170S Week 7 Discussion Notes

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Problem 1. Suppose the time it takes a company to fill an advertised open position is exponentially distributed with mean 20 (days), and suppose that the times are independent even when there are multiple advertised open positions.

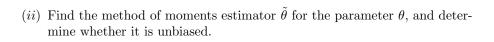
(i) The company needs at least a 90% chance that someone will be hired in the next 5 days. What is the smallest number n of open positions the company should advertise?

(ii) On the other hand, the company is worried that all n positions will be filled in the next 40 days. What is the probability of this happening?

(iii) If the company advertises three open positions, how long will it take on average for all of them to be filled?

Problem 2. Let $X \sim \text{Unif}(0, \theta)$ be a uniform random variable where $\theta > 0$ is an unknown parameter, and let x_1, \ldots, x_n be a sample drawn from X. In this problem, we will explore some estimators for θ .

(i) Find the MLE $\hat{\theta}$ for the parameter θ , and determine whether it is unbiased.



(iii) Show, by using an indicator function, that the sample maximum $x_{(n)}$ is a sufficient statistic for $\theta.$

(iv) Show that the sample mean \bar{x} is not a sufficient statistic for θ and hence that the sum of the samples $n\bar{x}$ is not either.

(v) However, in the case n=2, show that the sum $n\bar{x}=x_1+x_2$ along with the difference x_1-x_2 are jointly sufficient statistics for θ .