Please have *one* person from each group fill in your answers on this worksheet and turn it in at the end of class. You must **show your work**. Solutions should not be based on your "intuition" but rather on the rules of probability.

Names: _____, _____

Names: _____, _____

1. **Simpson's paradox**

   (a) What is the probability that the applicant is a woman, applies to department $b$ and gets accepted?

   **Solution:**

   $$\begin{aligned}
   &P(A = y \cap D = b \cap G = f) \\
   &= P(A = y | D = b \cap G = f) P(D = b \cap G = f) \\
   &= P(A = y | D = b \cap G = f) P(D = b | G = f) P(G = f) \\
   &= \frac{3}{4} \times \frac{1}{4} \times \frac{1}{2} = \frac{3}{32}
   \end{aligned}$$

   (b) What is the probability of being accepted given that the applicant is a woman?

   **Solution:**

   $$\begin{aligned}
   P(A = y | G = f) &= \frac{P(A = y, G = f)}{P(G = f)} \\
   &= \frac{\sum_{d \in \{a,b\}} P(A = y, D = d, G = f)}{P(G = f)} \\
   &= \frac{\sum_{d \in \{a,b\}} P(A = y | D = d) P(D = d | G = f) P(G = f)}{P(G = f)} \\
   &= \sum_{d \in \{a,b\}} P(A = y | D = d) P(D = d | G = f) \\
   &= P(A = y | D = a) P(D = a | G = f) + P(A = y | D = b) P(D = b | G = f) \\
   &= \frac{1}{2} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{4} = \frac{6}{16} + \frac{3}{16} = \frac{9}{16}
   \end{aligned}$$

   (c) What is the probability of being accepted given that the applicant is a man?

**Solution:**

$$P(A = y|G = m) = P(A = y|D = a)P(D = a|G = m) + P(A = y|D = b)P(D = b|G = m)$$
$$= \frac{1}{2} \times \frac{1}{4} + \frac{3}{4} \times \frac{3}{4} = \frac{2}{16} + \frac{9}{16} = \frac{11}{16}$$

(d) What is the probability of an applicant being accepted?

**Solution:** Let's write it in a way that lets us re-use answers from the previous two questions.

$$\begin{aligned} P(A = y) &= P(A = y \cap G = f) + P(A = y \cap G = m) \\ &= P(A = y|G = f)P(G = f) + P(A = y|G = m)P(G = m) \\ &= \frac{9}{16}P(G = f) + \frac{11}{16}P(G = m) \\ &= \frac{9}{16} \times \frac{1}{2} + \frac{11}{16} \times \frac{1}{2} = \frac{10}{16} \end{aligned}$$

(e) Suppose that this is, in fact, a population of Ferengi and suppose the probability that the applicant is male is $\frac{3}{4}$. How does this change your answer to Part (d)?

**Solution:** It only affects the $P(G = f)$ and $P(G = m)$ terms.

$$\begin{aligned} P(A = y) &= P(A = y \cap G = f) + P(A = y \cap G = m) \\ &= P(A = y|G = f)P(G = f) + P(A = y|G = m)P(G = m) \\ &= \frac{9}{16}P(G = f) + \frac{11}{16}P(G = m) \\ &= \frac{9}{16} \times \frac{1}{4} + \frac{11}{16} \times \frac{3}{4} = \frac{42}{64} = \frac{21}{32} = \frac{10.5}{16} \end{aligned}$$

So by increasing the percentage of males in the population, the overall acceptance rate goes up.