Final Exam Recommended Study

Colin Ni

March 14, 2025

The final exam will consist of 8 problems. Each problem will be worth 25 points, for a total of 200 points. Five of the problems will have an extra credit part worth 5 points, so theoretically one can earn 225/200 points.

Each problem on the final exam will strongly resemble a problem on the practice final. Thus, I will recommend additional things to study and things to put on your cheat sheet, based on each problem on the practice final. Note that I am not recommending that you write down all the usual distribution info, confidence intervals, and hypothesis tests, because the relevant ones will be given to you on the exam.

You should obviously start by doing the practice final, which have solutions.

Problem 1 (descriptive statistics)

- Repeat the problem with the sample $0, 0, 0, 0, 1, 1, 2, 2, 2, 2$ of nice numbers. You can use a website like this to check your work.
- This problem asked you to determine the sample mean and sample variance, given the sum of the samples and the sum of the squares. What if the problem instead gave the values of $\hat{M}_1$ and $\hat{M}_2$, the estimators for moments that we use in the method of moments?
- Review the statement of Bessel's correction from discussion section week 2.

Problem 2 (order statistics)

- This problem related certain order statistics of the power distribution to the power distribution and to the Kumaraswamy distribution. We have seen similar relationships such as the order statistics of uniform being Beta (discussion section week 1 and also HW) and the first order statistic of exponential being exponential (discussion section week 3, midterm, and also HW), and you should review these.
- You should absolutely put the formula for the pdf $f_{X_{(r)}}(x)$ of an order statistic $X_{(r)}$ on your cheat sheet.

Problem 3 (MLE's)

- This problem asked you to determine the MLE for a geometric random variable and to explain the intuition behind the answer. Do the same thing for the other common random variables, *e.g.* uniform, binomial, Poisson, the mean of normal, exponential, etc. You should do as many as you find helpful.
- You may as well make a table on your cheat sheet of the MLE's as well as their interpretations.

Problem 4 (method of moments)

- This problem asked you to determine the method of moments estimators for a random variable with two parameters, namely Gamma, and then explain why it may give nonsensical results. Do the same thing for the other common random variables with multiple parameters, *i.e.* binomial, uniform, normal, Beta.
- You may as well make a table on your cheat sheet of the method of moments estimators as well as examples of how they can be nonsensical.

Problem 5 (linear regression)

- This problem asked you to perform linear regression. You should practice with a different example (that you can easily make up yourself), and you can check your answer in several ways, *e.g.* this website or just plot it by hand.
- This problem has an extra credit problem that uses the usual matrix formulation to solve. If you are aiming to answer the corresponding (spoiler!) extra credit problem on the final exam, you should review discussion section week 5.
- You should absolutely write on your cheat sheet the formulas for linear regression, whichever ones you prefer (the ones in the lecture notes or the recipe I described in discussion section).

Problem 6 (sufficient statistics)

- Review the many examples from homework and lecture.
- You should write on your cheat sheet our definition of a sufficient statistic (the Fischer-Neyman factorization theorem), which will help with the analogous (spoiler!) extra credit problem, which will be extremely similar to the one on the practice final.

Problem 7 (Bayesian statistics)

- This problem asked you to determine the Bayes estimate for the mean of normal with a normal prior. You should review the other main examples of Bayes estimates, *e.g.* binomial-Beta from the book, Gamma-Gamma from homework.
- This problem also asked you to interpret the Bayes estimate. You should think about how to interpret it in the other main examples.
- You may as well make a table on your cheat sheet of your answers, similar to the one from discussion section week 6.

Problem 8 (confidence interval)

- This problem asked you to construct a confidence interval, namely for a difference of means, which is one of the harder ones to do. You should review how other confidence intervals are constructed in the book, *e.g.* mean of normal with known variance (easy), mean of normal with unknown variance (harder), or for proportions (which use a normal approximation).
- You should absolutely write in your cheat sheet the main things in discussion section week 8, *e.g.* our definition of chi-squared distributions, their Key Property, and our definition of a t-distribution. You need these to construct confidence intervals when variance is unknown.

Problem 9 (hypothesis test)

- This problem asked you to perform a hypothesis test using the confidence interval from Problem 8. You should make sure you can do this no matter what confidence interval you are given.