COSC 290 Discrete Structures

Lecture 35: Probability

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Monday, Dec. 1, 2017

Colgate University

Plan for today

1. Problem set, quick review

2. Probability problems

3. Key concepts: sample space, probability, event

Problem set, quick review

Problem set

Let’s go over a few problems from the problem set (please turn it in if you haven’t already).

Probability problems

Monty Hall Problem

Described in book (p. 1012). Should you switch doors?

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Yes!

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Yes!

Interesting backstory: Marilyn vos Savant (`http://www.nytimes.com/1991/07/21/us/behind-monty-hall-s-doors-puzzle-debate-and-answer.html`)

Poll: boy or girl paradox

Another question posed to Ms. vos Savant is similar to the following two part question:

- Mr. Jones has two children. The older child is a girl. What is the probability that both children are girls?
- Mr. Smith has two children. At least one of them is a boy. What is the probability that both children are boys?

What do you think the probabilities are?

A) $\frac{1}{2}$ and $\frac{1}{2}$ respectively
B) $\frac{1}{3}$ and $\frac{1}{2}$ respectively
C) $\frac{1}{2}$ and $\frac{1}{3}$ respectively
D) $\frac{1}{3}$ and $\frac{1}{3}$ respectively
E) None of the above
F) How should I know? We just started the probability unit!

Source: https://en.wikipedia.org/wiki/Boy˙or˙Girl˙paradox

Key concepts: sample space, probability, event

Strange dice

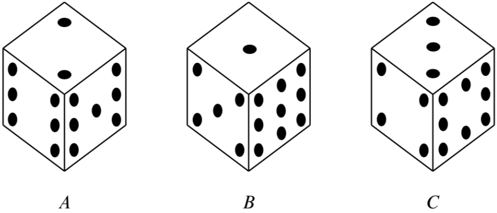

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each $1/3$.

A $100 bet. You choose a die, other player chooses die from among remaining two dice. Each player rolls his/her die. Highest roll wins. Should you take the bet?

Source: Mathematics for Computer Science, 2017, Eric Lehman, F Tom Leighton, Albert R Meyer.

Four steps

1. Find the sample space
2. Define events of interest
3. Determine outcome probabilities (possibly using tree diagrams)
4. Compute event probabilities

Sample space

An outcome of a probabilistic process is the sequence of results for all randomly determined quantities.

The sample space is the set of all outcomes.

Examples:

1. Flipping a coin.
2. Flipping two coins.
3. The birth month of a randomly chosen student.
4. Drawing a poker hand from a 52 card deck.

Sample space for a game of strange dice

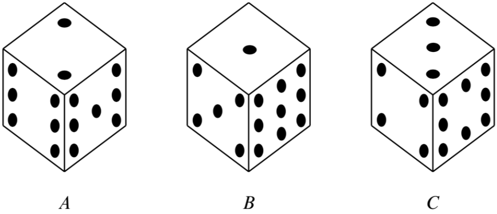

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each $1/3$.

What is sample space?

Sample space for a game of strange dice

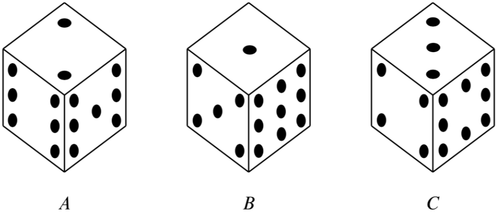

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each $1/3$.

What is sample space? Player choices are *not* random and therefore not part of sample space. Suppose you choose B and other player chooses A. Now what is sample space?

Source: Mathematics for Computer Science, 2017, Eric Lehman, F Tom Leighton, Albert R Meyer.

Four steps

1. Find the sample space
2. Define events of interest
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Events

Let S be a sample space.

An event is a subset of S.

Examples:

1. When flipping a coin: getting heads
2. When flipping two coins: getting at least one tails
3. When selecting birth month: the month ends in ‘y’
4. When drawing a poker hand from a 52 card deck: getting four of a kind.

Four steps

1. Find the sample space
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Probability

A probability function on sample space S is a function $Pr : S \to \mathbb{R}$ such that

$$\sum_{s \in S} Pr(s) = 1$$

and

$$\forall s \in S : Pr(s) \geq 0$$

(The probability function represents the fraction of the time the outcome would occur if the random process were repeated many, many times.)

Examples:

1. Flipping a coin.
2. Flipping two coins.
3. The birth month of a randomly chosen student.
4. Drawing a poker hand from a 52 card deck.

Tree diagrams

When the outcome is determined by a *sequence* of randomly determined quantities, it can be helpful to represent the process using a tree diagram.

Tree:

- Internal node: choice for a single random quantity
- Leaf: an outcome

Probabilities:

- Assign a probability to each edge in tree
- Compute probability of each outcome (leaf): equals product of edge probabilities on path from root to leaf.

Four steps

1. Find the sample space
2. Define events of interest
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Probability of an event

The probability of event E is

$$Pr(E) = \sum_{s \in E} Pr(s)$$

Examples: what is the probability of...

1. When flipping a coin: getting heads
2. When flipping two coins: getting at least one tails
3. When selecting birth month: the month ends in ‘y’
4. When drawing a poker hand from a 52 card deck: getting four of a kind.

Poll: strange dice

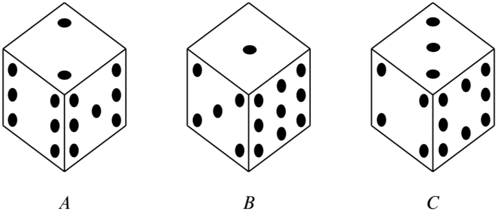

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each $1/3$.

Suppose you choose *A* and other player chooses *C*. What is the probability that you win (roll higher)?

A) $\frac{3}{9}$ B) $\frac{4}{9}$ C) $\frac{5}{9}$ D) $\frac{6}{9}$ E) I'm lost

Source: Mathematics for Computer Science, 2017, Eric Lehman, F Tom Leighton, Albert R Meyer.

Strange dice: new game

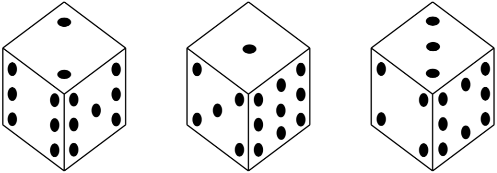

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each $1/3$.

New game: you pick die and roll twice; highest sum wins. Other player gets to pick first, and chooses *B*. You choose *A*. What is probability you win?

Source: Mathematics for Computer Science, 2017, Eric Lehman, F Tom Leighton, Albert R Meyer.