Position Control Experiment (MAE171a)

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this lecture and lab handouts will be available on the MAE171a canvas website for 2025: https://canvas.ucsd.edu/courses/62995

Main Objectives of Laboratory Experiment:

modeling and feedback control of a lumped mass system

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modeling and feedback control of a lumped mass system

Ingredients:

- modeling of dynamic behavior of lumped mass system
- estimation of model parameters of lumped mass system
- application of control theory for servo/positioning control
- design, implementation & verification of control
- sensitivity and error analysis

Background Theory:

- Kinematics and Newton's Law ($F = ma$),
- Ordinary Differential Equations (derivation & solutions)
- Linear System Theory (Laplace transform, Transfer function, poles, stablity, Bode plots)
- Proportional, Integral and Derivative (PID) control analysis and design (root-locus, loop gain, Nyquist stability criterion)

Outline of this lecture

- purpose of control & aim of lab experiment
- hardware description
  - schematics
  - hardware in the lab
- background theory on modeling
  - modeling a 2DOF system
  - step response of a 1DOF system
- outline of laboratory work
  - estimation of parameters: experiments
  - validation of model: simulation & experiments
  - design and implementation of controllers: P- & PD- & PID
- summary
- what should be in your report

Purpose of Control & Some Applications

Application of automatic control: to alter dynamic behavior of a system and/or reduce effect of disturbances.

- industrial processes
  - thickness control of steel plates in a rolling-mill factory
  - consistency control in papermaking machines
  - size and thickness control in glass production processes
  - control of chemical, distillation or batch reactors
- aerospace and aeronautical systems
  - gyroscope and altitude control of satellites
  - flight control of pitch, roll and angle-of-attack in aeroplanes
  - reduction of sound and vibration in helicopters and planes
- electromechanical systems
  - anti-lock brakes, cruise control and emission control
  - position control in optical or magnetic storage media
  - accurate path execution for robotic systems
  - vibration control in high precision mechanical systems

Aim of Lab Experiment

Focus on a (relatively simple) mechanical system (rotating or translating) masses connected by springs. Objective is to *create a stabilizing feedback system* to *position inertia/mass* at a *specified location* within a *certain time* and *accuracy*.

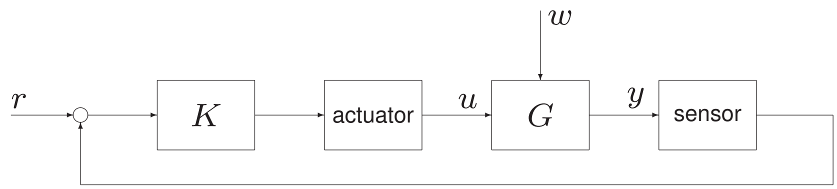
Control is needed to reduce:

- oscillatory behavior of mechanics
- the effect of external disturbances

Aim of the experiment:

- insight in control system principles
- design and implement control system
- evaluation of stability & performance
- robustness and error analysis

Schematics of Hardware Description – block diagram



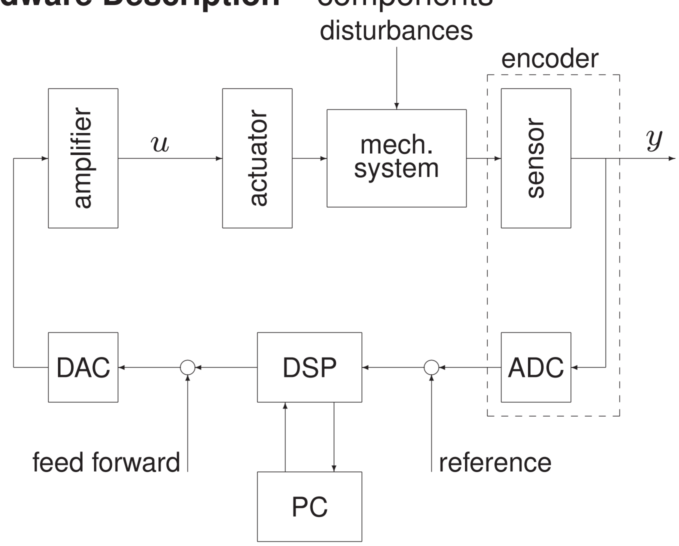
Feedback is essential in control to address *stability*, *disturbance rejection* and *robustness*.

For implementation of feedback: system G is equipped with *sensors* (to measure signals) and *actuators* (to activate system)

For flexibility of control system K: "computer control" or "digital control" and is a combination of:

- ADC (analogue to digital converter)
- DSP (digital signal processor)
- DAC (digital to analogue converter)

Detailed Hardware Description – components



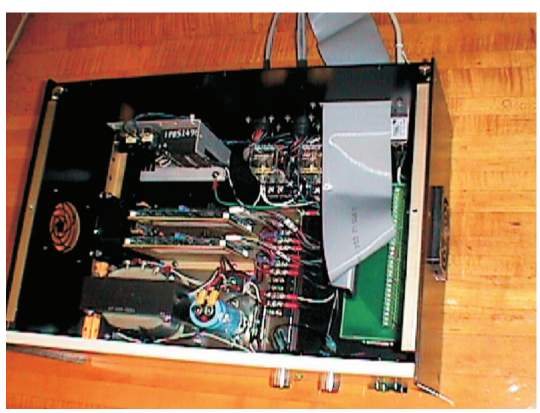
- **plant** (map from input u and output y)
  (actuator, lumped mass system and encoder)
- **real-time controller**
  (ADC, DSP, DAC, amplifier)
- **Personal Computer** (PC)
  (to run Matlab software and to program DSP)

Hardware in the Lab – mechanical systems

rectilinear system (left) and torsional plant (right)

- *control effort*: input $u =$ voltage V to servo motor (actuator)
- *measurement*: output $y =$ angular θ_i or rectilinear x_i position of a mass/inertia $m_i,\ i = 1, 2$. We limit ourselves to **2 masses only**.

Hardware in the Lab – real-time control system



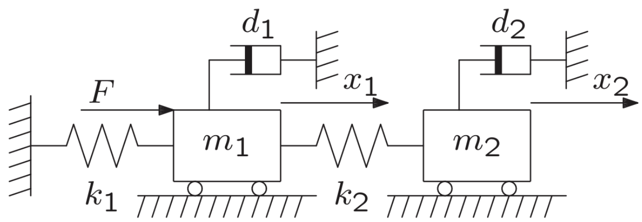
real time controller – now updated to in-house hardware

real-time controller: To implement control algorithm and perform digital signal processing. Contains ADC, DSP, DAC & amplifiers.

host-PC: To interact with DSP (run Matlab software)

Background theory: modeling a 2DOF system

Consider 2 mass/inertia system or 2 Degree Of Freedom system:

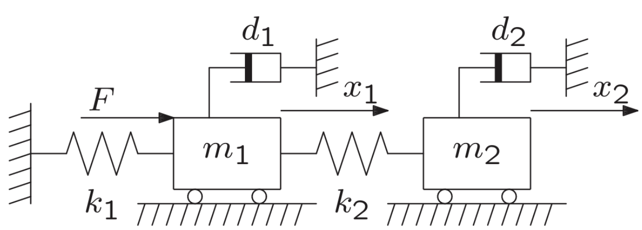


schematic view of rectilinear system with only two carts

schematics of model:

- 2 masses/inertia m_1, m_2
- each have a positioning freedom x_1, x_2: 2DOF system
- connected via spring elements k_1, k_2
- model damping: a viscous damping d_1, d_2
- input: *control effort* u (voltage V to servo motor), leading to a force F
- output: *measurement* $y = x_1$ or $y = x_2$ (both encoder counts)

Background theory: modeling a 2DOF system



2nd Newton's law $\sum F = ma$ to describe dynamic behavior:

$$\begin{aligned} m_1\ddot{x}_1 &= -k_1x_1 - d_1\dot{x}_1 - k_2(x_1 - x_2) + F \\ m_2\ddot{x}_2 &= k_2(x_1 - x_2) - d_2\dot{x}_2 \end{aligned}$$

Rearranging shows **coupled set of 2nd order ODEs**:

$$\begin{aligned} m_1\ddot{x}_1 + d_1\dot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 &= F \\ m_2\ddot{x}_2 + d_2\dot{x}_2 + k_2x_2 - k_2x_1 &= 0 \end{aligned}$$

Can be analyzed and even solved with **Laplace transform**!

Background theory: modeling a 2DOF system

Laplace Transform $\mathcal{L}\{\dot{x}(t)\} = sx(s),\ \mathcal{L}\{\ddot{x}(t)\} = s^2x(s)$ of

$$\begin{aligned} m_1\ddot{x}_1 + d_1\dot{x}_1 + (k_1+k_2)x_1 - k_2x_2 &= F \\ m_2\ddot{x}_2 + d_2\dot{x}_2 + k_2x_2 - k_2x_1 &= 0 \end{aligned}$$

yields

$$\underbrace{\left[\begin{array}{cc} m_1s^2 + d_1s + (k_1+k_2) & -k_2 \\ -k_2 & m_2s^2 + d_2s + k_2 \end{array}\right]}_{T(s)} \left[\begin{array}{c} x_1(s) \\ x_2(s) \end{array}\right] = \left[\begin{array}{c} F(s) \\ 0 \end{array}\right].$$

So short hand notation to relate input $u \sim F$ to output x_1 and output x_2:

$$T(s)\left[\begin{array}{c} x_1(s) \\ x_2(s) \end{array}\right] = \left[\begin{array}{c} 1 \\ 0 \end{array}\right] u(s)$$

All we need to do is **compute the inverse** $T^{-1}(s)$ **of** $T(s)$.

Background theory: modeling a 2DOF system

We known for a 2×2 matrix that

$$T = \left[\begin{array}{cc} a & b \\ c & d \end{array}\right] \Rightarrow T^{-1} = \frac{1}{\det(T)}\left[\begin{array}{cc} d & -b \\ -c & a \end{array}\right],\ \det(T) = ad - bc$$

With $T(s)$ on previous slide given by

$$T(s) = \left[\begin{array}{cc} m_1s^2 + d_1s + (k_1+k_2) & -k_2 \\ -k_2 & m_2s^2 + d_2s + k_2 \end{array}\right]$$

we see

$$d(s) := \det(T(s)) = (m_1s^2 + d_1s + k_1 + k_2)(m_2s^2 + d_2s + k_2) - k_2^2$$

As a result we can **compute the inverse of** $T(s)$ given by

$$T(s)^{-1} = \frac{1}{d(s)}\left[\begin{array}{cc} m_2s^2 + d_2s + k_2 & k_2 \\ k_2 & m_1s^2 + d_1s + (k_1+k_2) \end{array}\right]$$

Background theory: modeling a 2DOF system

With known $T^{-1}(s)$ we now have

$$\begin{bmatrix} x_1(s) \\ x_2(s) \end{bmatrix} = T^{-1}(s) \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(s)$$

If you are *only* using **encoder 1** as output $y = x_1$ for feedback, you have

$$y(s) = x_1(s) \;\Rightarrow\; y(s) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix} T^{-1}(s) \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{G(s)} u(s)$$

If you are *only* using **encoder 2** as output $y = x_2$ for feedback, you have

$$y(s) = x_2(s) \;\Rightarrow\; y(s) = \underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix} T^{-1}(s) \begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{G(s)} u(s)$$

Where

$$T(s)^{-1} = \frac{1}{d(s)} \begin{bmatrix} m_2 s^2 + d_2 s + k_2 & k_2 \\ k_2 & m_1 s^2 + d_1 s + (k_1 + k_2) \end{bmatrix}$$

Background theory: modeling a 2DOF system

When using **encoder 1 as output**, we have $y(s) = x_1(s)$ and

$$G(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} T^{-1}(s) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

resulting in

$$y(s) = G(s)u(s), \;\; G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

The transfer function $G(s)$ relates *control effort* $u(s)$ (typically in voltage) to the *measured position* $y(s)$ of **mass 1** (typically in encoder counts).

The coefficients a_i, b_i of $G(s)$ are given by

$$\begin{aligned} b_2 &= m_2 \\ b_1 &= d_2 \\ b_0 &= k_2 \end{aligned} \qquad \begin{aligned} a_4 &= m_1 m_2 \\ a_3 &= (m_1 d_2 + m_2 d_1) \\ a_2 &= (k_2 m_1 + (k_1 + k_2) m_2 + d_1 d_2) \\ a_1 &= ((k_1 + k_2) d_2 + k_2 d_1 \\ a_0 &= k_1 k_2 \end{aligned}$$

Background theory: modeling a 2DOF system

When using **encoder 2 as output**, we have $y(s) = x_2(s)$ and

$$G(s) = \begin{bmatrix} 0 & 1 \end{bmatrix} T^{-1}(s) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

resulting in

$$y(s) = G(s)u(s),\ G(s) = \frac{b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

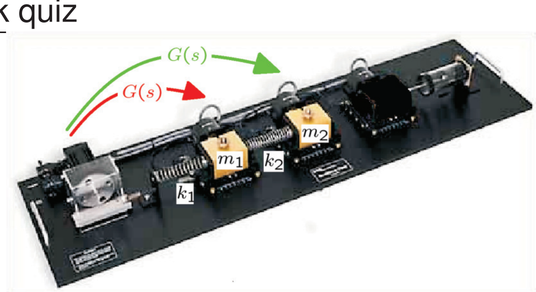
The transfer function $G(s)$ relates *control effort* $u(s)$ (typically in voltage) to the *measured position* $y(s)$ of **mass 2** (typically in encoder counts).

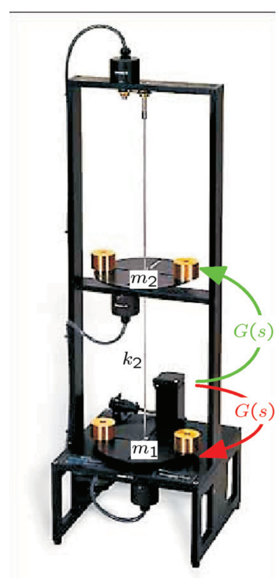
The coefficients a_i, b_i of $G(s)$ are given by

$$b_0 = k_2 \qquad \begin{aligned} a_4 &= m_1 m_2 \\ a_3 &= (m_1 d_2 + m_2 d_1) \\ a_2 &= (k_2 m_1 + (k_1 + k_2) m_2 + d_1 d_2) \\ a_1 &= ((k_1 + k_2) d_2 + k_2 d_1 \\ a_0 &= k_1 k_2 \end{aligned}$$

Background theory: modeling a 2DOF system

Quick quiz





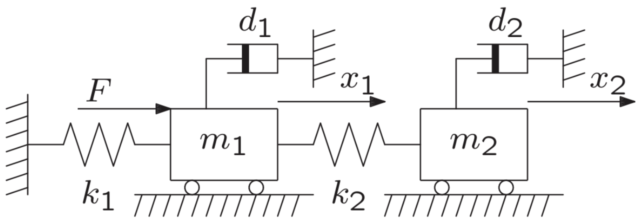
Given

$$G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

$$G(s) = \frac{b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

- What is the dynamic similarity and difference between $G(s)$ and $G(s)$?
- Which dynamic system is more difficult to control?
- what happens if stiffness $k_1 = 0$ (as in torsional system)?

Background theory: modeling a 2DOF system



schematic view of rectilinear system with two carts

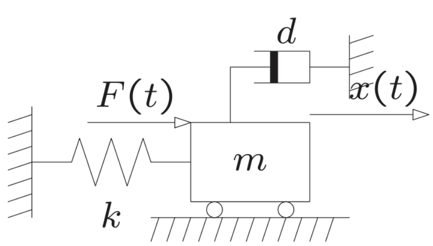
In summary, with transfer function coefficients:

$$
\begin{array}{rclcrcl}
 & & & \quad & a_4 & = & m_1 m_2 \\
b_2 & = & m_2 & & a_3 & = & (m_1 d_2 + m_2 d_1) \\
b_1 & = & d_2 & & a_2 & = & (k_2 m_1 + (k_1 + k_2) m_2 + d_1 d_2) \\
b_0 & = & k_2 & & a_1 & = & ((k_1 + k_2) d_2 + k_2 d_1 \\
 & & & & a_0 & = & k_1 k_2
\end{array}
$$

'All we need to do' is:
determine mass, spring and damper constants!

Background theory: modeling a 2DOF system

Alternative to simply determining mass, spring and damper constants:
dynamic experiments.

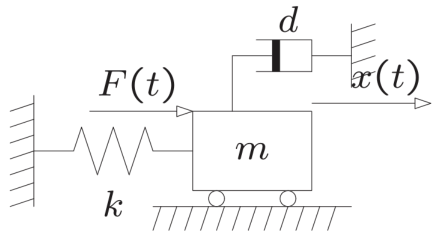


Main Idea:

- Restrict 2DOF system to (temporarily) become a 1DOF system with either only k_1, m_1 and d_1 or k_2, m_2 and d_2.
- Understand the step response of a 1DOF system as a function of k, m and d.
- From 1DOF step response experiments: estimate k, m, d to unravel all parameters k_1, m_1 and d_1 or k_2, m_2 and d_2.

Background theory: step response of a 1DOF system

Important observation: 2DOF system is built up from 2 single 1DOF or single mass/spring/damper systems:



Main Approach:

- Compute the step response as a function of k, m and d.
- Use step response *laboratory experiments* to determine values of k, m, d (will be posted on Google sites due to remote instruction).
- Relate these values back to parameters k_1, m_1 and d_1 or k_2, m_2 and d_2 to complete model.

Background theory: step response of a 1DOF system

RESULT:

Consider a 1DOF system with single mass m, damping d and stiffness k and let us define

with $\omega_n := \sqrt{\frac{k}{m}}$ (resonance) and $\beta := \frac{1}{2}\frac{d}{\sqrt{mk}}$ (damping ratio)

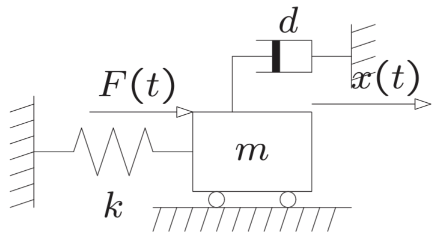
then a step input $u(t) = U,\ t \geq 0$ of size U on the 1DOF system results in the output response

$$y(t) = \frac{U}{k}\left[1 - \mathrm{e}^{-\beta\omega_n t}\sin(\omega_d t + \phi)\right]$$

where

$$\begin{aligned} \omega_d &= \omega_n\sqrt{1-\beta^2} && \text{damped resonance frequency in rad/s} \\ \phi &= \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta} && \text{phase shift of response in rad} \end{aligned}$$

Background theory: step response of a 1DOF system



DERIVATION:

$$m\ddot{x}(t) = F(t) - kx(t) - d\dot{x}(t)$$

Laplace transform:

$$ms^2x(s) + dsx(s) + kx(s) = F(s), \;\; \Rightarrow \; x(s) = \underbrace{\frac{1}{ms^2+ds+k}}_{G(s)}F(s),$$

The transfer function $G(s)$ written as standard 2nd order system:

$$G(s) = \frac{1}{ms^2+ds+k} = \frac{1}{k}\cdot\frac{\omega_n^2}{s^2+2\beta\omega_n s+\omega_n^2}$$

with $\omega_n := \sqrt{\frac{k}{m}}$ (resonance) and $\beta := \frac{1}{2}\frac{d}{\sqrt{mk}}$ (damping ratio)

Background theory: step response of a 1DOF system

Compute the dynamic response via inverse Laplace transform!

RESULT:
Consider a step input $u(t) = U, \; t \geq 0$ of size U. Then $u(s) = \frac{U}{s}$ and for the 1DOF system we have

$$y(s) = G(s)u(s) = \frac{1}{k}\frac{\omega_n^2}{s^2+2\beta\omega_n s+\omega_n^2}\frac{U}{s}$$

and the inverse Laplace transform is given by

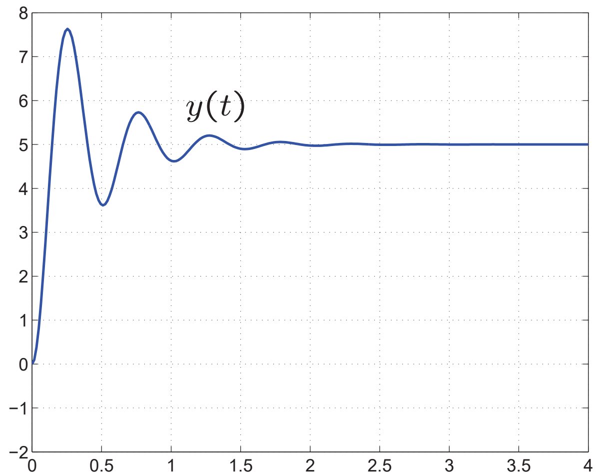
$$y(t) = \frac{U}{k}\left[1 - \mathrm{e}^{-\beta\omega_n t}\sin(\omega_d t + \phi)\right]$$

where

$$\omega_d = \omega_n\sqrt{1-\beta^2} \quad \text{damped resonance frequency in rad/s}$$
$$\phi = \tan^{-1}\frac{\sqrt{1-\beta^2}}{\beta} \quad \text{phase shift of response in rad}$$

Background theory: step response of a 1DOF system

Typical picture of $y(t) = \frac{U}{k}\left[1 - \mathrm{e}^{-\beta\omega_n t}\sin(\omega_d t + \phi)\right]$ for a step size of $U = 1$, stiffness $k = 1/5$, undamped resonance frequency $\omega_n = 2\cdot 2\pi \approx 12.566$ rad/s and damping ratio $\beta = 0.2$:

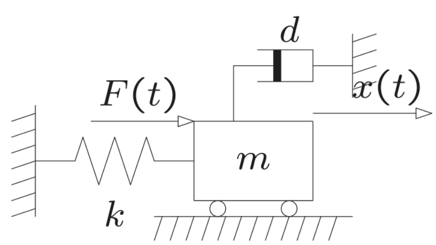


How about the reverse problem of finding m, k and d from $y(t)$?

Outline of Lab Work: estimation of model parameters

Alternative to simply determining mass, spring and damper constants: dynamic experiments.

Important observation: 2DOF system is built up from 2 single 1DOF or single mass/spring/damper systems:

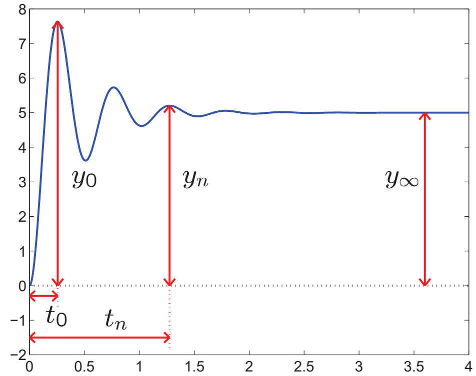


Main Idea:

- We know (how to compute) the step response of a 1DOF system as a function of k, m and d.
- Restrict 2DOF system to (temporarily) become a 1DOF system with either only k_1, m_1 and d_1 or k_2, m_2 and d_2.
- From 1DOF step response experiments to estimate k, m, d to determine k_1, m_1 and d_1 or k_2, m_2 and d_2.

Outline of Lab Work: estimation of model parameters

With the times t_0, t_n and the values y_0, y_n and y_∞ from step response:



Allows us to estimate:

$$\widehat{\omega}_d = 2\pi\frac{n}{t_n - t_0} \qquad \text{(damped resonance frequency)}$$

$$\beta\widehat{\omega}_n = \frac{1}{t_n - t_0}\ln\left(\frac{y_0 - y_\infty}{y_n - y_\infty}\right) \qquad \text{(exponential decay term)}$$

where $n =$ number of oscillations between t_n and t_0.

Outline of Lab Work: estimation of model parameters

With the estimates

$$\widehat{\omega}_d = 2\pi\frac{n}{t_n - t_0} \qquad \text{(damped resonance frequency)}$$

$$\beta\widehat{\omega}_n = \frac{1}{t_n - t_0}\ln\left(\frac{y_0 - y_\infty}{y_n - y_\infty}\right) \qquad \text{(exponential decay term)}$$

we can now compute:

$$\widehat{\omega}_n = \sqrt{\widehat{\omega}_d^2 + (\beta\widehat{\omega}_n)^2} \qquad \text{(undamped resonance frequency)}$$

$$\widehat{\beta} = \frac{\beta\widehat{\omega}_n}{\widehat{\omega}_n} \qquad \text{(damping ratio)}$$

Outline of Lab Work: estimation of model parameters

With the computed

$$\widehat{\omega}_n = \sqrt{\widehat{\omega}_d^2 + (\beta\widehat{\omega}_n)^2} \quad \text{(undamped resonance frequency)}$$

$$\widehat{\beta} = \frac{\beta\widehat{\omega}_n}{\widehat{\omega}_n} \quad \text{(damping ratio)}$$

we can now find the estimates

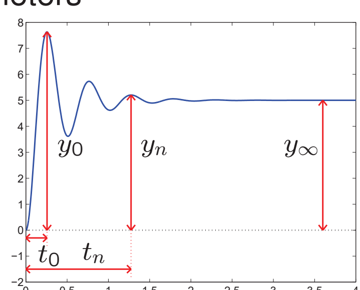
$$\widehat{k} = \frac{U}{y_\infty} \quad \text{(stiffness constant)}$$

$$\widehat{m} = \widehat{k} \cdot \frac{1}{\widehat{\omega}_n^2} \quad \text{(mass/inertia)}$$

$$\widehat{d} = \widehat{k} \cdot \frac{2\widehat{\beta}}{\widehat{\omega}_n} \quad \text{(damping constant)}$$

Outline of Lab Work: estimation of model parameters

EXAMPLE: step of $U = 0.5V$ on motor



Read from plot:
$t_0 = 0.25,\ t_n = 1.25,\ y_0 = 7.5,\ y_n = 5.25,\ y_\infty = 5$

$$\widehat{\omega}_d = 2\pi\frac{n}{t_n - t_0} = 2\pi\frac{2}{1.25 - 0.25} = 4\pi$$

$$\beta\widehat{\omega}_n = \frac{1}{t_n - t_0}\ln\left(\frac{y_0 - y_\infty}{y_n - y_\infty}\right) = \frac{1}{1.25 - 0.25}\ln\left(\frac{7.5 - 5}{5.25 - 5}\right) = \ln(10)$$

$$\widehat{\omega}_n = \sqrt{\widehat{\omega}_d^2 + (\beta\widehat{\omega}_n)^2} = \sqrt{16\pi^2 + \ln(10)^2} \approx 12.78$$

$$\widehat{\beta} = \frac{\beta\widehat{\omega}_n}{\widehat{\omega}_n} \approx \frac{\ln(10)}{12.78} \approx 0.18$$

creating

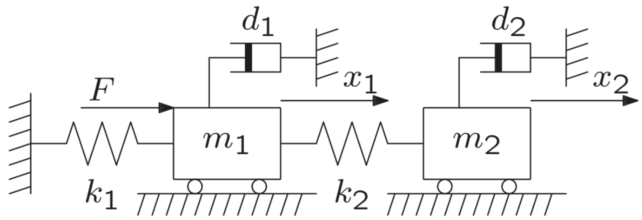
$$\widehat{k} = \frac{U}{y_\infty} = \frac{0.5}{5} = 0.10$$

$$\widehat{m} = \frac{\widehat{k}}{\widehat{\omega}_n^2} \approx \frac{0.10}{12.78^2} \approx 6.13 \cdot 10^{-4}$$

$$\widehat{d} = \widehat{k} \cdot \frac{2\widehat{\beta}}{\widehat{\omega}_n} \approx 0.10 \cdot \frac{2 \cdot 0.18}{12.78} \approx 2.82 \cdot 10^{-3}$$

Questions: What are the units of k, m and d? Does it matter?

Outline of Lab Work: estimation of model parameters



Parameter estimation:

- Reduce the 2DOF system to 2 1DOF systems!
- Estimate model parameters m_1, d_1, k_1 and m_2, d_2, k_2
- Verify the simulation of your step response with the measurement of a step response for each 1DOF system individually.
- Combine all parameter values to create your complte 2DOF system model $G(s)$ and validate.

Luckily, we have Matlab installed and a Matlab script/function called `maelab.m` to help you with his.

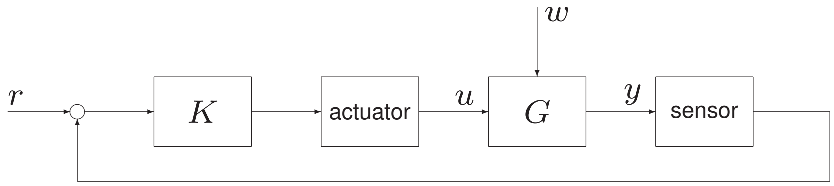
Simply enter your estimated parameters in `parameters.m`

Outline of Lab Work: estimation of model parameters

NOTE:

- Step response experiment and estimation of your model parameters m_1, d_1, k_1 and m_2, d_2, k_2 must be performed at least 5 times for statistical analysis!

Outline of Lab Work: model validation



Keep in mind:

- From parameter estimation experiments we know obtain a full 2DOF model specified as a transfer function
$$y(s) = G(s)u(s),\ G(s) = \frac{b_2 s^2 + b_1 s + b_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$
where $u(s) =$ is control input (motor Voltage or Force) and $y(t) =$ system output (position in encoder counts)
- Model $G(s)$ is created automatically for you via `maelab.m` script file by modifying the `parameters.m` file.
- You are going to use the model $G(s)$ to design a controller $K(s)$, so model $G(s)$ should be validated first!

Outline of Lab Work: model validation

What is suitable for model validation:

- Validate the estimation of each set of 1DOF parameters (e.g. m, k and d) by comparing actual experiments with a simulation of a 1DOF model $G(s)$.

  This can be done with the `maelab.m` script file.

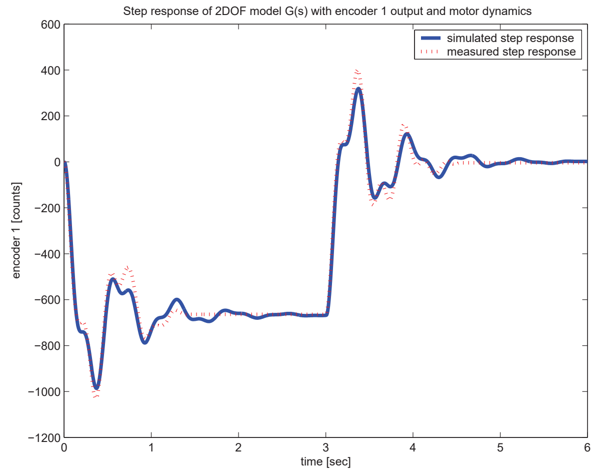
- Validate the estimation of the complete set of model parameters m_1, d_1, k_1 and m_2, d_2, k_2 of your 2DOF model $G(s)$ via a comparison of actual experiments with a simulation of your 2DOF model $G(s)$.

  Again, this can be done with the `maelab.m` script file.

- With a bad (unvalidated) model $G(s)$ you cannot do a proper model-based control $K(s)$ design!

Outline of Lab Work: model validation

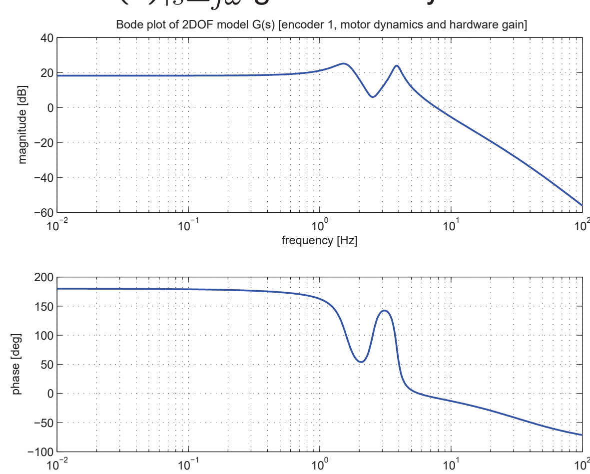
Typical picture of the validation of a 2DOF system based on step response experiments generated by `maelab.m`:



Validation: verify if both resonance modes of the 2DOF system (slow one and fast one) are matching (collect data).

Outline of Lab Work: model validation

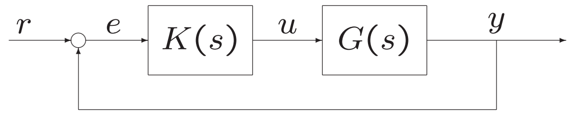
Validation of a 2DOF system based on sinusoidal experiments based on the Bode response of $G(s)|_{s=j\omega}$ generated by `maelab.m`:



Validation: excite the system with sinusoidal inputs and verify if both resonance modes and possible ‘anti-resonance’ mode of the 2DOF system have been modeled accurately (collect data).

Outline of Lab Work: design of controllers

Given model $G(s)$, construct feedback loop with $K(s)$:



Schematic view of closed loop configuration

Find a feedback controller $K(s)$ that satisfies:

- move a mass/inertia to a certain (angular) position as fast as possible
- limit overshoot during control/positioning to 25%
- no steady-state error e
- illustrate disturbance rejection when control is implemented

Trade off in design specifications:

high speed $\leftrightarrow$ overshoot
overshoot $\leftrightarrow$ robustness

Outline of Lab Work: design of controllers

Controller configurations to be implemented during the lab:

- P-control

$$\begin{aligned} u(t) &= k_p e(t),\ e(t) = r(t) - y(t) \text{ or} \\ u(s) &= K(s)[r(s) - y(s)],\ K(s) = K(s) = \frac{k_p}{\tau s + 1},\ 0 < \tau << 1 \end{aligned}$$

- PD-control

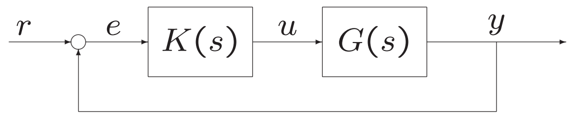
$$\begin{aligned} u(t) &= k_p e(t) + k_d \tfrac{d}{dt} e(t),\ e(t) = r(t) - y(t) \text{ or} \\ u(s) &= K(s)[r(s) - y(s)],\ K(s) = \frac{k_d s + k_p}{\tau s + 1},\ 0 < \tau << 1 \end{aligned}$$

- PID control

$$\begin{aligned} u(t) &= k_p e(t) + k_i \textstyle\int_{\tau=0}^{t} e(\tau) d\tau + k_d \tfrac{d}{dt} e(t),\ e(t) = r(t) - y(t) \text{ or} \\ u(s) &= K(s)[r(s) - y(s)],\ K(s) = \frac{k_d s^2 + k_p s + k_i}{s(\tau s + 1)},\ 0 < \tau << 1 \end{aligned}$$

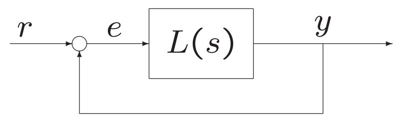
Outline of Lab Work: design of controllers (loop gain)

Model $G(s)$ of plant should be used for design of controller $K(s)$!



For design of controller $K(s)$, consider the loop gain:

$$L(s) := K(s)G(s) \;\Rightarrow\; L(s) \text{ depends on } k_p,\ k_i,\ k_d$$



loop gain: series connection of $K(s)$ and $G(s)$

Dynamics of loop gain $L(s)$ consists of fixed part $G(s)$ (plant dynamics) and to-be-designed part $K(s)$ (controller)

Outline of Lab Work: design of controllers (stability)

Loop gain $L(s) := K(s)G(s)$ important for:

- Stability
- Design specification

STABILITY:
With closed-loop poles found by those values of $s \in \mathbb{C}$ that satisfy

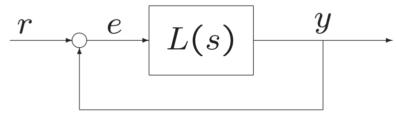
$$1 + L(s) = 0$$

all closed-loop poles should have negative real values (lie in the left part of the complex plane)

Stability can be checked by:

- Actually computing the solutions to $L(s) = -1$ as a function of $k_p,\ k_i,\ k_d$: Root Locus Method
- See if Nyquist plot of $L(s)$ encircles the point -1 as a function of $k_p,\ k_i,\ k_d$: Nyquist or Frequency Domain Method

Outline of Lab Work: design of controllers (design specs)



Error rejection transfer function:

$$E(s) = \frac{1}{1+L(s)} \quad \text{(map from } r \text{ to } e\text{)}$$

To avoid a steady state error $e(t)$ as $t \rightarrow \infty$, one specification for the loop gain can be found via the final value theorem. With $L(s) = G(s)K(s)$ we have:

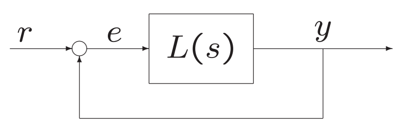
$$\lim_{t\rightarrow\infty} e(t) = \lim_{s\rightarrow 0} s \cdot \frac{1}{1+L(s)} r(s)$$

With $r(t) =$ (unit) step input, $r(s) = \frac{1}{s}$ and

$$\lim_{s\rightarrow 0} |L(s)| = \infty$$

is needed for zero steady-state behavior!

Outline of Lab Work: design of controllers (design specs)



Closed loop transfer function:

$$T(s) = \frac{L(s)}{1+L(s)} \quad \text{(map from } r \text{ to } y\text{)}$$

To make sure y follows r, we would like to make $T(s) = 1$ as close as possible.

Notice that with Error rejection transfer function:

$$E(s) = \frac{1}{1+L(s)} \quad \text{(map from } r \text{ to } e\text{)}$$

we have

$$T(s) + E(s) = 1$$

Hence, if you can make $|E(s)| \approx 0$ small, then $|T(s)| \approx 1$.

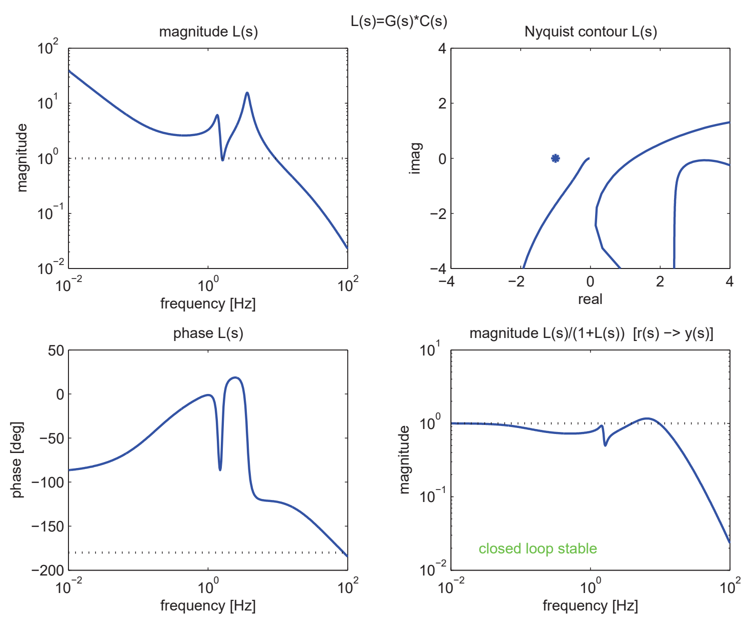
Outline of Lab Work: design of controllers (graphical design)

Computation of $K(s)$: translate design specifications to $L(s)$, $E(S)$ or $T(s)$ specifications.

- Use graphical analysis and design utilities (root locus or frequency domain methods) to shape loop gain $L(s)$ and design controller $K(s)$.
- Root-locus and frequency domain design method is implemented in Matlab via the `rltool` command.
- Frequency domain design method has also been implemented in a Matlab script file `maelab` provided during the lab.
- Stability via Nyquist criterion (do not encircle point -1)
- Phase and amplitude margin (stability and robustness) translate to shape Bode plot of loop gain $L(s) = G(s)K(s)$:
  phase margin: when $|L(s)| = 1, \angle L(s) > -\pi$ rad
  amplitude margin: when $\angle L(s) = -\pi$ rad, $|L(s)| < 1$

Outline of Lab Work: design of controllers (graphical design)

Example of figures produced by `maelab` script file



Outline of Lab Work: design of controllers (general trend)

Effects of control parameters: for PD-control and a standard 2nd order plant model this can be analyzed as follows:

$$T(s) = \frac{L(s)}{1+L(s)} = \frac{(k_p + k_d s)\frac{\omega_n^2}{s^2+2\beta\omega_n s+\omega_n^2}}{1+(k_p+k_d s)\frac{\omega_n^2}{s^2+2\beta\omega_n s+\omega_n^2}}$$

which yields the closed-loop transfer function

$$T(s) = \frac{\omega_n^2(k_p + k_d s)}{s^2 + 2\bar{\beta}\bar{\omega}_n + \bar{\omega}_n^2} \text{ with } \bar{\omega}_n = \omega_n\sqrt{1+k_p} \text{ and } \bar{\beta} = \frac{\beta + \omega_n k_d/2}{\sqrt{1+k_p}}$$

In this case $T(s)$ is also a second order system and with knowledge of the step response, we can conclude that the following influence of the controller parameters:

- $k_p \leftrightarrow$ speed of response
- $k_p \leftrightarrow$ damping
- $k_p \leftrightarrow$ steady-state error
- $k_d \leftrightarrow$ damping

Outline of Lab Work: design of controllers (summary)

- Model $G(s)$ of plant should be used for design of your controller $K(s)$!
- Increase complexity slowly. First design P, then PD and then PID control.
- Keep in mind the requirement of 25% overshoot, and no steady state error, e.g. $r(t) = y(t)$ as $t \rightarrow \infty$.
- Use graphical design tools to design your P, PD and PID control:
  - Root-locus and frequency domain design method is implemented in Matlab via the `rltool` command.
  - Frequency domain design method also implemented in a Matlab script file `maelab` provided during the lab.

Outline of Lab Work: design of controllers (summary)

- Consider how the frequency resp of P- and PD- and PID controller modifies the loop gain $L(s) = G(s)K(s)$. Look at asymptotes of Bode plot of controller

$$K(s) = \frac{k_d s^2 + k_p s + k_i}{s}$$

- Phase and amplitude margin (stability and robustness) translate to shape Bode plot of loop gain $L(s) = G(s)K(s)$:
phase margin: when $|L(s)| = 1, \angle L(s) > -\pi$ rad
amplitude margin: when $\angle L(s) = -\pi$ rad, $|L(s)| < 1$
- Argument and motive your control design in rapport.
- No trial-and-error control design results are accepted.

Summary

- *first week*
Study laboratory handout. Get familiar with the mechanical system(s), introduction to Matlab software used for experiments and controller implementation. Propose experiments to estimate (unknown) parameters in your model of the system.
- *second week*
Finish modeling of your system. Estimation and error (statistical) analysis of model parameters. Validation of model and model parameters via comparison of measured data and model simulation results. Preliminary design and implementation of a P-controller(s) using the available encoder measurements.
- *third week*
Choice to design and implement a PD-, PID-controller or state feedback controllers. Evaluation of the final control design. Sensitivity analysis of your designed controller via experiments with parameter variations.

What should be in your report (1-2)

- Abstract
  Standalone - make sure it contains clear statements w.r.t motivation, purpose of experiment, main findings (numerical) and conclusions.
- Introduction
  – Motivation (why are you doing this experiment)
  – Short description of the main engineering discipline (controls)
  – Answer the question: what is the aim of this experiment/report?
- Theory
  – Feedback system
  – Modeling
  – Parameter estimation
  – Control design

What should be in your report (2-2)

- Experimental Procedure
  – Short description of experiment
  – How are experiments done (detailed enough so someone else could repeat them)
- Results
  – Parameter estimation
  – Model validation
  – Controller Design and Implementation
- Discussion
  – Why are simulation results different from experiments?
  – Could the model be validated?
  – Are designed controller parameters O.K. from model?
- Conclusions
- Error Analysis
  – Mean, standard deviation and 99% confidence intervals of estimated parameters ω_n, β and K from data
  – How do errors in ω_n, β and K propagate to errors in m, d and k?