COSC 290 Discrete Structures

Lecture 36: Probability II

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Plan for today

1. Key concepts: sample space, probability, event

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Strange dice

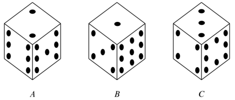

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each $1/3$.

A $100 bet. You choose a die, other player chooses die from among remaining two dice. Each player rolls his/her die. Highest roll wins. Should you take the bet? If so, which die do you choose?

Source for strange dice image: Mathematics for Computer Science, 2017, Eric Lehman, F Tom Leighton, Albert R Meyer.

Four steps

1. Find the sample space
2. Define events of interest
3. Determine outcome probabilities (possibly using tree diagrams)
4. Compute event probabilities

Sample space

An outcome of a probabilistic process is the sequence of results for all randomly determined quantities.

The sample space is the set of all outcomes.

Examples:

1. Flipping a coin.
2. Flipping two coins.
3. The birth month of a randomly chosen student.
4. Drawing a poker hand from a 52 card deck.

Sample space for a game of strange dice

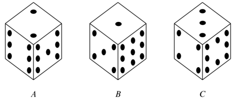

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each $1/3$.

Suppose you choose B and other player chooses A. What is sample space?

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Events

Let S be a sample space.

An event is a subset of S.

Examples:

1. When flipping a coin: getting heads
2. When flipping two coins: getting at least one tails
3. When selecting birth month: the month ends in 'y'
4. When drawing a poker hand from a 52 card deck: getting four of a kind.

Events of interest in strange dice

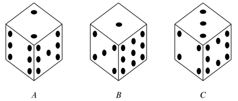

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each 1/3.

Assuming you choose B and other player chooses A, what is the event that you win?

Source for strange dice image: Mathematics for Computer Science, 2017, Eric Lehman, F Tom Leighton, Albert R Meyer.

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Probability

A probability function on sample space S is a function $Pr : S \to \mathbb{R}$ such that

$$\sum_{s \in S} Pr(s) = 1$$

and

$$\forall s \in S : Pr(s) \geq 0$$

One (but not the only!) intrepetation of Pr: it represents the fraction of the time the outcome would occur if the random process were repeated many, many times.

Examples:

1. Flipping a coin.
2. Flipping two coins.
3. The birth month of a randomly chosen student.
4. Drawing a poker hand from a 52 card deck.

Tree diagrams

When the outcome is determined by a *sequence* of randomly determined quantities, it can be helpful to represent the process using a tree diagram.

Tree:

- Internal node: choice for a single random quantity
- Leaf: an outcome

Probabilities:

- Assign a probability to each edge in tree.
- Compute probability of each outcome (leaf): equal to product of edge probabilities on path from root to leaf.

Examples:

- Flipping two coins
- Strange dice game

Four steps

1. Find the sample space
2. Define events of interest
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Probability of an event

The probability of event E is

$$Pr(E) = \sum_{s \in E} Pr(s)$$

Examples: what is the probability of...

1. When flipping a coin: getting heads
2. When flipping two coins: getting at least one tails
3. When selecting birth month: the month ends in 'y'
4. When drawing a poker hand from a 52 card deck: getting four of a kind.

Poll: strange dice

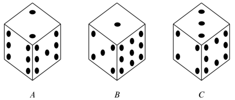

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each $1/3$.

Suppose you choose A and other player chooses C. What is the probability that you win (roll higher)?

A) $\frac{3}{9}$ B) $\frac{4}{9}$ C) $\frac{5}{9}$ D) $\frac{6}{9}$ E) I'm lost

Source for strange dice image: Mathematics for Computer Science, 2017, Eric Lehman, F Tom Leighton, Albert R Meyer.

Strange dice: new game

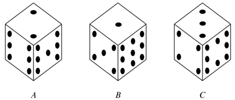

Figure 17.6 The strange dice. The number of pips on each concealed face is the same as the number on the opposite face. For example, when you roll die A, the probabilities of getting a 2, 6, or 7 are each 1/3.

New game: each player rolls twice and highest sum wins. Other player gets to pick first, and chooses B. You choose A. What is probability you win?

Source for strange dice image: Mathematics for Computer Science, 2017, Eric Lehman, F Tom Leighton, Albert R Meyer.

Poll: Tree diagram for new strange dice game

The game: Each player rolls twice and highest sum wins. Other player gets to pick first, and chooses B. You choose A.

Suppose we represent an outcome as a tuple $\langle A_1, A_2, B_1, B_2 \rangle$ where A_1 represents the outcome of the *first* roll of die A, and A_2 represents the *second* roll, and B_1 and B_2 are defined similarly.

How many leaves would the corresponding tree diagram contain?

1. 9
2. 16
3. 27
4. 81
5. I'm not sure.

Exercise: Cavs vs. Warriors

Cleveland Cavaliers play Golden State Warriors in a best of 3 series (first player to win two of three games wins). Suppose probability of Cavs winning a game is 3/5, regardless of results of previous games.

- What is the probability that three games are played?
- What is the probability that the winning team loses the first game?
- What is the probability the Cavs win?

Exercise: Monty Hall variant

Monty Hall problem with four doors.

- Contestant 1: always chooses door A. Probability of win?
- Contestant 2: always initially chooses door A, but then switches to randomly chosen door. Probability of win?