Starting today, your device will need to be registered at poll everywhere. (See problem set 0 for details.)

COSC 290 Discrete Structures

Lecture 4: Propositional Logic

Prof. Michael Hay
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Colgate University

Plan for today

1. Propositions
2. Thinking logically, or at least trying to
3. Syntax of propositional logic

Propositions

Poll: what is a proposition?

Propositional logic is based around the concept of a proposition.

Why isn't

"Where does COSC 290 lab meet?"

considered a proposition?

A) Because only "yes/no" questions can be propositions.
B) Because questions can never be propositions.
C) Because the answer changes over time (last semester it was held in a different room).
D) More than one of the above / None of the above.

Proposition

A proposition is a sentence that is either *true* or *false*.

- $3 + 4 = 6$
- My middle name is Gerald or my dog's name is Rufus.
- One of these three propositions is true.

These types of sentences are *not* propositions:

- *Questions*: Is $3 + 4 = 7$?
- *Imperatives*: You should major in computer science.
- *Opinions*: Beards look more sophisticated than mustaches.

Atomic vs. compound propositions

Example (Atomic propositions)

1. "Alice is over 21."
2. "Alice is drinking beer."
3. "Pigs can fly."

Example (Compound propositions)

1. "Either Alice is drinking beer and she is over 21 or she is under 21 and she is not drinking beer."
2. "If pigs can fly, then Alice is over 21."

Thinking logically, or at least trying to

Poll: deductive reasoning

Every card has a number on one side and a letter on the other.

Rule: "If D is on one side, then 3 is on the other side."

Which cards do you have to turn over to verify whether the rule does or does not hold on this deck of cards?

D	F	3	7

A) D alone
B) D and F
C) D and 3
D) D and 7
E) All of the cards

Credit: Example adapted from a lecture by Steven Pinker: `https://www.youtube.com/watch?v=1PXy3vWZiJo`

Humans and logic

Humans sometimes struggle with purely abstract logical reasoning.

Previous question: example of "confirmation bias"

(But, didn't humans, like, invent logic?)

Poll: deductive reasoning, again

Suppose you are a bouncer working at a restaurant in Hamilton... Your boss informs you, "If a person is drinking beer, then they must be over 21." Consider the following scenarios and actions:

i) A man at the bar with a mug full of beer. Action: check his id.
ii) A woman dining, drinking a Coke. Action: check her id.
iii) An elderly gentleman sipping a drink. Action: check the contents of his glass.
iv) A young woman, who looks clearly under 21, drinking at the bar. Action: check the contents of her glass.

For which scenarios must you take action to enforce your boss' rule?

A) i alone
B) i and ii
C) i and iii
D) i and iv
E) All of them

Credit: Example adapted from a lecture by Steven Pinker: `https://www.youtube.com/watch?v=1PXy3vWZiJo`

What's the point?

Previous two scenarios were *logically equivalent*.

Yet the second was "easier" in some sense.

Psychology: human reasoning is influenced by context ("content effect").

This course:

- Look at propositions in abstract form: $p \implies q$
- Learn systematic ways of reasoning about these abstract propositions
- Write code that automates this process (lab)

Syntax of propositional logic

Syntax

Propositional logic is a formal language.

Sentences in propositional logic must conform to the language's *syntax*.

Syntax defines what sentences are permissible in the language.

$$\underbrace{\neg(p \implies \neg(\neg q \lor r))}_{\text{valid syntax}} \qquad \underbrace{\neg(\land p \implies (\lor \neg q\ r))}_{\text{invalid syntax}}$$

Proposition variables

We can use variables like p, q, r, in place of *atomic* propositions.

Example

- Let $p :=$ "You study"
- Let $q :=$ "You already know the material"
- Let $r :=$ "You will earn an A in the class"

We can use the variables to construct more complex relationships:

Example

Let $s :=$ "If p or q, then r".

This sentence is either true or false—it's also a proposition!

Logical connectives

Rather than write, "If p or q, then r," we use mathematical symbols called logical connectives.

Name	Symbol	Meaning
Negation	$\neg p$	"not p"
Conjunction	$p \land q$	"p and q"
Disjunction	$p \lor q$	"p or q"
Exclusive or	$p \oplus q$	"either p or q but not both"
Implication	$p \implies q$	"if p, then q"
Mutual implication	$p \iff q$	"p if and only if q"

Example (continued...)

$s :=$ "If you study or you already know the material, you will earn an A in the class" can be written as $s := (p \lor q) \implies r$.

Implication

You are drinking alcohol legally $\implies$ You are at least 21.

In a natural language like English, this *same* implication can be expressed in *many* different forms.

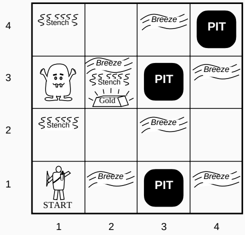
- If you are drinking legally, then you are at least 21.
- You are drinking legally only if you are at least 21.
- You are at least 21 if you are drinking legally.
- Being at least 21 is necessary for you to be drinking legally.
- Knowing that you are drinking legally is sufficient information to conclude you are at least 21.

What about this sentence? Is it a proposition? If so, is it true?

- If you are at least 21, then you are drinking alcohol legally.

Wumpus world

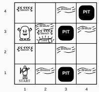
Goal: move around wumpus world, avoid pits (which are breezy) and the scary wumpus (who smells), find gold (which glitters).



Wumpus world sentences

Let atomic propositions be denoted as follows:

- w_{ij} denotes "there is a **w**umpus in [i, j]."
- s_{ij} denotes "there is a **s**tench in [i, j]."
- ...



Consider this sentence, "If the wumpus is in [1,3] or [2,4], then there is a stench in [1,4]."

Let's write that in propositional logic:

$$(w_{13} \vee w_{24}) \implies s_{14}$$

Poll: Wumpus world sentences

Let atomic propositions be denoted as follows:

- p_{ij} denotes "there is a **p**it in [i, j]."
- w_{ij} denotes "there is a **w**umpus in [i, j]."
- ...

Write this sentence in propositional logic: "There may be a pit in [1,2] or [2,2] and the wumpus is either in [2,1] or [3,1]."

A) $(p_{1,2} \oplus p_{2,2}) \wedge (w_{2,1} \oplus w_{3,1})$
B) $(p_{1,2} \vee p_{2,2}) \wedge (w_{2,1} \oplus w_{3,1})$
C) $(p_{1,2} \vee p_{2,2}) \wedge (w_{2,1} \vee w_{3,1})$
D) $(p_{1,2} \wedge p_{2,2}) \vee (w_{2,1} \wedge w_{3,1})$
E) More than one of the above / None of the above

Poll: Wumpus world sentences

Let atomic propositions be denoted as follows:

- p_{ij} denotes "there is a **p**it in [i, j]."
- b_{ij} denotes "there is a **b**reeze in [i, j]."
- ...

Suppose we are given the rule that "pits cause breezes in adjacent cells." Which of the following sentences is consistent with that rule?

A) $p_{2,1} \implies (b_{1,1} \land b_{2,2} \land b_{3,1})$
B) $b_{1,1} \implies (p_{1,2} \lor p_{2,1})$
C) $b_{1,1} \iff (p_{1,2} \lor p_{2,1})$
D) All of the above
E) None of the above unless we also know that $b_{1,1}$ is true or $p_{2,1}$ is true.

Hint: would your answer change if the rule said that pits are the *only* thing that causes breezes?