Calculus 2

Kinetic Energy

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Intro

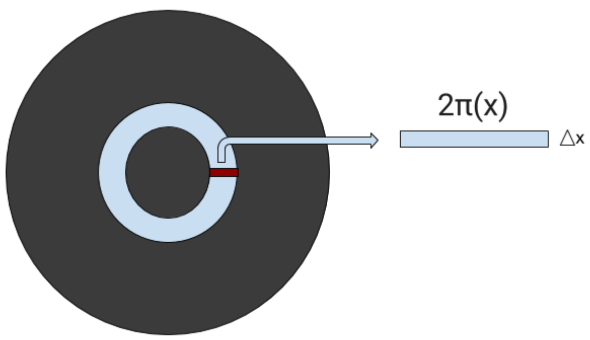
In this paper, we will be exploring the real world applications of calculating linear and angular velocities in different scenarios. We will model, a record, a helicopter, and the earth throughout the paper, in our equations.

Why cant we apply the kinetic energy formula directly?

We cannot use the formula for kinetic energy to directly estimate our models. This is because the formula for kinetic energy is linear, yet our models deal with angular velocity. In each model, the record, the helicopter blade, and the Earth, we are measuring the kinetic energy of an object about its axis. This requires us to use the slice and add modeling approach for integrals. By calculating the mass and velocity of individual slices we are able to estimate the total kinetic energy of an object.

The Record

To calculate the kinetic energy of a record as it plays on a record player we must slice the model into smaller parts. We have decided to think of the record as a collection of circular strips, radiating out from the center. This is convenient because our formula for angular velocity requires the object to be circular. By finding the angular velocity and mass of our slices, we are able to estimate the kinetic energy of the entire object. To start we will estimate the area of the record using an integral. By using the circumference of a circle, and thinking of our circular slice as a rectangle, we are able to derive the following integral.



$$\text{Area} = \int_0^R 2\pi(r)dr$$

Now that we have an integral to represent the area of the record, we can build upon it to estimate the mass. The formula for mass is:

Mass = V * d

V = Area * h

To estimate the mass we must first find the volume. We are going to assume that our record has constant height and density. By multiplying our area by our height we can compute volume, and multiplying that by our density leaves us with an estimate for mass.

$$\int_0^R hd2\pi(r)dr$$

We must again add to this integral. We are currently estimating the mass of the record and our next step is to estimate the linear velocity. The formula for linear velocity is:

Linear Velocity = ω r

To calculate linear velocity of each slice we must multiply angular velocity ω by the radius (r).

Linear Velocity = $\int_0^R \omega r dr$

Now that we have found integrals for mass and linear velocity we can apply our formula for kinetic energy.

K = $\frac{1}{2}mv^2$

K = $\int_0 R \frac{1}{2}(hd2\pi(r))(\omega r)^2 dr$

This integral should calculate the total kinetic energy of the record, evaluated for each slice. We can further simplify our integral by pulling out constants.

K = $hd\pi\omega^2 \int_0^R r^3 dr$

Finally we can integrate this equation to further simplify.

K = $hd\pi\omega^2(\frac{R^4}{4})$

Looking at the wikipedia page for LP record we see that a common 12 inch LP Record has the following characteristics.

Radius	Height	Density	Angular Velocity
0.1524 m (6 in)	0.001 m (1 mm)	$\frac{1350kg}{m^3}(\frac{1.35g}{cm^3})$	3.49065 $\frac{rad}{s}$ ($33\frac{1}{3}$rpm)

'many of the thinnest [LP Records] (0.9 to 1.0 mm) were the most uniform in thickness and some of the best sounding' -tketcham

We have chosen to go with 1 mm as our record thickness to make our calculations easier. We also convert this to meters to match the units of the kinetic energy formula.

$$1mm * \frac{1m}{1000mm} = 0.001m$$

We then found that LP Records were commonly made out of polyvinyl chloride which typically has a density of 1.35 g/cm^3. Again we convert this to kg/m^3 to match the units of kinetic energy.

$$(\frac{1.35g}{cm^3}) * (\frac{1kg}{1000g}) * (\frac{1000000cm^3}{1m^3}) = \frac{1350kg}{m^3}$$

Finally we found from the LP Record wikipedia page that 12 inch records typically spun at 33 $\frac{1}{3}$ rpms. We then converted this to radians per second to match the units of kinetic energy.

$$(33.3333\ \frac{rev}{min}) * (\frac{1min}{60s}) * (2\pi\frac{rad}{1rev}) = 3.49065\ \frac{rad}{s}$$

To confirm our previous work we are going to check the units of our formula given these values. Kinetic energy is typically measured in Joules:

$$J = \frac{(1kg * m^2)}{s^2}$$

The units for our formula are:

$$m * (\frac{kg}{m^3}) * (\frac{rad}{s})^2 * m^4$$

$$kg * \frac{m^2}{s^2}$$

As you can see the units for our formula model the units for Jules. This gives us confidence that the formula we derived for kinetic energy is accurate. Now we can plug assumed values into our formula to start estimating the kinetic energy of a record.

$$\frac{((0.001)(1350)\pi(3.49065^2)(0.1524^4)}{4}$$

$$=$$

$$0.00696909088(\frac{kg * m^2}{s^2})$$

$$K = 0.00696909088J$$

This is our estimate for the kinetic energy of a record rotating at 33 $\frac{1}{3}$ rpms, with a diameter of 12 inches, height of 1 mm, and density of $\frac{1.35g}{cm^3}$. To check our answer, we have found the average mass of a record of this size to range from 90g to 240g. Therefore we have decided to use 0.15kg (150g), which is the midpoint between the two values. We will plug this into a new integral we have derived that substitutes our calculation for mass with this specific value. This will allow us to compare our estimates that used volume and density with the average mass of a record.

$$K = \int_0^R (\frac{1}{2})(m)(\omega r)^2 dr$$

$$K = (\frac{1}{6}) m\omega^2 R^3$$

$$K = (\frac{1}{6})(0.15)(3.49065)^2(0.1524)^3$$

$$=$$

$$0.00107822033(\frac{(kg*m^2)}{s^2})$$

$$K = 0.00107822033J$$

To help confirm our answer we are going to calculate what our formula calculated the mass of the record to be given the height, density, and radius. If this value falls between the average mass of a record, 90g to 240g, then we can be more confident in our result.

$$Mass = \int_0^R hd2\pi r dr$$

$$Mass = hd\pi r^2$$

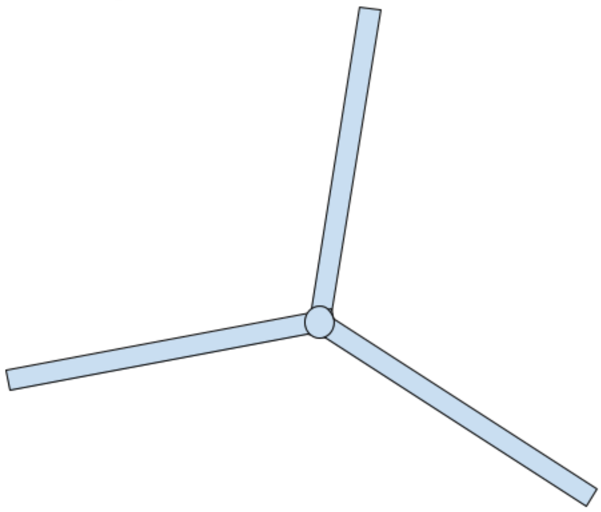
$$Mass = 0.001 * 1350 * \pi * 0.1524^2$$

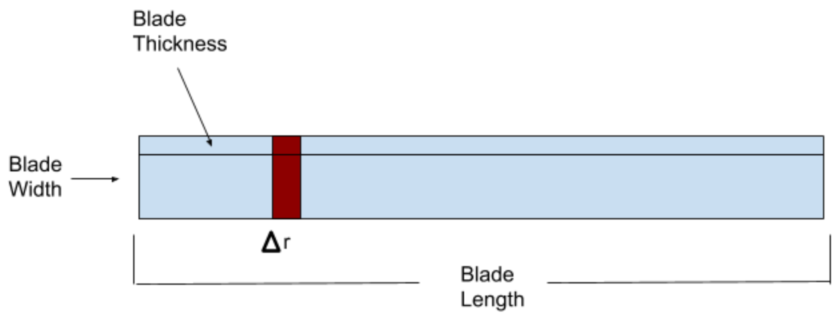
$$Mass = 0.0985kg = 98.5g$$

This calculation shows that our formula estimated the mass of the record to be 98.5 g which is between the average masses of 90g and 240g. This estimate still certainly has many flaws. We are unsure if LP Records are made completely out of polyvinyl chloride. We also did not account for the fact that a record would not have a constant density because of the grooves. We still have faith in our formula because the units correctly work out to give us the units for Joules, however our assumptions most likely introduced a significant amount of error.

Apache Helicopter Blades

The first problem asks us to examine a helicopter blade in flight. For this problem, we must use an integral to model and estimate the kinetic energy of the blade while in flight. We will need to consider this blades length, area, and rotational speed. The following equations demonstrate these calculations.





$$K = \frac{1}{2}\int_0^R (\text{blade width})(\text{black thickness})(\text{blade density})(\omega r)^2 dr$$

Earth

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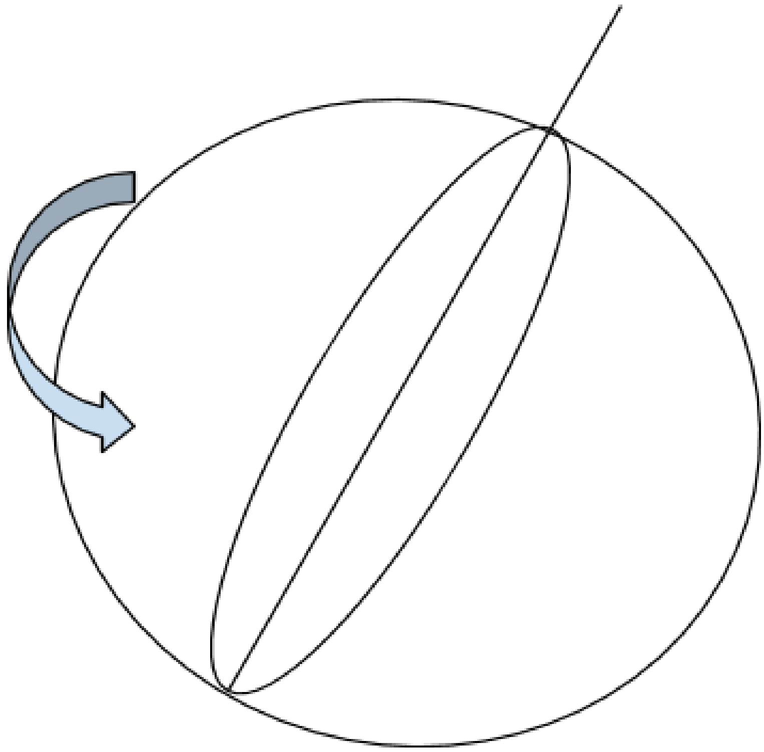

Layer	Density	Thickness
Crust	$\frac{2.5g}{cm^3}$	30 km
Upper Mantle	$\frac{4g}{cm^3}$	720 km
Lower Mantle	$\frac{5.1gm}{cm^3}$	2,171 km
Inner Core	$\frac{11.2gm}{cm^3}$	2,259 km
Outer Core	$\frac{12.9gm}{cm^3}$	1,221 km

$$\int_0^R \pi(\sqrt{r^2+R-r^2})^2 dr$$