COSC 290 Discrete Structures

Lecture 8: Nested quantifiers

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Plan for today

1. Practice: nested and negated quantifiers
2. Logistics

Practice: nested and negated quantifiers

True love, expressed mathematically

Predicate $loves(p_1, p_2)$ means "p_1 loves p_2." We can express the loves predicate visually using a graph.

Nodes are individuals. Edge from p_1 to p_2 indicates $loves(u, v)$.

Example

The proposition

$$loves(Alice, Darmesh) \wedge loves(Bob, Chloe) \wedge loves(Chloe, Darmesh)$$

can be shown as.

figs/loves0.pdf

Poll: nested quantifiers, part 1

Predicate *loves*(p_1, p_2) means "p_1 loves p_2," shown by an arrow from p_1 to p_2.

For which figure(s) is the following proposition true:

$$\forall p_1 \in P : \exists p_2 \in P : loves(p_1, p_2)$$

figs/loves1.pdf

(a)

figs/loves2.pdf

(b)

figs/loves3.pdf

(c)

More than one/ None of the above

(d)

Poll: nested quantifiers, part 2

Predicate *loves*(p_1, p_2) means "p_1 loves p_2," shown by an arrow from p_1 to p_2.

For which figure(s) is the following proposition true:

$$\exists p_2 \in P : \forall p_1 \in P : loves(p_1, p_2)$$

figs/loves1.pdf	figs/loves2.pdf	figs/loves3.pdf	More than one/ None of the above
(a)	**(b)**	**(c)**	**(d)**

Poll: showing a proposition is false

Consider the following proposition

$$\forall p_1 \in P : \exists p_2 \in P : \mathit{loves}(p_1, p_2)$$

How could we show that this proposition is false?

A) Show there is a person who loves everyone

B) Show there is a person who loves no one

C) Show there is a person who nobody loves

D) Show there is a person who everyone loves

E) Other/more/none

Poll: negating nested quantifiers

Consider the following proposition

$$\varphi \coloneqq \exists p_2 \in P : \forall p_1 \in P : loves(p_1, p_2)$$

What is the correct negation of φ? In other words, which of the following is logically equivalent to $\neg\varphi$?

A) $\forall p_2 \forall p_1 \neg loves(p_1, p_2)$

B) $\forall p_2 \exists p_1 \neg loves(p_1, p_2)$

C) $\exists p_2 \forall p_1 \neg loves(p_1, p_2)$

D) $\forall p_2 \exists p_1 \neg loves(p_2, p_1)$

E) Other/more/none

Logistics

Logistics

Next topic on schedule: proofs!

Reading does not align neatly with lecture plan.

We will spend some time (roughly 1 lecture each) on several proof techniques:

- direct proof
- proof by contrapositive
- proof by contradiction
- proof by cases

We will *return* to Ch. 4.2 to apply these techniques to Hamming codes.