170S Week 7 Discussion Notes

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Problem 1. Suppose the time it takes a company to fill an advertised open position is exponentially distributed with mean 20 (days), and suppose that the times are independent even when there are multiple advertised open positions.

(*i*) The company needs at least a 90% chance that someone will be hired in the next 5 days. What is the smallest number n of open positions the company should advertise?

(*ii*) On the other hand, the company is worried that all n positions will be filled in the next 40 days. What is the probability of this happening?

(*iii*) If the company advertises three open positions, how long will it take on average for all of them to be filled?

Problem 2. Let $X \sim \text{Unif}(0, \theta)$ be a uniform random variable where $\theta > 0$ is an unknown parameter, and let $x_1, \ldots, x_n$ be a sample drawn from X. In this problem, we will explore some estimators for θ.

(*i*) Find the MLE $\hat{\theta}$ for the parameter θ, and determine whether it is unbiased.

(*ii*) Find the method of moments estimator $\tilde{\theta}$ for the parameter θ, and determine whether it is unbiased.

(*iii*) Show, by using an indicator function, that the sample maximum $x_{(n)}$ is a sufficient statistic for θ.

(*iv*) Show that the sample mean $\bar{x}$ is not a sufficient statistic for θ and hence that the sum of the samples $n\bar{x}$ is not either.

(*v*) However, in the case $n = 2$, show that the sum $n\bar{x} = x_1 + x_2$ along with the difference $x_1 - x_2$ are jointly sufficient statistics for θ.