Theory of Computation (CS355) - Worksheet 1

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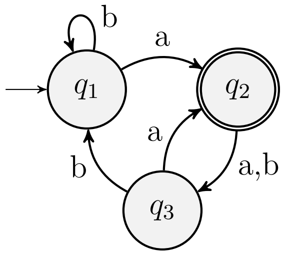
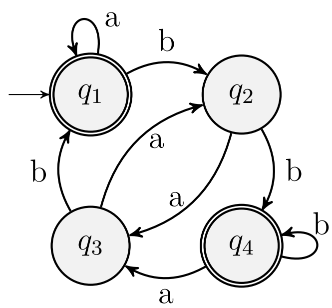
Deadline: 14:30, Mon 26-Feb-2024.
Submission: moodle (upload pdf or jpeg, scans/photos okay)

I: DFA Definition

Consider these state diagrams of two DFAs, M_1 and M_2:



M_1



M_2

1. Answer the following questions about each of these machines:
   (a) What is the start state?
   (b) What is the set of accept states?
   (c) What sequence of states does the machine go through on input *aabb* ?
   (d) Does the machine accept the string *aabb* ?
   (e) Does the machine accept the string ε ?
2. Give a formal description of machines M_1 and M_2.
3. The formal description of a DFA M_3 is $(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$ where δ is given by the following table.

q	$\delta(q,u)$	$\delta(q,d)$
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

Give the state diagram of this machine.

II: DFA Exercise

4. Enumerate all possible distinct functions mapping from the set $\{x, y\}$ to the set $\{1, 2, 3\}$.
5. How many DFA's exist with 4 states over the alphabet $\Sigma = \{\#, 0, 1\}$.

Give state diagrams of DFAs recognizing the following languages. In all parts, $\Sigma = \{0, 1\}$.

6. $\{w \mid w$ begins with a 1 and ends with a 0$\}$
7. $\{w \mid w$ contains at least three 1s$\}$
8. $\{w \mid w$ contains the substring 0101, i.e., $w = x0101y$ for some x and $y\}$
9. $\{w \mid w$ has length at least 3 and its third symbol is a 0$\}$
10. $\{w \mid w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$
11. $\{w \mid w$ does not contain the substring 110$\}$
12. $\{w \mid$ the length of w is at most 5$\}$
13. $\{w \mid$ is any string except 111$\}$

III: NFA/GNFA/Regular Language Exercise

Consider $L = \{abwba : w \in \Sigma^*$ but does not contain the substring $ba\}$ over $\Sigma = \{a, b, c, d\}$.

1. Show that L is a regular language by drawing a DFA for it, with the DFA having as few states as you can.
2. Draw an NFA for L with as few states as you can.
3. Convert the NFA to a GNFA, and show that.
4. Convert the GNFA to a regular expression, and show that, using the procedure discussed in class. Show the steps. You may, but are not required to, shorten intermediate regular expression using identities like $R\emptyset = \emptyset$ and $\varepsilon R = R$ and $R \cup \emptyset = R$ to reduce expression swell.