$trans\$tran

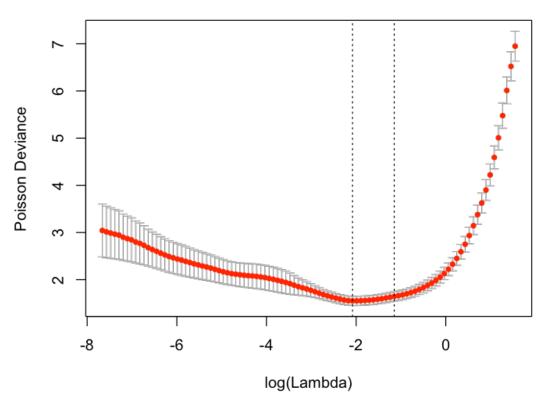
Transformation introduced infinite values in continuous y-axisWarning message: Removed 12 rows containing missing values (geom_point).



9 Slightly better: Generalized Linear model

- Model the outcome variable Penetration as having a poisson distribution
- Use cross validation to find optimal lambda, then make predictions with that lambda.
- GLM offsets advanced topic not covered here





9.1 Poisson-style glmnet model R-squared is comparable to linear-style glmnet model

```
In [37]: summary( lm( glmnet_poisson ~ y, all_pred_results) )
Call:
lm(formula = glmnet_poisson ~ y, data = all_pred_results)
```

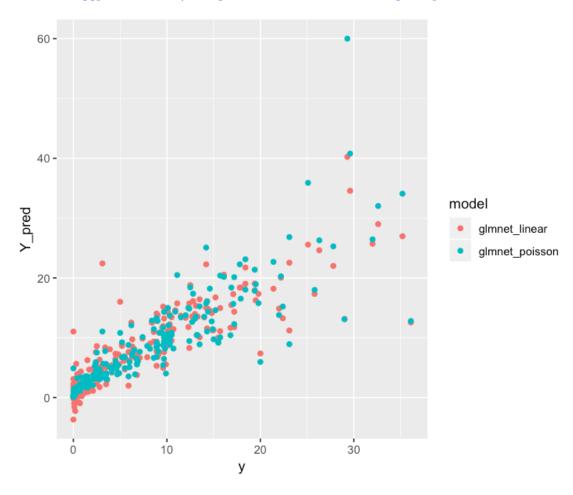
Residuals:

```
Min 1Q Median 3Q Max -20.227 -1.217 -0.325 1.229 32.886
```

Coefficients:

Residual standard error: 4.305 on 199 degrees of freedom

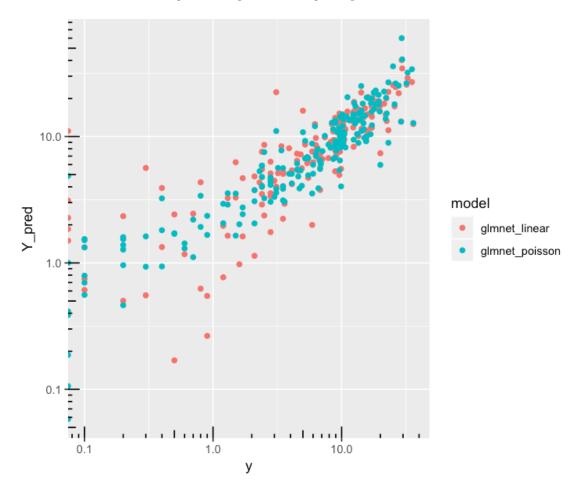
Multiple R-squared: 0.7196, Adjusted R-squared: 0.7182 F-statistic: 510.6 on 1 and 199 DF, p-value: < 2.2e-16



Warning message:

Transformation introduced infinite values in continuous x-axisWarning message in self\$trans\$tran

Transformation introduced infinite values in continuous y-axisWarning message: Removed 12 rows containing missing values (geom_point).



10 Use glmnet coefs for variable selection

```
In [41]: interesting_linear_vars
```

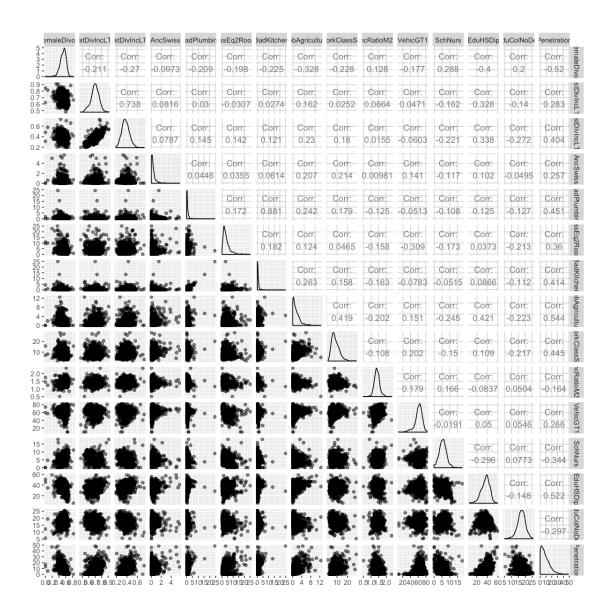
1. 'MarFemaleDivorcees' 2. 'CostDivIncLT25' 3. 'CostDivIncLT15' 4. 'AncSwiss' 5. 'BadPlumbing' 6. 'LessEq2Rooms' 7. 'BadKitchen' 8. 'JobAgriculture' 9. 'WorkClassSelf' 10. 'IncRatioM2F' 11. 'VehicGT1' 12. 'SchNurs' 13. 'EduHSDip' 14. 'EduColNoDeg' 15. 'Penetration'

```
In [42]: train_data_subset <- train_data %>% select( interesting_linear_vars )
```

10.1 Exploratory: Univariate and Bivariate dists

In [43]: skim_to_wide(train_data_subset) %>% select(-type, -n)

	. 11		1 .		1	0	25	5 0	
	variable	missing	complete	mean	sd	p0	p25	p50	p7
A tibble: 15 Œ 11	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<chr></chr>	<c< td=""></c<>
	AncSwiss	0	805	0.42	0.62	0	0.1	0.2	0.5
	BadKitchen	0	805	0.68	1.3	0	0.1	0.4	0.9
	BadPlumbing	0	805	0.74	1.33	0	0.2	0.4	0.9
	CostDivIncLT15	0	805	0.37	0.075	0.19	0.33	0.37	0.4
	CostDivIncLT25	0	805	0.7	0.063	0.47	0.66	0.7	0.7
	EduColNoDeg	0	805	17.24	2.92	5.2	15.6	17.4	19
	EduHSDip	0	805	36.23	7.89	6	31.4	37.2	41
	IncRatioM2F	0	805	1.39	0.18	0.5	1.28	1.38	1.5
	JobAgriculture	0	805	1.54	1.66	0	0.3	1.1	2.2
	LessEq2Rooms	0	805	3.84	3.34	0	1.7	3	4.8
	MarFemaleDivorcees	0	805	0.52	0.089	0	0.47	0.53	0.5
	Penetration	0	805	8.66	7.9	0	2.5	6.7	13
	SchNurs	0	805	5.58	2.36	0	4.1	5.6	7
	VehicGT1	0	805	60.93	9.87	9.6	56.8	62.3	67
	WorkClassSelf	0	805	8.61	3.65	1.6	6	7.9	10
	!	1							



11 Regular Linear Model

```
In [45]: model0 <- lm( Penetration ~ ., data=train_data_subset )
In [46]: summary( model0 )

Call:
lm(formula = Penetration ~ ., data = train_data_subset)

Residuals:
    Min    1Q    Median    3Q    Max
-12.3993    -2.3369    -0.2804    1.9329    15.2693</pre>
```

Coefficients:

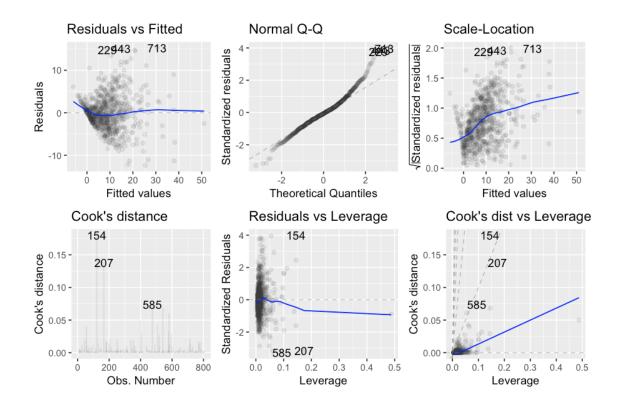
```
Estimate Std. Error t value Pr(>|t|)
(Intercept)
                  -22.52440
                               2.93277 -7.680 4.70e-14 ***
MarFemaleDivorcees -7.50162
                               1.89528 -3.958 8.24e-05 ***
CostDivIncLT25
                    5.32176
                               3.48363
                                        1.528 0.127000
CostDivIncLT15
                   11.31104
                               3.06880
                                        3.686 0.000244 ***
AncSwiss
                    0.98030
                               0.23534
                                        4.165 3.45e-05 ***
BadPlumbing
                    1.26269
                               0.22902
                                        5.514 4.76e-08 ***
LessEq2Rooms
                               0.04796 15.597 < 2e-16 ***
                    0.74801
BadKitchen
                               0.23727 1.633 0.102774
                    0.38757
JobAgriculture
                               0.10843 3.799 0.000157 ***
                    0.41190
                               0.04526 8.246 6.81e-16 ***
WorkClassSelf
                    0.37321
IncRatioM2F
                               0.85991 -2.901 0.003821 **
                   -2.49474
                               0.01651 14.906 < 2e-16 ***
VehicGT1
                    0.24610
SchNurs
                   -0.15866
                               0.06578 -2.412 0.016098 *
EduHSDip
                    0.29024
                               0.02195 13.223 < 2e-16 ***
                               0.05213 -2.623 0.008895 **
EduColNoDeg
                   -0.13671
```

Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1 1

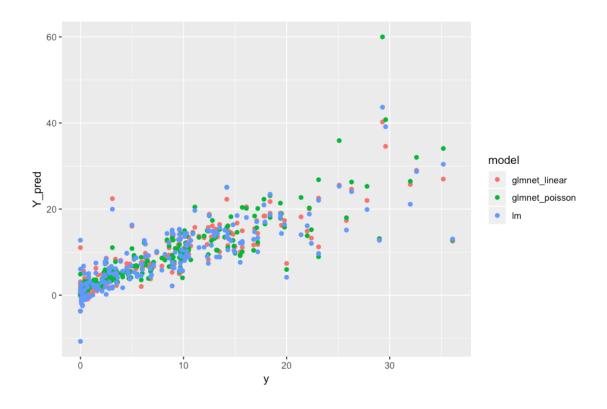
Residual standard error: 3.969 on 790 degrees of freedom Multiple R-squared: 0.7519, Adjusted R-squared: 0.7475 F-statistic: 171 on 14 and 790 DF, p-value: < 2.2e-16

In [47]: options(repr.plot.width=7.5, repr.plot.height=5)

In [48]: autoplot(model0, which = 1:6, ncol=3, alpha=0.1)



they will be dropped

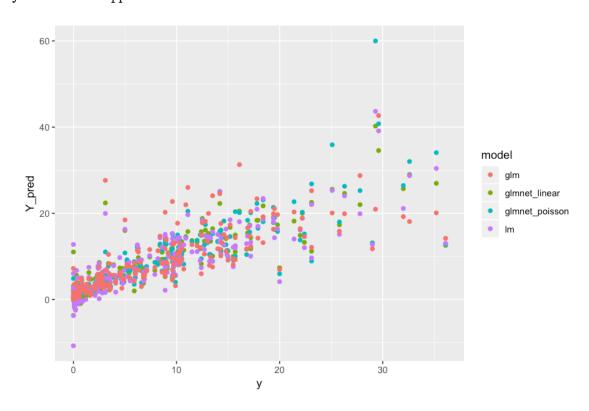


12 GLM model

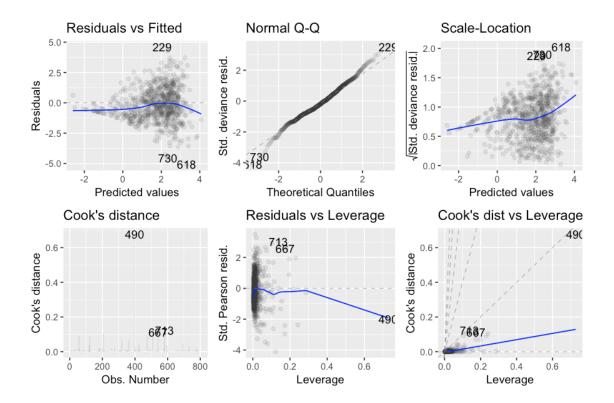
```
row | value
                               <chr>
                                       <dbl>
                  MarFemaleDivorcees
                                      -0.38459635
                                      0.38011921
                      CostDivIncLT25
                   LabFCivilEmployed
                                      0.25292790
                           (Intercept)
                                      0.21780932
                          AncSubSah
                                      -0.13735906
                      CostDivIncLT15
                                      0.13313636
  A df[,2]: 15 Œ 2
                            AncSwiss | 0.04909032
                          BadKitchen |
                                      0.02717750
                           AncItalian | -0.02633768
                    HouseMultiFamily | -0.02609654
                            AncCzech | 0.02494139
                        BadPlumbing | 0.02413368
                        WorkClassSelf
                                      0.02033181
                        JobAgriculture | 0.01565186
                           IncSupSec | 0.01512092
In [53]: # remove row marked '(Intercept)'
         interesting_poisson_vars <- c( setdiff( glm_coefs[1:15,'row'], '(Intercept)'), 'Pene'</pre>
In [54]: data_subset <- train_data %>% select( interesting_poisson_vars )
In [55]: model1 <- glm( Penetration ~ ., data=data_subset, family='quasipoisson' )</pre>
In [56]: summary( model1 )
Call:
glm(formula = Penetration ~ ., family = "quasipoisson", data = data_subset)
Deviance Residuals:
                   Median
    Min
              1Q
                                 3Q
                                         Max
-5.0784 -1.0049 -0.2477
                            0.6750
                                      4.6136
Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)
                    2.499317
                               0.457565 5.462 6.30e-08 ***
MarFemaleDivorcees -1.028500
                               0.178225 -5.771 1.13e-08 ***
CostDivIncLT25
                    0.848191
                               0.362827 2.338 0.019650 *
LabFCivilEmployed -0.207178
                               0.420219 -0.493 0.622133
AncSubSah
                   -0.357826
                               0.117860 -3.036 0.002476 **
CostDivIncLT15
                   -0.004742
                               0.315302 -0.015 0.988005
                                           4.581 5.38e-06 ***
AncSwiss
                    0.085180
                                0.018595
BadKitchen
                   -0.009841
                                0.019379 -0.508 0.611718
                               0.003932 -11.053 < 2e-16 ***
AncItalian
                   -0.043466
HouseMultiFamily
                   -0.044923
                                0.002607 -17.233 < 2e-16 ***
AncCzech
                    0.099778
                                0.029782
                                           3.350 0.000846 ***
BadPlumbing
                                0.019257 2.368 0.018138 *
                    0.045596
```

```
0.023411
                             0.004522
WorkClassSelf
                                        5.177 2.86e-07 ***
                             0.009138 5.820 8.53e-09 ***
JobAgriculture
                   0.053187
IncSupSec
                   Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1
(Dispersion parameter for quasipoisson family taken to be 1.740372)
   Null deviance: 5592.2 on 804 degrees of freedom
Residual deviance: 1424.6 on 790 degrees of freedom
AIC: NA
Number of Fisher Scoring iterations: 5
In [57]: glm_preds <- as.numeric( predict( model1, test_data, type='response' ) )</pre>
In [58]: all_pred_results <- all_pred_results %>%
            mutate( glm = glm_preds )
In [59]: summary( lm( glm ~ y, all_pred_results ) )
Call:
lm(formula = glm ~ y, data = all_pred_results)
Residuals:
    Min
              1Q
                  Median
                                3Q
                                       Max
-13.8162 -2.4198 -0.9994 1.2441 22.4002
Coefficients:
           Estimate Std. Error t value Pr(>|t|)
(Intercept) 3.10600
                       0.49328 6.297 1.91e-09 ***
                       0.04147 16.648 < 2e-16 ***
            0.69032
У
Signif. codes: 0 *** 0.001 ** 0.01 * 0.05 . 0.1
Residual standard error: 4.639 on 199 degrees of freedom
Multiple R-squared: 0.5821, Adjusted R-squared:
F-statistic: 277.1 on 1 and 199 DF, p-value: < 2.2e-16
In [60]: all_pred_results %>%
            gather( key='model', value='Y_pred', -y ) %>%
            ggplot( aes( y, Y_pred, color=model) ) + geom_point() #+
            \#scale_x\_continuous(trans = 'log10') +
            #scale_y_continuous(trans = 'log10') +
            #annotation_logticks()
```

Warning message: attributes are not identical across measure variables; they will be dropped



In [61]: autoplot(model1, which = 1:6, ncol=3, alpha=0.1)



12.1 What about log transforming the skewed vars?