Work in small groups to solve these problems. If you can find space, please write your answers on the whiteboard.

1. **Simpson's paradox** Recall the setting of Simpson's paradox: in 1973, UC Berkeley was sued for sex discrimination based on the fact that graduate school admission rates were 44% for men and 35% for women. The paradox was that if you look at the level of departments, women were being admitted at a higher rate than men. The resolution of the paradox is to observe that women were applying to competitive departments (acceptance rate was low for both genders) and men were applying to less competitive departments (acceptance rate was relatively high for both genders). We can model this with probability.

   Consider the following: let G denote the gender of the applicant (m or f), let D denote the department (a or b) and let A denote the acceptance (y or n) of an applicant to a department. Men and women are equally likely (this is a population of humans, not Ferengi). Women prefer department a: female applicants apply to a with probability $\frac{3}{4}$ and b with probability $\frac{1}{4}$. Men prefer department b: male applicants apply to a with probability $\frac{1}{4}$ and b with probability $\frac{3}{4}$. The departments are non-discriminating: department a admits any applicant with probability $\frac{1}{2}$; department b admits any applicant with probability $\frac{3}{4}$.

   (a) What is the probability that the applicant is a woman, applies to department b and gets accepted? (Hint: use the chain rule.)

   (b) What is the probability of being accepted given that the applican is a woman? (Hint: use the definition of conditional probability, your answer to Part (a). Also, use the total law of probability to sum over all assignments to D.)

   (c) What is the probability of being accepted given that the applican is a man?

   (d) What is the probability of an applicant being accepted? (Hint: you should be able to leverage your answers in the previous two parts.)

   (e) Suppose that this is, in fact, a population of Ferengi and suppose the probability that the applicant is male is $\frac{3}{4}$. How does this change your answer to Part (d)?

2. **Three prisoners** Three prisoners A, B, and C are on death row. The governor decides to pardon one of the prisoners and chooses one at random. He informs the warden of his choice but requests that the name be kept secret.

   The next day, A tries to get the warden to tell him who had been pardoned. The warden refuses. A then asks which of B and C will be executed. The warden thinks for a while and then tells A that B i s to be executed. A's sighs in relief and says, "The chance of me being pardoned has risen to $\frac{1}{2}$!" Is A correct? Explain why or why not using the rules of probability.[1]

[1]This problem adapted from Casella and Berger.

3. **Monty Hall** Welcome to *Let's Make a Deal*! You are on this famous game show hosted by Monty Hall. You have a chance to win a brand new car! Here's the rules of the game.[2]

> Assume that a room is equipped with three doors. Behind two are goats, and behind the third is a shiny new car. (The location of the car is chosen uniformly at random.) You are asked to pick a door, and will win whatever is behind it. Let's say you pick door 1. Before the door is opened, however, someone who knows what's behind the doors (Monty Hall) opens one of the other two doors, revealing a goat, and asks you if you wish to change your selection to the third door (i.e., the door which neither you picked nor he opened). The Monty Hall problem is deciding whether you do.

Suppose that you choose door 1, Monty chooses door 2.

(a) Use Bayes' rule to derive a probability distribution for the location of the car. (To simplify things, you can assume that you always choose door 1 and focus on two random variables, the location of the car C and Monty's choice M.)

(b) Is it significant that Monty knows what's behind the doors?

[2] from Mathworld `http://mathworld.wolfram.com/MontyHallProblem.html`